Pressure in Generalized Parton Distributions and Distribution Amplitudes

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Main Topics

- GPDs and (gravitational) formfactors: D-term, pressure and inflation
- Analyticity vs QCD factorization
- Crossing, Tomography, Holography
- D-term and pressure: GPDs/GDAs

Recent development: pressure

- Publihsed in Nature
- Talks of Maxim Polyakov, Barbara Pasquini,Qin-Tao Song, Simonetta Liuti (plenary)
- Based on previous work (Polyakov, OT, Anikin&OT, Pasquini, Vanderhaegen, Kumericky, Mueller, Goldstein&Liuti…
- First time the nice picture presented (Link with stability of stars (Poincare, v. Laue)

NATURE | www.nature.com/nature

https://doi.org/10.1038/s41586-018-0060-z

The pressure distribution inside the proton

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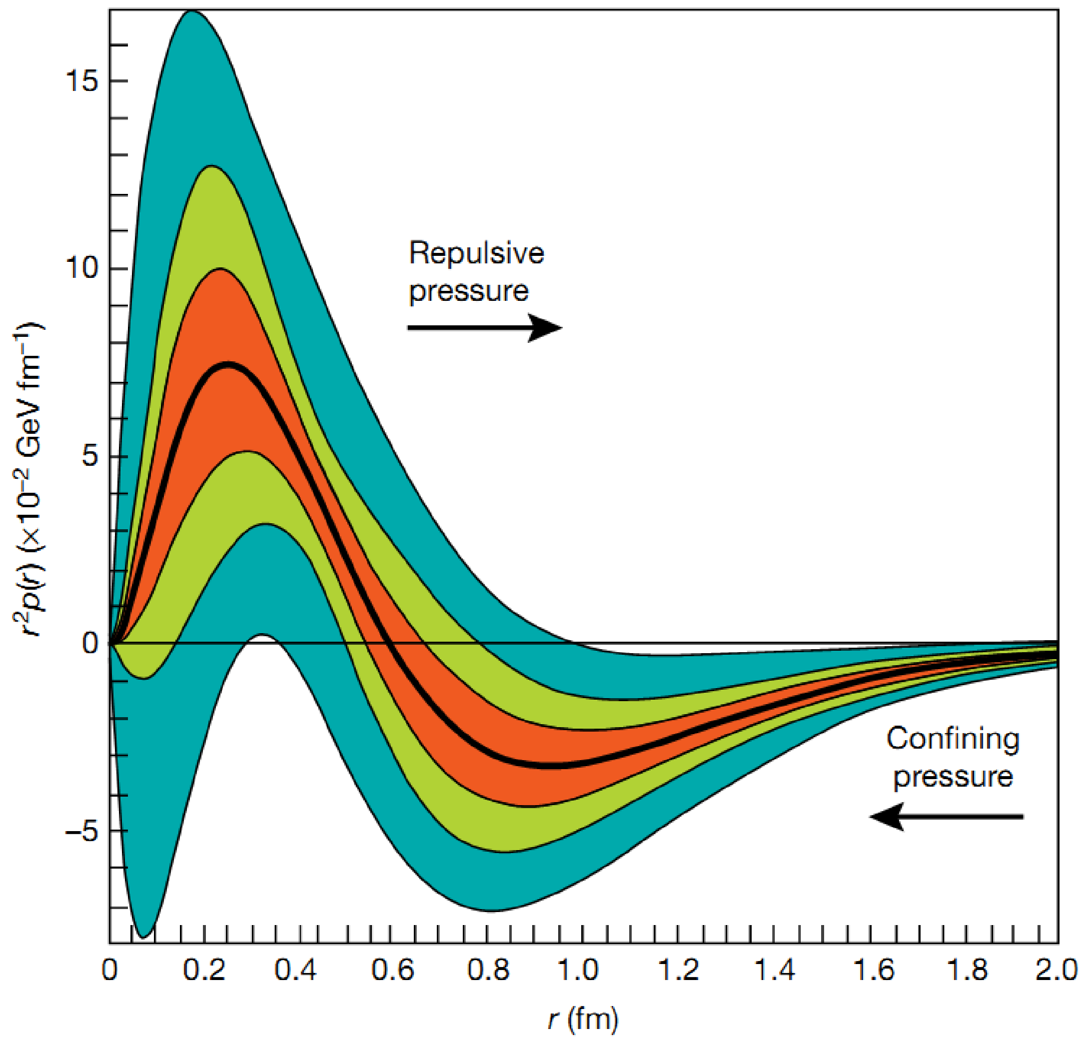

Gravitational Formfactors

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M\Big]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity

Generalized Parton Distributions (related to matrix elements of non local operators) – models for both EM and Gravitational Formfactors (Selyugin,OT ’09)

- Smaller mass square radius (attraction vs repulsion!?)

$$\rho(b) = \sum_q e_q \int dx q(x,b) \qquad = \int d^2q F_1(Q^2 = q^2) e^{i\vec{q}\,\vec{b}}$$

$$= \int_0^\infty \frac{qdq}{2\pi} J_0(qb) \frac{G_E(q^2) + \tau G_M(q^2)}{1+\tau}$$

$$\rho_0^{\mathrm{Gr}}(b) = \frac{1}{2\pi} \int_\infty^0 dq q J_0(qb) A(q^2)$$

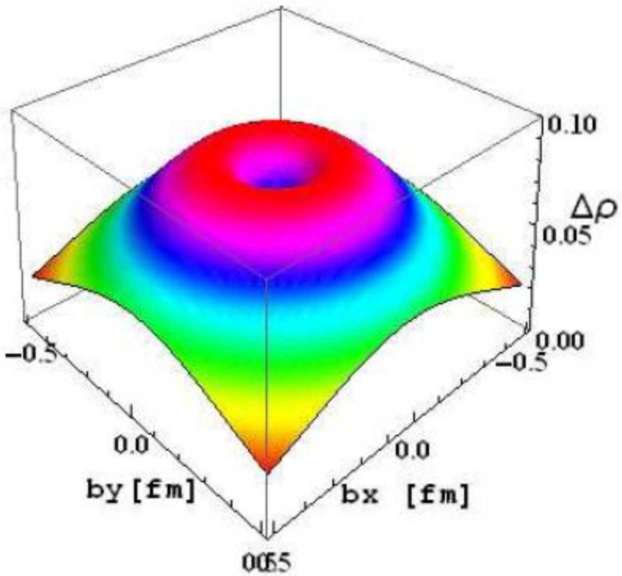

FIG. 17: Difference in the forms of charge density F_1^P and "matter" density (A)

Electromagnetism vs Gravity (OT'99)

- Interaction – field vs metric deviation

$$M = \langle P'|J_q^\mu|P\rangle A_\mu(q)$$

$$M = \frac{1}{2}\sum_{q,G}\langle P'|T_{q,G}^{\mu\nu}|P\rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P|J_q^\mu|P\rangle = 2e_q P^\mu$$

$$\sum_{q,G}\langle P|T_i^{\mu\nu}|P\rangle = 2P^\mu P^\nu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P|J_q^\mu|P\rangle A_\mu = 2e_q M\phi(q)$$

$$M_0 = \frac{1}{2}\sum_{q,G}\langle P|T_i^{\mu\nu}|P\rangle h_{\mu\nu} = 2M\cdot M\phi(q)$$

- Mass as charge – equivalence principle

Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from $M = \frac{1}{2}\sum_{q,G} \langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

  $\vec{H}_J = \frac{1}{2} rot\vec{g};\ \vec{g}_i \equiv g_{0i}$ spin dragging twice smaller than EM
- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$ Larmor frequency same as EM

  $\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L\ \vec{H}_L = rot\vec{g}$
- Orbital and Spin momenta dragging – the same - Equivalence principle

Equivalence principle

- Newtonian – "Falling elevator" – well known and checked (also for elementary particles)
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun'; ZhETF paper contains acknowledgment to Landau: probably his last contribution to theoretical physics before car accident); rederived from conservarion laws - Kobzarev and V.I. Zakharov
- Anomalous gravitomagnetic (and electric-CP-odd) moment iz ZERO or
- Classical and QUANTUM rotators behave in the SAME way

Experimental test of PNEP

- Reinterpretation of the data on G(EDM) search

PHYSICAL REVIEW LETTERS

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Search for a Coupling of the Earth's Gravitational Field to Nuclear Spins in Atomic Mercury

B. J. Venema, P. K. Majumder, S. K. Lamoreaux, B. R. Heckel, and E. N. Fortson

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(Received 25 September 1991)

- If (CP-odd!) GEDM=0 -> constraint for AGM (Silenko, OT'07) from Earth rotation – was considered as obvious (but it is just EP!) background

$$\mathcal{H} = -g\mu_N \boldsymbol{B}\cdot\boldsymbol{S} - \zeta\hbar\boldsymbol{\omega}\cdot\boldsymbol{S}, \quad \zeta = 1+\chi$$

$$|\chi(^{201}\mathrm{Hg}) + 0.369\chi(^{199}\mathrm{Hg})| < 0.042 \quad (95\%\mathrm{C.L.})$$

Indirect probe of spin-gravity coupling

- Matrix elements of energy-momentum tensors may be extracted from accurate high-energy experiments ("3D nucleon picture")
- Allow to probe the couplings to quarks and gluons separately

Equivalence principle for moving particles

- Compare gravity and acceleration: gravity provides EXTRA space components of metrics $h_{zz} = h_{xx} = h_{yy} = h_{00}$
- Matrix elements DIFFER
$$\mathcal{M}_g = (\epsilon^2 + p^2)h_{00}(q), \qquad \mathcal{M}_a = \epsilon^2 h_{00}(q)$$
- Ratio of accelerations: $R = \frac{\epsilon^2 + p^2}{\epsilon^2}$ - confirmed by explicit solution of Dirac equation (Silenko, OT, '05)
- Arbitrary fields – Obukhov, Silenko, OT '09,'11,'13

Gravity vs accelerated frame for spin and helicity

- Spin precession – well known factor 3 (Probe B; spin at satellite – probe of PNEP!) – smallness of relativistic correction (~$\mathbf{P}^2$) is compensated by 1/ $\mathbf{P}^2$ in the momentum direction precession frequency
- Helicity flip – the same!
- No helicity flip in gravitomagnetic field – another formulation of PNEP (OT'99) and
- Flip by "gravitoelectric" field: relic neutrino? Black hole?

$$\frac{d\sigma_{+-}}{d\sigma_{++}} = \frac{tg^2(\frac{\phi}{2})}{(2\gamma - \gamma^{-1})^2}$$

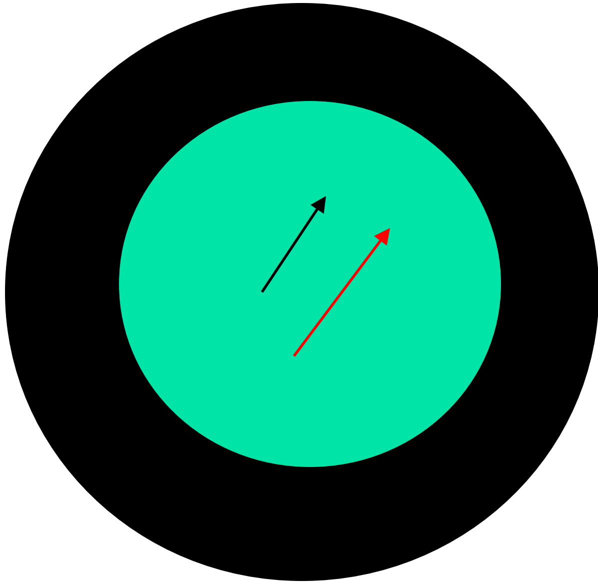
Gyromagnetic and Gravigyromagnetic ratios

- Free particles – coincide
- $<P+q|T^{mn}|P-q> = P^{\{m}<P+q|J^{n\}}|P-q>/e$ up to the terms linear in q
- Gravitomagnetic g=2 for any spin
- Special role of g=2 for ANY spin (asymptotic freedom for vector bosons)

- Should Einstein know about PNEP, the outcome of his and de Haas experiment would not be so surprising
- Recall also g=2 for Black Holes. Indication of "quantum" nature?!

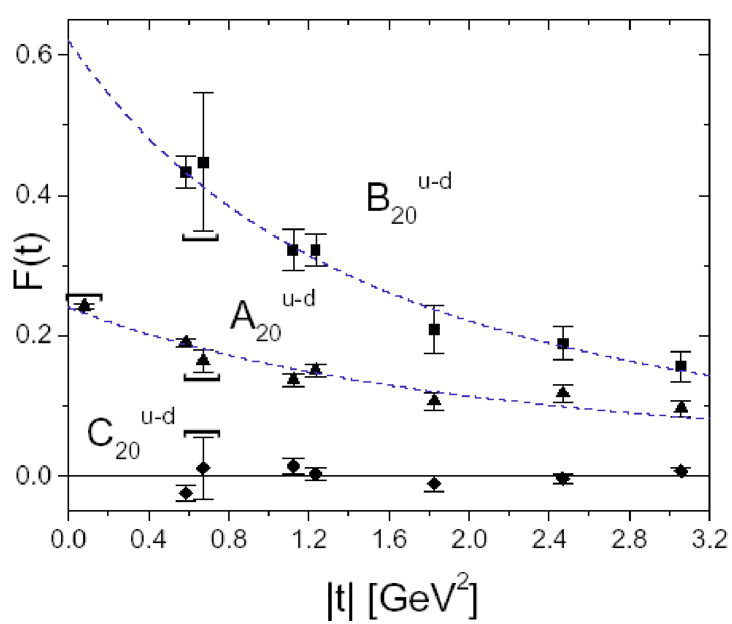
Cosmological implications of PNEP

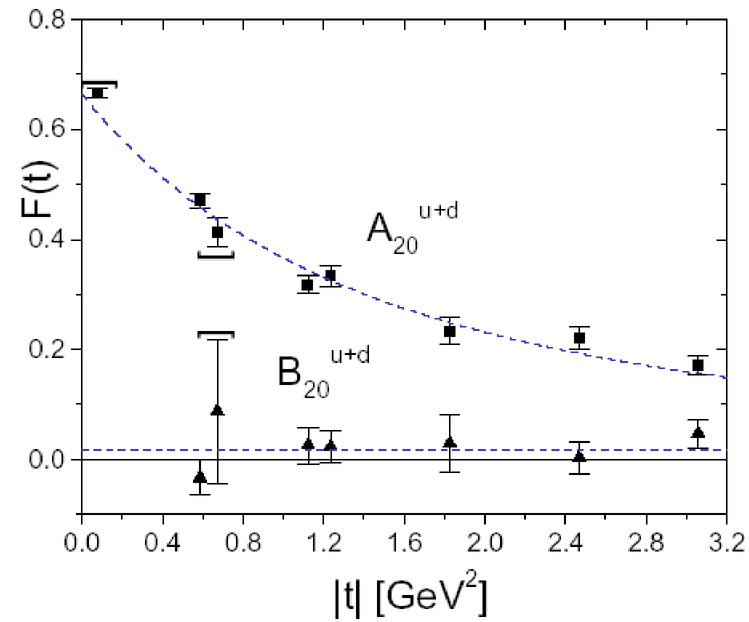
- Necessary condition for Mach's Principle (in the spirit of Weinberg's textbook) -
- Lense-Thirring inside massive rotating empty shell (=model of Universe)
- For flat "Universe" - precession frequency equal to that of shell rotation
- Simple observation-Must be the same for classical and quantum rotators – PNEP!
- More elaborate models - Tests for cosmology ?!



Generalization of Equivalence principle

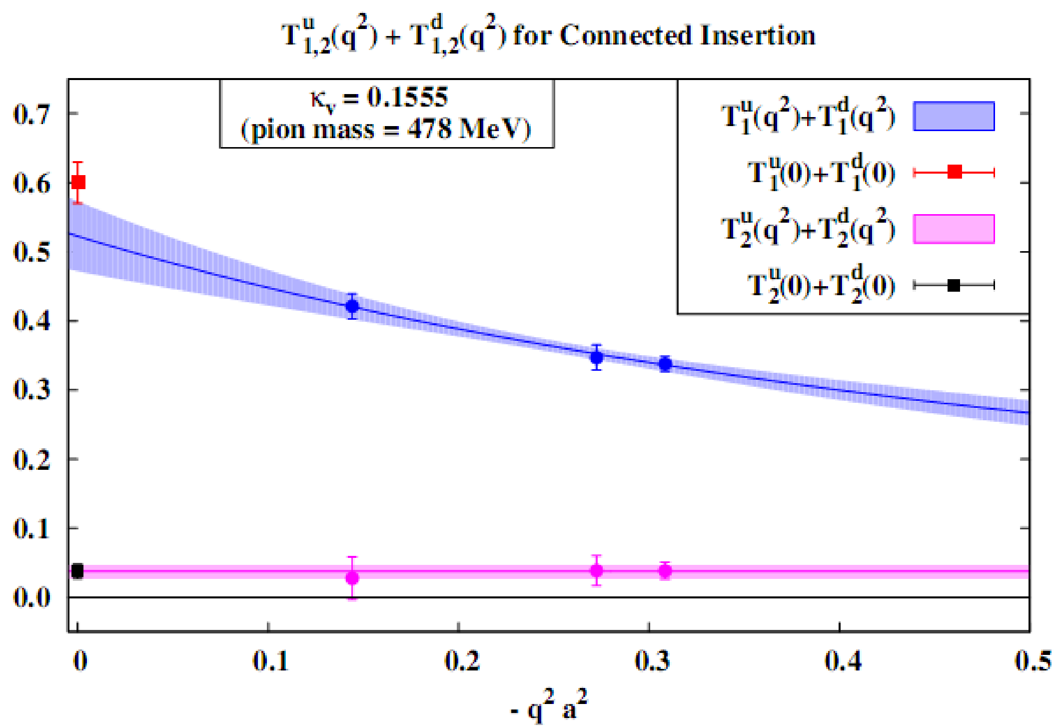
- Various arguments: AGM ≈ 0 separately for quarks and gluons – most clear from the lattice (LHPC/SESAM)





Recent lattice study (M. Deka et al. Phys.Rev. D91 (2015) no.1, 014505)

- Sum of u and d for Dirac (T1) and Pauli (T2) FFs



Extended Equivalence Principle=Exact EquiPartition

- In pQCD – violated
- Reason – in the case of ExEP- no smooth transition for zero fermion mass limit (Milton, 73)
- Conjecture (O.T., 2001 – prior to lattice data) – valid in NP QCD – zero quark mass limit is safe due to chiral symmetry breaking
- Gravityproof confinement? Nucleons do not break even by black holes?
- Support by recent observation of smalness of Cbar (talk of Maxim Polyakov)

One more gravitational formfactor

- Quadrupole

$$\langle P+q/2|T^{\mu\nu}|P-q/2\rangle = C(q^2)(g^{\mu\nu}q^2 - q^\mu q^\nu) + \ldots$$

- Cf vacuum matrix element – cosmological constant (vacuum pressure)

$$\langle 0|T^{\mu\nu}|0\rangle = \Lambda g^{\mu\nu}$$

$$\Lambda = C(q^2)q^2$$

- Inflation ~ annihilation ($q^2>0$) OT'15
- How to measure experimentally – Deeply Virtual Compton Scattering

D-term interpretation: Inflation and annihilation

- Quadrupole gravitational FF

$$\langle P + q/2 | T^{\mu\nu} | P - q/2 \rangle = C(q^2)(g^{\mu\nu} q^2 - q^\mu q^\nu) + \ldots$$

- Moment of D-term – positive
- Vacuum – Cosmological Constant $\langle 0 | T^{\mu\nu} | 0 \rangle = \Lambda g^{\mu\nu}$

- 2D effective CC – negative in scattering, positive in annihilation

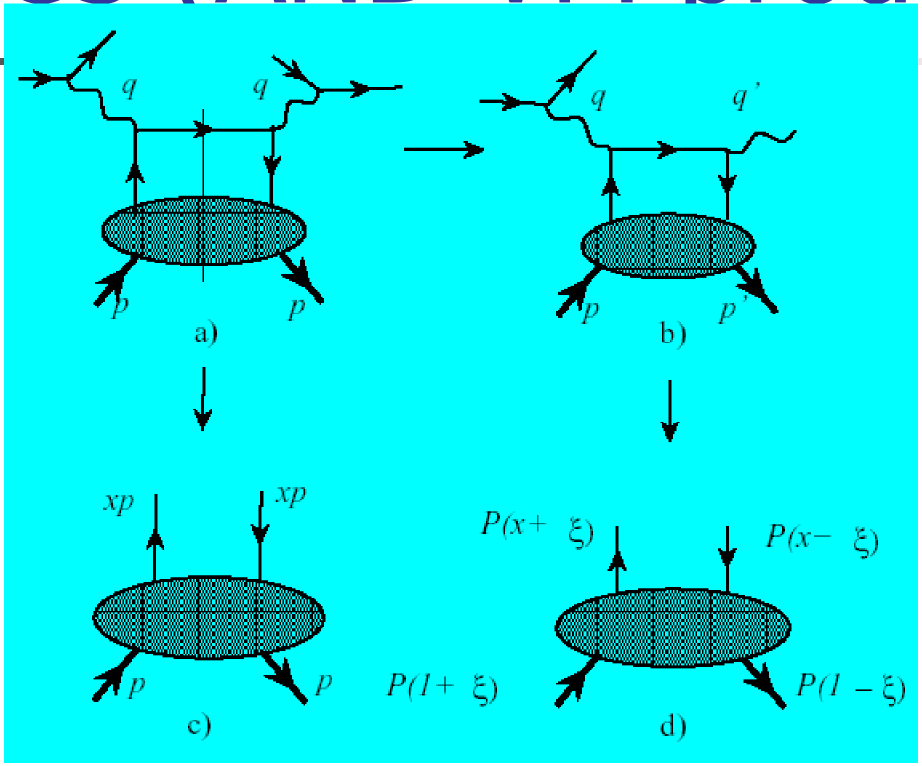
$$\Lambda = C(q^2) q^2$$

- Similarity of inflation and Schwinger pair production – Starobisnky, Zel'dovich
- Was OUR Big Bang resulting from one graviton annihilation at extra dimensions??! Version of "ekpyrotic" ("pyrotechnic") universe

C vs Cbar

- Cancellations of Cbars – negative pressure (cf Chaplygin gas)
- Cancellation in vacuum; Pauli (divergent), Zel’dovich (finite)
- Flavour structure of pressure: DVMP!

QCD Factorization for DIS and DVCS (AND VM production)



- Manifestly spectral

$$\mathcal{H}(x_B) = \int_{-1}^{1} dx \frac{H(x)}{x - x_B + i\epsilon}.$$

- Extra dependence on ξ

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x - \xi + i\epsilon},$$

Unphysical regions

- DIS : Analytical function – polynomial in $1/x_B$ if $1 \le |x_B|$

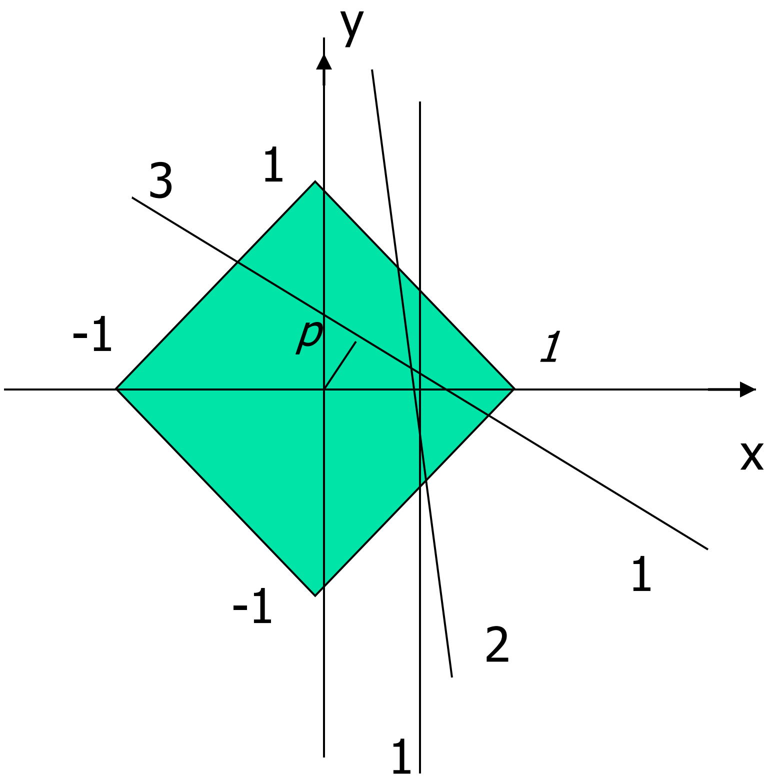
$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- DVCS – additional problem of analytical continuation of $H(x,\xi)$
- Solved by using of Double Distributions Radon transform

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

Double distributions and their integration

- Slope of the integration line- skewness
- Kinematics of DIS: $\xi = 0$
  ("forward") - vertical line (1)
- Kinematics of DVCS: $\xi < 1$
  - line 2
- Line 3: $\xi > 1$ unphysical region - required to restore DD by inverse Radon transform: tomography

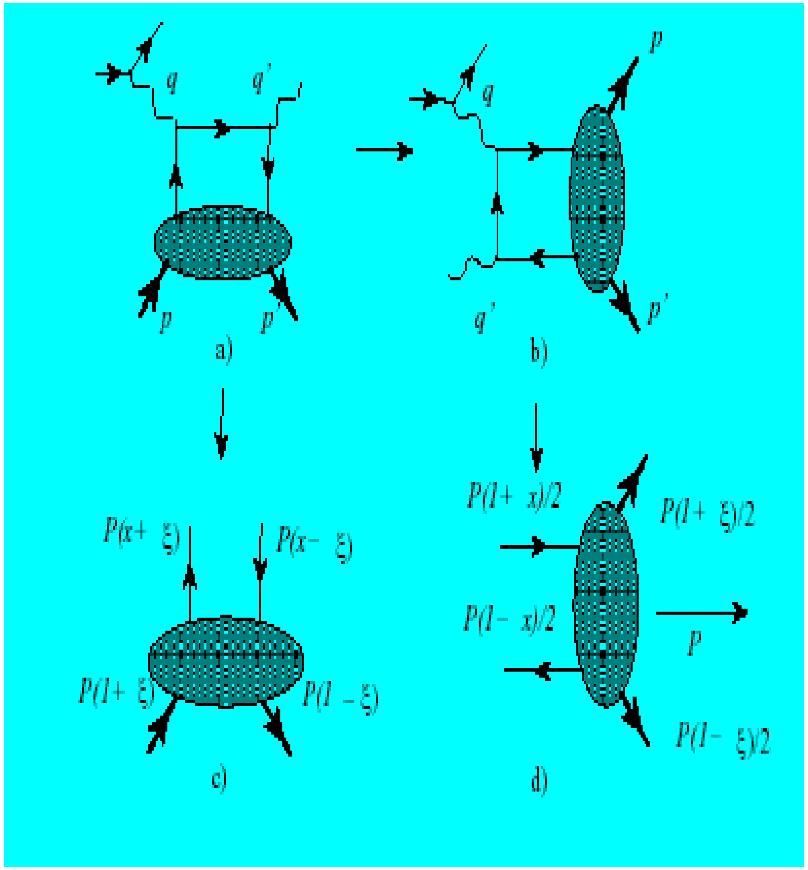


$$f(x,y) = -\frac{1}{2\pi^2}\int_0^\infty \frac{dp}{p^2}\int_0^{2\pi} d\phi|cos\phi|(H(p/cos\phi + x + ytg\phi, tg\phi) - H(x + ytg\phi, tg\phi)) =$$

$$= -\frac{1}{2\pi^2}\int_{-\infty}^\infty \frac{dz}{z^2}\int_{-\infty}^\infty d\xi(H(z + x + y\xi, \xi) - H(x + y\xi, \xi))$$

Crossing for DVCS and GPD

- DVCS -> hadron pair production in the collisions of real and virtual photons
- GPD -> Generalized Distribution Amplitudes
- Duality between s and t channels (Polyakov,Shuvaev, Guzey, Vanderhaeghen)



GDA -> back to unphysical regions for DIS and DVCS

- Recall DIS

$$H(x_B) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x) \frac{x^n}{x_B^{n+1}}$$

- Non-positive powers of x_B

- DVCS

$$H(\xi) = -\int_{-1}^{1} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

- Polynomiality (general property of Radon transforms): moments - integrals in *x* weighted with x^n - are polynomials in $1/\xi$ of power *n+1*
- As a result, analyticity is preserved: only non-positive powers of ξ appear

Holographic property (OT'05)

- Factorization Formula ->

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,\xi)}{x-\xi+i\epsilon}$$

- Analyticity -> Imaginary part -> Dispersion relation:

$$\mathcal{H}(\xi) = \int_{-1}^{1} dx \frac{H(x,x)}{x-\xi+i\epsilon}$$

- "Holographic" equation (DVCS AND VM)

$$\Delta\mathcal{H}(\xi) \equiv \int_{-1}^{1} dx \frac{H(x,x)-H(x,\xi)}{x-\xi+i\epsilon}$$

$$= \sum_{n=1}^{\infty} \frac{1}{n!} \frac{\partial^n}{\partial \xi^n} \int_{-1}^{1} H(x,\xi) dx (x-\xi)^{n-1} = const$$

Holographic property - II

- Directly follows from double distributions

$$H(z,\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy (F(x,y) + \xi G(x,y)) \delta(z - x - \xi y)$$

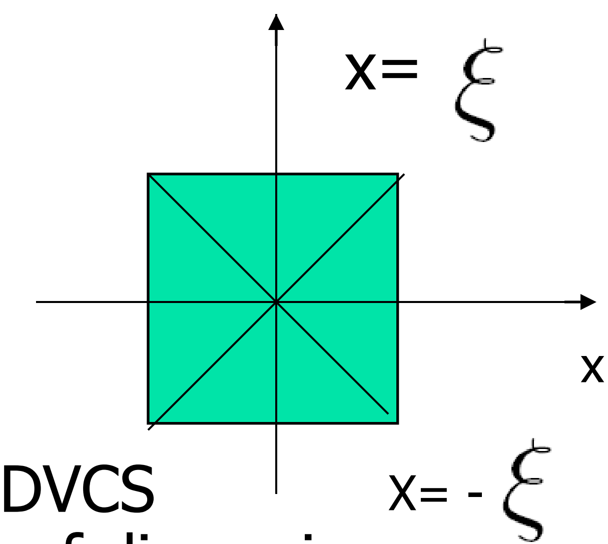
- Constant is the SUBTRACTION one - due to the (generalized) Polyakov-Weiss term G(x,y)

$$\Delta\mathcal{H}(\xi) = \int_{-1}^{1} dx \int_{|x|-1}^{1-|x|} dy \frac{G(x,y)}{1-y}$$

$$= \int_{-\xi}^{\xi} dx \frac{D(x/\xi)}{x - \xi + i\epsilon} = \int_{-1}^{1} dz \frac{D(z)}{z-1} = const$$

Holographic property - III

- 2-dimensional space -> 1-dimensional section!
- Momentum space: any relation to holography in coordinate space ?!



- Strategy (now adopted) of GPD's studies: start at diagonals (through SSA due to imaginary part of DVCS amplitude) and restore by making use of dispersion relations + subtraction constants

Holography vs NLO

- Depends on factorization scheme
- Special role of scheme preserving the coefficient function
- Nucleon as (scheme dependent) black hole – 3D information encoded in 2D

Pressure in hadron pairs production

- Back to GDA region
- -> moments of H(x,x) - define the coefficients of powers of cosine!– 1/ξ
- Higher powers of cosine in t-channel – threshold in s -channel
- Larger for pion than for nucleon pairs because of less fast decrease at x ->1
- Stability defines the sign of GDA and (via soft pion theorem) DA: work in progress

$$\mathcal{H}(\xi) = -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,\xi) \frac{x^n}{\xi^{n+1}}$$

$$= -\int_{-1/\xi}^{1/\xi} dx \sum_{n=0}^{\infty} H(x,x) \frac{x^n}{\xi^{n+1}} + \Delta\mathcal{H}.$$

Analyticity of Compton amplitudes in energy plane (Anikin,OT'07)

- Finite subtraction implied

$$\mathrm{Re}\mathcal{A}(\nu,Q^2)=\frac{\nu^2}{\pi}\mathcal{P}\int_{\nu_0}^{\infty}\frac{d\nu'^2}{\nu'^2}\frac{\mathrm{Im}\mathcal{A}(\nu',Q^2)}{(\nu'^2-\nu^2)}+\Delta \qquad \Delta=2\int_{-1}^{1}d\beta\,\frac{D(\beta)}{\beta-1}$$

$$\Delta^p_{\rm CQM}(2)=\Delta^n_{\rm CQM}(2)\approx 4.4, \qquad \Delta^p_{\rm latt}\approx\Delta^n_{\rm latt}\approx 1.1$$

- Numerically close to Thomson term for real proton (but NOT neutron) Compton Scattering!

- Duality (sum of squares vs square of sum; proton: 4/9+4/9+1/9=1)?!

From D-term to pressure

- *Inverse -> 1st moment (model)*
- *Kinematical factor – moment of pressure C~<p r^4> (<p r^2> =0)*
  M.Polyakov'03

$$T^Q_{\mu\nu}(\vec{r},\vec{s}) = \frac{1}{2E}\int \frac{d^3\Delta}{(2\pi)^3}\, e^{i\vec{r}\cdot\vec{\Delta}}\, \langle p', S'|\hat{T}^Q_{\mu\nu}(0)|p, S\rangle$$

$$T_{ij}(\vec{r}) = s(r)\left(\frac{r_i r_j}{r^2} - \frac{1}{3}\,\delta_{ij}\right) + p(r)\delta_{ij}$$

- *Stable equilibrium C>0:*

Loss of stability?

- D=0 -> extra node required (cf tensor distribution - Efremov,OT- mechanical analogy – c.m. and c.i.)
- Smooth decrease – two extra nodes
- ++++-------------------
- +++++++-------+++++---------------
- J=2 (Talk of Barbara Pasquini, comment by Maxim – zeros of Bessel functs?!)

CONCLUSIONS/OUTLOOK

- “Macroscopical” aspects of GPDs
- Pressure of quark flavours/gluons – DVMP
- Pressure from TMDs (TMD/GPD relations)?
- Comparison to QCD matter (HIC)

BACKUP

Is D-term independent?

- Fast enough decrease at large energy ->

$$\mathrm{Re}\,\mathcal{A}(\nu) = \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2 \frac{\mathrm{Im}\,\mathcal{A}(\nu')}{\nu'^2-\nu^2} + \mathrm{C}_0$$

$$\mathrm{C}_0 = \Delta - \frac{\mathcal{P}}{\pi}\int_{\nu_0}^{\infty} d\nu'^2 \frac{\mathrm{Im}\,\mathcal{A}(\nu')}{\nu'^2}$$

$$= \Delta + \mathcal{P}\int_{-1}^{1} dx \frac{H^{(+)}(x,x)}{x}.$$

- FORWARD limit of Holographic equation

$$\Delta = \mathcal{P}\int_{-1}^{1} dx \frac{H^{(+)}(x,0) - H^{(+)}(x,x)}{x}$$

$$= 2\mathcal{P}\int_{-1}^{1} dx \frac{H(x,0) - H(x,x)}{x},$$

$$\mathrm{C}_0(t) = 2\mathcal{P}\int_{-1}^{1} dx \frac{H(x,0,t)}{x}$$

"D – term" 30 years before…

- Cf Brodsky, Close, Gunion'72 (seagull ~ pressure) – but NOT DVMP
- D-term – a sort of renormalization constant
- May be calculated in effective theory if we know fundamental one
- OR
- Recover through special regularization procedure (D. Mueller)?

Vector mesons and EEP

- J=1/2 -> J=1. QCD SR calculation of Rho's AMM gives g close to 2.

- Maybe because of similarity of moments
- $g-2=<E(x)>$; $B=<xE(x)>$
- Directly for charged Rho (combinations like p+n for nucleons unnecessary!). Not reduced to non-extended EP:

EEP and AdS/QCD

- Recent development – calculation of Rho formfactors in Holographic QCD (Grigoryan, Radyushkin)
- Provides g=2 identically!
- Experimental test at time –like region possible

EEP and Sivers function

- Sivers function – process dependent (effective) one
- T-odd effect in T-conserving theory- phase
- FSI – Brodsky-Hwang-Schmidt model
- Unsuppressed by M/Q twist 3
- Process dependence- colour factors
- After Extraction of phase – relation to universal (T-even) matrix elements

EEP and Sivers function -II

- Qualitatively similar to OAM and Anomalous Magnetic Moment (talk of S. Brodsky)
- Quantification : weighted TM moment of Sivers PROPORTIONAL to GPD E (OT'07, **hep-ph/0612205**): $x f_{T}(x) : \; xE(x)$
- Burkardt SR for Sivers functions is then related to Ji's SR for E and, in turn, to Equivalence Principle

$$\sum_{q,G} \int dx\, x f_{T}(x) = \sum_{q,G} \int dx\, x E(x) = 0$$

EEP and Sivers function for deuteron

- EEP - smallness of deuteron Sivers function
- Cancellation of Sivers functions – separately for quarks (before inclusion gluons)
- Equipartition + small gluon spin – large longitudinal orbital momenta (BUT small transverse ones –Brodsky, Gardner)

Another relation of Gravitational FF and NP QCD (first reported at 1992: **hep-ph/9303228**)

- BELINFANTE (relocalization) invariance :

decreasing in coordinate –

$$M^{\mu,\nu\rho} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} J^5_{S\sigma} + x^\nu T^{\mu\rho} - x^\rho T^{\mu\nu}$$

smoothness in momentum space

$$M^{\mu,\nu\rho} = x^\nu T_B^{\mu\rho} - x^\rho T_B^{\mu\nu}$$

- Leads to absence of massless pole in singlet channel – U_A(1)

$$\epsilon_{\mu\nu\rho\alpha} M^{\mu,\nu\rho} = 0.$$

- Delicate effect of NP QCD

$$(g_{\rho\nu} g_{\alpha\mu} - g_{\rho\mu} g_{\alpha\nu}) \partial^\rho (J^\alpha_{5S} x^\nu) = 0.$$

- Equipartition – deeply related to relocalization invariance by QCD evolution

$$q^2 \frac{\partial}{\partial q^\alpha} \langle P|J^\alpha_{5S}|P+q\rangle = (q^\beta \frac{\partial}{\partial q^\beta} - 1) q_\gamma \langle P|J^\gamma_{5S}|P+q\rangle$$

$$\langle P,S|J^5_\mu(0)|P+q,S\rangle = 2MS_\mu G_1 + q_\mu (Sq) G_2,$$

$$q^2 G_2|_0 = 0$$