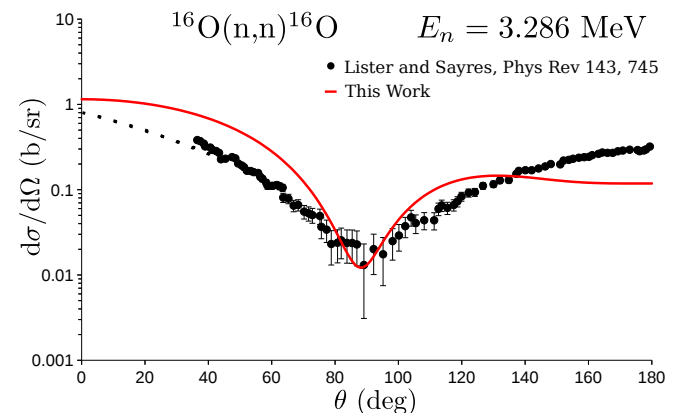
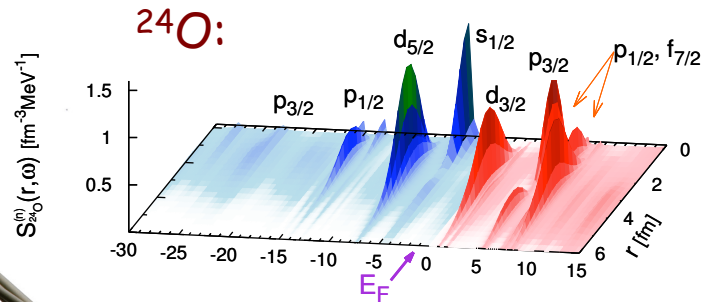
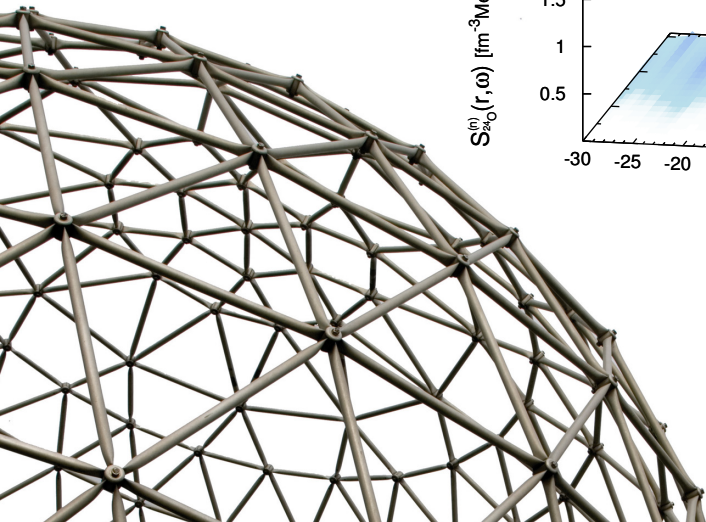


Concept of spectral function and applications to scattering in nuclear physics

Carlo Barbieri — University of Surrey



Current Status of low-energy nuclear physics

Composite system of interacting fermions

Binding and limits of stability

Coexistence of individual and collective behaviors

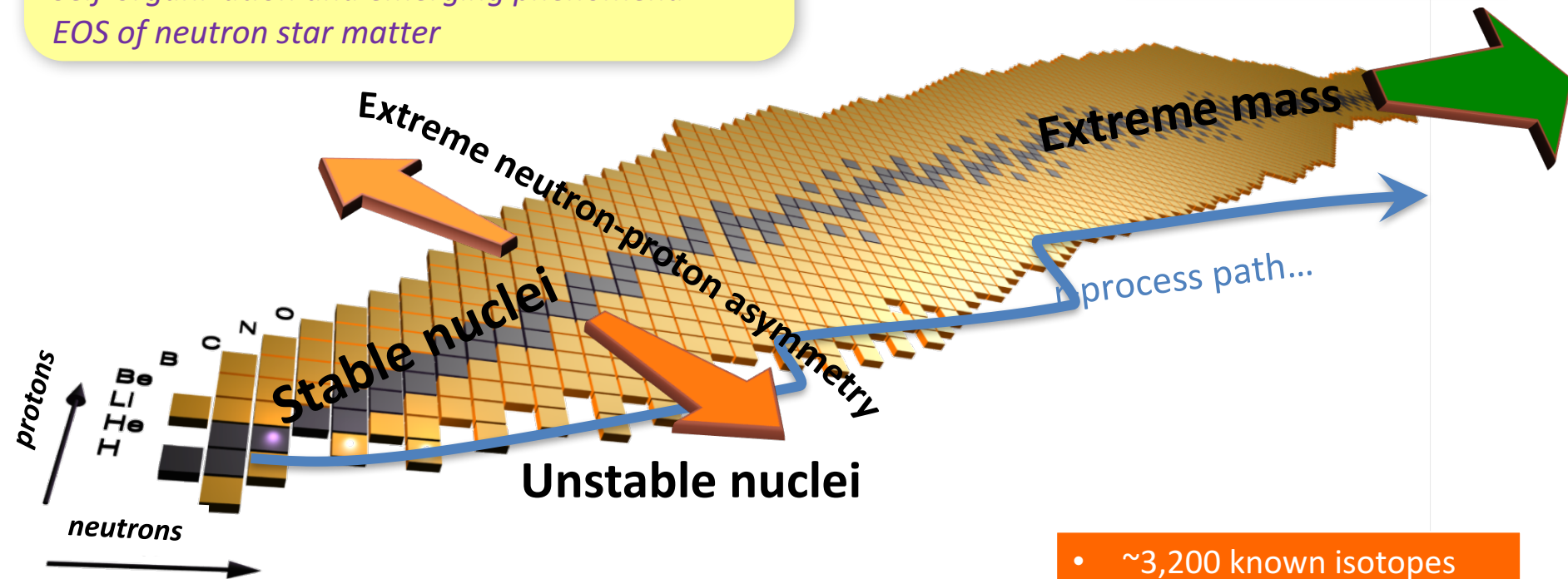
Self-organization and emerging phenomena

EOS of neutron star matter

Experimental

programs

RIKEN, FAIR, FRIB



- ~3,200 known isotopes
- ~7,000 predicted to exist
- Correlation characterised in full for ~283 stable

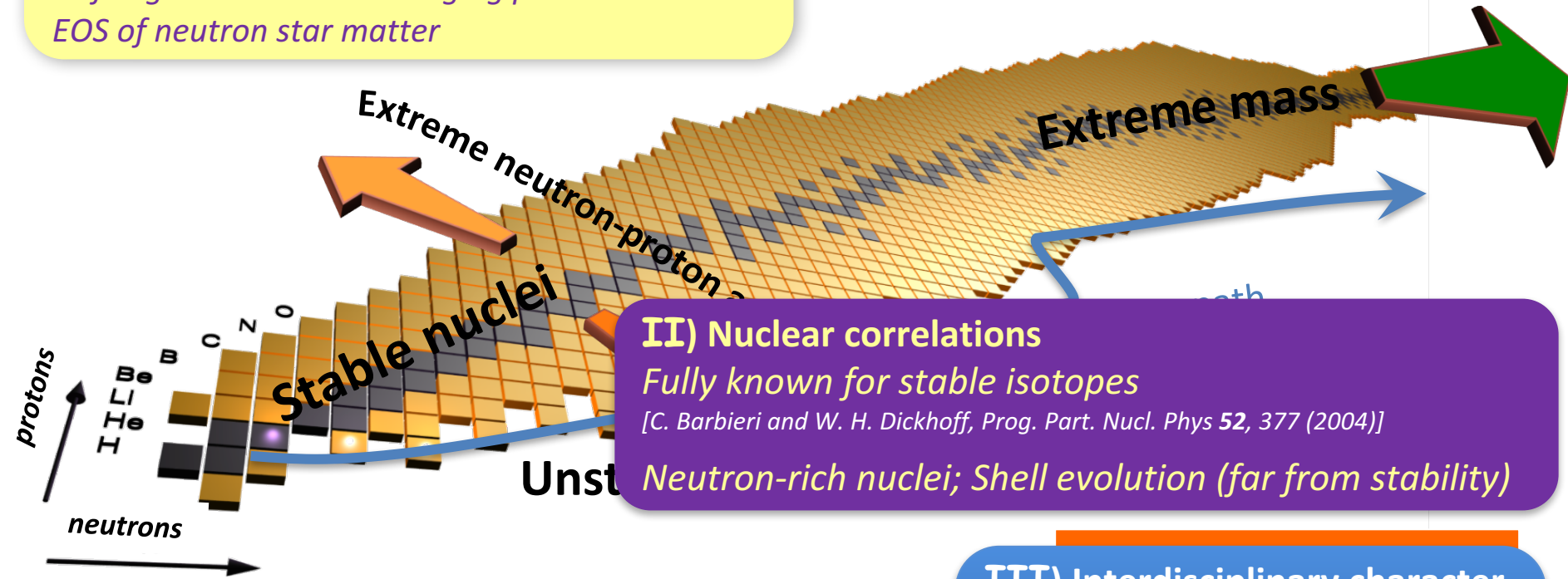
Nature **473**, 25 (2011); **486**, 509 (2012)

Current Status of low-energy nuclear physics

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Coexistence of individual and collective behaviors
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Experimental
programs
RIKEN, FAIR, FRIB



II) Nuclear correlations

Fully known for stable isotopes

[C. Barbieri and W. H. Dickhoff, Prog. Part. Nucl. Phys 52, 377 (2004)]

Neutron-rich nuclei; Shell evolution (far from stability)

I) Understanding the nuclear force

QCD-derived; 3-nucleon forces (3NFs)

First principle (ab-initio) predictions

III) Interdisciplinary character

Astrophysics

Tests of the standard model

Other fermionic systems:

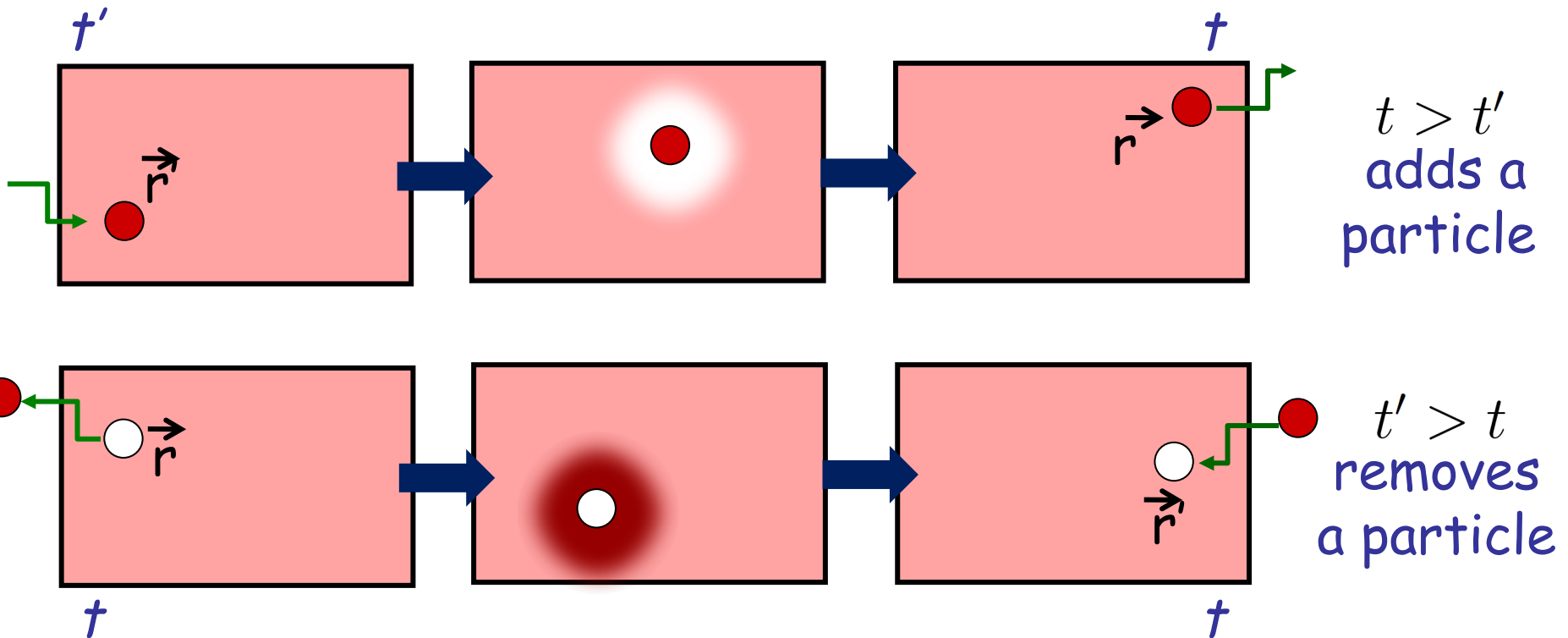
ultracold gasses; molecules;

Definition of one-body GF

With explicit time dependence:

$$g_{ss'}(\mathbf{r}, \mathbf{r}'; t - t') = -\frac{i}{\hbar} \theta(t - t') \langle \Psi_0^N | \psi_s(\mathbf{r}) e^{-i(H - E_0^N)(t - t')/\hbar} \psi_{s'}^\dagger(\mathbf{r}') | \Psi_0^N \rangle$$

$$\mp \frac{i}{\hbar} \theta(t' - t) \langle \Psi_0^N | \psi_{s'}^\dagger(\mathbf{r}') e^{i(H - E_0^N)(t - t')/\hbar} \psi_s(\mathbf{r}) | \Psi_0^N \rangle .$$



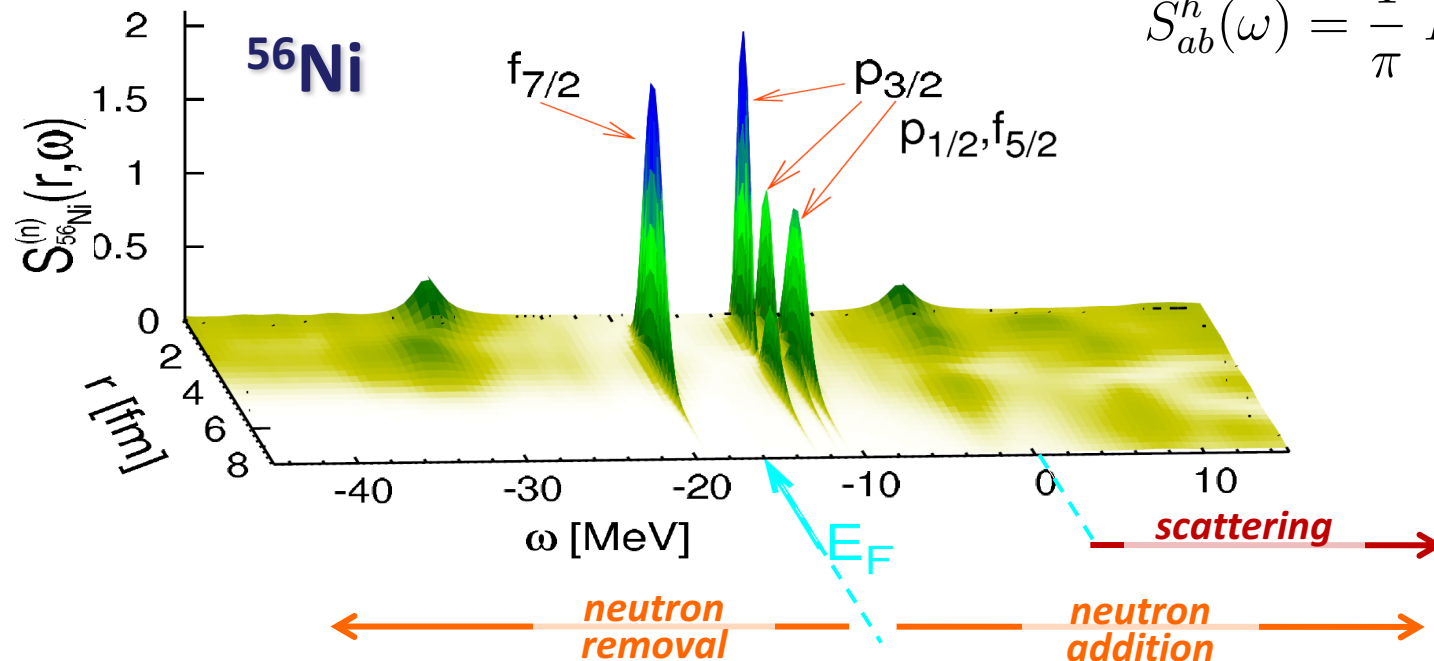
Example of spectral function ^{56}Ni

One-body Green's function (or propagator) describes the motion of quasi-particles and holes:

$$g_{\alpha\beta}(E) = \sum_n \frac{\langle \Psi_0^A | c_\alpha | \Psi_n^{A+1} \rangle \langle \Psi_n^{A+1} | c_\beta^\dagger | \Psi_0^A \rangle}{E - (E_n^{A+1} - E_0^A) + i\eta} + \sum_k \frac{\langle \Psi_0^A | c_\beta^\dagger | \Psi_k^{A-1} \rangle \langle \Psi_k^{A-1} | c_\alpha | \Psi_0^A \rangle}{E - (E_0^A - E_k^{A-1}) - i\eta}$$

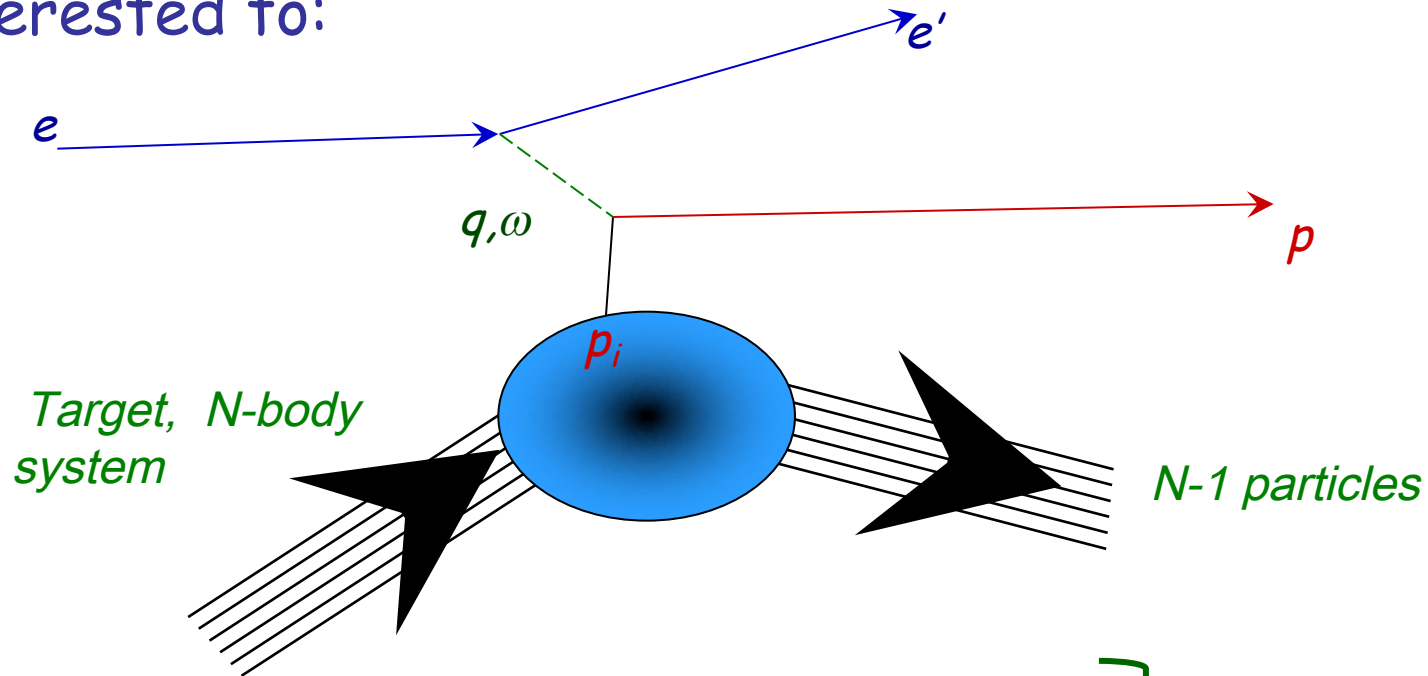
...this contains all the structure information probed by nucleon transfer (spectral function):

$$S_{ab}^h(\omega) = \frac{1}{\pi} \text{Im} g_{ab}(\omega)$$



Spectroscopy via knock out reactions - *basic idea*

Use a probe (ANY probe) to eject the particle we are interested to:



Basic idea:

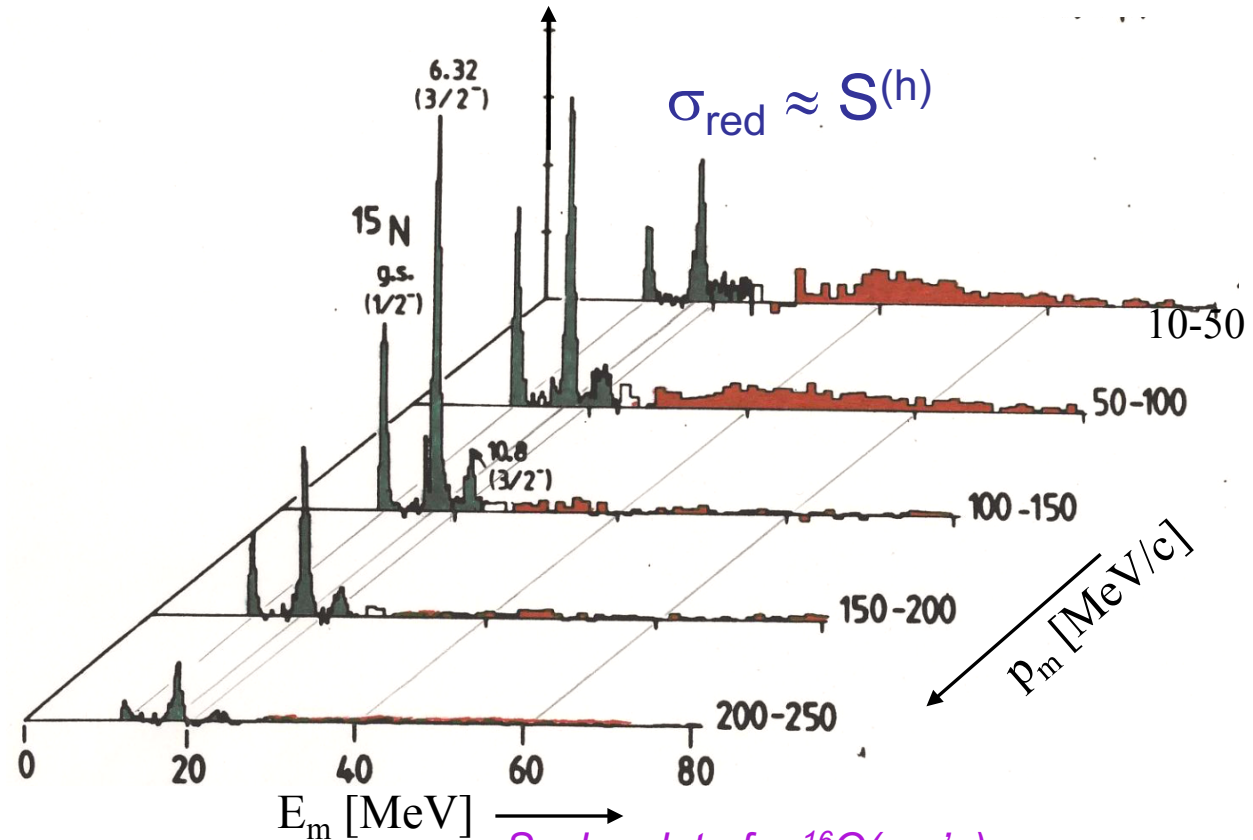
- we know, e , e' and p

- "get" *energy and momentum* of p_i :
$$p_i = k_{e'} + k_p - k_e$$
$$E_i = E_{e'} + E_p - E_e$$

Better to choose
large transferred
momentum and weak
probes!!!

Concept of correlations

Spectral function: distribution of momentum (p_m) and energies (E_m)



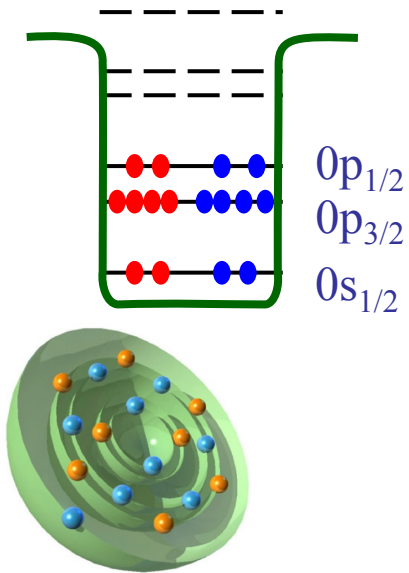
Saclay data for $^{16}\text{O}(e, e'p)$

[Mougey et al., Nucl. Phys. A335, 35 (1980)]

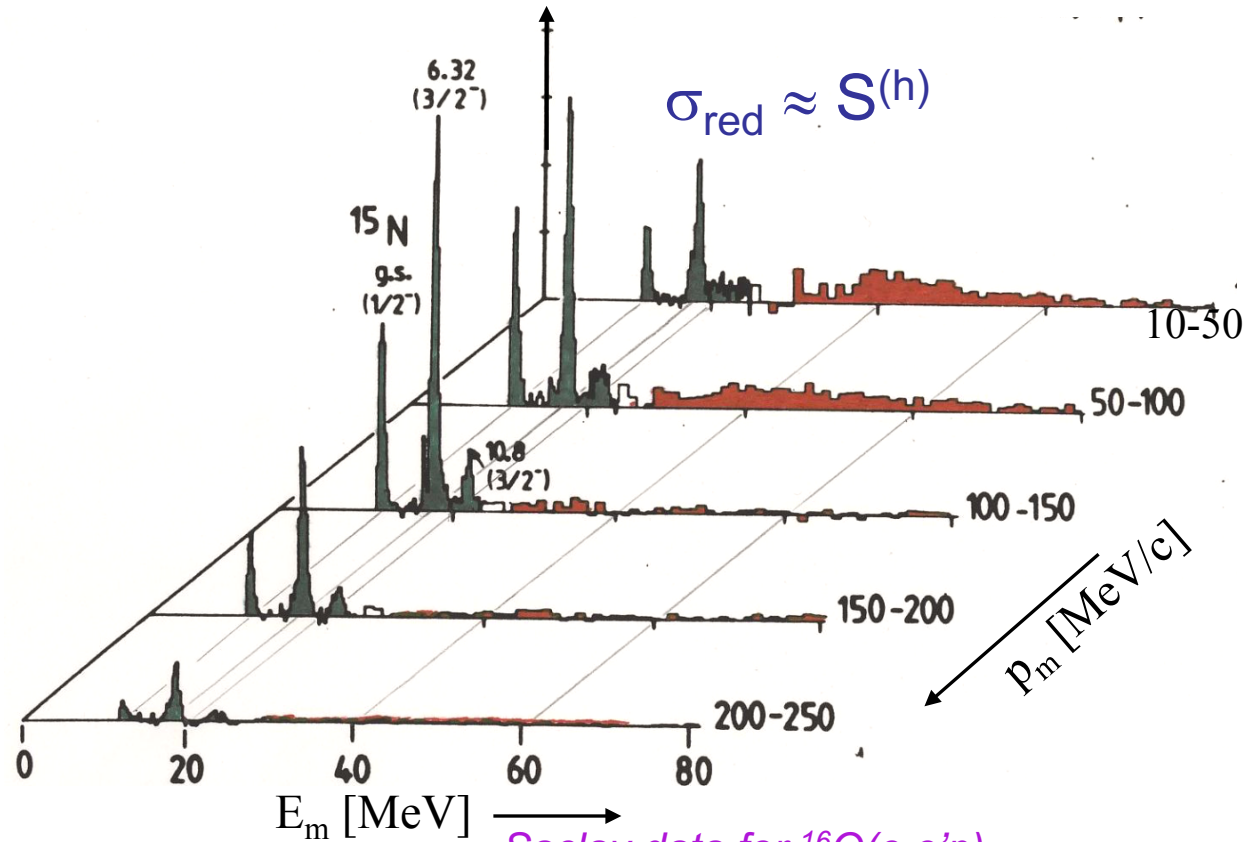
$$S^{(h)}(p_m, E_m) = \sum_n \left| \langle \Psi_n^{A-1} | c_{p_m}^- | \Psi_0^A \rangle \right|^2 \delta(E_m - (E_0^A - E_n^{A-1}))$$

Concept of correlations

independent
particle picture



Spectral function: distribution of
momentum (p_m) and energies (E_m)



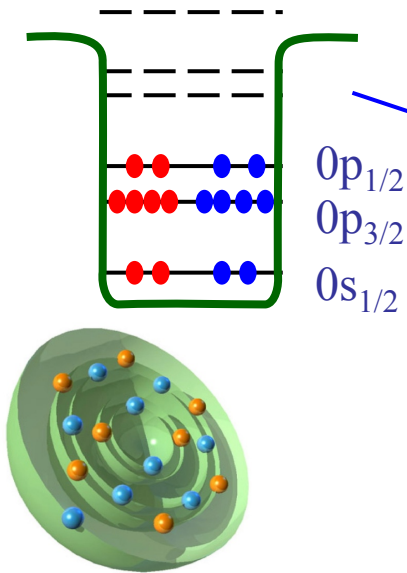
Saclay data for $^{16}\text{O}(e, e'p)$

[Mougey et al., Nucl. Phys. A335, 35 (1980)]

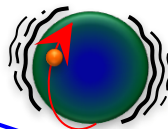
$$S^{(h)}(p_m, E_m) = \sum_n \left| \langle \Psi_n^{A-1} | c_{p_m}^- | \Psi_0^A \rangle \right|^2 \delta(E_m - (E_0^A - E_n^{A-1}))$$

Concept of correlations

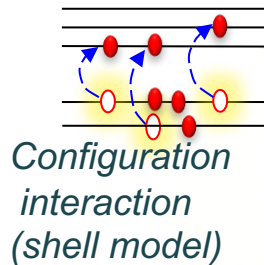
independent
particle picture



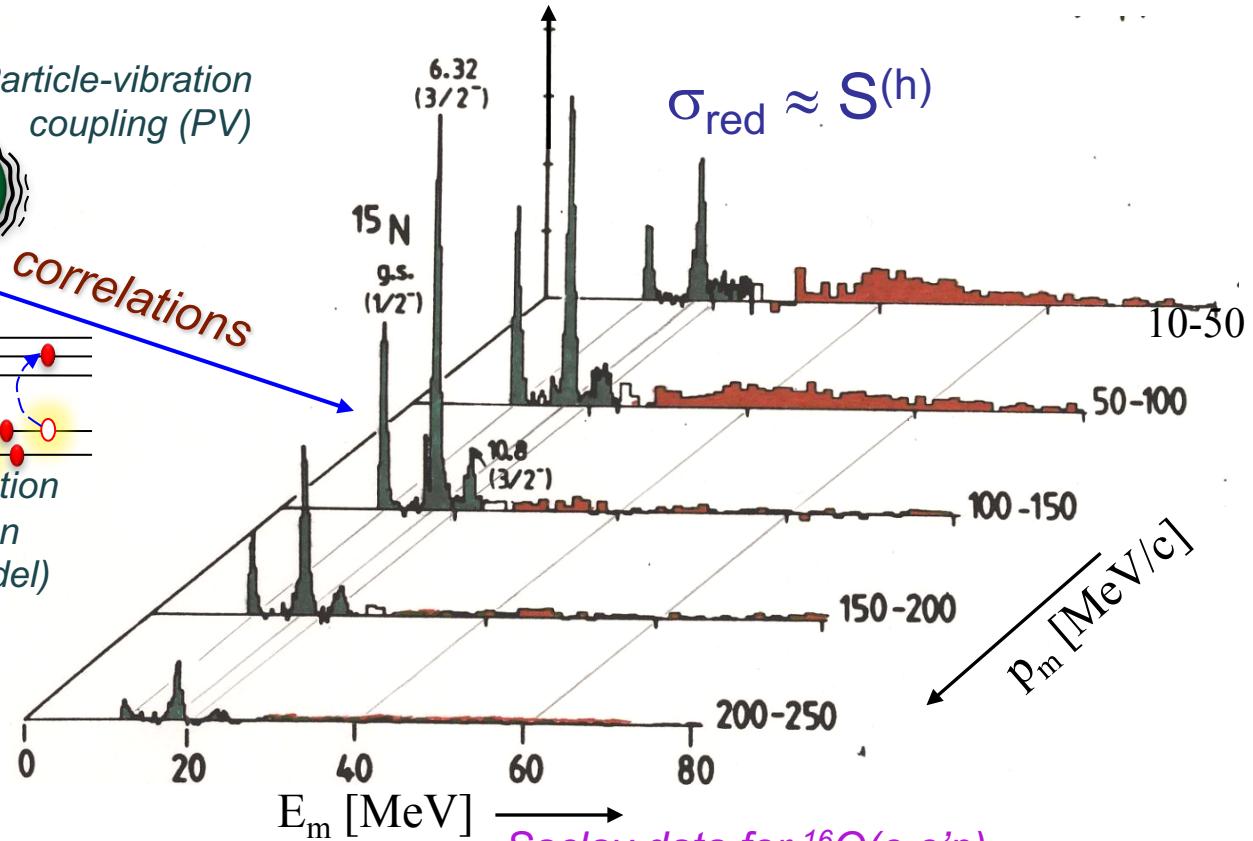
Particle-vibration
coupling (PV)



correlations



Spectral function: distribution of
momentum (p_m) and energies (E_m)



Saclay data for $^{16}\text{O}(e, e'p)$

[Mougey et al., Nucl. Phys. A335, 35 (1980)]

$$S^{(h)}(p_m, E_m) = \sum_n \left| \langle \Psi_n^{A-1} | c_{p_m}^- | \Psi_0^A \rangle \right|^2 \delta(E_m - (E_0^A - E_n^{A-1}))$$

Concept of correlations

independent
particle picture

Spectral function: distribution of
momentum (p_m) and

Particle-vibration
coupling

Want to understand structure and nuclear forces
directly from first principles (ab initio).

So far, fully characterised only for closed-shell and
stable isotopes... (!)

[W. Dickhoff, CB, Prog. Part. Nucl. Phys. **52**, 377 (2004)]

E_m [MeV] → 60 80 200-250

Saclay data for $^{16}\text{O}(e,e'p)$

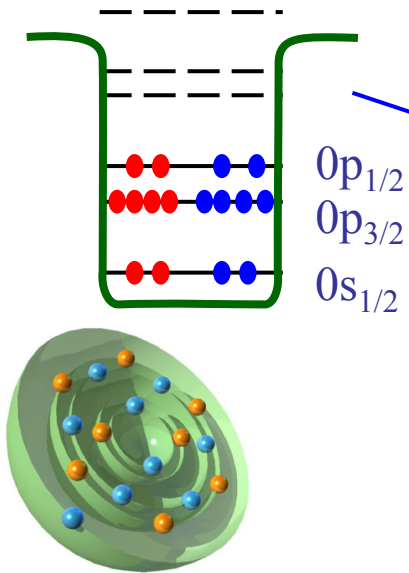
[Mougey et al., Nucl. Phys. A335, 35 (1980)]

Understand for a few stable closed shells:

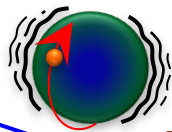
[CB and W. H. Dickhoff, Prog. Part. Nucl. Phys **52**, 377 (2004)]

Concept of correlations

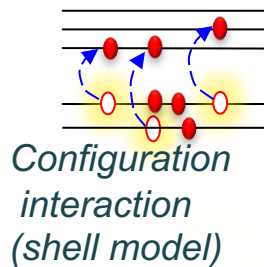
independent
particle picture



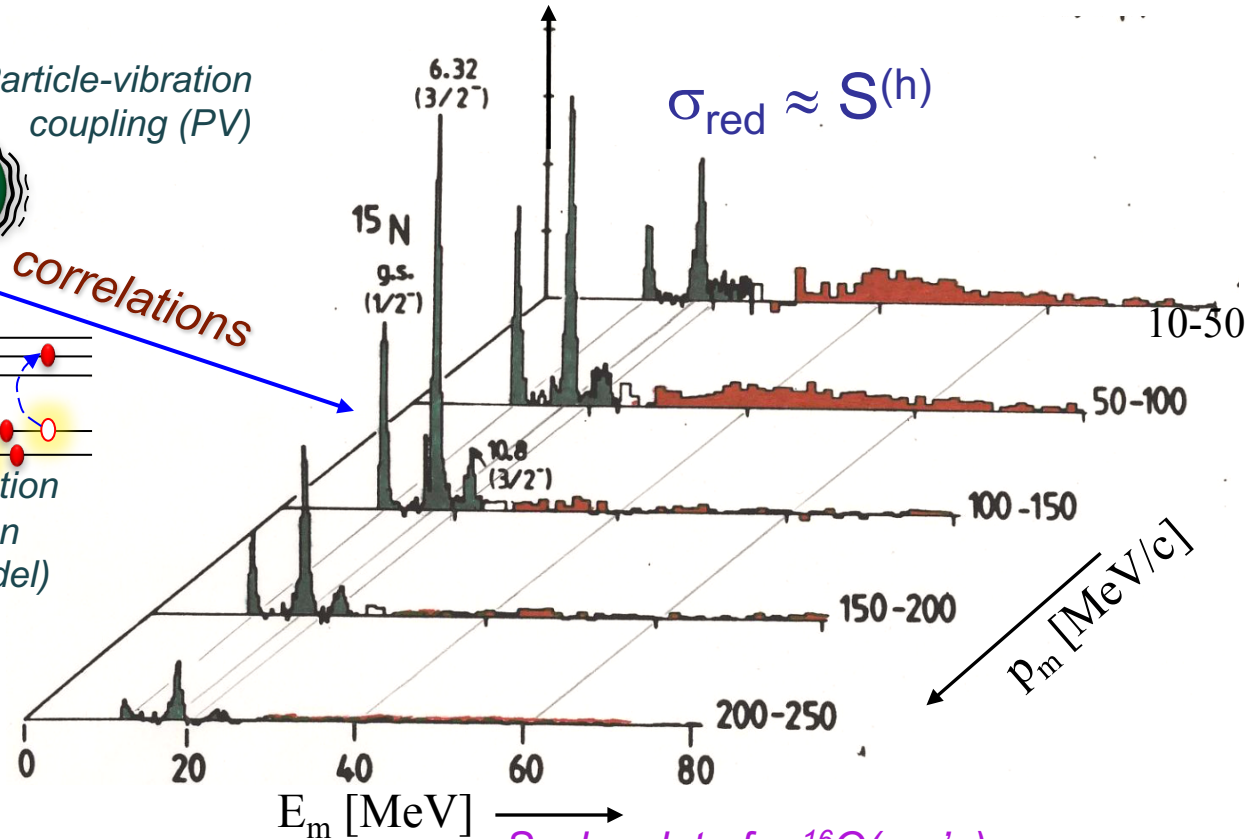
Particle-vibration
coupling (PV)



correlations



Spectral function: distribution of
momentum (p_m) and energies (E_m)



Saclay data for $^{16}O(e,e'p)$

[Mougey et al., Nucl. Phys. A335, 35 (1980)]

Understood for a few stable closed shells:

[CB and W. H. Dickhoff, Prog. Part. Nucl. Phys 52, 377 (2004)]

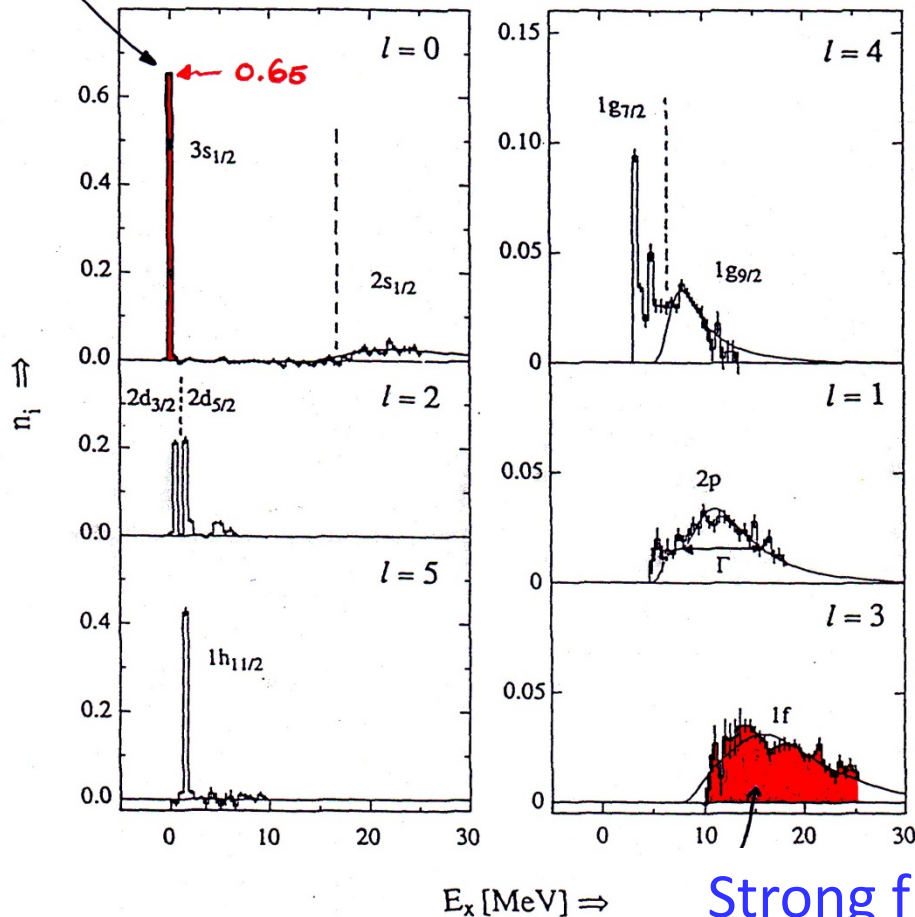
Fragmentation of ^{208}Pb [from $(e, e'p)$]

$$|\langle ^{207}_{g.s.}\text{Te} | a_{3s_{1/2}} | ^{208}_{g.s.}\text{Pb} \rangle|^2 \Rightarrow 0.65 (-0.70)$$

$^{208}\text{Pb}(e, e'p) ^{207}\text{Tl}$

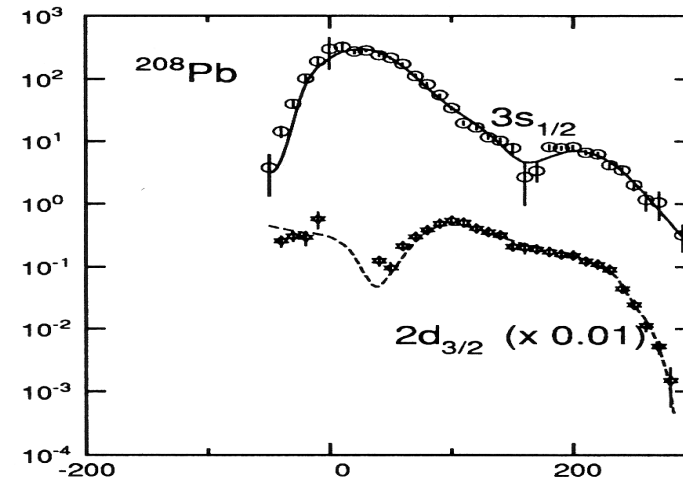
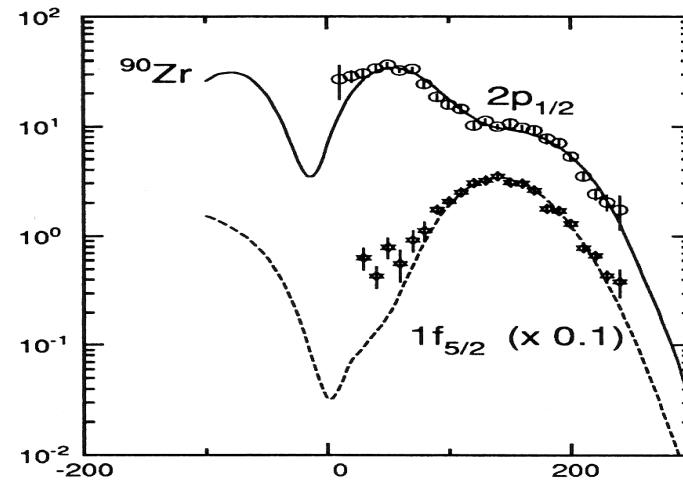
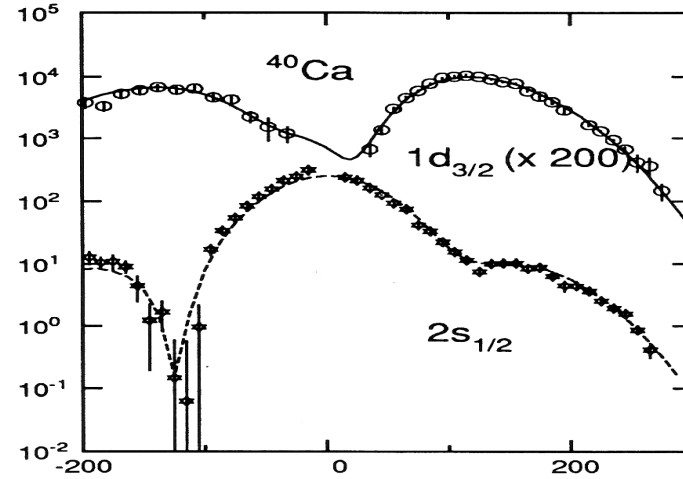
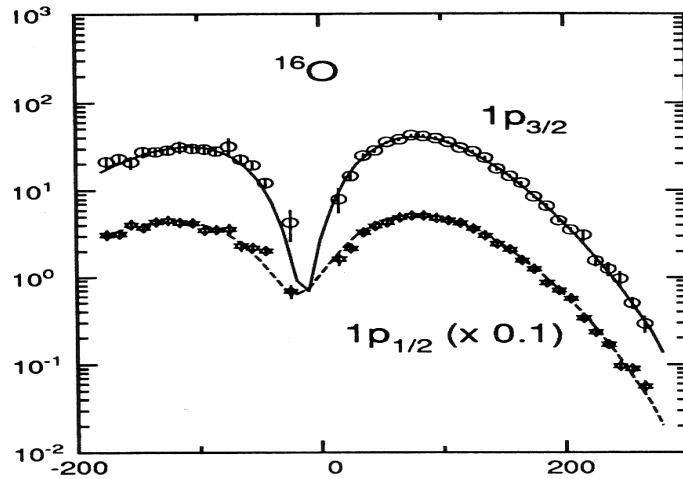
also $n(3s_{1/2}) = 0.75$

NIKHEF, 1988

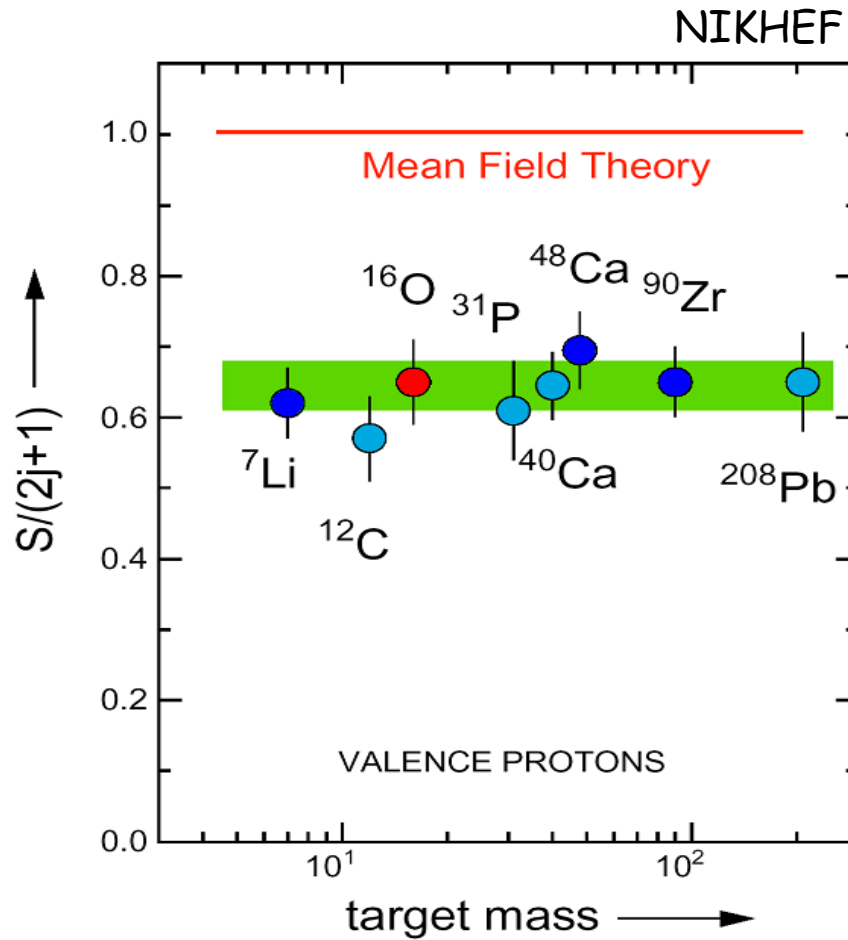


Strong fragmentation of
deeply-bound states

Mean field orbits in nuclei [from $(e, e'p)$]



Experimental spectroscopic factors

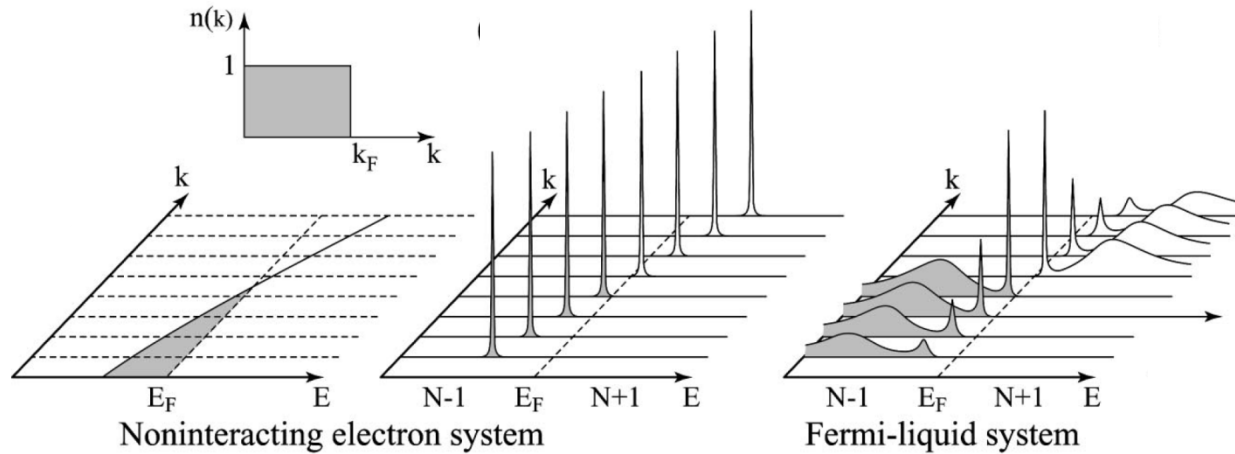


Stable nuclei,
From (e,e'p)

Nucl. Phys. A553 (1993) 297c

One-hole spectral function

Spectral function of infinite fermion systems

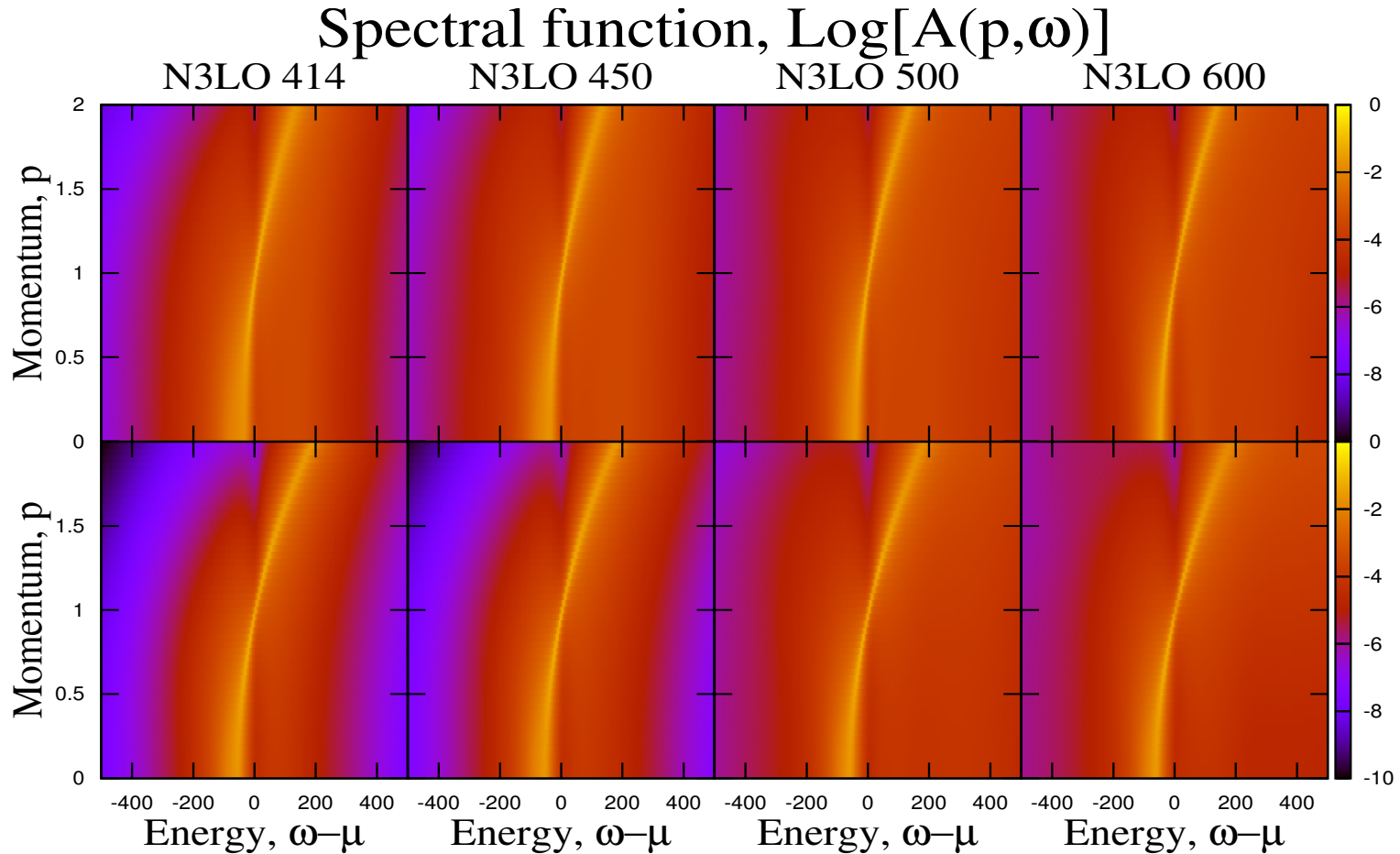


$$S(\mathbf{p}, \omega) = \theta(|\mathbf{p}| - k_F) \delta\left(\hbar\omega - \frac{p^2}{2m}\right) + \theta(k_F - |\mathbf{p}|) \delta\left(\hbar\omega - \frac{p^2}{2m}\right)$$

$$S^h(\mathbf{p}, \omega) = \sum_k \left| \langle \Psi_k^{N-1} | \psi_k(\mathbf{p}) | \Psi_0^N \rangle \right|^2 \delta\left(\hbar\omega - (E_0^N - E_k^{N-1})\right)$$

[Picture credit: A. Damascelli, Rev. Mod. Phys. 75, 473 (2003)]

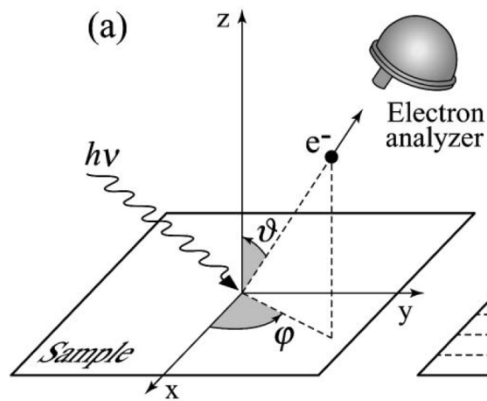
Spectral function in asymm. matter



A. Carbone, priv. comm.

Angle Resolved Photon Emission Spectroscopy (ARPES)

An ARPES setup - spectroscopy at the Fermi surface



Photoemission geometry

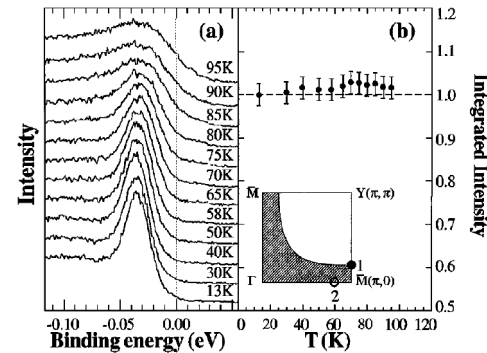


FIG. 4. Temperature dependence of the photoemission data from $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ ($T_c = 87$ K): (a) ARPES spectra measured at $\mathbf{k} = \mathbf{k}_F$ (point 1 in the Brillouin-zone sketch); (b) integrated intensity. From Randeria *et al.*, 1995.

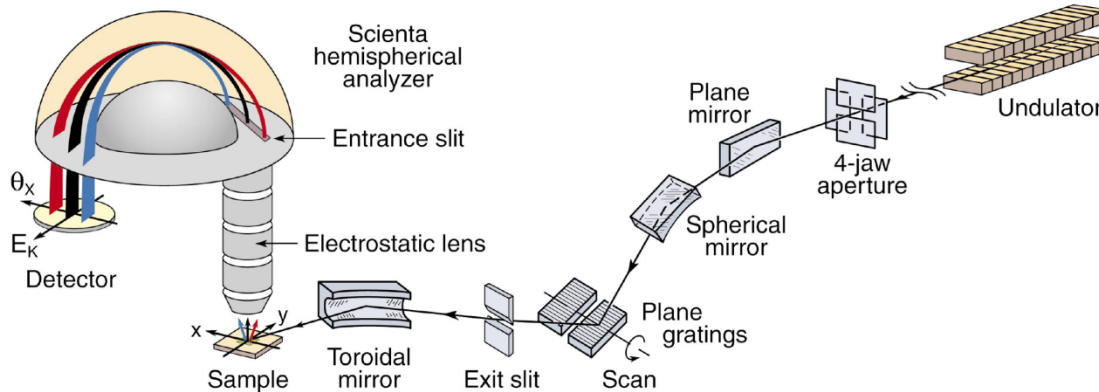


FIG. 6. Generic beamline equipped with a plane grating monochromator and a Scienta electron spectrometer (Color).

- Incoming beam of real photons
- Measure the emitted electron
- From angle and energy recover the momentum of the ejected particle + separation energy

[Pictures credit: A. Damascelli, *et. al*, Rev. Mod. Phys. 75, 473 (2003)]

Angle Resolved Photon Emission Spectroscopy (ARPES)

An ARPES setup - spectroscopy at the Fermi surface

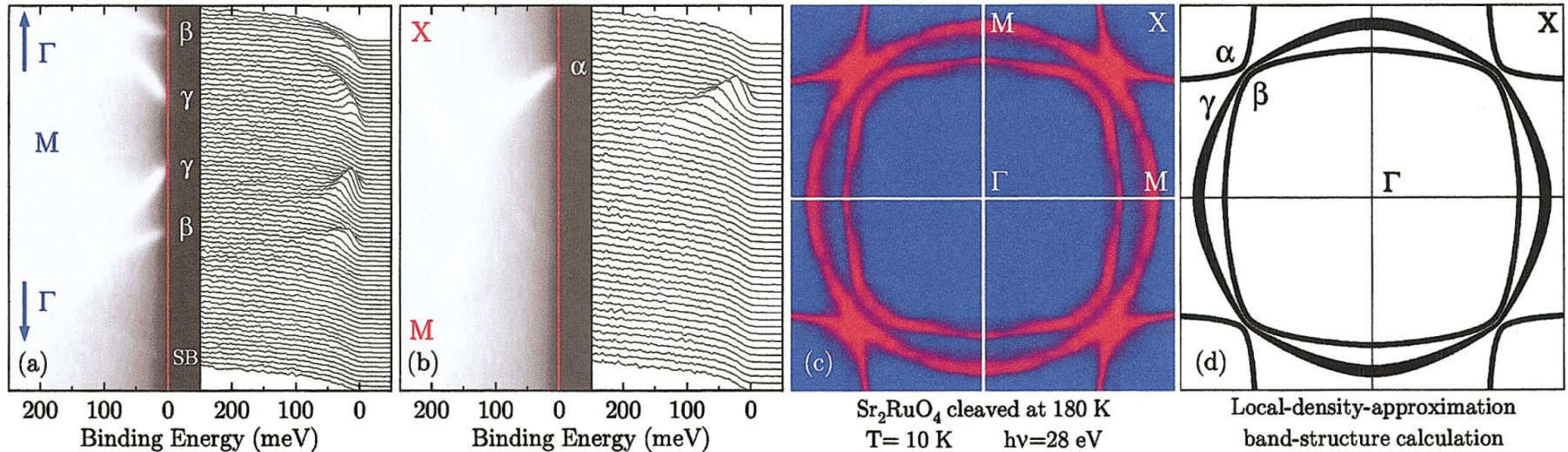


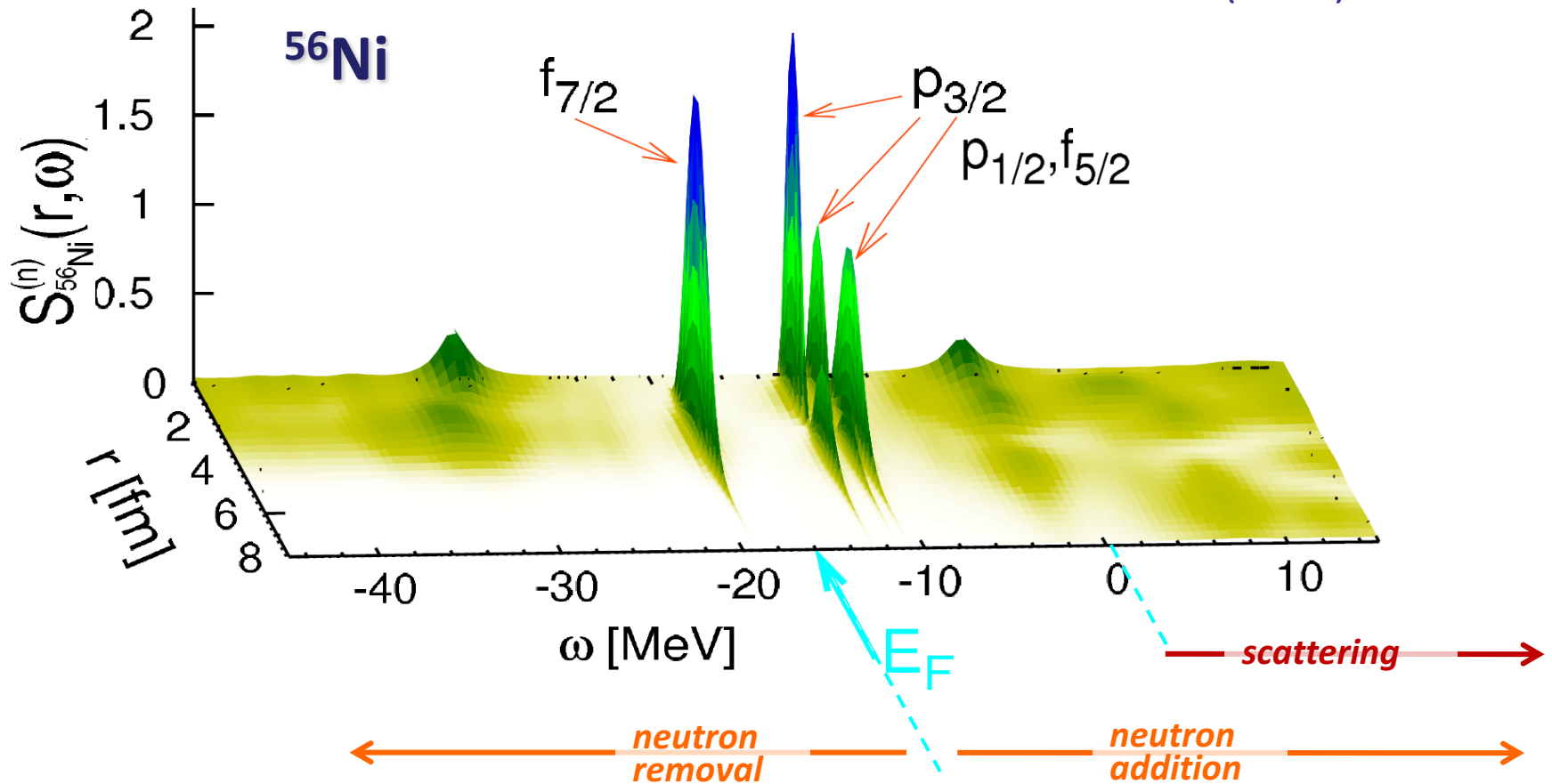
FIG. 9. Photoemission results from Sr_2RuO_4 : ARPES spectra and corresponding intensity plot along (a) Γ -M and (b) M-X; (c) measured Fermi surface; (d) calculated Fermi surface (Mazin and Singh, 1997). From Damascelli *et al.*, 2000 (Color).

→ can "see" the Fermi surface!!

Calculating spectral functions in finite (and exotic) nuclei

Spectral Function of ^{56}Ni

Faddeev-RPA (FRPA) calculations



[CB, M.Hjorth-Jensen, Pys. Rev. C79, 064313 (2009)]

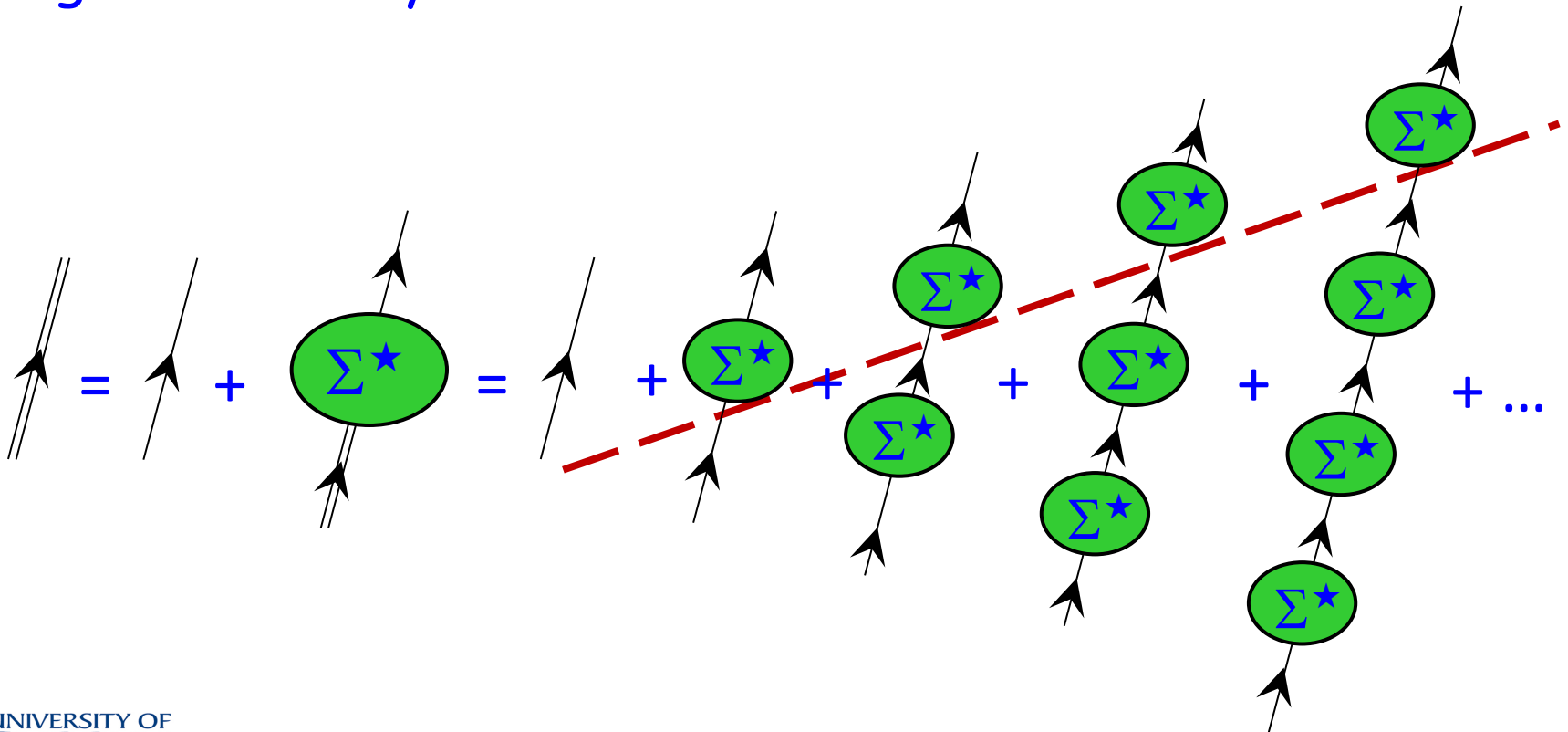
CB, Phys. Rev. Lett. 103, 202502 (2009)]

Dyson equation

Dyson equation:

$$g_{\alpha\beta}(t-t') = g_{\alpha\beta}^{(0)}(t-t') + g_{\alpha\gamma}^{(0)}(t-t_\gamma) \Sigma_{\gamma\delta}^*(t_\gamma, t_\delta) g_{\delta\beta}(t_\gamma-t')$$

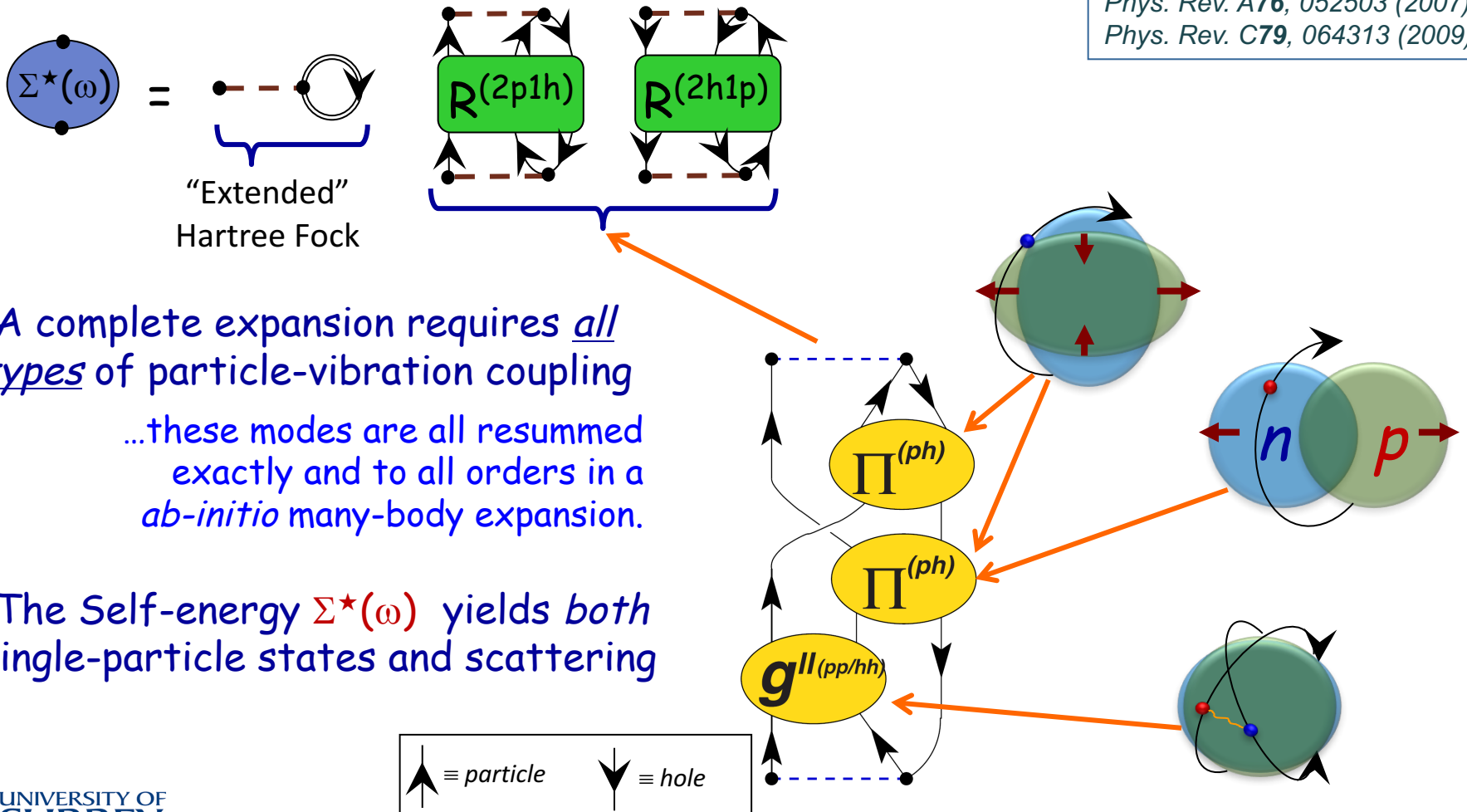
Diagrammatically:



The FRPA Method in Two Words

Particle vibration coupling is the main cause driving the distribution of particle strength—on both sides of the Fermi surface...

CB et al.,
 Phys. Rev. C63, 034313 (2001)
 Phys. Rev. A76, 052503 (2007)
 Phys. Rev. C79, 064313 (2009)

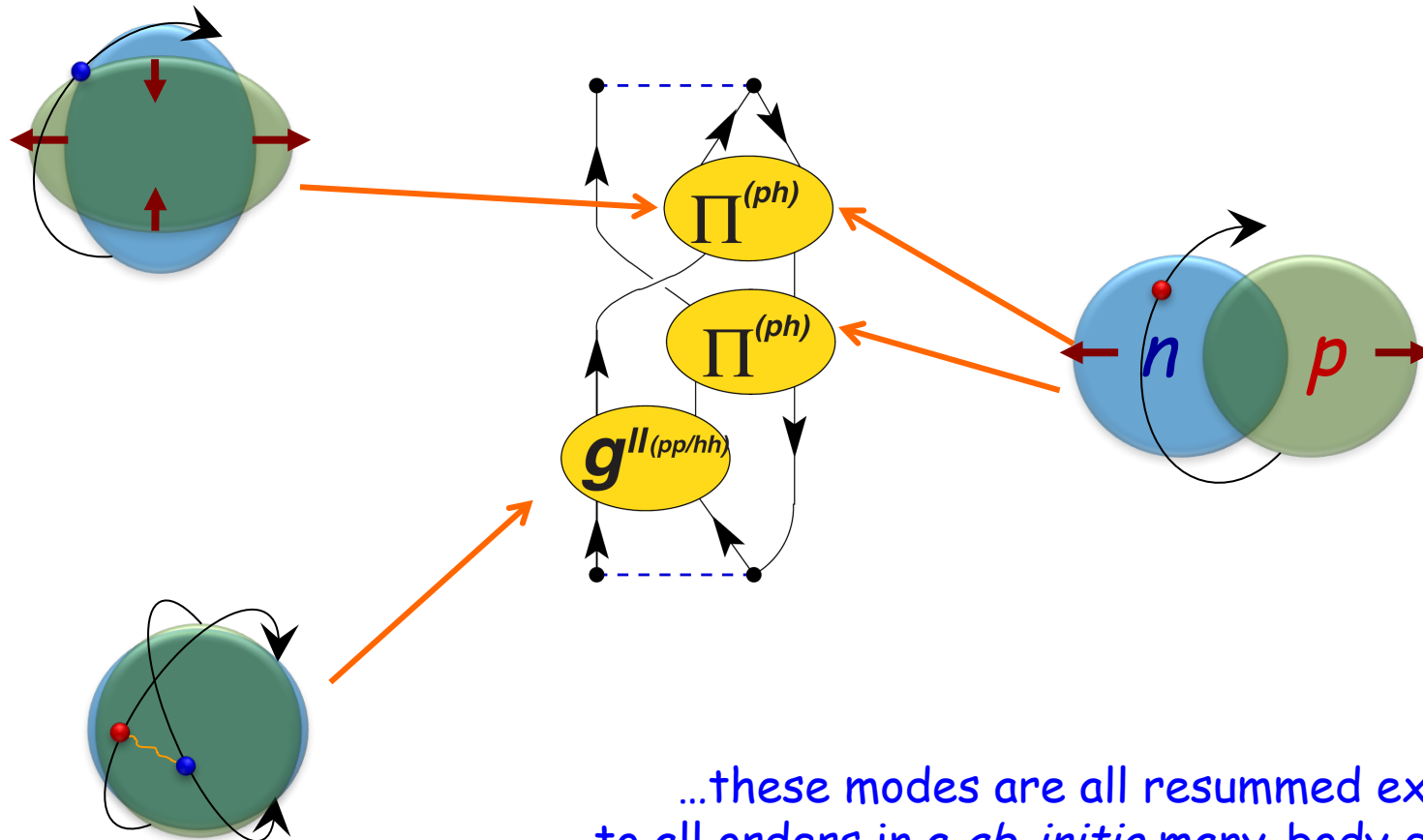
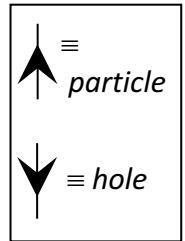


- A complete expansion requires all types of particle-vibration coupling
 ...these modes are all resummed exactly and to all orders in a *ab-initio* many-body expansion.

- The Self-energy $\Sigma^*(\omega)$ yields both single-particle states and scattering

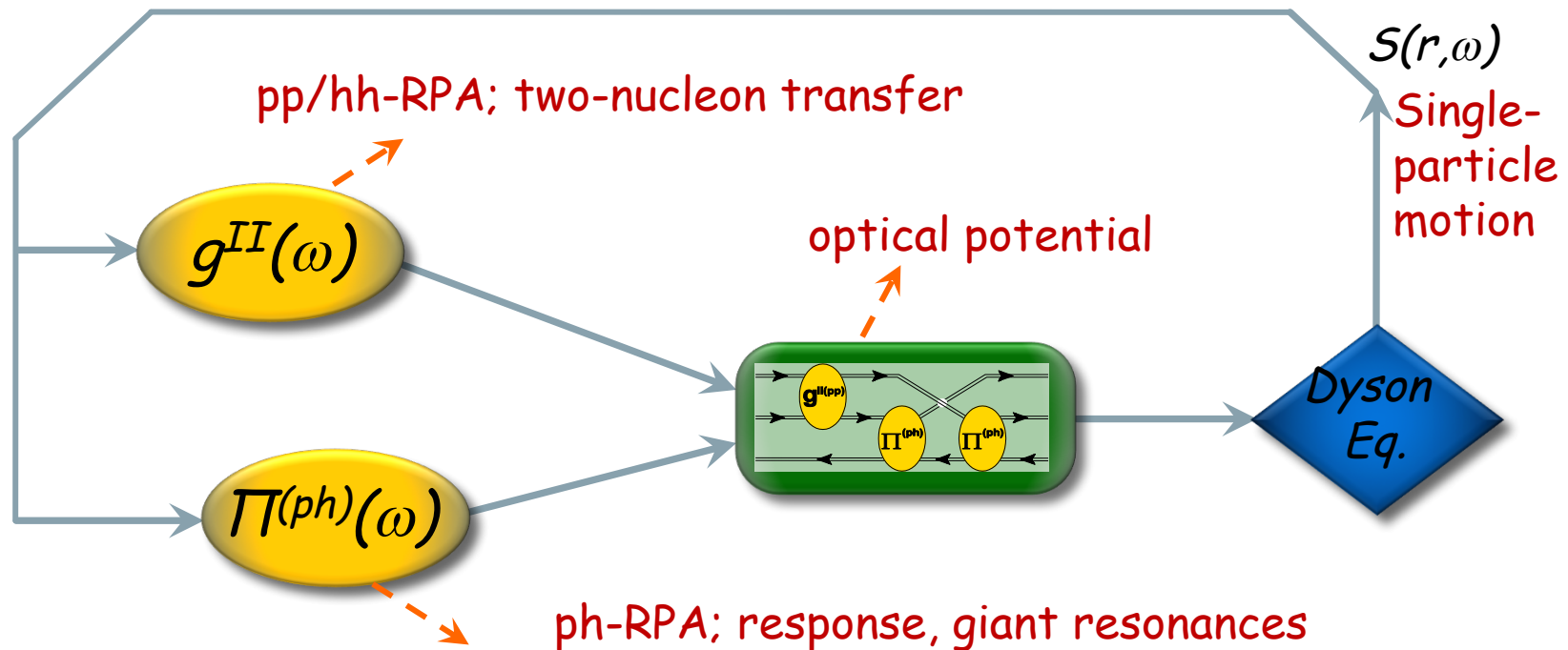
Faddeev-RPA in two words...

Particle vibration coupling is the main cause driving the distribution of particle strength—a least close to the Fermi surface...



...these modes are all resummed exactly and to all orders in a *ab-initio* many-body expansion.

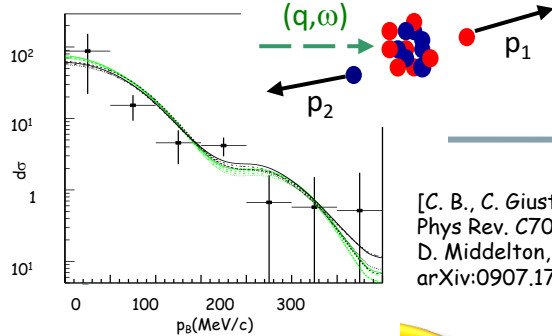
Self-Consistent Green's Function Approach



- Global picture of nuclear dynamics
- Reciprocal correlations among effective modes
- Guaranties *macroscopic conservation laws*

Self-Consistent Green's Function Approach

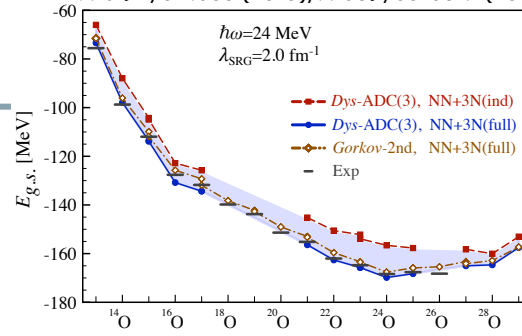
$^{16}\text{O}(e,e'pn)^{14}\text{N}$ @ MAINZ



[C. B., C. Giusti, et al. Phys Rev. C70, 014606 (2004)
D. Middleton, et al. arXiv:0907.1758; EPJA in print]

Binding energies

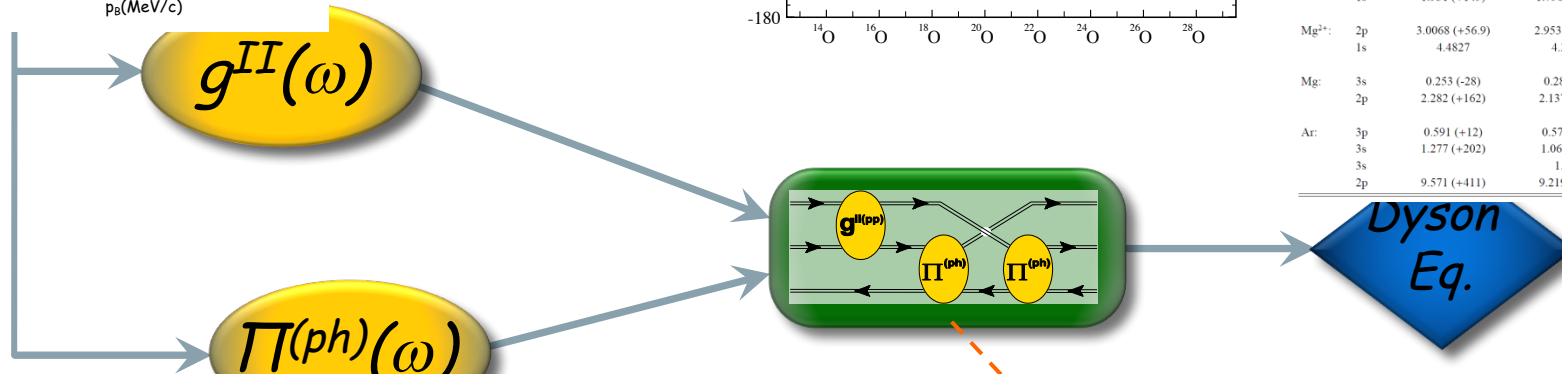
[PRL. 111, 062501 (2013),
PRC 92, 014306 (2015), PRC89, 061301R (2014)]



Ionization energies/
affinities, in atoms

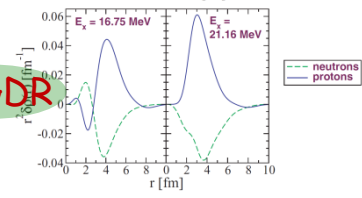
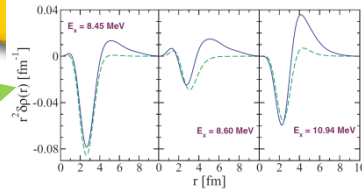
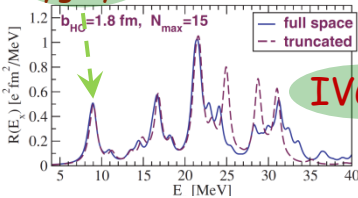
[CB, D. Van Neck,
AIP Conf.Proc.1120,104 ('09) & in prep]

		Hartree-Fock	FRPAc	Experiment [16, 17]
He:	1s	0.918 (+14)	0.9008 (-2.9)	0.9037
Be ²⁺ :	1s	5.6672 (+116)	5.6551 (-0.5)	5.6556
Be:	2s	0.3093 (-34)	0.3224 (-20.2)	0.3426
	1s	4.733 (+200)	4.5405 (+8)	4.533
Ne:	2p	0.852 (+57)	0.8037 (+11)	0.793
	1s	1.931 (+149)	1.7967 (+15)	1.782
Mg ²⁺ :	2p	3.0068 (+56.9)	2.9537 (+3.8)	2.9499
	1s	4.4827	4.3589	
Mg:	3s	0.253 (-28)	0.280 (-1)	0.281
	2p	2.282 (+162)	2.137 (+17)	2.12
Ar:	3p	0.591 (+12)	0.579 (±0)	0.579
	3s	1.277 (+202)	1.065 (-10)	1.075
	3s		1.544	
	2p	9.571 (+411)	9.219 (+59)	9.160



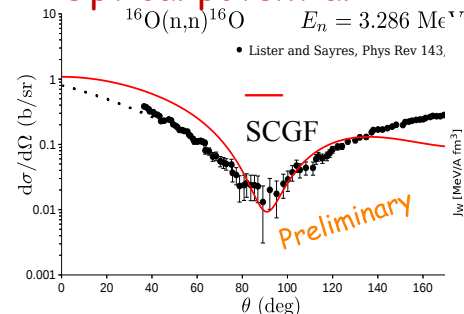
Isovector response
for ^{32}Ar , ^{34}Ar

Proton
Pygmy

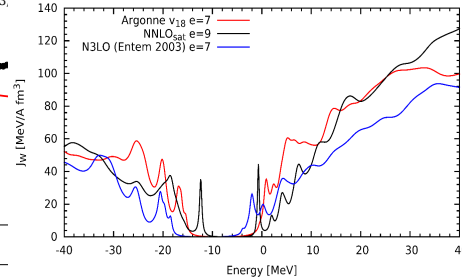


IVGDR

Optical potential



arXiv:1612.01478 [nucl-th]



Approaches in GF theory

Truncation scheme:

1st order:

2nd order:

...

3rd and all-orders sums, P-V coupling

Dyson formulation (closed shells)

Hartree-Fock

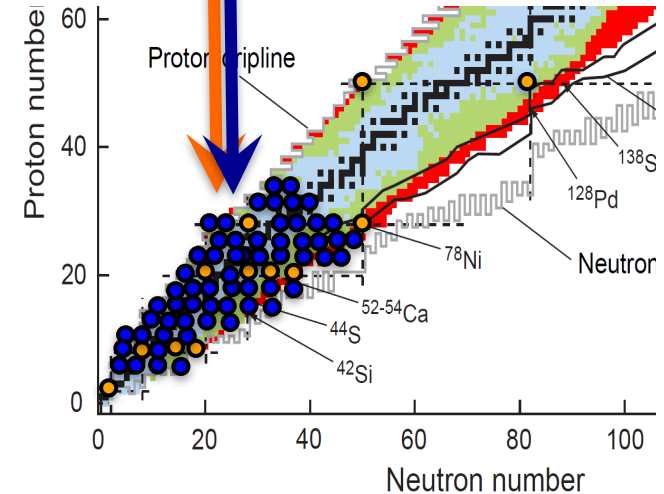
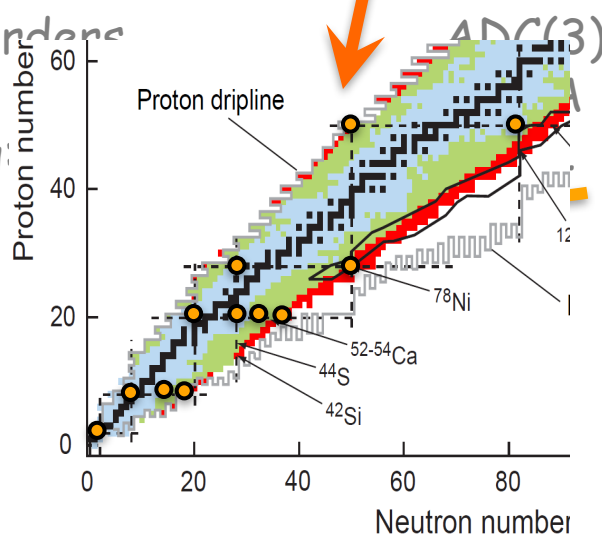
2nd order

...

Gorkov formulation (semi-magic)

HF-Bogoliubov

2nd order (w/ pairing)



Ab-initio Nuclear Computation & BcDor code

BoccaDorata code:

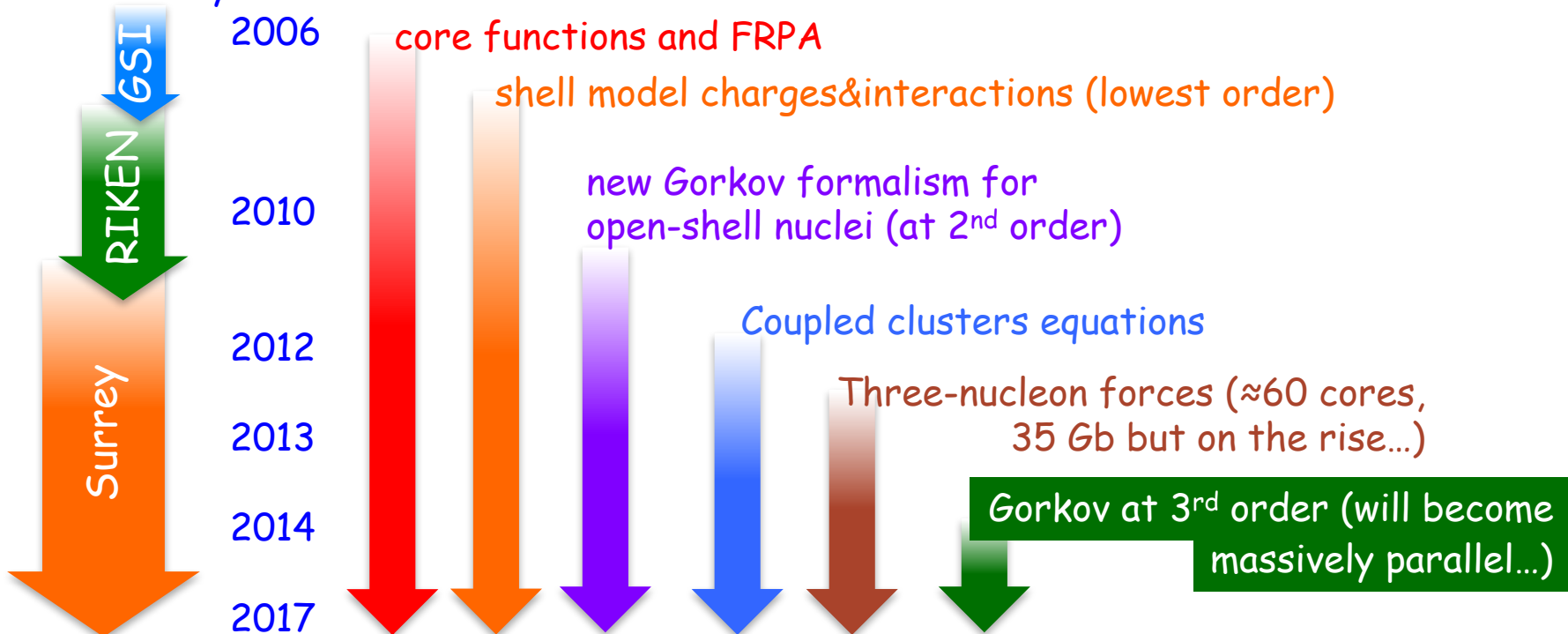
(C. Barbieri 2006-16

V. Somà 2010-15

A. Cipollone 2011-14)

- Provides a C++ class library for handling many-body propagators ($\approx 40,000$ lines, MPI&OpenMP based).
- Allows to solve for nuclear spectral functions, many-body propagators, RPA responses, coupled cluster equations and effective interaction/charges for the shell model.

Code history:

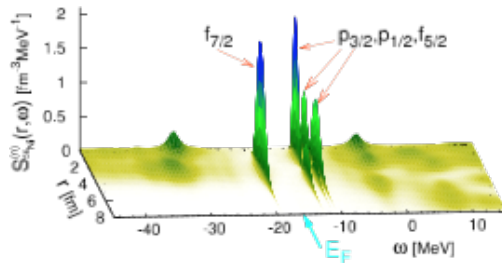


... applications ...

Ab-initio Nuclear Computation & BcDor code

<http://personal.ph.surrey.ac.uk/~cb0023/bcdor/>

Computational Many-Body Physics



Welcome

From here you can download a public version of my self-consistent Green's function (SCGF) code for nuclear physics. This is a code in J-coupled scheme that allows the calculation of the single particle propagators (a.k.a. one-body Green's functions) and other many-body properties of spherical nuclei.

This version allows to:

- Perform Hartree-Fock calculations.
- Calculate the correlation energy at second order in perturbation theory (MBPT2).
- Solve the Dyson equation for propagators (self consistently) up to second order in the self-energy.
- Solve coupled cluster CCD (doubles only!) equations.

When using this code you are kindly invited to follow the creative commons license agreement, as detailed at the weblinks below. In particular, we kindly ask you to refer to the publications that led the development of this software.

Relevant references (which can also help in using this code) are:

- Prog. Part. Nucl. Phys. 52, p. 377 (2004),
- Phys. Rev. A76, 052503 (2007),
- Phys. Rev. C79, 064313 (2009),
- Phys. Rev. C89, 024323 (2014)

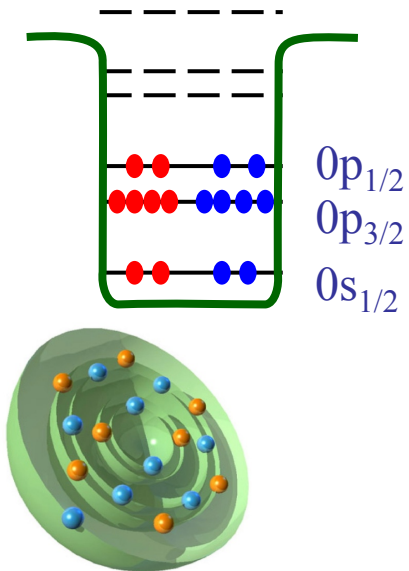
Download

Documentation

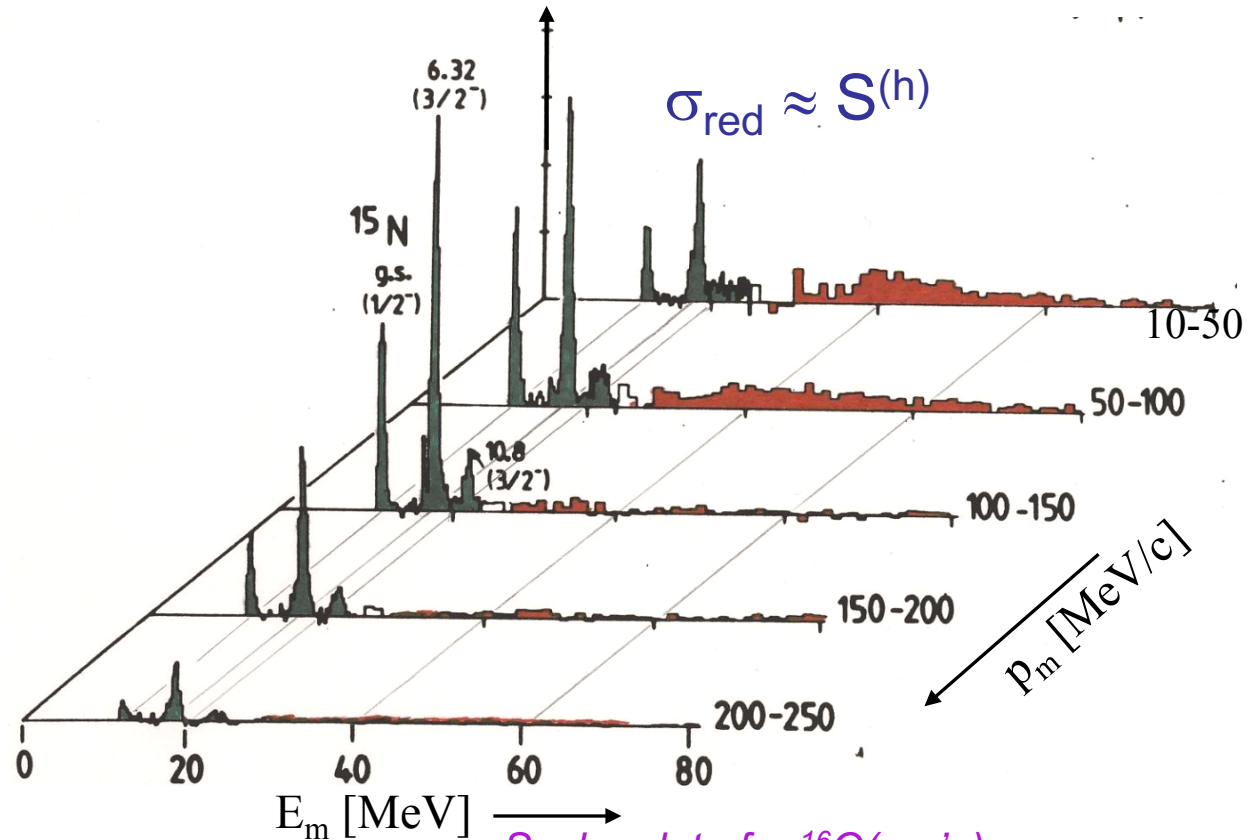
Spectroscopic factors

Concept of correlations

independent
particle picture



Spectral function: distribution of
momentum (p_m) and energies (E_m)



Saclay data for $^{16}\text{O}(e, e'p)$

[Mougey et al., Nucl. Phys. A335, 35 (1980)]

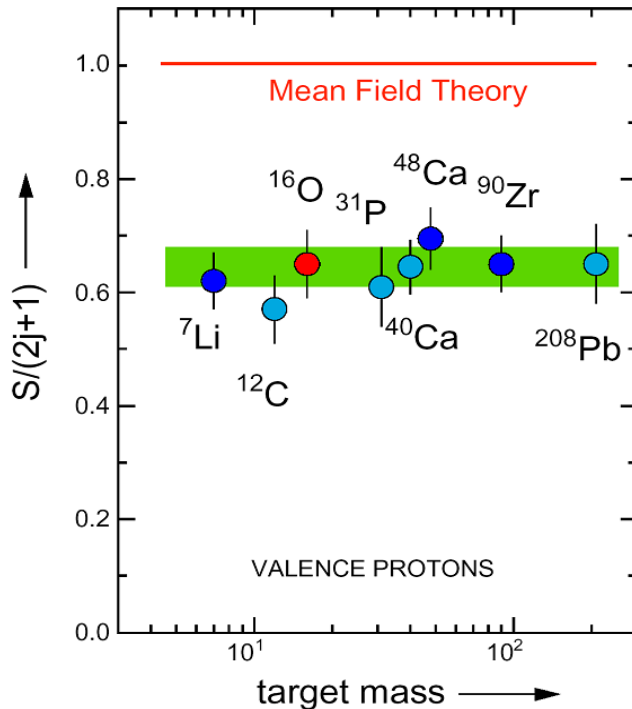
Understood for a few stable closed shells:

[CB and W. H. Dickhoff, Prog. Part. Nucl. Phys 52, 377 (2004)]

Quenching of SF in stable nuclei

Nucl. Phys. A553 (1993) 297c

NIKHEF:



A common misconception about SRC:

"The quenching is constant over all stable nuclei, so it must be a short-range effect"



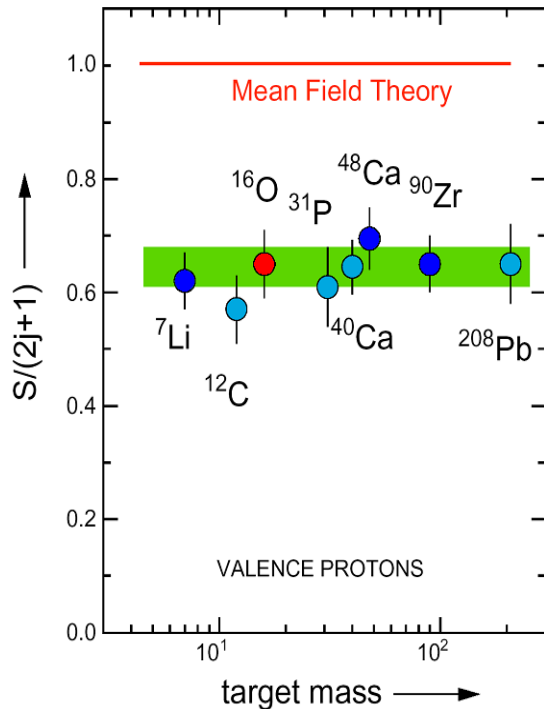
*Actually, **NO!***

All calculations show that SRC have just a small effect at the Fermi surface. And the correlation to the experimental p-h gap is much more important.

[W. Dickhoff, CB, Prog. Part. Nucl. Phys. **52**, 377 (2004)]

Quenching of SF in stable nuclei

NIKHEF:
Nucl. Phys. A553 (1993) 297c



- Short-range correlations oriented methods:

- VMC [Argonne, '94]
- GF(SRC) [St.Louis-Tübingen '95]
- FHNC/SOC [Pisa '00]

- Including particle-phonon couplings:

- GF(FRPA) [St.Louis '01]
[CB et al., Phys. Rev. C65, (02)]

- Experiment:

$S_{p1/2}$

$S_{p3/2}$

0.90

0.91

0.90

0.89

0.77

0.63

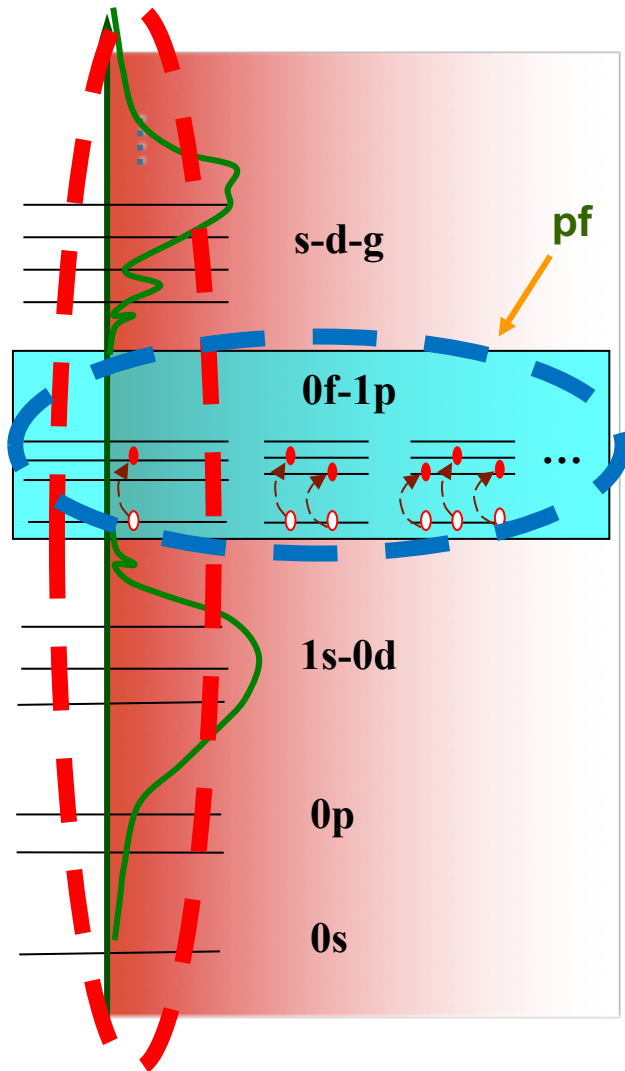
0.72

0.67 ± 0.07
(estimated uncertainty)

SRC are present and verified experimentally

BUT they are **NOT** the dominant mechanism for quenching SF !!!

Quenching of absolute spectroscopic factors



Particle-vibration coupling *dominates* the quenching of spectroscopic factors

Relative strength among fragments *requires* shell-model approach

Quenching of absolute spectroscopic factors

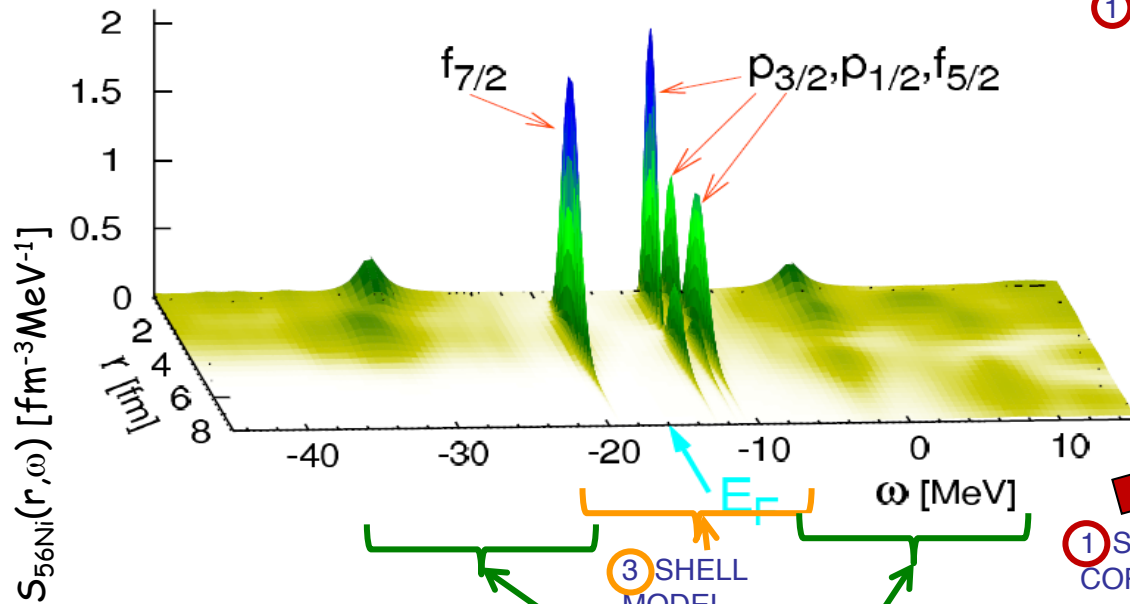
[CB, Phys. Rev. Lett. **103**, 202520

...with ~~analogous~~ **analogous conclusions for ^{48}Ca**

Overall quenching of *spectroscopic factors* is driven by:

- SRC* → ~10%
- part-vibr. coupling* → dominant
- "shell-model"* → in open shell

	10 osc. shells		Exp. [30]	1p0f space		
	FRPA (SRC)	full FRPA		FRPA	SM	ΔZ_α
^{57}Ni :						
$\nu 1p_{1/2}$	0.96	0.63	0.61	0.79	0.77	-0.02
$\nu 0f_{5/2}$	0.95	0.59	0.55	0.79	0.75	-0.04
$\nu 1p_{3/2}$	0.95	0.65	0.62	0.58(11)	0.79	-0.03
^{55}Ni :						
$\nu 0f_{7/2}$	0.95	0.72	0.69	0.89	0.86	-0.03



$$Z_\alpha = \int d^3r |\psi_\alpha^{overlap}(\mathbf{r})|^2 = \frac{1}{1 - \left. \frac{\partial \Sigma_{\hat{a}\hat{a}}(\omega)}{\partial \omega} \right|_{\omega=\epsilon_\alpha}}$$

① SHORT RANGE CORRELATIONS

② PARTICLE-VIBRATION COUPLING

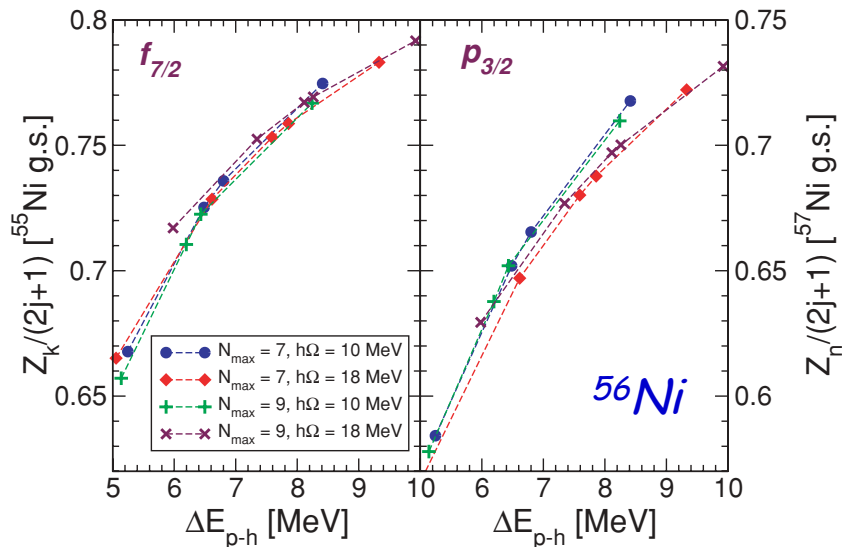
③ SHELL MODEL

Z/N asymmetry dependence of SFs - Theory

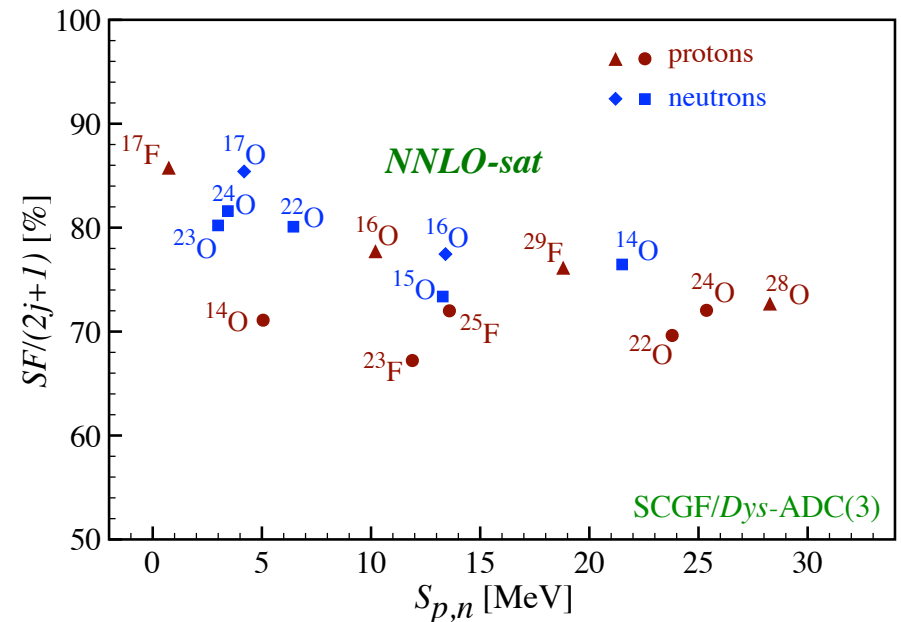
Ab-initio calculations explain (a very weak) the Z/N dependence but the effect is much lower than suggested by direct knockout

Rather the quenching is high correlated to the gap at the Femi surface.

Spectroscopic factor are strongly correlated to p-h gaps:



CB, M. Hjorth-Jensen,
Phys. Rev. C **79**, 064313 (2009)



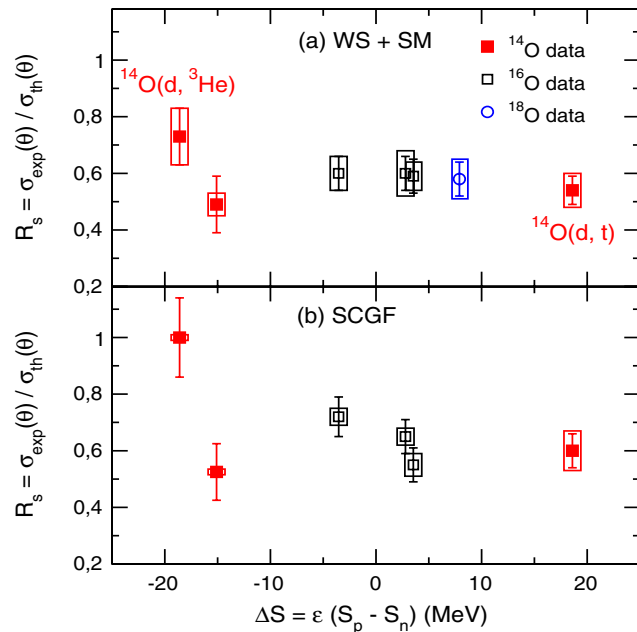
A. Cipollone, CB, P Navrátil, Phys. Rev. C **92**, 014306 (2015)
and CB, unpublished (2016)

Z/N asymmetry dependence of SFs

Calculated spectroscopic factors are found to be:

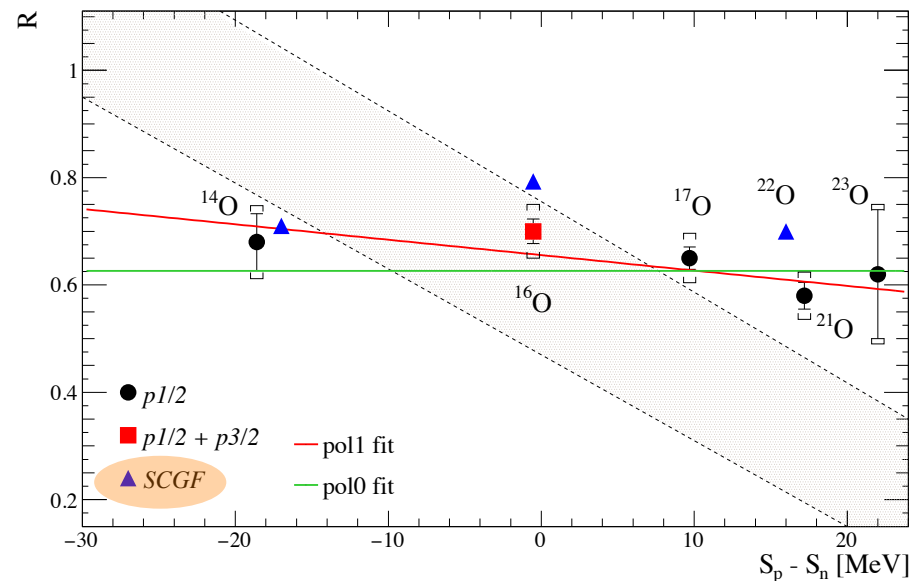
- correlated to p-h gaps
- independent of asymmetry
- consistent with experimental data

$^{14}\text{O}(d,t)^{13}\text{O}$ and $^{14}\text{O}(d,^3\text{He})^{13}\text{N}$
transfer reactions @ SPIRAL



[F. Flavigny et al, PRL110, 122503 (2013)]

$A\text{O}(p,2p)^{A-1}\text{N}$ at GSI (R³B-LAND)



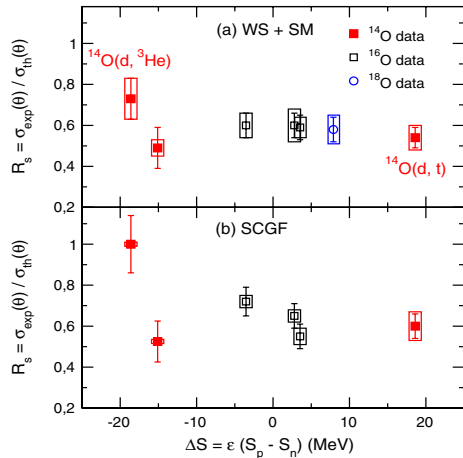
Proton SF for $^{16}\text{O} \rightarrow ^{15}\text{N}$:

$p_{1/2}$: 0.78 (SCGF) 0.80 (exp.)
 $p_{3/2}$: 0.80 (SCGF) 0.65 (exp. – up to cont.)

L. Atar, et al., in preparation (2017) – see talk by T. Aumann

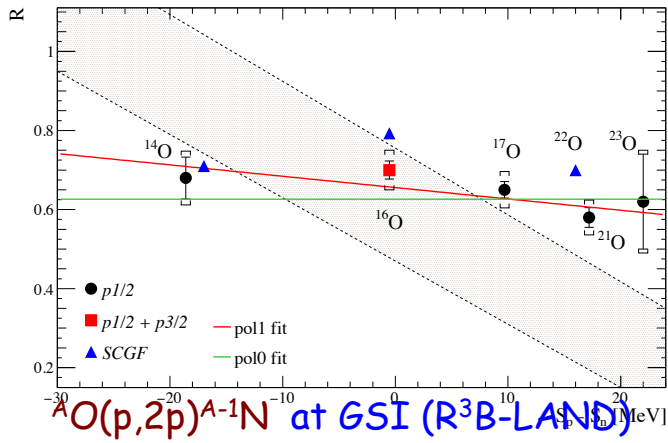
Spectroscopic factor Asymmetry puzzle

$^{14}\text{O}(d,t)^{13}\text{O}$ and $^{14}\text{O}(d,^3\text{He})^{13}\text{N}$ transfer reactions @ SPIRAL

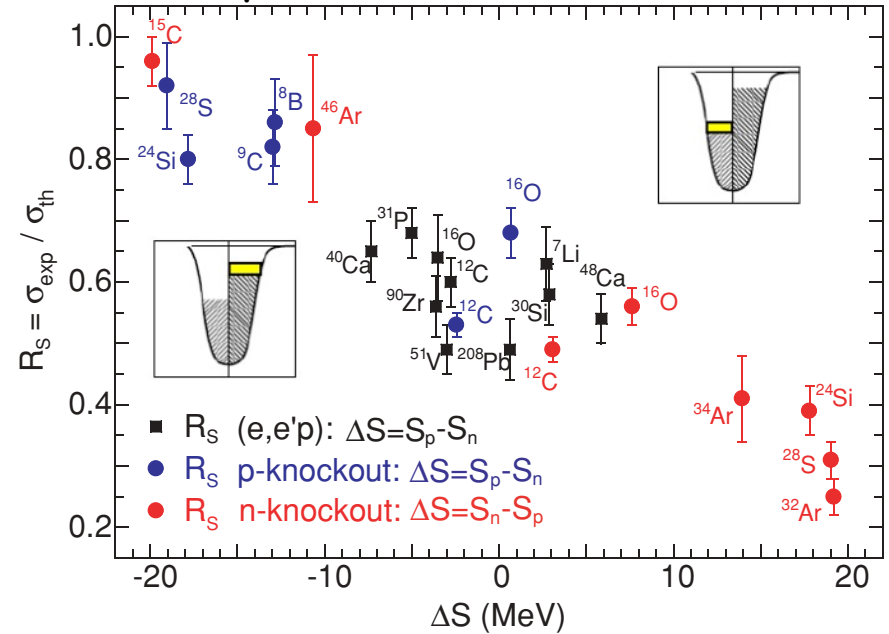


Dependence on proton-neutron difference is still unresolved...

- Missing many-body correlations?
- Reaction mechanism?



Peripheral knockout on ^9Be



Phys. Rev. C77, 044306 (2008)

Short-range correlations (SRC)

Are there signatures??

High momentum components - where are they?

Momentum distribution:

$$n(k) = \int_{-\infty}^{\bar{\varepsilon}_F} d\omega S^{(h)}(k, \omega)$$

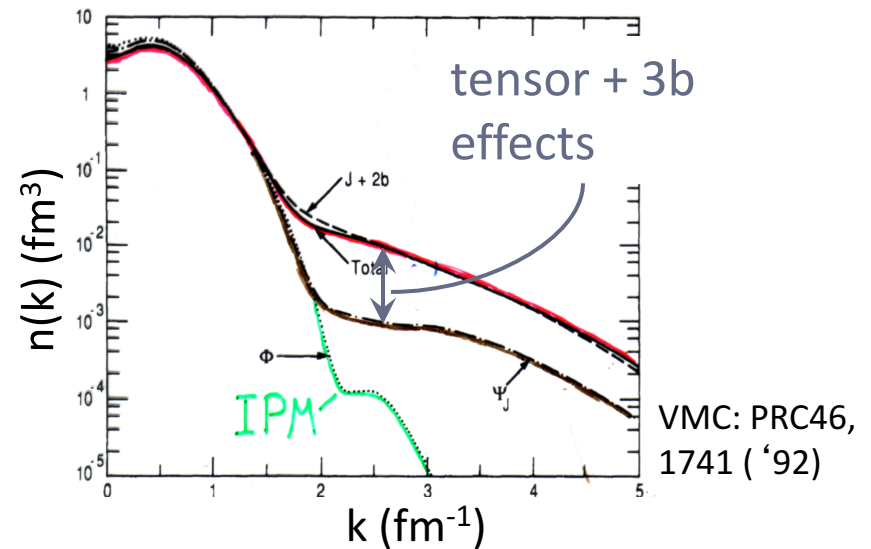
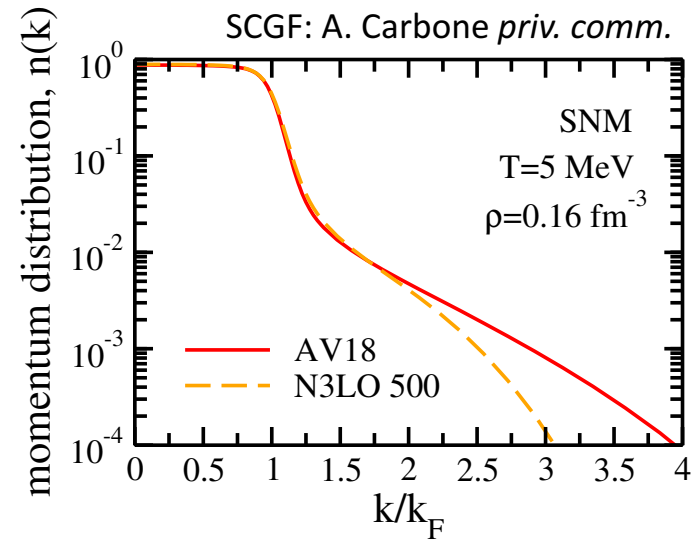
- High k components are found at high missing energies

- Short-range repulsion in r -space
 \leftrightarrow strong potential at large momenta

- A complication: the nuclear interaction includes also a tensor term (from Yukawa's meson meson exchange):

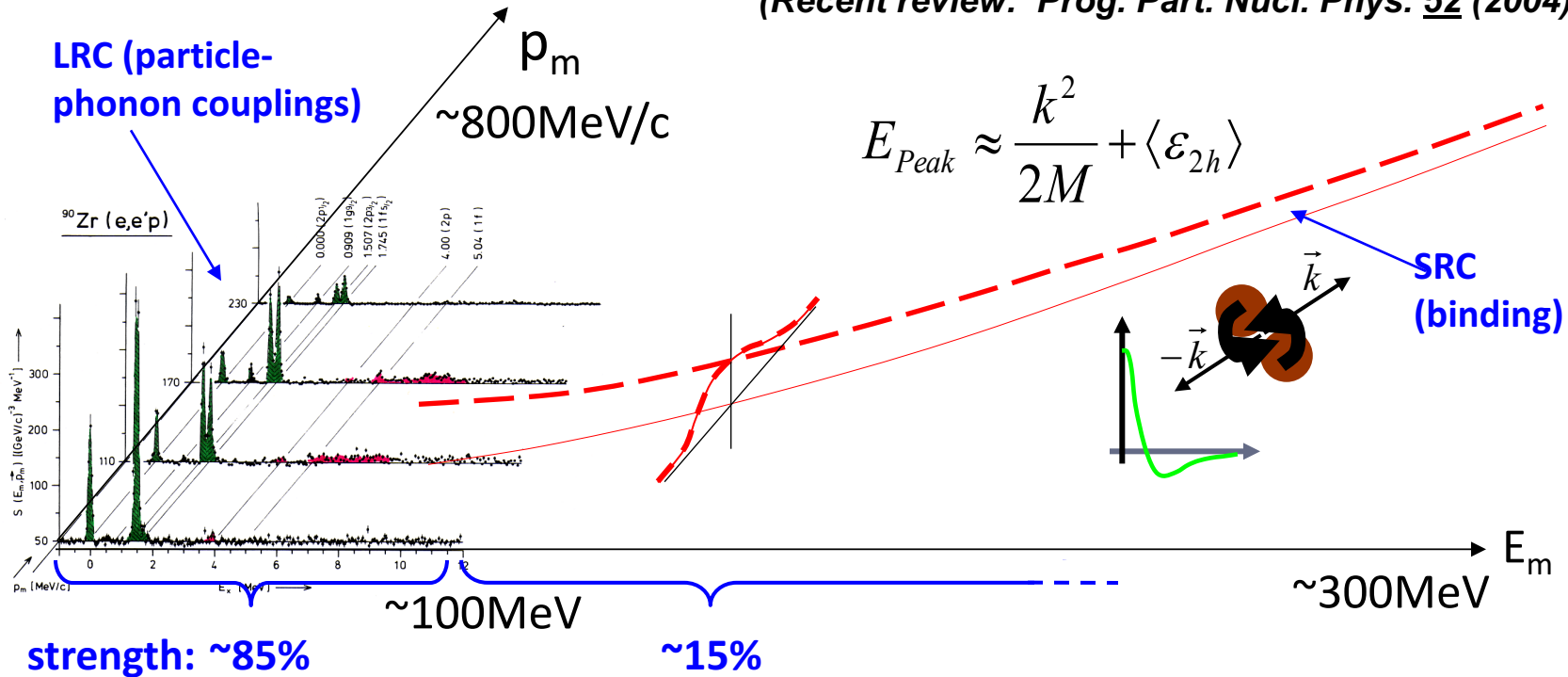
$$S_{12} = 3(\vec{\sigma}_1 \cdot \hat{r})(\vec{\sigma}_2 \cdot \hat{r}) - 1$$

\rightarrow interaction among 2 dipoles!!!!!!



Distribution of (All) the Nuclear Strength

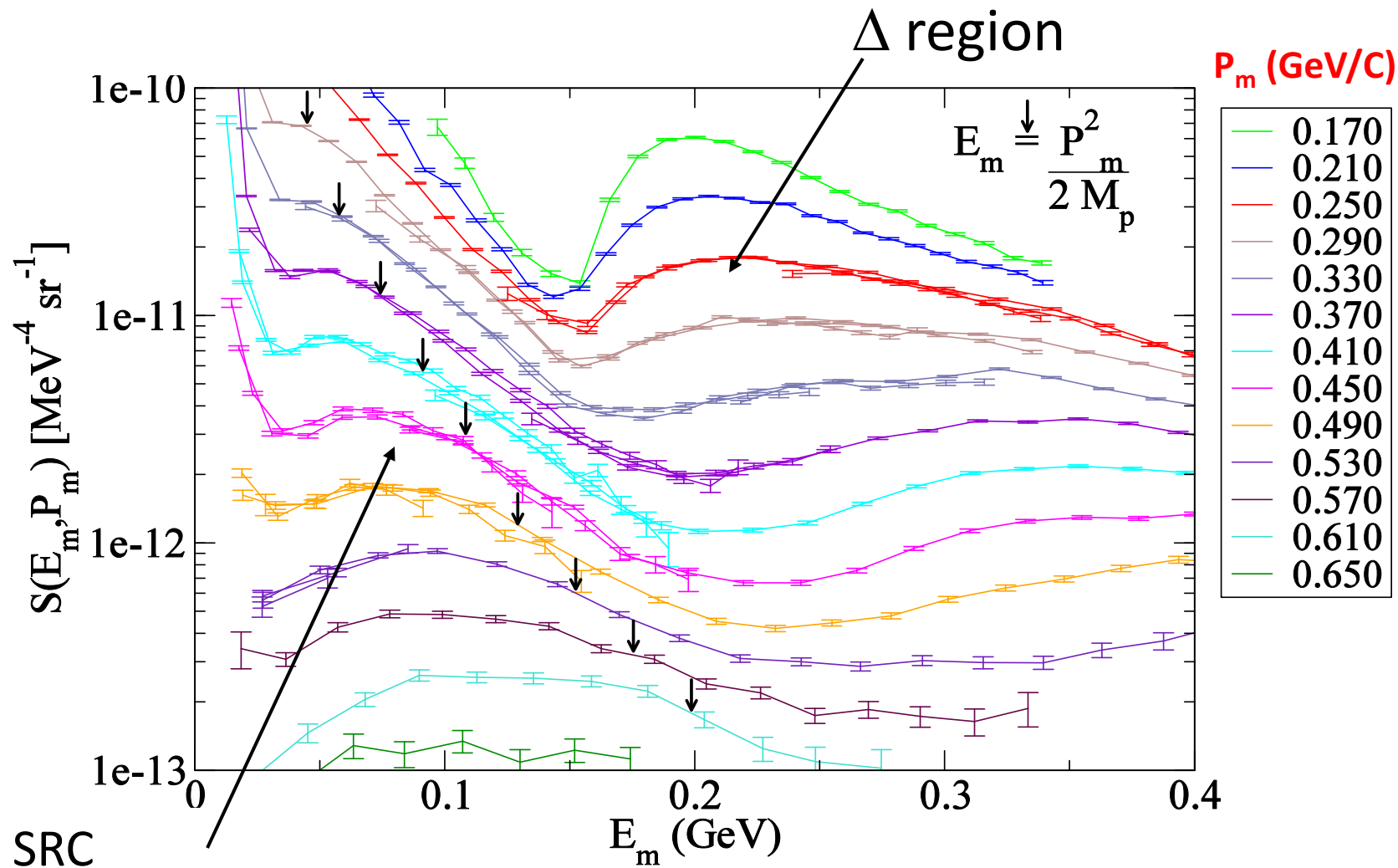
(Recent review: *Prog. Part. Nucl. Phys.* **52** (2004) 337.)



Interest in short range correlations:

- a fraction of the total number of nucleons:
 - $\sim 10\%$ in light nuclei (VMC, FHNC, Green's function)
 - 15-20% in heavy systems (CBF, Green's function)
- can explain up to **2/3 of the binding energy** [see ex. PRC51, 3040 ('95) for ^{16}O]
- influence NM saturation properties [see ex. PRL90, 152501 ('03)]

Spectral strength of ^{12}C from exp. E97-006



D.Rohe, et. al, Eur. Phys. J. A17, 349 (2003),
Phys Rev. Lett. 93 182501 (2004).

Theory vs. measured strength - I

- About 0.6 protons are found in the correlated region:

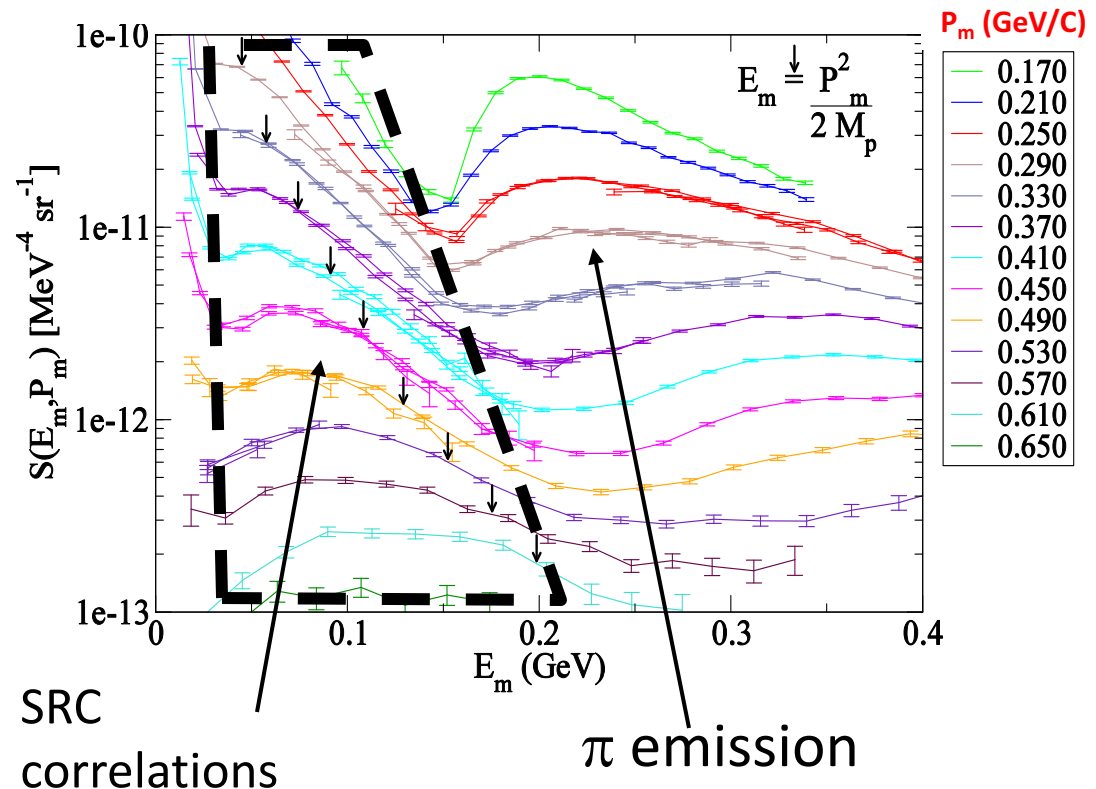
TABLE I. Correlated strength, integrated over shaded area of Fig. 2 (quoted in terms of the number of protons in ^{12}C .)

Experiment	0.61 ± 0.06
Greens Function Theory [28]	0.46
CBF Theory [3]	0.64

D.Rohe, et. Al,
Eur. Phys. J.
A17, 349 (2003)
PRL93 182501 (2004)

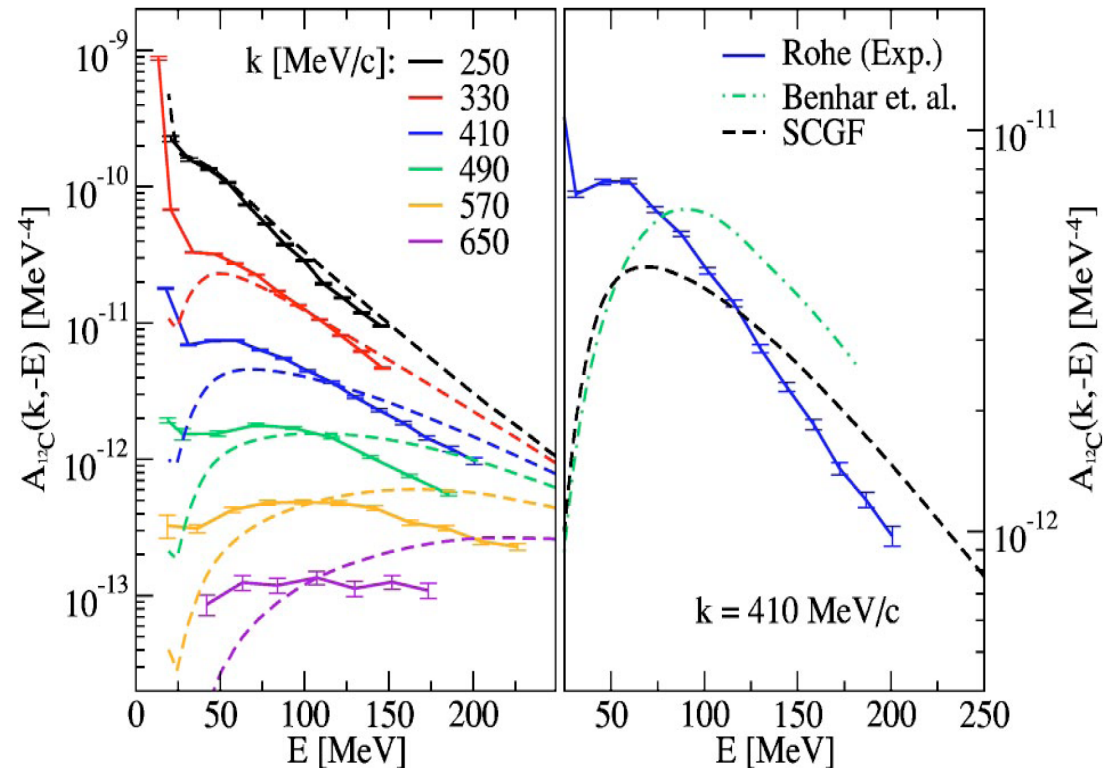
→ in good agreement with early theoretical predictions!

- what about the position of the peak?



Theory vs. measured strength - II

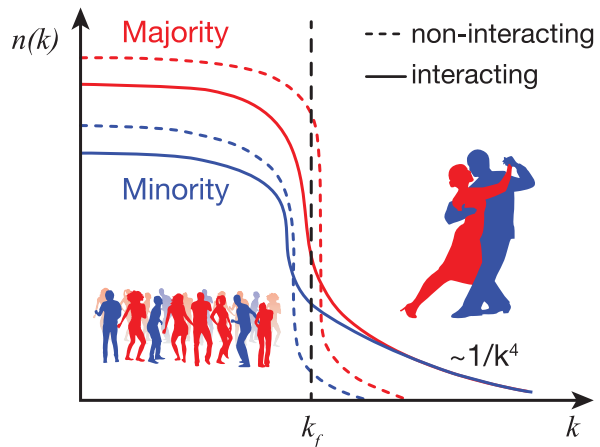
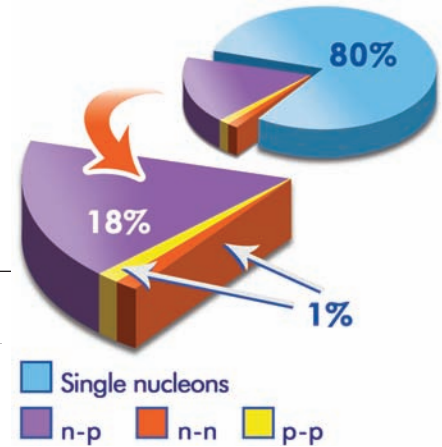
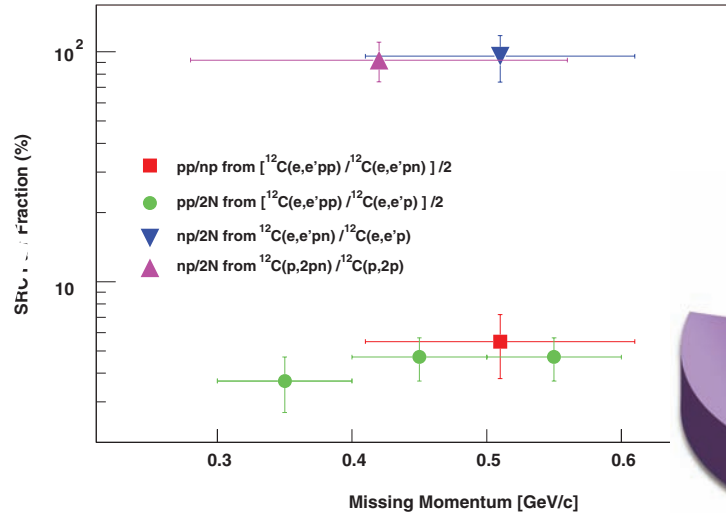
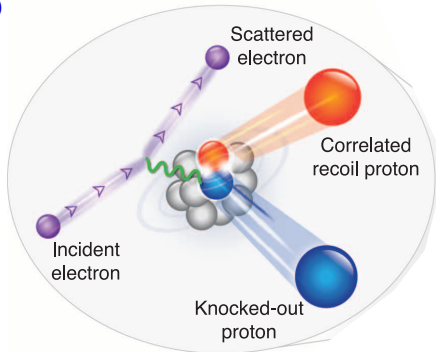
- Theory reproduces the total amount of correlated strength and its shape
- The exact position of the correlated peak depends on the particular many-body approach and (NN interaction?) used.



Phys. Rev. C70, 0243909 (2004)

Two-nucleon pair and SRC in nuclei

Two-nucleon emission at Jlab



High-momentum proton-neutron pairs dominate over p-p and p-n...

High-k protons even in asymmetric nuclei?

Science **320**, 1476 (2008)
Science **346**, 614 (2014)

Ab initio studies along the oxygen chain

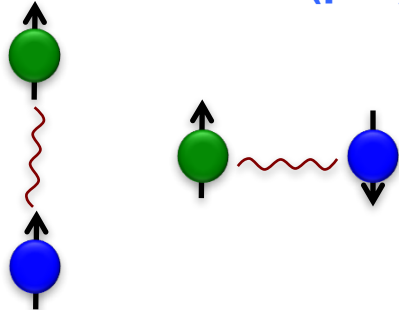
Nuclear forces in exotic nuclei

Nucleon interactions are very complex and difficult to handle

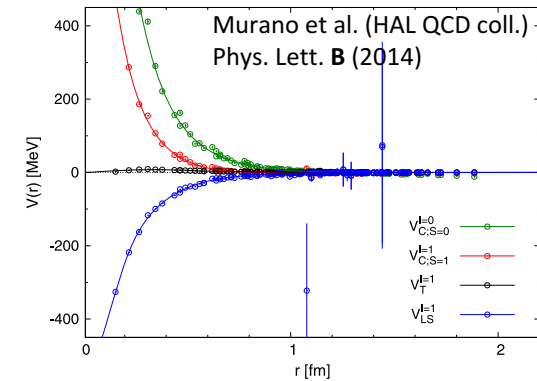
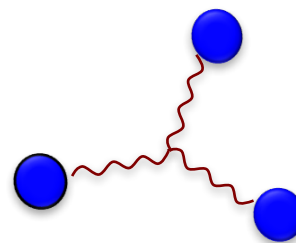
Change of regime from stable to dripline isotopes !

Symmetric matter:
 $N \approx Z$

Tensor force (p-n)



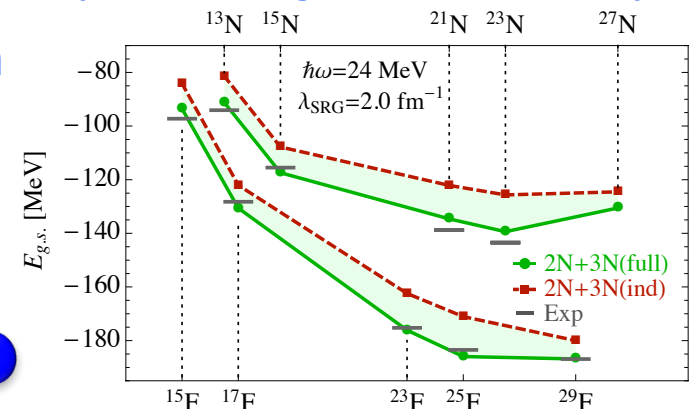
Three-nucleon Force (3NF)



Neutron-rich matter ($N \gg Z$):

- Neutron star matter EoS
- Symmetry energy
- New shell closures

Dripelines of nitrogen and fluorine isotopes



[A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)]

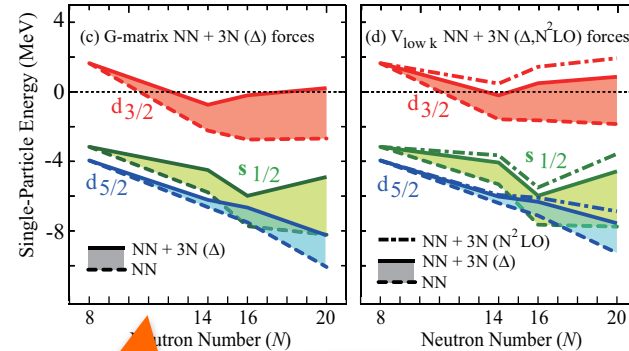
Modern realistic nuclear forces

Chiral EFT for nuclear forces:

	2N forces	3N forces	4N forces
LO $\mathcal{O}\left(\frac{Q^0}{\Lambda^0}\right)$			
NLO $\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$			
N ² LO $\mathcal{O}\left(\frac{Q^3}{\Lambda^3}\right)$			
N ³ LO $\mathcal{O}\left(\frac{Q^4}{\Lambda^4}\right)$			

(3NFs arise naturally at N2LO)

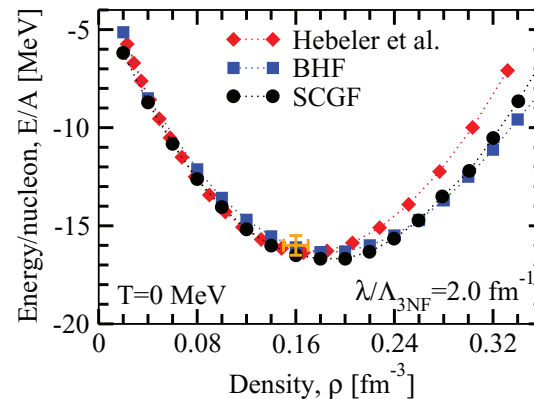
Single particle spectrum at E_{fermi} :



[T. Otsuka et al., Phys. Rev. Lett. **105**, 032501 (2010)]

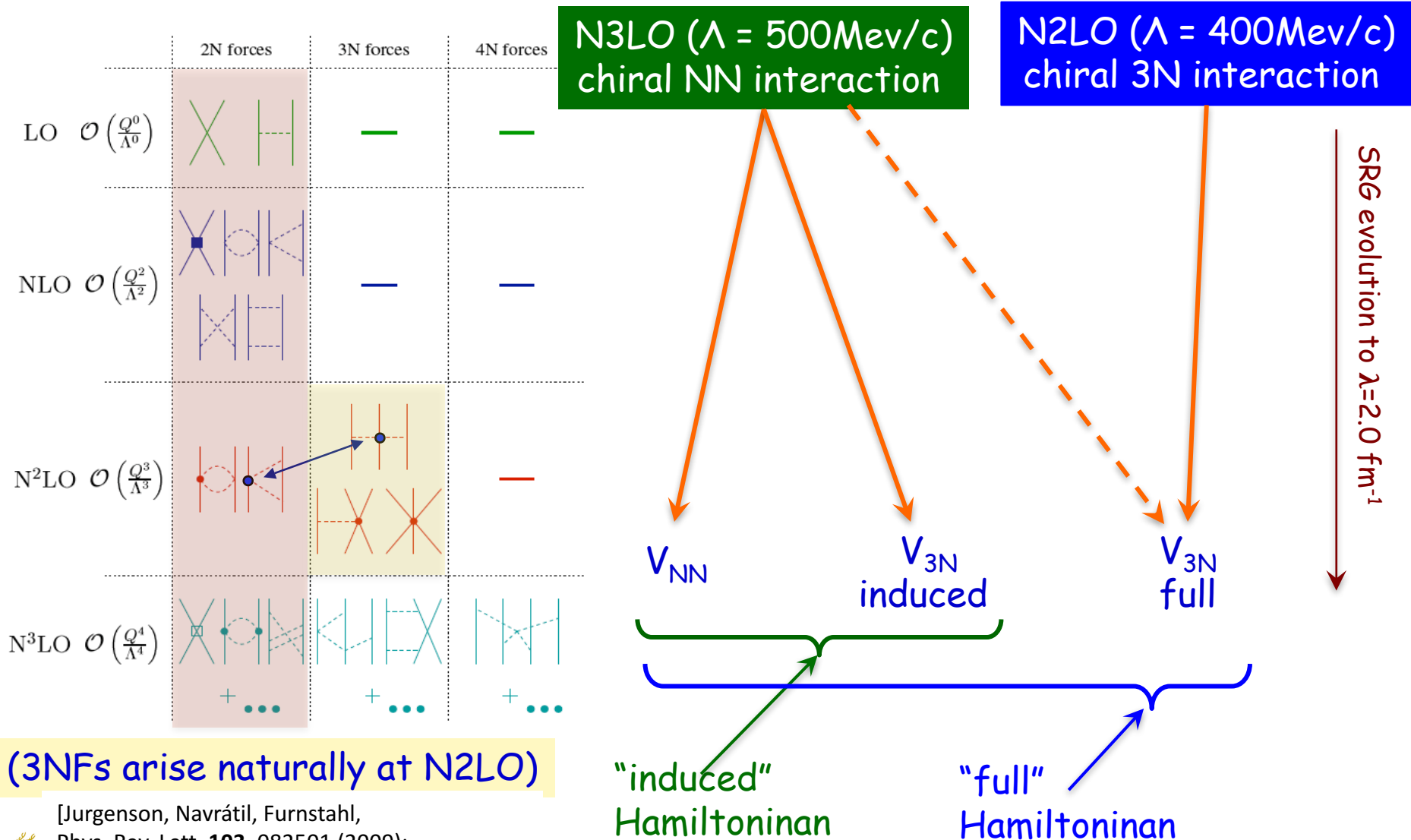
Need at LEAST 3NF!!!
("cannot" do RNB physics without...)

Saturation of nuclear matter:



[A. Carbone et al., Phys. Rev. C **88**, 044302 (2013)]

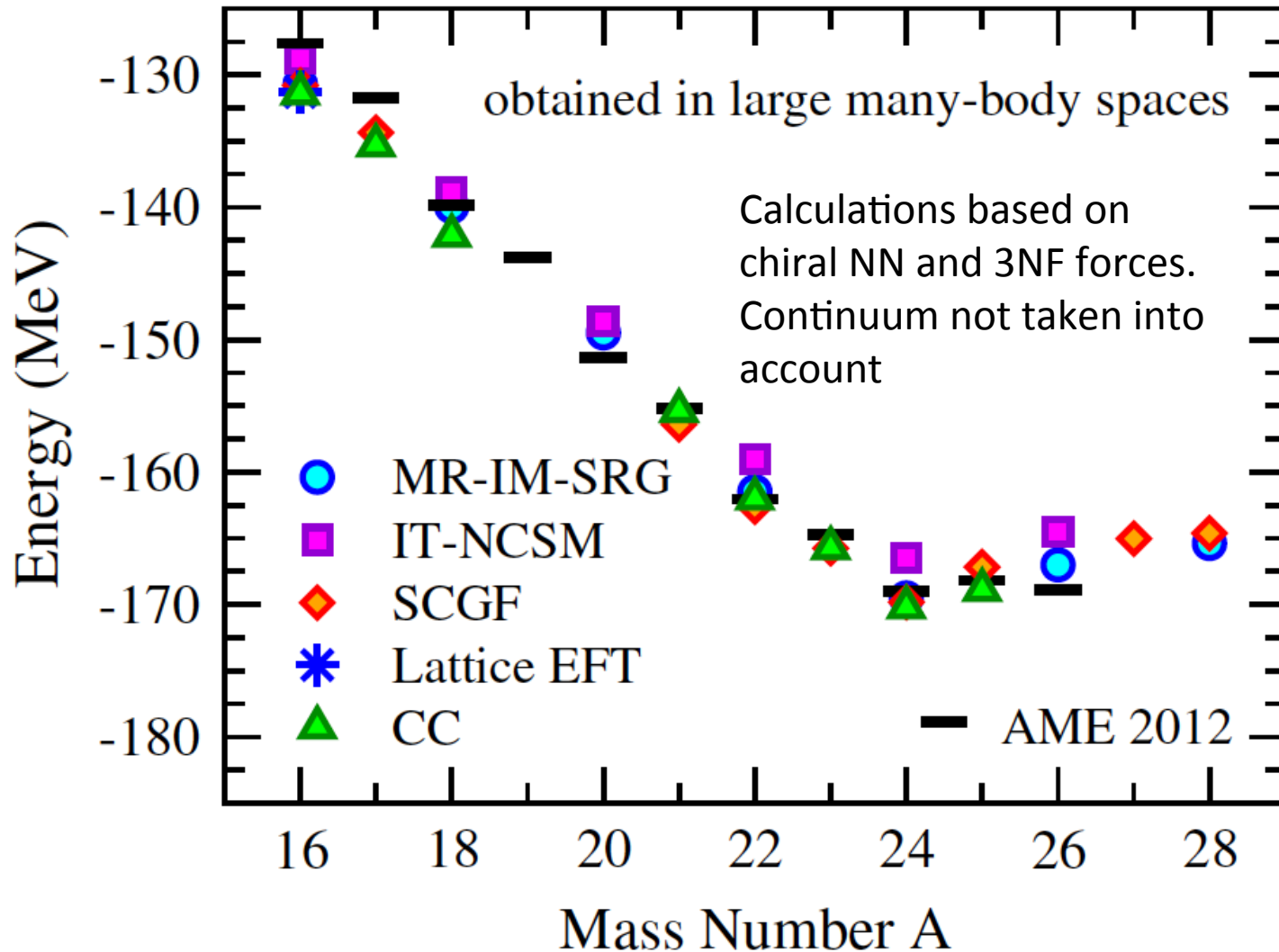
Chiral Nuclear forces - SRG evolved



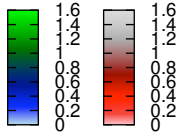
[Jurgenson, Navrátil, Furnstahl,
Phys. Rev. Lett. **103**, 082501 (2009);
Hebeler, Phys. Rev. C **85**, 021002 (2012)]



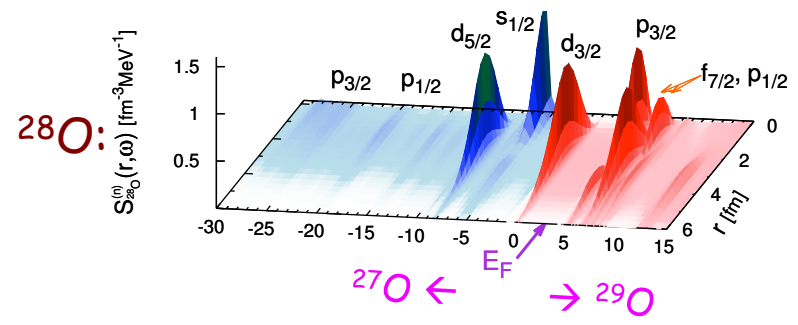
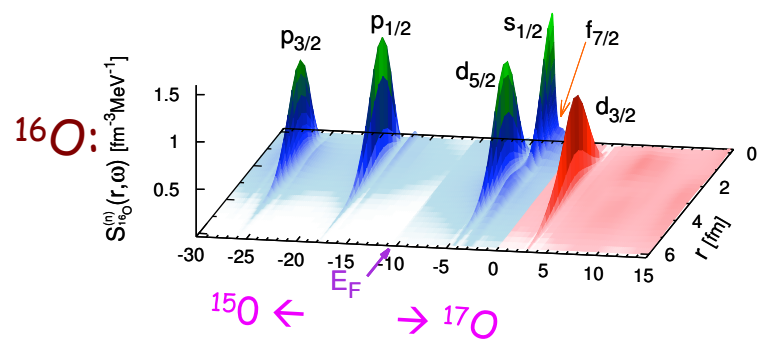
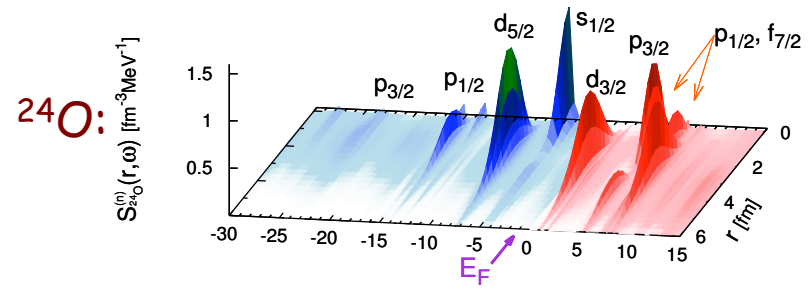
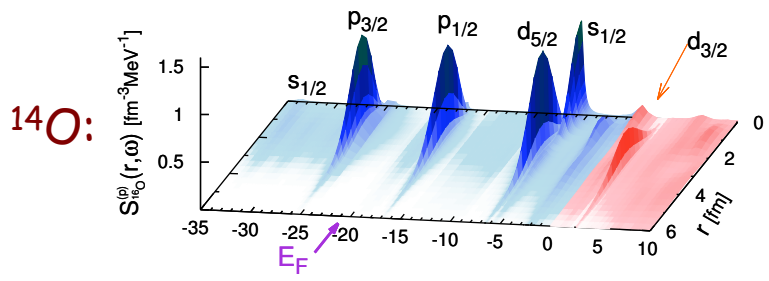
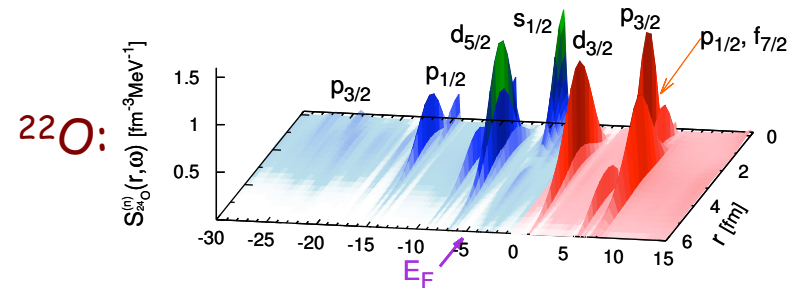
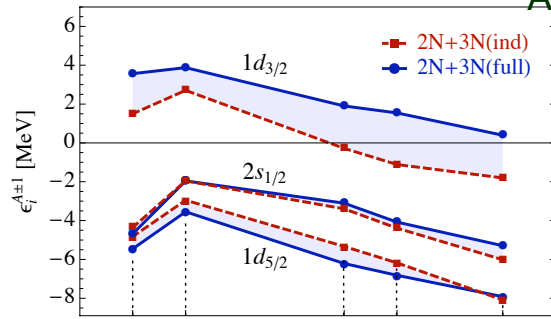
Benchmark of *ab-initio* methods in the oxygen isotopic chain



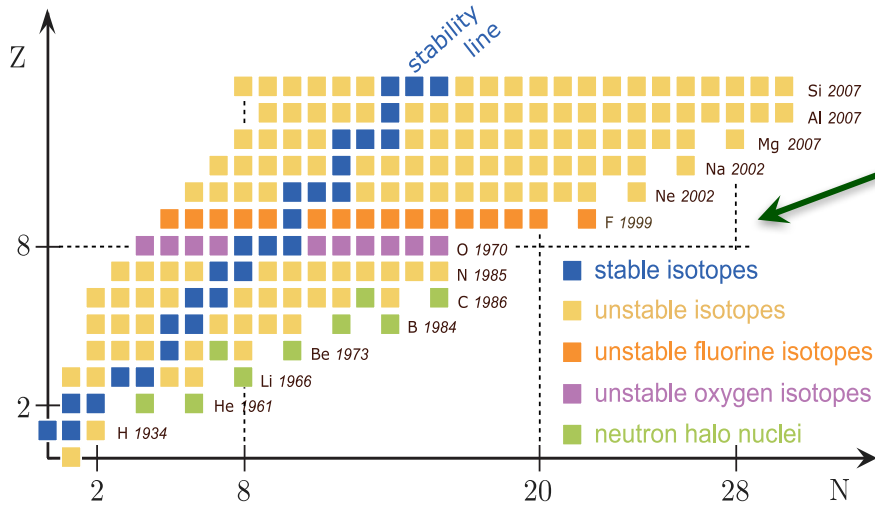
Neutron spectral function of Oxygens



A. Cipollone, CB, P. Navrátil, *Phys. Rev. C* **92**, 014306 (2015)



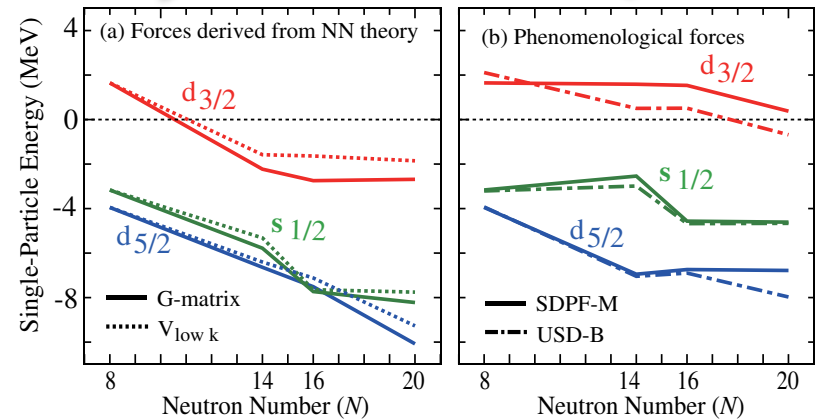
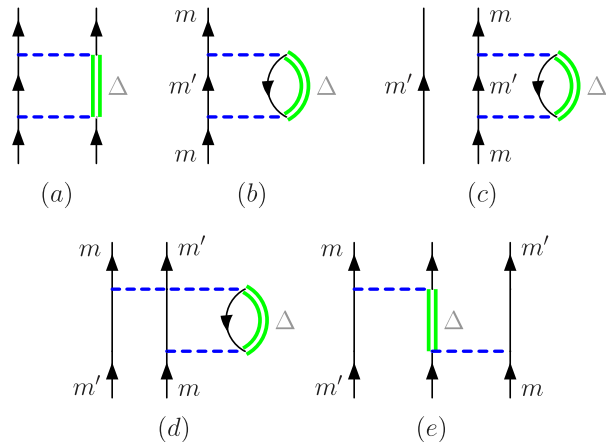
Oxygen puzzle...



The oxygen dripline is at ^{24}O , at odds with other neighbor isotope chains.

Phenomenological shell model interaction reflect this in the s.p. energies but no realistic NN interaction alone is capable of reproducing this...

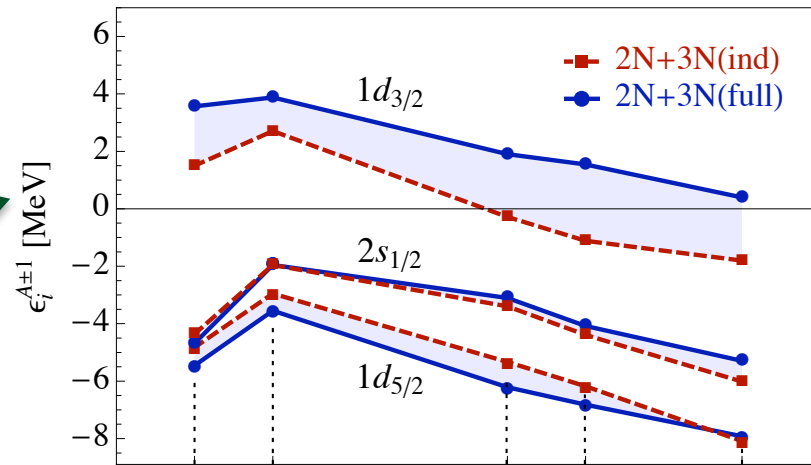
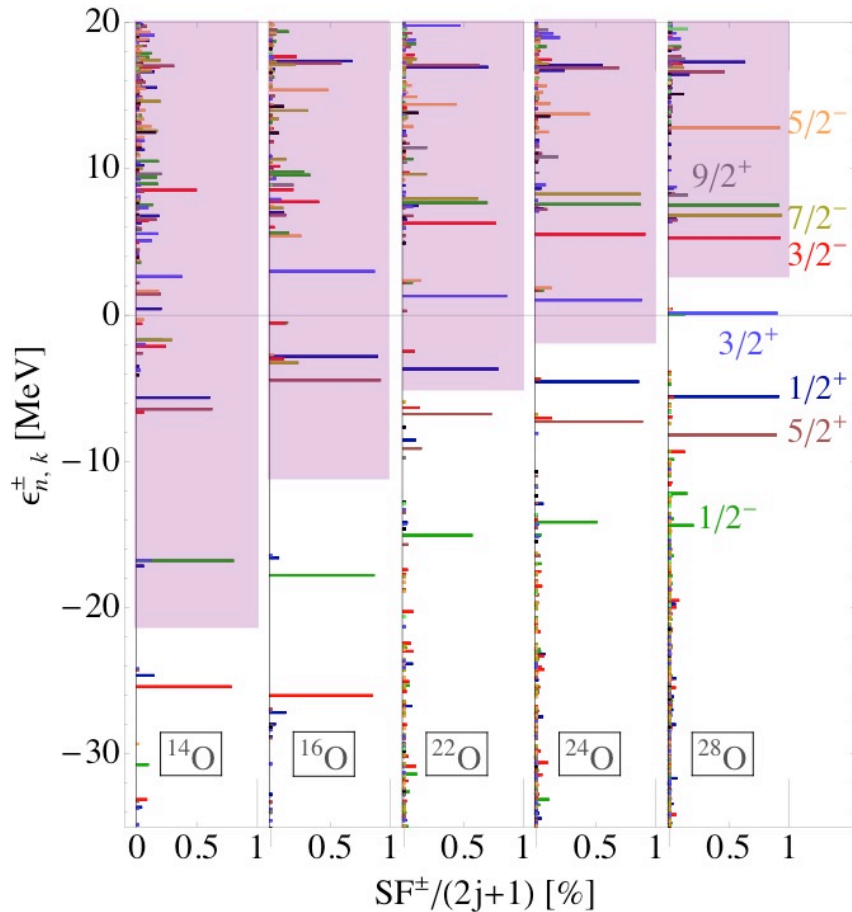
The Fujita-Miyazawa 3NF provides repulsion through Pauli screening of other 2NF terms:



[T. Otsuka et al., Phys Rev. Lett **105**, 32501 (2010)]

Results for the N-O-F chains

A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)
and Phys. Rev. C **92**, 014306 (2015)

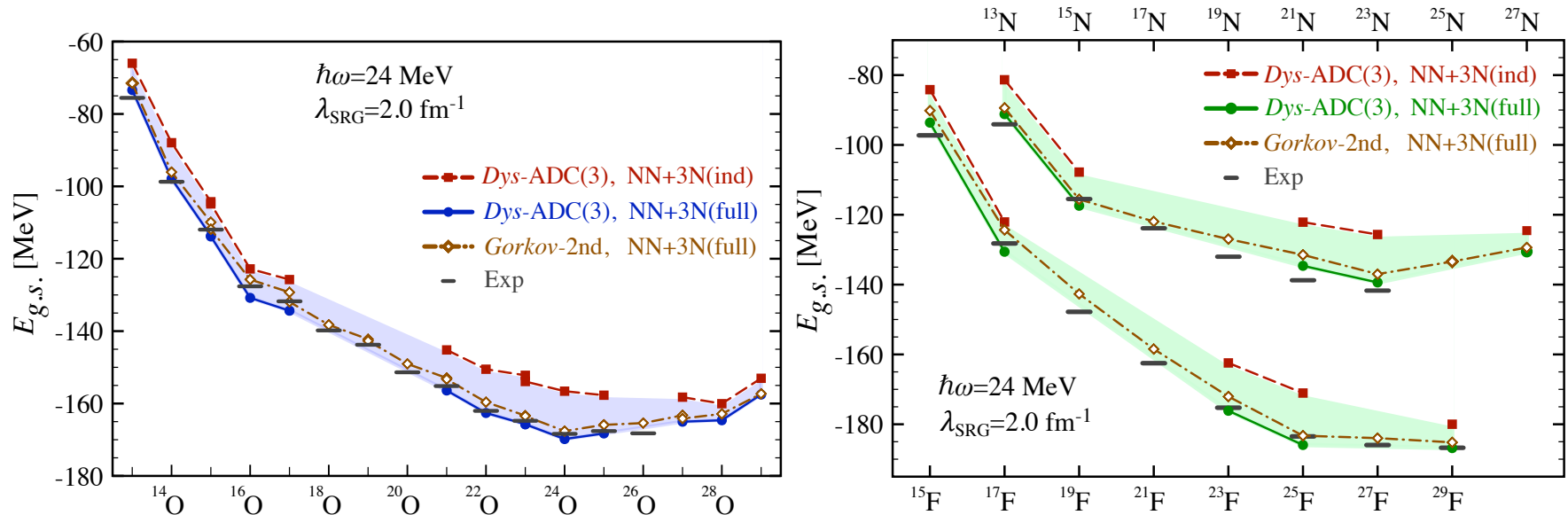


→ $d_{3/2}$ raised by genuine 3NF

→ cf. microscopic shell model [Otsuka et al, PRL**105**, 032501 (2010).]

Results for the N-O-F chains

A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)
and Phys. Rev. C **92**, 014306 (2015)

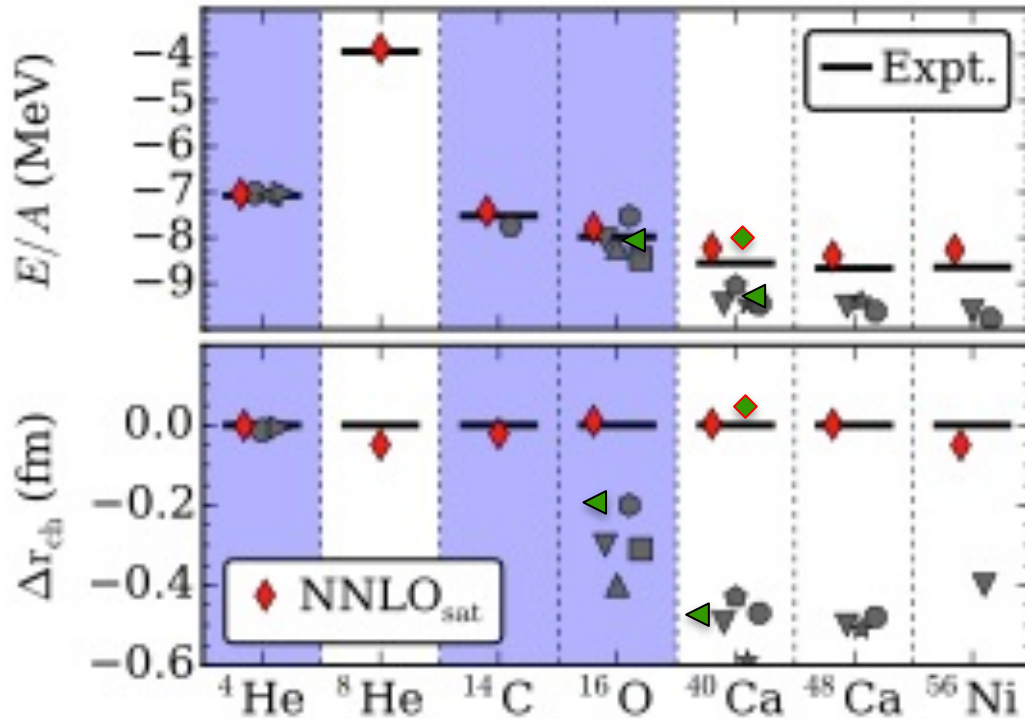


→ 3NF crucial for reproducing binding energies and driplines around oxygen

→ cf. microscopic shell model [Otsuka et al, PRL**105**, 032501 (2010).]

NNLO-sat : a global fit up to $A \approx 24$

A. Ekström *et al.* Phys. Rev. C **91**, 051301(R) (2015)



- Constrain NN phase shifts

- Constrain radii and energies up to $A \leq 24$

→ Provides saturation up to large masses!

◆ NNLOsat (V2 + W3) -- *Grkv 2nd ord.*

From **SCGF**:

◀ { V2-N3LO(500) + W3-NNLO(400MeV/c) w/ SRG at 2.0 fm^{-1}
 A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)
 V. Somà, CB *et al.* Phys. Rev. C **89**, 061301R (2014)



Radii and Binding Energies in Oxygen Isotopes: A Challenge for Nuclear Forces

V. Lapoux,^{1,*} V. Somà,¹ C. Barbieri,² H. Hergert,³ J.D. Holt,⁴ and S.R. Stroberg⁴

- New fits of chiral interactions (NNLO_{sat}) highly improve comparison to data

- Deficiencies remain for neutron rich isotopes

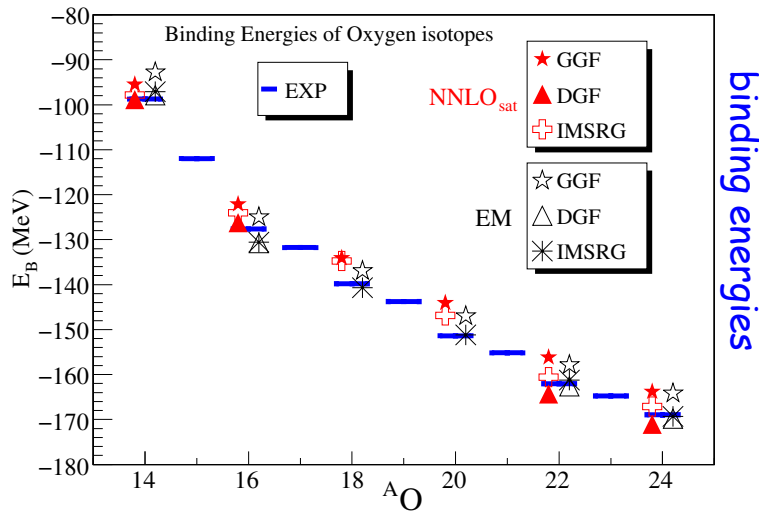
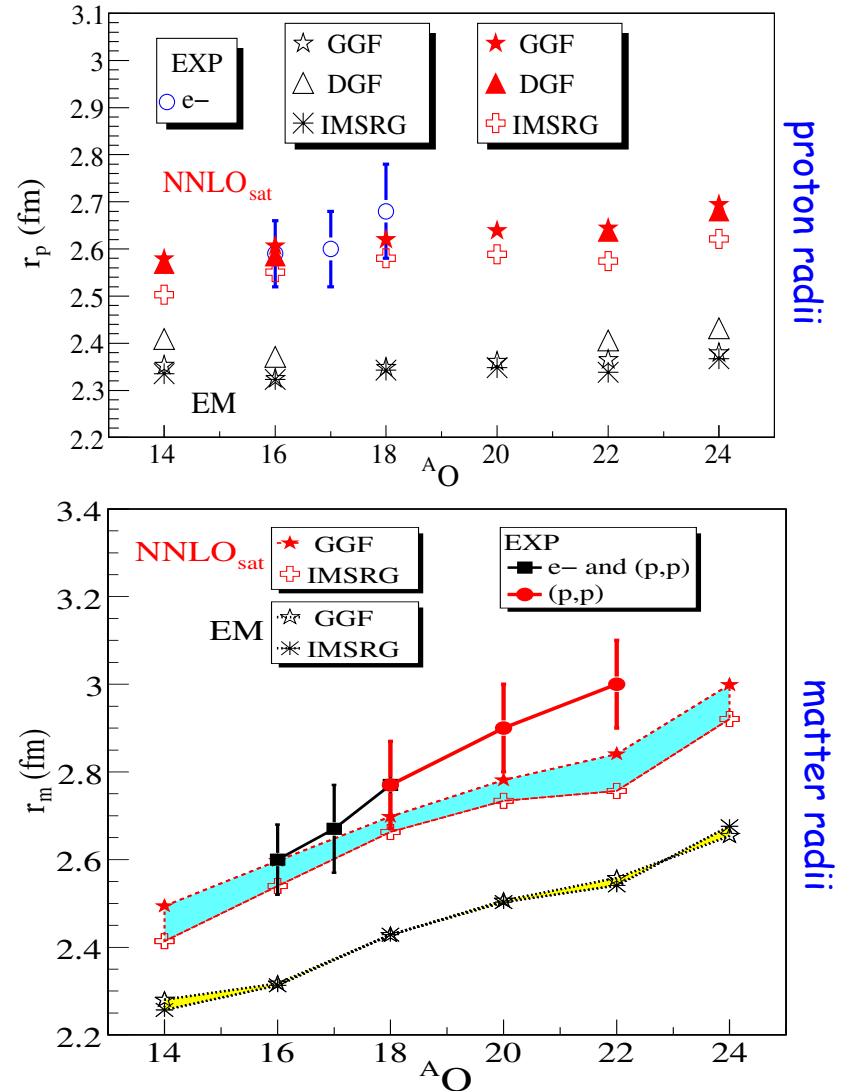


FIG. 1. Oxygen binding energies. Results from SCGF and IMSRG calculations performed with EM [20–22] and NNLO_{sat} [26] interactions are displayed along with available experimental data.



Elastic scattering

Elastic scattering of one particle

- The self-energy is an optical potential for elastic scattering acting on both particle and hole spaces. See for example:
 - F. Capuzzi and C. Mahaux, Ann. Phys. (NY) **245**, 147 (1996) (for proof).
 - L. S. Cederbaum, Ann. Phys. (NY) **291**, 169 (2001) (for extensions to inelastic scattering).
- One can use the knowledge of the self energy (in particular the dispersive relation) to constrain optical models.
- For the "dispersive optical model" see:
 - C. Mahaux and R. Sartor, Adv. Nucl. Phys. **20**, 1 (1991).
 - R. J. Charity et al., Phys. Rev. Lett. **97**, 162503 (2006).
 - R. J. Charity et al., Phys. Rev. C **76**, 044314 (2007).
 - Chapter 23, of Dickhoff and Van Neck book (2nd edition).

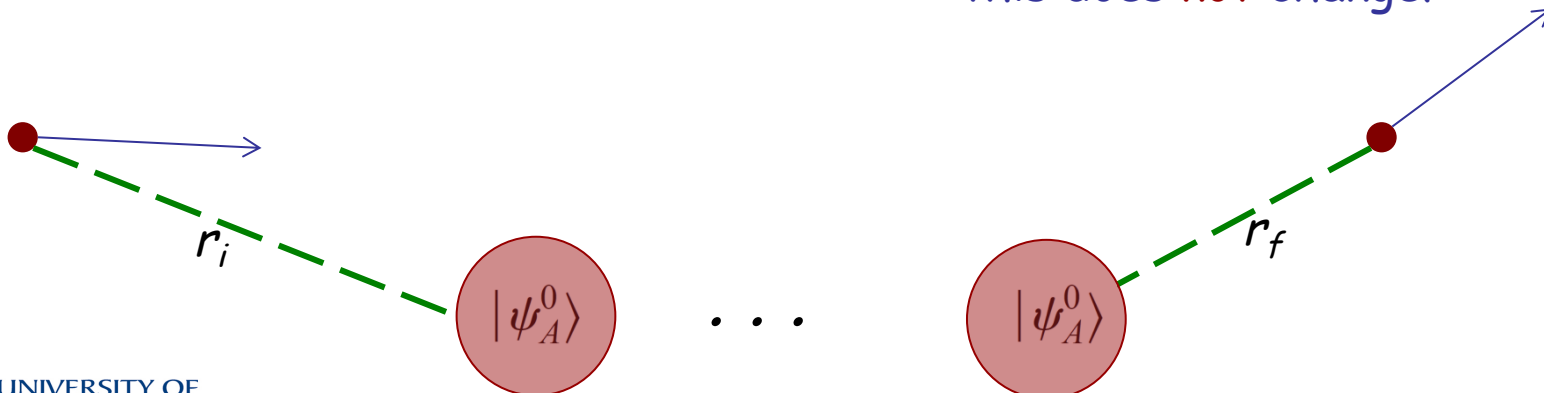
Elastic scattering of one particle

- Feshbach projection formalism:
- At time $t \rightarrow -\infty$ and $t \rightarrow +\infty$ the system is a particle separated and far away from the rest of the system.
- Then, one is interested in initial and final states that look like:

$$a^\dagger(\mathbf{r}) |\psi_A^0\rangle$$

$$a^\dagger(\mathbf{r}') |\psi_A^0\rangle$$

$|\psi_A^0\rangle$ is the state n in which the target system is prepared (usually the ground state).
→ we look at elastic scattering, so this does **not** change!



Elastic scattering of one particle

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$$a^\dagger(\mathbf{r}') |\psi_A^0\rangle$$

$|\psi_A^0\rangle$ is the state n in which the target system is prepared (usually the ground state).
→ we look at elastic scattering, so this does not change!

but these do not cover the full $N+1$ body Hilbert space!

→ must work in a subspace

Elastic scattering of one particle

- Feshbach projection formalism:
- At time $t \rightarrow -\infty$ and $t \rightarrow +\infty$ the system is a particle separated and far away from the rest of the system.
- Then, one is interested in initial and final states that look like:

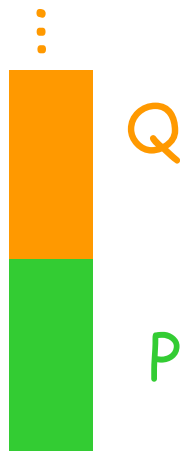
$$\begin{array}{l} a^\dagger(\mathbf{r}) |\psi_A^0\rangle \\ a^\dagger(\mathbf{r}') |\psi_A^0\rangle \end{array}$$

$|\psi_A^0\rangle$ is the state n in which the target system is prepared (usually the ground state).
→ we look at elastic scattering, so this does not change!

One-body subset 'P' of the whole space

but these do not cover the full $N+1$ body Hilbert space!

→ must work in a subspace



Elastic scattering of one particle

Feshbach projection formalism:

After some Math:

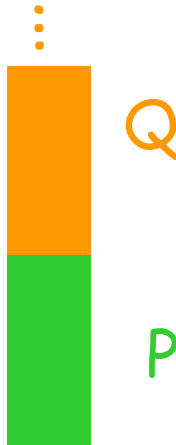
$$E_m^{A+1} \phi_{nm}^{A+1}(\mathbf{r}) = \int d\mathbf{r}' d\mathbf{r}'' \langle \Psi_A^n | a(\mathbf{r}) \left(H + HQ_n^p \frac{1}{E_m^{A+1} - Q_n^p H Q_n^p} Q_n^p H \right) a^\dagger(\mathbf{r}') | \Psi_A^n \rangle \\ \times \mathcal{N}^A(n, \mathbf{r}', n, \mathbf{r}'')^{-1} \phi_{nm}^{A+1}(\mathbf{r}'')$$

$$E_m^{A+1} \phi_{nm}^{A+1}(\mathbf{r}) = \int d\mathbf{r}' \mathcal{H}_n^p(\mathbf{r}, \mathbf{r}'; E_m^{A+1}) \phi_{nm}^{A+1}(\mathbf{r}') \rightarrow \text{Equation for the overlap amplitudes!!}$$

where:

$$\langle \Psi_A^n | a(\mathbf{r}) P_n^p | \Psi_{A+1}^m \rangle = \langle \Psi_A^n | a(\mathbf{r}) | \Psi_{A+1}^m \rangle = \phi_{nm}^{A+1}(\mathbf{r})$$

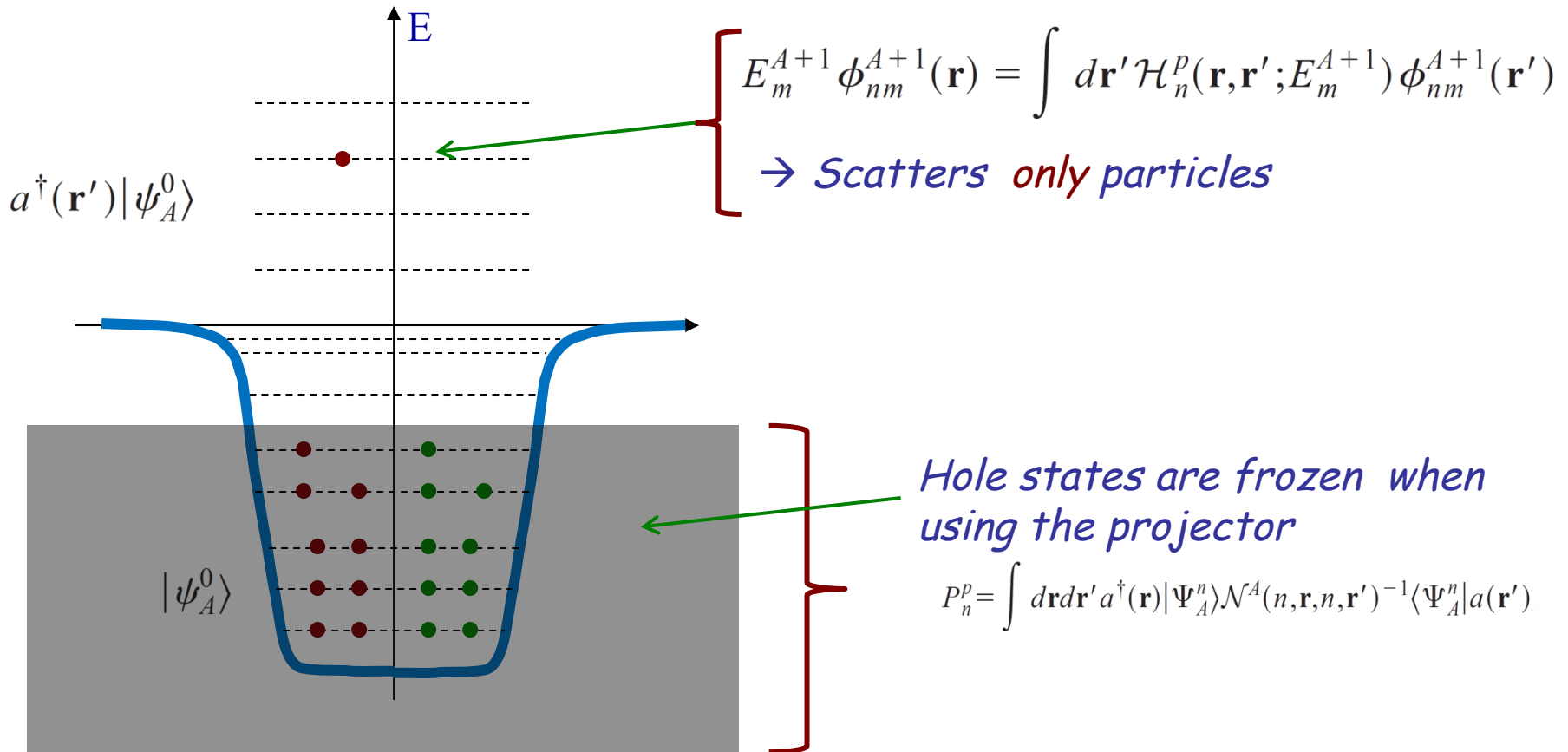
target (usually n=0)
scattering state
overlap function!!



Note: this is **not** the Dyson equation, it only has particles.

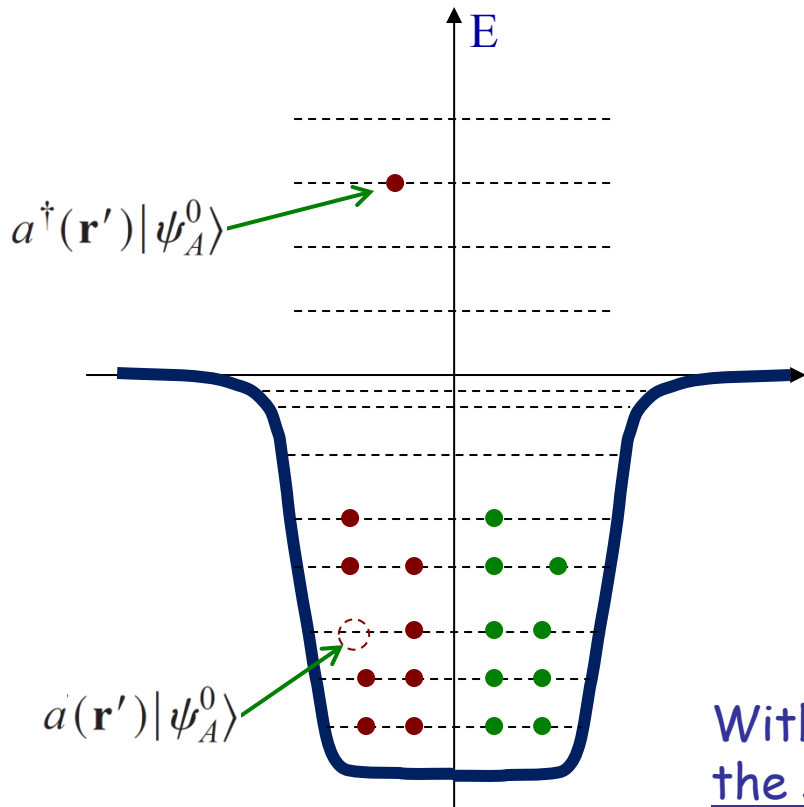
Elastic scattering of one particle

Feshbach projection formalism:



Elastic scattering of one particle

Feshbach projection formalism:



In order to open the full single particle space, one needs to project on **particles** and **holes** at the *same time*:

$$\underbrace{[a(\mathbf{r}) + a^\dagger(\mathbf{r})]}_{\text{Chose the one-body 'P' so that it includes both 'particle' and 'hole' states.}} |\Psi_A^n\rangle$$

Chose the one-body 'P' so that it includes both 'particle' and 'hole' states.

With this choice, one can prove that **Feshbach** is the same as the mass operator for **Dyson's Eq.**

→ One can use **ab initio** theory to do scattering.

Ab initio optical potentials from propagator theory

Relation to Feshbach theory:

Mahaux & Sartor, Adv. Nucl. Phys. 20 (1991)

Escher & Jennings Phys. Rev. C**66**, 034313 (2002)

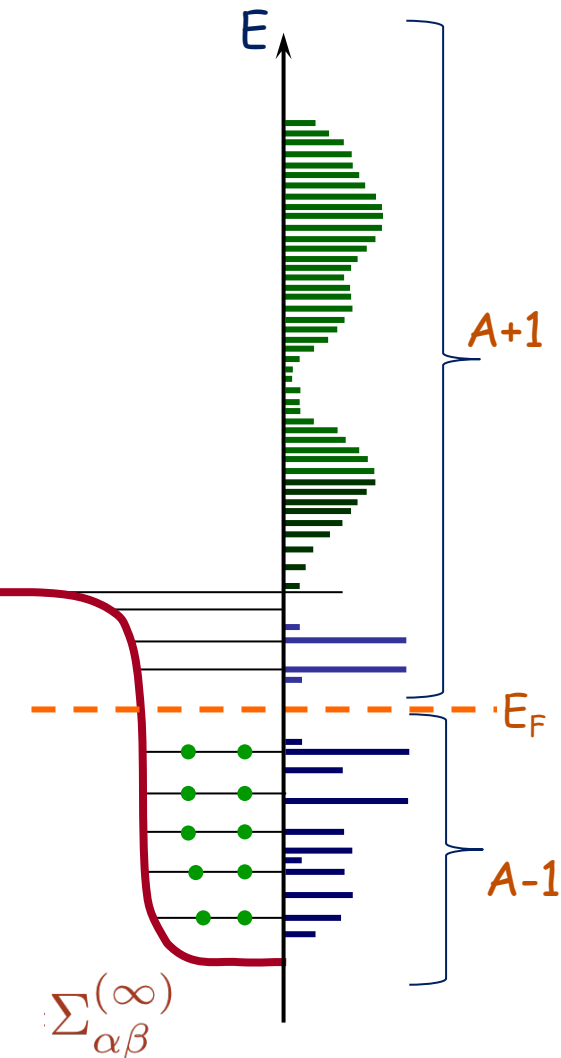
Previous SCGF work:

CB, B. Jennings, Phys. Rev. C**72**, 014613 (2005)

S. Waldecker, CB, W. Dickhoff, Phys. Rev. C**84**, 034616 (2011)

A. Idini, CB, P. Navrátil, arXiv:1612.01478v1 [nucl-th] and in prep.

Dispersive Optical Model (DOM)



Nuclear self-energy $\Sigma^*(\mathbf{r}, \mathbf{r}'; \varepsilon)$:

- contains *both particle* and *hole* props.
- it is proven to be a **Feshbach opt. pot**
→ in general it is *non-local*!
- *must* satisfy the **dispersion relation**:

$$\Sigma^*(\mathbf{r}, \mathbf{r}'; \varepsilon) = \Sigma_{\alpha\beta}^{HF} - \frac{1}{\pi} \int_{\varepsilon_T^>}^{\infty} dE' \frac{\text{Im} \Sigma^*(\mathbf{r}, \mathbf{r}'; E')}{\varepsilon - E' + i\eta} + \frac{1}{\pi} \int_{-\infty}^{\varepsilon_T^<} dE' \frac{\text{Im} \Sigma^*(\mathbf{r}, \mathbf{r}'; E')}{\varepsilon - E' - i\eta}$$

$$\frac{1}{x \pm i\eta} = \mathcal{P} \frac{1}{x} \mp i\pi\delta(x)$$

$$\theta(\pm\tau) = \mp \lim_{\eta \rightarrow 0^+} \frac{1}{2\pi i} \int_{-\infty}^{+\infty} d\omega \frac{e^{-i\omega\tau}}{\omega \pm i\eta}$$

proper boundary conditions are driven by the causality principle

Dispersive Optical Model (DOM)

The **DOM** is a (for now local) parameterization of the self-energy that satisfy dispersion (i.e. parameterize ONLY $\mathcal{V}_{\text{HF}}(r, E)$ and $\mathcal{W}(r, E)$!!)

$$U(r, E) = \mathcal{V}(r, E) + i\mathcal{W}(r, E)$$

$$\mathcal{V}(r, E) = \mathcal{V}_{\text{HF}}(r, E) + \Delta\mathcal{V}(r, E)$$

$$\Delta\mathcal{V}(r, E) = \frac{1}{\pi} P \int \mathcal{W}(r, E') \left(\frac{1}{E' - E} - \frac{1}{E' - E_F} \right) dE'$$

Developed by Mahaux and collaborators, in the 80s (^{208}Pb , etc):

• C. Mahaux and R. Sartor, Adv. Nucl. Phys. 20, 1 (1991).

Recent developments: global model around ^ACa chain (St.Louis):

• R. J. Charity et al., Phys. Rev. Lett. 97, 162503 (2006); Phys. Rev. C 76, 044314 (2007).

DOM - more recent work

Present application of DOM to nuclei:

- Fit: $^{40-48}\text{Ca}$ isotopes chain ($Z=20$, $N=20-28$)
- 81 data sets, 3569 points
 - up to 200 MeV scattering
 - information on radii, spectroscopic factor, etc...
- 25 parameters
- Extrapolation to ^{60}Ca \rightarrow not fully determined: need more information from neutron scattering...
- Extension to other Z s ...

DOM - more recent work

Most important are radii and volume integrals of the potential:

$$R_{\text{rms}}^V = \sqrt{\frac{\int r^2 \mathcal{V}(r) dr}{J_V}}$$

$$J_V = \int \mathcal{V}(r) dr$$

and similarly for $W...$

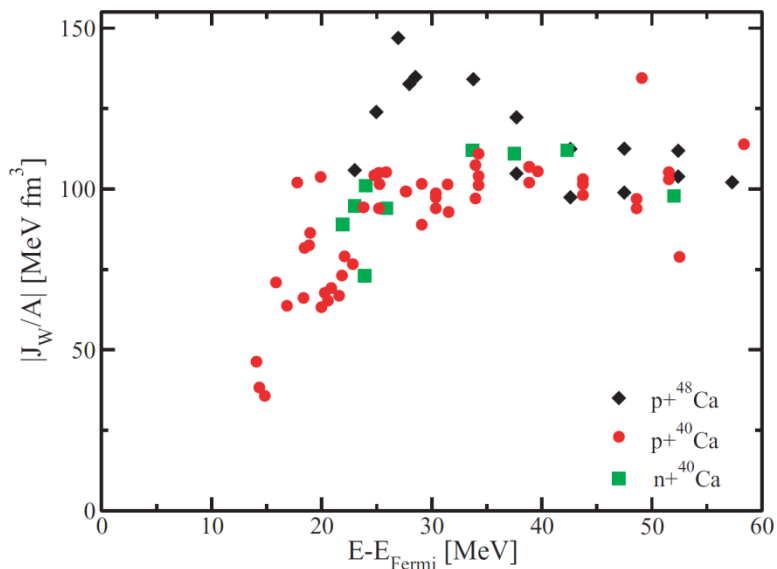


FIG. 2. (Color online) Energy dependence of the integrated imaginary potential determined from published optical-model fits to $p+^{40}\text{Ca}$, $p+^{48}\text{Ca}$, and $n+^{48}\text{Ca}$ elastic-scattering data.

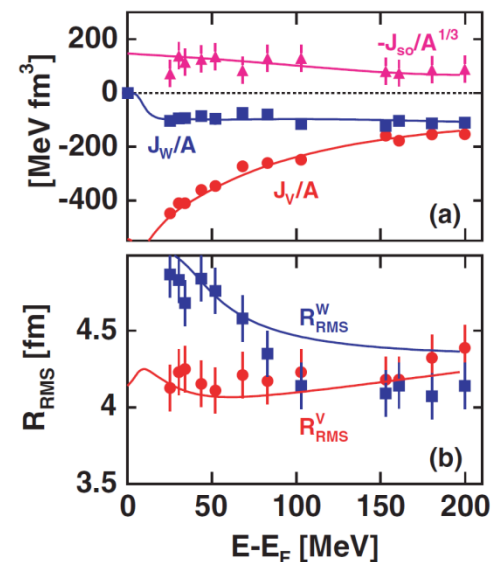
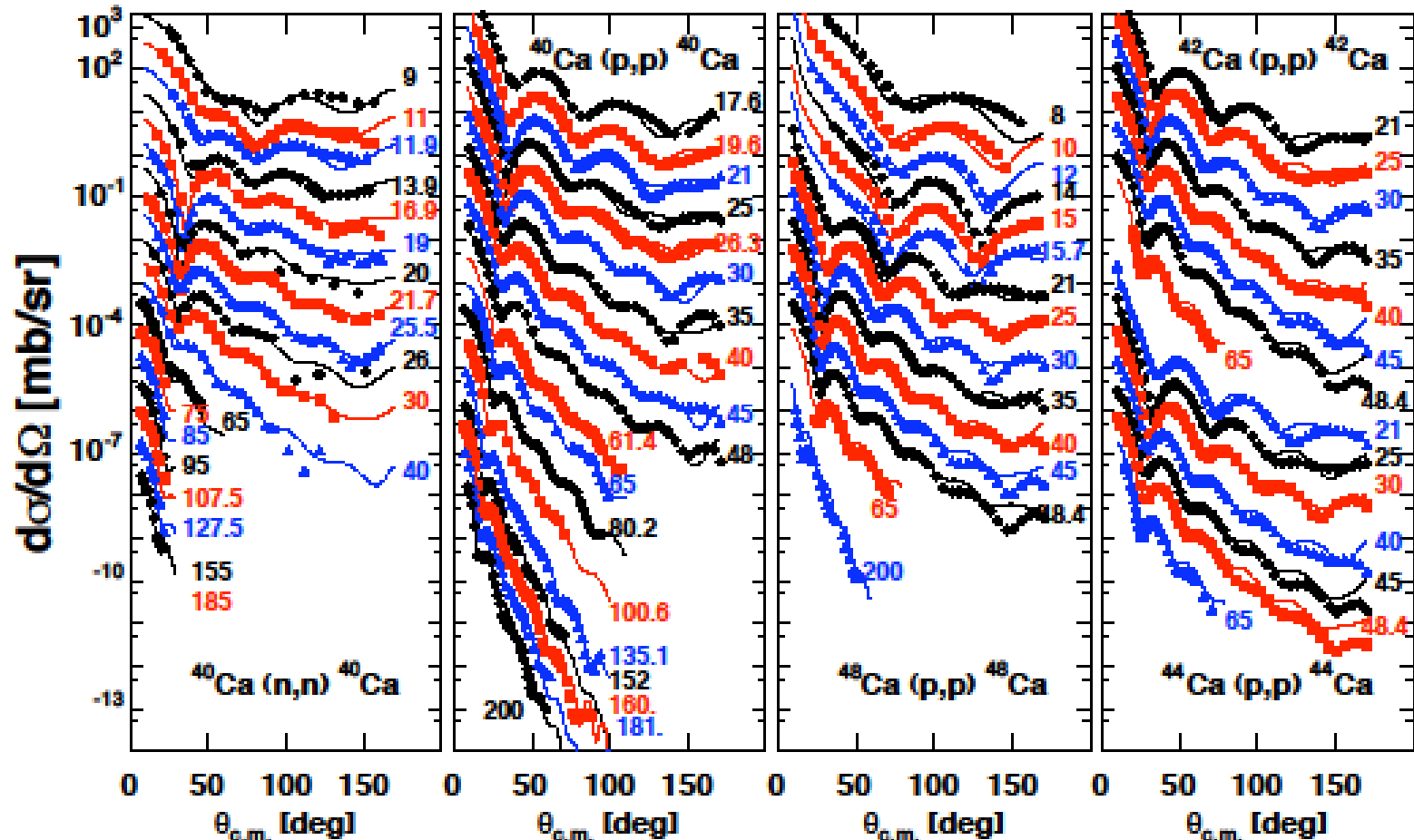


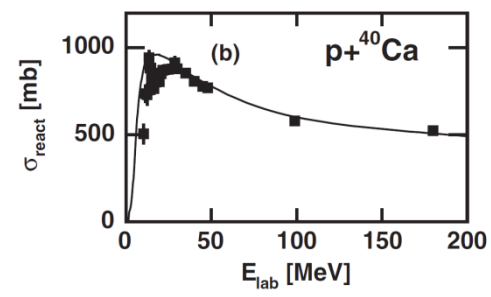
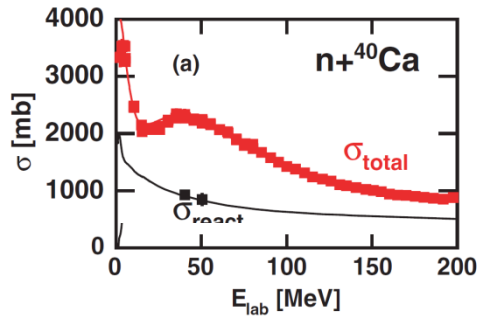
FIG. 3. (Color online) (a) Integrated potentials and (b) rms radii obtained from combined fits to $p+^{40}\text{Ca}$ elastic-scattering data and $n+^{40}\text{Ca}$ total cross sections.

DOM - more recent work

Fitted differential cross sections:



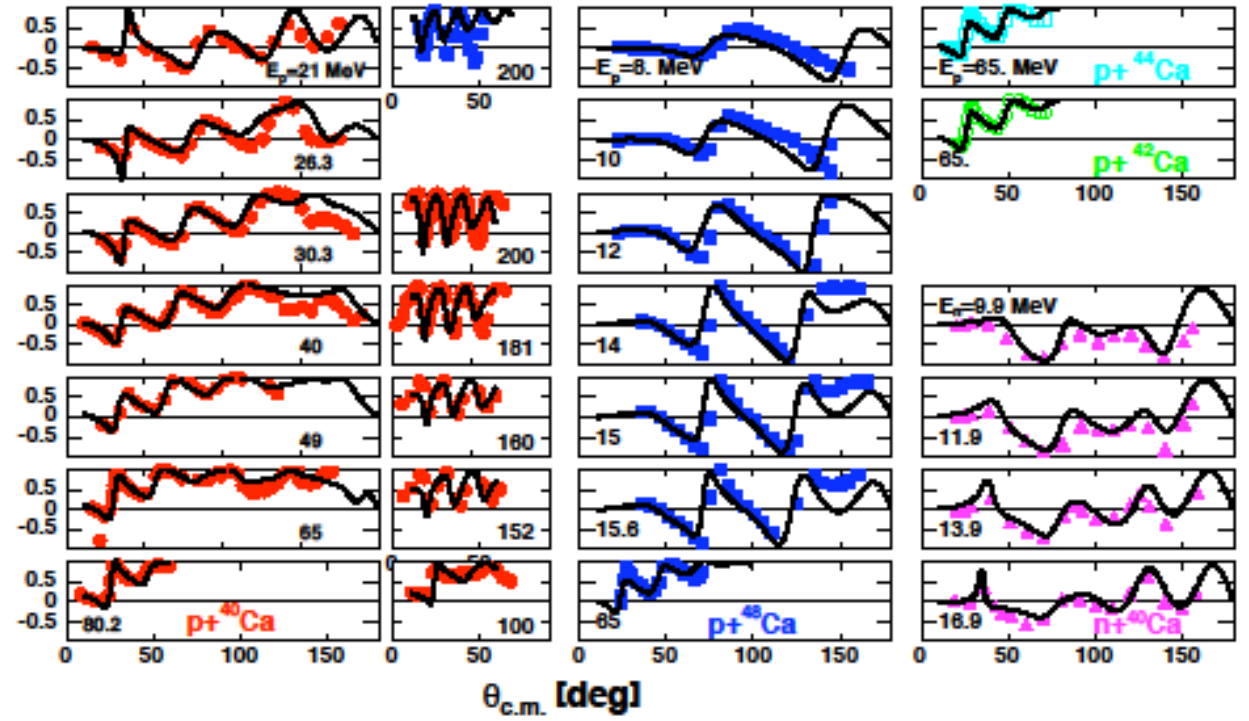
DOM - more recent work



← Total cross sections

Fitted polarization observables →

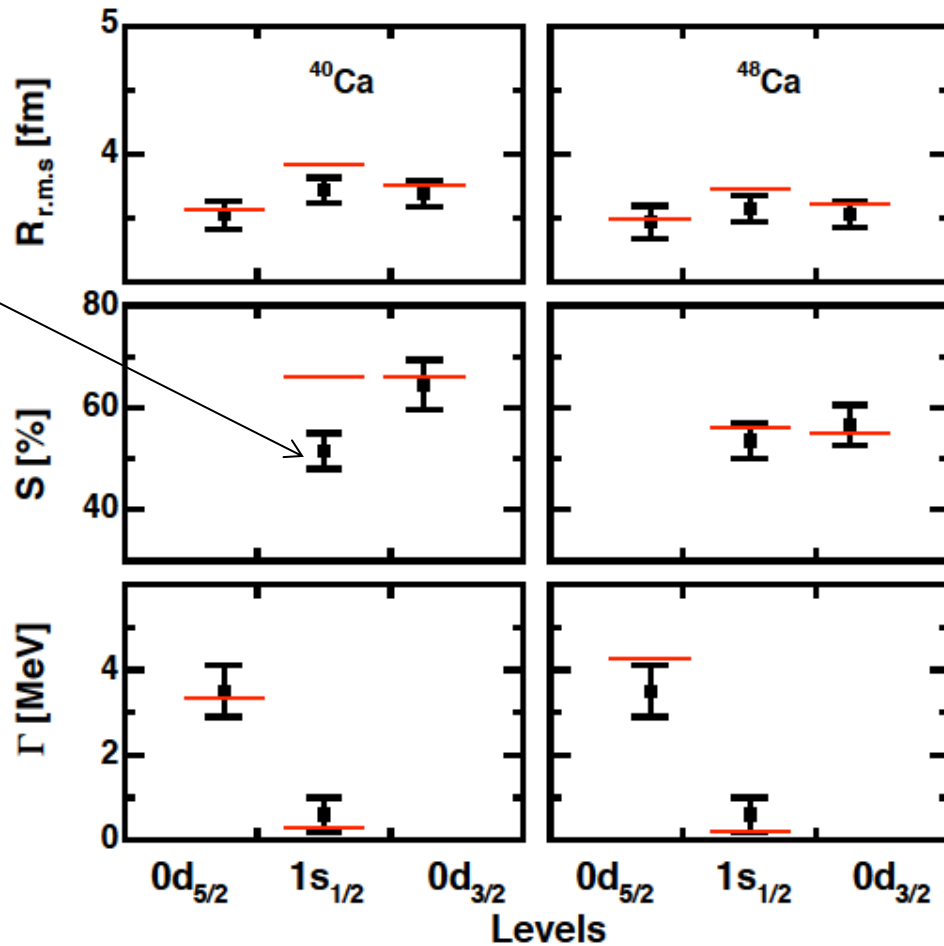
A_y



Fit to $(e, e'p)$ data

Fit to $(e, e'p)$ reaction data...

One point could not be fitted...



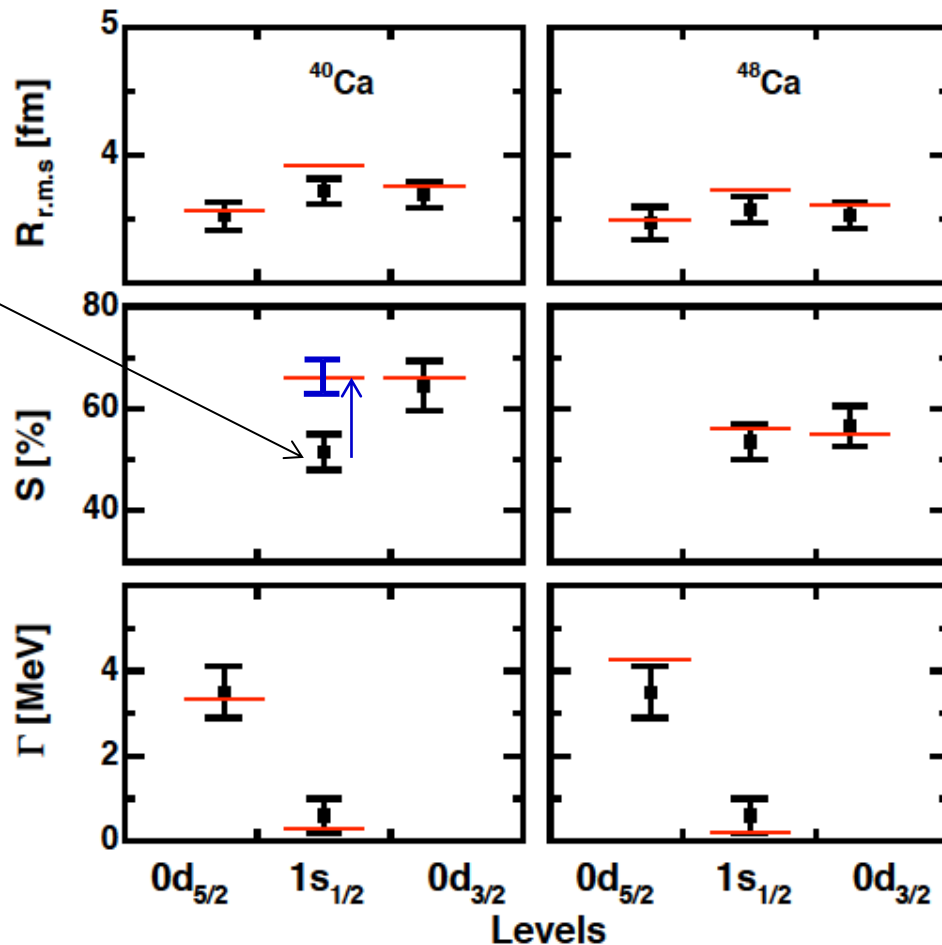
Fit to $(e, e'p)$ data

Fit to $(e, e'p)$ reaction data...

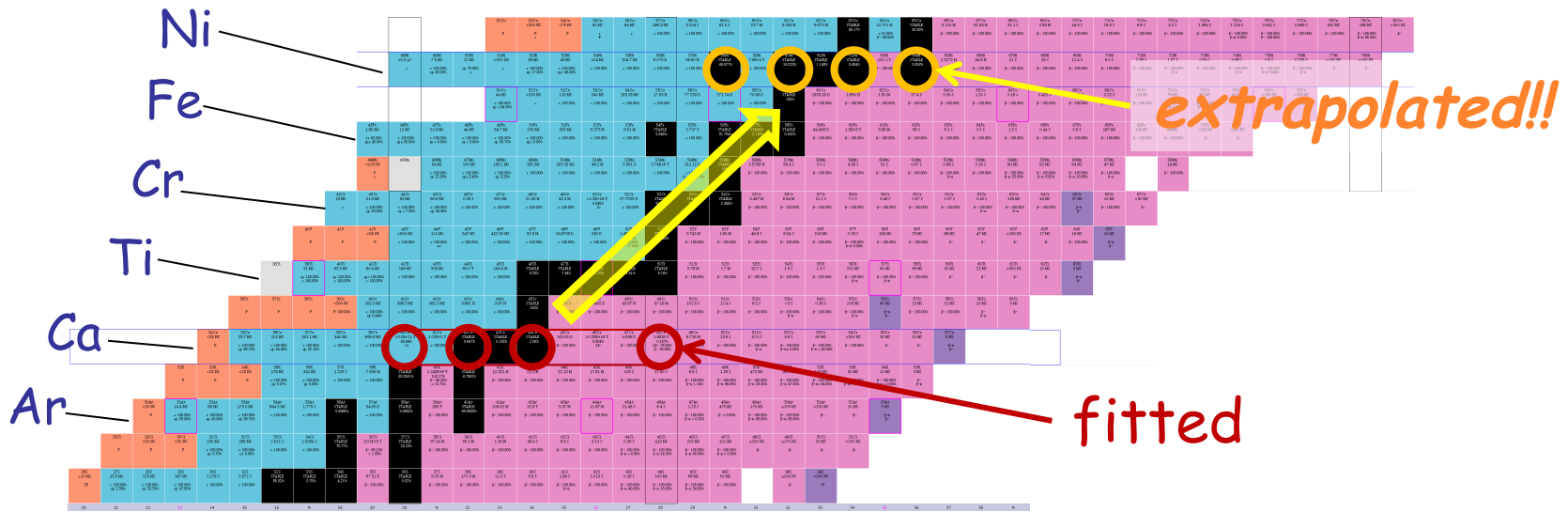
One point could not be fitted...

An *independent* re-analysis of $^{40}\text{Ca}(e, e'p)$ brings the SF in agreement with the DOM fit!

[L. Lapikas, priv. comm.]

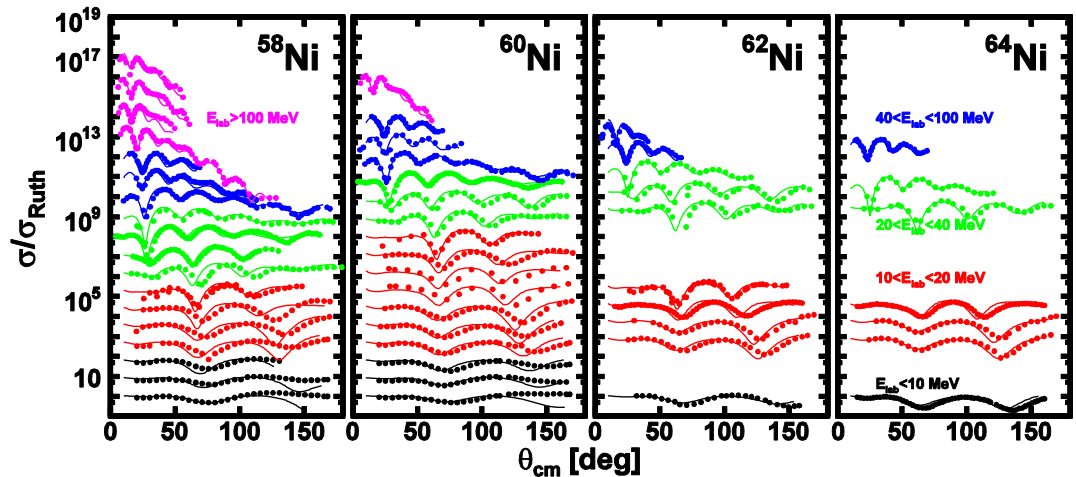


DOM - more recent work

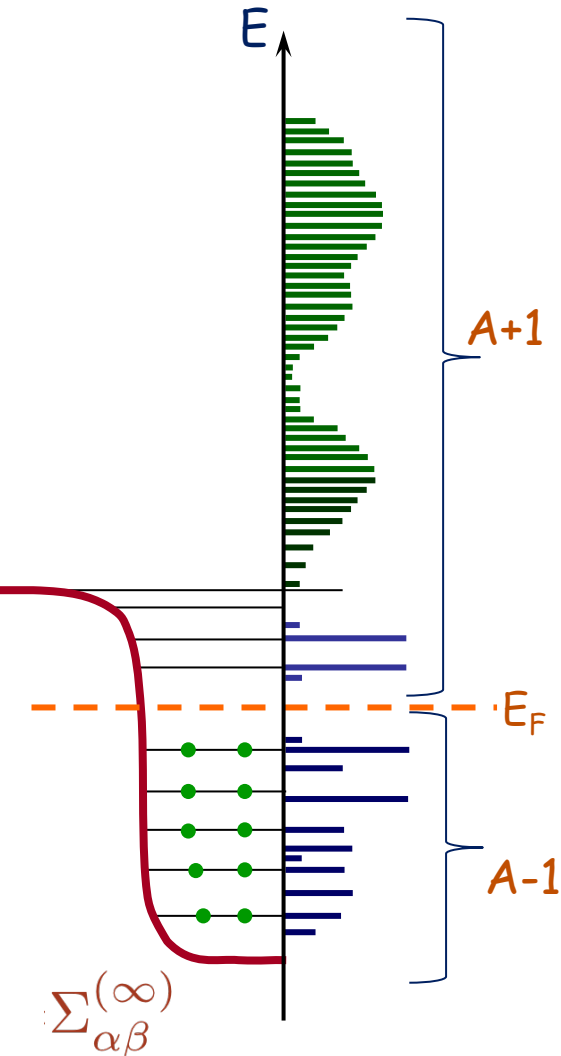


The fit made for Ca isotopes gives good predictions for Ni...

→ NO refitting !!



Microscopic optical potential

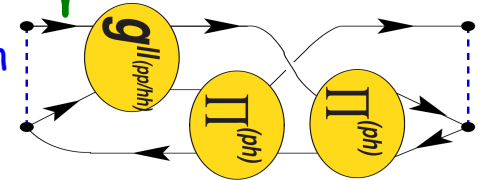


Nuclear self-energy $\Sigma^*(\mathbf{r}, \mathbf{r}'; \varepsilon)$

- contains *both* *particle* and *hole* props.
- it is proven to be a *Feshbach opt. pot*
 \rightarrow in general it is *non-local* !

$$\Sigma_{\alpha\beta}^*(\omega) = \underbrace{\Sigma_{\alpha\beta}^{(\infty)}}_{\text{mean-field}} + \underbrace{\sum_{i,j} \mathbf{M}_{\alpha,i}^\dagger \left[\frac{1}{E - (\mathbf{K}^> + \mathbf{C}) + i\Gamma} \right]_{i,j} \mathbf{M}_{j,\beta} + \sum_{r,s} \mathbf{N}_{\alpha,r} \left[\frac{1}{E - (\mathbf{K}^< + \mathbf{D}) - i\Gamma} \right]_{r,s} \mathbf{N}_{s,\beta}^\dagger}_{\text{Particle-vibration couplings}}$$

Particle-vibration couplings:



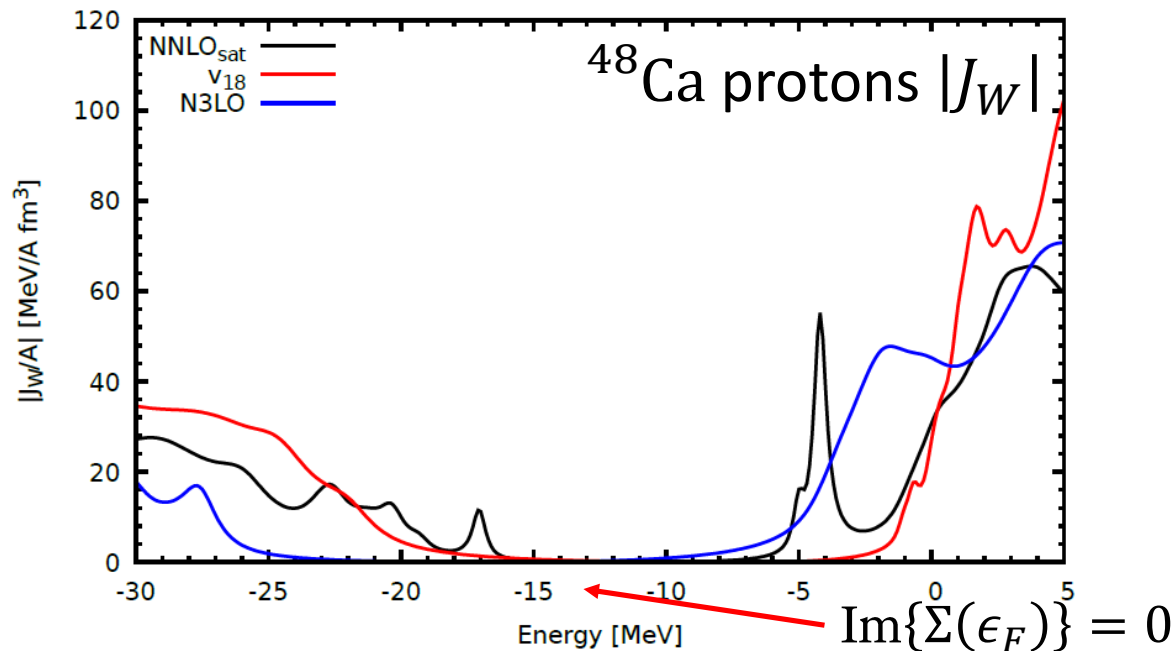
Solve scattering and overlap functions directly in momentum space:

$$\Sigma^{*l,j}(k, k'; E) = \sum_{n, n'} R_{nl}(k) \Sigma_{n, n'}^{*l,j} R_{nl}(k')$$

$$\frac{k^2}{2\mu} \psi_{l,j}(k) + \int dk' k'^2 \Sigma^{*l,j}(k, k'; E_{c.m.}) \psi_{l,j}(k') = E_{c.m.} \psi_{l,j}(k)$$

Overall absorption of opt. mod.

$$J_W(E) = 4\pi \int dr r^2 \int dr' r'^2 \sum_{l,j} \Im m \{ \Sigma^{*l,j}(r, r'; E) \}$$



[S. Waldecker, CB, W. Dickhoff, PRC84, 034616 (2011)]

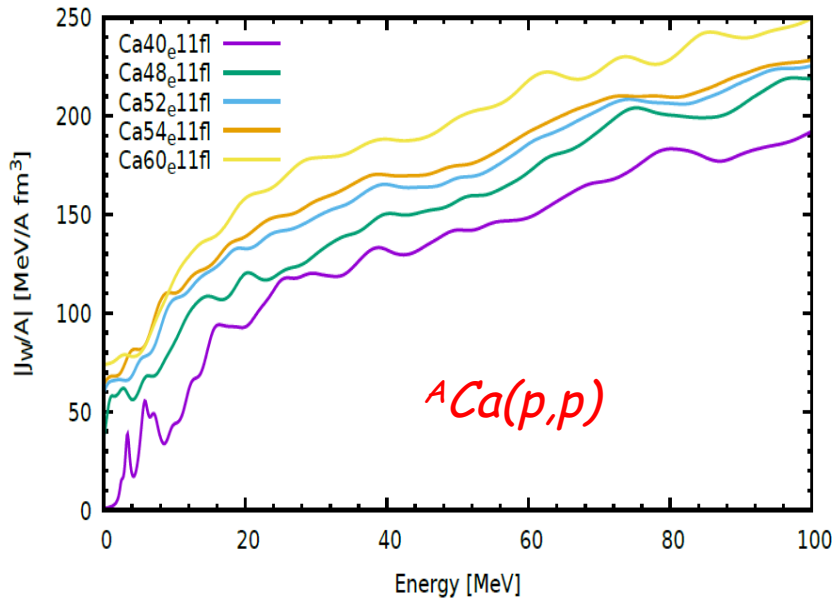
[A. Idini, CB, Navratil, in prep.]

$$\Sigma_{\alpha\beta}^*(\omega) = \Sigma_{\alpha\beta}^{(\infty)} + \sum_{i,j} \mathbf{M}_{\alpha,i}^\dagger \left[\frac{1}{E - (\mathbf{K}^> + \mathbf{C}) + i\Gamma} \right]_{i,j} \mathbf{M}_{j,\beta} + \sum_{r,s} \mathbf{N}_{\alpha,r} \left[\frac{1}{E - (\mathbf{K}^< + \mathbf{D}) - i\Gamma} \right]_{r,s} \mathbf{N}_{s,\beta}^\dagger$$

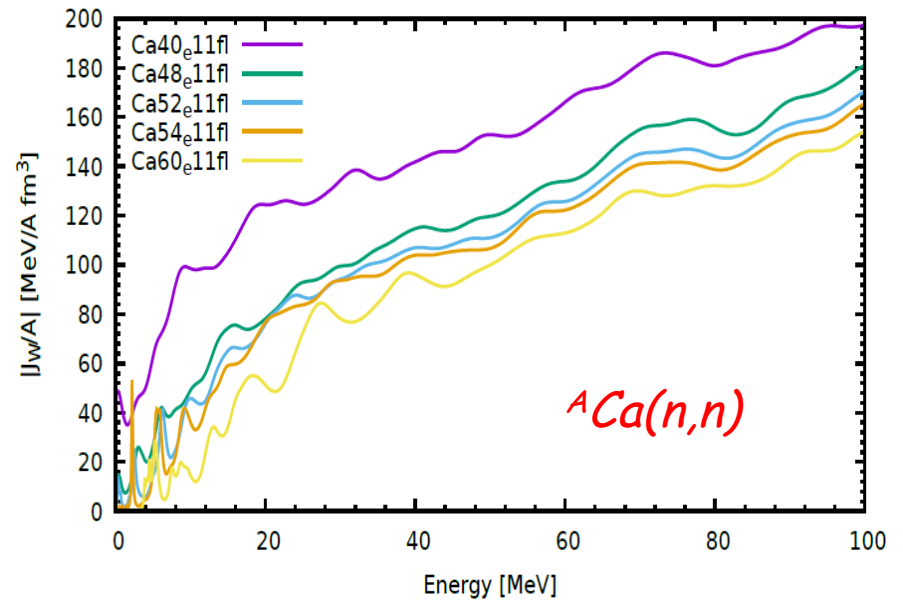
Overall absorption of opt. mod.

$$J_W(E) = 4\pi \int dr r^2 \int dr' r'^2 \sum_{l,j} \Im m \{ \Sigma^{*l,j}(r, r'; E) \}$$

NNLO_{sat} proton comparison



NNLO_{sat} neutron comparison



[A. Idini, CB, Navrátil, in prep.]

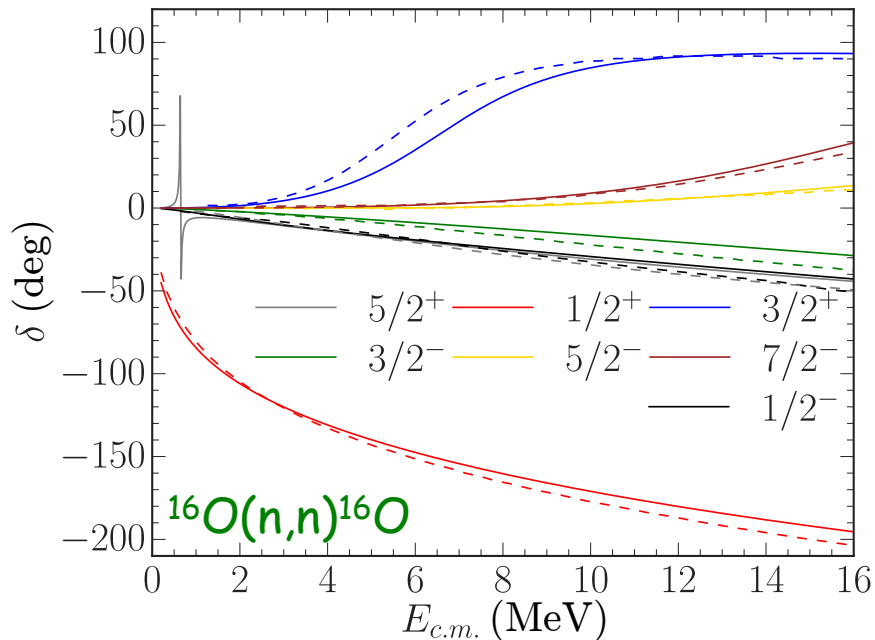
Low energy scattering - from SCGF

[A. Idini, CB, Navratil, in prep.]

Benchmark with NCSM-based scattering.

NN-only interaction at $\lambda_{\text{SRG}} = 2.66 \text{ fm}^{-1}$

Scattering from mean-field only:

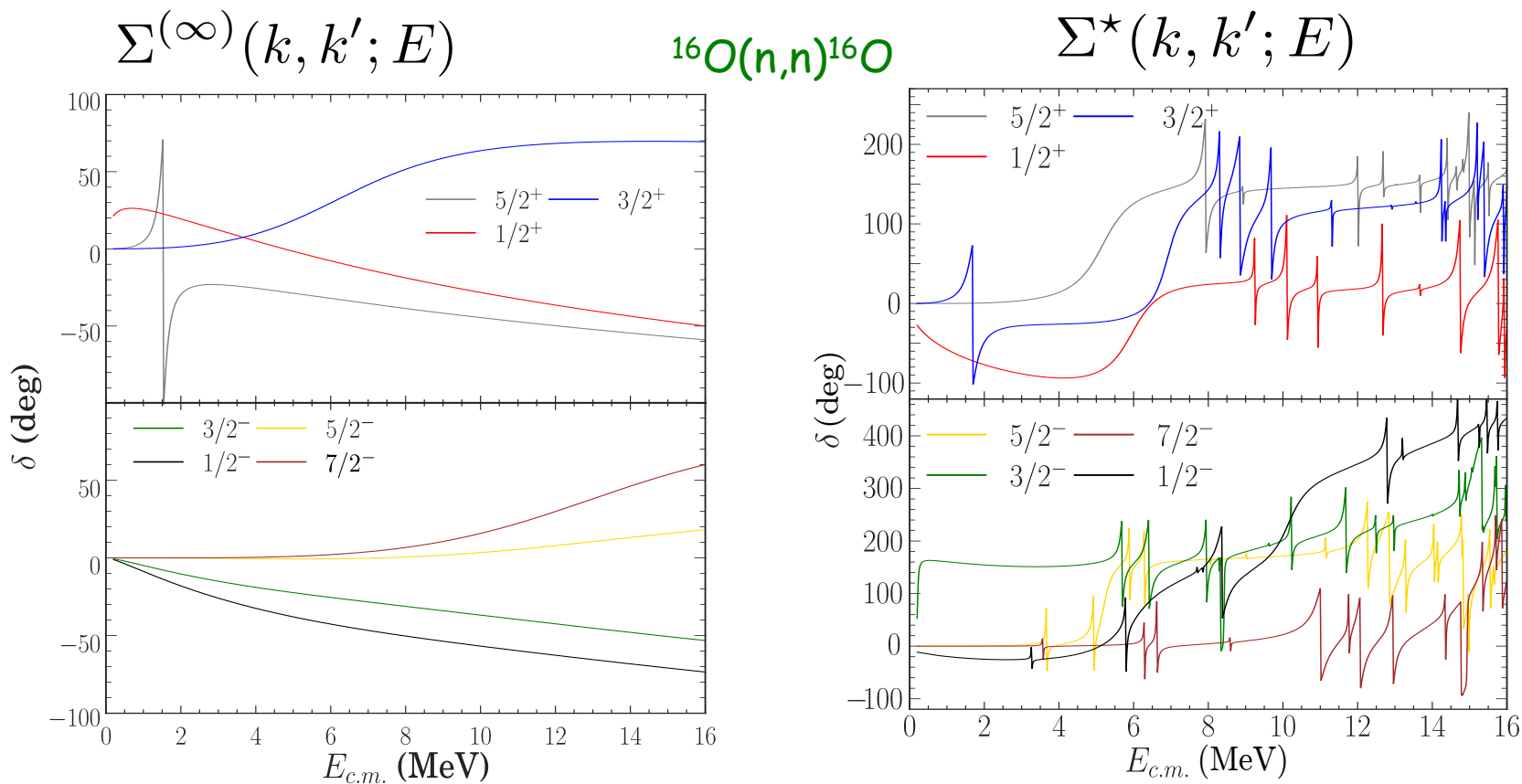


--- NCSM/RGM [no core excitations]
PRC82, 034609 (2010)

— SCGF [$\Sigma^{(\infty)}$ only]

Low energy scattering - from SCGF

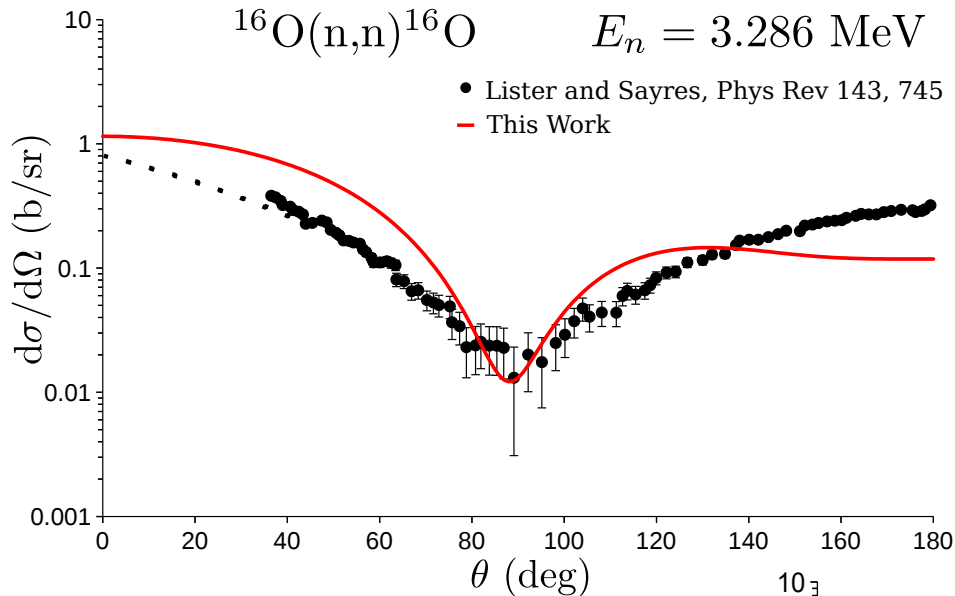
[A. Idini, CB, Navratil, in prep.]



→ Dynamic correlations have a strong impact on shifting the single-particle spectrum.

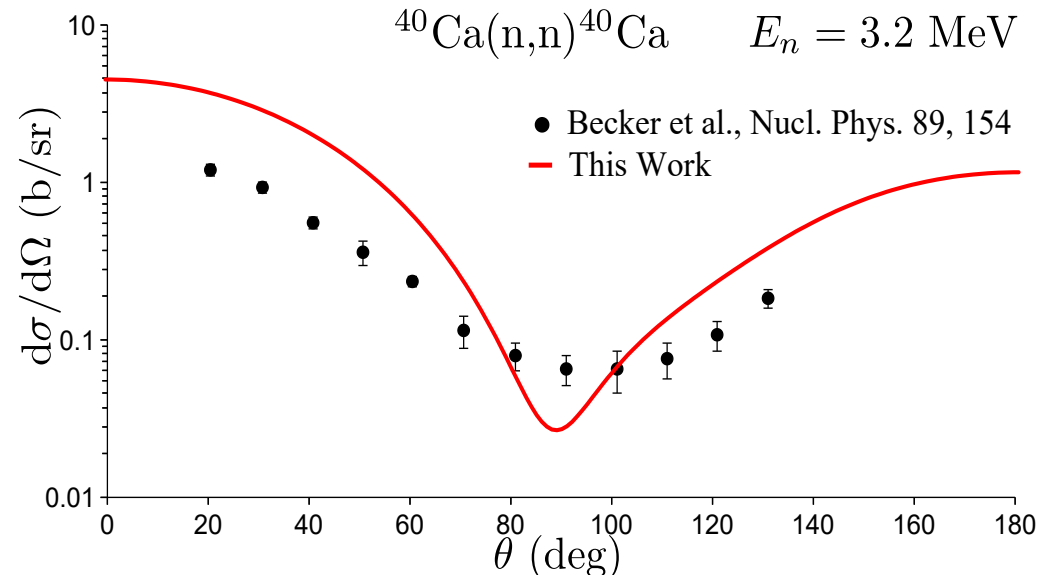
Low energy scattering - from SCGF

[A. Idini, CB, Navratil, in prep.]



- Elastic neutron scattering derived from first principle calculations (no fitting!)
- Can be extended to radioactive isotopes and large masses

Works decently at low energies but improvement in theory and interaction are still in need...



Thank you for
your attention!!!