Forces inside hadrons: pressure, shear forces, mechanical radius and all that

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- Interaction of the nucleon with gravity, EMT form factors
- Pressure and shear forces distribution in the nucleon
- Normal and tangential forces inside nucleon. Stability conditions.
- Mechanical radius and surface tension - shaping hadrons
- First experimental results on gravitational form factors
- Forces between quark and gluon subsystems inside the nucleon
- Conclusion and outlook.

See recent review: P. Schweitzer, MVP 1805.06596

Interaction of the nucleon with gravity

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\; R + \int d^4x\sqrt{-g}\; T_{\mu\nu}(x)g^{\mu\nu}(x)$$

Let

$$g^{\mu\nu}(x) = \eta^{\mu\nu} + \delta g^{\mu\nu}(\vec{r}) \qquad\qquad \lambda_{\rm grav} \gg \frac{1}{M_N}$$

Than the response of the nucleon to the static change of the space-time metric is characterised by static EMT (Breit frame):

$$T_{\mu\nu}(\vec{r}) = \int \frac{d^3\Delta}{(2\pi)^3 2E}\; e^{-i\vec{r}\vec{\Delta}}\langle p'|T_{\mu\nu}(0)|p\rangle,$$

MVP '2003

$$T_q^{\mu\nu} = \frac{1}{4}\overline{\psi}_q\Big(-i\overleftarrow{\mathcal{D}}^{\mu}\gamma^{\nu} - i\overleftarrow{\mathcal{D}}^{\nu}\gamma^{\mu} + i\overrightarrow{\mathcal{D}}^{\mu}\gamma^{\nu} + i\overrightarrow{\mathcal{D}}^{\nu}\gamma^{\mu}\Big)\psi_q - g^{\mu\nu}\overline{\psi}_q\Big(-\frac{i}{2}\overleftarrow{\not{\mathcal{D}}} + \frac{i}{2}\overrightarrow{\not{\mathcal{D}}} - m_q\Big)\psi_q,$$

$$T_g^{\mu\nu} = F^{a,\mu\eta}\,F^{a,}{}_{\eta}{}^{\nu} + \frac{1}{4}\,g^{\mu\nu}F^{a,\kappa\eta}\,F^{a,}{}_{\kappa\eta}.$$

$$\langle p'|T^a_{\mu\nu}(0)|p\rangle = \bar{u}'\Big[A^a(t)\,\frac{P_\mu P_\nu}{M_N} + J^a(t)\,\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2M_N} + D^a(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M_N} + M_N\,\bar{c}^a(t)g_{\mu\nu}\Big]u$$

Kobazarev, Okun '1963 , Pagels '1966

$$P = (p'+p)/2,\; \Delta = p'-p.$$

Interaction of the nucleon with gravity

$$\langle p'|T^a_{\mu\nu}(0)|p\rangle = \bar{u}'\left[A^a(t)\,\frac{P_\mu P_\nu}{M_N} + J^a(t)\,\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2M_N} + D^a(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M_N} + M_N\,\bar{c}^a(t)g_{\mu\nu}\right]u$$

δg^{00} — Mass — $\sum_a A^a(0) = 1$

δg^{0i} — Spin — $\sum_a J^a(0) = \frac{1}{2}$

δg^{ij} — $\mathrm{deformation\ of\ space} =$ $\mathrm{elastic\ properties\ of}\ N$

$\mathrm{non-conservation\ of\ EMT\ pieces}$ — $\sum_a \bar{c}^a(t) = 0$

Elasticity stress tensor (Landau & Lifshitz vol. 7)

Shear forces distribution

Pressure distribution

$$T^a_{ij}(\vec{r}) = \left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right)s^a(r) + \delta_{ij}\,p^a(r)$$

MVP ‘2003

$$s^a(r) = -\frac{1}{4M_N}\;r\,\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\widetilde{D}^a(r), \qquad p^a(r) = \frac{1}{6M_N}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\widetilde{D^a}(r) - M_N\int\frac{d^3\Delta}{(2\pi)^3}\;e^{-i\vec{\Delta}\vec{r}}\;\bar{c}^a(-\vec{\Delta}^2).$$

$$\widetilde{D}^a(r) = \int\frac{d^3\Delta}{(2\pi)^3}\;e^{-i\vec{\Delta}\vec{r}}\;D^a(-\vec{\Delta}^2)$$ D-term

Weiss, MVP ‘1999

The D-term

last global unknown: How do we learn about hadrons?

$|N\rangle$ = **strong** interaction particle. Use other forces to probe it!

em:	$\partial_\mu J^\mu_{\rm em} = 0$	$\langle N'\|J^\mu_{\rm em}\|N\rangle$	$\longrightarrow$	$Q,\ \mu,\ \dots$
weak:	PCAC	$\langle N'\|J^\mu_{\rm weak}\|N\rangle$	$\longrightarrow$	$g_A,\ g_p,\ \dots$
gravity:	$\partial_\mu T^{\mu\nu}_{\rm grav} = 0$	$\langle N'\|T^{\mu\nu}_{\rm grav}\|N\rangle$	$\longrightarrow$	$M,\ J,\ D,\ \dots$

global properties:

$$
\begin{aligned}
Q_{\rm prot} &= 1.602176487(40) \times 10^{-19}\,\mathrm{C}\\
\mu_{\rm prot} &= 2.792847356(23)\mu_N\\
g_A &= 1.2694(28)\\
g_p &= 8.06(0.55)\\
M &= 938.272013(23)\ \mathrm{MeV}\\
J &= \tfrac{1}{2}\\
\boldsymbol{D} &= \ ??
\end{aligned}
$$

and more:
t-dependence ... ...
parton structure, etc ...

$\hookrightarrow$ $\boldsymbol{D} =$ *"last" global unknown*

which value does it have?

what does it mean?

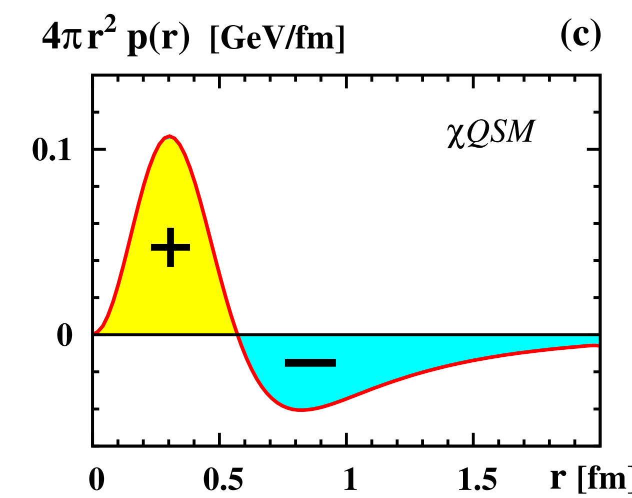
Total p(r) and s(r), normal and tangential forces, stability conditions

The force acting on the area element $d\vec{S} = dS_r\vec{e}_r + dS_\theta\vec{e}_\theta + dS_\phi\vec{e}_\phi$ $\quad (dF_i = T_{ij}dS_j)$

$$\frac{dF_r}{dS_r} = \frac{2}{3}s(r) + p(r), \quad \frac{dF_\theta}{dS_\theta} = \frac{dF_\phi}{dS_\phi} = -\frac{1}{3}s(r) + p(r).$$ Eigenvalues of stress tensor

Normal forces

Tangential forces



Goeke et al. '2007

Stability condition $\int d^3r\; p(r) = 0$ von Laue '1911

Local stability condition $\frac{dF_r}{dS_r} = \frac{2}{3}s(r) + p(r) \geq 0$

D-term $D(0) = -\frac{4m}{15}\int d^3r\; r^2\, s(r) = m\int d^3r\; r^2\, p(r) \leq 0$

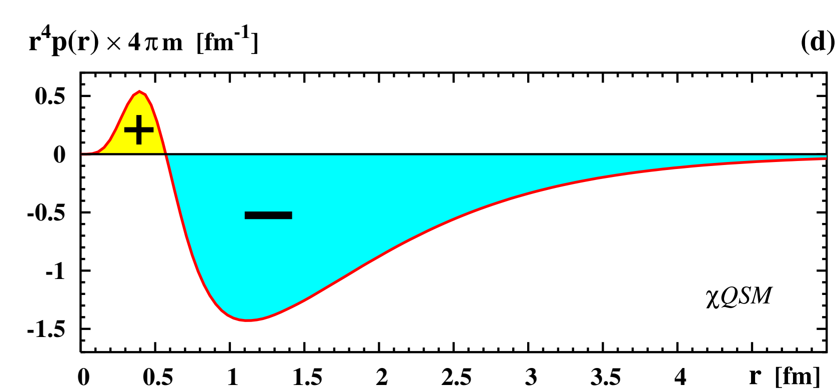
All calculations of the D-term in various approaches give negative value for it.

For some systems the D-term is fixed by general principles:

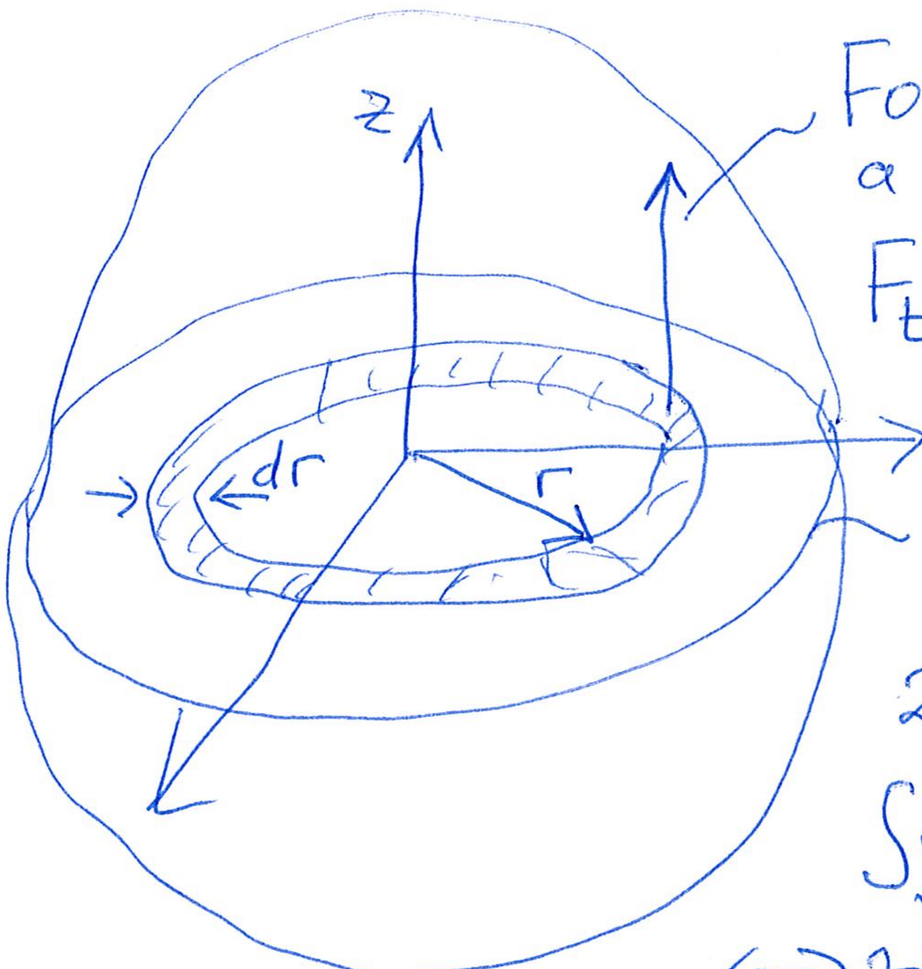
$D(0) = -1$ Goldstone bosons (pions etc.) Novikov, Shifman '1980

$D(0) = 0$ Free fermions Hudson, Schweitzer '2017

What are the nucleon D-term and the forces inside the nucleon?.



Stability conditions for low dimensional sub-systems in the nucleon



Force acting on a ring

$$F_t = \underbrace{\left(-\frac{1}{3}s(r)+p(r)\right)}_{p^{(2D)}(r)}\underbrace{2\pi r\,dr}_{dV^{(2D)}}$$

$\theta = \frac{\pi}{2}$ plane

2D von Laue condition

$$\int dV^{(2D)}\, p^{(2D)}(r) = 0 \Leftrightarrow$$

$$\Leftrightarrow 2\pi\int_0^\infty dr\cdot r\left(-\frac{1}{3}s(r)+p(r)\right)=0$$

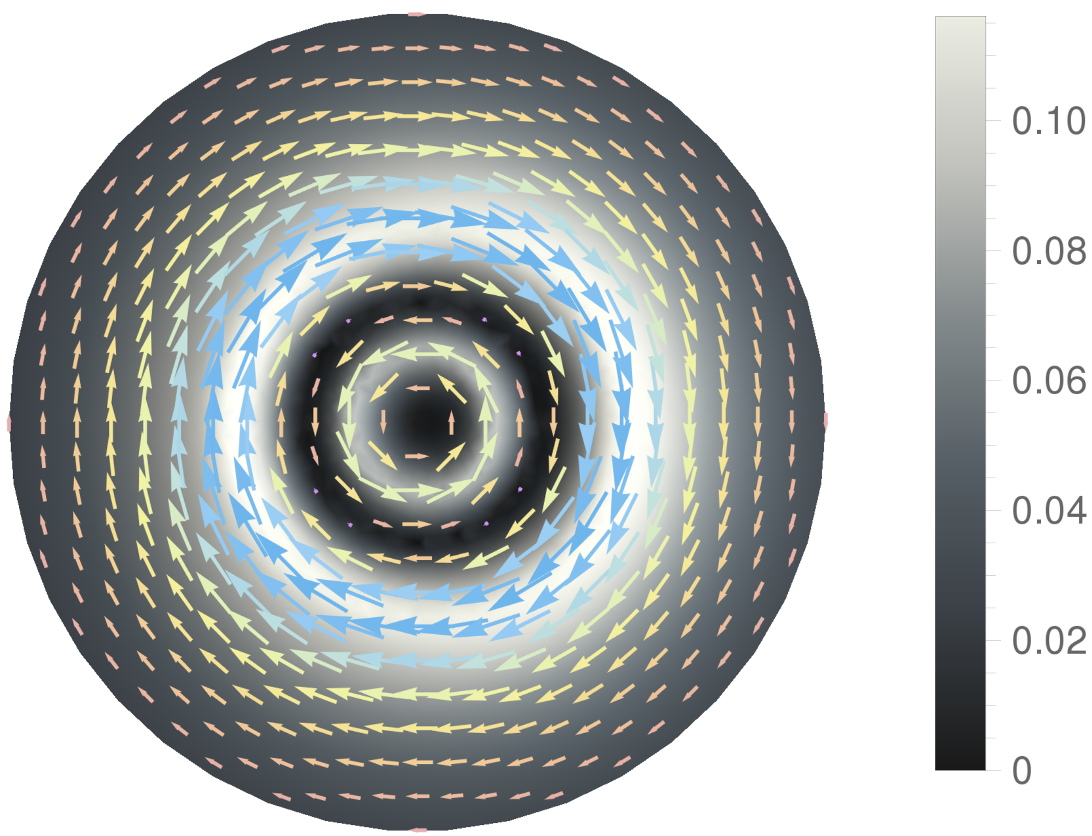
Generically, the dimensional reduction:

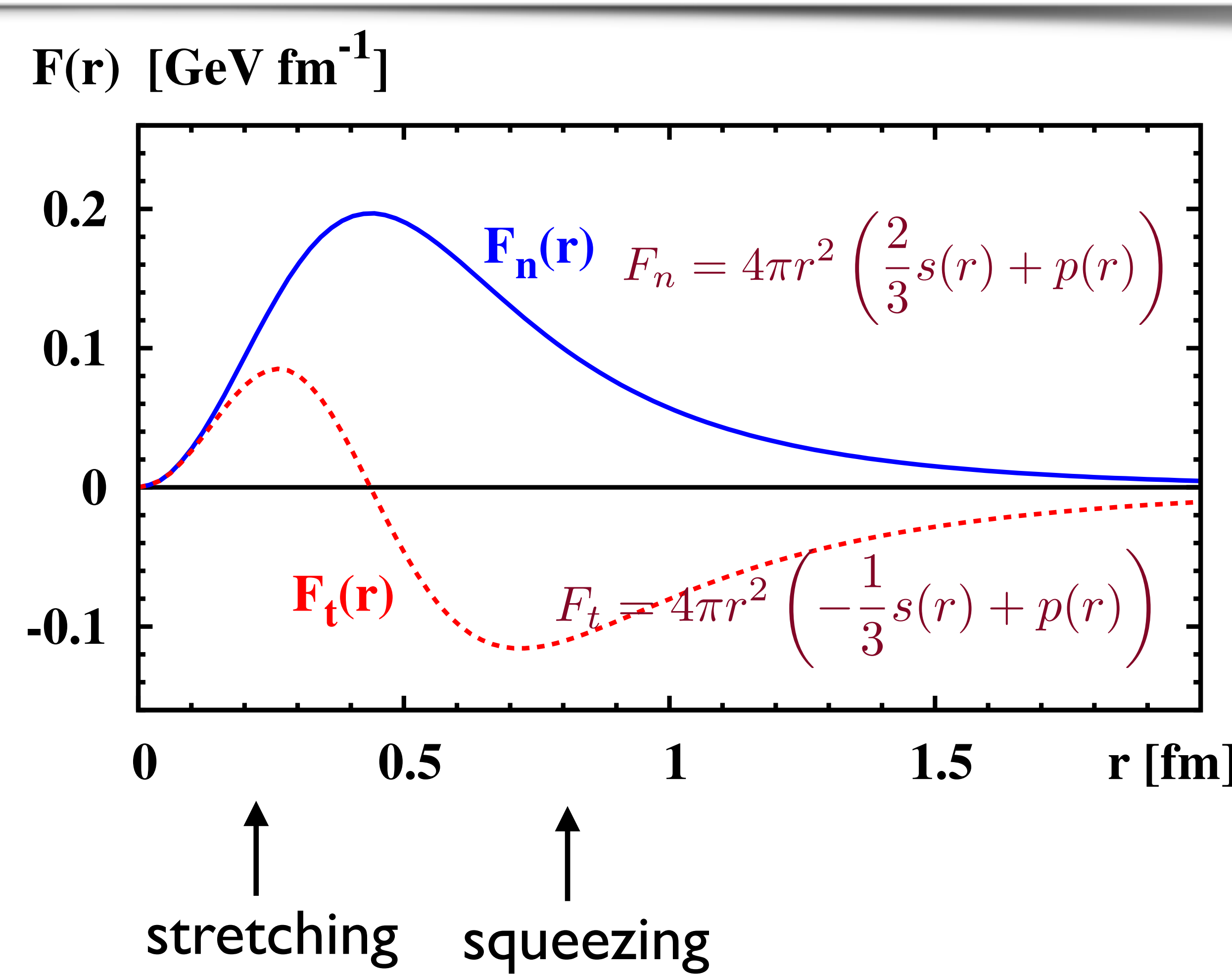
$$p^{(\mathrm{n-1})\mathrm{D}}(r) = -\frac{1}{n}\, s^{(\mathrm{nD})}(r) + p^{(\mathrm{nD})}(r), \quad s^{(\mathrm{n-1})\mathrm{D}}(r) = \frac{n-1}{n-2}\, s^{(\mathrm{nD})}(r)$$

The von Laue stability conditions for n-dimensional subsystem is: $\int dV^{(\mathrm{nD})}\, p^{(\mathrm{nD})}(r) = 0$

We can view the nucleon as a dimensional reduction of an object in higher dimensions!
AdS/QCD ?

Size of the forces in the nucleon. Comparison with confinement forces





Compare with the linear potential force of ~1 GeV/fm !

What does it imply for pictures of the confinement?

Values of D-term for the nucleon:

$-2 \geq D(0) \geq -4$ Chiral Quark Soliton model Boffi, Radici, Schweitzer '2001 Goeke et al. '2007

$D^Q(0) \approx -1.56$ at $\mu = 4\ \mathrm{GeV}^2$ Dispersion relations Pasquini, Vanderhaeghen, MVP '2014

Details in talk by Barbara Pasquini tomorrow

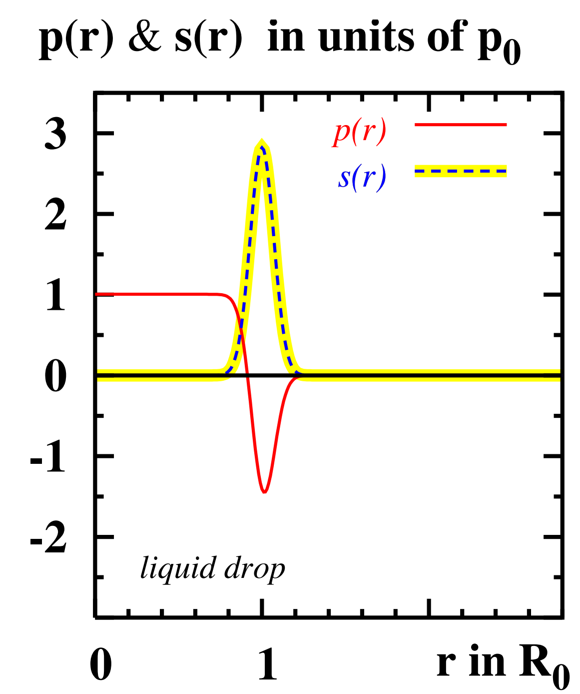
Mechanical radius and surface tension

$$\frac{dF_r}{dS_r} = \frac{2}{3}s(r) + p(r) \geq 0$$

Positive quantity - allows to define the mechanical radius

$$\langle r^2 \rangle_{\rm mech} = \frac{\int d^3r\ r^2\ \left[\frac{2}{3}s(r) + p(r)\right]}{\int d^3r\ \left[\frac{2}{3}s(r) + p(r)\right]} = \frac{6D(0)}{\int_{-\infty}^{0} dt\ D(t)}$$

Note that mech radius is NOT the slope of D(t)



For a liquid drop

$$p(r) = p_0\theta(r-R) - \frac{p_0 R}{3}\delta(r-R), \quad s(r) = \gamma\delta(r-R),$$

$p_0 = 2\gamma/R$ Relation between pressure in the drop and the surface tension Lord Kelvin '1858

Hence for a liquid drop $\displaystyle \frac{dF_r}{dS_r} = \frac{2}{3}s(r) + p(r) = p_0\theta(r-R)$

mechanical radius has the intuitive clear value

For general systems one can obtain the generalisation of the Kelvin relation $\displaystyle p(0) = \int_0^\infty dr\ \frac{2s(r)}{r}$

s(r) can be called surface tension for the system

Mechanical radius and surface tension

The surface tension energy $$\int d^3r\ s(r) = -\frac{3}{8m}\int_{-\infty}^{0} dt\ D(t)$$

This energy must be less than the total energy of the system $\int d^3r\ s(r) \leq m$ this implies

$\langle r^2\rangle_{\mathrm{mech}} \geq -9D/(4m^2)$ we checked that for stable systems (stable solitons) is always satisfied. Violated for unstable systems!

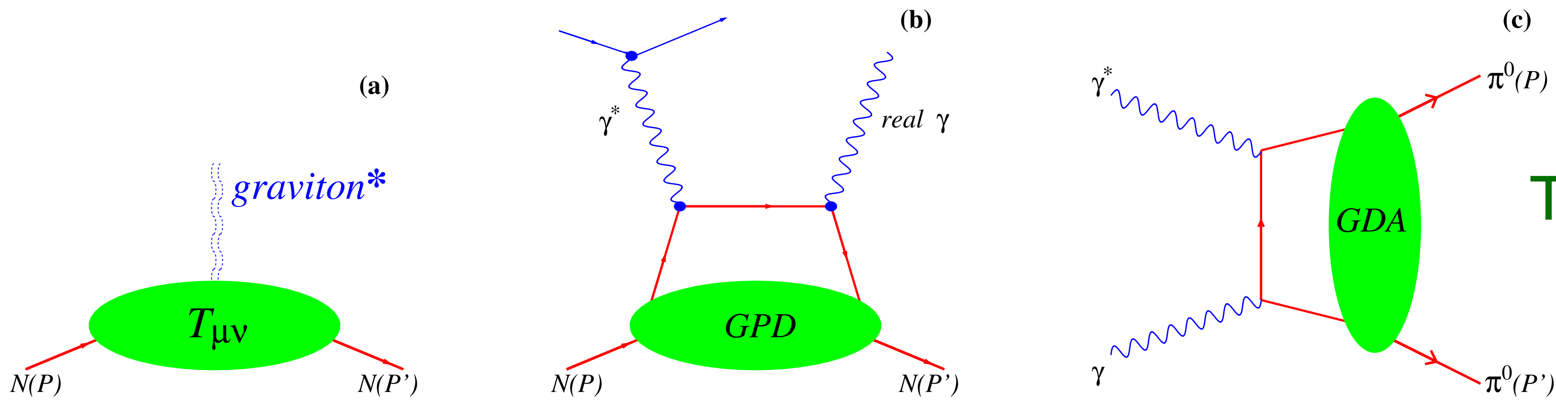
$\langle r^2\rangle_{\mathrm{mech}} \approx 0.75\ \langle r^2\rangle_{\mathrm{charge}}$ in chiral soliton picture of the nucleon

Shear forces distribution s(r) is important for forming the shape of the hadron.
For s(r)=0 the hadron corresponds to homogeneous, isotropic fluid. Hence has infinite mechanical radius. Non-zero s(r) is responsible for *hadron structure formation*!

Interestingly the pressure anisotropy (shear forces distribution) plays an essential role in astrophysics, see the review [Herrera:1997plx] on the role of pressure asymmetry for self-gravitating systems in astrophysics and cosmology.

Cedric Lorce, privat communication

Accessing p(r) and s(r) in hard exclusive processes



Talk by Oleg Teryaev on Wednesday

$$\int_{-1}^{1} \mathrm{d}x\, x\, H^a(x,\xi,t) = A^a(t) + \xi^2 D^a(t)\,, \qquad \int_{-1}^{1} \mathrm{d}x\, x\, E^a(x,\xi,t) = 2J^a(t) - A^a(t) - \xi^2 D^a(t)\,.$$

Unfortunately the Mellin moments are not observable in model independent way. However, D(t) is related to subtraction constant in dispersion relations

$$\mathcal{H}(\xi,t) = \int_{-1}^{1} dx \left(\frac{1}{\xi - x - i0} - \frac{1}{\xi + x - i0}\right) H(x,\xi,t)$$

$$\mathrm{Re}\mathcal{H}(\xi,t) = \Delta(t) + \frac{1}{\pi}\mathrm{vp}\int_0^1 d\xi'\ \mathrm{Im}\mathcal{H}(\xi',t)\left(\frac{1}{\xi-\xi'} - \frac{1}{\xi+\xi'}\right)$$

MVP '2003
Teryaev '2005
Anikin, Teryaev '2007
Diehl, Ivanov '2007

$$\Delta(t) = \frac{4}{5}\sum_q e_q^2\, D^q(t) + \sum_q e_q^2\, d_3^q(t) + \ldots$$

D(t) is more easy access than J(t). It is possible model independent extraction of D(t) in contrast to J(t)

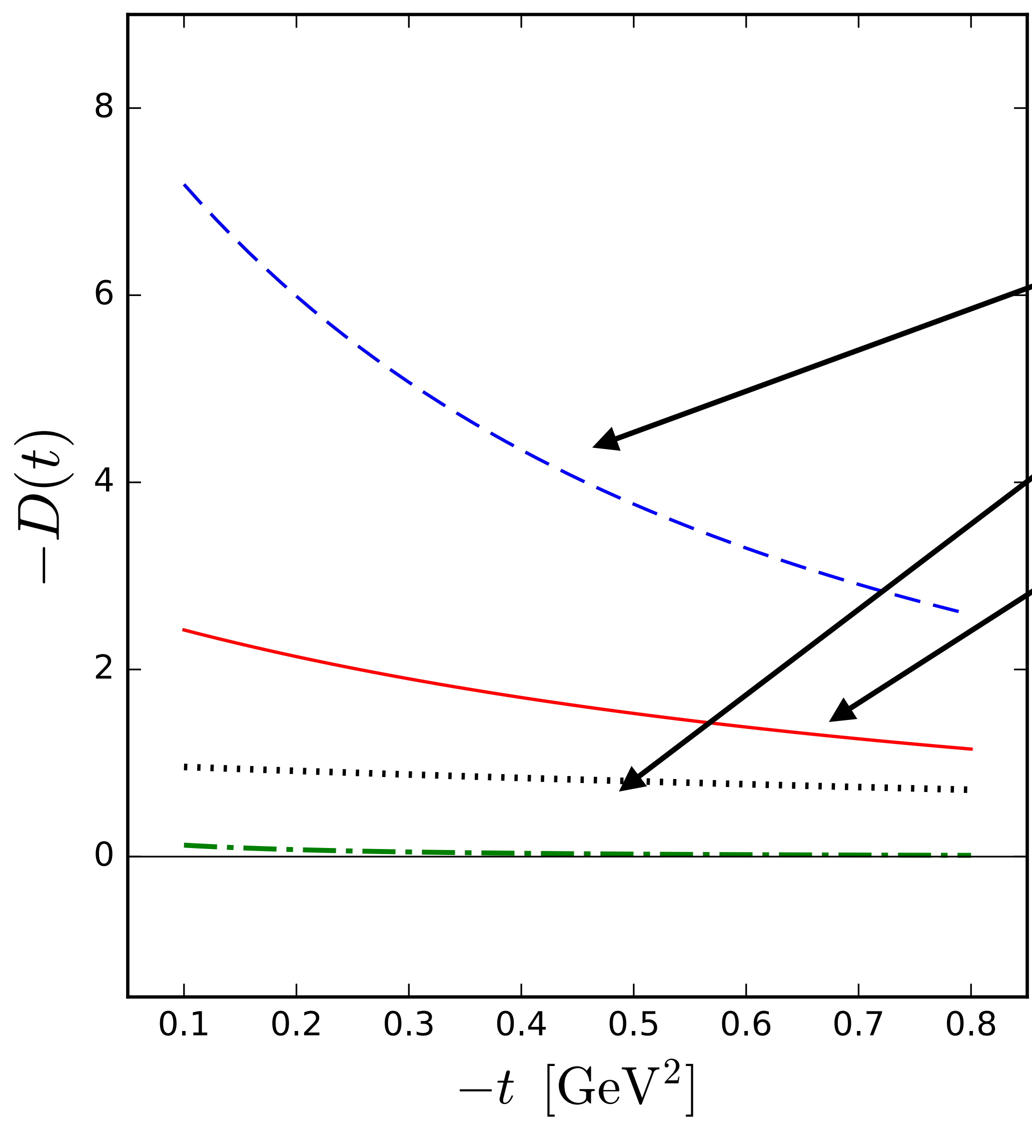
Accessing p(r) and s(r) in hard exclusive processes

Simplifying assumptions (for present state of art of the experiment):

1) d3(t), d5(t), … much smaller than D(t). It is so at large normalisation scale.
2) Flavour singlet D(t) is dominant. Justified in large Nc limit. Can be relaxed for more precise data.

Under these assumptions we obtain:

$$D^Q(t) = \frac{4}{5}\,\frac{1}{2(e_u^2+e_d^2)}\Delta(t) = \frac{18}{25}\,\Delta(t).$$



The first determination of D(t) from DVCS
Kumericki, Mueller Nucl. Phys. B841 (2010) 1

KM10, statistical accuracy 70%

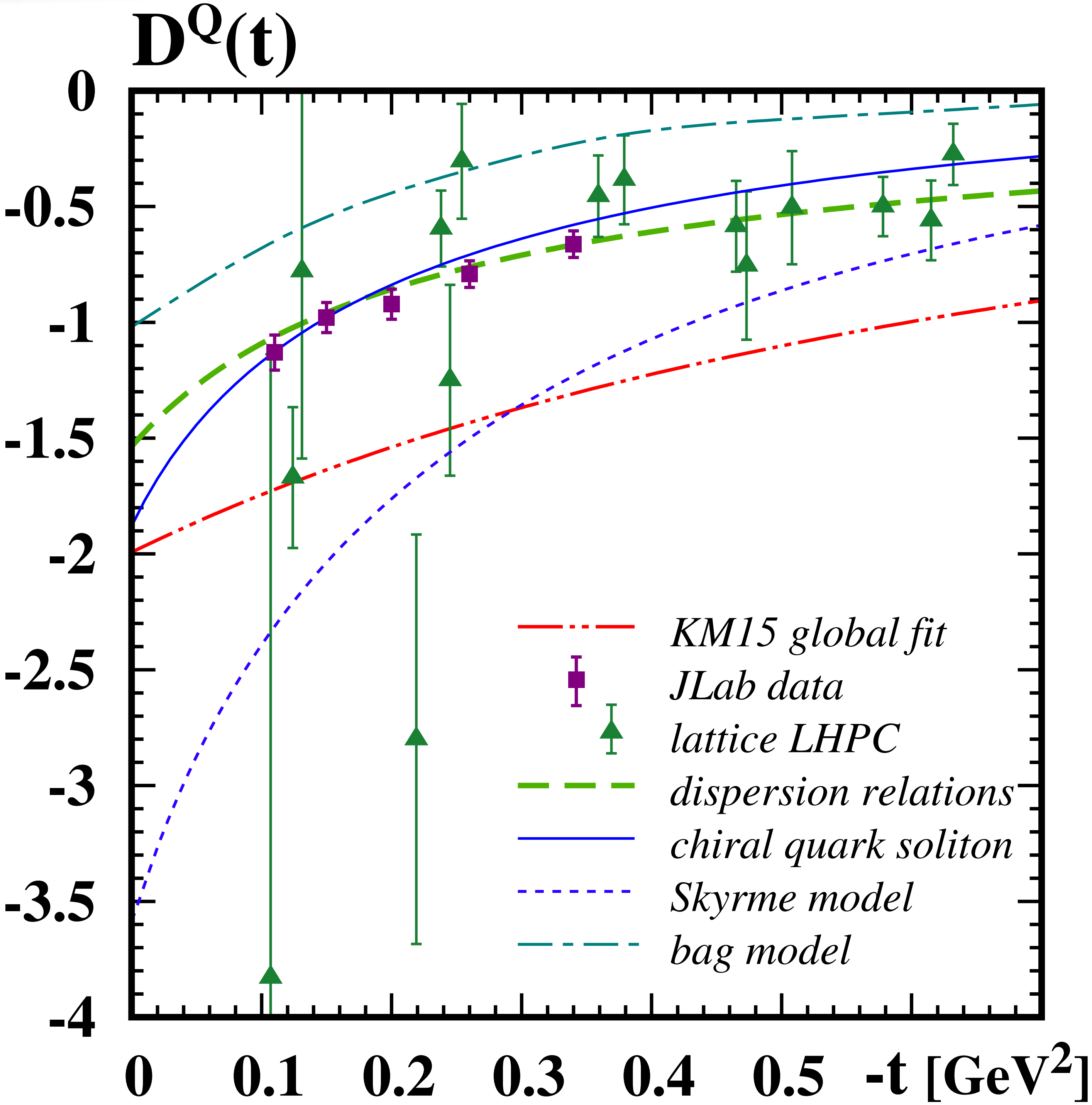
KM12, statistical accuracy 50%

KM15, statistical accuracy 20%

The D-term is *negative*, statistical accuracy is increasing with new data added.

The systematic uncertainty remains unestimated !
Kresimir Kumericki, privat commuication

Accessing p(r) and s(r) in hard exclusive processes



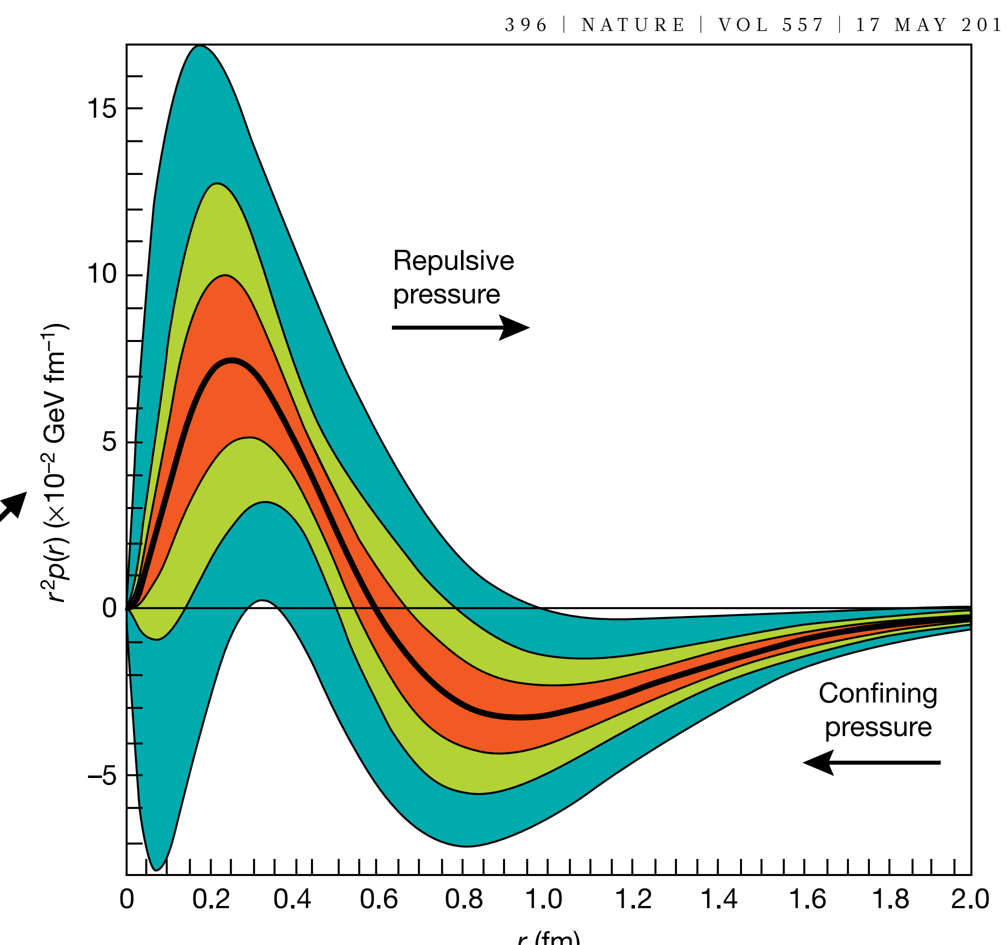
Recent analysis of CLAS data

Burkert, Elouadrhiri, Girod, **Nature 557, 396 (2018)**

1) D-term negative and sizeable
2) Agrees with chiral quark soliton model

However:

1) Systematic uncertainty much smaller than in KM15, although the same exp. data used
2) In extraction of p(r) strong model assumptions are used.



Caution: 1) scale of the main figure is not correct by ~10

2) several equation in the Nature article are not correct

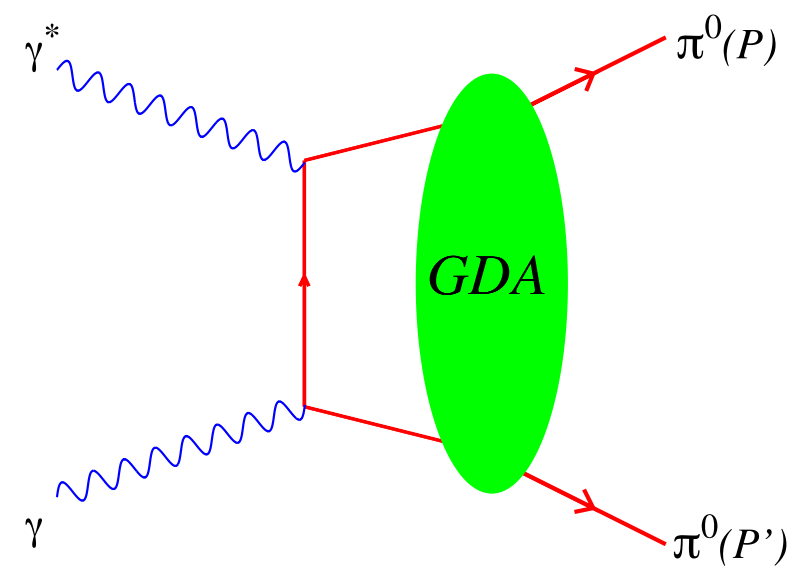
Pion D-term

Details in talk by Qin-Tao Song tomorrow

- D-term of π^0

access EMT form factors of unstable particles through generalized distribution amplitudes (analytic continuation of GPDs) via $\gamma\gamma^* \to \pi^0\pi^0$ in e^+e^-

Masuda et al (Belle), PRD 93, 032003 (2016)



best fit to Belle data $\to$ $D^Q_{\pi^0} \approx -0.7$ at $\langle Q^2 \rangle = 16.6\ \mathrm{GeV}^2$
compatible with soft pion theorem $D_{\pi^0} \approx -1$ (if gluons contribute the rest)

Kumano, Song, Teryaev, PRD97, 014020 (2018)

Slopes obtained:

$$\frac{1}{A^Q(0)}\frac{d}{dt}A^Q(0) = 1.33 \sim 2.02\ \mathrm{GeV}^{-2}, \quad \frac{1}{D^Q(0)}\frac{d}{dt}D^Q(0) = 8.92 \sim 10.35\ \mathrm{GeV}^{-2}.$$

Considerably larger than estimates in chiral effective theory! Why?

Schweitzer, MVP '2018

$$-D'(0) = \frac{N_c}{48\pi^2 f_\pi^2} + \frac{\ln\left(\mu^2/m_\pi^2\right)}{24\pi^2 f_\pi^2} = (0.73 + 1.66)\ \mathrm{GeV}^{-2} = 2.40\ \mathrm{GeV}^{-2}$$

Interaction of the gluon and quark subsystems inside the nucleon

$$\langle p'|T^a_{\mu\nu}(0)|p\rangle = \bar{u}'\left[A^a(t)\,\frac{P_\mu P_\nu}{M_N} + J^a(t)\,\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2M_N} + D^a(t)\,\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M_N} + \underline{M_N\,\bar{c}^a(t)g_{\mu\nu}}\right]u$$

$$\Delta^\beta M_N\;\bar{c}^Q(t)\;\bar{u}'u = \langle p'|ig\bar{\psi}G^{\beta\alpha}\gamma_\alpha\psi|p\rangle$$

In QCD: $\partial_\mu T^Q_{\mu\nu} = -g\;\bar{\psi}G_{\mu\nu}\gamma_\mu\psi$ $\qquad \partial_\mu T^g_{\mu\nu} = -\frac{1}{2}\;\mathrm{tr}\left(G_{\nu\alpha}\left[\mathcal{D}^\sigma, G_{\sigma\alpha}\right]\right)$

$\partial_\mu(T^Q_{\mu\nu} + T^g_{\mu\nu}) = 0$ due to EOM $\quad [\mathcal{D}^\sigma, G_{\sigma\alpha}] = j^a_\alpha t^a \quad \text{with} \quad j^a_\alpha = -g\;\bar{\psi}\gamma_\alpha t^a\psi$

$\partial_\mu T^Q_{\mu\nu} = G^a_{\mu\nu}\;j^a_\mu \longleftarrow$ Expression for the Lorentz force experienced by a quark in external gluon field. We may expect that Cbar(t) is related to forces between quark and gluon subsystems.

$$\bar{c}^Q(t)|_\mu = \bar{c}^Q(t)|_{\mu_0}\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{\gamma/b}$$

$b = \frac{11}{3}N_c - \frac{2}{3}N_f$ coeff. of QCD beta-function

$\gamma = \frac{4(N_c^2-1)}{3N_c} + \frac{2}{3}N_f$ anomalous dimension

With increasing of the QCD scale this FF logarithmically disappear. The interaction of quark and gluon subsystems decreasing due to the asymptotic freedom.

Interaction of the gluon and quark subsystems inside the nucleon

$$\frac{\partial T^{Q}_{ij}(\boldsymbol{r})}{\partial r_j} + f_i(\boldsymbol{r}) = 0. \tag{5.5}$$

Landau, Lifshitz, vol. 7

This equation can be interpreted (see e.g §2 of [28]) as equilibrium equation for quark internal stress and external force (per unit of the volume) $f_i(\boldsymbol{r})$ from the side of the gluons. This gluon force can be computed in terms of EMT form factor $\bar{c}^Q(t)$ as:

$$f_i(\boldsymbol{r}) = M_N \frac{\partial}{\partial r_i} \int \frac{d^3\Delta}{(2\pi)^3}\, e^{-i\boldsymbol{\Delta r}}\; \bar{c}^Q(-\boldsymbol{\Delta}^2) \tag{5.6}$$

H.-D. Son, MVP '2018

For $\bar{c}^Q(0) > 0$) the corresponding force is directed towards the nucleon centre, therefore we call it squeezing (compression) force. For opposite sign the corresponding force is stretching.

The total squeezing gluon force acting on quarks in the nucleon is equal to

$$F_{\rm total} = \frac{2M_N}{\pi} \int_{-\infty}^{0} \frac{dt}{\sqrt{-t}}\; \bar{c}^Q(t).$$

Cbar(t) FF important to know what are (compressing or stretching) forces experienced by quarks from side of gluons inside the nucleon. Size of this forces?

Interaction of the gluon and quark subsystems inside the nucleon from instantons.

Instantons form a dilute liquid in the QCD vacuum. They provide a mechanism of spontaneous breakdown of chiral symmetry in QCD.

Shuryak '1982
Diakonov, Petrov '1983

$$\Delta^\beta M_N\ \bar{c}^Q(t)\ \bar{u}'u = \langle p'|ig\bar{\psi}G^{\beta\alpha}\gamma_\alpha\psi|p\rangle$$

Computed in QCD vacuum using the method of Diakonov, Weiss, MVP '1996 Balla, Weiss, MVP '1997

We found a strong suppression by the instanton packing fraction

$$\bar{c}^Q(t) = \frac{\bar{c}_{\rm quark}}{(1-t/\Lambda^2)^2} \qquad \bar{c}_{\rm quark} \sim \frac{1}{6}\frac{\bar{\rho}^4}{\bar{R}^4}\ \ln\left(\frac{\bar{R}}{\bar{\rho}}\right) \qquad \bar{c}^Q(0) = \bar{c}_{\rm quark} \simeq 1.4\cdot 10^{-2}.$$

H.-D. Son, MVP '2018

We obtained small and *positive* value at a low normalisation point of ~0.5 GeV^2.
This corresponds to rather *small* compression forces experienced by quarks!

$$F_{\rm total} = \bar{c}_{\rm quark}\ M_N\Lambda \simeq 5.9\cdot 10^{-2}\frac{\rm GeV}{\rm fm}$$

it looks like the two systems decouple.
Justification of Teryaev's equipartition conjecture ?

We estimate that the contribution of $\bar{c}^Q(t)$ to the pressure distribution inside the nucleon is in the range of $1-20\%$ relative to the contribution of the quark D-term.

negative

Interaction of the gluon and quark subsystems inside the nucleon from lattice

Yi-Bo Yang et al. '2018

(χQCD Collaboration)

$\bar{c}^Q(0) = -A^Q(0)/4 + \text{mass term}$ obtained using $T^Q_{\mu\mu} = 0$ in chiral limit

Frequently used in several other papers!

Where is contribution of the dilatation anomaly? Is QCD scale dependence (in)consistent?

the parton $i = u, d, s, g\ldots$ satisfying the . We have $p_{u+d} = 0.105(12)(12)$, $\bar{p}_s = 0.002(8)(2)$, and $\bar{p}_g = -\bar{C}_{u+d+s} = -0.107(15)(12)$.

They obtained that the Cbar contribution to quark pressure is positive and quarks experience stretching forces from the side of gluon. Picture opposite to ours!

Conclusions

- gravitational D-form factor is related to "elastic properties" of the nucleon, and gives access to details of strong forces inside the nucleon.

- D(0) (the D-term) is the last unknown global (in the same sense as mass and spin) property of the nucleon

- First experimental results for D(t) of the nucleon and of the pion are obtained. It is negative, as expected from stability conditions.

- Cbar(t) FF is important to understand forces between quark and gluon subsystems inside hadrons. Instanton picture of QCD vacuum predicts small positive value of the FF. That corresponds to compression forces experienced by quark subsystem (at variance with lattice results)

Outlook

- knowledge of the D-term can be important to understand hadron interaction in gravitational field relevant to BH or NS mergers (LIGO events). MVP in preparation

- the pressure distribution inside hadrons important to understand the physics of hadro-charmonia (LHCb pentaquarks, tetraquarks with hidden charm) Eides, Petrov, MVP '2016, Perevalova, Schweitzer, MVP '2017, Panteleeva, Perevalova, Schweitzer, MVP '2018

- several theoretical issues - relation between pressure and energy density (elastic waves in hadrons?), analogies with cosmology, hadrons as projection of higher dimensional objects, etc.