TMD evolution as a double-scale evolution

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in collaboration with Ignazio Scimemi
based on [1803.11089]

TMD evolution
is
a double-scale evolution

$$F(x, b; \boldsymbol{\mu}, \boldsymbol{\zeta})$$

This aspect has been completely overlooked. Its account reveals completely novel picture of TMD evolution.

Outlook

- Review of TMD evolution status
- Evolution plane and the general solution
- Induced path dependence of the solution
- Evolution potential
- ζ-prescription and optimal TMD

TMD evolution is used for two practical purposes

- Compare different experiments
- Modeling TMD distribution

$$\frac{d\sigma}{dX} \sim \int d^2 b\, e^{i(bq_T)} H_{ff'}(Q,\mu) F_{f\leftarrow h}(x_1, b; \mu, \zeta_1) F_{f'\leftarrow h}(x_2, b; \mu, \zeta_2)$$

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Minimize $\ln(Q/\mu)$
$\mu = Q$

$\zeta_1\zeta_2 = Q^4$
or
$\zeta_1 = \zeta_2 = Q^2$

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Minimize $\mathbf{L}_\mu$, $\mathbf{L}_{\sqrt{\zeta}}$
$\mu \sim \sqrt{\zeta} \sim b^{-1}$

$$F(x,b;\mu,\zeta) \sim C(x,b;\mu,\zeta) \otimes \mathrm{PDF}(x,\mu)$$

Typical model for TMD includes matching

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$$F(x, b; \mu_f, \zeta_f) = R[b, (\mu_f, \zeta_f) \to (\mu_i, \zeta_i)] F(x, b; \mu_i, \zeta_i)$$

Final scale $\longmapsto$ TMD evolution factor $\longmapsto$ Initial scale

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$\mu \sim \sqrt{\zeta} \sim b^{-1}$

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Typical model for TMD includes matching

TMD evolution equations

$$\mu^2 \frac{d}{d\mu^2} F_{f\leftarrow h}(x,b;\mu,\zeta) = \frac{\gamma_F^f(\mu,\zeta)}{2} F_{f\leftarrow h}(x,b;\mu,\zeta), \quad (1)$$

$$\zeta \frac{d}{d\zeta} F_{f\leftarrow h}(x,b;\mu,\zeta) = -\mathcal{D}^f(\mu,b) F_{f\leftarrow h}(x,b;\mu,\zeta), \quad (2)$$

Solution: $F(x,\mathbf{b};\mu_f,\zeta_f) = R[\mathbf{b};(\mu_f,\zeta_f)\to(\mu_i,\zeta_i)]F(x,\mathbf{b};\mu_i,\zeta_i)$

- γ_F – TMD anomalous dimension
- $\mathcal{D}$ – rapidity anomalous dimension ($= -\frac{\tilde{K}}{2}$[Collins' book], $= K$[Bacchetta, at al,1703.10157])
- Anomalous dimensions are *universal*, i.e. depend only on flavor (gluon/quark).

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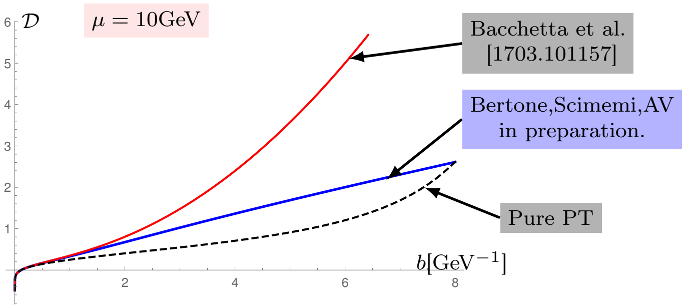
$$\mathcal{D}(\mu,b) = \mathcal{D}_{\text{perp}}(\mu,b^*) + d_{NP}(b)$$

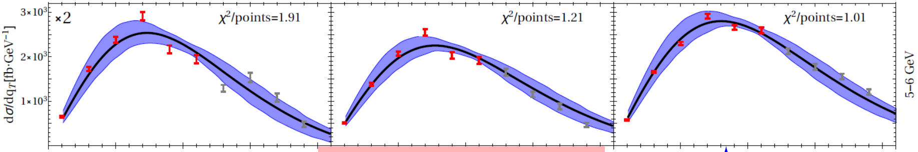
Perturbative part

- Soft/rapidity correspondence $\mathcal{D} \leftrightarrow \gamma_s$ [AV,PRL 118(2017)]
- Everything at NNLO (+Γ-cusp at N^{3}LO [Vogt et al.,1808.08981])



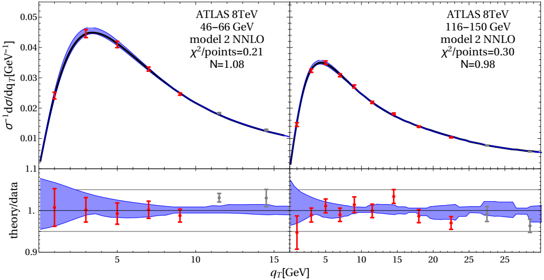
d_{NP} is a universal non-perturbative function. In many aspects more fundamental then TMDs.

Global extraction of F_1 at NNLO by [Scimemi & AV,1706.01473]



Drell-Yan at $Q = 5 - 6\text{GeV}$

Drell-Yan at $Q = 116 - 150\text{GeV}$



TMD evolution is a key element

$$\frac{\chi^2_{\text{global}}}{d.o.f.} \simeq 1.25$$

Here:

- 3-loop evolution
- 2-loop coefficient function
- 2-loop matching
- **ζ-prescription**

plots from [1706.01473]

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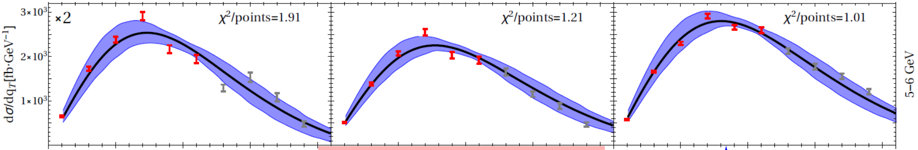

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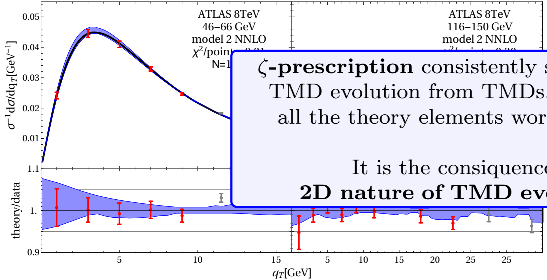

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$$\frac{\chi^2_{\text{global}}}{d.o.f.} \simeq 1.25$$

Here:

- -loop evolution
- -loop coefficient unction
- -loop matching
- **-prescription**

plots from [1706.01473]

ζ-prescription consistently separates the TMD evolution from TMDs, and makes all the theory elements work together.

It is the consiquence of **2D nature of TMD evolution.**

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TMD evolution is two-dimensional

$$\begin{pmatrix} \mu^2 \dfrac{d}{d\mu^2} \\ \zeta \dfrac{d}{d\zeta} \end{pmatrix} F = \begin{pmatrix} \dfrac{\gamma_F}{2} \\ -\mathcal{D} \end{pmatrix} F \quad \longleftrightarrow \quad \vec{\boldsymbol{\nabla}} F = \vec{\mathbf{E}} F$$

$\vec{\mathbf{E}}$ is 2D evolution field in $\vec{\boldsymbol{\nu}} = (\ln \mu^2, \ln \zeta)$ coordinates

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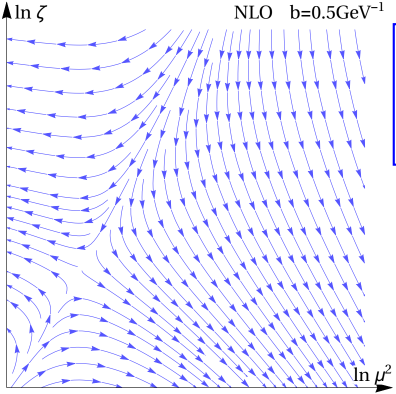

Solution

$$R[(\mu_f, \zeta_f) \to (\mu_i, \zeta_i)] = \exp\left(\int_P d\vec{\boldsymbol{\nu}} \cdot \vec{\mathbf{E}} \right)$$

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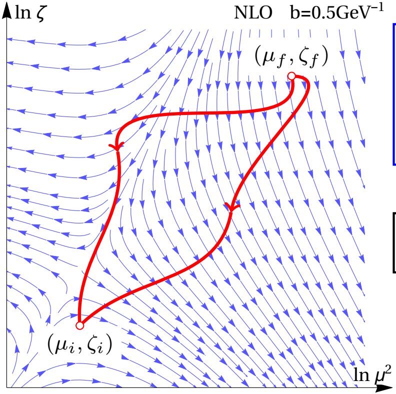
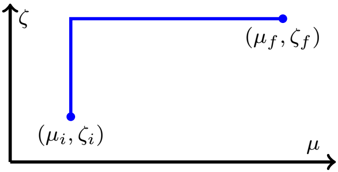


Solution

$$R[(\mu_f,\zeta_f)\to(\mu_i,\zeta_i)] = \exp\left(\int_P d\vec{\boldsymbol{\nu}}\cdot\vec{\mathbf{E}}\right)$$

The integration path is unimportant!

Examples



Solution 1

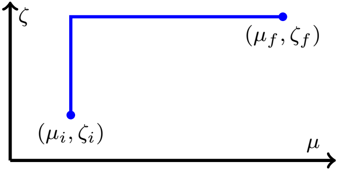
$$\ln R = \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F(\mu, \zeta_f) - \mathcal{D}(\mu_i, b) \ln\left(\frac{\zeta_f}{\zeta_i}\right)$$

[Collins' textbook],[Aybat,Rogers,1101.5057],...

99% popular

$$\times \exp\left\{ \ln \frac{\sqrt{\zeta_A}}{\mu_b} \tilde{K}(b_*; \mu_b) + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_D(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_A}}{\mu'} \gamma_K(g(\mu')) \right] \right\}. \tag{13.70}$$

Examples



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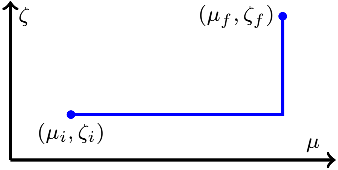

Solution 2

$$\ln R = \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F(\mu, \zeta_i) - \mathcal{D}(\mu_f, b) \ln\left(\frac{\zeta_f}{\zeta_i}\right)$$

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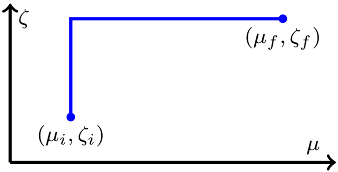
Examples



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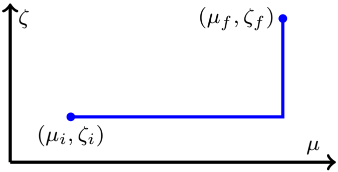

Solution 2

$$\ln R = \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \gamma_F(\mu, \zeta_i) - \mathcal{D}(\mu_f, b) \ln\left(\frac{\zeta_f}{\zeta_i}\right)$$

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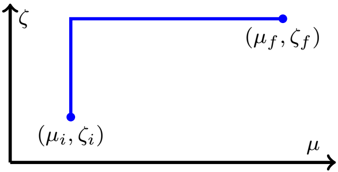

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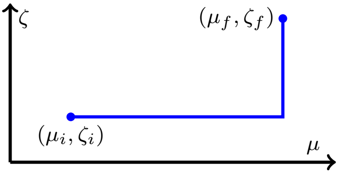

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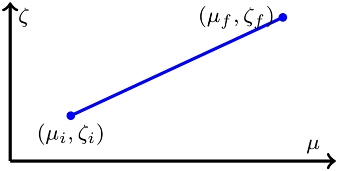

Solution 3

$$\ln R = \int_0^1 \Big(\gamma_F(\mu(t), \zeta(t)) \frac{\mu_f - \mu_i}{(\mu_f - \mu_i)t + \mu_i} - \mathcal{D}(\mu(t), b) \frac{\zeta_f - \zeta_i}{(\zeta_f - \zeta_i)t + \zeta_i} \Big) dt$$

Unique solution

Solution exist only if *integrability condition* holds $$\zeta \frac{d\gamma_F}{d\zeta} = -\mu^2 \frac{d\mathcal{D}}{d\mu^2}$$	$\longleftrightarrow$	$$\vec{\boldsymbol{\nabla}} \times \vec{\mathbf{E}} = 0$$ $\vec{\mathbf{E}}$ is conservative field

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Integrability condition is trivially satisfied due to collinear overlap of divergences

$$\zeta \frac{d}{d\zeta}\gamma_F(\mu,\zeta) = -\Gamma(\mu), \qquad \mu^2 \frac{d}{d\mu^2}\mathcal{D}(\mu,b) = \Gamma(\mu)$$

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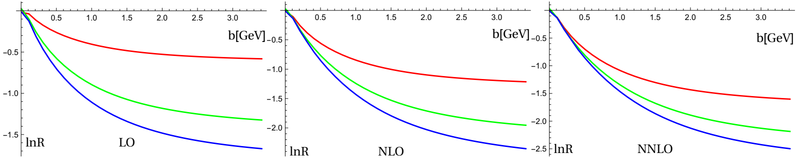
$$\zeta \frac{d}{d\zeta}\gamma_F(\mu,\zeta) = -\Gamma(\mu), \qquad \mu^2 \frac{d}{d\mu^2}\mathcal{D}(\mu,b) \neq \Gamma(\mu)$$

In fixed order PT **integrability condition is violated.**
The restoration procedure is ambigous
(large impact at large-b)
See extended dicussion in [Scimemi,AV;1803.11089]

Numerical effect of path dependence

Evolution from M_Z to $1/b^*$

Without Log-resummation $\mathcal{O}(a_s^{n+1}L^n)$



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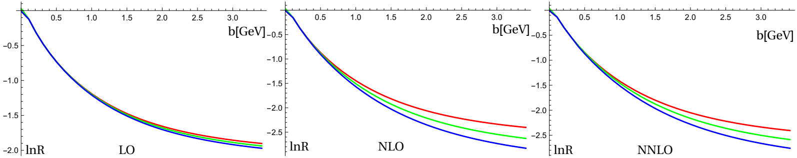

There are methods to eliminate path-dependence by adding higher-PT terms in anomalous dimension. For detailed discussion see [1803.11089].

Evolution potential

Solution exist only if *integrability condition* holds

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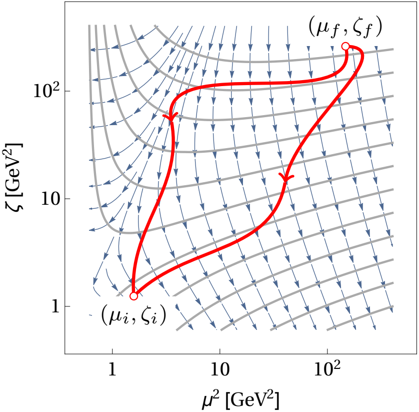
$\vec{\mathbf{E}}$ is conservative field



Conservative field is determined by a potential

$$\vec{\mathbf{E}} = \vec{\boldsymbol{\nabla}} U$$

Evolution is a difference between potentials

$$R[(\mu_f, \zeta_f) \to (\mu_i, \zeta_i)] = \exp\left(U_f - U_i\right)$$

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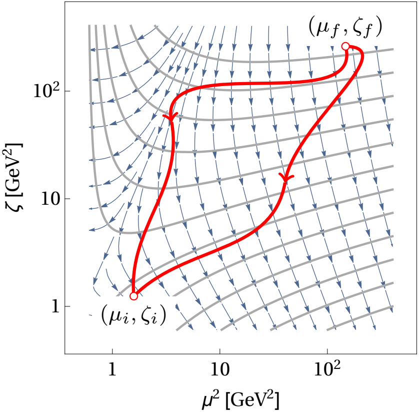

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This absolutely standard picture contains an important message.

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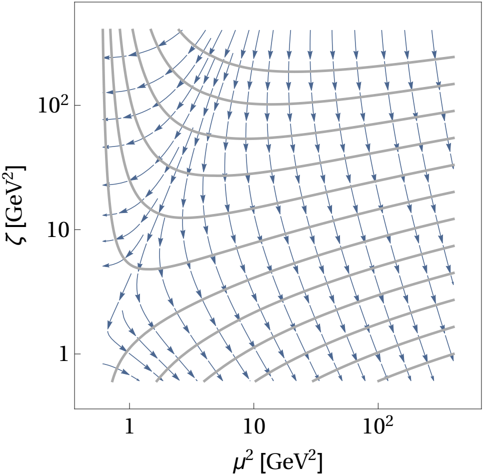
TMD distribution is not defined by a scale (μ, ζ)
It is defined by an equipotential line.



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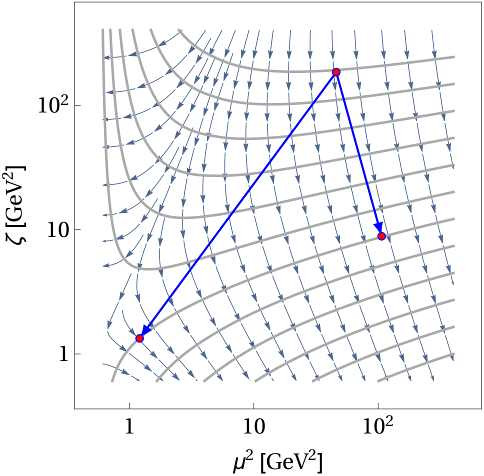
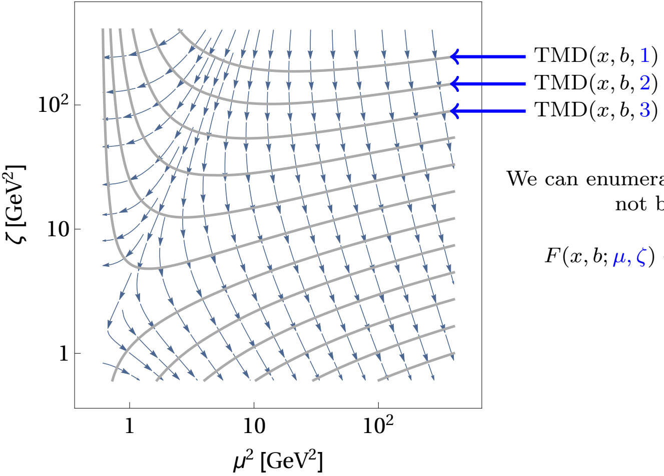


The scaling is defined by
~~a difference between scales~~
a difference between potentials

Evolution factor to both points
is the same
although the scales are
different by 10^2GeV^2

TMD distributions on the same equipotential line are equivalent.



We can enumerate them by a lines not by (μ, ζ)

$$F(x, b; \mu, \zeta) \to F(z, b; \text{line})$$

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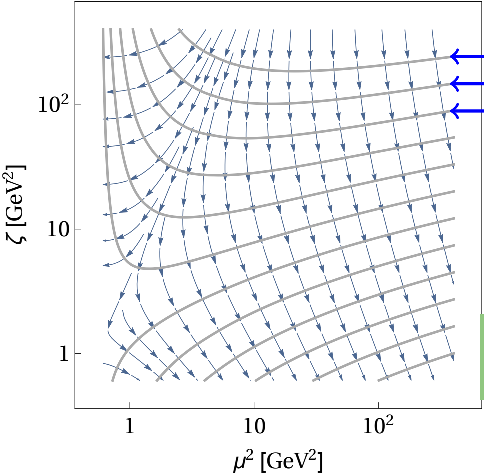

TMD$(x, b, 1)$
TMD$(x, b, 2)$
TMD$(x, b, 3)$

We can enumerate them by a lines not by (μ, ζ)

$$F(x, b; \mu, \zeta) \to F(z, b; \text{line})$$

Initially in [Scimemi,AV,1706.01473] we call it ζ-prescription, which is, probably, not the best name.

In ζ-prescription we set
$\zeta \to \zeta_\mu(\boldsymbol{\nu})$

- TMDs are "enumerated" by $\boldsymbol{\nu}$ (the number of line)
- TMDs are "naive" scale-independent

$$\mu \frac{d}{d\mu} F(x, b; \mu, \zeta_\mu) = 0 \qquad \Rightarrow \text{No double-logs in the matching.}$$

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TMD distribution depends only on the "number" of equipotential line

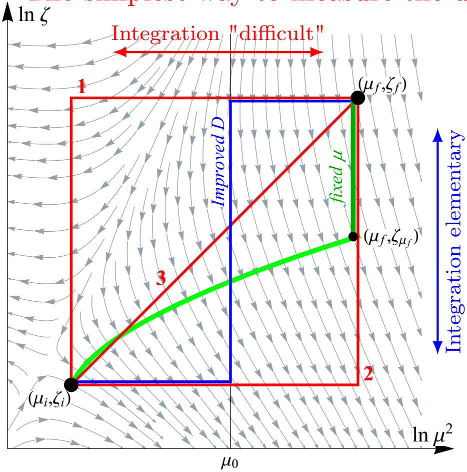
$$F(x, \mathbf{b}; \mu, \zeta) \to F(x, \mathbf{b}; \nu)$$

$$\frac{dF(x, \mathbf{b}; \nu)}{d\nu} = \frac{dU(\mathbf{b}; \nu)}{d\nu} F(x, \mathbf{b}; \nu)$$

$$\Updownarrow$$

$$F(x, \mathbf{b}; \nu) = e^{U(\mathbf{b};\nu) - U(\mathbf{b};\nu_0)} F(x, \mathbf{b}; \nu_0)$$

The simplest way to measure the difference between potentials



$$R = \left(\frac{\zeta_f}{\zeta_{\mu_f}}\right)^{-\mathcal{D}(\mu_f,b)}$$

- Numerically simple (and fast). Compare to

$$\times \exp\left\{\ln\frac{\sqrt{\zeta_A}}{\mu_b}\tilde{K}(b_*;\mu_b) + \int_{\mu_b}^{\mu}\frac{d\mu'}{\mu'}\left[\gamma_D(g(\mu');1) - \ln\frac{\sqrt{\zeta_A}}{\mu'}\gamma_K(g(\mu'))\right]\right\}. \quad (13.70$$

- $\mu_f = Q$ thus a_s is small
- **It is different representation of the Sudakov exponent.**

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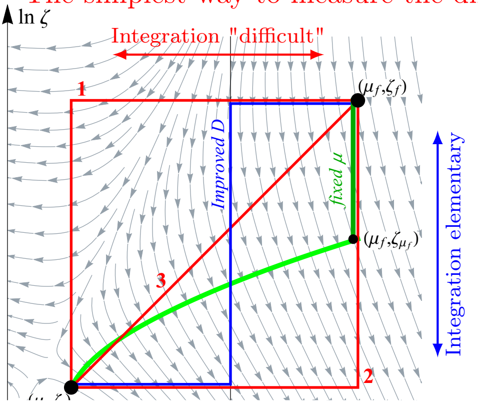

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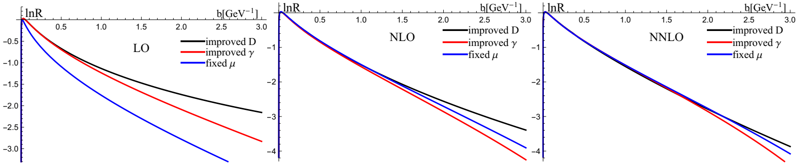
Different solutions converge with increase of PT order



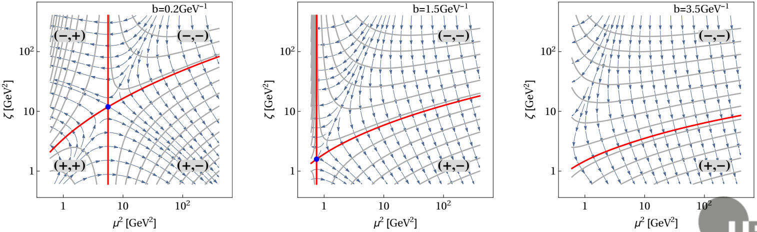
Universal scale-independent TMD

There is a unique line which passes though all μ's

The optimal TMD distribution

$$F(x,b) = F(x,b;\mu,\zeta_\mu)$$

where ζ_μ is the special line.



TMD cross-section

$$\frac{d\sigma}{dX} = \sigma_0 \sum_f \int \frac{d^2 b}{4\pi} e^{i(b\cdot q_T)} H_{ff'}(Q)\{\tilde{R}^f[b;Q]\}^2 \tilde{F}_{f\leftarrow h}(x_1, b)\tilde{F}_{f'\leftarrow h}(x_2, b),$$

with $\zeta_f = \mu_f^2 = Q^2$

$$\tilde{R}^f[b;Q] \quad = \quad (Qb)^{-\mathcal{D}^f(Q,b)} \exp\{-\mathcal{D}^f(Q,b) v^f(Q,b)\}$$

- v is given by the perturbative series, $v = \frac{3}{2} + a_s \ldots$
- $\tilde{F}$ is TMD in the "naive" ζ-prescription

- There are only (μ_f, ζ_f) scales and no solution dependence.
- **Clear separation of TMD evolution from the model for TMD distribution.**

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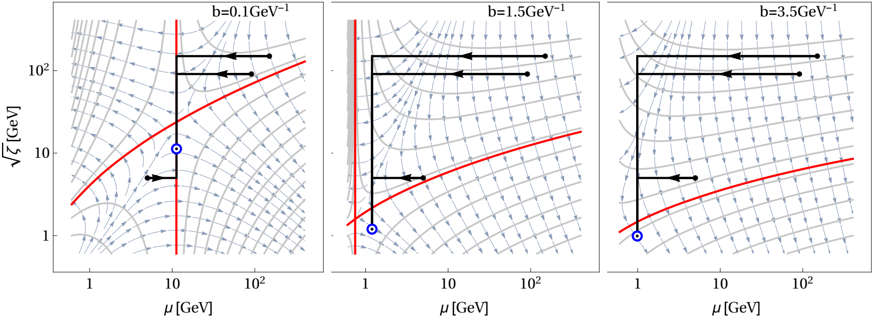
Evolution with b-dependent scale (CSS-like)
$(Q,Q^2)\to(\mu_b,\mu_b^2)$



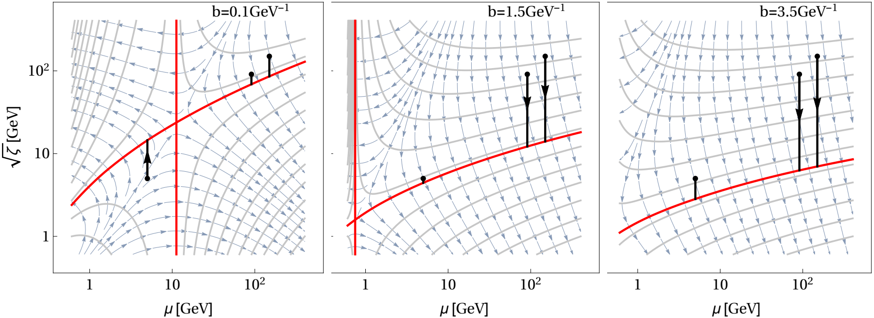
Here $\mu_b = \dfrac{C_0}{b^*}$ with $b_{\max} = 1.2\text{GeV}^{-1}$

Analogy in DIS

Scale depends on paremeter $\leftrightarrow$ $d\sigma = C(Q)R[Q\to \text{ch}(x)]f(x,\text{ch}(x))$

PDF $f(x,\text{ch}(x))$ has no interpretation, no sense, and **depends on the order of evolution in use.**

Optimal version
$(Q, Q^2) \to (Q, \zeta_Q)$



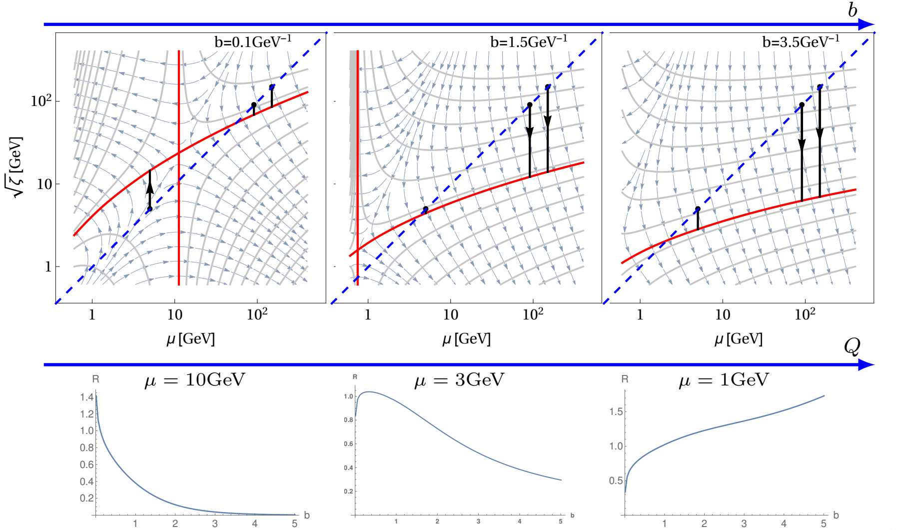
Analogy in DIS

Scale (potential) is fixed $\leftrightarrow$ $d\sigma = C(Q)R[Q \to 2\text{GeV}]f(x, 2\text{GeV})$

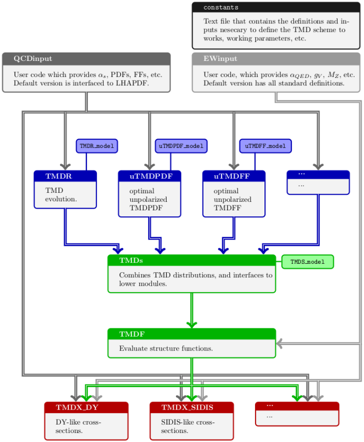
PDF $f(x, 2\text{GeV})$ is just a model and

is dependent on the order of evolution in use.

The evolution potential depends on b.
Relative position of its elements (saddle-point, special lines) dictates the shape of evolution factor.



arTeMiDe v1.3



- Variety of evolutions
- LO, NLO, NNLO
- No restriction for NP models
- Fast code
- DY cross-sections
- SIDIS cross-sections (not tuned yet)
- Theory uncertainty bands

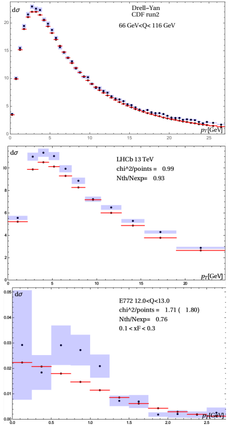
https://teorica.fis.ucm.es/artemide/

Conclusion

Main message:
TMD evolution is a double scale evolution.
Therefore, it should be considered with care, and then it grants many simplifications.

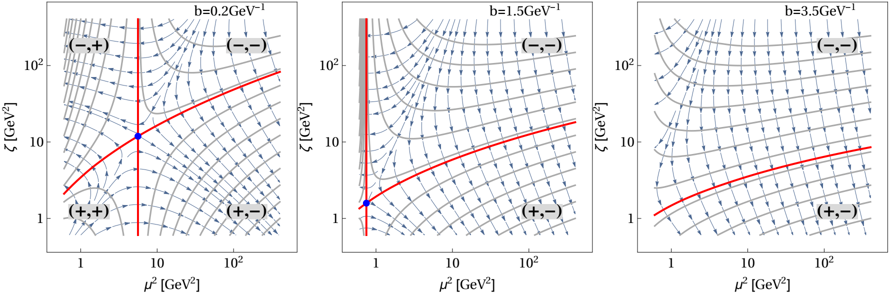
TMD distributions on a same equipotential line are equivalent. Enumerate them with lines!

- Universal for all quantum numbers
- Very simple practical formula (no integrations!)
- Guarantied absence of (large) logarithms in the matching coefficient
- TMD model is independent on evolution order.
  E.g You can use NNLO unpolarized and LO Sivers together, without theory tensions



Double-scale evolution is not unique for TMD case. It also appears in k_T-resummation, joint resummation, DPDs, etc.

Which line is the best?



- Some non-interesting singularities at $\mu, \zeta \to \infty$
- Landau pole at $\mu = \Lambda$
- Saddle point (blue dot)

$$\mathcal{D}(\mu_{\text{saddle}}, b) = 0, \qquad \gamma_M(\mu_{\text{saddle}}, \zeta_{\text{saddle}}, b) = 0$$

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