Status of neutrinoless $\beta\beta$ decay nuclear matrix elements

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Nuclear physics and fundamental symmetries

Neutrinos, dark matter... can be studied in high-energy experiments

Nuclear physics offers an alternative:
Nuclei are abundant in huge numbers \( N_A = 6.02 \times 10^{23} \) nuclei in A grams!

Lots of material over long times provides access to detect very rare decays and very small cross-sections!

Limit background: underground

KamLAND-Zen, GERDA...
Nuclear physics and neutrinoless $\beta\beta$ decay

Neutrinos, dark matter studied in experiments using nuclei

Nuclear matrix elements depend on nuclear structure crucial to anticipate reach and fully exploit experiments

$0\nu\beta\beta$ decay: $\left( T_{1/2}^{0\nu\beta\beta} \right)^{-1} \propto |M^{0\nu\beta\beta}|^2 m_{\beta\beta}^2$

Dark matter: $\frac{d\sigma_{\chi N}}{dq^2} \propto \sum_i c_i \zeta_i |F_i|^2$

$M^{0\nu\beta\beta}$: Nuclear matrix element

$F_i$: Nuclear structure factor
Neutrinoless double-beta decay

Lepton-number violation, Majorana nature of neutrinos

Second order process only observable in rare cases with $\beta$-decay energetically forbidden or hindered by $\Delta J$

Best limit: $^{130}\text{Te}$ (CUORE), $^{76}\text{Ge}$ (GERDA), $^{136}\text{Xe}$ (EXO, KamLAND-Zen)
Next generation experiments: inverted hierarchy

The decay lifetime is

\[ T_{1/2}^{0\nu\beta\beta} (0^+ \rightarrow 0^+)^{-1} = G_{01} |M_{0\nu\beta\beta}|^2 m_{\beta\beta}^2 \]

sensitive to absolute neutrino masses, \( m_{\beta\beta} = |\sum U_{ek}^2 m_k| \), and hierarchy

Matrix elements needed to make sure next generation ton-scale experiments fully explore "inverted hierarchy"

KamLAND-Zen, PRL117 082503(2016)
Outline

Present status of $0\nu\beta\beta$ decay nuclear matrix elements

What can we learn from other nuclear experimental data?

Future prospects for $0\nu\beta\beta$ nuclear matrix element calculations
Calculating nuclear matrix elements

Nuclear matrix elements needed to study fundamental symmetries

$$\langle \text{Final} | L_{\text{leptons-nucleons}} | \text{Initial} \rangle = \langle \text{Final} | \int dx \, j_\mu(x) J_\mu(x) | \text{Initial} \rangle$$

- **Nuclear structure calculation of the initial and final states:**
  - Shell model Retamosa, Poves, JM, Horoi...
  - Energy-density functional Rodríguez, Yao...
  - QRPA Vogel, Faessler, Šimkovic, Suhonen...
  - Interacting boson model Iachello, Barea...
  - Ab initio many-body methods
    - Green’s Function MC, Coupled-cluster, IM-SRG...

- **Lepton-nucleus interaction:**
  - Study hadronic current in nucleus:
    - phenomenological approaches,
    - effective theory of QCD
$0\nu\beta\beta$ nuclear matrix elements: last 5 years

Comparison of nuclear matrix element calculations: 2012 vs 2017

What have we learned in the last 5 years?


## Configuration space

Nuclear shell model configuration space only keep essential degrees of freedom

- High-energy orbits: always empty
- Configuration space: where many-body problem is solved
- Inert core: always filled

\[
H |\psi\rangle = E |\psi\rangle \rightarrow H_{\text{eff}} |\psi\rangle_{\text{eff}} = E |\psi\rangle_{\text{eff}}
\]

\[
|\psi\rangle_{\text{eff}} = \sum_{\alpha} c_{\alpha} |\phi_{\alpha}\rangle, \quad |\phi_{\alpha}\rangle = a_{i_1}^+ a_{i_2}^+ ... a_{i_A}^+ |0\rangle
\]

### Shell model codes (1 major oscillator shell)
\sim 10^{10} \text{ Slater dets.} \quad \text{Caurier et al. RMP77 (2005)}

### QRPA calculations suggest
larger spaces (\sim 2 \text{ major shells}) needed

### Dimension
\sim \left( \frac{(p + 1)(p + 2)^N}{N} \right)^{\frac{(p + 1)(p + 2)}{Z}}
Shell model: enlarging configuration space

For $^{48}$Ca enlarge configuration space from $pf$ to $sdpf$
4 to 7 orbitals, dimension $10^5$ to $10^9$
increases matrix elements but only moderately 30%
Iwata et al. PRL116 112502 (2016)

Contributions dominated by pairing 2 particle – 2 hole excitations enhance the $\beta\beta$ matrix element,
Contributions dominated by 1 particle – 1 hole excitations suppress the $\beta\beta$ matrix element
Pairing correlations and $0\nu\beta\beta$ decay

$0\nu\beta\beta$ decay favoured by proton-proton, neutron-neutron pairing, but it is disfavored by proton-neutron pairing.

Ideal case: superfluid nuclei reduced with high-seniorities

Addition of isoscalar pairing reduces matrix element value

Related to approximate $SU(4)$ symmetry of the $\sum H(r)\sigma_i \sigma_j \tau_i \tau_j$ operator
Large configuration space calculations in 2 major oscillator shells
Include all relevant correlations: isovector/isoscalar pairing, deformation
Many-body approach: generating coordinate method (GCM)

GCM approximates shell model calculation

Degrees of freedom, or generating coordinates, validated against exact shell model in small configuration space

Jiao et al. arXiv:170703940

\(^{76}\text{Ge}\) nuclear matrix element in 2 major shells very similar to shell model nuclear matrix element in 1 major shell
IBM matrix elements with proton-neutron pairing

Energy-density functional (EDF) theory and interating boson model (IBM) calculated nuclear matrix elements do not include explicitly proton-neutron pairing correlations. This effect (partially) accounted for by other degrees of freedom present in these approaches.

Include $p$-boson ($L = 1$) to IBM in addition to $s$ and $d$ bosons ($L = 0, 2$).

First IBM results in calcium region suggest nuclear matrix elements could be somewhat reduced.

van Isacker et al. arXiv:1708.05925
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Tests of nuclear structure

Spectroscopy well described: masses, spectra, transitions, knockout...

Schiffer et al. PRL100 112501(2009)
Kay et al. PRC79 021301(2009)
...
Szwec et al., PRC94 054314 (2016)
Rodríguez et al. PRL105 252503 (2010)
...
Vietze et al. PRD91 043520 (2015)
Gamow-Teller strength distributions

Gamow-Teller (GT) distributions well described by theory (quenched)

\[ \langle 1^+ \mid \sum_i g_A^{\text{eff}} \sigma_i T_i^\pm \mid 0^+_{\text{gs}} \rangle, \quad g_A^{\text{eff}} \approx 0.7 g_A \]

Freckers et al.
NPA916 219 (2013)

Iwata et al. JPSCP 6 03057 (2015)

\[ M^{2\nu\beta} = \sum_k \frac{\langle 0^+_f \mid \sum_n \sigma_n T_n^- \mid 1^+ \rangle \langle 1^+_k \mid \sum_m \sigma_m T_m^- \mid 0^+_i \rangle}{E_k - (M_i + M_f)/2} \]

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Double Gamow-Teller strength distribution

Measurement of Double Gamow-Teller (DGT) resonance in double charge-exchange reactions $^{48}$Ca(pp,nn)$^{48}$Ti proposed in 80’s
Auerbach, Muto, Vogel... 1980’s, 90’s
Recent experimental plans in RCNP, RIKEN ($^{48}$Ca), INFN Catania
Capuzzello et al. EPJA 51 145 (2015)

Promising connection to $\beta\beta$ decay, two-particle-exchange process, specially the (tiny) transition to ground state of final state

Two-nucleon transfers related to $0\nu\beta\beta$ decay matrix element
Brown et al. PRL113 262501 (2014)
**48 Ca Double Gamow-Teller distribution**

Calculate with shell model $^{48}\text{Ca} \, 0^+_{gs}$ Double Gamow-Teller distribution

\[ B(\text{DGT}^-; \lambda; i \rightarrow f) = \frac{1}{2J_i + 1} \left| \left\langle \frac{^{48}\text{Ti}}{\sum_i \sigma_i \tau_i^- \times \sum_j \sigma_j \tau_j^-} \right\rangle^{(\lambda)} ^{48}\text{Ca}_{gs} \right|^2 \]

Add/remove pairing

\[ H' = H + G^{JT} P^{JT} \]

like-particle (T=1) or proton-neutron (T=0)

The properties of the DGT giant resonance very sensitive to pairing

Similar to $0\nu\beta\beta$ decay!

Shimizu, JM, Yako, arXiv:1709.01088
48Ca DGT resonance and $0\nu\beta\beta$ decay

Correlation btw 48Ca DGT resonance and 0νββ decay matrix element

Energy of DGT resonance, to $\sim 1\text{MeV}$, which may be feasible in the near future can give insight on the value of 0νββ decay matrix element

$$E_{av} = \frac{\sum_f E_f B(DGT^{-}, i \rightarrow f)}{\sum_f B(DGT^{-}, i \rightarrow f)}$$

Shimizu, JM, Yako, arXiv:1709.01088
DGT to ground state and $0^{\nu}\beta\beta$ decay

Very good linear correlation between DGT and $0^{\nu}\beta\beta$ decay nuclear matrix elements

$$M^{\text{DGT}} = \sqrt{B(DGT\; ; 0; 0^{+}_{gs} \rightarrow 0^{+}_{gs})}$$

Correlation holds across wide range of nuclei, from Ca to Ge and Xe

Common to nuclear shell model and energy-density functional

$0.5 \lesssim M^{0^{\nu}\beta\beta} \lesssim 5$

Explained by dominance of short internucleon distances btw exchanged / decaying neutrons

Shimizu, JM, Yaco, arXiv:1709.01088

Bogner et al. PRC86 064304 (2012)
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Nuclear matrix elements: theoretical uncertainty

Systematic uncertainty hard to estimate for phenomenological matrix elements

Effective theory for $\beta\beta$ decay:
spherical core coupled to one nucleon

Couplings adjusted to experimental data, uncertainty given by breakdown scale (systematic order by order expansion)

Use data on $\beta$ decay to predict $2\nu\beta\beta$ decay

Good agreement to data with large error bars

$0\nu\beta\beta$ decay: no data to fit to... use nuclear model instead

Coello-Pérez, JM, Schwenk, arXiv:1708.06140
Chiral Effective Field Theory

Chiral EFT: low energy approach to QCD, nuclear structure energies

Approximate chiral symmetry: pion exchanges, contact interactions

Systematic expansion: nuclear forces and electroweak currents

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Park, Gazit, Klos, Baroni...

Short-range couplings fitted to experiment once

Weinberg, van Kolck, Kaplan, Savage, Epelbaum, Kaiser, Meißner...
Ab initio many-body methods

Oxygen dripline using chiral NN+3N forces correctly reproduced ab-initio calculations treating explicitly all nucleons excellent agreement between different approaches

No-core shell model (Importance-truncated)
In-medium SRG
Hergert et al. PRL110 242501(2013)

Self-consistent Green’s function
Cipollone et al. PRL111 062501(2013)

Coupled-clusters
Jansen et al. PRL113 142502(2014)
Calcium isotopes with NN+3N forces

Calculations with NN+3N forces predict shell closures at \(^{52}\text{Ca}\), \(^{54}\text{Ca}\)

\[ S_{2n} (\text{MeV}) \]

\[ 2^+ \text{ Energy (MeV)} \]

\(^{51-54}\text{Ca}\) masses [TRIUMF/ISOLDE]
\(^{54}\text{Ca} 2^+_1\) excitation energy [RIBF, RIKEN]

Hebeler et al. ARNPS 65 457 (2015)
Ab initio $0\nu\beta\beta$ decay matrix elements?

Nuclei up to $A \sim 70$
explored with ab initio approaches
Limited by good nuclear force

Success of ab initio methods
(unitary) transformation: $H' = U^\dagger HU$
Likewise for operators: $O' = U^\dagger OU$
First EM transition results published

Ab initio single-$\beta$, $2\nu\beta\beta$ decay
calculations in light nuclei
give insight in "quenching" puzzle

$^{48}\text{Ca}$ ab initio $0\nu\beta\beta$ decay
nuclear matrix element
ready very soon: stay tuned!

Simonis et al. PRC96 014303 (2017)
Parzuchowski et al. arXiv 1705.05511
Chiral effective field theory

Chiral EFT: low energy approach to QCD, nuclear structure energies

Approximate chiral symmetry: pion exchanges, contact interactions

Systematic expansion: nuclear forces and electroweak currents

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*Weinberg, van Kolck, Kaplan, Savage, Epelbaum, Kaiser, Meißner...

Park, Gazit, Klos, Hoferichter...

2b currents applied to $\nu d$ scattering (SNO), $^3H$ $\beta$-decay, $\mu$ moment...
Gamow-Teller transitions: quenching

Single $\beta$ decays well described by nuclear structure (shell model)

\[ \langle F | \sum_i g^\text{eff}_A \sigma_i \tau_i^- | I \rangle \]

\[ g^\text{eff}_A = qg_A, \quad q \sim 0.7 - 0.8. \]

Theory needs to “quench“ Gamow-Teller operator to reproduce Gamow-Teller lifetimes: problem in nuclear many-body wf or operator?

This puzzle has been the target of many theoretical efforts:

Arima, Rho, Towner, Bertsch and Hamamoto, Wildenthal and Brown...
2b currents in medium-mass nuclei

Normal-ordered 2b currents modify GT operator
JM, Gazit, Schwenk PRL107 062501 (2011)

\[ J_{n,2b}^{\text{eff}} \simeq -\frac{g_A \rho}{f_\pi^2} \tau_n^- \sigma_n \left[ I(\rho, P) \frac{(2c_4 - c_3)}{3} \right] - \frac{g_A \rho}{f_\pi^2} \tau_n^- \sigma_n \frac{2}{3} c_3 \frac{p^2}{m_\pi^2 + p^2}, \]

2b currents predict \( g_A \) quenching \( q = 0.85 \ldots 0.66 \)
Quenching reduced at \( p > 0 \), relevant for \( 0\nu\beta\beta \) decay where \( p \sim m_\pi \)
Nuclear matrix elements with 1b+2b currents

2b currents reduce matrix elements
\( \sim 20\% - 50\% \)

Momentum transfer \( p \sim m_\pi \), reduces quenching \( \downarrow \)

Smaller quenching \( q = 0.96...0.92 \)
Ekström et al. PRL113 262504 (2014)
Improved (ab initio) calculations needed
Summary

Reliable nuclear matrix elements needed to plan and fully exploit impressive experiments looking for neutrinoless double-beta decay

- Matrix element differences between present calculations, factor 2 – 3
- New $^{48}$Ca and $^{76}$Ge matrix elements in larger configuration spaces moderate $\sim 30\%$ increase or less
- Double Gamow-Teller transitions pursued in different labs very useful insight on value of $0\nu\beta\beta$ decay matrix elements
- Ab initio calculations on the way, consistent 2b currents (quenching?) evaluation of theoretical uncertainties
Collaborators

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