General relativity experiment with frozen spin rings

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(based on: A.László,Z.Zimborás:*Class.Quant.Grav.***35**(2018)175003)

Spin2018, Ferrara

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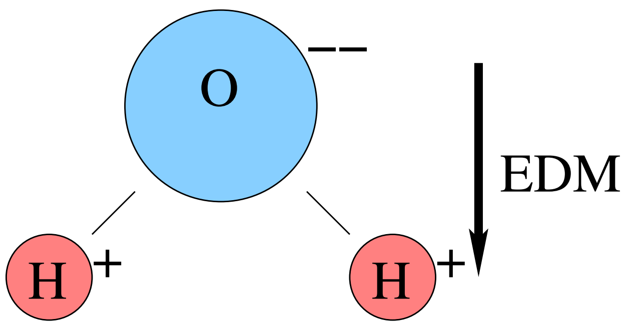
Introduction

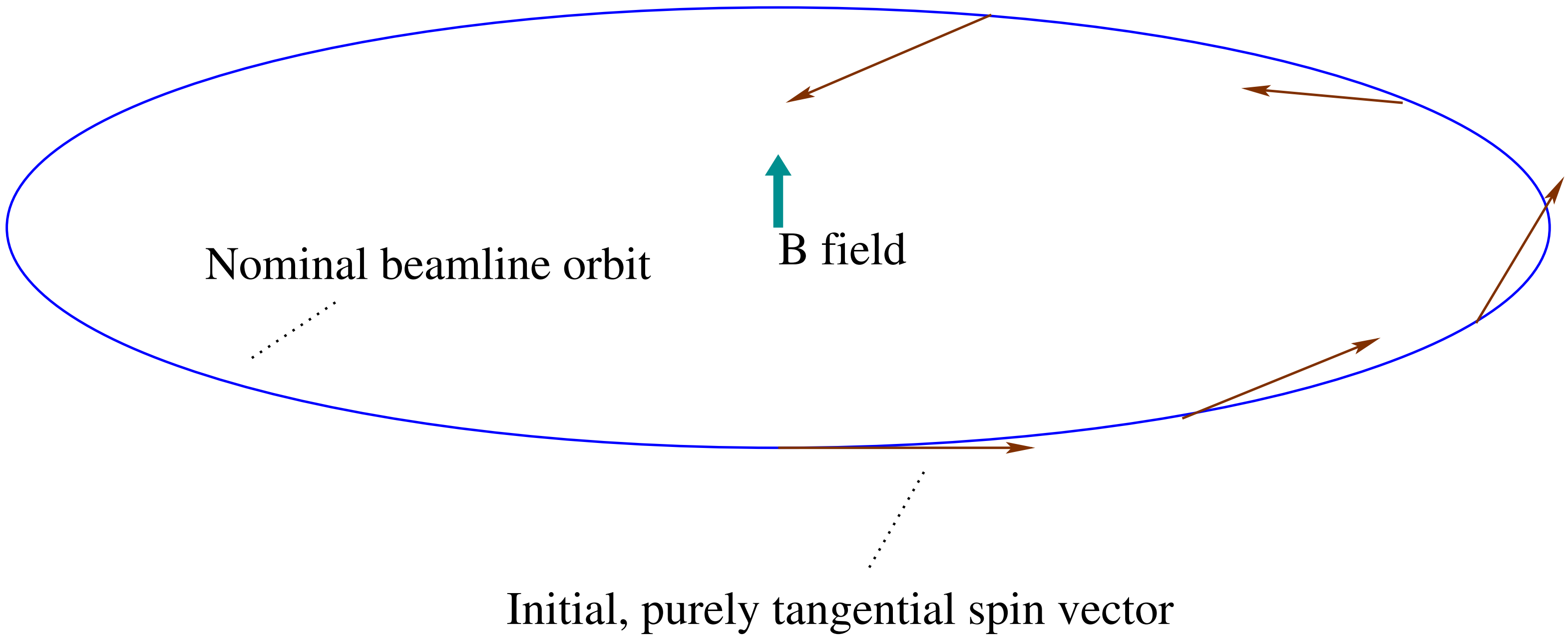
- Magnetic dipole moment is an important quantity: sensitive to radiative corrections.
  For elementary particle obeying classical Dirac equation: $\mathrm{g} := \frac{2\,m\,\mu}{q\,s} = 2$.
  QFT: $\mathrm{g} \approx 2$ but $\mathrm{g} \neq 2$ due to radiative corrections, sensitive to model details.
  For composite particles: g grossly deviates from 2, due to internal structure.
- So, $\mathrm{g}-2$ is a sensitive probe for SM / BSM physics ($\Rightarrow$ $\mathrm{g}-2$ experiment).
  E.g. for muons: $\frac{(\mathrm{g}-2)}{2} \approx 0.0011659209\ldots$ in $3\,\sigma$ tension with SM.
- Electric Dipole Moment (EDM) is also sensitive to internal structure.
  Crude analogy: water molecule. (Or at the level of elementary Lagrangian: CP violation.)



- SM gives negligible EDM, so can be SM / BSM discriminator ($\Rightarrow$ CPEDM collaboration).
  Planned sensitivity: $10^{-29}\,\mathrm{e\,cm}$, can see new physics up to $\approx 3000\,\mathrm{TeV}$ mass.

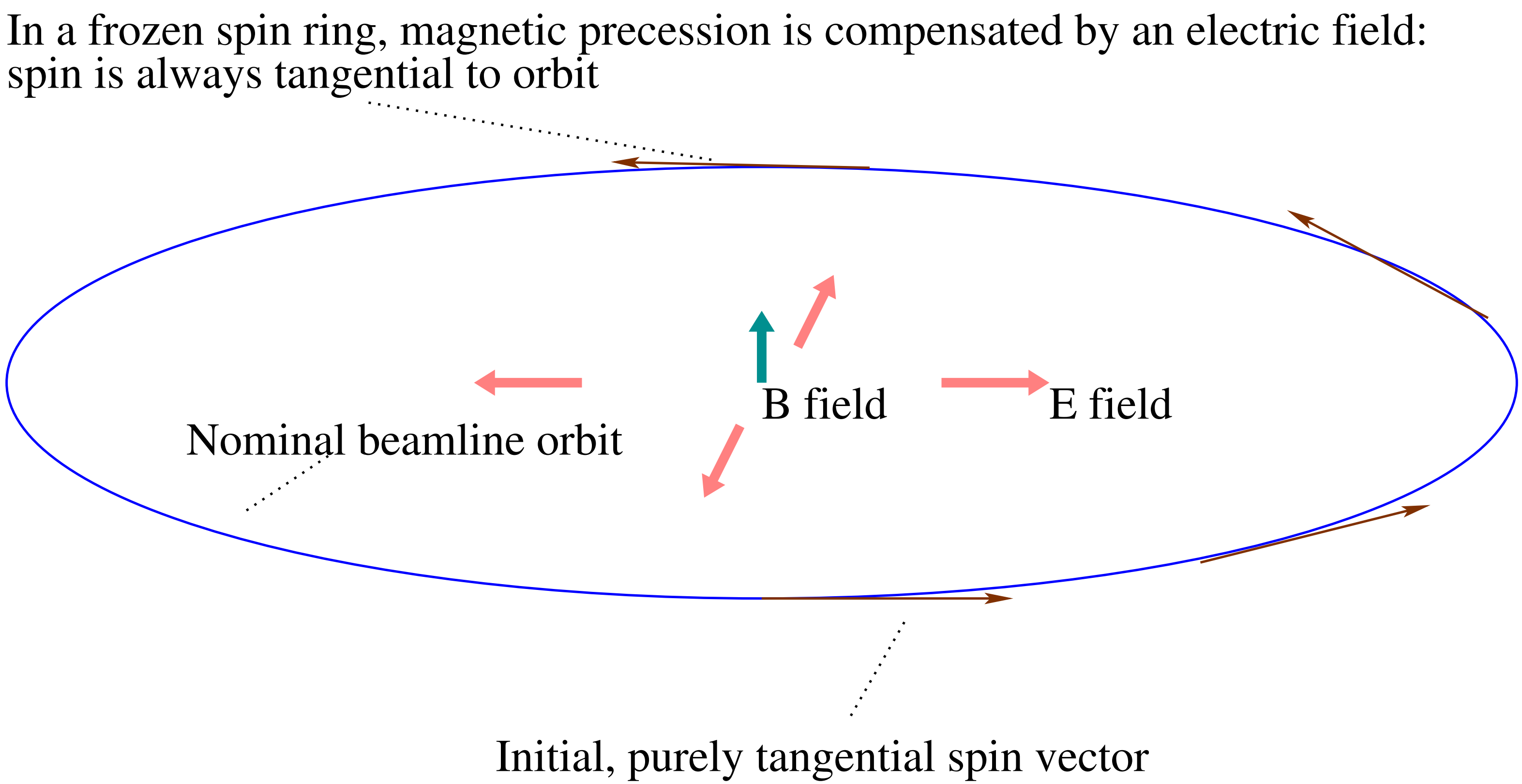
How the magnetic moment anomaly $a := \frac{g-2}{2}$ is measured?

Principle of g–2 measuring storage rings:
in vertical magnetic field, spin precesses in orbital plane at a rate $a\gamma\omega$



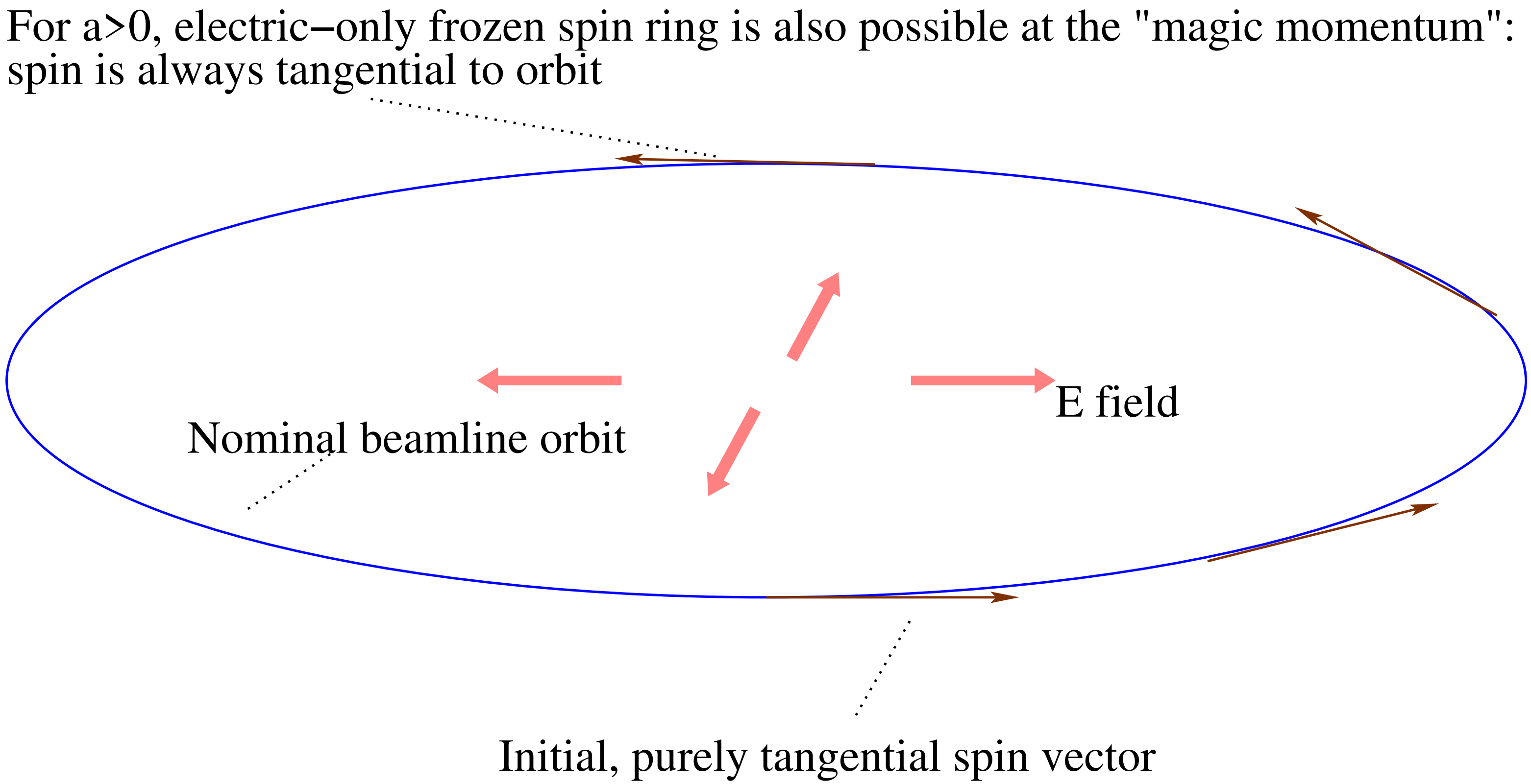
If the particle circular velocity is ω, the spin vector is ahead of momentum vector by $a\,\gamma\,\omega$.

How EDM is measured?



Magnetic precession is compensated by an electric field (“frozen spin”): spin$||$momentum.

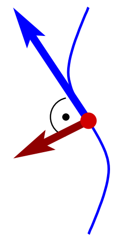
If EDM existed, it would slowly elevate spin out of the orbital plane.



For $a > 0$, electric-only frozen spin ring can be made at “magic momentum” $\beta\gamma = \frac{1}{\sqrt{a}}$.
If EDM existed, it would slowly elevate spin out of the orbital plane.

Relativistic motion of particle with spin in electromagnetic field:

u^a denotes the four velocity of the particle at points of trajectory.



w^a denotes the spin direction four vector at points of trajectory.

(For quantum mechanical reasons, always: $u_a\, w^a = 0$.)

Then the equation of motion is Newton + Thomas-Bargmann-Michel-Telegdi equation.

$$u^a \nabla_a u^b = -\frac{q}{m}\, F^{bc}\, u_c \qquad (\leftarrow \text{ Newton equation with Lorentz force}),$$

$$D_u^F w^b = -\frac{\mu}{s}\left(F^{bc} - u^b u_d\; F^{dc} - \; F^{bd}\, u_d u^c \right) w_c \qquad (\leftarrow \text{ TBMT equation})$$
$$+\frac{d}{s}\left({}^\star F^{bc} - u^b u_d\, {}^\star F^{dc} - {}^\star F^{bd}\, u_d u^c \right) w_c.$$

$D_u^F w^b := u^a \nabla_a w^b + w^a\, u^b u^c \nabla_c u_a - w^a\, u_a u^c \nabla_c u^b$ is the Fermi-Walker derivative.
(Conserves the constraint $u_a\, w^a = 0$. Free gyroscope equation would be $D_u^F w^b = 0$.)

GR corrections?!

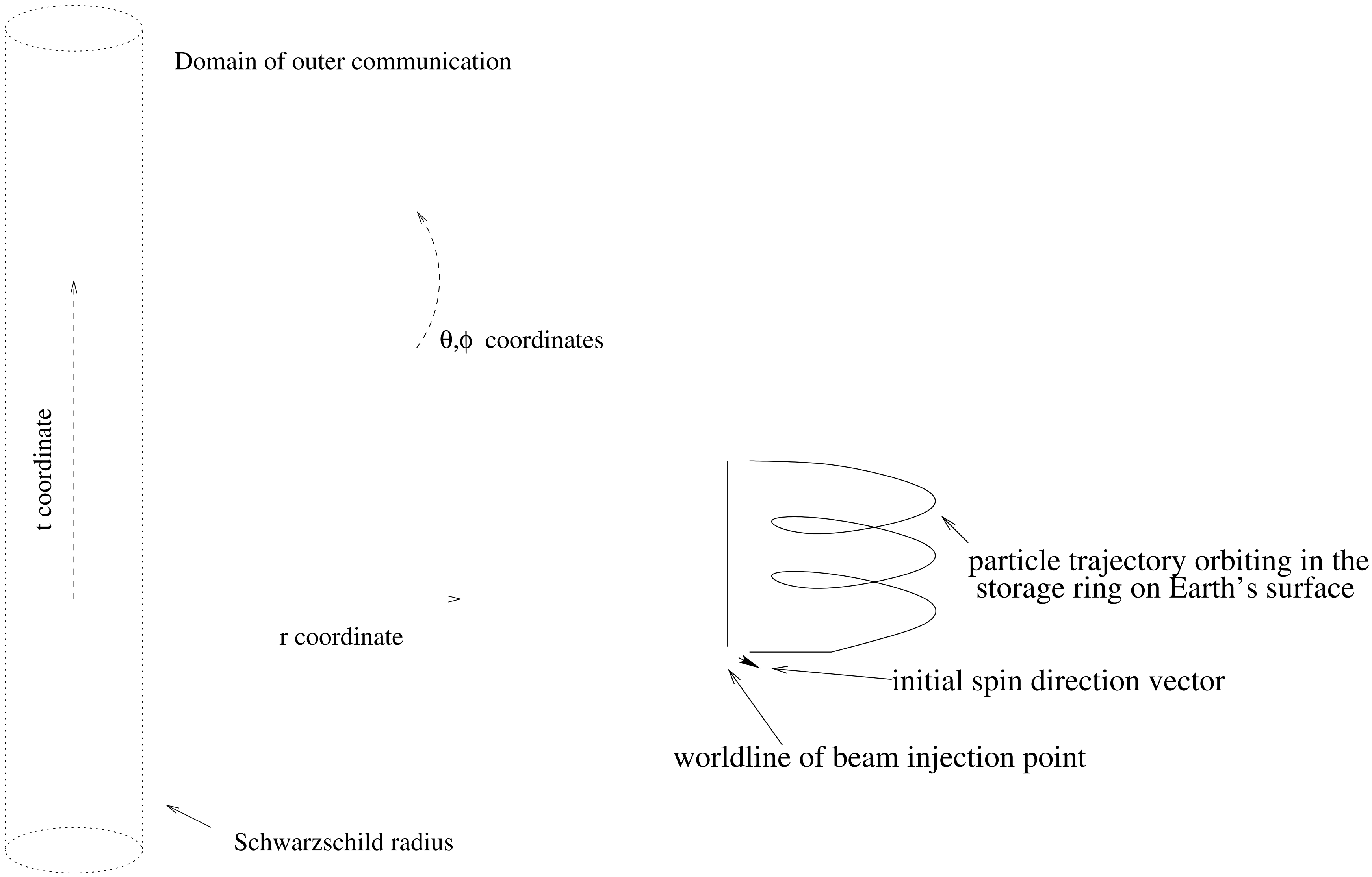
For $\mathbf{g-2}$:

- In February, a series of preprints appeared on arXiv, claiming that GR gives an unaccounted systematic error to $g-2$ experiment, resolving the $3\,\sigma$ tension against SM. T. Morishima, T. Futamase, H. M. Shimizu: `arXiv:1801.10244`, `arXiv:1801.10245`, `arXiv:1801.10246`.
- Other authors responded: GR correction is much smaller than exp.sensitivity 10^{-7}. M. Visser: `arXiv:1802.00651`, P. Guzowski: `arXiv:1802.01120`.
- Further authors: the effect is exactly zero. H. Nikolic: `arXiv:1802.04025`.
- Which is true? From first principles it is difficult to judge.

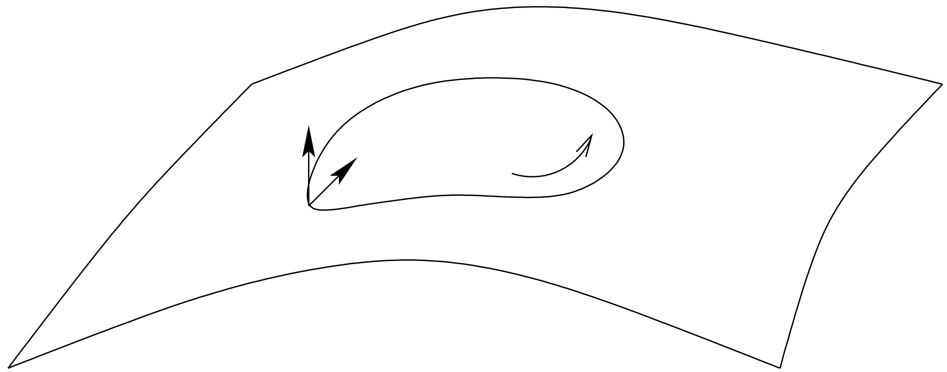
For EDM:

- Earlier papers already warned about possibility for a GR systematics on precession! NPB911(2016)206, PRD76(2007)061101, PRD94(2016)044019, ... Perturbative. (See also: talk by prof. Nikolai Nikolaev.)
- Explicitely first calculated for electric-only frozen spin ring in PLA376(2012)2822. Perturbative. Quantitatively OK!!
- One thing for sure from first principles: only Earth can contribute (equivalence principle).

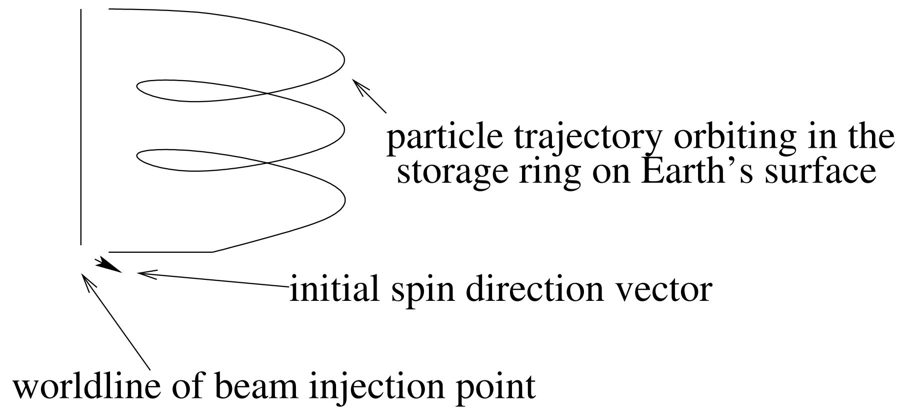
The kinematic configuration in GR setting



Since vectors are transported along closed curves, GR effect is very likely nonzero.



difference is expected due to curvature



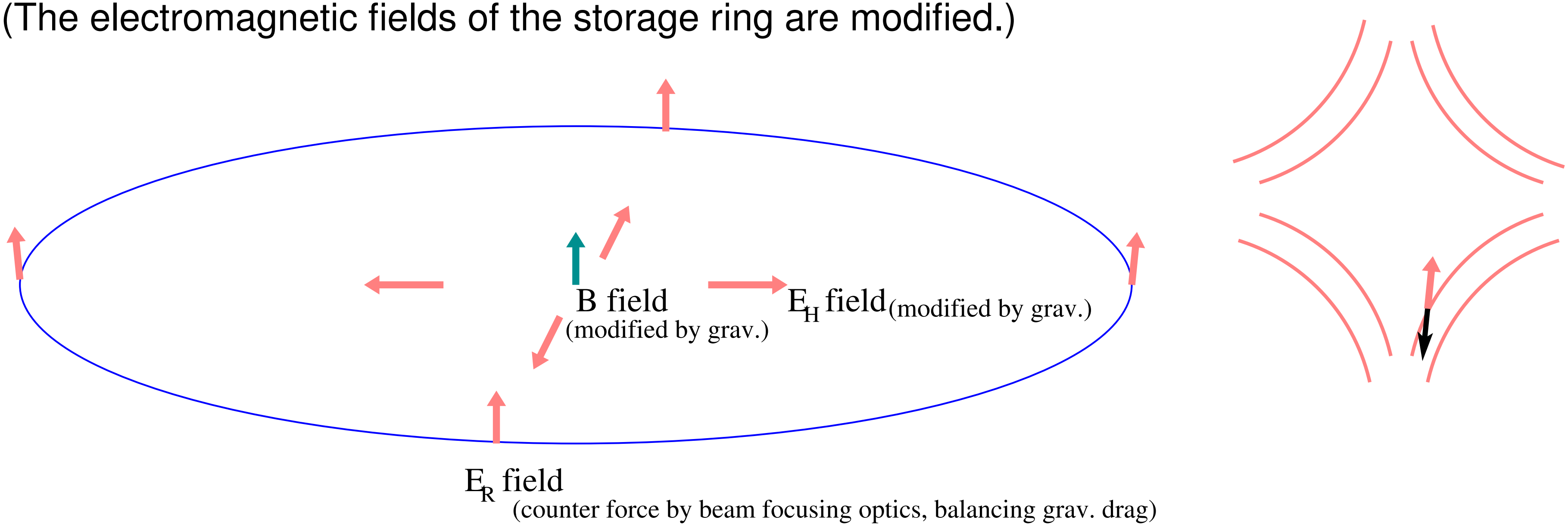
Because curvature just measures that.

But the effect can be small.

Question is: can this be substantially large?

What does the GR modify?

- The spacetime metric, and thus the parallel transport ∇_a, Fermi-Walker derivative D^F. (Newton and TBMT equations of motion are modified.)
- The Maxwell equations $\nabla_a F^{ab} = 0,\ \nabla_a {}^{\star}F^{ab} = 0$. (The electromagnetic fields of the storage ring are modified.)



Notion of “vertical homogeneous magnetic field”, “horizontal cylindrical electric field”, and “Earth-radial electric field” makes sense and calculable over Schwarzschild.

These electomagnetic fields are necessary for modeling fields in idealized storage ring.

Results

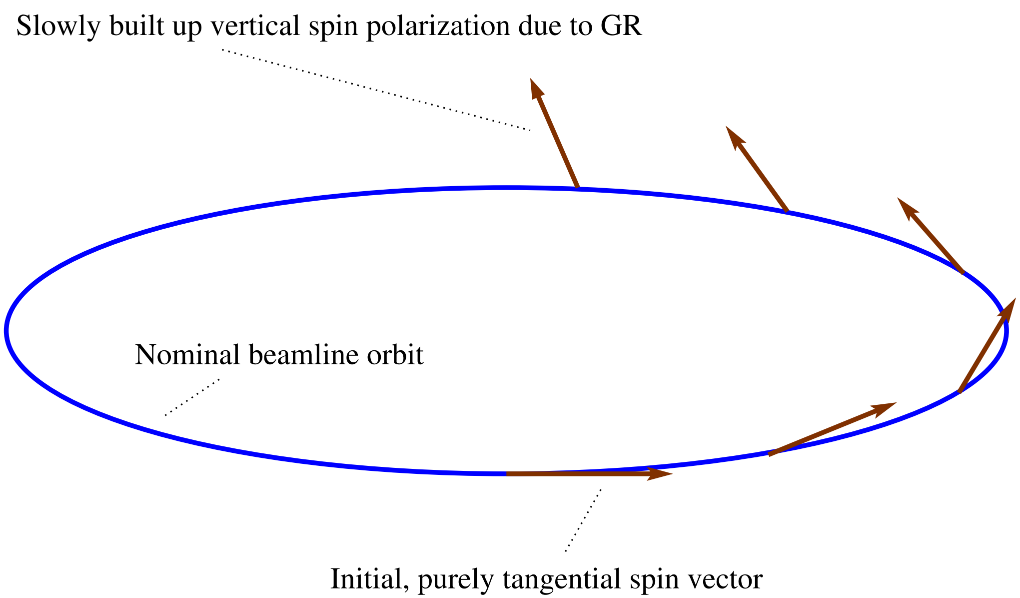
A. László, Z. Zimborás: *Class.Quant.Grav.***35**(2018)175003.

(R: Earth radius, r_S: Schwarzschild radius, L: storage ring radius, g: grav.accel.)

- **g − 2:** Systematic errors by GR is $\sim \frac{r_S}{R}\frac{L^2}{R^2} \approx 10^{-21}$, pretty much negligible. Qualitative reason: precession due to magnetic moment anomaly is relatively large effect, plus GR mainly modifies precession in the other direction.

- **EDM:** In a "frozen spin" storage ring GR torques the spin vector out of the orbital plane, at a rate $-a\,\beta\gamma\,\underbrace{g/c}_{\approx 33\,\mathrm{nrad/sec}}$. This fake EDM signal is $\approx 10\times$ foreseen sensitivity.

(What about optimizing for specific GR experiment?)



Optimalization

We need to optimize for large $|a\,\beta\gamma|$. Grows unboundedly with γ, but becomes expensive.

- Given $a := \frac{g-2}{2}$ and $a\,\beta\gamma$, the Newton equation for circular motion and the “frozen spin” condition uniquely determines the B and E_H.
  (2 equations, for 2 variables.)

$$
\begin{aligned}
E_H\,L &= -\,\mathrm{sign}(a)\,\frac{m\,c^2}{q}\,\frac{(a\,\beta\gamma)^2\sqrt{a^2+(a\,\beta\gamma)^2}}{a^2\,(1+a)},\\
B\,L &= \frac{m\,c}{q}\,\frac{(a\,\beta\gamma)(a-(a\,\beta\gamma)^2)}{a^2\,(1+a)}
\end{aligned}
$$

- Observe:
  The necessary $|E_H|$ grows monotonically as $\sim|a\,\beta\gamma|^3$, for large $|a\,\beta\gamma|$.
  The necessary $|E_H|$ decreases as $\sim|a|^{-2}$, for large $|a|$.

  Experimental limitation is in $|E_H|$: above $8\,\mathrm{MV/m}$, essentially impossible.
- Experimental idea:
  Use large $|a|$ particle (nucleus), so that too large $|E_H|$ can be avoided.

Experimental / financial constraints:

ring radius L maximum $\sim 10\,\mathrm{m}$,

magnetic field $|B|$ maximum $\sim 1\,\mathrm{Tesla}$,

electric field $|E_H|$ maximum $\sim 8\,\mathrm{MV/m}$.

Let us aim for a GR signal strength $|a\,\beta\gamma| = 0.4 \quad (13.1\,\mathrm{nrad/sec})$.

Assume a surely realistic electric field $|E_H| = 4.10\,\mathrm{MV/m}$.

Possible settings:

particle	a $(\approx)$	L [m]	$\lvert B\rvert$ [Tesla]	p [MeV/c]	$\mathcal{E}_{\mathrm{kin}}$ [MeV]
triton	7.92	**1.55**	0.0335	141.9	3.58
helion3	−4.18	**4.13**	0.0353	268.5	12.8
proton	1.79	**7.50**	0.0304	209.7	23.1

Not realistic settings:

particle	a $(\approx)$	L [m]	$\lvert B\rvert$ [Tesla]	p [GeV/c]	$\mathcal{E}_{\mathrm{kin}}$ [GeV]
deuteron	−0.142	1796	0.0243	5.283	3.731
electron	0.00116	5942	0.0136	0.1765	0.1760
muon	0.00116	1228520	0.0136	36.497	36.391

Summary

- GR gives substantial contribution to "frozen spin" EDM experiments, of magnitude $-a\,\beta\gamma\,\dfrac{g}{c}$.
  A.László,Z.Zimborás:*Class.Quant.Grav.***35**(2018)175003
  (Y.Orlov,E.Flanagan,Y.Semertzidis:*Phys.Lett.***A376**(2012)2822 for purely electrostatic)
- Dedicated GR experiment via maximizing this contribution? The signal grows unboundedly with γ.
- Technical limiting factor is the necessary electric field. This can be decreased via using large $|a|$ particle.
- With modest energy triton, helion3, or proton beams, seems to be OK (?).
- For electric-only frozen spin ring: with triton, $4\times$ less electric bending power is enough in comparison to proton. Also GR effect would be $2\times$ bigger.

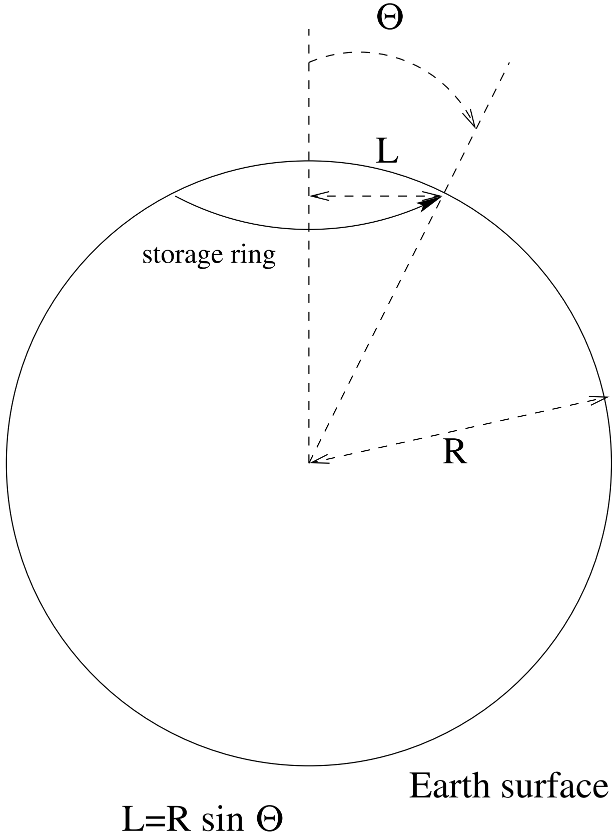
Backup

Observation: triton, helion3, or proton beam can be OK.
For triton beams, it can be "tabletop" experiment!

Open questions:

- spin polarized ion source (seems to be feasible),
- acceleration without depolarization (seems to be feasible),
- precision storage ring with large spin coherence time (seems to be feasible),
- polarimetry (probably doable, is carbon polarimetry OK at these low energies?)

Coordinate conventions: Schwarzschild metric:



$$g_{\mathsf{ab}}(t,r,\vartheta,\varphi) = \begin{pmatrix} 1-\frac{r_S}{r} & 0 & 0 & 0 \\ 0 & -\frac{1}{1-\frac{r_S}{r}} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\,\sin^2\vartheta \end{pmatrix}$$

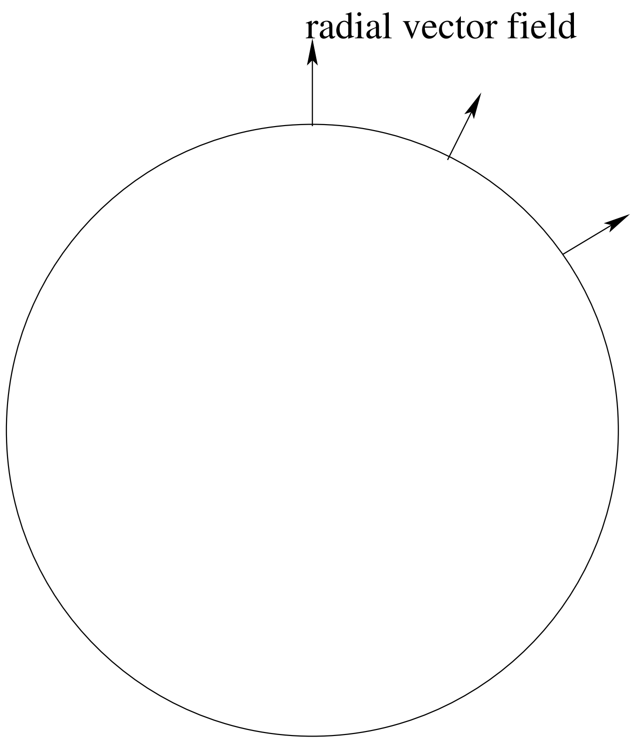
Schwarzschild metric is time translation (t) and rotationally (ϑ, φ) invariant.

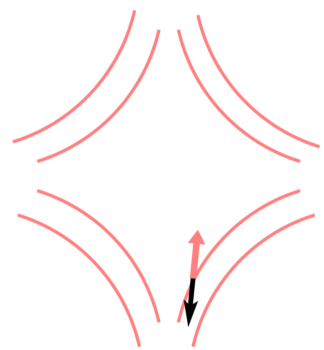
Earth surface at: $r = R = const$.

The storage ring: $r = R = const$, $\vartheta = \Theta = const$.

Time: in terms of proper time along curves (along laboratory and particle trajectory).

Earth-radial electrostatic field effectively exerted by beam focusing optics:

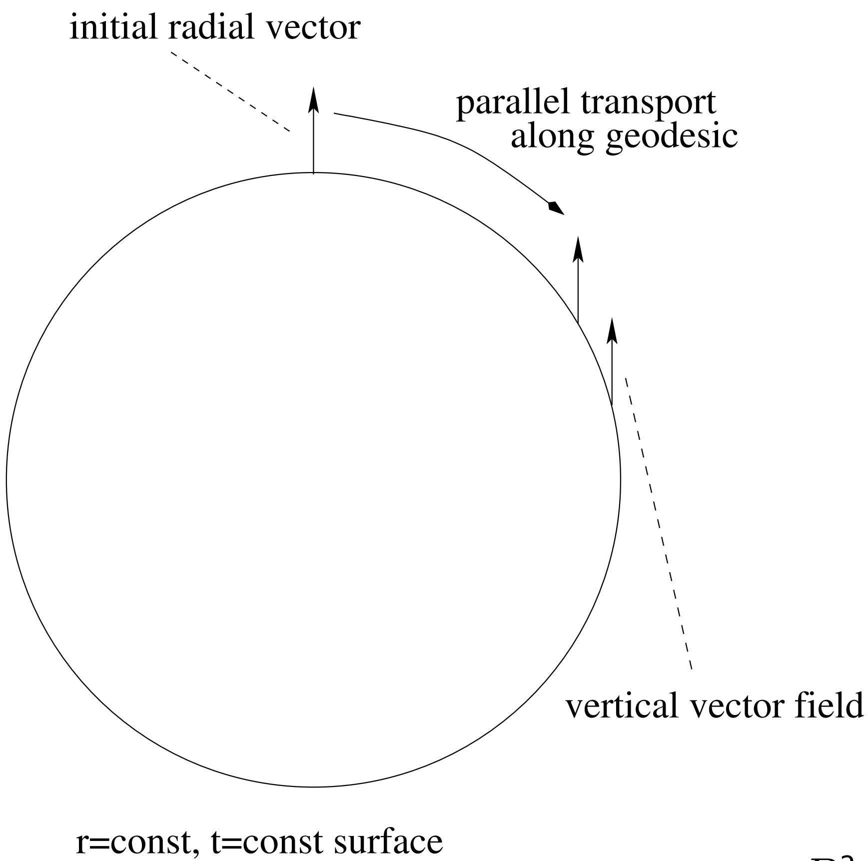




$$E_R{}^a(t, r, \vartheta, \varphi) = E_R \, \frac{R^2}{r^2} \begin{pmatrix} 0 \\ \sqrt{1 - \frac{r_S}{r}} \\ 0 \\ 0 \end{pmatrix}$$

Holds the beam against falling. (Field of charged spherical shell around graviting center.)

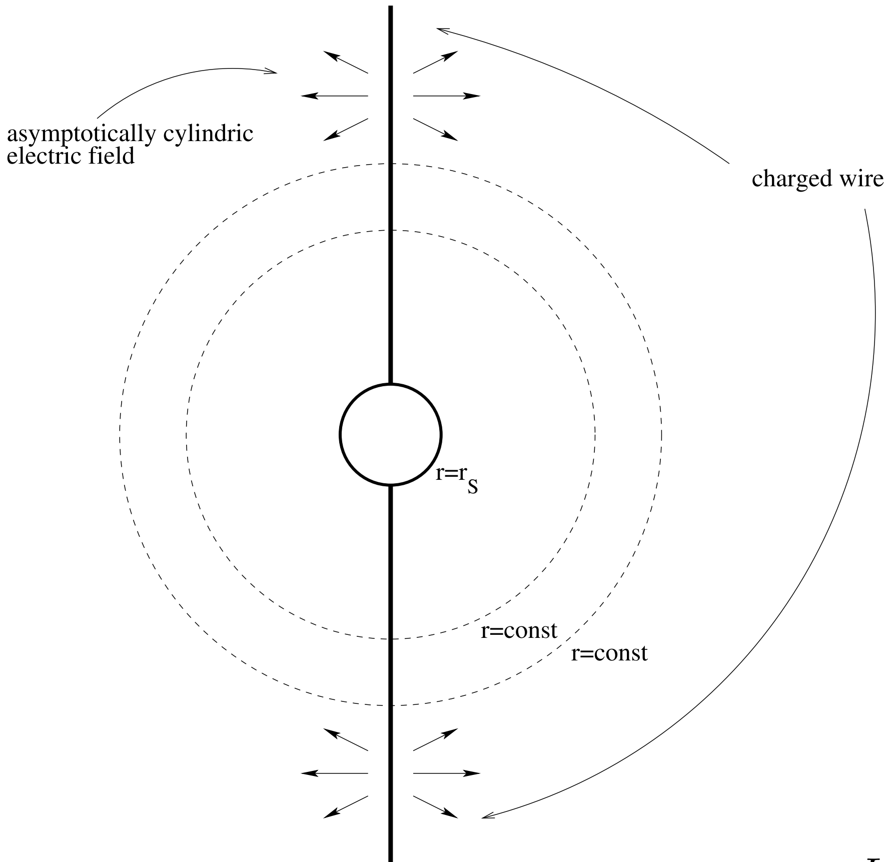
Vertical magnetic field (field inside infinite solenoid):



$$B^{\mathsf{a}}(t,r,\vartheta,\varphi) = B\sqrt{\frac{1-\frac{r_S}{r}}{1-\frac{r_S}{R}\left(\frac{L}{R}\right)^2}}\begin{pmatrix}0\\ \cos\vartheta\\ -\frac{1}{r}\ \sin\vartheta\\ 0\end{pmatrix}$$

Bending field.

Horizontal electrostatic field (field of infinite uniformly charged suspended wire):



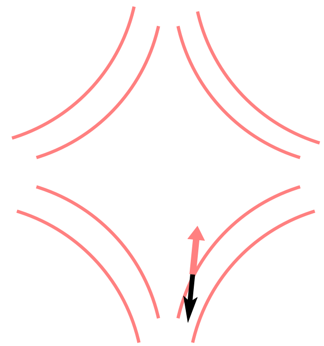
$$E_H{}^{a}(t,r,\vartheta,\varphi) = E_H \, \frac{L}{r\,\sin\vartheta}\,\sqrt{1-\frac{r_S}{r}}\,\mathcal{N}_{r_S}\begin{pmatrix} 0 \\ \sin\vartheta\,\left(1+\frac{r_S}{r}\,\ln(\frac{1}{2}\sin\vartheta)\right) \\ \frac{1}{r}\,\cos\vartheta \\ 0 \end{pmatrix},$$

Bending field.

$$\text{with } \mathcal{N}_{r_S} := \left(\left(\frac{L}{R}\right)^2\left(1+\frac{r_S}{R}\ln\left(\frac{L}{2R}\right)\right)^2 + \left(1-\left(\frac{L}{R}\right)^2\right)\left(1-\frac{r_S}{R}\right)\right)^{-\frac{1}{2}}$$

Part of the contribution is coming merely from:

$$-\frac{\mu}{s}\left(F_{bc}^{E_R} - u_b u^d F_{dc}^{E_R} - F_{bd}^{E_R} u^d u_c\right) \qquad (\leftarrow \text{ Larmor precession by } E_R)$$



E_R merely compensates the gravitational drag of Earth:

Little to do with GR, kind of “classical” effect.

What is the “real” GR contribution, other than E_R ?

Answer:

$$\text{“real” GR} \;:\; E_R \qquad = \qquad 1 \;:\; (1+a)$$

for all $\beta\gamma$.

To what extent it is GR modification of kinematics vs Larmor precession?

$$-\overbrace{\frac{\mathrm{g}-2}{2}}^{a}\;\beta\gamma\;\; g/c \qquad (\leftarrow \text{vertical polarization buildup rate by GR})$$

$$\underbrace{D^F_u\, w^b}_{\substack{\text{kinematics}\\ \text{(Thomas)}}} \;=\; \underbrace{-\,\mathrm{g}\,\frac{q}{2\,m}\left(\,F^{bc} - u^b u_d\;F^{dc} - \;F^{bd}\,u_d u^c\right)\,w_c}_{\substack{\text{electrodynamics}\\ \text{(Larmor)}}} \qquad (\leftarrow \text{TBMT equations})$$

In “classical” notation (3+1 split in lab), over flat spacetime:

$$\frac{\mathrm{d}\vec{S}_{\mathrm{lab,corot.}}}{\mathrm{d}t_{\mathrm{lab}}} \quad = \quad -\frac{q}{m} \left(a\,\vec{B} + \left(\frac{1}{(\beta\gamma)^2} - a \right) \vec{\beta} \times \vec{E} + \frac{1}{2}\,\eta \left(\vec{E} + \vec{\beta} \times \vec{B} \right) \right) \;\times\; \vec{S}_{\mathrm{lab,corot.}}$$

($a := \frac{\mathrm{g}-2}{2}$ is magnetic moment anomaly, $\eta := \frac{2\,m\,c\,d}{\hbar\,q}$ is the “g” of EDM.)

Within a, one has kinematics (inertial torque $\rightarrow$ relativistically: Thomas), plus Larmor.