Neutron Electric Dipole Moment

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Neutron EDM and CP Violation

• Measures separation between centers of (+) and (-) charges
  \[ \delta H = d_n \hat{S} \cdot \vec{E} \]

• Current bound:
  \[ |d_n| < 2.9 \times 10^{-26} \text{ e}\cdot\text{cm} \]

• Nonzero nEDM violates P and T (CP if CPT holds)
Neutron EDM Searches

- Predictions
  - Standard Model
    \[ |d_n| \sim 10^{-31} \text{ e}\cdot\text{cm} \]
  - Supersymmetry
    \[ |d_n| \sim 10^{-25} - 10^{-28} \text{ e}\cdot\text{cm} \]

- Experiments targeting \( 5 \times 10^{-28} \text{ e}\cdot\text{cm} \) precision
  - PSI EDM
  - Munich FRMII
  - RCNP/TRIUMF
  - SNS nEDM
  - JPARC
  - LANL nEDM
Impacts

- **New source of CP violation**
  - CPV in SM is not sufficient to explain observed baryon asymmetry

- **Test of **Supersymmetry** and other BSM models**
  - In many BSM theories, nEDM is predicted to be in the range $10^{-26} - 10^{-28} \, e \cdot cm$
Effective Lagrangian at 1 GeV

\[ \mathcal{L}_{\text{CPV}}^{d=6} = -\frac{g_s^2}{32\pi^2} \bar{\theta} G\tilde{G} \]

\[ -\frac{i}{2} \sum_{q=u,d,s} d_q \bar{q} (\sigma \cdot F)\gamma_5 q \]

\[ -\frac{i}{2} \sum_{q=u,d,s} \tilde{d}_q g_s \bar{q} (\sigma \cdot G)\gamma_5 q \]

\[ + d_w \frac{g_s}{6} G\tilde{G} \]

\[ + \sum_i C_i^{(4q)} O_i^{(4q)} \]

• \( \bar{\theta} \leq O(10^{-9} - 10^{-11}) \): Strong CP problem
• effectively \( \text{dim}=5 \) suppressed by \( d_q \approx v/\Lambda_{\text{BSM}}^2 \)
• Dim=6 terms

Lattice QCD calculations of matrix elements can play an important role
Spinor transformation under Parity

<table>
<thead>
<tr>
<th>P, CP-even</th>
<th>P, CP-violating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dirac Eq.</td>
<td>((ip_\mu \gamma_\mu + m)u = 0)</td>
</tr>
<tr>
<td>Parity Op.</td>
<td>(\gamma_4)</td>
</tr>
<tr>
<td></td>
<td>(u_\bar{p} \rightarrow \gamma_4 u_{-\bar{p}})</td>
</tr>
</tbody>
</table>

• **CPV interactions** ➔ phase in neutron mass term \(\gamma_4\) no longer parity op of neutron state

• Introduce new parity operator or

• **Rotate neutron state** so that \(\gamma_4\) remains the parity op:

\[
\tilde{u} = e^{i\alpha\gamma_5} u, \quad \tilde{\bar{u}} = \bar{u} e^{i\alpha\gamma_5}
\]
$F_3$: The CP Violating Form Factor

Expanding the matrix element in terms of form factors

$$
\langle N | J^{EM}_\mu | N \rangle_{CPV} = e^{i\alpha(q^2)\gamma_5} \bar{u} \left[ \gamma_\mu F_1(q^2) + (2i m_N \gamma_5 q_\mu - \gamma_\mu \gamma_5 q^2) \right] \frac{F_A(q^2)}{m_N^2} + i\sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{2m_N} + \sigma_{\mu\nu} q_\nu \gamma_5 \frac{F_3(q^2)}{2m_N} \right] u e^{i\alpha(q^2)\gamma_5}
$$

With

$$
\sum_S u(p,s)\bar{u}(p,s) = \frac{(E\gamma_4-ip\cdot\gamma+m)}{2E}
$$

The contribution to nEDM is given by

$$
d_N = \frac{F_3(q^2=0)}{2m_N}
$$
Two equally important challenges

• Signal in the CP violating form factor $F_3$
  • Needs very high statistics

• Renormalization and divergent mixing between operators
  • Needs non-perturbative calculations of mixing coefficients in order to obtain results that are finite in the continuum limit
$- \frac{g_s^2}{32\pi^2} \theta \bar{G} \tilde{G}$
QCD $\theta$-term

• Calculate $d_N$ in presence of CP violating $\theta$-term

$$ S = S_{QCD} + S_\theta $$

$$ S_\theta = -i\theta \int d^4x G\tilde{G} / 32\pi^2 = -i\theta Q_{top} $$

• Lattice calculation strategies
  – Expansion in $\theta$
  – External electric field method
  – Simulation with imaginary $\theta$
Expansion in $\theta$

$$\langle O(x) \rangle_{\theta} = \frac{1}{Z_{\theta}} \int d[U,q,\bar{q}] \, O(x) \, e^{-S_{QCD} + i\theta Q_{\text{top}}}$$

$$= \langle O(x) \rangle_{\theta=0} + i\theta \langle O(x) Q_{\text{top}} \rangle_{\theta=0} + O(\theta^2)$$

- Measurements performed on regular ($\theta=0$) lattices
- Nucleon interpolating operator $N = \epsilon^{abc} (d^T a C \gamma_5 u^b) d^c$
- $O(x) = \langle N(\tau)V_\mu N(0) \rangle$ nucleon 3-pt fn with insertion of vector current
- $\langle O(x) Q_{\text{top}} \rangle$ “reweights” the nucleon 3-point fn $O(x)$ by $Q_{\text{top}}$
- $d_n$ extracted from form-factor $F_3$ extrapolated to $q^2=0$
Correlation of $G\tilde{G}$ with nucleon 3-point function with $V_\mu$ insertion
• Otherwise Phase $e^{i\alpha\gamma_5}$ mixes $F_2$ and $F_3$

$$F_2 = \cos(2\alpha) \tilde{F}_2 - \sin(2\alpha) \tilde{F}_3$$

$$F_3 = \sin(2\alpha) \tilde{F}_2 + \cos(2\alpha) \tilde{F}_3$$

[ETMC 2016]

<table>
<thead>
<tr>
<th>Type</th>
<th>$m_\pi$ [MeV]</th>
<th>$m_N$ [GeV]</th>
<th>$F_2$</th>
<th>$\alpha$</th>
<th>$\tilde{F}_3$</th>
<th>$F_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>n</td>
<td>373</td>
<td>1.216(4)</td>
<td>-1.50(16) $^a$</td>
<td>-0.217(18)</td>
<td>-0.555(74)</td>
<td>0.094(74)</td>
</tr>
<tr>
<td>n</td>
<td>530</td>
<td>1.334(8)</td>
<td>-0.560(40) $^b$</td>
<td>-0.247(17) $^b$</td>
<td>-0.325(68)</td>
<td>-0.048(68)</td>
</tr>
<tr>
<td>n</td>
<td>690</td>
<td>1.575(9)</td>
<td>-1.715(46)</td>
<td>-0.070(20)</td>
<td>-1.39(1.52)</td>
<td>-1.15(1.52)</td>
</tr>
<tr>
<td>n</td>
<td>605</td>
<td>1.470(9)</td>
<td>-1.698(68)</td>
<td>-0.160(20)</td>
<td>0.60(2.98)</td>
<td>1.14(2.98)</td>
</tr>
<tr>
<td>n</td>
<td>465</td>
<td>1.246(7)</td>
<td>-1.491(22) $^c$</td>
<td>-0.079(27) $^d$</td>
<td>-0.375(48)</td>
<td>-0.130(76) $^d$</td>
</tr>
<tr>
<td>n</td>
<td>360</td>
<td>1.138(13)</td>
<td>-1.473(37) $^c$</td>
<td>-0.092(14) $^d$</td>
<td>-0.248(29)</td>
<td>0.020(58) $^d$</td>
</tr>
</tbody>
</table>

[Shintani et al 2005]

[Guo et al 2015]

No signal in data generated prior to 2017 post correction
Noise reduction

RBC: 4-d cylinder about the correlator
XQCD: 4-d sphere around the sink
MSU: in time around source
Shintani: in time around current
$\Theta$ induced $F_3 (M_\pi = 330 \text{ MeV})$
STATUS $\Theta$ induced $d_n$

$$d_N = a \, M_{\pi}^2 + b \, M_{\pi}^2 \log M_{\pi}^2 + \cdots$$

RBC/LHP ($M_\pi = 330$ MeV)

$$|2M_n \, d_n| = |F_{3n}(0)| \approx 0.05 \cdot \theta \, e$$

$$d_n \approx 0.005 \cdot \theta \, e \, fm$$

Need much higher statistics as $M_\pi \rightarrow 135$ MeV

MSU/Juelich (lattice 2018)

$$M_\pi = 411, 570, 701 \, MeV$$

$$d_N = 0.0029(21) \, \Theta \, e \, fm$$
Quark EDM

\[-\frac{i}{2} \sum_{q=u,d,s} d_q \bar{q} (\sigma \cdot F) \gamma_5 q\]

- nEDM from qEDMs given by the tensor charges \( g_T \)

\[ d_N = d_u g_T^u + d_d g_T^d + d_s g_T^s \]

\[ \langle N | \bar{q} \sigma_{\mu\nu} q | N \rangle = g_T^q \bar{u}_N \sigma_{\mu\nu} u_N \]

- \( d_q \propto m_q \) in many models; \( m_u/m_d \approx 1/2, \ m_s/m_d \approx 20 \)

Precise determination of \( g_T^s \) is important
Contribution of quark EDM to neutron EDM

\[ g_T^q = \langle n(0)|\bar{q}\sigma_{\mu\nu}q|n(0)\rangle \]

Disconnected

\[ g_T^l = -0.0064(32) \]

\[ g_T^s = -0.0027(16) \]

Connected + Disconnected for the proton for neutron \( u \leftrightarrow d \)

\[ g_T^u = 0.784(28); \quad g_T^d = -0.204(11); \quad g_T^s = -0.0027(16) \]

ArXiv:1808.07597
Contribution of quark EDM to neutron EDM

\[ g_T^d = 0.784(28); \quad g_T^u = -0.204(11); \quad g_T^s = -0.0027(16) \]

2015 results: \( g_T^d = 0.774(66); \quad g_T^u = -0.233(28); \quad g_T^s = -0.008(9) \)

Relation between charges \( g_T^q \), couplings \( d_q^\gamma \), and the neutron EDM \( d_n \)

\[ d_n = d_u^\gamma g_T^u + d_d^\gamma g_T^d + d_s^\gamma g_T^s + \cdots \]

This is the only result so far on nEDM from lattice QCD
Contribution of quark EDM to neutron EDM

\[ g_T^d = 0.784(28); \quad g_T^u = -0.204(11); \quad g_T^s = -0.0027(16) \]

\[ g_T^d = 0.774(66); \quad g_T^u = -0.233(28); \quad g_T^s = -0.008(9) \]

Constraint on \( d_n \) in Split SUSY

ArXiv:1808.07597
Quark Chromo EDM (cEDM)

\[-\frac{i}{2} \sum_{q=u,d,s} \tilde{d}_q g_s \bar{q}(\sigma \cdot G)\gamma_5 q\]
Quark Chromo EDM

- Calculate $d_N$ in presence of CP violating cEDM term

$$S = S_{QCD} + S_{cEDM}$$

$$S_{cEDM} = -\frac{i}{2} \int d^4 x \, \tilde{d}_q g_s \bar{q} (\sigma \cdot G) \gamma_5 q$$

- Three methods explored
  - Expansion in $\tilde{d}_q$
  - External electric field method
  - Schwinger source method
Expansion in $\tilde{d}_q$

$$\langle NV_\mu \bar{N}\rangle_{\text{CPV}} = \langle NV_\mu \bar{N}\rangle + \tilde{d}_q \left\langle NV_\mu \bar{N} \cdot \sum_x O_{\text{cEDM}}(x) \right\rangle + O(\tilde{d}_q^2)$$

$$O_{\text{cEDM}} = \frac{i}{2} g_s \bar{q} (\sigma \cdot G) \gamma_5 q$$

Needs calculation of four-point correlator

$$\left\langle NV_\mu \bar{N} \sum_x O_{\text{cEDM}}(x) \right\rangle$$

$$d_n = \frac{F_3(0)}{2M_N} \Theta e \text{ with } F_3 \text{ obtained from } \langle NV_\mu \bar{N}\rangle_{\text{CPV}}$$

$$\langle NV_\mu \bar{N}\rangle_{\text{CPV}} = \bar{u} \left[ F_1(q^2) \gamma_\mu + i \frac{F_2(q^2)}{2m_N} \sigma_{\mu\nu} q^\nu - \frac{F_3(q^2)}{2m_N} \sigma_{\mu\nu} q^\nu \gamma_5 \right] u$$
Expansion in $\tilde{d}_q$ (RBC/LHP)

Connected Diagrams

• Four-point correlator is evaluated using
  Regular and backward props ($F$, $B$),
  cEDM sequential prop ($C$) and doubly-sequential props ($E$, $G$)

Abramczyk, et al., PRD96 (2017) 014501
Expansion in $\tilde{d}_q$

- DWF
- $a = 0.11\text{fm}$
- $M_\pi = 340 \text{ MeV}$

Abramczyk, et al., PRD96 (2017) 014501

Syritsyn, Lattice 2018
Expansion in $\tilde{d}_q$

Syritsyn, et al., for RBC/LHP Lattice 2018

- DWF
- $a = 0.11\text{fm}$
- $M_\pi = 340\text{ MeV}$
**Schwinger Source Method**

- Quark chromo EDM operator is a quark bilinear
  \[ i \bar{q} (\sigma \cdot G) \gamma_5 q \]
- Include cEDM term in valence quark propagators by changing Dirac op inversion routine
  \[ D_{\text{clov}} \rightarrow D_{\text{clov}} + i \epsilon \sigma^{\mu \nu} \gamma_5 G_{\mu \nu} \]
- Effectively
  \[ c_{sw} \sigma^{\mu \nu} G_{\mu \nu} \rightarrow \sigma^{\mu \nu} (c_{sw} + i \epsilon \gamma_5) G_{\mu \nu} \]
- No four-point correlators; \( d_N \) extracted from \( F_3 \)
- Fermion determinant gives reweighting factor
  \[ e^{i \epsilon \text{Tr} \left( \sigma^{\mu \nu} \gamma_5 G_{\mu \nu} D_{\text{clov}}^{-1} \right)} \]
The full calculation requires

\[
\frac{\text{Det}[\mathcal{D} + m - \frac{r}{2} D^2 + \Sigma^{\mu\nu} (c_{SW} G_{\mu\nu} + i\epsilon \tilde{G}_{\mu\nu})]}{\text{Det}[\mathcal{D} + m - \frac{r}{2} D^2 + c_{SW} \Sigma^{\mu\nu} G_{\mu\nu} ]}
\]

\[
= \exp\{Tr \ Ln[1 + i\epsilon \Sigma^{\mu\nu} \tilde{G}_{\mu\nu} (\mathcal{D} + m - \frac{r}{2} D^2 + c_{SW} \Sigma^{\mu\nu} G_{\mu\nu})^{-1}]\}
\]

\[
\approx \exp\{Tr \ i\epsilon \Sigma^{\mu\nu} \tilde{G}_{\mu\nu} (\mathcal{D} + m - \frac{r}{2} D^2 + c_{SW} \Sigma^{\mu\nu} G_{\mu\nu})^{-1}\}
\]

Calculate \(e^{i\epsilon}\)

Reweight factor for the configurations

\[
\sum_{\text{seq}} \exp\{\text{Tr} i\epsilon \frac{1}{2} (\mathcal{D} + m - \frac{r}{2} D^2 + c_{SW} \Sigma^{\mu\nu} G_{\mu\nu})^{-1}\}
\]
Schwinger Source Method

- Calculation performed at small $\varepsilon$ so that results are linear in $\varepsilon$
- cEDM mixes with $\gamma_5$, so investigated both operators
- Test at $a = 0.09$ fm, $m_\pi = 310$ MeV
Define $X^{imp}_\epsilon = X_\epsilon - X_{\epsilon=0}$ and exploit correlations.
Renormalization

• Renormalization of cEDM Operators are studied
  – 1-loop perturbation on twisted-mass fermion
    [Constantinou, et al, 2015]
  – Nonperturbative RI-ŠMOM
    [Bhattacharya, et al, 2015]

• Mixing with lower-dimensional operator
  \[ O_{cEDM} = a^2 \bar{q} \sigma^{\mu\nu} \gamma_5 G_{\mu\nu} q \]
  \[ O_p = \bar{q} \gamma_5 q \]

  – Divergent $1/a^2$ mixing
Ongoing Work

• Weinberg Three-gluon Operator
  \[ d_w \frac{g_s}{6} \tilde{G} G G \]

• Renormalization and mixing
  – Gradient Flow
Summary

- **QCD $\theta$-term**
  Actively being calculated and progress at $M_\pi > 330$ MeV;
  need better variance reduction to get precision at $M_\pi = 135$ MeV

- **Quark EDM**
  Calculated: $g_T^d = 0.784(28); g_T^u = -0.204(11); g_T^s = -0.0027(16)$

- **Quark Chromo EDM**
  Exploratory studies show signal in connected contribution;
  next step: disconnected diagrams & renormalization/mixing

- **Weinberg Three-gluon Operator**
  Exploratory studies just started

- **Four-quark Operators**
  Not yet explored

*Should have better estimate of accuracy achievable in 1-2 years*