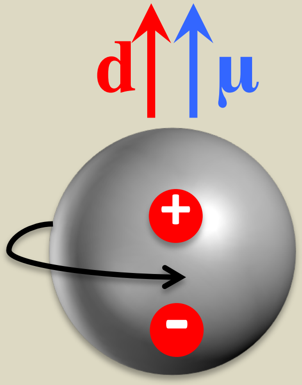
Neutron Electric Dipole Moment



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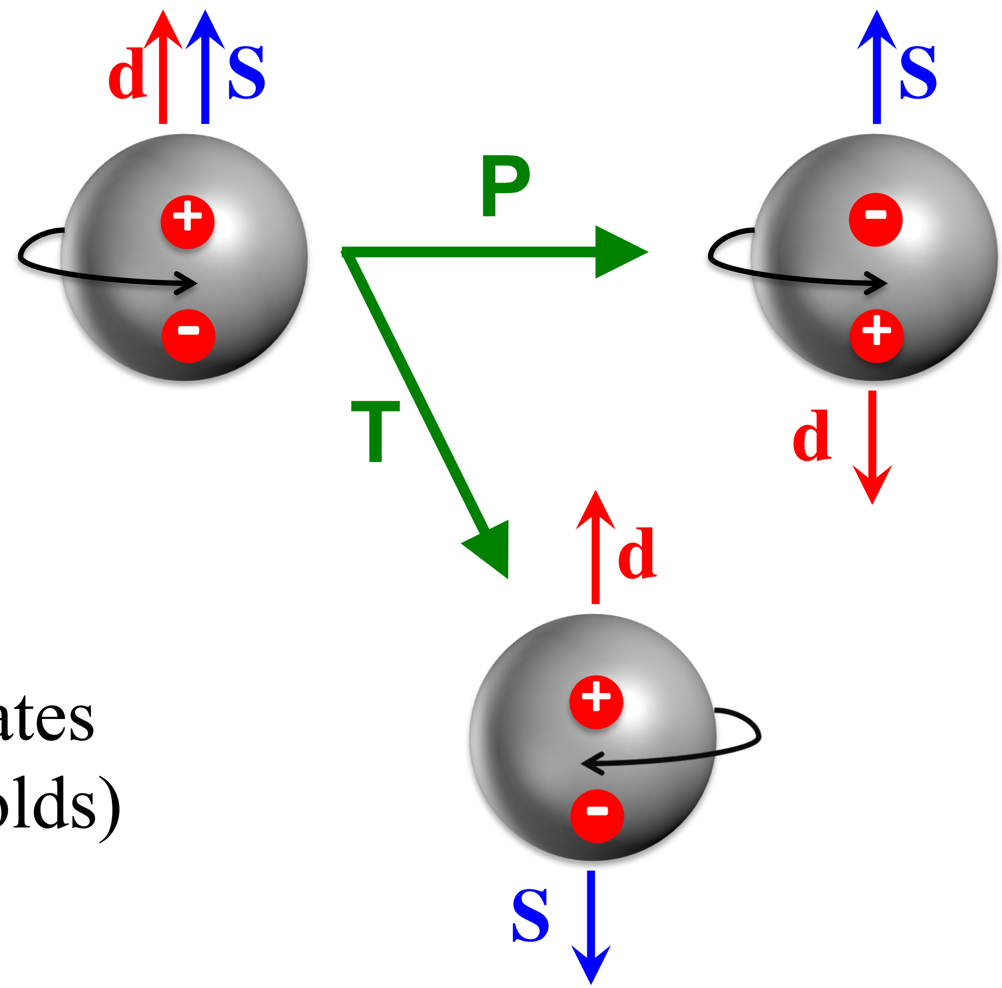
PNDME collaboration:

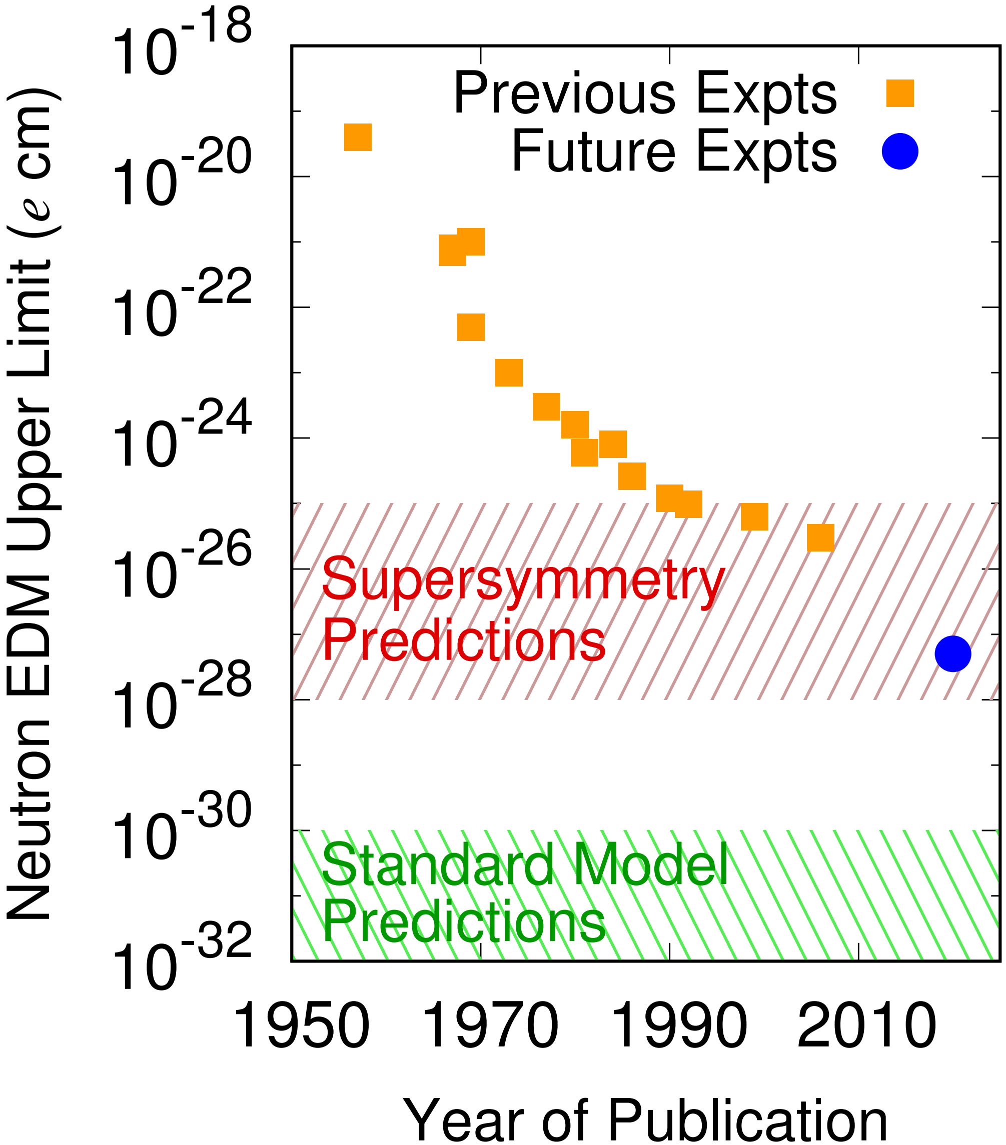
- Tanmoy Bhattacharya
- Vincenzo Cirigliano
- Boram Yoon

Neutron EDM and CP Violation

- Measures separation between centers of (+) and (-) charges
$$\delta H = d_N \hat{S} \cdot \vec{\mathcal{E}}$$
- Current bound: $|d_n| < 2.9 \times 10^{-26}\, e \cdot \mathrm{cm}$
- Nonzero nEDM violates P and T (CP if CPT holds)



Neutron EDM Searches



- Predictions
  - Standard Model
    $|d_n| \sim 10^{-31}\ e{\cdot}\mathrm{cm}$
  - Supersymmetry
    $|d_n| \sim 10^{-25} - 10^{-28}\ e{\cdot}\mathrm{cm}$
- Experiments targeting $5 \times 10^{-28}\ e{\cdot}\mathrm{cm}$ precision
  - PSI EDM
  - Munich FRMII
  - RCNP/TRIUMF
  - SNS nEDM
  - JPARC
  - LANL nEDM

Impacts

- New source of CP violation
  - CPV in SM is not sufficient to explain observed baryon asymmetry
- Test of Supersymmetry and other BSM models
  - In many BSM theories, nEDM is predicted to be in the range 10^{-26}–10^{-28} $e\cdot$cm

Effective Lagrangian at 1 GeV

$$\mathcal{L}_{\mathrm{CPV}}^{d\leq 6} = -\frac{g_s^2}{32\pi^2}\bar{\theta}G\tilde{G}$$ dim=4 QCD θ-term

$$-\frac{i}{2}\sum_{q=u,d,s} d_q \bar{q}(\sigma\cdot F)\gamma_5 q$$ dim=5 Quark EDM (qEDM)

$$-\frac{i}{2}\sum_{q=u,d,s} \tilde{d}_q g_s \bar{q}(\sigma\cdot G)\gamma_5 q$$ dim=5 Quark Chromo EDM (CEDM)

$$+d_w \frac{g_s}{6} G\tilde{G}G$$ dim=6 Weinberg 3g operator

$$+\sum_i C_i^{(4q)} O_i^{(4q)}$$ dim=6 Four-quark operators

- $\bar{\theta} \leq O(10^{-9} - 10^{-11})$: Strong CP problem
- effectively dim=5 suppressed by $d_q \approx v/\Lambda_{\mathrm{BSM}}^2$
- Dim=6 terms

Lattice QCD calculations of matrix elements can play an important role

Spinor transformation under Parity

	P, CP-even	P, CP-violating
Dirac Eq.	$\left(ip_\mu\gamma_\mu + m\right)u = 0$	$\left(ip_\mu\gamma_\mu + me^{-2i\alpha\gamma_5}\right)\tilde{u} = 0$
Parity Op.	γ_4 $u_{\vec{p}} \rightarrow \gamma_4 u_{-\vec{p}}$	$e^{2i\alpha\gamma_5}\gamma_4$ $\tilde{u}_{\vec{p}} \rightarrow e^{2i\alpha\gamma_5}\gamma_4\tilde{u}_{-\vec{p}}$

- CPV interactions ➔ phase in neutron mass term
  γ_4 no longer parity op of neutron state
- Introduce new parity operator or
- Rotate neutron state so that γ_4 remains the parity op:

$$\tilde{u} = e^{i\alpha\gamma_5}u, \qquad \bar{\tilde{u}} = \bar{u}e^{i\alpha\gamma_5}$$

Abramczyk, et al., PRD96 (2017) 014501

F_3: The CP Violating Form Factor

Expanding the matrix element in terms of form factors

$$\langle N|J_\mu^{EM}|N\rangle_{CPV} = e^{i\alpha(q^2)\gamma_5}\,\bar{u}\,[\gamma_\mu F_1(q^2) + \left(2im_N\gamma_5 q_\mu - \gamma_\mu\gamma_5 q^2\right)\frac{F_A(q^2)}{m_N^2}$$

$$+i\sigma_{\mu\nu}q_\nu \frac{F_2(q^2)}{2m_N} + \sigma_{\mu\nu}q_\nu\gamma_5 \frac{F_3(q^2)}{2m_N}]\,u\,e^{i\alpha(q^2)\gamma_5}$$

With $\sum_s u(p,s)\bar{u}(p,s) = \frac{(E\gamma_4 - ip\cdot\gamma + m}{2E}$

The contribution to nEDM is given by $d_N = \dfrac{F_3(q^2=0)}{2m_N}$

Two equally important challenges

- **Signal in the CP violating form factor F_3**
  - **Needs very high statistics**
- **Renormalization and divergent mixing between operators**
  - **Needs non-perturbative calculations of mixing coefficients in order to obtain results that are finite in the continuum limit**

QCD θ-term

$$-\frac{g_s^2}{32\pi^2}\bar{\theta}G\tilde{G}$$

QCD θ-term

- Calculate d_N in presence of CP violating θ-term

$$S = S_{QCD} + S_\theta$$

$$S_\theta = -i\theta \int d^4x\, G\tilde{G} / 32\pi^2 = -i\theta\, Q_{\mathrm{top}}$$

- Lattice calculation strategies
  - Expansion in θ
  - External electric field method
  - Simulation with imaginary θ

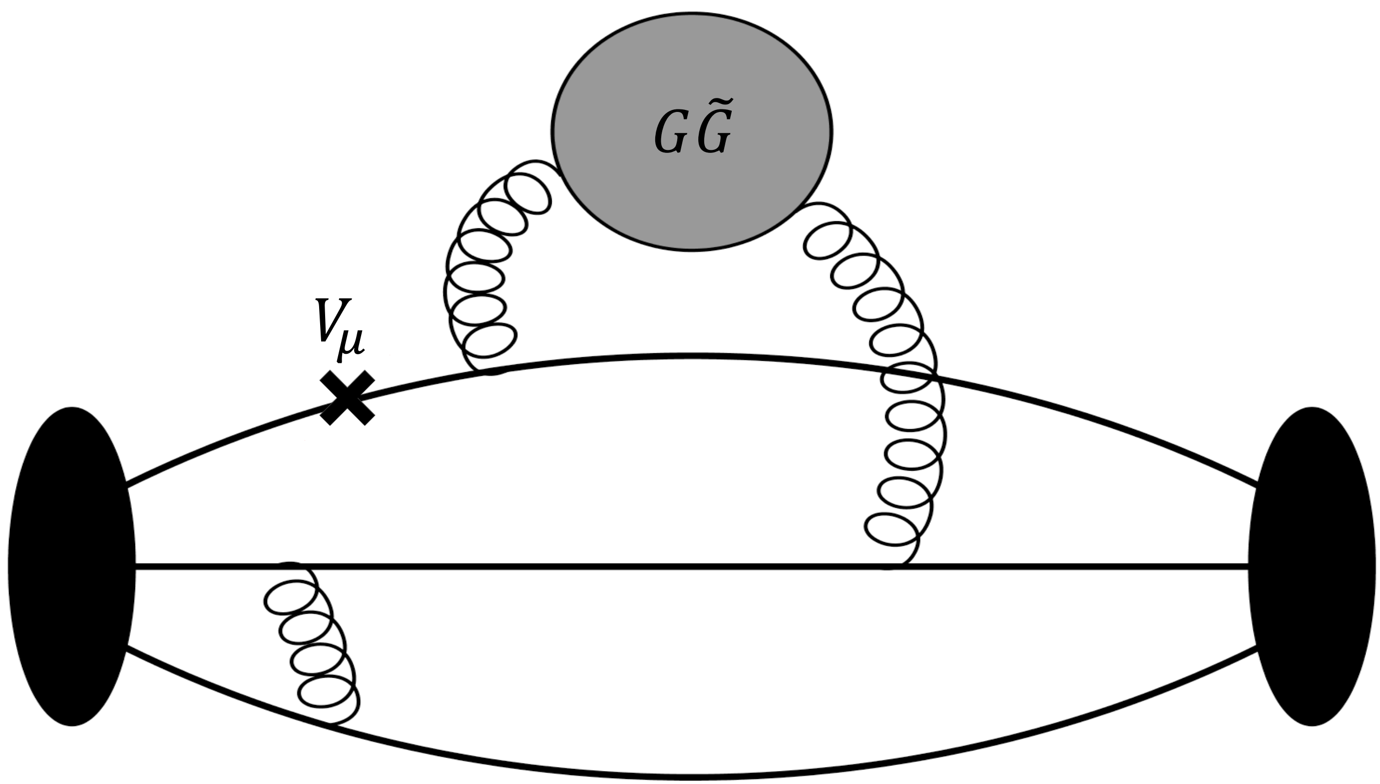
Expansion in θ

$$\langle \mathcal{O}(x) \rangle_\theta = \frac{1}{Z_\theta} \int d[U,q,\overline{q}]\, \mathcal{O}(x)\, e^{-S_{QCD} + \boxed{i\theta Q_{\text{top}}}}$$

$$= \langle \mathcal{O}(x) \rangle_{\theta=0} + i\theta \langle \mathcal{O}(x) Q_{\text{top}} \rangle_{\theta=0} + O\left(\theta^2\right)$$

- Measurements performed on regular (θ=0) lattices
- Nucleon interpolating operator $N = \varepsilon^{abc}\left(d^{Ta} C\gamma_5 u^b\right) d^c$
- $O(x) = \langle N(\tau) V_\mu N(0) \rangle$ nucleon 3-pt fn with insertion of vector current
- $\langle O(x) Q_{top} \rangle$ "reweights" the nucleon 3-point fn $O(x)$ by Q_{top}
- d_n extracted from form-factor F_3 extrapolated to $q^2=0$

Correlation of $G\tilde{G}$ with nucleon 3-point function with V_μ insertion



Form Factors with Parity Mixing

Abramczyk, et al., **Phys.Rev. D96 (2017) 014501**

- Otherwise Phase $e^{i\alpha\gamma_5}$
  mixes F_2 and F_3

$$F_2 = \cos(2\alpha)\tilde{F}_2 - \sin(2\alpha)\tilde{F}_3$$
$$F_3 = \sin(2\alpha)\tilde{F}_2 + \cos(2\alpha)\tilde{F}_3$$

[M.Abramczyk, S.Aoki, S.N.S., *et al*, (2017)]

		m_π [MeV]	m_N [GeV]	F_2	α	$\tilde{F}_3$	F_3
[ETMC 2016]	n	373	1.216(4)	−1.50(16)[a]	−0.217(18)	−0.555(74)	0.094(74)
[Shintani et al 2005]	n	530	1.334(8)	−0.560(40)	−0.247(17)[b]	−0.325(68)	−0.048(68)
	p	530	1.334(8)	0.399(37)	−0.247(17)[b]	0.284(81)	0.087(81)
[Berruto et al 2006]	n	690	1.575(9)	−1.715(46)	−0.070(20)	−1.39(1.52)	−1.15(1.52)
	n	605	1.470(9)	−1.698(68)	−0.160(20)	0.60(2.98)	1.14(2.98)
[Guo et al 2015]	n	465	1.246(7)	−1.491(22)[c]	−0.079(27)[d]	−0.375(48)	−0.130(76)[d]
	n	360	1.138(13)	−1.473(37)[c]	−0.092(14)[d]	−0.248(29)	0.020(58)[d]

No signal in data generated prior to 2017 post correction

Noise reduction

RBC: 4-d cylinder about the correlator
XQCD: 4-d sphere around the sink
MSU: in time around around source
Shintani: in time around current

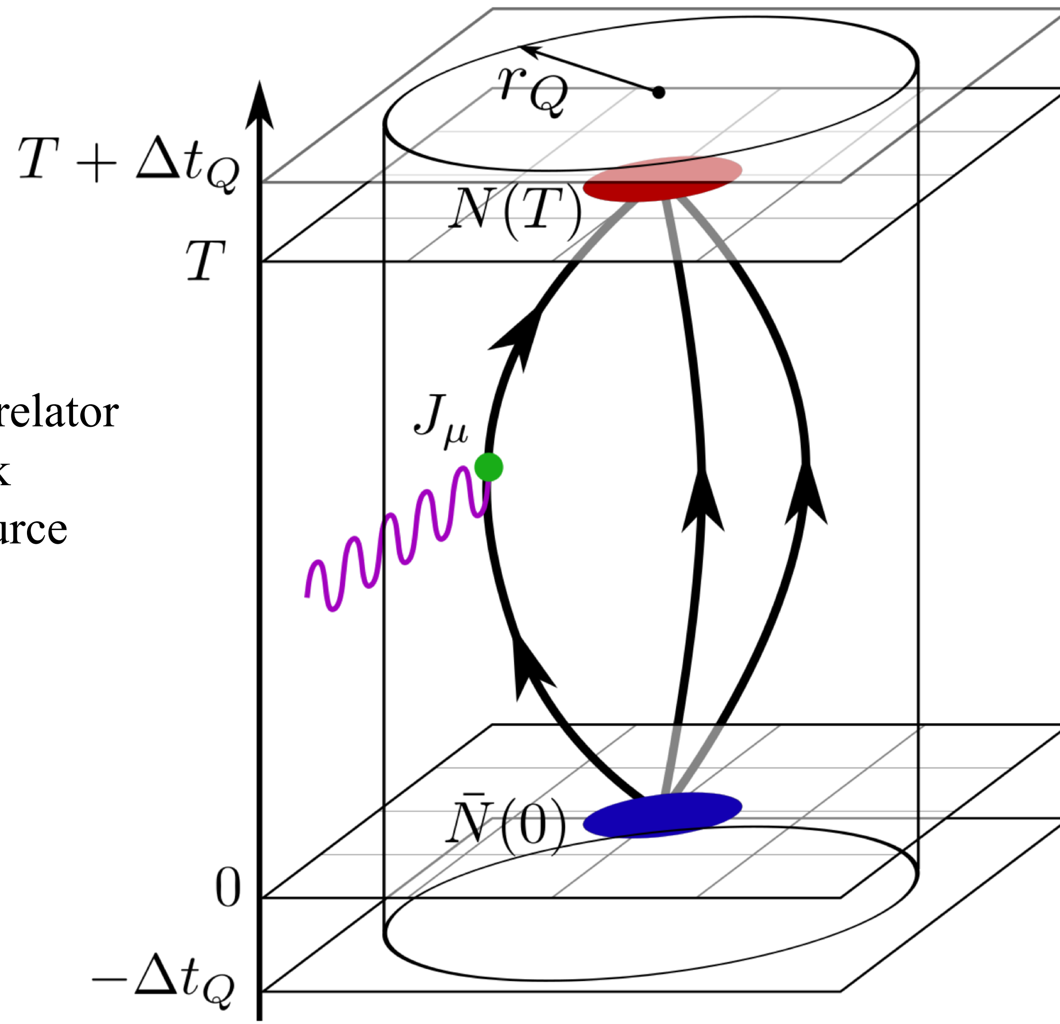
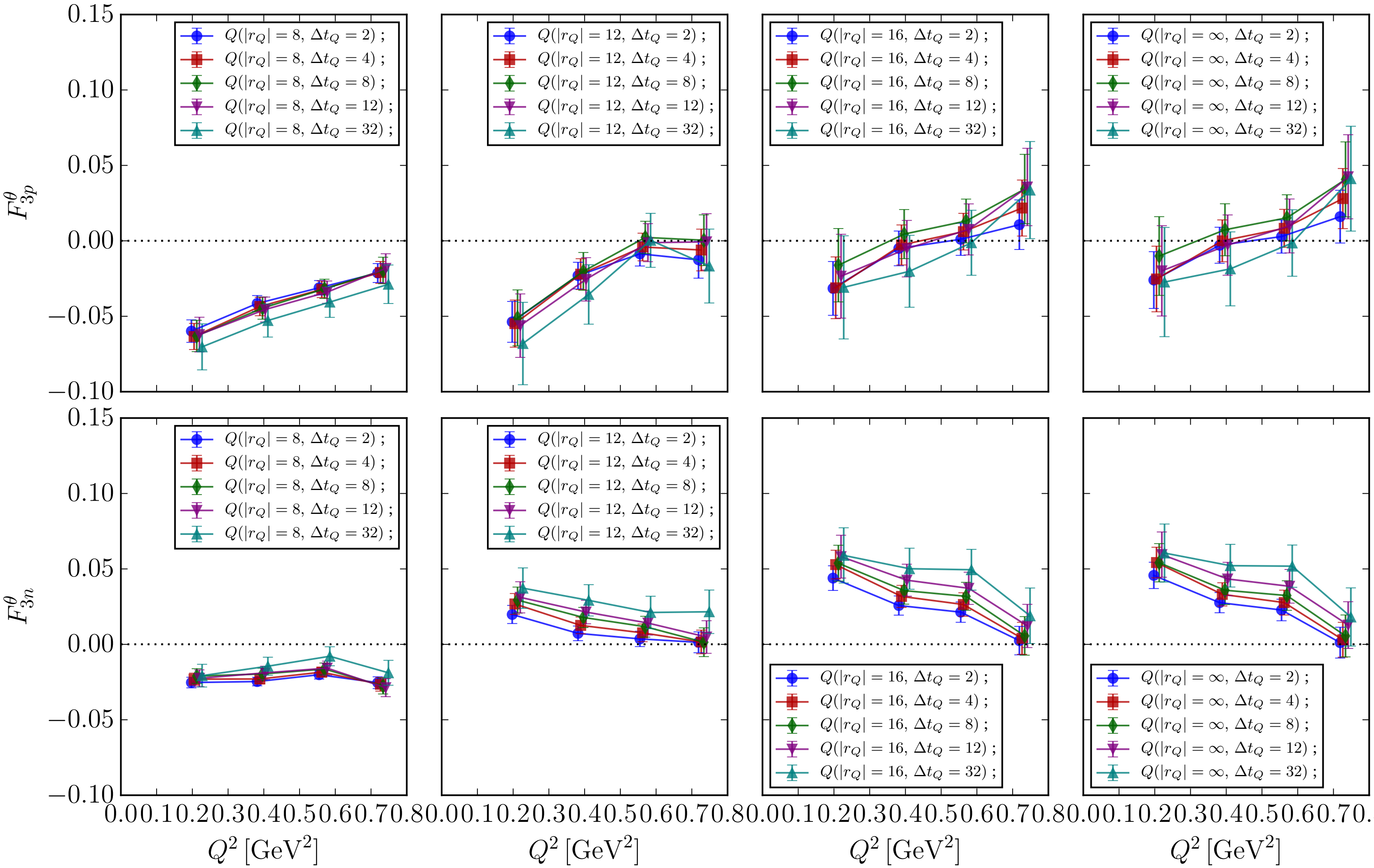


Figure courtesy Syritsyn

Θ induced F_3 (M_π = 330 MeV)



Syritsyn et al., 2018

STATUS Θ induced d_n

$$d_N = a\, M_\pi^2 + b\, M_\pi^2 \log M_\pi^2 + \cdots$$

Mereghetti et al, PLB696 (2011) 97

RBC/LHP ($M_\pi = 330$ MeV)

$$|2M_n\, d_n| = |F_{3n}(0)| \approx 0.05\cdot\theta\, e$$

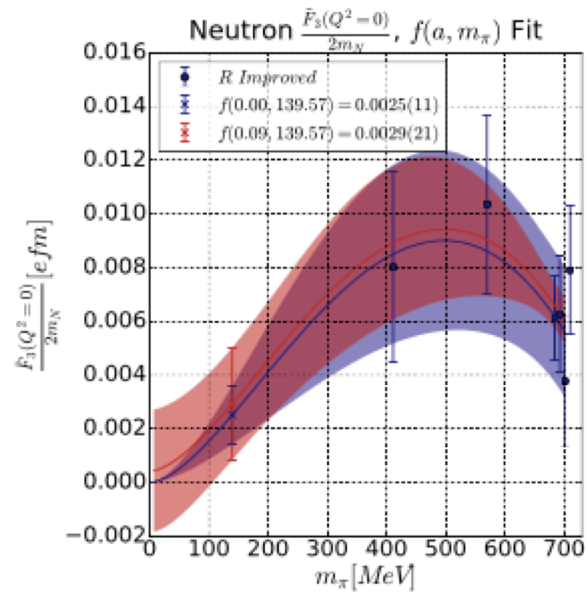
$$d_n \approx 0.005\cdot\theta\, e\, fm$$

Need much higher statistics as $M_\pi \to 135$ MeV

MSU/Juelich (lattice 2018)

$M_\pi =$ 411, 570, 701 MeV

$$d_N = 0.0029(21)\ \Theta\, e\, fm$$



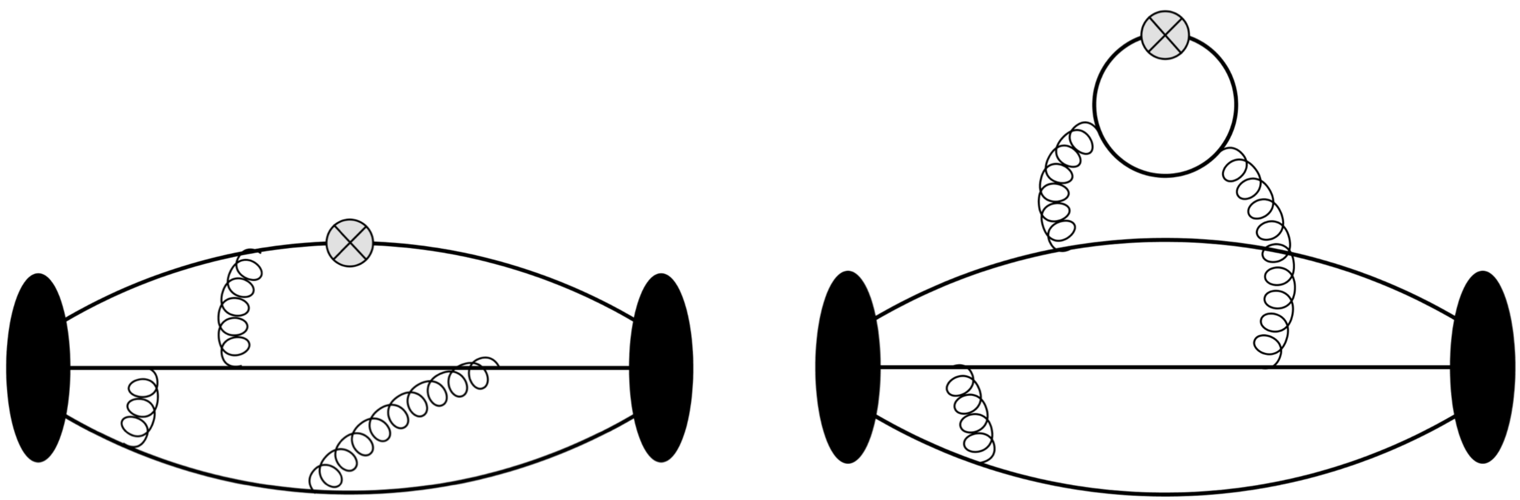
Quark EDM $-\frac{i}{2}\sum_{q=u,d,s} d_q \bar{q}(\sigma\cdot F)\gamma_5 q$

- nEDM from qEDMs given by the tensor charges g_T

$$d_N = d_u g_T^u + d_d g_T^d + d_s g_T^s$$

$$\langle N|\bar{q}\sigma_{\mu\nu}q|N\rangle = g_T^q\ \bar{u}_N\sigma_{\mu\nu}u_N$$

- $d_q \propto m_q$ in many models; $m_u/m_d \approx 1/2$, $m_s/m_d \approx 20$
  Precise determination of g_T^s is important



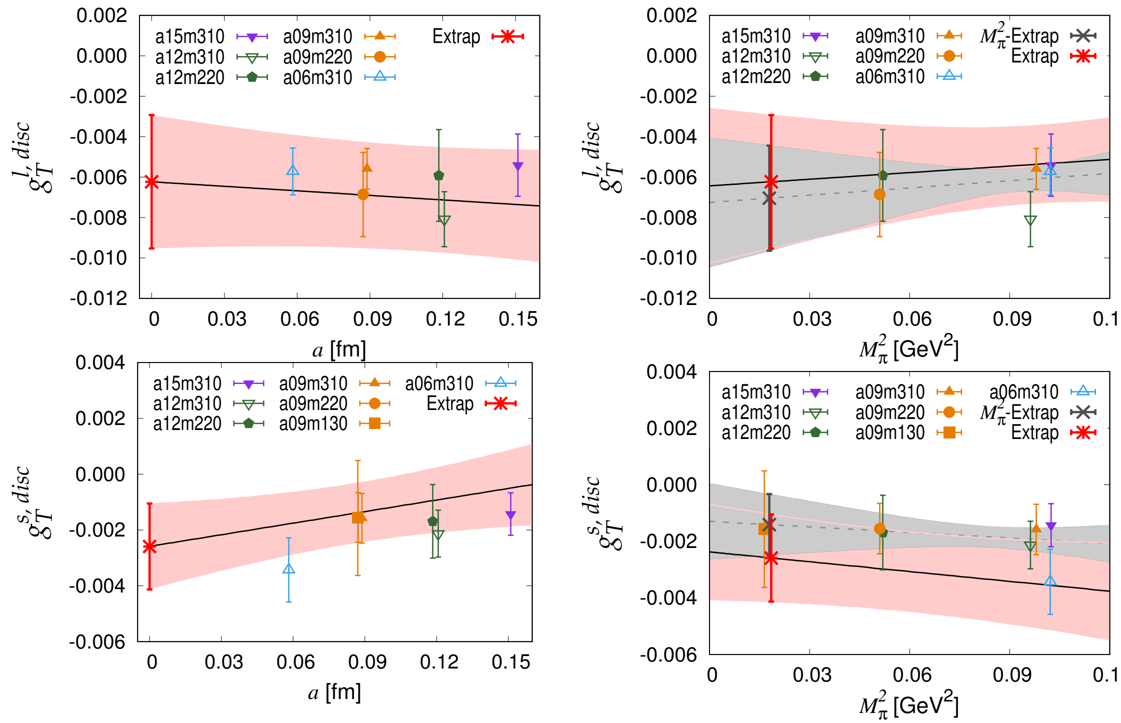
Contribution of quark EDM to neutron EDM

$$g_T^q = \langle n(0)|\bar{q}\sigma_{\mu\nu}q|n(0)\rangle$$

Disconnected

$$\boldsymbol{g_T^l = -0.0064(32)}$$

$$\boldsymbol{g_T^s = -0.0027(16)}$$



Connected + Disconnected for the proton

for neutron $u \leftrightarrow d$

$$\boldsymbol{g_T^u = 0.784(28);\quad g_T^d = -0.204(11);\quad g_T^s = -0.0027(16)}$$

ArXiv:1808.07597

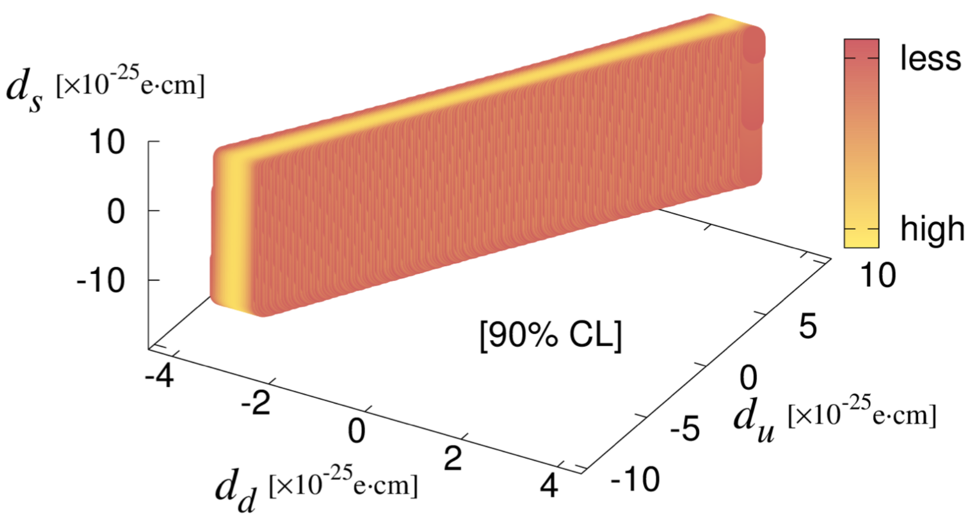
Contribution of quark EDM to neutron EDM

$$g_T^d = 0.784(28); \quad g_T^u = -0.204(11); \quad g_T^s = -0.0027(16)$$

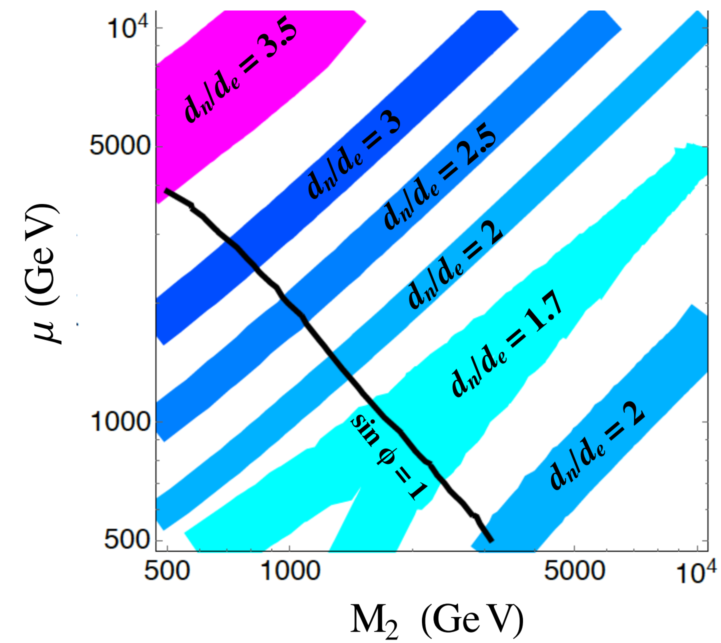
$$2015\ results: \ g_T^d = 0.774(66); \quad g_T^u = -0.233(28); \quad g_T^s = -0.008(9)$$

Relation between charges g_T^q, couplings d_q^γ, and the neutron EDM d_n

$$d_n = d_u^\gamma g_T^u + d_d^\gamma g_T^d + d_s^\gamma g_T^s + \cdots$$



Constraint on d_n in Split SUSY



This is the only result so far on nEDM from lattice QCD

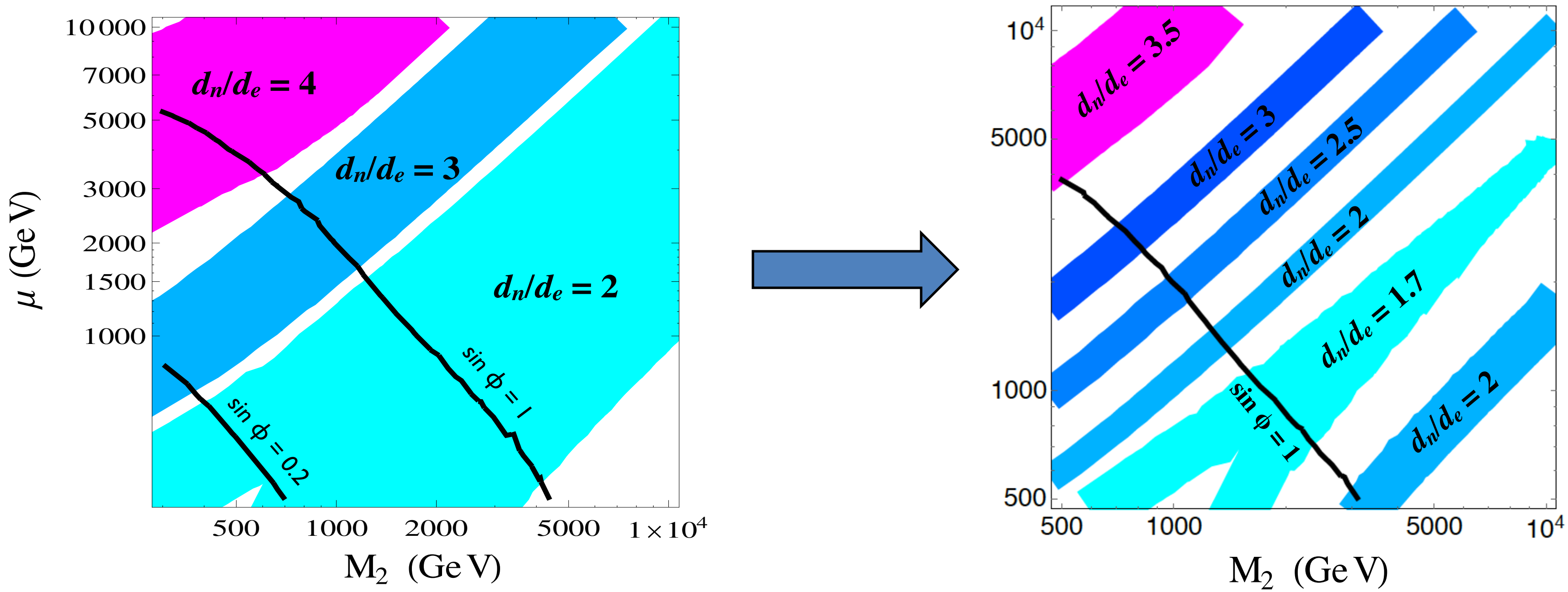
PhysRevLett.115.212002; PhysRevD.92.094511; PhysRevD.92.114026 ; ArXiv:1808.07597

Contribution of quark EDM to neutron EDM

$$g_T^d = 0.784(28); \quad g_T^u = -0.204(11); \quad g_T^s = -0.0027(16)$$

$$g_T^d = 0.774(66); \quad g_T^u = -0.233(28); \quad g_T^s = -0.008(9)$$

Constraint on d_n in Split SUSY



ArXiv:1808.07597

Quark Chromo EDM (cEDM)

$$-\frac{i}{2}\sum_{q=u,d,s}\tilde{d}_q g_s \overline{q}(\sigma\cdot G)\gamma_5 q$$

Quark Chromo EDM

- Calculate d_N in presence of CP violating cEDM term

$$S = S_{QCD} + S_{cEDM}$$

$$S_{cEDM} = -\frac{i}{2}\int d^4x \;\; \tilde{d}_q g_s \overline{q}(\sigma \cdot G)\gamma_5 q$$

- Three methods explored
  - Expansion in $\tilde{d}_q$
  - External electric field method
  - Schwinger source method

Expansion in $\tilde{d}_q$

$$\langle N V_\mu \bar{N} \rangle_{CPV} = \langle N V_\mu \bar{N} \rangle + \tilde{d}_q \left\langle N V_\mu \bar{N} \cdot \sum_x O_{\mathrm{cEDM}}(x) \right\rangle + O(\tilde{d}_q^2)$$

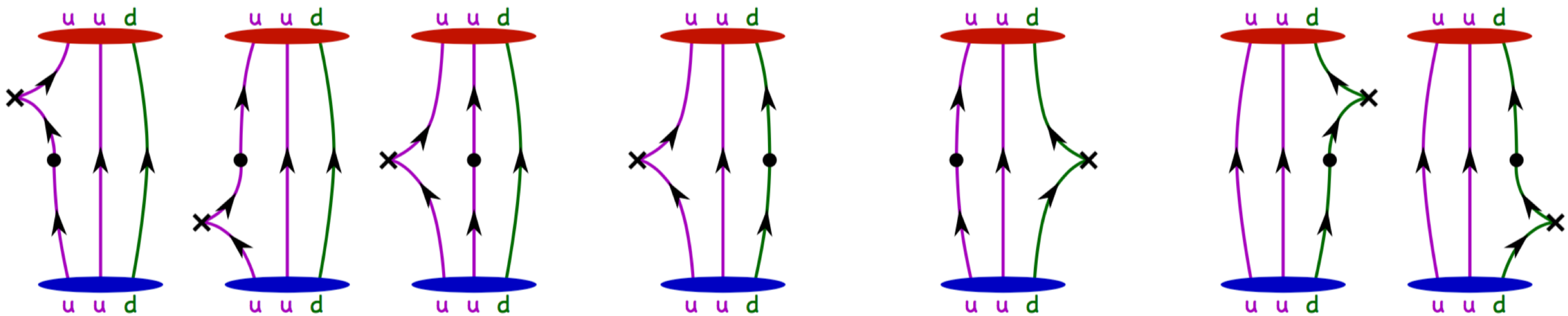
$$O_{\mathrm{cEDM}} = \frac{i}{2} g_s \bar{q} (\sigma \cdot G) \gamma_5 q$$

Needs calculation of **four-point correlator** $\left\langle N V_\mu \bar{N} \sum_x O_{\mathrm{cEDM}}(x) \right\rangle$

$$d_n = \frac{F_3(0)}{2M_N} \Theta \, \mathrm{e} \; \textit{with } F_3 \textit{ obtained from } \langle N V_\mu \bar{N} \rangle_{CPV}$$

$$\langle N V_\mu \bar{N} \rangle_{\mathrm{CPV}} = \bar{u} \left[F_1(q^2) \gamma_\mu + i \frac{F_2(q^2)}{2m_N} \sigma_{\mu\nu} q^\nu - \frac{F_3(q^2)}{2m_N} \sigma_{\mu\nu} q^\nu \gamma_5 \right] u$$

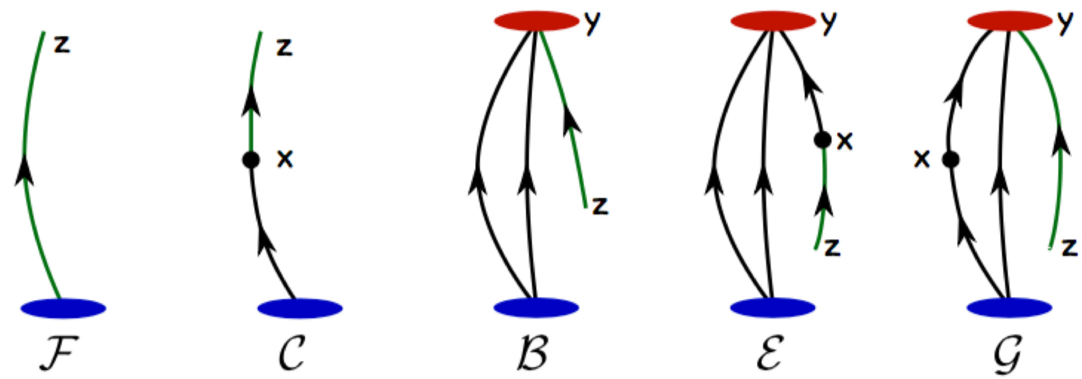
Expansion in $\tilde{d}_q$ (RBC/LHP)



$\langle J_u\,(\mathrm{cEDM})_u \rangle$ $\langle J_d\,(\mathrm{cEDM})_u \rangle$ $\langle J_u\,(\mathrm{cEDM})_d \rangle$ $\langle J_d\,(\mathrm{cEDM})_d \rangle$

Connected Diagrams

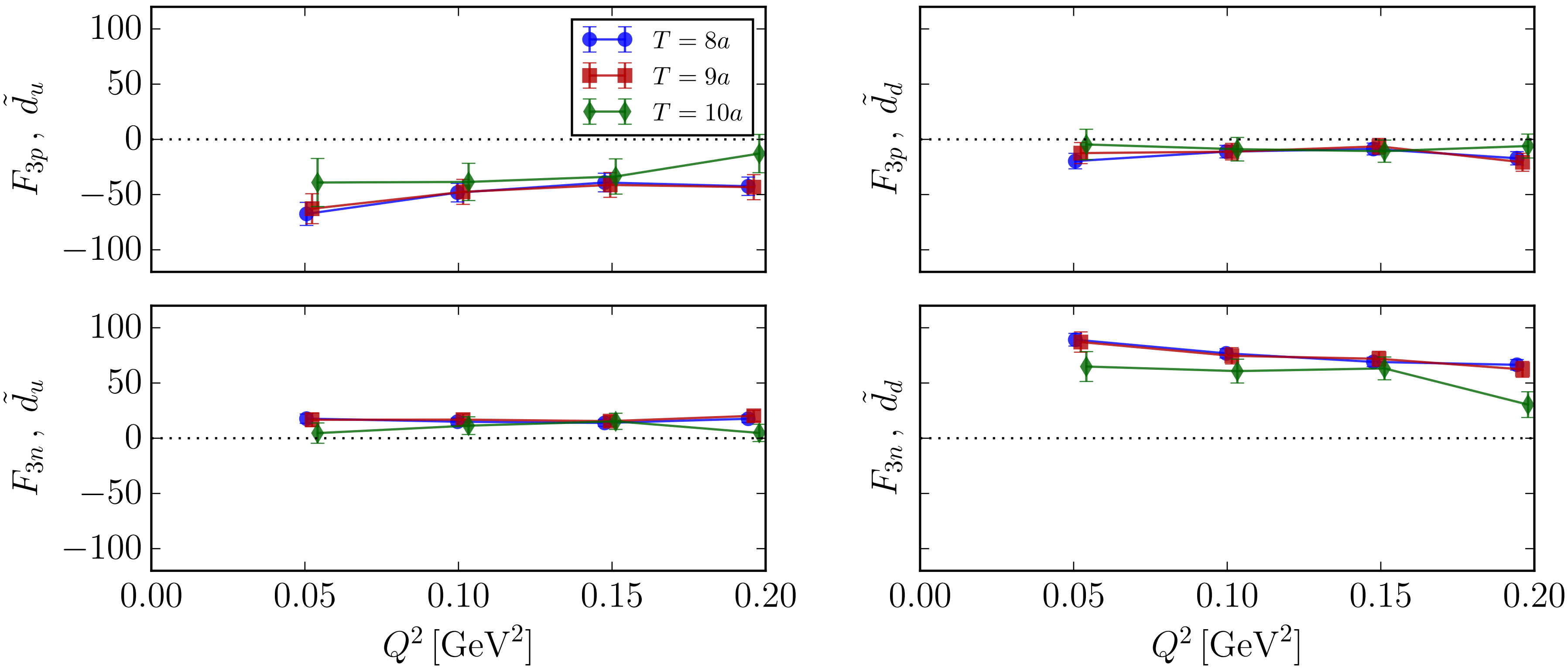
Propagators Needed



- Four-point correlator is evaluated using Regular and backward props (F, B), cEDM sequential prop (C) and doubly-sequential props (E, G)

Abramczyk, et al., PRD96 (2017) 014501

Expansion in $\tilde{d}_q$

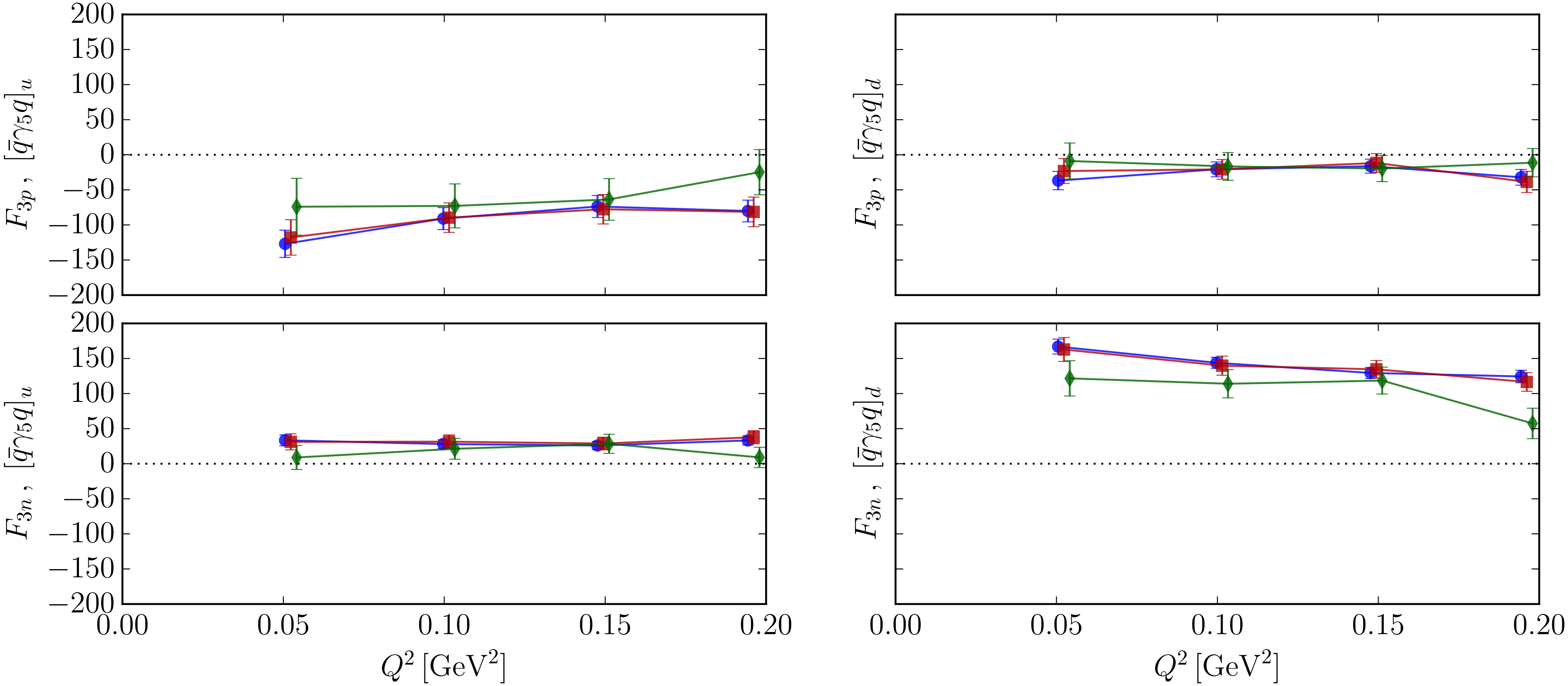


Abramczyk, et al., PRD96 (2017) 014501

Syritsyn, Lattice 2018

- DWF
- a = 0.11fm
- M_π=340 MeV

Expansion in $\tilde{d}_q$



- DWF
- a = 0.11fm
- M_π=340 MeV

Syritsyn, et al., for RBC/LHP Lattice 2018

Schwinger Source Method

- Quark chromo EDM operator is a quark bilinear

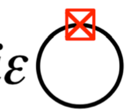
$$i\bar{q}(\sigma\cdot G)\gamma_5 q$$

- Include cEDM term in valence quark propagators by changing Dirac op inversion routine

$$D_{\text{clov}} \to D_{\text{clov}} + i\varepsilon\sigma^{\mu\nu}\gamma_5 G_{\mu\nu}$$

Effectively

$$c_{sw}\sigma^{\mu\nu}G_{\mu\nu} \to \sigma^{\mu\nu}(c_{sw} + i\varepsilon\gamma_5)G_{\mu\nu}$$

- No four-point correlators; d_N extracted from F_3
- Fermion determinant gives reweighting factor $e^{i\varepsilon}$ 

$$\frac{\det\left(D_{\text{clov}} + i\varepsilon\sigma^{\mu\nu}\gamma_5 G_{\mu\nu}\right)}{\det\left(D_{\text{clov}}\right)} \approx \exp\left[i\varepsilon\,\mathrm{Tr}\left(\sigma^{\mu\nu}\gamma_5 G_{\mu\nu}\, D_{\text{clov}}^{-1}\right)\right]$$

The full calculation requires

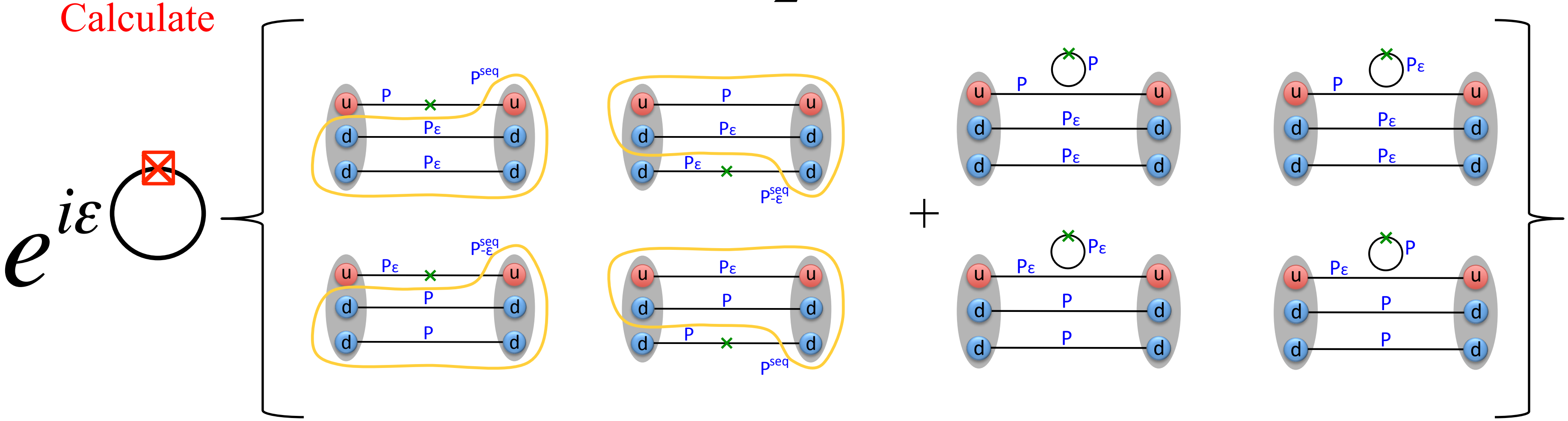
Reweight factor for the configurations

$$\frac{Det[\not{D} + m - \frac{r}{2}D^2 + \Sigma^{\mu\nu}(c_{SW}G_{\mu\nu} + i\varepsilon\tilde{G}_{\mu\nu})]}{Det[\not{D} + m - \frac{r}{2}D^2 + c_{SW}\Sigma^{\mu\nu}G_{\mu\nu}]}$$

$$= \exp\{Tr\ Ln[1 + i\varepsilon\Sigma^{\mu\nu}\tilde{G}_{\mu\nu}(\not{D} + m - \frac{r}{2}D^2 + c_{SW}\Sigma^{\mu\nu}G_{\mu\nu})^{-1}]\}$$

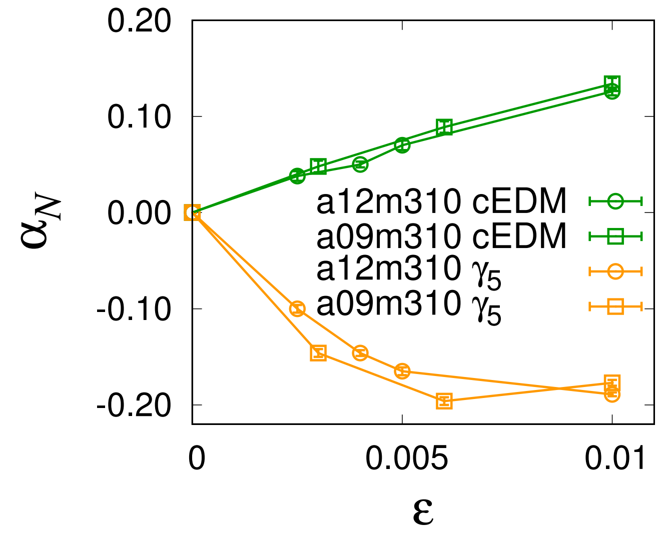
$$\approx \exp\{Tr\ i\varepsilon\Sigma^{\mu\nu}\tilde{G}_{\mu\nu}(\not{D} + m - \frac{r}{2}D^2 + c_{SW}\Sigma^{\mu\nu}G_{\mu\nu})^{-1}\}$$

Calculate

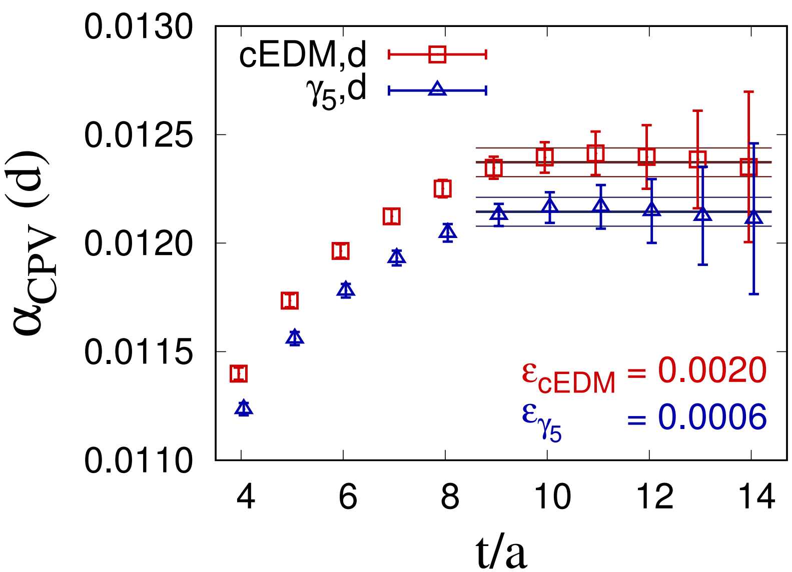


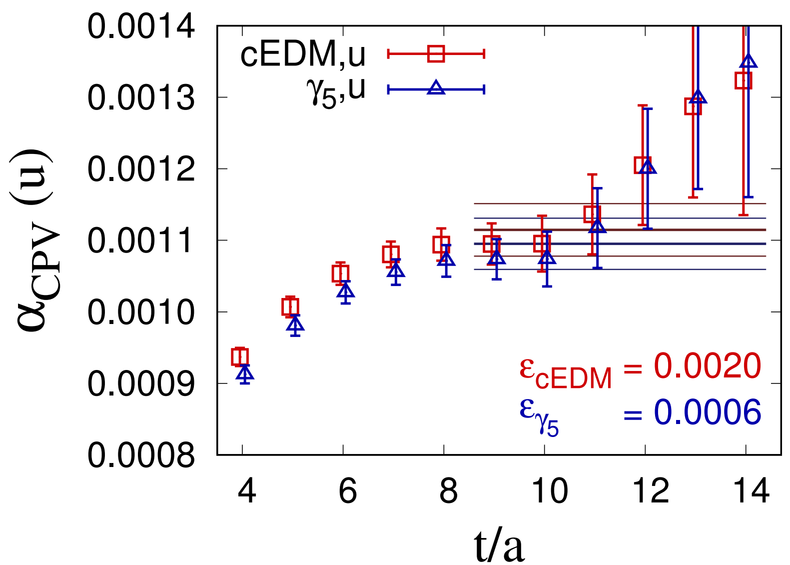
Schwinger Source Method

- Calculation performed at small ε so that results are linear in ε
- cEDM mixes with γ_5, so investigated both operators



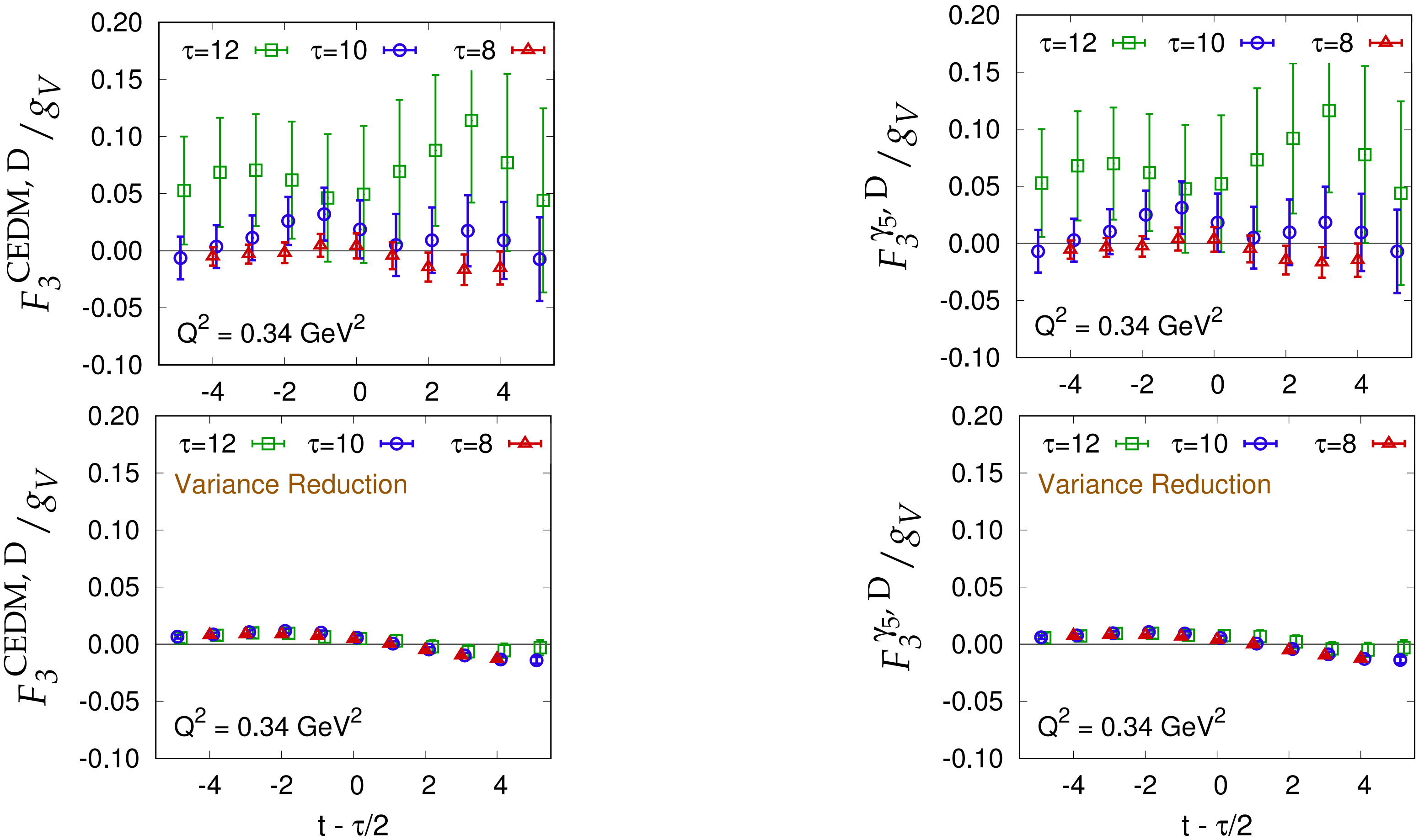
- Test at $a = 0.09$ fm, $m_\pi = 310$ MeV





Variance reduction

Define $X_{\epsilon}^{imp} = X_{\epsilon} - X_{\epsilon=0}$ and exploit correlations



Renormalization

- Renormalization of cEDM Operators are studied
  - 1-loop perturbation on twisted-mass fermion

    [Constantinou, et al, 2015]
  - Nonperturbative RI-$\tilde{\mathrm{S}}$MOM

    [Bhattacharya, et al, 2015]

- Mixing with lower-dimensional operator

$$O_{\mathrm{cEDM}} = a^2 \bar{q} \sigma^{\mu\nu} \gamma_5 G_{\mu\nu} q$$

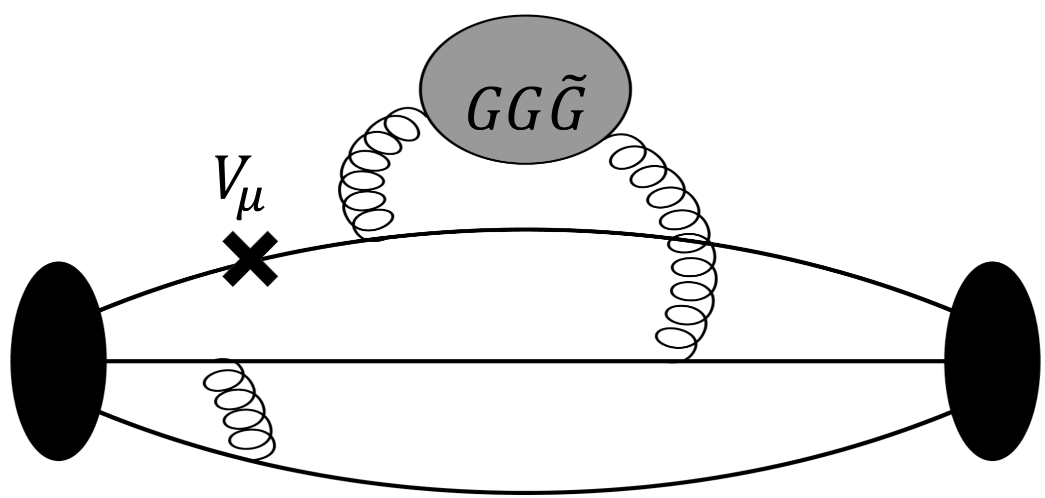
$$O_{\mathrm{P}} = \bar{q} \gamma_5 q$$

  - Divergent $1/a^2$ mixing

Ongoing Work

- Weinberg Three-gluon Operator $d_w \frac{g_s}{6} G\tilde{G}G$

- Renormalization and mixing
  - Gradient Flow



Summary

- **QCD θ-term**

  Actively being calculated and progress at $M_\pi > 330$ MeV;
  need better variance reduction to get precision at $M_\pi = 135$ MeV

- **Quark EDM**

  Calculated: $g_T^d = 0.784(28)$; $g_T^u = -0.204(11)$; $g_T^s = -0.0027(16)$

- **Quark Chromo EDM**

  Exploratory studies show signal in connected contribution;
  next step: disconnected diagrams & renormalization/mixing

- **Weinberg Three-gluon Operator**

  Exploratory studies just started

- **Four-quark Operators**

  Not yet explored

Should have better estimate of accuracy achievable in 1-2 years