



Possible effect of mixed phase and deconfinement upon spin correlations in the $\Lambda\bar{\Lambda}$ pairs generated in relativistic heavy-ion collisions

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Introduction

- Spin correlations for the $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$ pairs, generated in one act of collision of hadrons or nuclei, and respective angular correlations at the joint registration of hadronic decays of two hyperons give the important information about the character of multiple processes.
- The advantage of $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$ systems is due to the fact that the P -odd decays $\Lambda \rightarrow p + \pi^-$ and $\bar{\Lambda} \rightarrow \bar{p} + \pi^+$ serve as effective analyzers of the spin states of Λ and $\bar{\Lambda}$ particles.

1. General structure of the spin density matrix of the pairs of $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$

- The spin density matrix of the $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$ pairs, just as the spin density matrix of two **spin-1/2** particles in general, can be presented in the following form :

$$\hat{\rho}^{(1,2)} = \frac{1}{4} \left[\hat{I}^{(1)} \otimes \hat{I}^{(2)} + (\hat{\sigma}^{(1)} \mathbf{P}_1) \otimes \hat{I}^{(2)} + \hat{I}^{(1)} \otimes (\hat{\sigma}^{(2)} \mathbf{P}_2) + \sum_{i=1}^3 \sum_{k=1}^3 T_{ik} \hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_k^{(2)} \right]$$

in doing so, $tr_{(1,2)} \hat{\rho}^{(1,2)} = 1$.

Here \hat{I} is the two-row unit matrix, $\hat{\sigma} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)$ is the vector Pauli operator ($x, y, z \rightarrow 1, 2, 3$),

\mathbf{P}_1 and \mathbf{P}_2 are the polarization vectors of the first and second particle ($\mathbf{P}_1 = \langle \hat{\sigma}^{(1)} \rangle$, $\mathbf{P}_2 = \langle \hat{\sigma}^{(2)} \rangle$), $T_{ik} = \langle \hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_k^{(2)} \rangle$ are the correlation tensor components . In the general case $T_{ik} \neq P_{1i} P_{2k}$. The tensor with components $C_{ik} = T_{ik} - P_{1i} P_{2k}$ describes the spin correlations of two particles .

2. Spin correlations and angular correlations at joint registration of decays of two Λ particles into the channel $\Lambda \rightarrow p + \pi^-$

- Any decay with the space parity nonconservation may serve as an analyzer of spin state of the unstable particle .

The normalized angular distribution at the decay $\Lambda \rightarrow p + \pi^-$ takes the form:

$$\frac{dw(\mathbf{n})}{d\Omega_{\mathbf{n}}} = \frac{1}{4\pi} (1 + \alpha_{\Lambda} \mathbf{P}_{\Lambda} \cdot \mathbf{n}) .$$

Here \mathbf{P}_{Λ} is the polarization vector of the Λ particle, \mathbf{n} is the unit vector along the direction of proton momentum in the rest frame of the Λ particle, α_{Λ} is the coefficient of P -odd angular asymmetry ($\alpha_{\Lambda} = 0.642$).

The decay $\Lambda \rightarrow p + \pi^-$ selects the projections of spin of the Λ particle onto the direction of proton momentum; the analyzing power equals $\xi = \alpha_{\Lambda} \mathbf{n}$.

- Now let us consider the double angular distribution of flight directions for protons formed in the decays of two Λ particles into the channel $\Lambda \rightarrow p + \pi^-$, normalized by unity (the analyzing powers are $\xi_1 = \alpha_\Lambda \mathbf{n}_1$, $\xi_2 = \alpha_\Lambda \mathbf{n}_2$). It is described by the following formula :

$$\frac{d^2 w(\mathbf{n}_1, \mathbf{n}_2)}{d\Omega_{\mathbf{n}_1} d\Omega_{\mathbf{n}_2}} = \frac{1}{16 \pi^2} \left[1 + \alpha_\Lambda \mathbf{P}_1 \mathbf{n}_1 + \alpha_\Lambda \mathbf{P}_2 \mathbf{n}_2 + \alpha_\Lambda^2 \sum_{i=1}^3 \sum_{k=1}^3 T_{ik} n_{1i} n_{2k} \right]$$

where \mathbf{P}_1 and \mathbf{P}_2 are polarization vectors of the first and second Λ particle, T_{ik} are the correlation tensor components, \mathbf{n}_1 and \mathbf{n}_2 are unit vectors in the respective rest frames of the first and second Λ particle, defined in the common (unified) coordinate axes of the c.m. frame of the pair ($i, k = \{1, 2, 3\} = \{x, y, z\}$).

The polarization parameters can be determined from the angular distribution of decay products by the method of moments -- as a result of averaging combinations of trigonometric functions of angles of proton flight over the double angular distribution :

$$P_{1i} = \frac{3}{\alpha_{\Lambda}} \langle n_{1i} \rangle, \quad P_{2k} = \frac{3}{\alpha_{\Lambda}} \langle n_{2k} \rangle, \quad T_{ik} = \frac{9}{\alpha_{\Lambda}^2} \langle n_{1i} n_{2k} \rangle$$

Here

$$\langle \dots \rangle \equiv \int (\dots) \left(\frac{d^2 w(\mathbf{n}_1, \mathbf{n}_2)}{d\Omega_{\mathbf{n}_1} d\Omega_{\mathbf{n}_2}} \right) d\Omega_{\mathbf{n}_1} d\Omega_{\mathbf{n}_2} \quad ;$$

$$n_{1x} = \sin \theta_1 \cos \varphi_1; \quad n_{1y} = \sin \theta_1 \sin \varphi_1; \quad n_{1z} = \cos \theta_1; \\ n_{2x} = \sin \theta_2 \cos \varphi_2; \quad n_{2y} = \sin \theta_2 \sin \varphi_2; \quad n_{2z} = \cos \theta_2,$$

where θ_1 and φ_1 , θ_2 and φ_2 are the polar and azimuthal angles of emission of protons in the rest frames of the first and second Λ particle, respectively – with respect to the unified system of coordinate axes of c.m. frame of pair;

$$d\Omega_{\mathbf{n}_1} = \sin \theta_1 d\theta_1 d\varphi_1 \quad \text{and} \quad d\Omega_{\mathbf{n}_2} = \sin \theta_2 d\theta_2 d\varphi_2$$

are the elements of solid angles of proton emission .

- The angular correlation, integrated over all angles except the angle θ between the vectors \mathbf{n}_1 and \mathbf{n}_2 and described by the formula :

$$dw(\cos \theta) = \frac{1}{2} \left(1 + \frac{1}{3} \alpha_{\Lambda}^2 T \cos \theta \right) \sin \theta d\theta ,$$

is determined only by the “trace” of the correlation tensor $T = T_{11} + T_{22} + T_{33} = W_t - 3W_s$, and it does not depend on the polarization vectors (single-particle states may be unpolarized).

So, finally we have :

$$dw(\cos \theta) = \frac{1}{2} \left(1 - \alpha_{\Lambda}^2 \left(W_s - \frac{W_t}{3} \right) \cos \theta \right) \sin \theta d\theta ,$$

W_s and W_t are relative fractions of the singlet state and triplet states, respectively .

3. Correlations at the joint registration of the decays

$$\Lambda \rightarrow p + \pi^- \text{ and } \bar{\Lambda} \rightarrow \bar{p} + \pi^+$$

- Due to CP invariance, the coefficients of P -odd angular asymmetry for the decays $\Lambda \rightarrow p + \pi^-$ and $\bar{\Lambda} \rightarrow \bar{p} + \pi^+$ have equal absolute values and opposite signs: $\alpha_{\bar{\Lambda}} = -\alpha_{\Lambda} = -0.642$. The double angular distribution for this case is as follows :

$$\frac{d^2w(\mathbf{n}_1, \mathbf{n}_2)}{d\Omega_{\mathbf{n}_1} d\Omega_{\mathbf{n}_2}} = \frac{1}{16\pi^2} \left[1 + \alpha_{\Lambda} \mathbf{P}_{\Lambda} \cdot \mathbf{n}_1 - \alpha_{\Lambda} \mathbf{P}_{\bar{\Lambda}} \cdot \mathbf{n}_2 - \alpha_{\Lambda}^2 \sum_{i=1}^3 \sum_{k=1}^3 T_{ik} n_{1i} n_{2k} \right]$$

(here $-\alpha_{\Lambda} = +\alpha_{\bar{\Lambda}}$ and $-\alpha_{\Lambda}^2 = +\alpha_{\Lambda} \alpha_{\bar{\Lambda}}$) .

- Thus, the angular correlation between the proton and antiproton momenta in the rest frames of the Λ and $\bar{\Lambda}$ particles is described by the expression :

$$dw(\cos \theta) = \frac{1}{2} \left(1 - \frac{1}{3} \alpha_{\Lambda}^2 T \cos \theta \right) \sin \theta d\theta = \frac{1}{2} \left(1 + \alpha_{\Lambda}^2 \left(W_s - \frac{W_t}{3} \right) \cos \theta \right) \sin \theta d\theta ,$$

- where θ is the angle between the proton and antiproton momenta .

4. Spin correlations at the generation of $\Lambda\bar{\Lambda}$ pairs in multiple processes

- Further we will use the model of one-particle sources, which is the most adequate one in the case of collisions of relativistic ions .
- Spin and angular correlations at the decays of two Λ particles, being identical particles, with taking into account Fermi statistics and final-state interaction , were considered previously .
- In the present report we are interested in spin correlations in the decays of $\Lambda\bar{\Lambda}$ pairs . In the framework of the model of independent one-particle sources, spin correlations in the $\Lambda\bar{\Lambda}$ system arise only on account of the difference between the interaction in the final triplet state ($S = 1$) and the interaction in the final singlet state. At small relative momenta, the s -wave interaction plays the dominant role as before, but, contrary to the case of identical particles ($\Lambda\Lambda$) , in the case of non-identical particles ($\Lambda\bar{\Lambda}$) the total spin may take both the values $S = 1$ and $S = 0$ at the orbital momentum $L = 0$. In doing so, the interference effect, connected with quantum statistics, is absent .

If the sources emit unpolarized particles, then, in the case under consideration, the correlation function describing momentum-energy correlations has the following structure

(in the c.m. frame of the $\Lambda\bar{\Lambda}$ pair) :

$$R(\mathbf{k}, \mathbf{v}) = 1 + \frac{3}{4} B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k}, \mathbf{v}) + \frac{1}{4} B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k}, \mathbf{v}) .$$

Here $B_t^{(\Lambda\bar{\Lambda})}$ and $B_s^{(\Lambda\bar{\Lambda})}$ -- contributions of interaction of Λ and $\bar{\Lambda}$ in the final triplet (singlet) state, which are expressed through the amplitudes of scattering of non-identical particles Λ and $\bar{\Lambda}$, and depend on space-time dimensions of the generation region of

$\Lambda\bar{\Lambda}$ -pair.

R.Lednicky, V.L.Lyuboshitz (1982) // Yad. Fiz. 35, 1316

\mathbf{k} – momentum of $\bar{\Lambda}$ in the c.m. frame of the pair, \mathbf{v} – velocity of the pair .

The spin density matrix of the $\Lambda\bar{\Lambda}$ pair is given by the formula :

$$\hat{\rho}^{(\Lambda\bar{\Lambda})} = \hat{I}^{(1)} \otimes \hat{I}^{(2)} + \frac{B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k}, \mathbf{v}) - B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k}, \mathbf{v})}{4 R(\mathbf{k}, \mathbf{v})} \hat{\sigma}^{(1)} \otimes \hat{\sigma}^{(2)} ,$$

and the components of the correlation tensor are as follows:

$$T_{ik} = \frac{B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k}, \mathbf{v}) - B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k}, \mathbf{v})}{4 + 3B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k}, \mathbf{v}) + B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k}, \mathbf{v})} \delta_{ik}$$

- At sufficiently large values of k , one should expect that :

$$B_s^{(\Lambda\bar{\Lambda})}(\mathbf{k}, \mathbf{v}) = 0, \quad B_t^{(\Lambda\bar{\Lambda})}(\mathbf{k}, \mathbf{v}) = 0 \quad .$$

In this case the angular correlations in the decays $\Lambda \rightarrow p + \pi^-$ and $\bar{\Lambda} \rightarrow p + \pi^+$, connected with the final-state interaction, are absent :

$$T_{ik} = 0, \quad T = 0 .$$

5. Angular correlations in the decays $\Lambda \rightarrow p + \pi^-$ and $\bar{\Lambda} \rightarrow \bar{p} + \pi^+$ and the “mixed phase”

- Thus, at sufficiently large relative momenta (for $k \gg m_\pi$) one should expect that the angular correlations in the decays $\Lambda \rightarrow p + \pi^-$ and $\bar{\Lambda} \rightarrow \bar{p} + \pi^+$, in the framework of the model of one-particle sources are absent. In this case two-particle (and multiparticle) sources may be, in principle, the cause of the spin correlations. Such a situation may arise, if at the considered energy the dynamical trajectory of the system passes through the so-called “mixed phase”; then the two-particle sources, consisting of the free quark and antiquark, start playing a noticeable role. For example, the process $s\bar{s} \rightarrow \Lambda\bar{\Lambda}$ may be discussed. The CP parity of the fermion-antifermion pair is $CP = (-1)^{S+1}$.
- In the case of one-gluon exchange, $CP = 1$, and then $S = 1$, i.e. the $\Lambda\bar{\Lambda}$ pair is generated in the triplet state; in doing so, the “trace” of the correlation tensor $T = 1$.

- Even if the frames of one-gluon exchange are overstepped, the quarks s and \bar{s} , being ultrarelativistic, interact in the triplet state ($S = 1$). In so doing, the primary CP parity $CP = 1$, and, due to the CP parity conservation, the $\Lambda\bar{\Lambda}$ pair is also produced in the triplet state. Let us denote the contribution of two-quark sources by x . Then at large relative momenta $T = x > 0$.
- Apart from the two-quark sources, there are also two-gluon sources being able to play a comparable role. Analogously with the annihilation process $\gamma\gamma \rightarrow e^+e^-$, in this case the “trace” of the correlation tensor is described by the formula (here the process $gg \rightarrow \Lambda\bar{\Lambda}$ is implied) :

$$T = 1 - \frac{4(1 - \beta^2)}{1 + 2\beta^2 \sin^2 \theta - \beta^4 - \beta^4 \sin^4 \theta} ,$$

where β is the velocity of Λ (and $\bar{\Lambda}$) in the c.m. frame of the $\Lambda\bar{\Lambda}$ pair, θ is the angle between the momenta of one of the gluons and Λ in the c.m. frame . At small β ($\beta \ll 1$) the $\Lambda\bar{\Lambda}$ pair is produced in the singlet state (total spin $S = 0$, $T = -3$), whereas at $\beta \approx 1$ – in the triplet state ($S = 1$, $T = 1$) . Let us remark that at ultrarelativistic velocities β (i.e. at extremely large relative momenta of Λ and $\bar{\Lambda}$) both the two-quark and two-gluon mechanisms lead to the triplet state of the $\Lambda\bar{\Lambda}$ pair ($T = 1$) .

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6. Summary

So, it is surely advisable to investigate the spin correlations of $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$ pairs generated in relativistic heavy-ion collisions.

In doing so, the best way of studying the spin correlations is the method of angular correlations – method of moments.

In the general case, the appearance of angular correlations in the decays $\Lambda \rightarrow p + \pi^-$ and $\bar{\Lambda} \rightarrow \bar{p} + \pi^+$ with the nonzero values of the “trace” of the correlation tensor T at large relative momenta of the Λ and $\bar{\Lambda}$ particles may testify to the passage of the system through the “mixed phase”.



Thank you !