

Proton and ^3He polarimetry with new forward pp helicity flip amplitudes

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A theoretical study of elastic collisions with spin at momentum transfers in the CNI region in the context of proton and ^3He polarimetry

A description of the HJET data analysis and its implications for the energy behavior of amplitudes and asymmetries is given by Andrei Poblaguev

OUTLINE

- Polarized proton and ^3He beams provide polarized up and down quarks
Musgrave *et al.*, PoS PSTP **2017** (2018) 020
- Polarimeters for energetic polarized beams need a study of amplitudes
 - Examine helicity amplitudes for fermions scattering on p, ^3He , C, ^{13}C
 - The spin physics programme requires precision polarimetry: $< 5\%$
- Review asymmetries in the electromagnetic hadronic interference region
- Express A_N and A_{NN} in terms of values of the hadronic ratios r_5 and r_2
- Analysis of exchanges at 100 and 255 GeV leads to asymmetry prediction

Amplitudes and Asymmetries for Elastic Collisions

$$\begin{aligned}
 \phi_1 &= \langle ++ | M | ++ \rangle, & \phi_4 &= \langle +- | M | -+ \rangle \propto -t \\
 \phi_2 &= \langle ++ | M | -- \rangle, & \phi_5 &= \langle ++ | M | +- \rangle \propto \sqrt{-t} \\
 \phi_3 &= \langle +- | M | +- \rangle, & \phi_6 &= \langle ++ | M | -+ \rangle \propto \sqrt{-t}
 \end{aligned}$$

Hadronic ϕ_1, ϕ_2, ϕ_3 are nonzero at $t = 0$, and $\phi_6 = -\phi_5$ for $pp \rightarrow pp$

$$(k\sqrt{s} / 2\pi) \sigma_{\text{tot}} = \text{Im} [\phi_1(s, 0) + \phi_3(s, 0)]$$

$$\frac{2k^2 s}{\pi} \frac{d\sigma}{dt} = |\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 2|\phi_5|^2 + 2|\phi_6|^2$$

$$A_N \frac{2k^2 s}{\pi} \frac{d\sigma}{dt} = \text{Im} [(\phi_1 + \phi_2 + \phi_3 - \phi_4)^* \phi_5]$$

$$A_{\text{NN}} \frac{2k^2 s}{\pi} \frac{d\sigma}{dt} = \text{Re} [\phi_1^* \phi_2 - \phi_3^* \phi_4 - 2\phi_5^* \phi_6]$$

Scattering of Identical and Non-identical Fermions

- For the elastic reactions $p p \rightarrow p p$ and ${}^3\text{He } {}^3\text{He} \rightarrow {}^3\text{He } {}^3\text{He}$, $\phi_6 = -\phi_5$
- For non-identical $p {}^3\text{He} \rightarrow p {}^3\text{He}$ or $p {}^{13}\text{C} \rightarrow p {}^{13}\text{C}$, in general, $\phi_6 \neq -\phi_5$
NHB, Gotsman, Leader, Phys Rev D18 (1978) 694
- The p-C polarimeter uses a thin carbon ribbon with a 2 MHz event rate
- Here $p {}^{12}\text{C} \rightarrow p {}^{12}\text{C}$ and ${}^3\text{He } {}^{12}\text{C} \rightarrow {}^3\text{He } {}^{12}\text{C}$ have just two amplitudes

The proton carbon polarimeter requires calibration from a H-jet polarimeter operating at a 90 Hz rate to achieve a $\delta P \approx 2\%$ statistical accuracy for an 8-hour RHIC store.

A. Poblaguev *et al.*, PoS PSTP **2017** (2018) 022

Interference Region Elastic Amplitudes

Amplitudes including Coulomb contributions and the Bethe-Soloviev phase

$$\delta_C = -Z^2\alpha \left(\ln |Bt/2 + 4t/\Lambda^2| - \gamma \right), \quad \text{for } t \text{ close to } t_c = -8\pi Z^2\alpha/\sigma_{\text{tot}}$$

$$\phi_1 \approx \phi_3 = \frac{k\sqrt{s}}{4\pi} \sigma_{\text{tot}} \left(i + \rho - \frac{t_c}{t} e^{i\delta_C} \right)$$

$$\phi_2 = \frac{k\sqrt{s}}{4\pi} \sigma_{\text{tot}} \left(2r_2 - \frac{\kappa^2 t_c}{4m_p^2} e^{i\delta_C} \right)$$

$$\frac{m_p}{\sqrt{-t}} \phi_5 = \frac{k\sqrt{s}}{4\pi} \sigma_{\text{tot}} \left(r_5 - \frac{\kappa t_c}{2t} e^{i\delta_C} \right)$$

The hadronic amplitude $\phi_4 \propto t$ is ignored, but $\phi_4^{\text{em}} = -\phi_2^{\text{em}}$ is kept below.

The CM (TS) momentum is given by $k^2 = s - 2m^2 - 2\tilde{m}^2 + (m^2 - \tilde{m}^2)^2$

The reason for significant interference at low momentum transfers may be found by observing that the following terms in the above expression for A_N

$$\text{Im} (\phi_1 + \phi_3)^* \phi_5$$

indicate that the spin averaged amplitude $\phi_1 + \phi_3$ is mainly EM and real below the CNI region. The analyzing power A_N highlights $\text{Im} r_5$ at low $-t$. By contrast, above the CNI region, A_N responds largely to $\text{Re} r_5$.

The opposite is true for the double spin asymmetry A_{NN} where the important amplitudes from the expression given above are

$$\text{Re} (\phi_1^* \phi_2)$$

The electromagnetic interaction particularly emphasises $\text{Re} r_2$ at low $(-t)$. Beyond the CNI region, hadronic $\text{Im} \phi_1$ interferes with $\text{Im} r_2$ most strongly.

The logarithm of the hadronic pp differential cross section as function of t near $t = 0$ has slope B (GeV^2). The EM form factor F_1 has expansion in t

$$F_1(t) = \frac{1 - \mu_p t/4m_p^2}{(1 - t/4m_p^2)(1 - t/\Lambda^2)^2} = 1 + \left(\frac{2}{\Lambda^2} - \frac{\kappa}{4m_p^2} \right) t + \dots$$

where $\Lambda^2 = 0.71 \text{ GeV}^2$ in the case of elastic pp collisions. More generally, to include the case of $^3\text{He } ^3\text{He}$ elastic scattering and its EM form factors,

$$F_1(t) = 1 + F_1' t + \dots$$

leading to a small correction term in the pp cross section below of the form (though larger for the $^3\text{He } ^3\text{He}$ differential cross section, since B is greater)

$$\epsilon = B/2 - 2F_1'$$

The t dependence of hadronic (encapsulated by slope B) and electromagnetic (encoded by a form factor) amplitudes is more significant for reactions involving ${}^3\text{He}$ and Carbon ions by comparison with collisions using protons. The Coulomb phases typically take values

$$\begin{aligned}\delta_C(\text{p p}) &= 2.4\%, & \delta_C({}^3\text{He } {}^3\text{He}) &\approx 9\% \\ \delta_C(\text{p } {}^{12}\text{C}) &= 11\%, & \delta_C({}^3\text{He } {}^{12}\text{C}) &\approx 16\%\end{aligned}$$

the latter being a guess based upon an unknown value for the ${}^3\text{He } {}^{12}\text{C}$ total cross section. It is clear that Coulomb phases need to be taken account of when evaluating a possible ${}^3\text{He}$ carbon polarimeter for polarized ${}^3\text{He}$ beams.

Differential cross section close to $t = t_c$ with hadronic slope parameter B

$$\frac{16\pi}{\sigma_{\text{tot}}^2} \frac{d\sigma}{dt} e^{-Bt} = \left(\frac{t_c}{t}\right)^2 - 2(\rho + \delta_C + \epsilon) \frac{t_c}{t} + 1 + \rho^2$$

Spin dependent observables in interference region of momentum transfer

$$\frac{m_p A_N}{\sqrt{-t}} \frac{8\pi}{\sigma_{\text{tot}}^2} \frac{d\sigma}{dt} e^{-Bt} = \left[\frac{\kappa}{2} (1 + \text{Im } r_2 - \delta_C \rho) - \text{Im } r_5 + \delta_C \text{Re } r_5 \right] \frac{t_c}{t} - (1 + \text{Im } r_2) \text{Re } r_5 + (\rho + \text{Re } r_2) \text{Im } r_5$$

$$A_{\text{NN}} \frac{8\pi}{\sigma_{\text{tot}}^2} \frac{d\sigma}{dt} e^{-Bt} = - \left[\text{Re } r_2 + \delta_C \text{Im } r_2 \right] \frac{t_c}{t} + (\kappa t_c / m_p^2) \text{Re } r_5 + \text{Im } r_2 + \rho (\text{Re } r_2 - \kappa^2 t_c / 4m_p^2)$$

For protons, $\kappa = \mu_p - 1$; for He-3 (h), $\kappa = \mu_h / Z - m_p / m_h$, with $Z = 2$.

Leading contributions to $(\rho + i)\sigma_{\text{tot}}$ at high energy often have even (+) and odd (−) signature terms of the form FMS, IJMP A **32**, 1750184

$$i(\ln s - i\pi/2)^{\beta_+}, \quad (\ln s - i\pi/2)^{\beta_-}$$

Exponents β_+ and β_- are constrained by the Froissart bound $\beta_{\pm} \leq 2$ and by the ‘Cornille plot’ H. Cornille, Lett Nuovo Cimento 4 (1970) 267

$$\sigma_{\text{tot}} \geq \sigma_{\text{el}} \implies \beta^+ + 2 \geq 2\beta^-$$

$$\sigma_{\text{tot}} \geq 0 \implies \beta^+ + 1 \geq \beta^-$$

Spin independent pp data suggest $\beta_+ = 2$, Martynov et al, arXiv:1808.08580
 and possibly $\beta_- = 1$, an Odderon, Joyson et al, Nuovo Cim. A **30** (1975) 345
 a feature of A_{NN} for pp, Leader and Trueman, Phys. Rev. D **61** (2000) 077504

We may write $(\rho + i) \sigma_{\text{tot}}$ as a linear combination, with real coefficients, of Pomeron terms and (f2/a2), Odderon, (ω/ρ) terms of the form

$$iC + i(\ln z)^{\beta_+}, \quad iz^{a_+}, \quad (\ln z)^{\beta_-}, \quad z^{a_-}, \quad \text{where } z = se^{-i\pi/2}/s_0$$

i.e., even signature terms, odd signature terms, where s_0 is typically $4m_p^2$. Values of even and odd exponents often used in the above expressions are

$$\beta_+ = 2, \quad a_+ \approx -0.4, \quad \beta_- = 1, \quad a_- \approx -0.6$$

Spin dependent amplitudes, $r_j \sigma_{\text{tot}}$, $j \in \{5, 2\}$, may also be written as the same combination, but weighted by factors f_j^R , where $R \in \{P, +, O, -\}$,

$$if_j^P(C + \ln^2 z), \quad if_j^+ z^{-0.4}, \quad f_j^O \ln z, \quad f_j^- z^{-0.6}$$

The four real factors f_j^R may be obtained from values of the complex ratios, r_j , $j \in \{5, 2\}$, resulting from measurements at two energies. **A. Poblaguev**

A polarimeter requires a process with nonvanishing high energy polarization

- Spin one photon exchange suggests the Primakoff or a Coulomb effect
- Helium-3 scattering reaches about -3% asymmetry in the CNI region

A spin half hadron of mass m , charge Ze , magnetic moment μ scattering elastically off a charge $Z'e$ has an asymmetry that involves an interference

$$2 \operatorname{Im} \left[\frac{Z Z'}{137 t} + (\rho + i) \frac{\sigma_{\text{tot}}}{8 \pi} \right]^* \frac{\kappa \sqrt{-t}}{2 m_p} \left[\frac{Z Z'}{137 t} + (\operatorname{Re} r_5 + i \operatorname{Im} r_5) \frac{\sigma_{\text{tot}}}{8 \pi} \right]$$

of helicity nonflip and flip amplitudes with electromagnetic and hadronic elements and σ_{tot} relates to the hadronic particles of charges Ze and $Z'e$.

Including the spin averaged denominator, the asymmetry is proportional to

$$A_N \propto \frac{\sqrt{x}}{x^2 + 3}, \quad x = \frac{t_e}{t}, \quad t_e = -\frac{8\pi\sqrt{3}|ZZ'|}{137\sigma_{\text{tot}}(s)} = \sqrt{3}t_c$$

the extremum value of which occurs at $x = 1$, that is, at transfer $t = t_e$.

The optimum value of 3% to 4% varies slowly with energy s as $1/\sqrt{\sigma_{\text{tot}}(s)}$

It is either a maximum or minimum depending on the sign of constant κ

$$A_N^{\text{opt}} = \frac{\kappa}{4m_p} \sqrt{-3t_e}, \quad \kappa = \frac{\mu}{Z} - \frac{m_p}{m}$$

The value of κ is 1.793 (anomalous μ) for protons and -1.398 for helions.

Hadronic helicity flip amplitudes and two photon exchange are ignored here.

Quantities related to proton carbon collisions may be compared to those of the more familiar proton proton case with the same incident fermion, viz,

$$\frac{t_e^{\text{pC}}}{t_e^{\text{pp}}} = \frac{6 \sigma_{\text{tot}}^{\text{pp}}}{\sigma_{\text{tot}}^{\text{pC}}} \approx 0.74, \quad \frac{A_N^{\text{pC}}}{A_N^{\text{pp}}} = \left(\frac{t_e^{\text{pC}}}{t_e^{\text{pp}}} \right)^{1/2} \approx 0.86$$

With distinct incident fermions, by contrast, helion carbon and proton carbon scattering have extremum momentum transfer and asymmetry ratios

$$\frac{t_e^{\text{hC}}}{t_e^{\text{pC}}} = \frac{2 \sigma_{\text{tot}}^{\text{pC}}}{\sigma_{\text{tot}}^{\text{hC}}} \approx 1.0, \quad \frac{A_N^{\text{hC}}}{A_N^{\text{pC}}} = \frac{\kappa_{\text{h}}}{\kappa_{\text{p}}} \left(\frac{t_e^{\text{hC}}}{t_e^{\text{pC}}} \right)^{1/2} \approx -0.78$$

The same would be approximately true if the target carbon particle C here were replaced throughout by another ion such as a proton p or a helion ${}^3\text{He}$

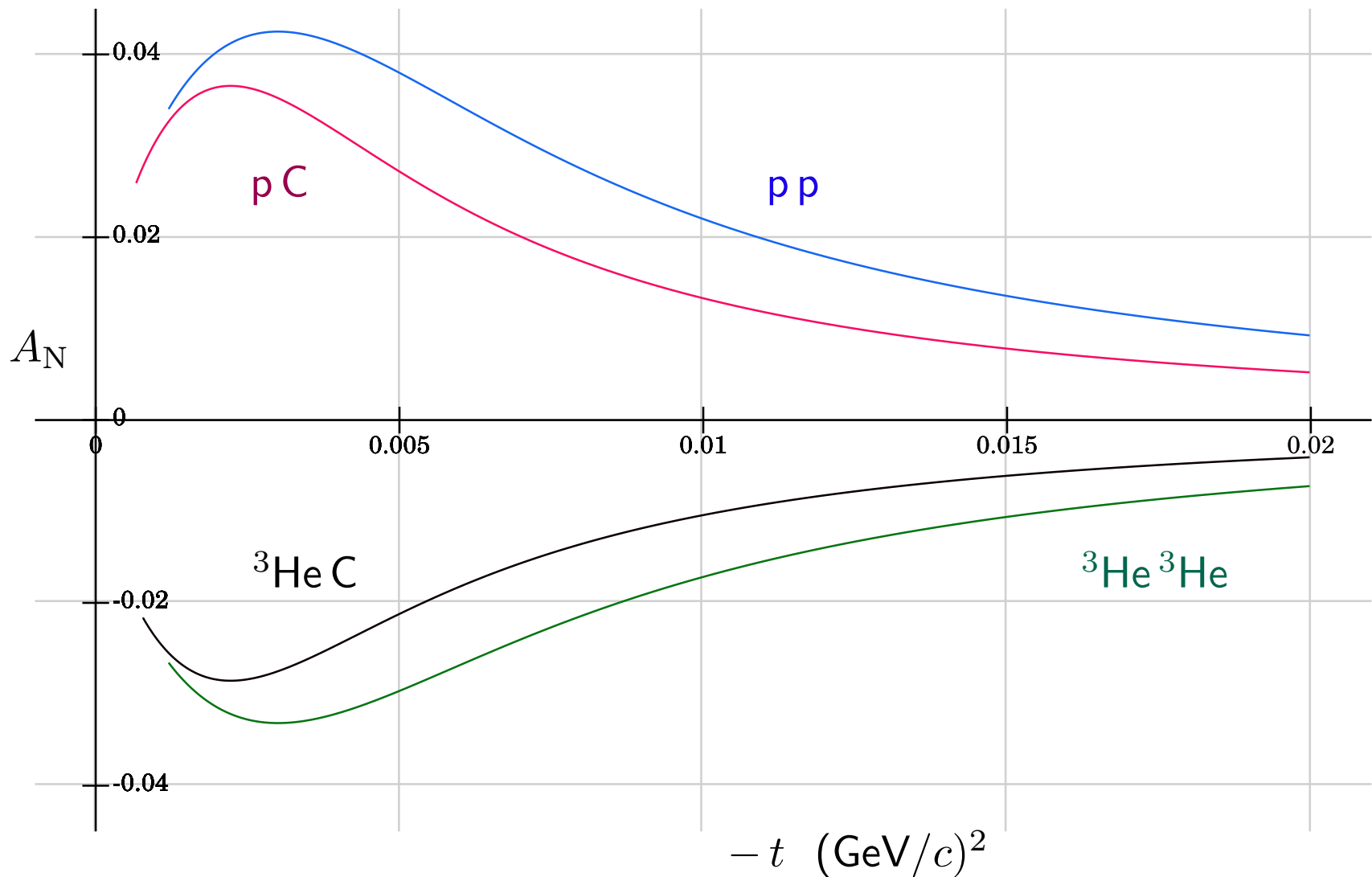


Figure 1: Analyzing power A_N versus invariant momentum transfer $(-t)$ in $(\text{GeV}/c)^2$ for (1) pp and ph scattering, (2) pC scattering, (3) h C scattering, (4) hh and hp scattering

The extremum value of t has first order corrections in the Coulomb phase δ_C , the hadronic non-flip real part ratio ρ , and the helicity-flip ratio r_5

$$t_e : 1 - (\rho + \delta_C)/\sqrt{3} - (\text{Re } r_5 - \rho \text{ Im } r_5) 4/\sqrt{3}$$

Another factor with small items δ , ρ , r_5 , multiplies the extremum of A_N

$$A_N : 1 + (\rho + \delta_C)\sqrt{3}/2 - (\sqrt{3} \text{Re } r_5 + \text{Im } r_5)$$

The extremum value of A_{NN} occurs at $t = t_c = t_e/\sqrt{3}$, approximately. Polarized proton nucleus scattering has been studied over a range of momentum transfers [Kopeliovich and Trueman, Phys Rev D **64** \(2001\) 034004](#)

Hadronic spin flip and Coulomb phase effects have been treated in detail in

NB, Kopeliovich, Leader, Soffer, Trueman, Phys Rev D59 (1999) 114010

The pp2pp experiment at STAR (RHIC BNL) has shown that the elastic pp

- hadronic single helicity-flip amplitude is small at $\sqrt{s} = 200$ GeV

L. Adamczyk *et al.* [STAR Collaboration], Phys Lett B 719, 62 (2013)

The acceleration of Helium-3 nuclei to high energy has been discussed.

W. W. MacKay, AIP Conference Proceedings 980, 191 (2008)

Helium-3 ions have been accelerated in the AGS at BNL (Haixin Huang).

The Helium-3 carbon cross section at the AGS appears to be twice that for proton carbon scattering.

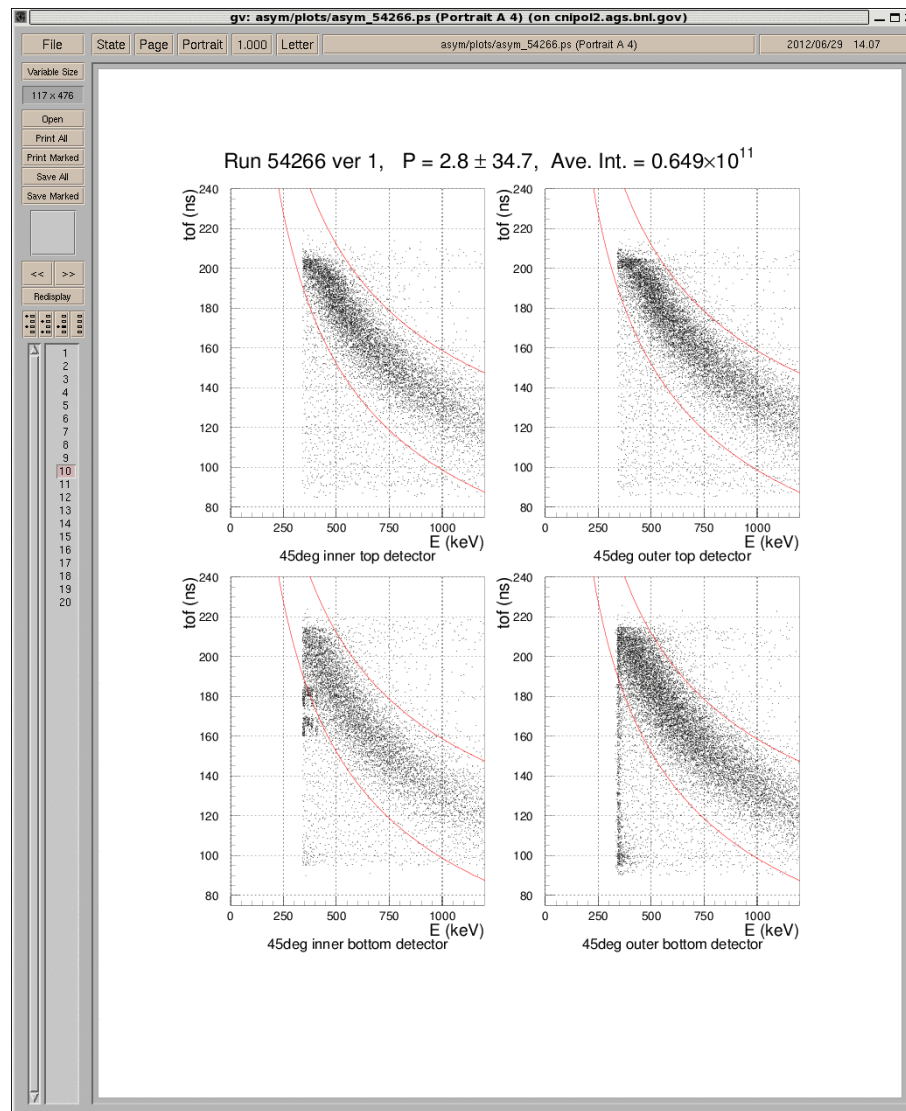


Figure 2: Time of flight of carbon recoils (on y -axis) versus the recoil kinetic energy of Helium-3 (on x -axis) as measured at the AGS. The ^3He -C events are double those of p-C.

CONCLUSIONS

Probing the spin structure of hadrons increases an understanding of QCD

There is great potential for studies using polarized up and down quarks

Proton polarimetry is mature now and polarized ^3He may be forthcoming

The $^3\text{He-C}$ analyzing power is $\approx -78\%$ of A_N for p-C in the CNI region

A polarized ^3He jet and beam would enable an absolute ^3He polarimeter

Single and double helicity flip pp amplitudes are known at many energies

Extrapolation of amplitudes to other high energies is becoming possible