Spin-dependent PDFs from Lattice QCD

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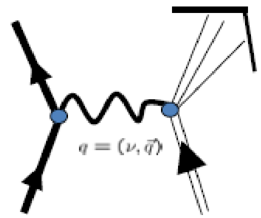
Quark distributions and quasi-distributions

Cross sections are measured



Cross sections written in terms of structure functions: $F_1(x,Q^2), F_2(x,Q^2), g_1(x,Q^2), g_2(x,Q^2), \cdots$

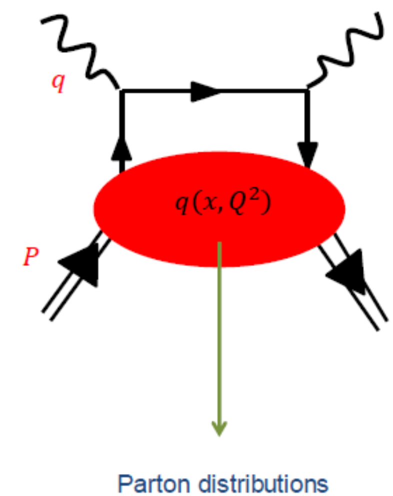
QCD + OPE:

$$\int_0^1 dx x^{n-2} F_2(x,Q^2) = \sum_i a_n^{(i)} C_n^{(i)}(Q^2)$$

$$\langle P | \mathcal{O}_{\mu_1 \cdots \mu_n} | P \rangle = a_n P_{\mu_1} \cdots P_{\mu_2}$$

Moments of the parton distributions: $a_n = \int dx\, x^{n-1} q(x)$

At leading order (LO) in pQCD:, $F_2(x,Q^2) = x \sum_q e_q^2 q(x,Q^2)$



Light-cone quark distributions

The most general form of the matrix element is:

$$\langle P|O^{\mu_1\mu_2\cdots\mu_n}|P\rangle = 2a_n^{(0)}\Pi^{\mu_1\mu_2\cdots\mu_n}$$

$$\Pi^{\mu_1\mu_2\cdots\mu_n} = \sum_{j=0}^{k}(-1)^j\frac{(2k-j)!}{2^j(2k)!}\{g\cdots gP\cdots P\}_{k,j}(P^2)^j$$

We use the following four-vectors

$$P = (P_0,0,0,P_3) \quad \lambda = (1,0,0,-1)/\sqrt{2} \quad \longrightarrow \quad \boxed{\lambda\cdot P = (P_0+P_3)/\sqrt{2} = P_+}$$

$$\lambda_{\mu_1}\lambda_{\mu_2}\left\langle P\middle|O^{\mu_1\,\mu_2}\middle|P\right\rangle = 2a_n^{(0)}\left(P^+P^+ - \lambda^2\frac{M^2}{4}\right) = 2a_n^{(0)}P^+P^+$$

In general, we have

$$\lambda_{\mu_1}\cdots\lambda_{\mu_n}\Pi^{\mu_1\cdots\mu_n} = (P^+)^n \quad \longrightarrow \quad \boxed{\langle P|O^{+\cdots+}|P\rangle = 2a_n^{(0)}(P^+)^n}$$

Matrix elements projected on the light-cone are protected from target mass corrections

Taking the inverse Mellin transform

$$a_n^{(0)} = \int dx\, x^{n-1} q(x) \qquad\qquad q(x) = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} dn\, x^{-n} a_n^{(0)}$$

Using $\quad a_n^{(0)} = \langle P|O^{+\cdots+}|P\rangle / 2(P^+)^n$

$$\Rightarrow \quad q(x) = \int_{-\infty}^{+\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle P|\bar{\psi}(\xi^-)\gamma^+ W(\xi^-,0)\psi(0)|P\rangle$$

$$W(\xi^-,0) = e^{-ig\int_0^{\xi^-} A^+(\eta^-)d\eta^-}$$ (Wilson line)

- Light cone correlations
- Equivalent to the distributions in the Infinite Momentum Frame
- Light cone dominated $\quad \xi^2 = t^2 - z^2 \sim 0$
- Not calculable on Euclidian lattice $\quad t^2 + z^2 \sim 0$
- Moments, however, can be calculated

Moments of the distributions

- If a sufficient number of moments are calculated, one can reconstruct the x dependence of the distributions;
- Hard to simulate high order derivatives on the lattice;
- Nevertheless, the first few moments can be calculated

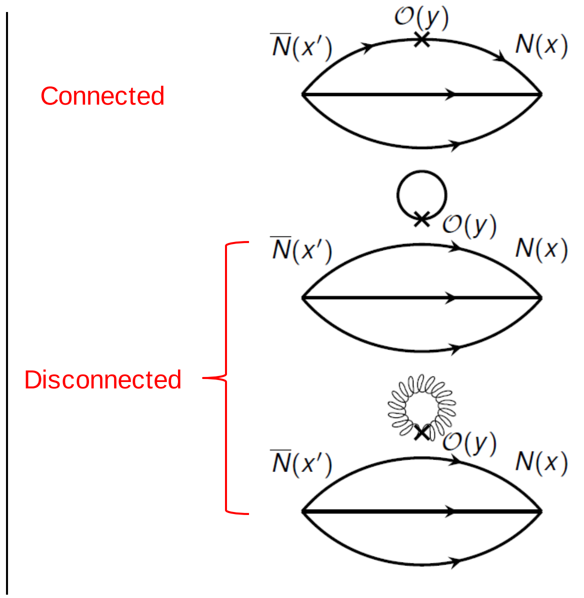
Extracting the moments

$$C^{2pt}(\vec{P},t,t') = \frac{e^{-E_0(t-t')}}{2E_0}\langle\Omega|N(P)|0\rangle\langle 0|\bar{N}(P)|\Omega\rangle, \quad t \gg t'$$

(the two point function)

E_0 → Nucleon mass

$$\frac{C^{3pt}_{\Gamma}(t,\tau,t';\vec{P},\vec{P})}{C^{2pt}_{\Gamma}(t,t';\vec{P})} = \frac{Tr\left(\Gamma(\gamma_\mu P_0^\mu + m)O_{00}(\gamma_\mu P_0^\mu + m)\right)}{2E\,Tr\left(\Gamma'(\gamma_\mu P_0^\mu + m)\right)}, \quad t \gg \tau \gg t'$$



Example: Proton spin decomposition

$$\langle N(p',s')|\mathcal{O}_A^{\mu,q}|N(p,s)\rangle = \bar{u}_N(p',s')g_A^q(Q^2)\gamma^\mu\gamma_5\, u_N(p,s)$$

$$\Delta\Sigma = g_A^{(0)} = \sum_q g_A^q(0) = \Delta u + \Delta d + \Delta s + \cdots$$

Total helicity carried by quarks

$$\langle N(p',s')|\mathcal{O}_V^{\mu\nu}|N(p,s)\rangle = \bar{u}_N(p',s')\Lambda_q^{\mu\nu}(Q^2)u_N(p,s)$$

$$\Lambda_q^{\mu\nu}(Q^2) = A_{20}^q(Q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}^q(Q^2)\frac{\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}^q(Q^2)\frac{Q^{\{\mu}Q^{\nu\}}}{m}$$

$$\langle x\rangle^q = A_{20}^q(0)$$

Average fraction x of the nucleon momentum carried by quark q

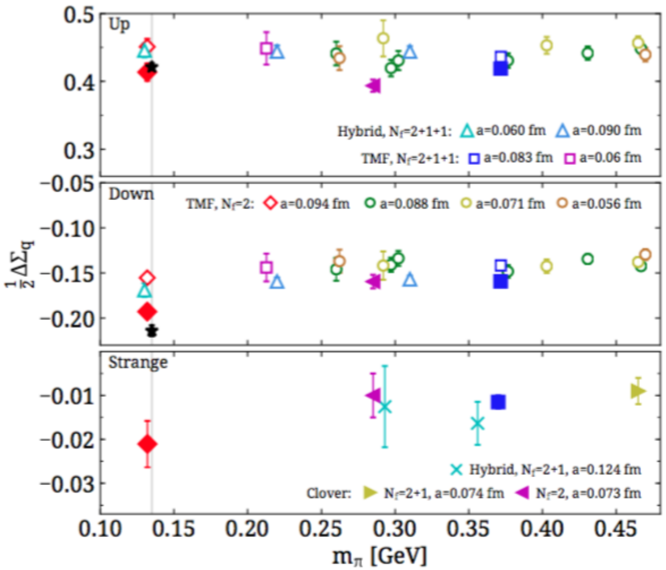
The total quark angular momentum is given by

$$J^{quark} = \frac{1}{2}\sum_q \left(A_{20}^q(0) + B_{20}^q(0)\right) = \frac{1}{2}\Delta\Sigma + L^{quarks}$$

Orbital angular momentum carried by quarks

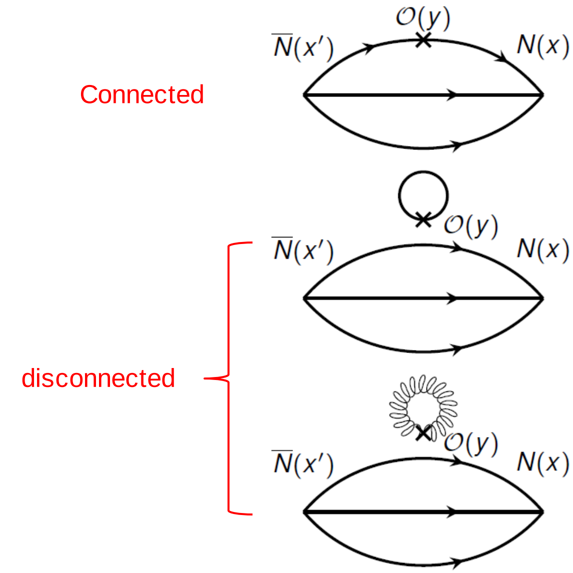
Similar expression can be obtained for the total angular momentum of gluons, J^{gluon}

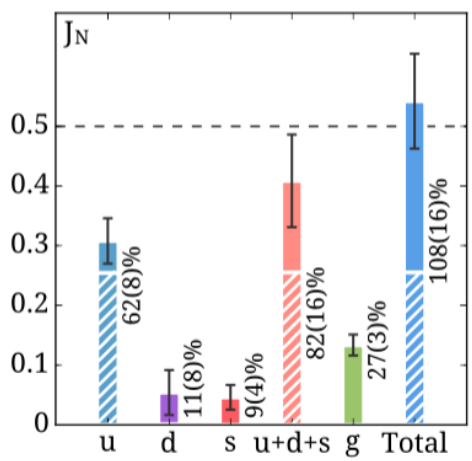
Results for $\mu = 2$ GeV

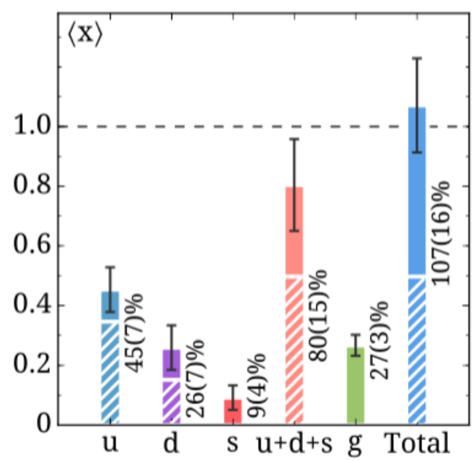


Open symbols: only connected contributions

Filled symbols: both connected and disconnected contributions







	$\frac{1}{2}\Delta\Sigma$	J	L	$\langle x\rangle$
u	0.415(13)(2)	0.308(30)(24)	-0.107(32)(24)	0.453(57)(48)
d	-0.193(8)(3)	0.054(29)(24)	0.247(30)(24)	0.259(57)(47)
s	-0.021(5)(1)	0.046(21)(0)	0.067(21)(1)	0.092(41)(0)
g	-	0.133(11)(14)	-	0.267(22)(27)
tot.	0.201(17)(5)	0.541(62)(49)	0.207(64)(45)	1.07(12)(10)

Total angular momentum

Average x: $\langle x\rangle$

- First ever results at the physical point;
- Spin sum rule satisfied;
- Momentum sum rule satisfied;
- Slightly negative polarized strangeness;
- Still, we need to go beyond the moments to a deeper understanding of the parton dynamics

C. Alexandrou et al., arXiv: 1706.02973, PRL 119 (2017) 034503

Quasi Distributions

X. Ji, “Parton Physics on a Euclidean Lattice,” PRL 110 (2013) 262002.

Suppose we project outside the light-cone:

$$\lambda = (0,0,0,-1) \qquad P = (P_0, 0,0, P_3) \qquad \lambda \cdot P = P_3$$

For example, for n=2

= -1

$$\langle P|O^{33}|P\rangle = 2\tilde{a}_n^{(0)}(P^3P^3 - \lambda^2 P^2/4) = 2\tilde{a}_n^{(0)}((P^3)^2 + P^2/4)$$

Mass terms contribute

After the inverse Mellin transform,

$$\tilde{q}(x, P_3) = \int_{-\infty}^{+\infty} \frac{dz}{4\pi} e^{-izxP_3} \langle P|\bar{\psi}(z)\gamma^3 W(z,0)\psi(0)|P\rangle + \mathcal{O}\left(\frac{M^2}{P_3^2}, \frac{\Lambda_{QCD}^2}{P_3^2}\right)$$

Higher twist

- Nucleon moving with finite momentum in the z direction
- Pure spatial correlation
- Can be simulated on a lattice

The light cone distributions:

$$x = \frac{k^+}{P^+}$$

$$0 \leq x \leq 1$$

Distributions can be defined in the infinite momentum frame: $P_3, P^+ \rightarrow \infty$

Quasi distributions:

P_3 large but finite

Usual partonic interpretation is lost

$x < 0$ or $x > 1$ is possible

But they can be related to each other!

Extracting quark distributions from quark quasi-distributions

Infrared region untouched when going from finite to infinite momentum

Infinite momentum: $p_3 \to \infty$ (before integrating over the quark transverse momentum k_T)

$$q(x,\mu) = q_{bare}(x)\left\{1 + \frac{\alpha_s}{2\pi} Z_F(\mu)\right\} + \frac{\alpha_s}{2\pi}\int_x^1 \Gamma\left(\frac{x}{y},\mu\right) q_{bare}(y)\frac{dy}{y} + \mathcal{O}(\alpha_s^2)$$

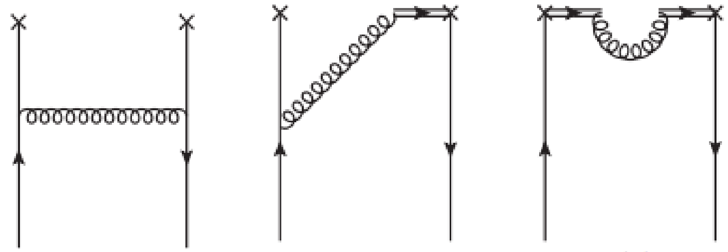
Finite momentum: p_3 fixed

$$\tilde{q}(x,P_3) = q_{bare}(x)\left\{1 + \frac{\alpha_s}{2\pi} \tilde{Z}_F\,(P_3)\right\} + \frac{\alpha_s}{2\pi}\int_{x/y_c}^1 \tilde{\Gamma}\left(\frac{x}{y},P_3\right) q_{bare}(y)\frac{dy}{y} + \mathcal{O}(\alpha_s^2)$$

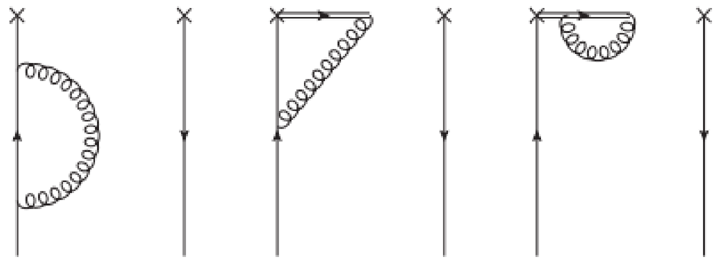
$$\tilde{q}(\pm y_c) = 0$$

In principle, $y_c \to \infty$

Perturbative QCD in the continuum



Vertex: Γ or $\tilde{\Gamma}$



Self-energy: Z_F **or** $\tilde{Z}_F$

$$q(x,\mu) = \tilde{q}(x,p_3) - \frac{\alpha_s}{2\pi}\tilde{q}(x,p_3)\delta Z_F\left(\frac{\mu}{p_3},x_c\right) - \frac{\alpha_s}{2\pi}\int_{-x_c}^{-|x|/y_c}\delta\Gamma\left(y,\frac{\mu}{p_3}\right)\tilde{q}\left(\frac{x}{y},p_3\right)\frac{dy}{|y|} - \frac{\alpha_s}{2\pi}\int_{+|x|/y_c}^{+x_c}\delta\Gamma\left(y,\frac{\mu}{p_3}\right)\tilde{q}\left(\frac{x}{y},p_3\right)\frac{dy}{|y|}$$

$$\delta\Gamma = \tilde{\Gamma} - \Gamma$$

Matching equation

$$\delta Z_F = \tilde{Z}_F - Z_F$$

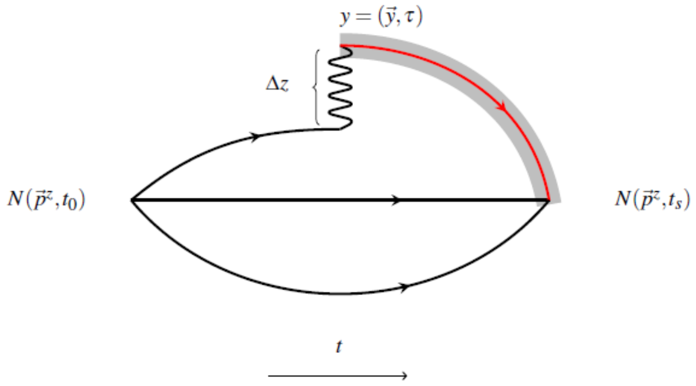
X. Xiong, X. Ji, J. H. Zhang and Y. Zhao, PRD 90 014051 (2014)
C.Alexandrou, K.Cichy, V.Drach, E.Garcia-Ramos, K.Hadjiyiannakou, K.Jansen, F.Steffens and C.Wiese, PRD 92 014502 (2015)
W. Wang, S. Zhao and R. Zhu, Eur. Phys. J. C78 (2018) 147;
W. Stewart, Y. Zhao, PRD 97 054512 (2018
T.Izubuchi, X.Ji, L.Jin, I.W.Stewart and Y.Zhao, arXiv:1801.03917
C.Alexandrou, K.Cichy, M.Constantinou, K.Jansen, A.Scapellato and F.Steffens, arXiv:1803.02685, to appear in PRL

Main steps of the procedure:

1. Compute the matrix elements between proton states with finite P_3;
2. Non-perturbative renormalization of the matrix elements;
3. Fourier transform to obtain the quasi-PDF $\tilde{q}(x, P_3, \mu)$;
4. Matching procedure to obtain the light-cone PDF $q(x, \mu)$;
5. Apply Target Mass Corrections (TMCs) to correct for the powers of M^2/P_3^2 .

Computation of matrix elements

$$\frac{C^{3pt}(T_s,\tau,0;P_3)}{C^{2pt}(T_s,0;P_3)} \propto \Delta h(P_3,z), \qquad 0 \ll \tau \ll T_s$$



With the 3 point function given by:

$$C^{3pt}(t,\tau,0) = \langle N_\alpha(\vec{P},t)\mathcal{O}(\tau)\overline{N_\alpha}(\vec{P},0)\rangle$$

And

$$\mathcal{O}(z,\tau,Q^2=0) = \sum_{\vec{y}} \bar{\psi}(y+z)\gamma^3\gamma^5 W(y+z,y)\psi(y)$$

Where the matrix elements (ME) are: $\Delta h(P_3,z) = \langle P|\bar{\psi}(z)\gamma^3\gamma^5 W(z,0)\psi(0)|P\rangle$

Setup: $N_f = 2, \qquad \beta = \frac{6}{g_0^2} = 2.10, \qquad a = 0.0938(3)(2)\,fm$

$$48^3 \times 96, \qquad L = 4.5 fm, \qquad m_\pi = 0.1304(4)\ GeV, \qquad m_\pi L = 2.98(1)$$

$$P_3 = \frac{6\pi}{L}, \frac{8\pi}{L}, \frac{10\pi}{L} = 0.84, 1.11, 1.38\ \text{GeV}$$

6 directions of Wilson line: $\pm x, \pm y, \pm z$

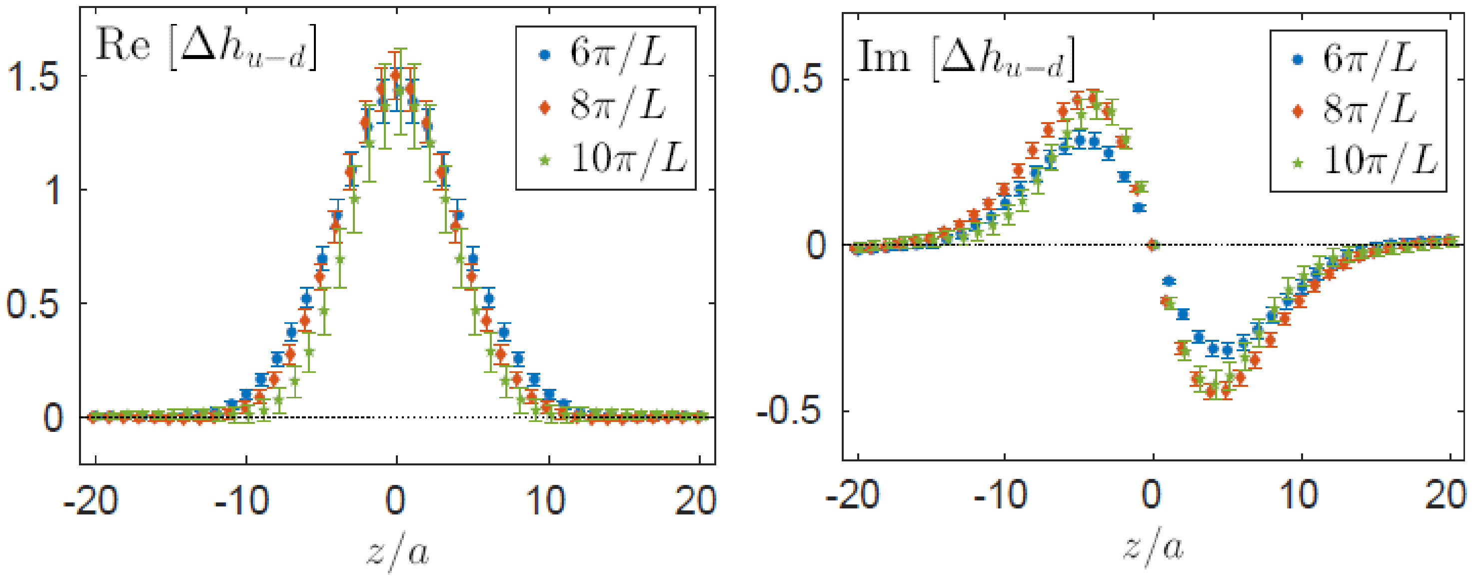
16 source positions

Separation $T_s \approx 1.1$ fm as the lowest safe choice

$P_3 = \frac{6\pi}{L}$			$P_3 = \frac{8\pi}{L}$			$P_3 = \frac{10\pi}{L}$		
Ins.	N_{conf}	N_{meas}	Ins.	N_{conf}	N_{meas}	Ins.	N_{conf}	N_{meas}
$\gamma_5\gamma_3$	65	6240	$\gamma_5\gamma_3$	425	38250	$\gamma_5\gamma_3$	655	58950

With these configurations, we compute the corresponding matrix elements

C. Alexandrou et al., 1803.02685



Helicity

The bare matrix elements $\Delta h_{u-d}(P_3, z) = \langle P | \bar{\psi}(z)\gamma^3\gamma^5 W(z,0)\tau^3\psi(0) | P \rangle$, however, contain divergences:

Next step: Renormalization!

Renormalization

$$\Delta h^{R,u-d} = Z_{\Delta h} M \Delta h^{u-d} = (Re[Z_{\Delta h}] + i\, Im[Z_{\Delta h}])\left(Re\left[\Delta h^{u-d}\right] + i\, Im\left[\Delta h^{u-d}\right]\right)$$

$Z_{\Delta h}$ renormalizes both the usual log divergence
and the linear divergence associated with the Wilson line

Nonperturbative renormalization using the RI'-MOM to remove both divergences

C. Alexandrou et al., NPB 923 (2017) 394 (Frontier Article)
J-W. Chen et al., PRD 97 014505 (2018)
C. Alexandrou et al., 1807.00232

Convert the ME from RI'-MOM to $\overline{MS}$ using 1-loop perturbation theory

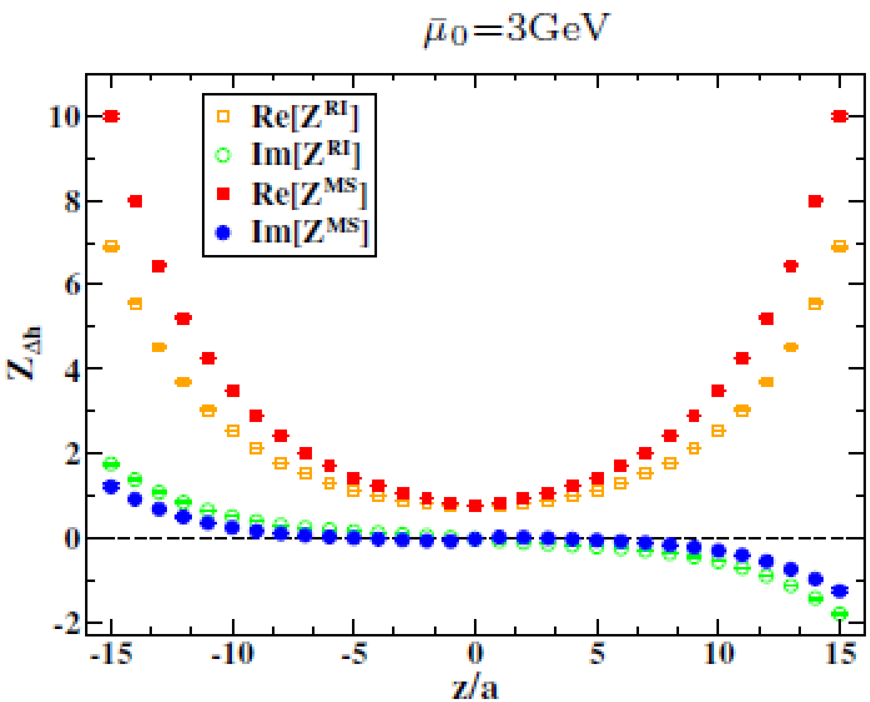
M. Constantinou, H. Panapaulos, PRD (2017)054506

We present results for the $\overline{MS}$ scheme

Renormalization factor for helicity

RI'-MOM scheme at the scale $\bar{\mu}_0 = 3$ GeV

Perturbative conversion to $\overline{MS}$ scheme at the scale 2 GeV



$$Z_q^{-1} Z_{\mathcal{O}} \frac{1}{12} Tr[\nu(p,z)(\nu^{Born}(p,z))^{-1}]|_{p^2=\bar{\mu}_0^2} = 1$$

$$Z_q = \frac{1}{12} Tr[\left(S(p)\right)^{-1} S^{Born}(p)]|_{p^2=\bar{\mu}_0^2}$$

The vertex function ν contains the same divergences as the nucleon matrix elements

The factor $Z_{\mathcal{O}}$ subtracts both the linear and log divergences.

The linear divergence associated with the Wilson line makes $Z_{\mathcal{O}}$ to grow very fast for large z;

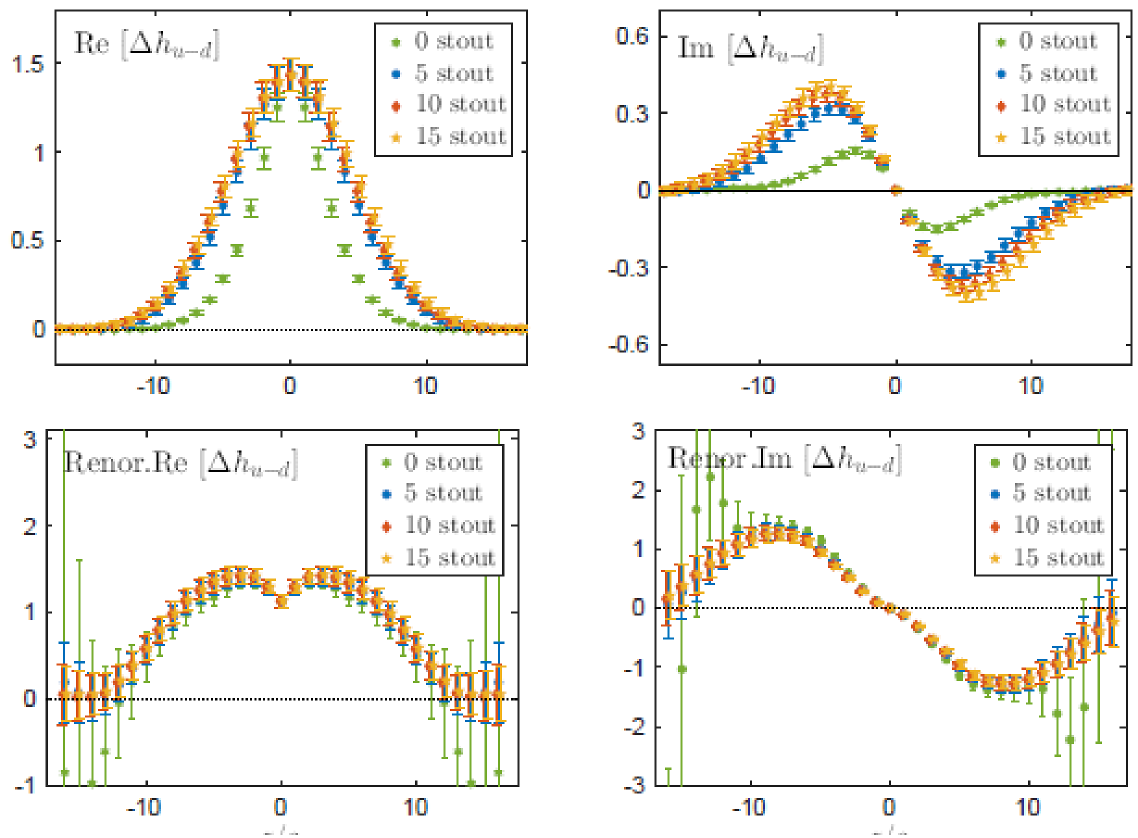
That makes the renormalized ME to have amplified errors at large z;

We thus apply smearing to the Wilson lines only in order to smooth the divergence;

In the end, if the procedure is consistent, the resulting renormalized ME should be the same, independent of the smearing applied

Renormalized ME for the helicity case

$P_3 \approx 0.83\ GeV$



Bare ME

Renormalized ME

$Re[\Delta h^{u-d}]\ Re[Z_{\Delta h}] - Im[\Delta h^{u-d}]\ Im[Z_{\Delta g}]$

$Re[\Delta h^{u-d}]\ Im[Z_{\Delta h}] + Im[\Delta h^{u-d}]\ Re[Z_{\Delta h}]$

ME sit on top of each other after renormalization

Renormalization is doing its job!

The x dependence of $\Delta u(x) - \Delta d(x)$

Once we have the ME, we compute the qPDF:

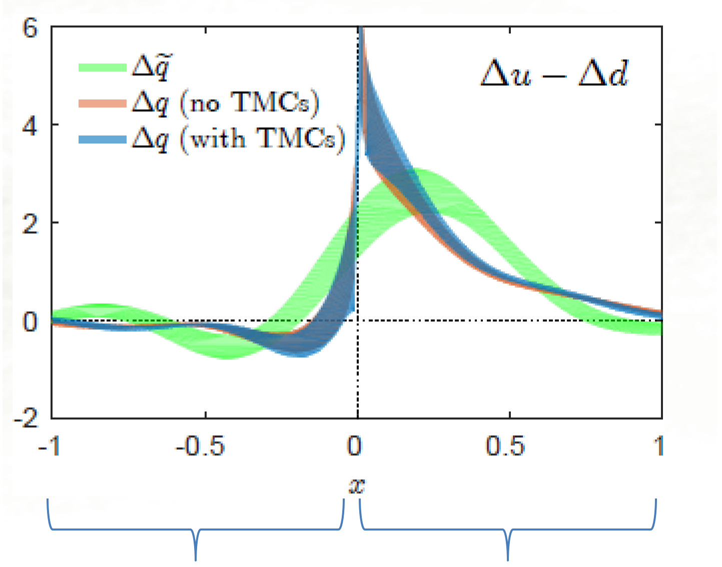
$$\Delta\tilde{q}(x,\mu^2,P_3) = \int \frac{dz}{4\pi} e^{-ixP_3 z}\langle P|\bar{\psi}(z)\gamma^3\gamma^5 W(z,0)\psi(0)|P\rangle$$

Continuum Euclidean qPDF = continuum Minkowski qPDF: Carlson, Freid, PRD 95 (2017) 094504
Briceño *et al.*, PRD 96 (2017) 014502

And then apply the matching plus target mass corrections to obtain the light-cone PDF:

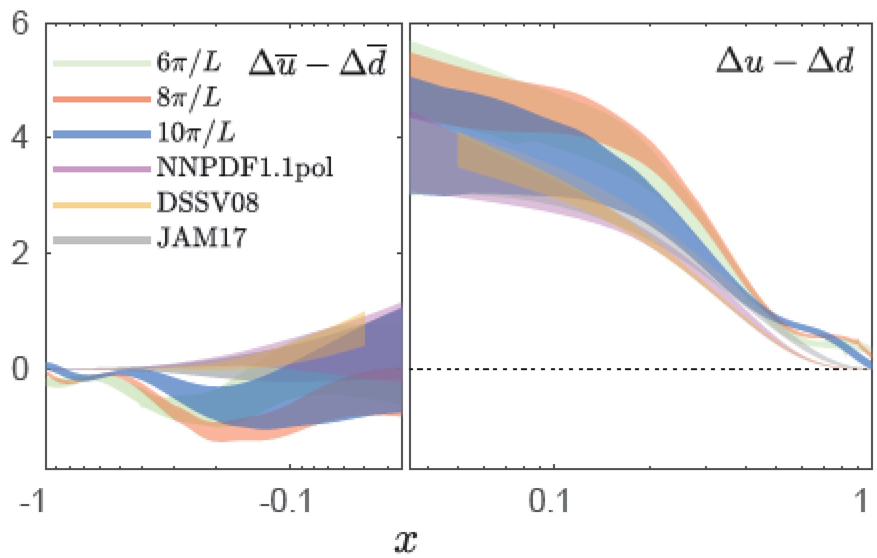
$$\Delta q(x,\mu) = \int_{-\infty}^{+\infty} \frac{d\xi}{\xi} C\left(\xi, \frac{\mu}{xP_3}\right) \Delta\tilde{q}\left(\frac{x}{\xi},\mu,P_3\right)$$

Helicity iso-vector quark distribution



$P_3 = \frac{10\pi}{L} \approx 1.38$ GeV

Helicity iso-vector quark distribution



C. Alexandrou et al., 1803.02685, to appear in PRL

Remarkable qualitative agreement

For the values of P_3 used here, the ME do not decay fast enough, that is, before e^{-ixP_3z} becomes negative

When doing the Fourier transform, unphysical oscillations appear, remarkably for $x > 0.5$, and an unphysical minimum at $x \approx -0.2$

Summary

Proton spin decomposition was presented at the physical pion mass. Spin and momentum sum rules are satisfied;

We have also shown an *ab initio* computation of the x dependence of the iso-vector PDF at the physical point;

No input nor any assumption on their functional dependence, this was unthinkable of just few years ago;

Enormous progress over the last couple of years:

- a complete non-perturbative prescription for the ME has emerged
- the matching equations relating the qPDFs to the light-cone PDFs have been improved

Still, many challenges remain:

- How to go to higher values of P_3?
- Unphysical oscillations
- Discretization and volume effects
- Higher twist

Physical point computation also presented in Huey-Wen Lin et all., 1807.07431

Quasi-PDFs are intrinsically related to pseudo-PDFs, see Radyushkin, PRD 96 (2017) 034025