
# On the spin correlations of muons and tau leptons produced in the high-energy annihilation processes $e^+e^- \to \mu^+\mu^-, \tau^+\tau^-$

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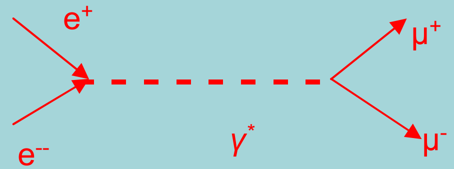
In the first approximation over the constant $e^2/\hbar c$ , the process of conversion of the electron-positron pair into the muon one (or $\tau^+\tau^-$ ) is described by the one-photon diagram :




The virtual photon with time-like momentum transfers angular momentum J=1 and negative parity. The internal parities of $\mu^+$ and $\mu^-$ are opposite: the ( $\mu^+\mu^-$ ) pair is generated in triplet states (total spin *S*=1 ), with total angular momentum J=1 and negative space parity .

Helicity amplitudes:

$$f_{\Lambda'\Lambda}(\theta,\phi) = R_{\Lambda'\Lambda}(E)\, d^{(1)}_{\Lambda'\Lambda}(\theta)\, \exp\,(i\Lambda\phi)$$

*d*- functions

θ and φ – polar and azimuthal angles of flight direction of $\mu^+$ with respect to positron momentum in the c.m. frame of the reaction; Λ' – difference of helicities of $\mu^+$ and $\mu^-$ , Λ - difference of helicities of $e^+$ and $e^-$ ; *E* – total energy in the c.m. frame .

Factorization: $$R_{\Lambda'\Lambda}(E) = r^{(\mu)}_{\Lambda'}(E)\, r^{(e)}_{\Lambda}(E)$$

Parity conservation: $$r^{(\mu)}_{+1}(E) = r^{(\mu)}_{-1}(E), \quad r^{(e)}_{+1}(E) = r^{(e)}_{-1}(E)$$

As follows from the structure of electromagnetic current for pairs $(e^{+}\, e^{-})$ and $(\mu^{+}\, \mu^{-})$ :

$$r^{(\mu)}_{0}(E) = \frac{m_\mu}{E}\, r^{(\mu)}_{1}(E) = \sqrt{1-\beta^2_\mu}\, r^{(\mu)}_{1}(E), \quad r^{(e)}_{0}(E) = \frac{m_e}{E}\, r^{(e)}_{1}(E)$$

$m_\mu$ and $m_e$ - muon and electron masses , $\beta_\mu$ - muon velocity .

Since always $E \geq m_\mu >> m_e$ , the contribution of states of electron and positron with antiparallel spins ( equal helicities ) is negligibly small : $r_0^{(e)}(E) \approx 0$, $R_{\Lambda' 0}(E) \approx 0$.

At the annihilation of electron and positron being totally polarized in the direction parallel to positron momentum in the reaction c.m. frame, the $(\mu^{+}\, \mu^{-})$ system is generated in the triplet state :

$$|\Psi\rangle^{(+1)} = \frac{\sqrt{2}}{\sqrt{2-\beta^2_\mu \sin^2\theta}} \left( \frac{1+\cos\theta}{2}\,|+1\rangle - \sqrt{1-\beta^2_\mu}\, \frac{\sin\theta}{\sqrt{2}}\,|0\rangle + \frac{1-\cos\theta}{2}\,|-1\rangle \right)$$

$$|+1\rangle = |+1/2\rangle^{(\mu^+)} \otimes |+1/2\rangle^{(\mu^-)}, \quad |-1\rangle = |-1/2\rangle^{(\mu^+)} \otimes |-1/2\rangle^{(\mu^-)},$$

$$|0\rangle = \frac{1}{\sqrt{2}} \left( |+1/2\rangle^{(\mu^+)} \otimes |-1/2\rangle^{(\mu^-)} + |-1/2\rangle^{(\mu^+)} \otimes |+1/2\rangle^{(\mu^-)} \right)$$

- states with projections of total spin of $(\mu^{+}\, \mu^{-})$ pair onto the $\mu^{+}$ momentum direction in the c.m.s. +1, -1 and 0

- If the electron and positron are totally polarized in the direction being antiparallel to the positron momentum in the c.m. frame:

$$|\Psi\rangle^{(-1)} = \frac{\sqrt{2}}{\sqrt{2-\beta_\mu^2\sin^2\theta}}\left(\frac{1-\cos\theta}{2}|+1\rangle + \sqrt{1-\beta_\mu^2}\,\frac{\sin\theta}{\sqrt{2}}|0\rangle + \frac{1+\cos\theta}{2}|-1\rangle\right)$$

- When the primary electron and positron are not polarized, the nonfactorizable states $|\psi\rangle^{(+1)}$ and $|\psi\rangle^{(-1)}$ are generated with equal probabilities .

- Spin states of two particles with spin ½ are characterized by the polarization vectors $\vec{P}_1 = \langle\hat{\vec{\sigma}}^{(1)}\rangle$ , $\vec{P}_2 = \langle\hat{\vec{\sigma}}^{(2)}\rangle$ and correlation tensor $T_{ik} = \langle\hat{\sigma}_i^{(1)} \otimes \hat{\sigma}_k^{(2)}\rangle$ ;

$\hat{\vec{\sigma}} = \{\hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}_3\} \equiv \{\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z\}$ - vector Pauli operator ;

$\langle\ldots\rangle$ - sign of averaging .

- In the one-photon approximation, the generated muons are unpolarized but their spins are strongly correlated .
- Correlation tensor components at the choice of axis *z* along the relative momentum of muons in the c.m. frame and axis *y* – along the normal to the reaction plane :

$$T_{zz}^{(\mu^+\mu^-)} = \frac{2\cos^2\theta + \beta_\mu^2 \sin^2\theta}{2 - \beta_\mu^2 \sin^2\theta},$$

$$T_{xx}^{(\mu^+\mu^-)} = \frac{(2-\beta_\mu^2)\sin^2\theta}{2 - \beta_\mu^2 \sin^2\theta}, \quad T_{yy}^{(\mu^+\mu^-)} = -\frac{\beta_\mu^2 \sin^2\theta}{2 - \beta_\mu^2 \sin^2\theta},$$

$$T_{xz}^{(\mu^+\mu^-)} = T_{zx}^{(\mu^+\mu^-)} = -\frac{(1-\beta_\mu^2)^{1/2}\sin 2\theta}{2 - \beta_\mu^2 \sin^2\theta},$$

$$T_{xy}^{(\mu^+\mu^-)} = T_{yx}^{(\mu^+\mu^-)} = T_{yz}^{(\mu^+\mu^-)} = T_{yz}^{(\mu^+\mu^-)} = 0.$$

The trace of correlation tensor : $$T^{(\mu+\mu^-)} = T_{xx}^{(\mu+\mu^-)} + T_{yy}^{(\mu^+\mu^-)} + T_{zz}^{(\mu^+\mu^-)} = 1,$$ ,

just as it should hold for any triplet states .

The trace of correlation tensor $T$ determines the angular correlation between the flight directions ( $\vec{n}_1$ and $\vec{n}_2$ ) of products of decay of two unstable particles in the case when space parity is not conserved .

R. Lednicky, V. L. Lyuboshitz (2001) // Phys.Lett., B508, 146
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- Angular distributions at parity nonconservation :

$$dW_1 = \frac{1}{4\pi}(1+\alpha_1 \vec{P}_1 \vec{n}_1) d\Omega_1, \qquad dW_2 = \frac{1}{4\pi}(1+\alpha_2 \vec{P}_2 \vec{n}_2) d\Omega_2,$$

$\vec{P}_1, \vec{P}_2$ – polarization vectors, $\alpha_1$ and $\alpha_2$ – coefficients of $P$ – odd asymmetry ;
decay is the spin analyzer for an unstable particle .

Double angular distribution :

$$d^2W(\vec{n}_1, \vec{n}_2) = \frac{1}{16\pi^2}\left(1 + \alpha_1 \vec{P}_1 \vec{n}_1 + \alpha_2 \vec{P}_2 \vec{n}_2 + \sum_{i=1}^{3} T_{ik} n_{1i} n_{2k}\right) d\Omega_1 d\Omega_2$$

correlation tensor components

The unit vectors $\vec{n}_1$ , $\vec{n}_2$ are defined in the rest frames of the unstable particles 1 and 2 , in the coordinate axes of the c.m. frame of the particle pair .

- Angular correlations between the directions $\vec{n}_1$ , $\vec{n}_2$ at the decays of two unstable particles are integrated over all angles except the angle β between $\vec{n}_1$ and $\vec{n}_2$ ;

$$dN(\beta) = \frac{1}{2}\left(1 + \frac{\alpha_1\alpha_2}{3} T\cos\beta\right)\sin\beta\, d\beta$$

irrespective of the polarization vectors $\vec{P}_1$ and $\vec{P}_2$ , which may be equal to zero $(\cos\beta = \vec{n}_1\vec{n}_2)$.

$$T = \sum_{i=1}^{3} T_{ii} = \rho_t - 3\rho_s$$ - trace of the correlation tensor ;

$\rho_s$ and $\rho_t$ - relative fractions of the singlet and triplet states, respectively .

Coefficient of *P* – odd asymmetry of electron emission at the decay $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ , averaged over the electron energy spectrum: α = -⅓ .

At the decay $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$ : α = +⅓ .

- Angular correlations between the flight directions for the electron and positron at the decay of muon pair ( $\mu^+ \mu^-$ ) :

$$dN(\beta) = \frac{1}{2}\left(1 - \frac{1}{27} T\cos\beta\right)\sin\beta\, d\beta$$

In the process $e^+e^- \rightarrow \mu^+\mu^-$ the ( $\mu^+ \mu^-$ ) system is produced in the triplet state, thus, *T* =1 : $dN(\beta) = \frac{1}{2}\left(1 - \frac{1}{27}\cos\beta\right)\sin\beta\, d\beta$ .

- As it was shown previously, R. Lednicky, V. L. Lyuboshitz (2001) // Phys.Lett., B508, 146

  in the case of incoherent mixtures of factorizable states of two particles with spin ½ the modulus of sum of any two ( and three ) diagonal components of the correlation tensor cannot exceed unity :

$$|T| = |T_{xx} + T_{yy} + T_{zz}| \leq 1 \quad |T_{xx} + T_{yy}| \leq 1 \quad |T_{xx} + T_{zz}| \leq 1 \quad |T_{yy} + T_{zz}| \leq 1$$

In the case of non-factorizable coherent superpositions of two-particle states , the "incoherence" inequalities may be violated .

For the singlet state all the inequalities are violated :

$$T_{xx} + T_{yy} = T_{xx} + T_{zz} = T_{yy} + T_{zz} = -2 , \quad T = -3 \quad .$$

In the case of the non-factorizable triplet state with zero projection of total spin onto the axis $z$ , one of the restrictions is not satisfied : instead of the inequality $|T_{xx} + T_{yy}| <1$ , the equality $T_{xx} + T_{yy} = 2$ (>1) holds .

In the process of ( $e^+e^-$ ) annihilation , the ( $\mu^+\mu^-$ ) system is generated in the non-factorizable triplet states : $|\psi\rangle^{(+1)}$ and $|\psi\rangle^{(-1)}$.

- One of the “incoherence” inequalities is always violated in the process $e^+ e^- \to \mu^+ \mu^-$ at $\theta \neq 0$ :

$$T_{xx}^{(\mu^+\mu^-)} + T_{zz}^{(\mu^+\mu^-)} = \frac{2}{2 - \beta_\mu^2 \sin^2\theta} > 1.$$

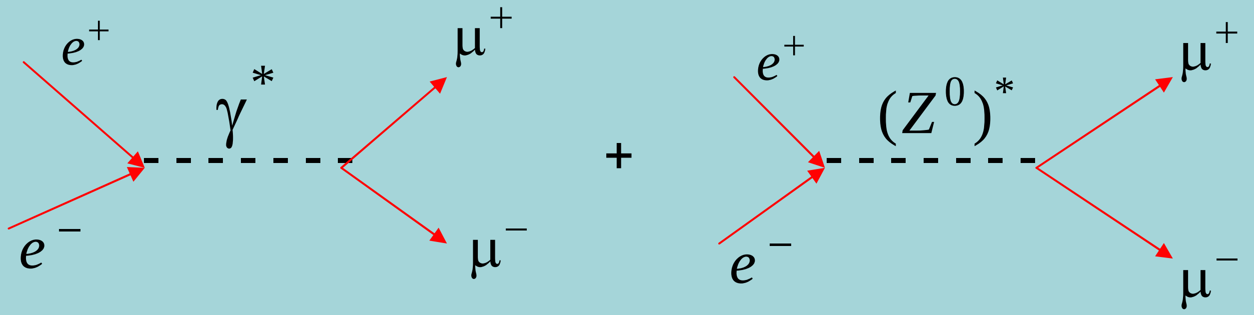
- Analogous consideration -- for the process $e^+e^- \to \tau^+\tau^-,$ ( $m_\mu \to m_\tau, \quad \beta_\mu \to \beta_\tau$ ) .

- At very high energies ( $\beta_\mu \to 1$, $\beta_\tau \to 1$ ) , the nonzero components of the correlation tensor are as follows :

$$T_{xx}^{(\mu^+\mu^-)} = -T_{yy}^{(\mu^+\mu^-)} = \frac{\sin^2\theta}{1+\cos^2\theta}, \quad T_{zz}^{(\mu^+\mu^-)} = 1. \qquad T_{xx}^{(\mu^+\mu^-)} + T_{zz}^{(\mu^+\mu^-)} = \frac{2}{1+\cos^2\theta} > 1$$

- At high energies the annihilation processes

$e^+e^- \rightarrow \mu^+\mu^-$, $e^+e^- \rightarrow \tau^+\tau^-$ are conditioned not only by electromagnetic interaction but also by the weak interaction of neutral currents through the virtual $Z^0$ boson :




- Interference of the amplitudes of the purely electromagnetic and weak interaction → leads to the charge asymmetry in lepton emission and to the space parity violation.

$\mu^+\mu^-$, $\tau^+\tau^-$ $\rightarrow$ generated in the triplet states : ${}^3S_1$, ${}^3D_1$ and ${}^3P_1$

due to weak interaction

($J$ = 1, positive $CP$ parity)

It follows from the structure of "left" and "right" components of neutral currents that the nonzero helicity amplitudes of the processes $e^+e^- \to \mu^+\mu^-,\ e^+e^- \to \tau^+\tau^-$ in the c.m. frame take the form :

$$R_{11}(E) = \frac{e^2}{2E}\left[1 + x\left(\xi - \frac{1}{2}\right)^2\right]; \qquad R_{-1-1}(E) = \frac{e^2}{2E}\left[1 + x\,\xi^2\right].$$

$$R_{-11}(E) = R_{1-1}(E) = \frac{e^2}{2E}\left[1 + x\,\xi\left(\xi - \frac{1}{2}\right)\right];$$

( $R_{0\lambda}$ , $R_{\lambda'0}$ → turn practically to zero at high energies ) .

Here: $\xi = \sin^2\theta_W$ , where $\theta_W$ is the Weinberg angle ( angle of gauge boson mixing ) ; parameter $x$ → determines the relative contribution of weak interaction :

$$x = \frac{1}{\sin^2\theta_W \cos^2\theta_W}\,\frac{s}{s - \left(M_{Z^0} - i\frac{\Gamma_{Z^0}}{2}\right)^2} \qquad \left(s = (2E)^2\right)$$

According to the standard model :

$$\frac{1}{\sin^2\theta_W \cos^2\theta_W} = \frac{1}{\xi(1-\xi)} \approx 6 = \frac{\sqrt{2}\,G_F\,M_{Z^0}^2}{\pi\,\alpha}$$

$G_F = 1.166\cdot 10^{-5}\ GeV^{-2}$ → universal Fermi constant of weak interaction, $\alpha = 1/137$

Finally, performing the further analysis, we obtain, in particular, that :

1) Due to the weak interaction through the $Z^0$ boson, the final leptons, generated at the annihilation of the unpolarized electron and positron, acquire the longitudinal polarization ( whereas, if the weak interaction contribution is neglected, the final leptons are correlated but unpolarized ) .

At the energies below and above the resonance energy, the average helicities of the final leptons have different signs :

$$E < \frac{M_{Z^0}}{2} \;\rightarrow\; x < 0,\; \overline{\lambda}_+ < 0,\; \overline{\lambda}_- > 0 \;;$$

$$E > \frac{M_{Z^0}}{2} \;\rightarrow\; x > 0,\; \overline{\lambda}_+ > 0,\; \overline{\lambda}_- < 0 \;.$$

2) Structure of the correlation tensor of the final leptons is, on the whole, similar to that for the case of purely electromagnetic annihilation at high energies .

In doing so, $T_{zz} = 1$ as before ;

the expression for $T_{xx}$ changes , but, as before, $T_{xx} = -T_{yy}$ :

$$T_{xx} = -T_{yy} = \sin^2\theta \frac{[\,1 + x\,(\,\xi^2 - \frac{1}{2}\xi + \frac{1}{8}\,)\,]\,[\,1 + x\,\xi\,(\xi - \frac{1}{2}\,)\,]}{a_+(E)(1+\cos^2\theta) + 2\,a_-(E)\cos\theta}$$

where

$$a_+(E) = 1 + \frac{1}{2}x\left(\frac{1}{2} - 2\xi\right)^2 + \frac{1}{4}x^2\left[\left(\frac{1}{2} - \xi\right)^2 + \xi^2\right]^2 \quad ,$$

$$a_-(E) = \frac{1}{8}x + \frac{1}{4}x^2\left(\frac{1}{4} - \xi\right)^2 \quad .$$

Again, one of the incoherence inequalities for the correlation tensor components is violated : $T_{xx} + T_{zz} > 1$ .

# Main conclusions

- On the basis of the technique of helicity amplitudes, the process $e^+ e^{-} \rightarrow \mu^+ \mu^{-}$ is theoretically investigated in the one-photon approximation . The structure of triplet states of the $(\mu^+ \mu^{-})$ system is found .
- It is shown that, if the primary electron and positron are not polarized, the final muons $\mu^+$ и $\mu^{-}$ are not polarized as well but their spins are strongly correlated . Explicit expressions for the components of correlation tensor of the final $(\mu^+ \mu^{-})$ system are derived .
- The formula for the angular correlation at the decays of muons $\mu^+$ and $\mu^{-}$, generated in the annihilation process $e^+ e^{-} \rightarrow \mu^+ \mu^{-}$, into the channels $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$ and $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ , is obtained .
- It is established that in the process $e^+ e^- \rightarrow \mu^+ \mu^{-}$ one of the “incoherence” inequalities for the correlation tensor components is always violated .
- The obtained results remain qualitatively valid with the account of weak interaction of neutral currents through the exchange by $Z^0$ boson .

# Thank you !