The Generalized Polarizabilities of the proton

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Proton Polarizabilities

Fundamental structure constants (such as mass, size, shape, ...)

Response of internal structure & dynamics to external EM field

Sensitive to the full excitation spectrum of the nucleon

Accessed experimentally through Compton Scattering processes

Virtual Compton Scattering:

Virtuality of photon gives access to the Generalized Polarizabilities $\alpha_E(Q^2)$ & $\beta_M(Q^2)$ (+ 4 spin GPs)

$\Rightarrow$ mapping out the spatial distribution of the polarization densities

Fourier transform of densities of electric charges and magnetization of a nucleon deformed by an applied EM field

$$I(\mu^p) = \frac{1}{2}(\frac{1}{2}^+)$$

<table>
<thead>
<tr>
<th>PDG</th>
<th>Baryon Summary Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>$N$ BARYONS</td>
</tr>
<tr>
<td></td>
<td>$(S = 0, l = 1/2)$</td>
</tr>
<tr>
<td></td>
<td>$p, N^+ = uud, n, N^0 = udd$</td>
</tr>
</tbody>
</table>

| $p$  | Mass $m = 1.00727646681 \pm 0.00000000009 \ u$ |
|      | Mass $m = 938.272046 \pm 0.000021 \ MeV \ [6]$ |
|      | $|m_p - m_{\text{phot}}|/m_p < 7 \times 10^{-10}, \ CL = 90\% \ [6]$ |
|      | $|Q_E^p|/(Q_{\text{phot}}) = 0.99999999991 \pm 0.0000000009$ |
|      | $|\alpha_p + \alpha_s|/e < 7 \times 10^{-10}, \ CL = 90\% \ [6]$ |
|      | $|\beta_p + \beta_s|/e < 1 \times 10^{-21} \ [6]$ |
|      | Magnetic moment $\mu = 2.792847356 \pm 0.000000023 \mu_N$ |
|      | $(\mu_p + \mu_s)/\mu_p = (0 \pm 5) \times 10^{-6}$ |
|      | Electric dipole moment $d < 0.54 \times 10^{-33} \text{ cm}$ |
|      | Electric polarizability $\alpha = (11.2 \pm 0.4) \times 10^{-4} \text{ fm}^3$ |
|      | Magnetic polarizability $\beta = (2.5 \pm 0.4) \times 10^{-4} \text{ fm}^3$ |
|      | Charge radius, $\mu p$ Lamb shift $= 0.84087 \pm 0.00039 \text{ fm} \ [6]$ |
|      | Charge radius, $e p$ CODATA value $= 0.8773 \pm 0.0051 \text{ fm} \ [6]$ |
|      | Magnetic radius $= 0.777 \pm 0.018 \text{ fm}$ |
|      | Mean life $\tau > 2.1 \times 10^{39} \text{ years}, \ \text{CL} = 90\% \ [6]$ (
|      | Mean life $\tau > 10^{31}$ to $10^{33} \text{ years} \ [6]$ (mode dependent) |
Proton GPs

Intense experimental effort on $\alpha_E(Q^2)$ & $\beta_M(Q^2)$:

- currently facing a puzzle with respect to the electric GP
- new results are coming up
- new experiments are coming up

Spin polarizabilities:

- They have been measured in RCS (A2/MAMI): PRL 114, 112501 (2015)
- VCS: only one measurement (A1/MAMI) of a structure function that is a combination of the electric GP and two spin GPs

<table>
<thead>
<tr>
<th>$\mathcal{P}_{LT}$ (GeV$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>This experiment</td>
</tr>
<tr>
<td>DR model [16]</td>
</tr>
<tr>
<td>HBChPT $\mathcal{O}(p^3)$ [17]</td>
</tr>
</tbody>
</table>

Beam-recoil polarization measurement at $Q^2=0.33$ (GeV/c)$^2$
Scalar Polarizabilities

Response of internal structure to an applied EM field
Scalar Polarizabilities

Response of internal structure to an applied EM field

“stretchability”

\[ \vec{d}_E \text{ induced} \sim \alpha \vec{E} \]

External field deforms the charge distribution

“alignability”

\[ \vec{d}_M \text{ induced} \sim \beta \vec{B} \]

\[ \beta_{\text{para}} > 0 \]
\[ \beta_{\text{diam}} < 0 \]

Paramagnetic: proton spin aligns with the external magnetic field

Diamagnetic: \( \pi \)-cloud induction produces field counter to the external one
Virtual Compton Scattering

- REACTION PLANE
- SCATTERING PLANE

\[ \begin{align*}
\text{Bethe-Heitler} & \quad \text{VCS Born} \\
\text{VCS non-Born} &
\end{align*} \]
Virtual Compton Scattering

**DR**
- valid below & above Pion threshold
- Dispersive integrals for Non Born amplitudes
- Spin GPs are fixed
- Scalar GPs have an unconstrained part
- Fit to the experimental cross section at each $Q^2$

**LEX**
- valid only below Pion threshold
- Structure functions
  \[ d\hat{\sigma} = d\hat{\sigma}^{BB+B_{\text{Born}}} + q_{\text{em}} \cdot \phi \cdot \Psi_0 + \mathcal{O}(q_{\text{em}}^2) \]
  \[ \Psi_0 = v_1 \cdot (P_{LL} - \frac{1}{\epsilon}P_{TT}) + v_2 \cdot P_{LT} \]
- Subtract the spin part
- utilize DR

**scalar GPs $\alpha_\varepsilon$ and $\beta_M$**
Virtual Compton Scattering


Sensitivity to the GPs grows with the photon energy
Early Experiments

**MIT-Bates @ $Q^2=0.06$ GeV$^2$**

**MAMI-A1 @ $Q^2=0.33$ GeV$^2$**

**Jlab-Hall A @ $Q^2=0.9 \& 1.8$ GeV$^2$**
The data suggest non-trivial $Q^2$ evolution of $a_E$. Current theoretical calculations are not able to describe the enhancement at low $Q^2$. $Q^2 = 0.33 \text{ (GeV/c)}^2$ was measured twice at MAMI:


$\beta_M$ small $\Leftrightarrow$ cancellation of competing mechanisms

Large uncertainties

Higher precision measurements needed

Quantify the balance between diamagnetism and paramagnetism

Current situation unsatisfactory:

- More measurements needed (vs $Q^2$)
- Higher precision measurements needed
Theoretical Landscape

HBChPT
NRQCM
Effective Lagrangian Model
Linear Sigma Model

All theoretical calculations predict a smooth fall off for $\alpha_E$
None of the models can account for the non trivial structure of $\alpha_E$ suggested by the data

Lattice QCD
Currently: Q$^2$=0 calculations exist but at unphysical quark masses
Near Future: calculations at the physical point for Q$^2$=0
first calculations for Q$^2\neq0$
Spatial dependence of induced polarizations on an external EM field

Nucleon form factor data $\rightarrow$ light-front quark charge densities

Formalism extended to the deformation of these quark densities when applying an external e.m. field:

GPs $\rightarrow$ spatial deformation of charge & magnetization densities under an applied e.m. field

Induced polarization in a proton when submitted to an e.m. field

Light (dark) regions $\rightarrow$ largest (smaller) values
(photon polarization along x-axis, as indicated)

Induced polarization along $b_y=0$

M. Gorchtein, C. Lorce, B. Pasquini, M. Vanderhaeghen
Ongoing Experimental Efforts
MAMI

MAMI A1/1-09 (vcsq2) below threshold
MAMI A1/3-12 (vcsdelta) above threshold

Both experiments utilized the A1 setup at MAMI

Preliminary results were recently released

Analysis is ongoing
vcsq2 @ MAMI

~ 1.0 GeV beam

\[ Q^2 = 0.1 \text{ (GeV/c)}^2, \ 0.2 \text{ (GeV/c)}^2, \text{ and } 0.45 \text{ (GeV/c)}^2 \]

Figure from PhD thesis of L. Correa, Mainz / Cl. Ferrand, 2016

Figure 5.8: Setting INP: measured \( ep \rightarrow e\gamma \) cross section at fixed \( q'_{cm} = 112.5 \text{ MeV/c} \) with respect to \( \varphi_{cm} \) for all the \( \cos(\theta_{cm}) \)-bins. The curves follow the convention of figure 5.6.

GP effect typically 0 - 15% of the cross section

Polarizability fits:

- **DR fit:**
  DR calculation includes full dependency in \( q'_{cm} \)

- **LEX fit:**
  truncated in \( q'_{cm} \). Suppress contribution from higher order terms
For LEX the higher order terms have to be negligible

\[ d^5\sigma = d^5\sigma^{BH+Born} + q'_{cm} \cdot \phi \cdot \Psi_0 + \mathcal{O}(q'^2_{cm}) \]

A phase space masking has to be applied to keep these terms smaller than the 2\%-3\% level.

Figure 3.13: (Left) behavior of \( \mathcal{O}^{DR}(q'^2_{cm}) \) in the \((\cos(\theta_{cm}),\varphi_{cm})\)-plane at \( q'_{cm} = 87.5 \, MeV/c \) and (right) two-dimensional representation of the angular region where \( \mathcal{O}^{DR}(q'^2_{cm}) < 2\% \) (blue), the red squares correspond to the two areas of interest to perform the GP extraction.

Figure from PhD thesis of L. Correa, Mainz / Cl. Ferrand, 2016
Blue bins = where the higher-order estimator is < 3% 
(LEX truncation « valid »)

VCS expt:

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Bates</th>
<th>MAMI</th>
<th>MAMI</th>
<th>MAMI</th>
<th>MAMI</th>
<th>JLab</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q^2$ (GeV$^2$)</td>
<td>0.06</td>
<td>0.10</td>
<td>0.20</td>
<td>0.33</td>
<td>0.45</td>
<td>0.92</td>
</tr>
</tbody>
</table>

New « vcsq2 » data:

- OOP kinematics (to access the blue region)
- LEX Fit done with bin selection at $Q^2 = 0.1$ and $0.2$ GeV$^2$.
- was found not necessary at $Q^2 = 0.45$ GeV$^2$.

(material from H. Fonville)
Goal 2-fold:

1) Measurement of the electric GP $\alpha_E$

2) First measurement of N-$\Delta$ transition form factors through the $\gamma$ channel

1.1 GeV beam

Measurement at $Q^2 = 0.2 \ (GeV/c)^2$
\[ H(e,e'p)^{\pi^0} \approx 66\% \]
\[ H(e,e'\pi^+)n \approx 33\% \]
\[ H(e,e'p)^{\gamma} \approx 0.6\% \]
\[ \alpha = 4 \times 10^{-4} \text{ fm}^3 \]

\[ \alpha = 8 \times 10^{-4} \text{ fm}^3 \]
Data analyzed by 4 PhD students

Jure Bericic (Ljubljana Univ.)
Loup Correa (Clermont-Fd Univ.)
Meriem BenAli (Clermont-Fd Univ.)
Adam Blomberg (Temple Univ.)

2 independent measurements at $Q^2=0.20 \ (GeV/c)^2$
MAMI Preliminary Results

Preliminary A1/1-09 (vcsq2)

(material from H. Fonvieille)
First measurement of the $N-\Delta C2$ amplitude through the photon channel

Important for cross check to the world data and for cross checking & constraining the model uncertainties

MAMI Preliminary Results

Sato Lee
Revisiting the $Q^2=0.33$ GeV$^2$ data

$Q^2 = 0.33 \text{ (GeV/c)}^2$ measured twice at MAMI - two different experiments


(Revised H.O-cut)

The $a_E$ puzzle still holds

LEX and DR

(material from H. Fonvieille)
Ongoing Experimental Efforts
JLab

New Experiment
E12-15-001 (JLab)

Going from $\varepsilon = 0.6 \to 0.9$ doubles the sensitivity to the GPs

$\varepsilon = 0.97$ (Jlab)
$\varepsilon = 0.62$ (MAMI)

Beam energy x 4
Beam current x 5
JLab Hall C with 12 GeV upgrade

- Super High Momentum Spectrometer
  - HB, 3 Quads, Dipole
  - $P \rightarrow 2 \text{ - } 11 \text{ GeV}$
  - Resolution: $\delta < 0.1\%$
  - Acceptance: $\delta \rightarrow 30\%, \, 4 \text{ msr}$
  - $5.5^\circ < \theta < 40^\circ$
  - Good $e/\pi/K/p$ PID
- High Momentum Spectrometer
  - 3 Quads, Dipole
  - $P \rightarrow 7.5 \text{ GeV}$
  - Resolution: $\delta < 0.1\%$
  - Acceptance: $\delta \rightarrow 18\%, \, 6.5 \text{ msr}$
  - $10.5^\circ < \theta < 90^\circ$
  - Good $e/\pi/K/p$ PID
- Minimum opening angle $\sim 17^\circ$
- Well shielded detector huts
- 2 beam line polarimeters
- Ideal facility for:
  - Rosenbluth (L/T) separations
  - Exclusive reactions
  - Low cross sections (neutrino level)
Hall C HMS and SHMS

**SHMS:**
- 11-GeV Spectrometer
- Partner of existing 6-GeV HMS

**MAGNETIC OPTICS:**
- Point-to-Point QQGD for easy calibration and wide acceptance.
- Horizontal bend magnet allows acceptance at forward angles (5.5°)

**Detector Package:**
- Drift Chambers
- Hodoscopes
- Cerenkovs
- Calorimeter
- All derived from existing HMS/SOS detector designs

**Well-Shielded Detector Enclosure**

**Rigid Support Structure**
- Rapid & Remote Rotation
- Provides Pointing Accuracy & Reproducibility demonstrated in HMS

**SHMS:**
- (New)
- dQQGD

**HMS:**
- (Original)
- QQGD

**SHMS = Super High Momentum Spectrometer**

**HMS = High Momentum Spectrometer**
Hall C: SHMS, HMS
4.4 GeV
40-85 μA
Liquid hydrogen 15 cm

Experimental Setup

Photon: missing mass, e & p detection in coincidence

Cross sections
In-plane azimuthal asymmetries

\[ A_{(\phi, \gamma=0, \pi)} = \frac{\sigma_{\phi, \gamma=0} - \sigma_{\phi, \gamma=180}}{\sigma_{\phi, \gamma=0} + \sigma_{\phi, \gamma=180}} \]

Sensitivity to GPs
Suppression of systematic uncertainties
Projected Measurements

\[ Q^2 = 0.43 \text{ (GeV/c)}^2 \]

- \[ \Phi_{\gamma\gamma} = 0^\circ, 180^\circ \]
- \[ \delta \beta_M = 2 \times 10^{-4} \text{ fm}^3 \]
- \[ \alpha_E = 5.8 \times 10^{-4} \text{ fm}^3 \]
- \[ \alpha_E = 2.4 \times 10^{-4} \text{ fm}^3 \]

**Graphs:**
- BH peaks
- Avoid BH peaks
- Stay at \( \theta_{\gamma\gamma} > 120^\circ \)
# Kinematical Settings

SHMS: one change of setting through Part I
same position & momentum through out Part II

<table>
<thead>
<tr>
<th>Kinematical Setting</th>
<th>$\theta_{\gamma'\gamma}$</th>
<th>$\theta_\phi$</th>
<th>$P_e'(MeV/c)$</th>
<th>$P_p'(MeV/c)$</th>
<th>S/N</th>
<th>beam time (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part I</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kin Ia</td>
<td>155</td>
<td>7.97</td>
<td>3884.4</td>
<td>37.20</td>
<td>1.1</td>
<td>0.5</td>
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<tr>
<td>Kin Ib</td>
<td>155</td>
<td>7.97</td>
<td>3884.4</td>
<td>51.26</td>
<td>2.7</td>
<td>0.5</td>
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<tr>
<td>Kin IIa</td>
<td>140</td>
<td>7.97</td>
<td>3884.4</td>
<td>33.08</td>
<td>1</td>
<td>0.45</td>
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<td>Kin IIb</td>
<td>140</td>
<td>7.97</td>
<td>3884.4</td>
<td>55.38</td>
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<td>Kin IIIa</td>
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<tr>
<td>Kin IVa</td>
<td>165</td>
<td>9.39</td>
<td>3820.5</td>
<td>40.85</td>
<td>1.3</td>
<td>0.5</td>
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<tr>
<td>Kin IVb</td>
<td>165</td>
<td>9.39</td>
<td>3820.5</td>
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<td>2.4</td>
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<td>3820.5</td>
<td>50.96</td>
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<td>Kin VIa</td>
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<td>9.39</td>
<td>3820.5</td>
<td>31.84</td>
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<td>0.95</td>
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<td>57.46</td>
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<td>Part II</td>
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<td></td>
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<tr>
<td>Kin VIIa</td>
<td>165</td>
<td>11.54</td>
<td>3708.6</td>
<td>40.81</td>
<td>2.6</td>
<td>1.5</td>
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<tr>
<td>Kin VIIb</td>
<td>165</td>
<td>11.54</td>
<td>3708.6</td>
<td>47.35</td>
<td>5</td>
<td>2</td>
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<tr>
<td>Kin VIIIa</td>
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<td>11.54</td>
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<td>39.73</td>
<td>2.2</td>
<td>1.5</td>
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<td>Kin VIIIb</td>
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<td>11.54</td>
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<td>48.43</td>
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<td>2</td>
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<td>35.52</td>
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<td>1.5</td>
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<td>11.54</td>
<td>3708.6</td>
<td>52.64</td>
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<td>2</td>
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SHMS: one change of setting through Part I
same position & momentum through out Part II

<table>
<thead>
<tr>
<th>Part</th>
<th>I</th>
<th>I</th>
<th>I</th>
<th>II</th>
<th>II</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q^2$</td>
<td>0.33 (GeV/c)</td>
<td>0.43 (GeV/c)^2</td>
<td>0.52 (GeV/c)^2</td>
<td>0.65 (GeV/c)^2</td>
<td>0.75 (GeV/c)^2</td>
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</tbody>
</table>
Phase Space

Part I

Φ_{\gamma*\gamma} = 0° and 180° at \( \theta_{\gamma*\gamma} = 165° \)

Part II

Φ_{\gamma*\gamma} = 0° and 180° at \( \theta_{\gamma*\gamma} = 165° \)

Phase space binned in \( Q^2, W, \theta_{\gamma*\gamma}, \Phi_{\gamma*\gamma} \)

Cross section: DR calculation, B. Pasquini


<table>
<thead>
<tr>
<th>Part</th>
<th>I</th>
<th>I</th>
<th>I</th>
<th>II</th>
<th>II</th>
</tr>
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<tbody>
<tr>
<td>( Q^2 )</td>
<td>0.33 (GeV/c)</td>
<td>0.43 (GeV/c)^2</td>
<td>0.52 (GeV/c)^2</td>
<td>0.65 (GeV/c)^2</td>
<td>0.75 (GeV/c)^2</td>
</tr>
</tbody>
</table>
**Projected Measurements**

\[ \alpha_E = 4.8 \times 10^{-4} \text{ fm}^3 \]
\[ \alpha_E = 1.5 \times 10^{-4} \text{ fm}^3 \]

\[ \beta_M = 1.6 \times 10^{-4} \text{ fm}^3 \]
\[ \beta_M = 0.4 \times 10^{-4} \text{ fm}^3 \]

\[ Q^2 = 0.65 \text{ (GeV/c)}^2 \]

Statistical < ±1.3%
Beam energy / scat. Angle ±1-2.5%
Target density ±0.5%
Detector efficiency ±0.5%
Acceptance ±0.5%
Target cell backgr. ±0.5%
Target length ±0.3%
Beam charge ±0.3%
Dead time ±0.3%
Pion contamination in MM ±0.3%
Rad. Corr. ±1.5%
Other ±0.5%

\[ \sigma < ±1.3\% \text{ (stat)} \]
\[ \sigma < ±3.3\% \text{ (syst)} \]

\[ A \approx ±0.7\% \text{ (stat)} \]
\[ A \approx ±1.1\% \text{ (syst)} \]
Projected Measurements

\[ a_e (10^{-4} \text{fm}^3) \]

\[ \beta_M (10^{-4} \text{fm}^3) \]

\[ Q^2 (\text{GeV}^2) \]
Status of E12-15-001

Part I approved in summer 2016 (Jlab PAC 44): (4.4 GeV, 85 µA, Hall C)

Current plan is to take data in June 2019
Other ongoing efforts

E08-010 (Hall-A/Jlab): $\gamma$-channel

parasitic access to VCS - data analysis ongoing

$Q^2 = 0.04 \text{ (GeV/c)}^2 \text{ to } 0.13 \text{ (GeV/c)}^2$
Summary

Intense experimental effort focusing on the measurement of the electric and magnetic GPs
• fundamental structure constants
• internal structure and dynamics of the nucleon
• complementary information to elastic & transition FFs, GPDs, TMDs, ...

New results (MAMI) and an upcoming new experiment (Jlab) in a region very sensitive to the nucleon dynamics
• improve the precision of $a_E$ and $\beta_M$ by a factor of 2
• GPs $Q^2$ signature
• explore mechanism for the non trivial $Q^2$ dependence of $a_E$
• quantify the balance between paramagnetism and diamagnetism through $\beta_M$
• provide, with high precision, the spatial deformation of charge & magnetization densities under an applied e.m. field (currently a profound structure is suggested in the region 0.5 fm - 1 fm)
• Lattice QCD results will be emerging in the next few years - very important to cross check these calculations

Puzzle w.r.t. $a_E$

Thank you!