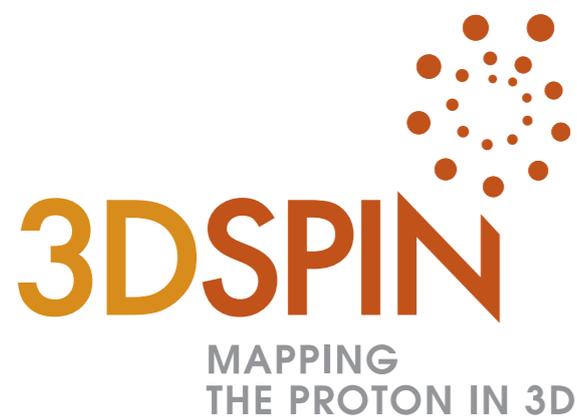


Drell-Yan at low energy and high transverse momentum

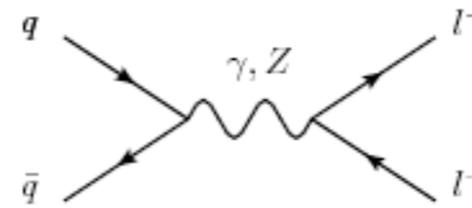
Fulvio Piacenza

in collaboration with A.Bacchetta, G.Bozzi

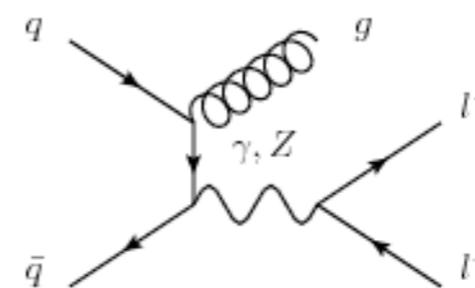
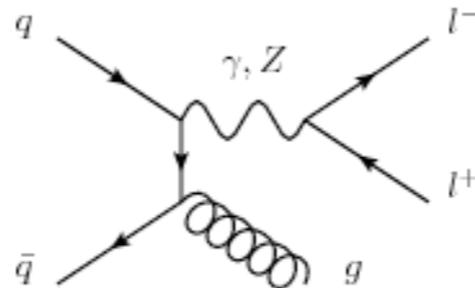
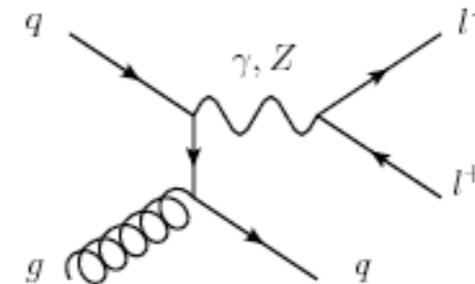
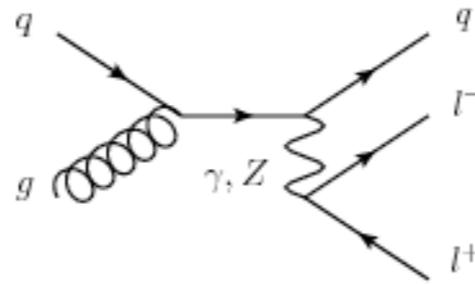


Transverse momentum of Drell-Yan pairs

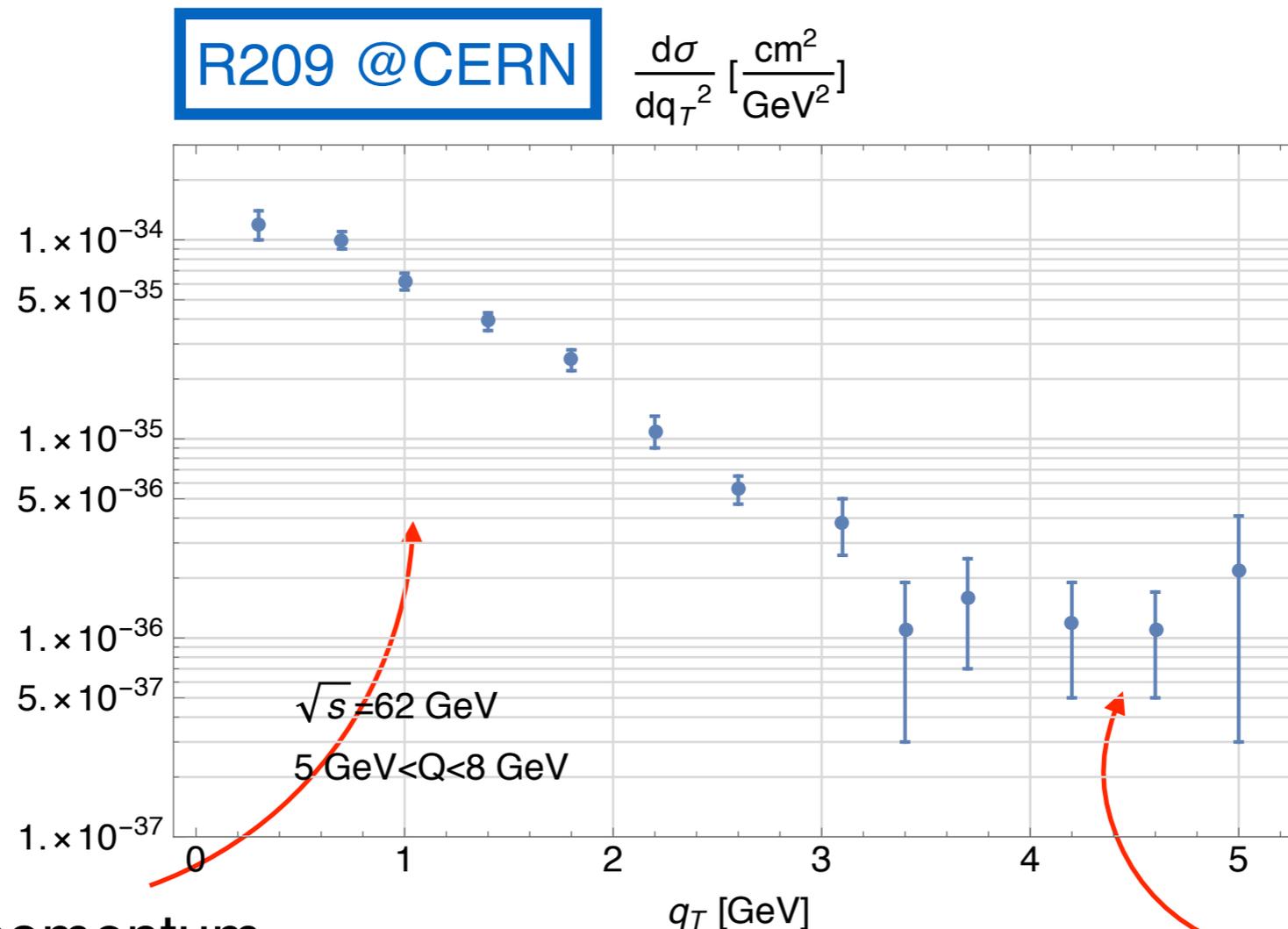
- No transverse momentum at $\mathcal{O}(\alpha_s^0)$



- LO = $\mathcal{O}(\alpha_s^1)$



Transverse momentum of Drell-Yan pairs



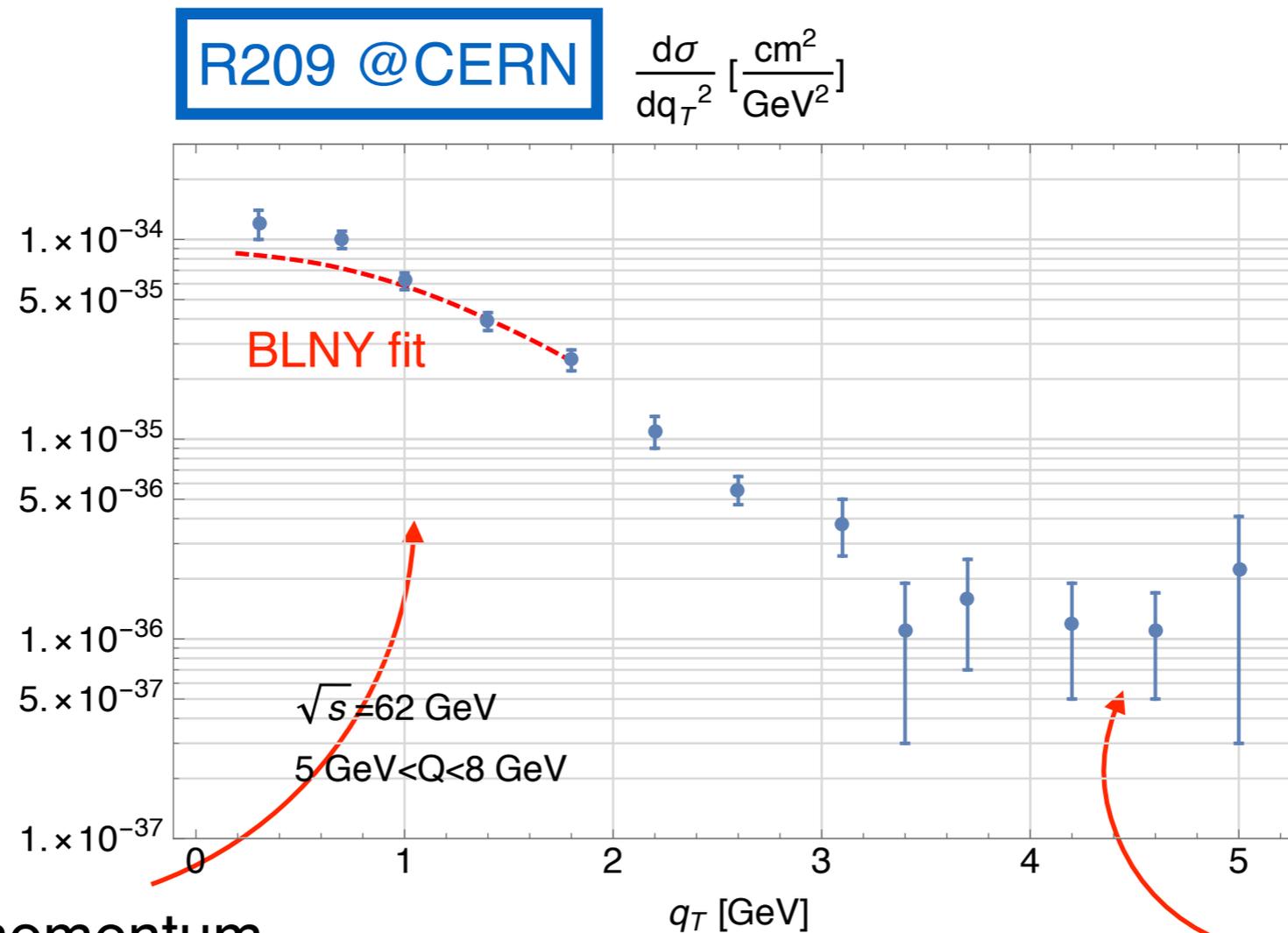
Transverse momentum
resummation

(q_T -logs) $\alpha_s^n \ln^m \frac{q_T^2}{Q^2}$

+ non perturbative
effects

Fixed order
collinear factorization

Transverse momentum of Drell-Yan pairs



Transverse momentum
resummation

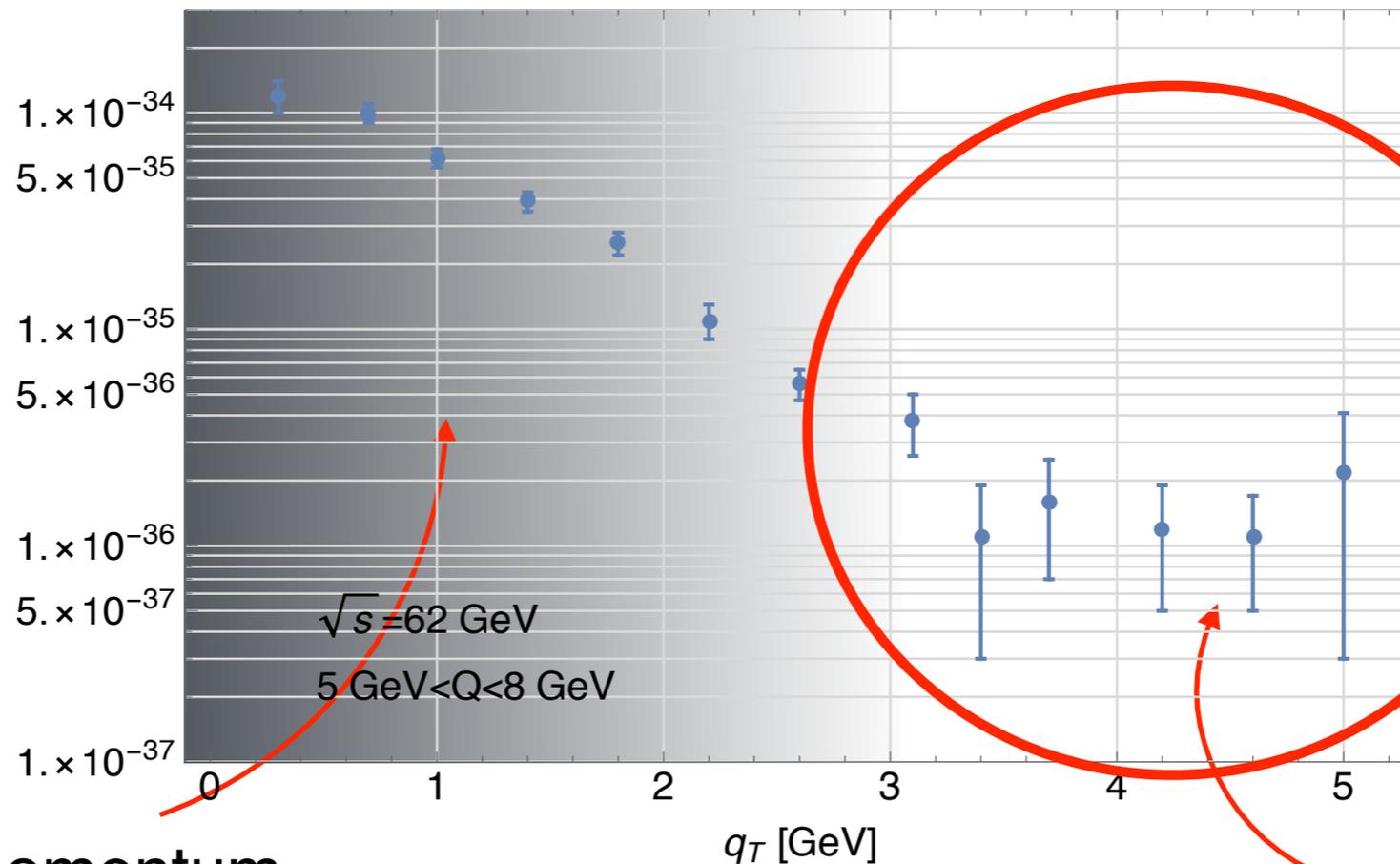
$$(q_T\text{-logs}) \quad \alpha_s^n \ln^m \frac{q_T^2}{Q^2}$$

+ non perturbative
effects

Fixed order
collinear factorization

Transverse momentum of Drell-Yan pairs

R209 @CERN $\frac{d\sigma}{dq_T^2} \left[\frac{\text{cm}^2}{\text{GeV}^2} \right]$



Can we describe data here?

Transverse momentum resummation

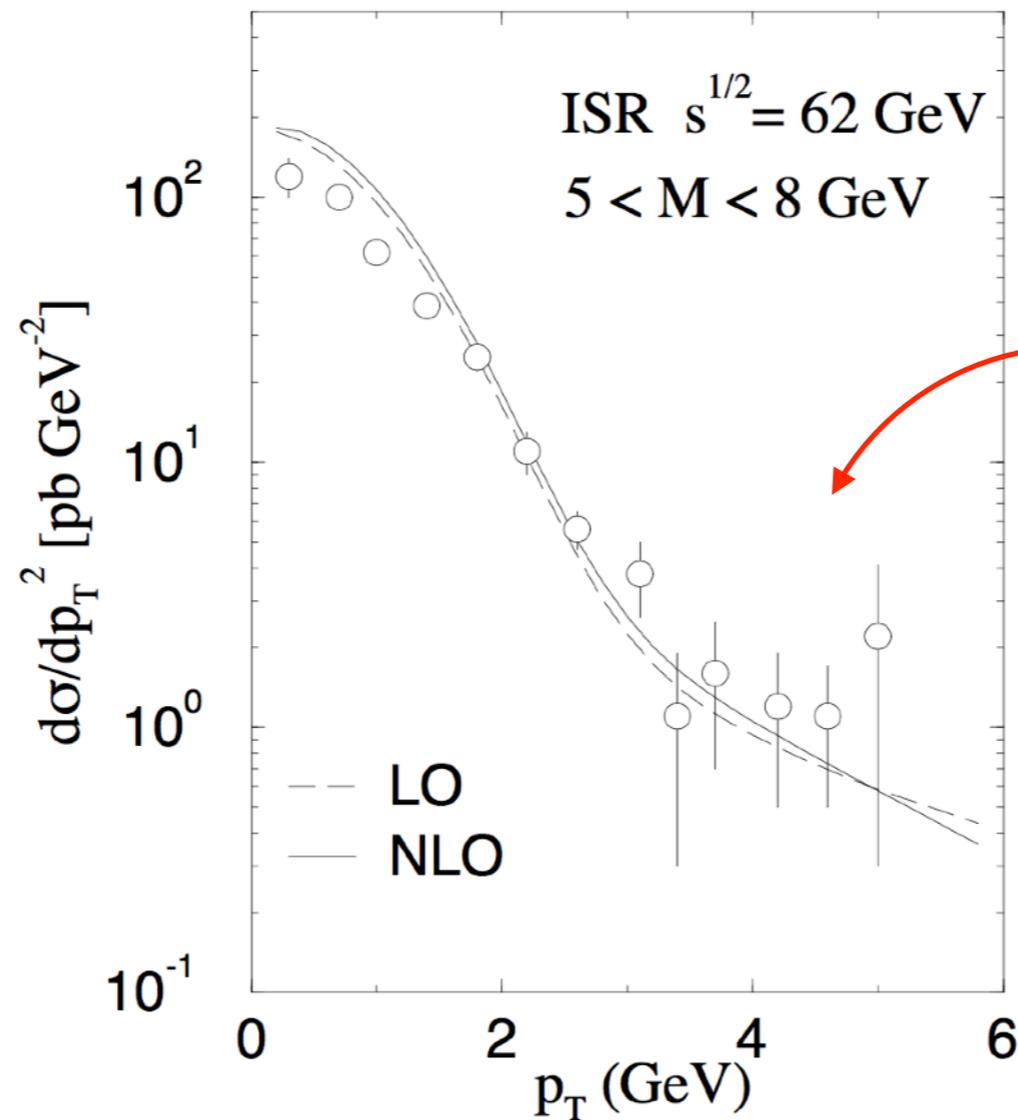
(q_T -logs) $\alpha_s^n \ln^m \frac{q_T^2}{Q^2}$

+ non perturbative effects

Fixed order collinear factorization

Matching resummation and fixed order

$$\frac{d\sigma}{dq_T^2}(\text{matched}) = \underbrace{\frac{d\sigma}{dq_T^2}(\text{resum})_{NLL}}_W - \underbrace{\frac{d\sigma}{dq_T^2}(\text{expanded})}_{\mathcal{O}(\alpha_s)} + \frac{d\sigma}{dq_T^2}(\text{LO})_Y$$



W + Y

Gavin et al
 I.J.Mod.Phys. A10 (1995)

Arnold & Kauffman's rule

$$\frac{d\sigma}{dq_T^2}(\text{matched}) = \underbrace{\frac{d\sigma}{dq_T^2}(\text{resum})_{NLL}}_W - \underbrace{\frac{d\sigma}{dq_T^2}(\text{expanded})}_{\mathcal{O}(\alpha_s)} + \frac{d\sigma}{dq_T^2}(\text{LO})$$

W Y

When $W < 0$ switch to fixed order!

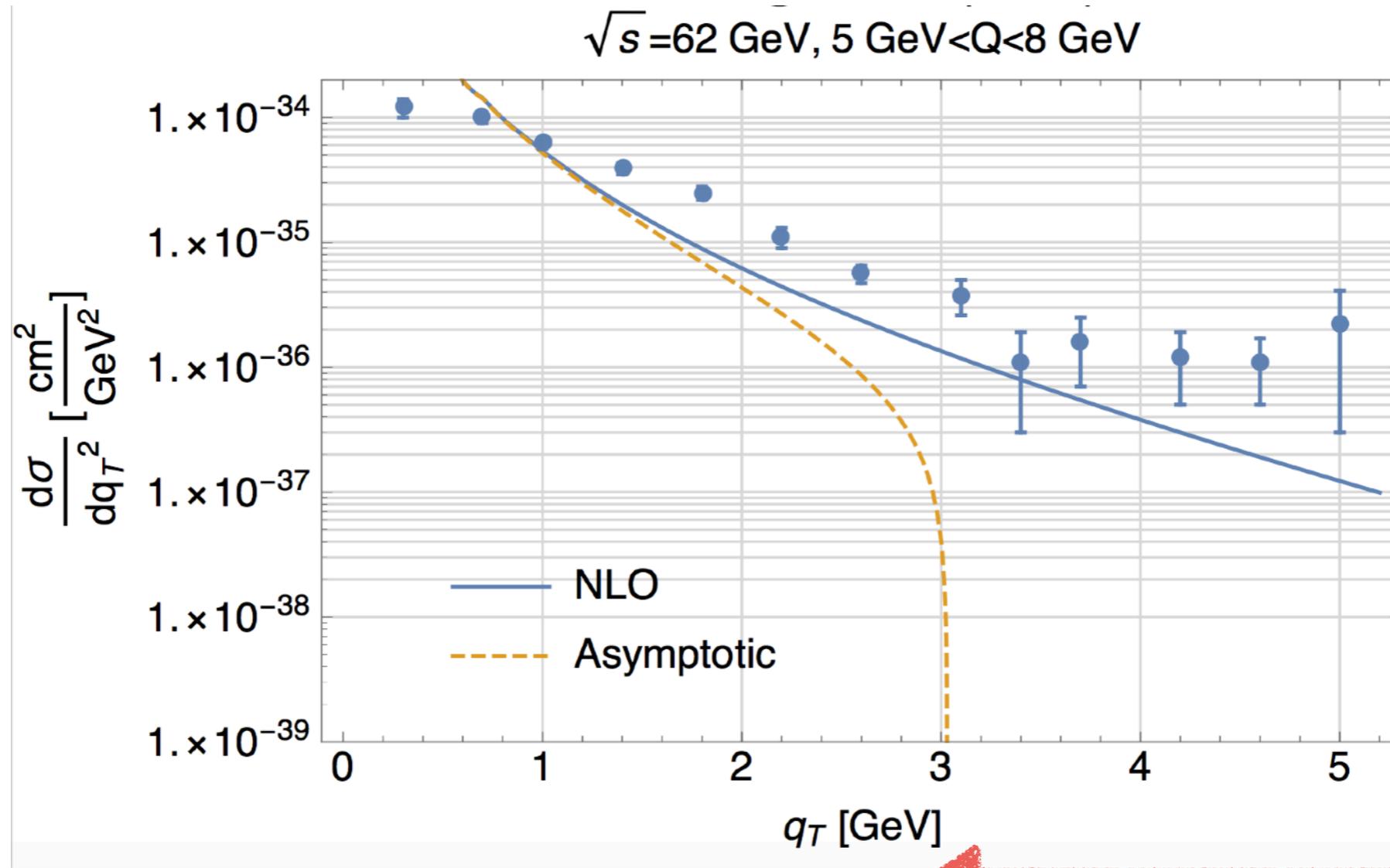
(usually happens at $q_T = Q/2$)

Arnold & Kauffman NP B349 381

Collins et al PRD94 034014

Transverse momentum of Drell-Yan pairs

NLO collinear factorization: $\mathcal{O}(\alpha_s^2)$



Asymptotic =
expansion of resummed result
at fixed order

here collinear factorization
should be reliable

low energy DY data

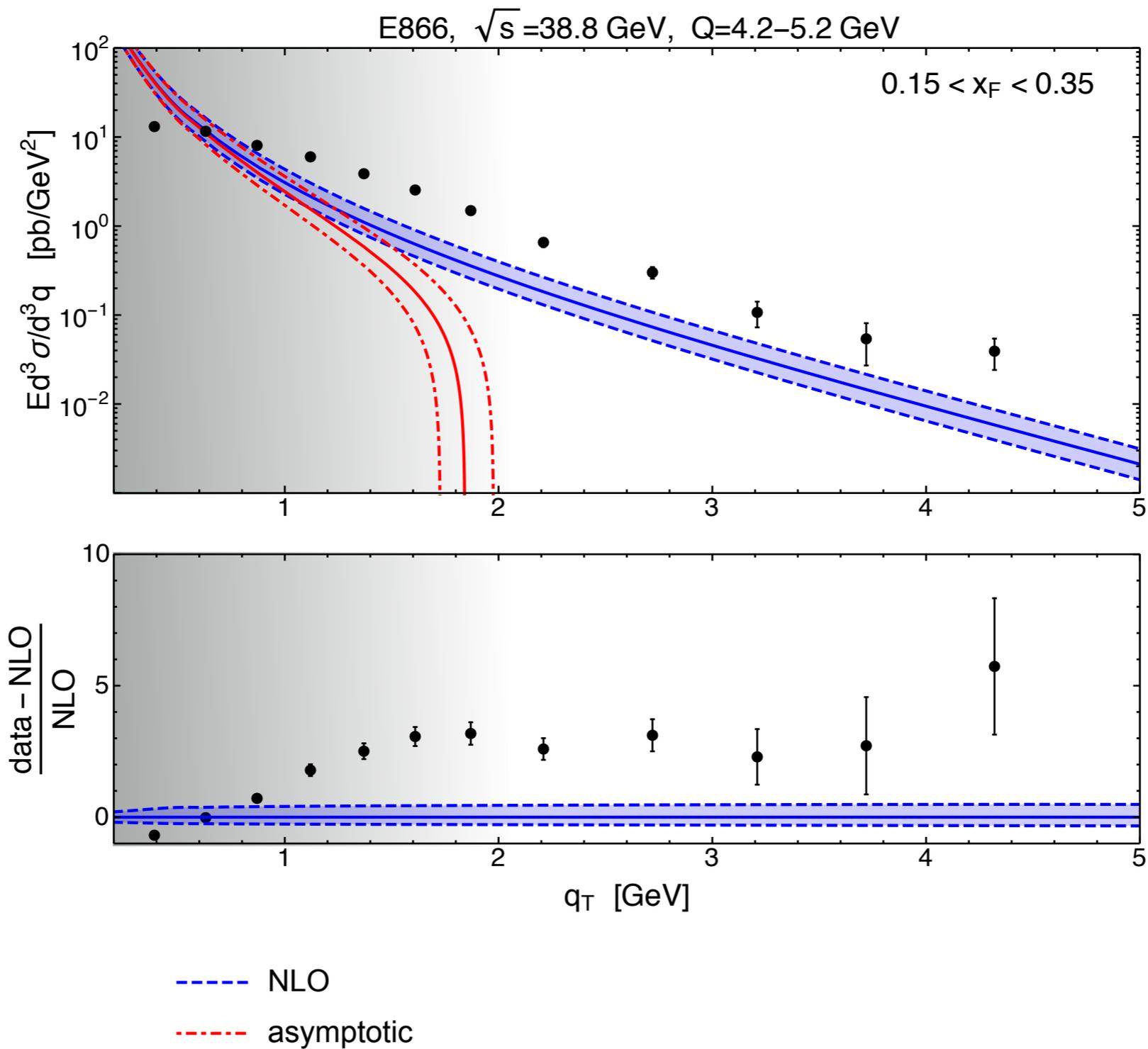
| Experiment | Reaction | Year | TMD fits | PDF fits |
|------------|---------------|------|----------|----------|
| R209 | p-p | 1981 | ✓ | ✗ |
| E288 | p-Cu, p-Pt | 1981 | ✓ | ✗ |
| E605 | p-Cu | 1991 | ✓ | ✓ |
| E866 | p-p, p-d | 2003 | ✗ | ✓ |

$$20 \text{ GeV} \lesssim \sqrt{s} \lesssim 60 \text{ GeV}$$

E866/NuSea

$$pp \rightarrow \mu^+ \mu^- X$$

$$\sqrt{s} = 38.8 \text{ GeV}$$

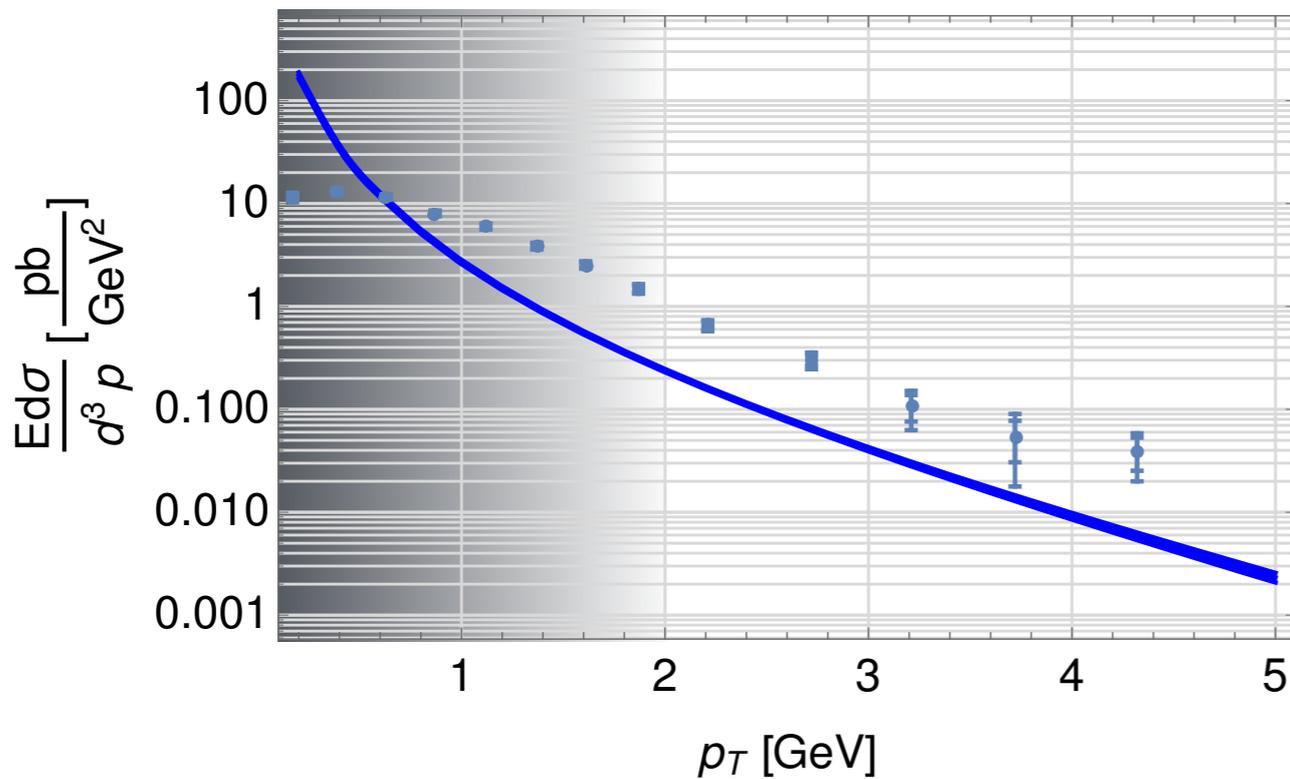


E866/NuSea

$$pp \rightarrow \mu^+ \mu^- X$$

$$\sqrt{s} = 38.8 \text{ GeV}$$

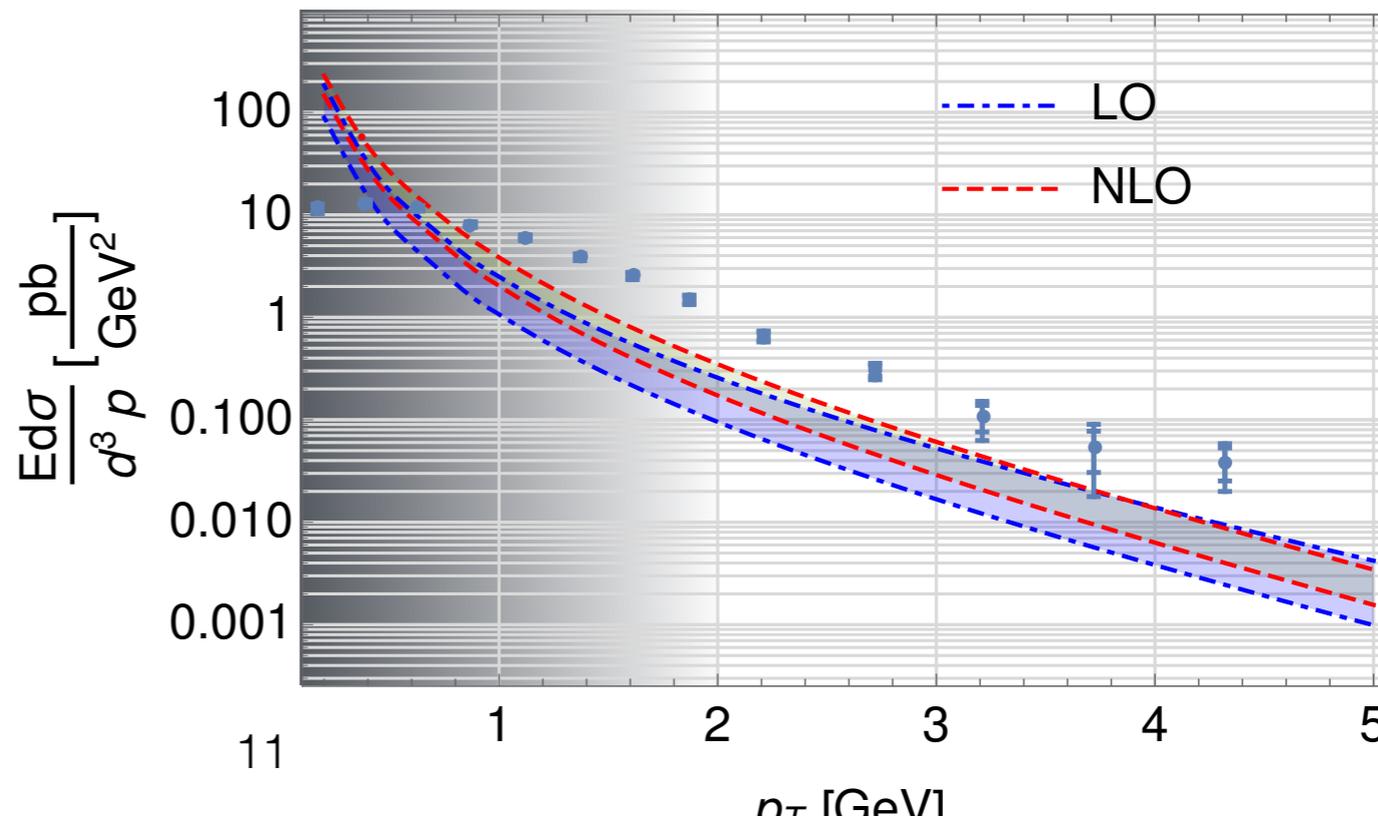
$Q=4.2-5.2 \text{ GeV}, x_F=0.15-0.35$



NLO $\mathcal{O}(\alpha_s^2)$
+PDF uncertainty

scale variations

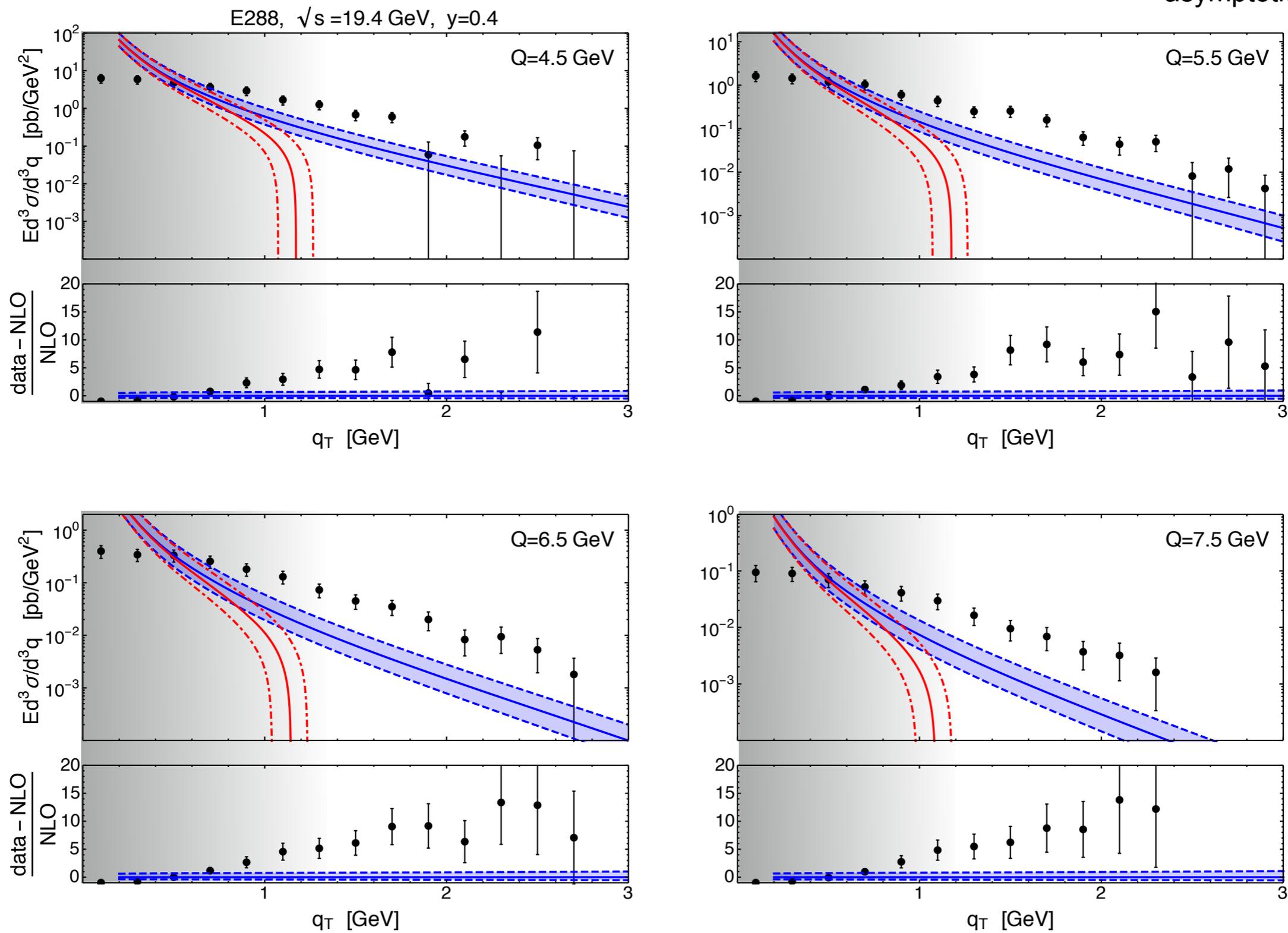
$Q=4.7 \text{ GeV}, x_F=\{0.15,0.35\}, \text{target}=p$



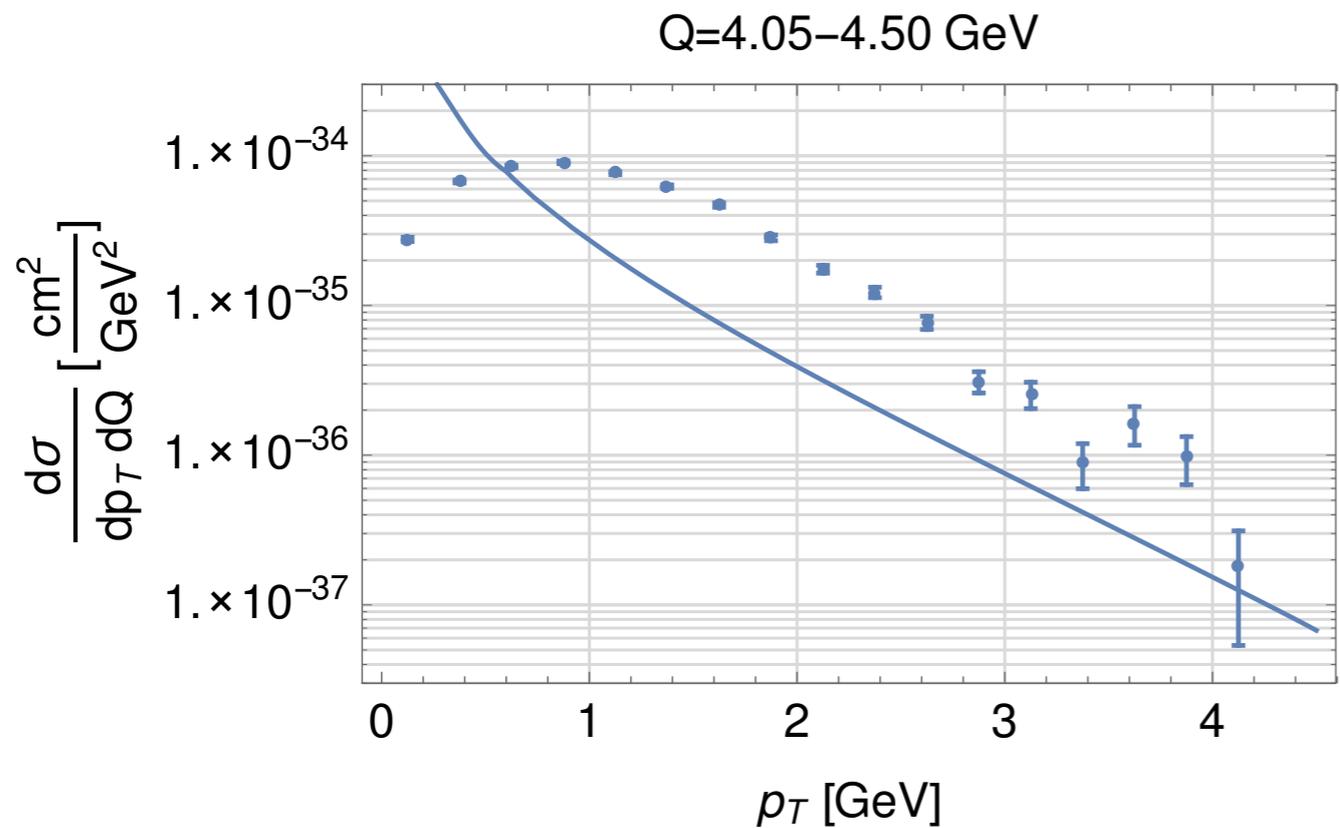
E288

$$p \text{ Cu}, p \text{ Pt} \rightarrow \mu^+ \mu^- X$$

--- NLO
- - - asymptotic

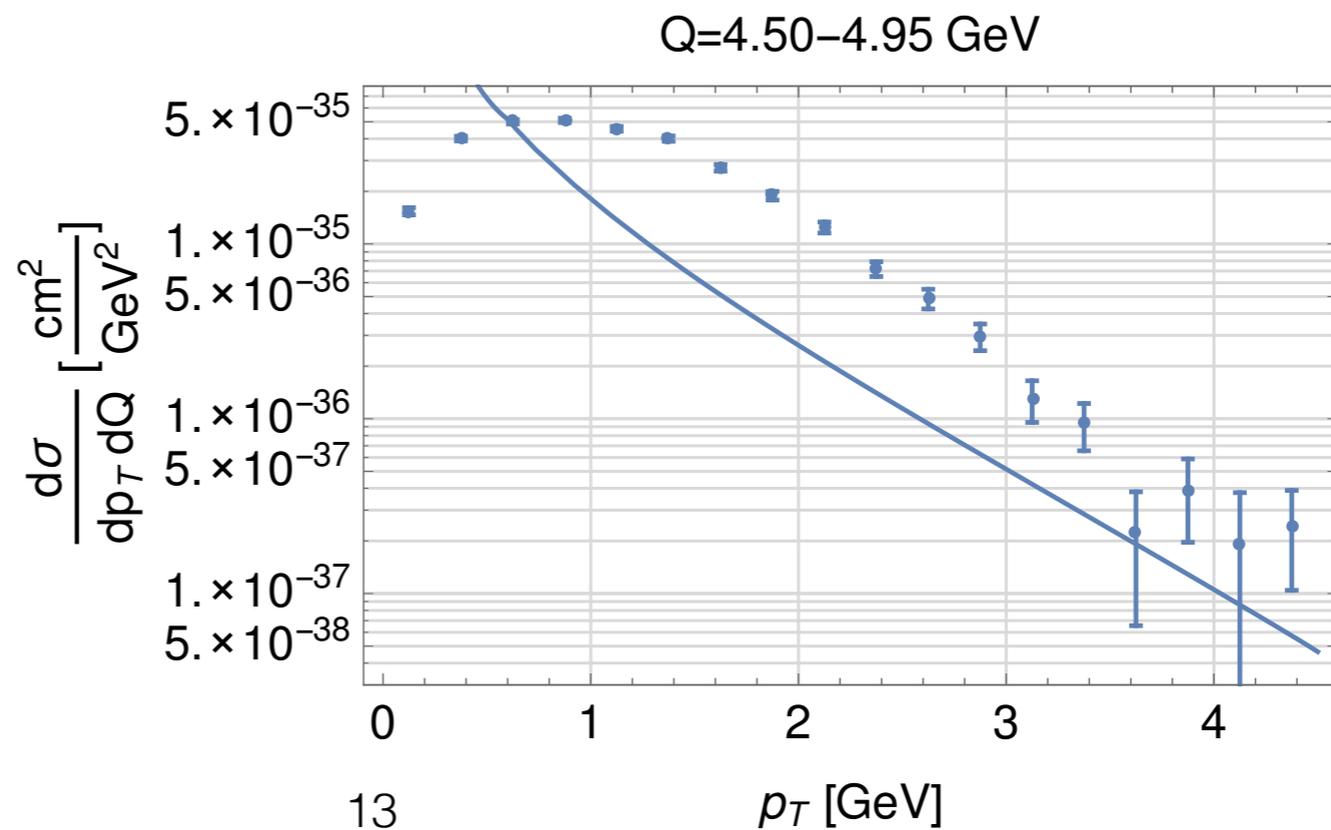


pion-nucleus Drell-Yan



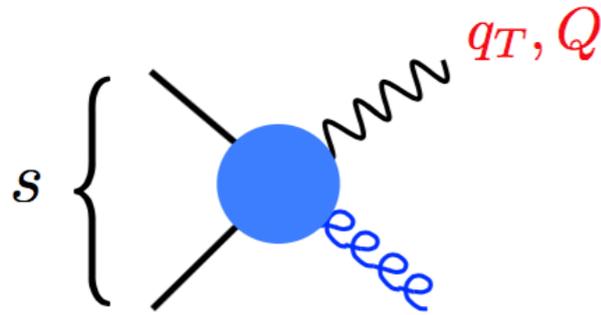
NLO $\mathcal{O}(\alpha_s^2)$

E615
 $\pi W \rightarrow \mu^+ \mu^- X$
 $\sqrt{s} = 21.8 \text{ GeV}$



threshold resummation

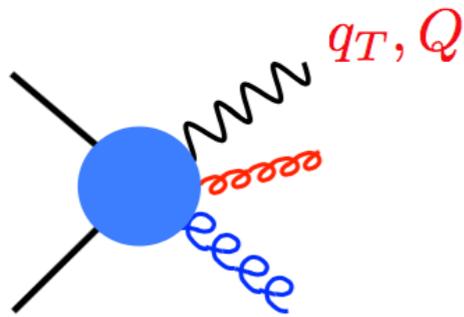
• LO :



$$\sqrt{s} \geq q_T + \sqrt{Q^2 + q_T^2}$$

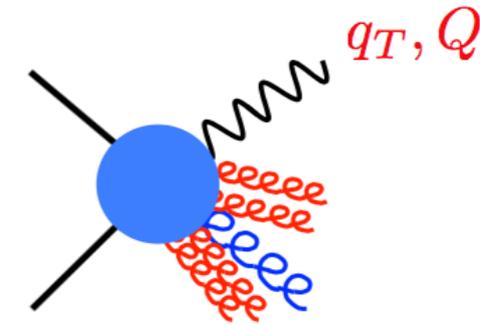
$$y_T \equiv \frac{q_T + \sqrt{q_T^2 + Q^2}}{\sqrt{s}} \leq 1$$

• NLO :



$$\frac{d\hat{\sigma}^{\text{NLO}}}{dq_T} \propto \alpha_s [A \log^2(1 - y_T^2) + B \log(1 - y_T^2) + C]$$

• N^kLO :

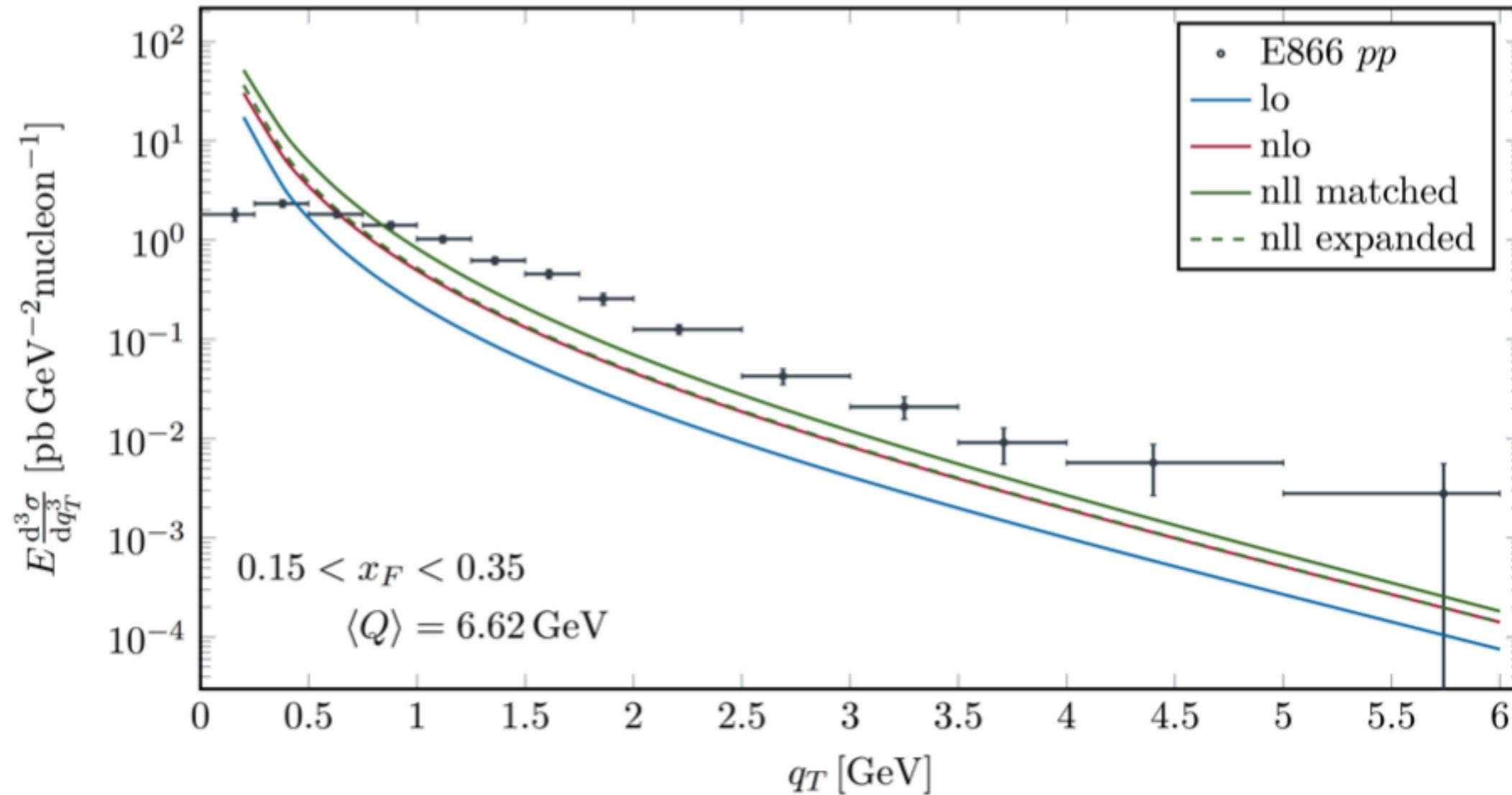


$$\frac{d\hat{\sigma}^{\text{N}^k\text{LO}}}{dq_T} \propto \alpha_s^k \log^{2k}(1 - y_T^2) + \dots$$

• threshold logarithms

W. Vogelsang @Transversity 2017

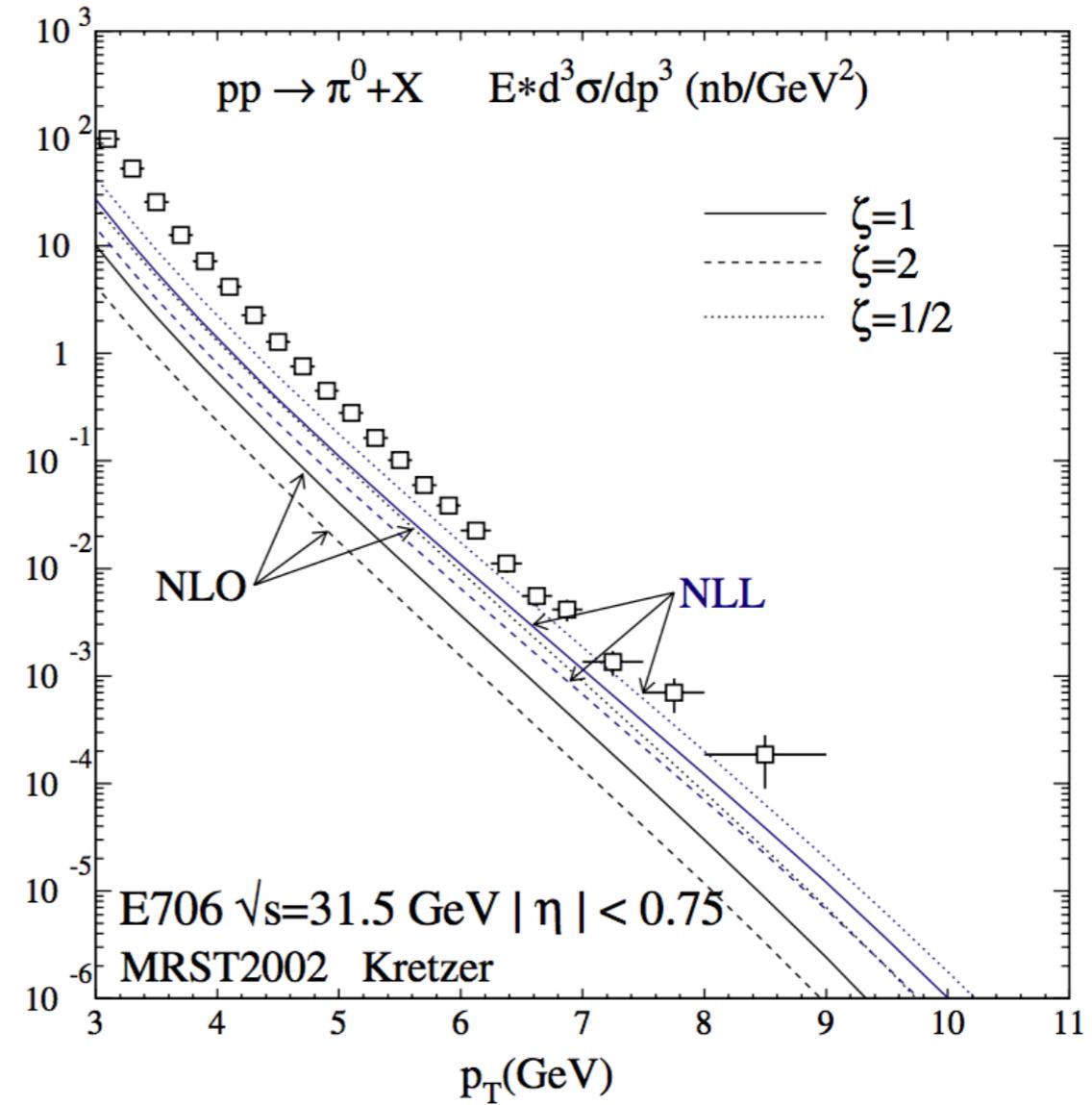
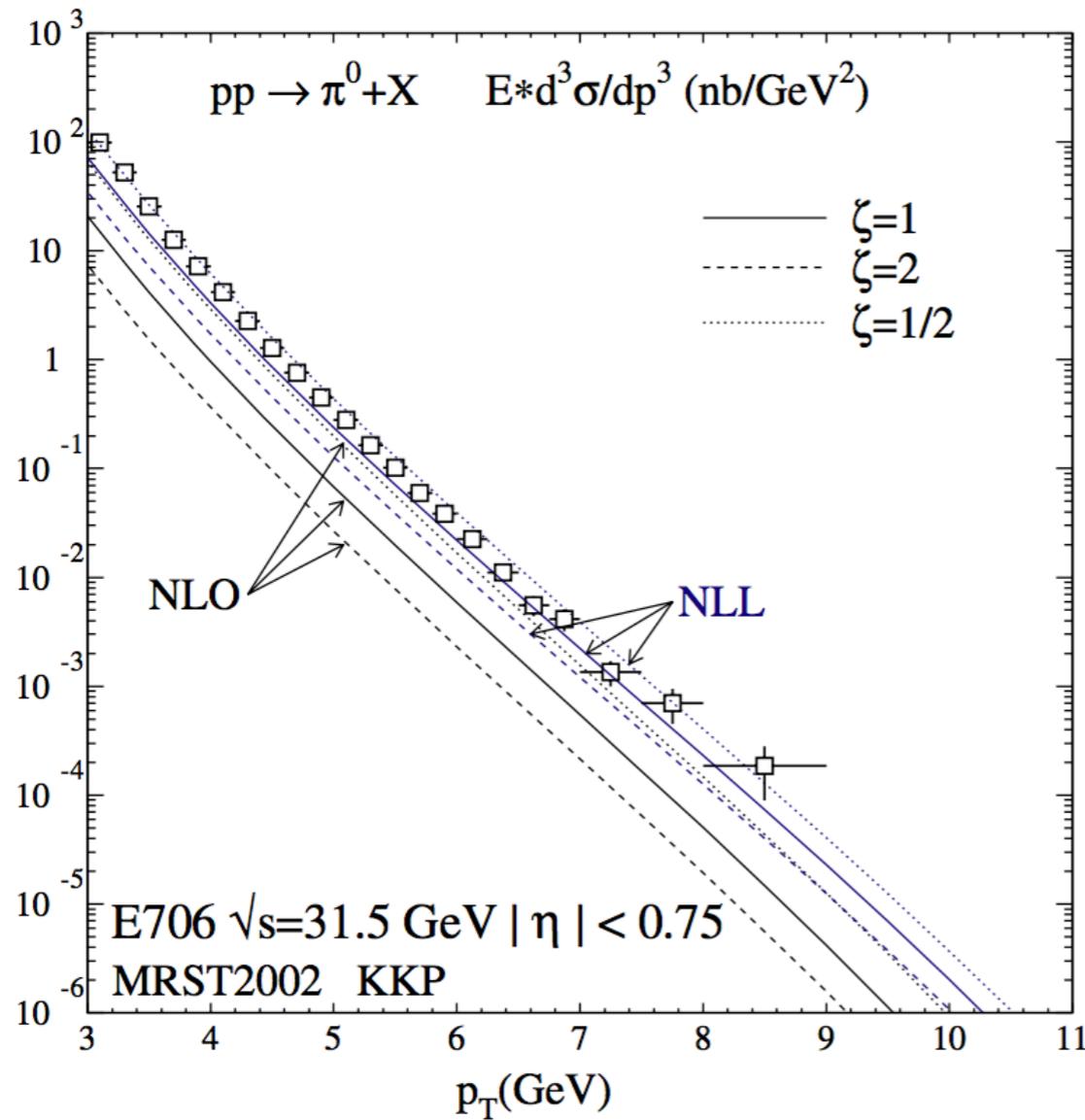
threshold resummation



W. Vogelsang @Transversity 2017

known similar cases

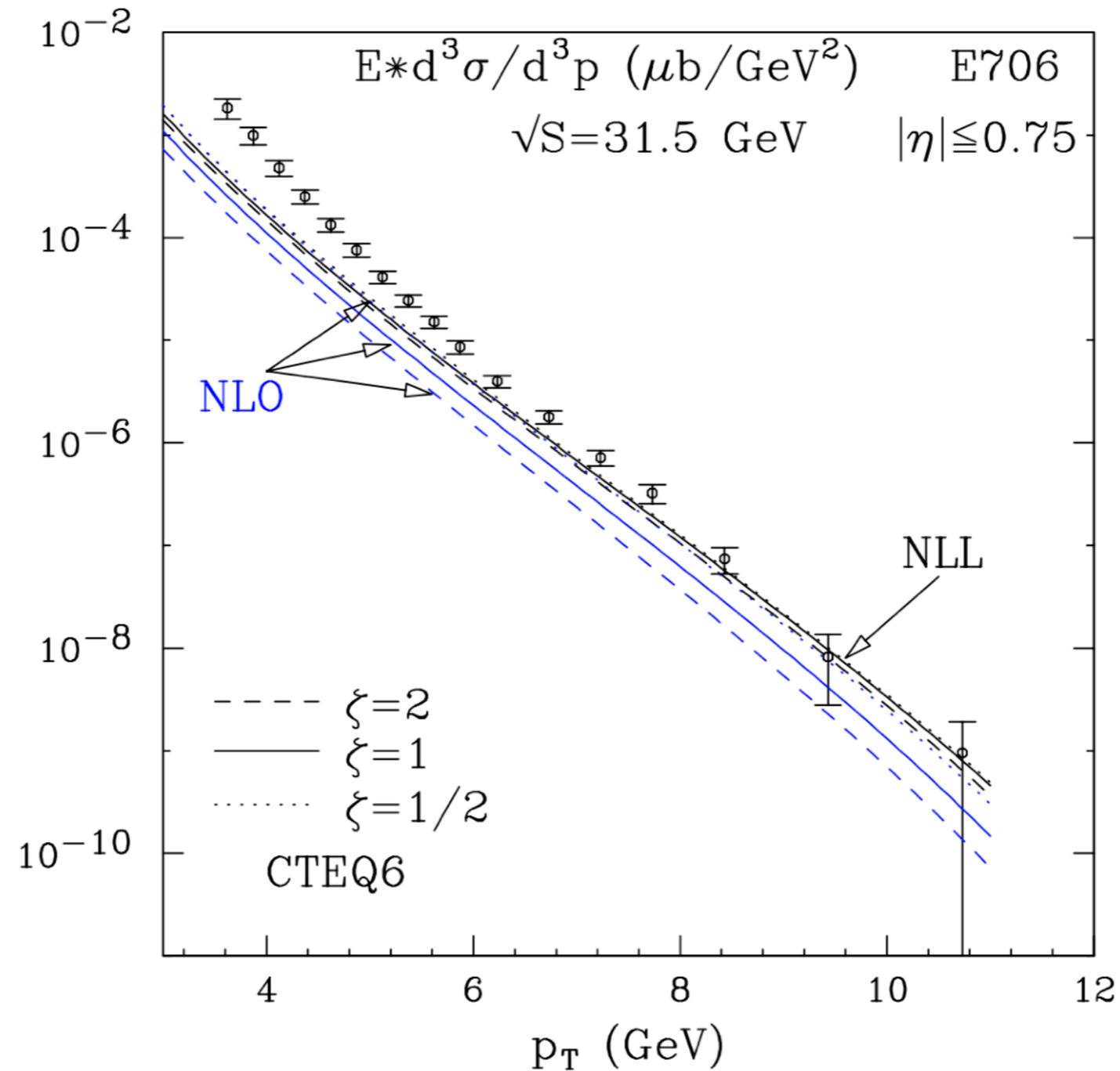
pion production



de Florian Vogelsang PRD71 114004 (2005)

known similar cases

prompt photon



de Florian Vogelsang PRD72 014014 (2005)

intrinsic k_T smearing

- take collinear factorization formula

$$d\sigma = \sum_{ab} \int dx_a dx_b f_{a/A}(x_a) f_{b/B}(x_b) d\hat{\sigma}^{ab \rightarrow l^+ l^-}$$

- give the incoming partons a small k_T

$$d\sigma = \sum_{ab} \int dx_a d^2\mathbf{k}_{Ta} dx_b d^2\mathbf{k}_{Tb} \\ \times f_{a/A}(x, \mathbf{k}_{Ta}) f_{b/B}(x, \mathbf{k}_{Tb}) \frac{\hat{s}}{x_a x_b s} d\sigma^{ab \rightarrow l^+ l^-}$$

intrinsic k_T smearing

intrinsic k_T has a long history...
(for prompt photon and pion production)

Owens RMP59, 465 (1987)

Sivers PRD41, 83 (1990)

D'Alesio Murgia PRD70, 074009 (2004)

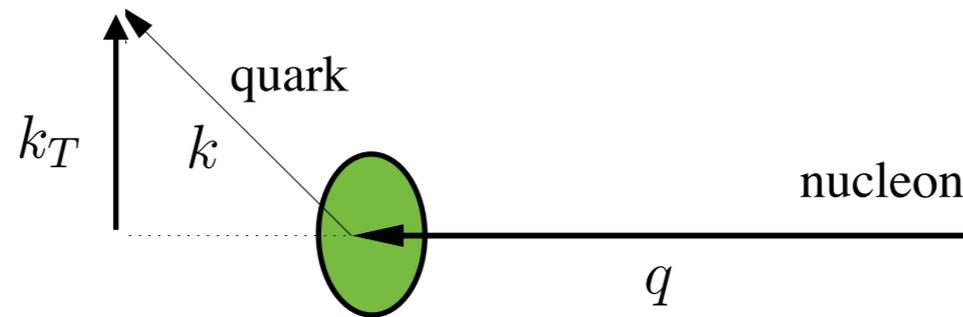
...

- give the incoming partons a small k_T

$$d\sigma = \sum_{ab} \int dx_a d^2\mathbf{k}_{Ta} dx_b d^2\mathbf{k}_{Tb} \\ \times f_{a/A}(x, \mathbf{k}_{Ta}) f_{b/B}(x, \mathbf{k}_{Tb}) \frac{\hat{s}}{x_a x_b S} d\sigma^{ab \rightarrow l^+ l^-}$$

intrinsic k_T smearing

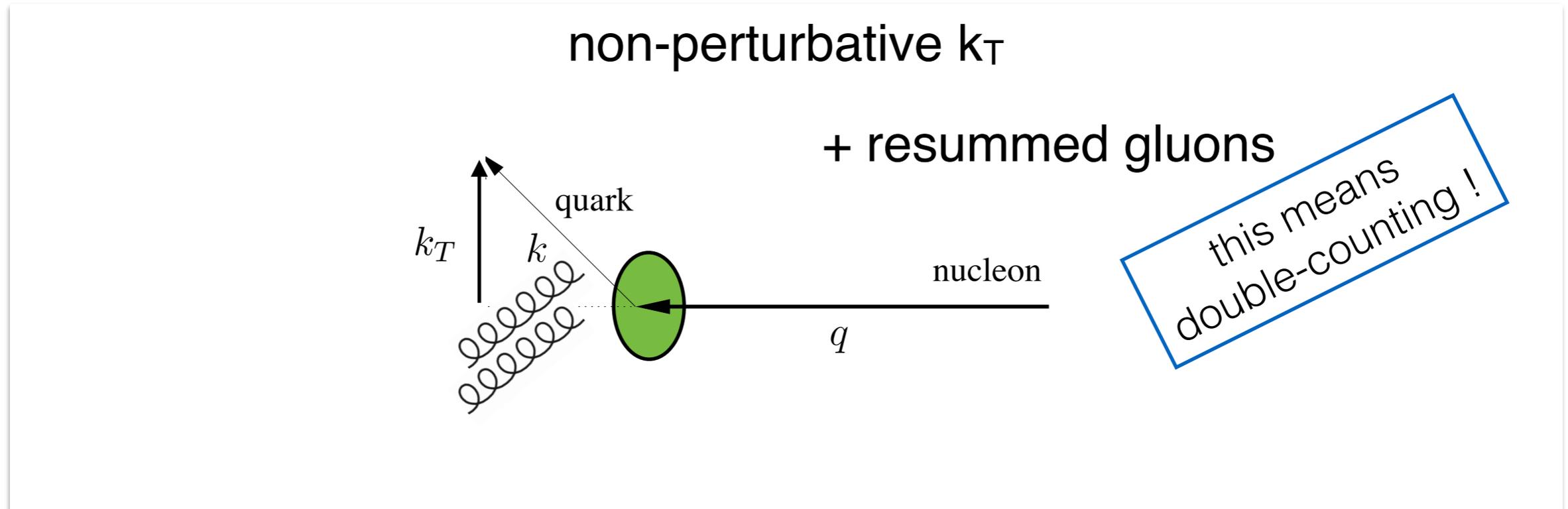
non-perturbative k_T



- give the incoming partons a small k_T

$$d\sigma = \sum_{ab} \int dx_a d^2\mathbf{k}_{Ta} dx_b d^2\mathbf{k}_{Tb} \\ \times f_{a/A}(x, \mathbf{k}_{Ta}) f_{b/B}(x, \mathbf{k}_{Tb}) \frac{\hat{s}}{x_a x_b s} d\sigma^{ab \rightarrow l^+ l^-}$$

intrinsic k_T smearing



- give the incoming partons a small k_T

$$d\sigma = \sum_{ab} \int dx_a d^2\mathbf{k}_{Ta} dx_b d^2\mathbf{k}_{Tb}$$

$$\times f_{a/A}(x, \mathbf{k}_{Ta}) f_{b/B}(x, \mathbf{k}_{Tb}) \frac{\hat{s}}{x_a x_b s} d\sigma^{ab \rightarrow l^+ l^-}$$

intrinsic k_T smearing

$$d\sigma = \sum_{ab} \int dx_a d^2\mathbf{k}_{Ta} dx_b d^2\mathbf{k}_{Tb} \\ \times f_{a/A}(x, \mathbf{k}_{Ta}) f_{b/B}(x, \mathbf{k}_{Tb}) \frac{\hat{s}}{x_a x_b S} d\sigma^{ab \rightarrow l^+ l^-}$$

parton momentum:

$$p_a^\mu \doteq (p_a^0, \mathbf{p}_a^T, p_a^3) = \left(x_a P_A + \frac{k_{Ta}^2}{4x_a P_A}, \mathbf{k}_{Ta}, x_a P_A - \frac{k_{Ta}^2}{4x_a P_A} \right)$$

this enforces: $p_a^\mu p_{a\mu} = 0 \quad x_a = p_a^+ / P_A^+$

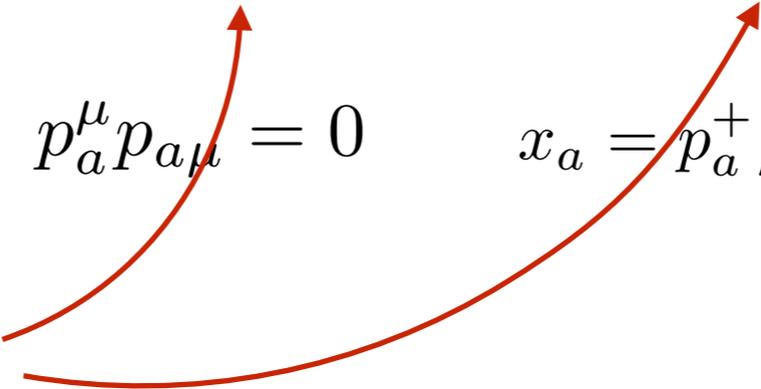
intrinsic k_T smearing

$$d\sigma = \sum_{ab} \int dx_a d^2\mathbf{k}_{Ta} dx_b d^2\mathbf{k}_{Tb} \\ \times f_{a/A}(x, \mathbf{k}_{Ta}) f_{b/B}(x, \mathbf{k}_{Tb}) \frac{\hat{s}}{x_a x_b s} d\sigma^{ab \rightarrow l^+ l^-}$$

parton momentum:

$$p_a^\mu \doteq (p_a^0, \mathbf{p}_a^T, p_a^3) = \left(x_a P_A + \frac{k_{Ta}^2}{4x_a P_A}, \mathbf{k}_{Ta}, x_a P_A - \frac{k_{Ta}^2}{4x_a P_A} \right)$$

this enforces: $p_a^\mu p_{a\mu} = 0$ $x_a = p_a^+ / P_A^+$



must make sure

- quark energy < proton energy
- quark direction = proton direction

$$k_{Ta} < \sqrt{x_a (1 - x_a)} \sqrt{s}$$

$$k_{Ta} < x_a \sqrt{s}$$

better to stay far from the bounds!

intrinsic k_T smearing

$$d\sigma = \sum_{ab} \int dx_a d^2\mathbf{k}_{Ta} dx_b d^2\mathbf{k}_{Tb} \\ \times f_{a/A}(x, \mathbf{k}_{Ta}) f_{b/B}(x, \mathbf{k}_{Tb}) \frac{\hat{s}}{x_a x_b S} d\sigma^{ab \rightarrow l^+ l^-}$$

parton momentum:

$$p_a^\mu \doteq (p_a^0, \mathbf{p}_a^T, p_a^3) = \left(x_a P_A + \frac{k_{Ta}^2}{4x_a P_A}, \mathbf{k}_{Ta}, x_a P_A - \frac{k_{Ta}^2}{4x_a P_A} \right)$$

$$\hat{t} = (q_\gamma - p_a)^2 = Q^2 - 2q_\gamma^- p_a^+ - 2q_\gamma^+ p_a^- + 2\mathbf{q}_T \cdot \mathbf{p}_{Ta}$$

make sure

\hat{t}, \hat{u}

not too small

Fixed Order must be valid!

intrinsic k_T smearing

$$d\sigma = \sum_{ab} \int dx_a d^2\mathbf{k}_{Ta} dx_b d^2\mathbf{k}_{Tb}$$
$$\times f_{a/A}(x, \mathbf{k}_{Ta}) f_{b/B}(x, \mathbf{k}_{Tb}) \frac{\hat{s}}{x_a x_b S} d\sigma^{ab \rightarrow l^+ l^-}$$

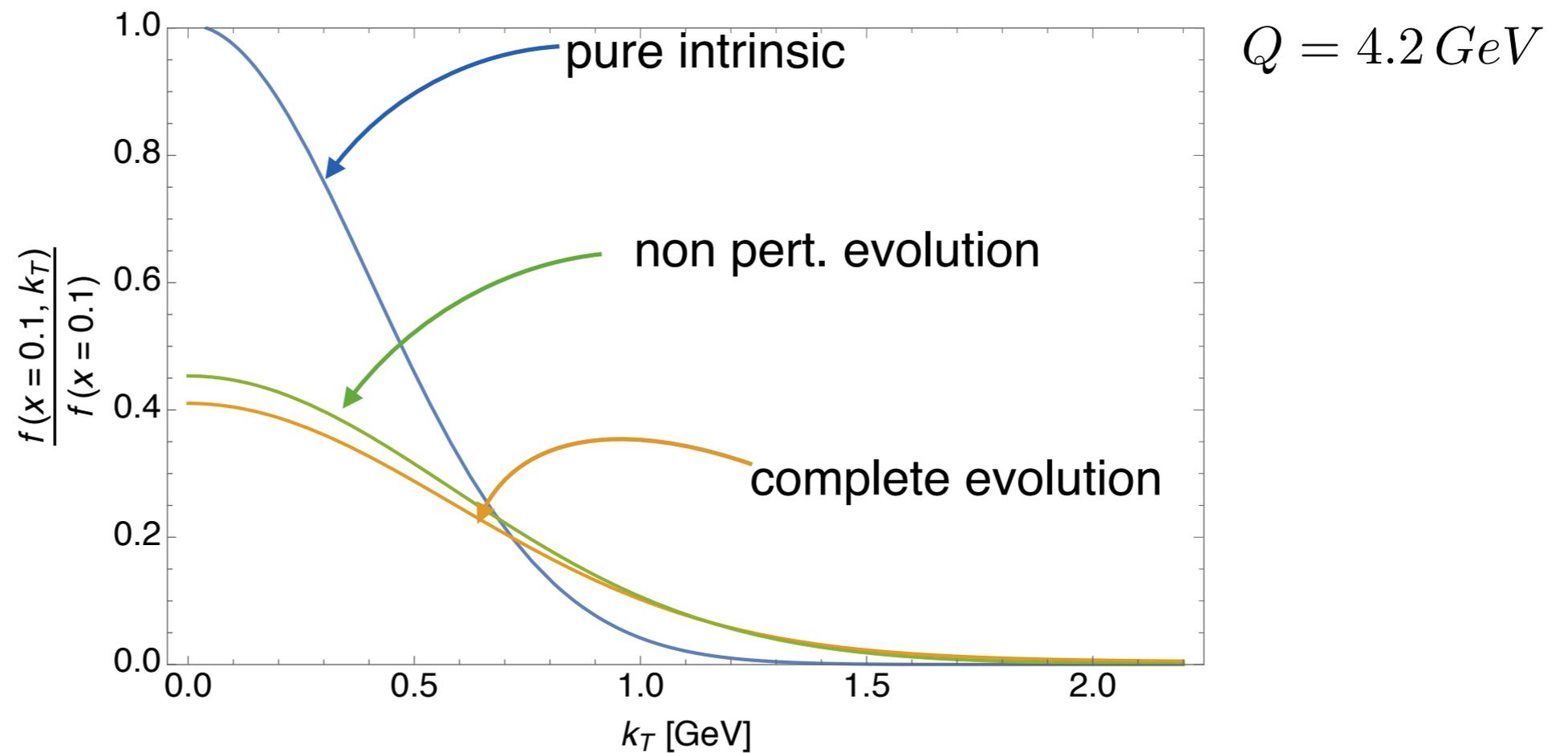
$$\int_0^{k_{Tmax}^2} \pi dk_T^2$$

check independence from cutoff!

intrinsic k_T smearing

$$\frac{f_{a/A}(x_a, \mathbf{k}_{T a})}{f_{a/A}(x_a)}$$

Q evolution from TMD extraction:

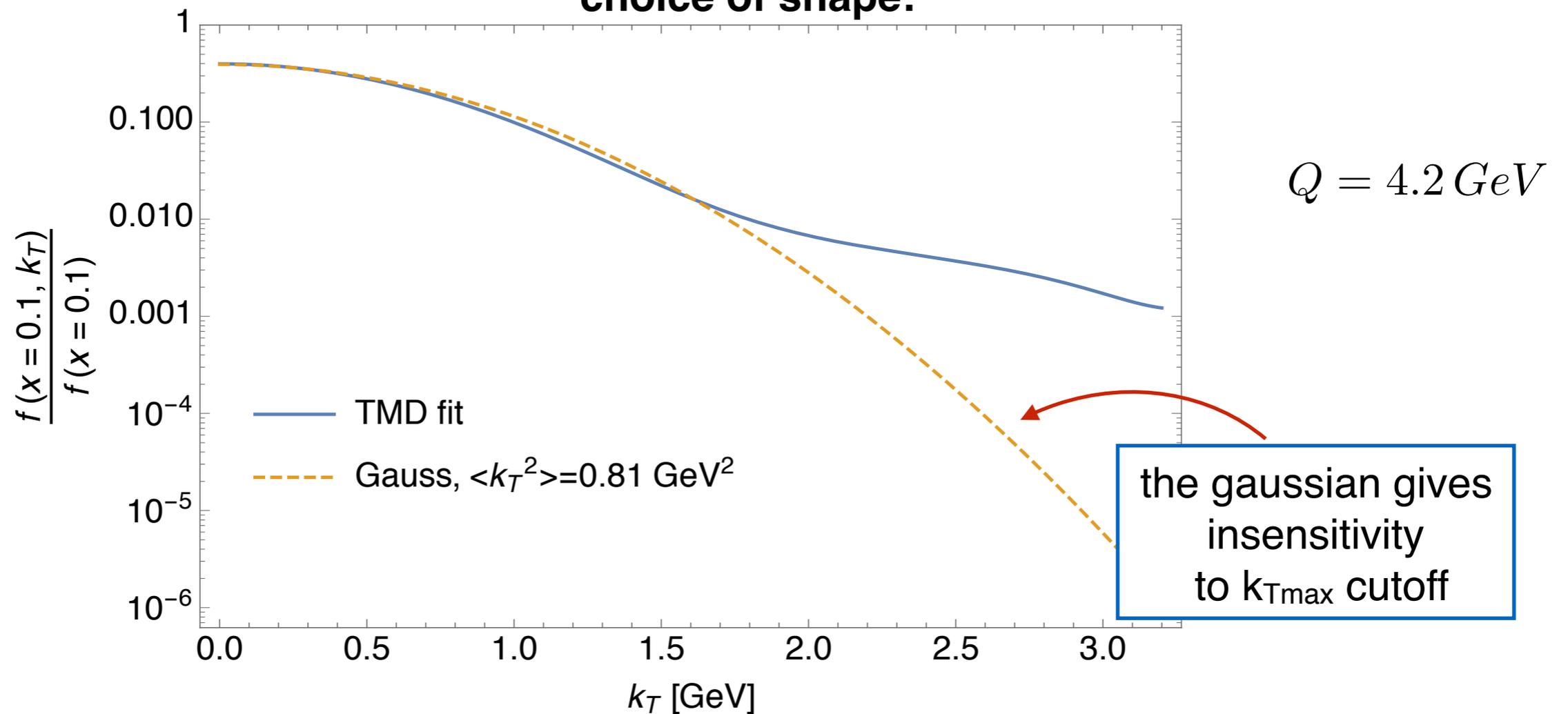


Bacchetta et al JHEP 1706 081

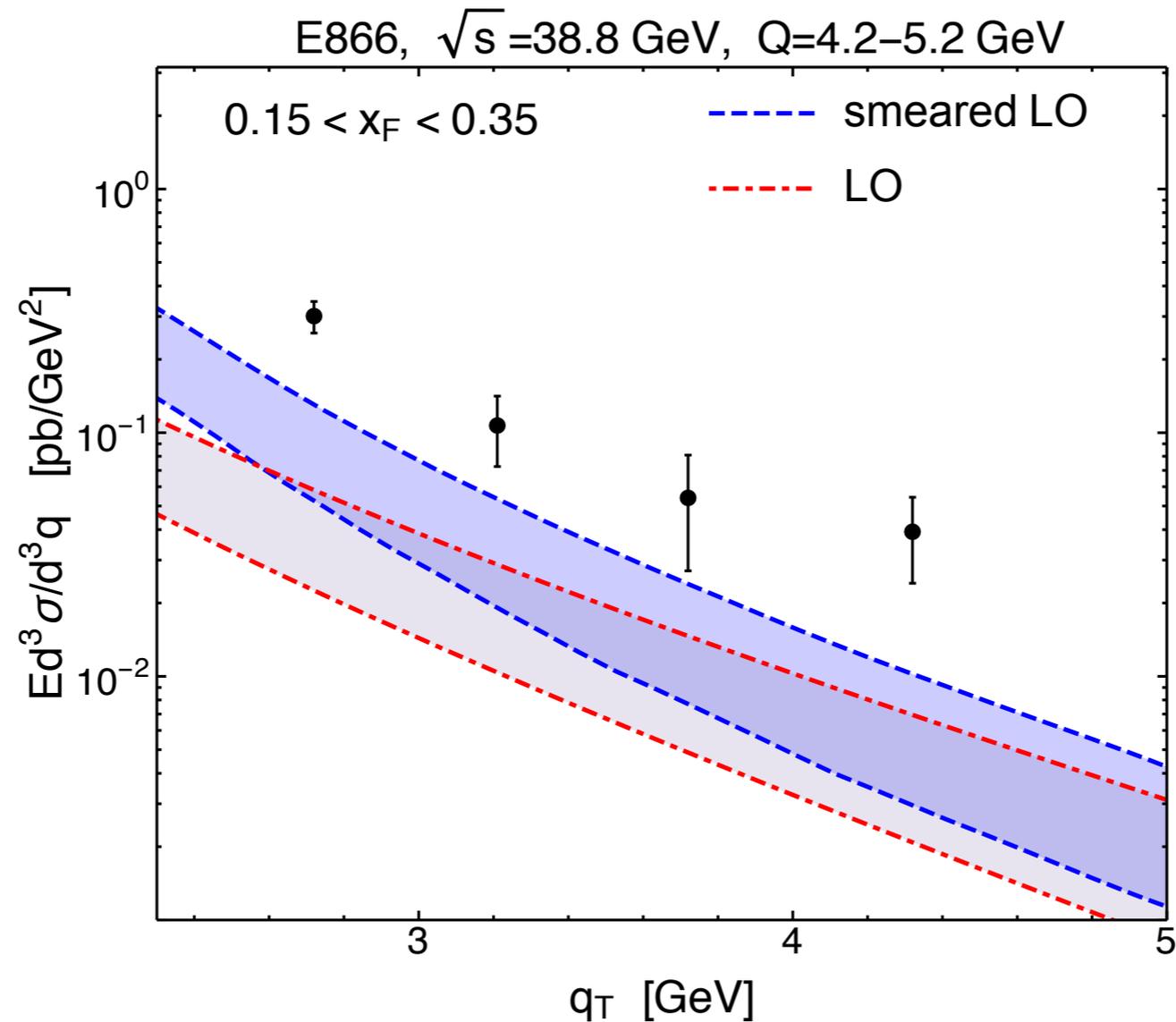
intrinsic k_T smearing

$$\frac{f_{a/A}(x_a, \mathbf{k}_{Ta})}{f_{a/A}(x_a)}$$

choice of shape:



intrinsic k_T smearing



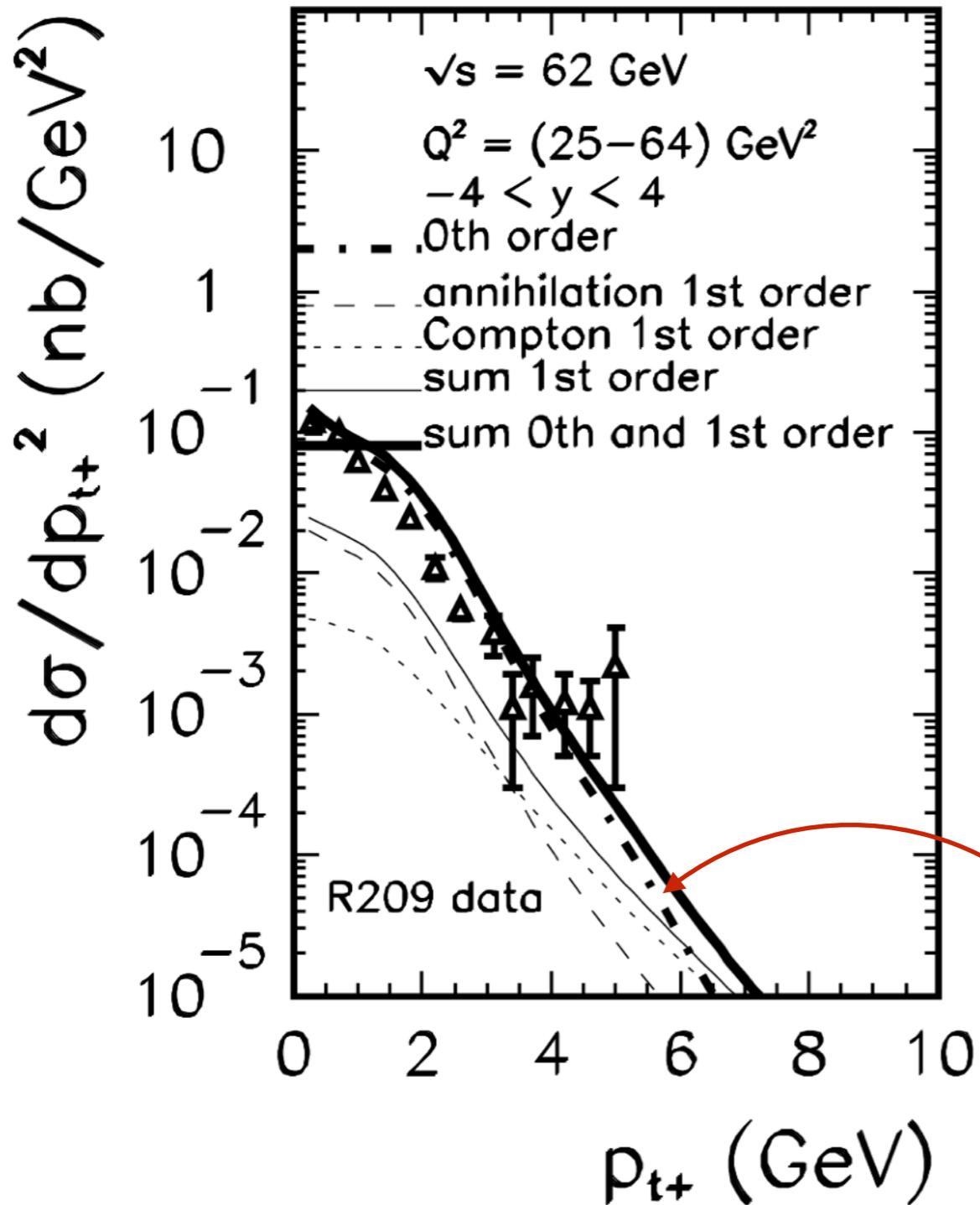
$$f_q(x, \mathbf{k}_T) = f(x) \frac{1}{\pi \langle k_T^2 \rangle} e^{-\frac{k_T^2}{\langle k_T^2 \rangle}}$$

$$\langle k_T^2 \rangle = 0.81 \text{ GeV}^2 \quad \text{same for any } x \text{ and flavor} \\ \dots \text{and for gluons!!}$$

Summary

- fixed-order pQCD largely underestimates low-energy DY data at high q_T
- nor threshold resummation nor intrinsic- k_T model seem to help
- more high q_T data needed (Compass ?)

... "k_T-factorization" formalism?

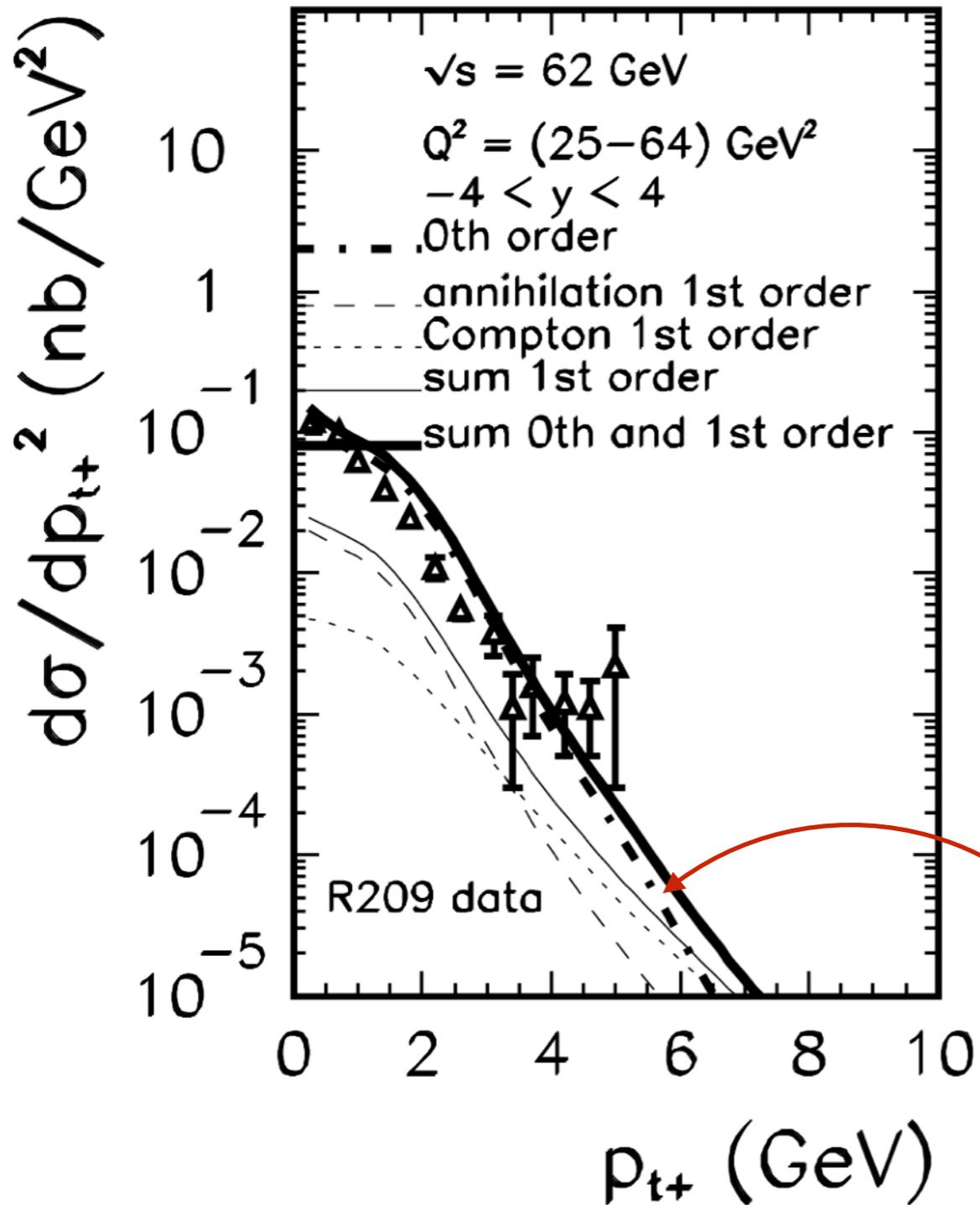


$$d\sigma = \sum_{ab} \int dx_a d^2\mathbf{k}_{Ta} dx_b d^2\mathbf{k}_{Tb} \times F_{a/A}^u(x_a, \mathbf{k}_{Ta}, \mu_F) F_{b/B}^u(x_b, \mathbf{k}_{Tb}, \mu_F) \frac{\hat{s}}{x_a x_b} d\hat{\sigma}^{ab \rightarrow l^+ l^-}$$

“unintegrated parton distributions”
(from extension of CCFM equations)

almost all of the cross-section
given by soft gluons +
intrinsic k_T

... "k_T-factorization" formalism?



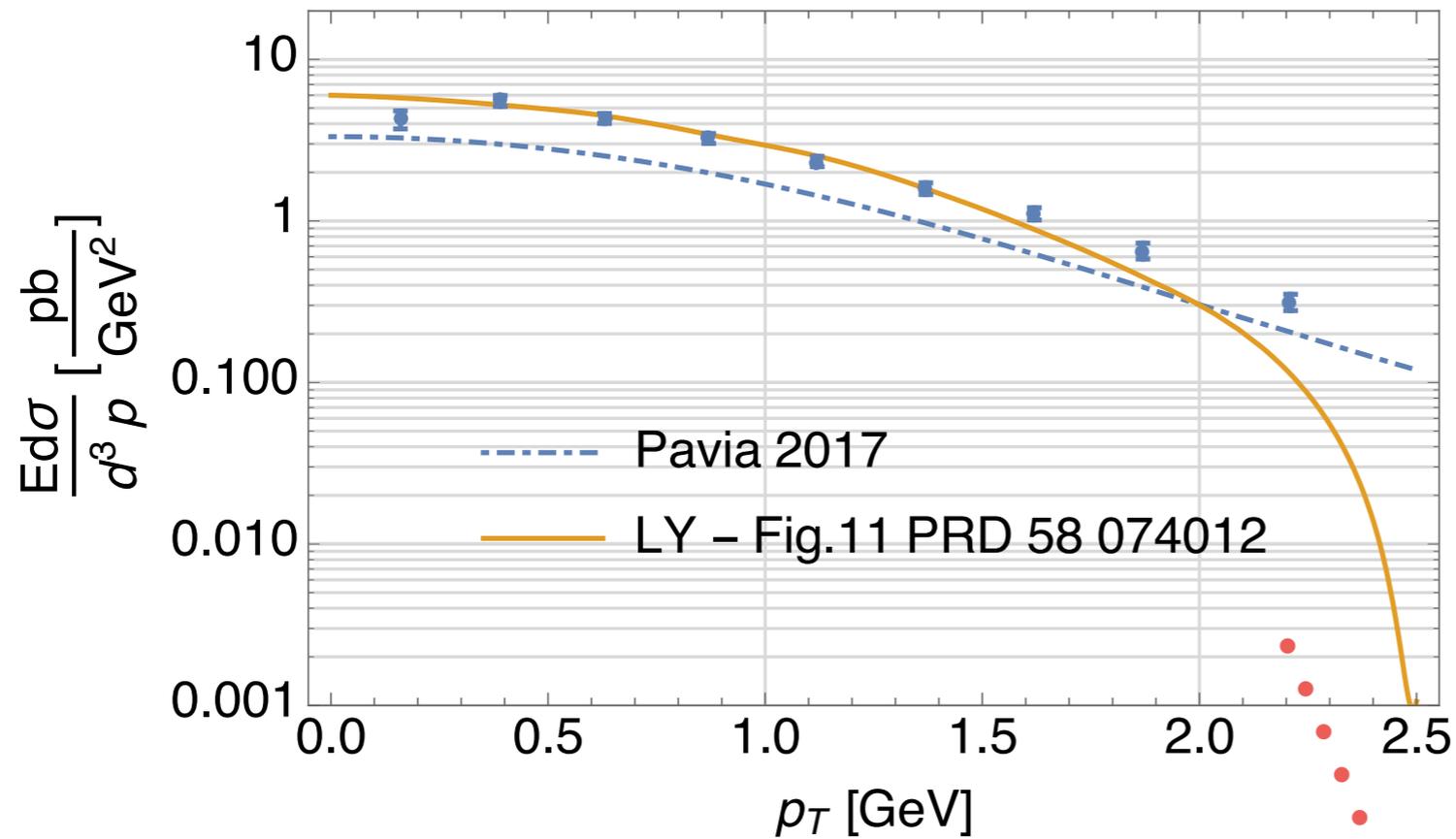
$$d\sigma = \sum_{ab} \int dx_a d^2\mathbf{k}_{Ta} dx_b d^2\mathbf{k}_{Tb} \times F_{a/A}^u(x_a, \mathbf{k}_{Ta}, \mu_F) F_{b/B}^u(x_b, \mathbf{k}_{Tb}, \mu_F) \frac{\hat{s}}{x_a x_b} d\hat{\sigma}^{ab \rightarrow l^+ l^-}$$

**apparently difficult to find
 a formal proof**
Avsar, Collins arXiv:1209.1675

**not clear why
 it should differ from "traditional"
 formalism at this kinematics
 (not small-x)**

almost all of the cross-section
 given by soft gluons +
 intrinsic k_T

Q=5.2–6.2 GeV, $x_F=0.15-0.35$

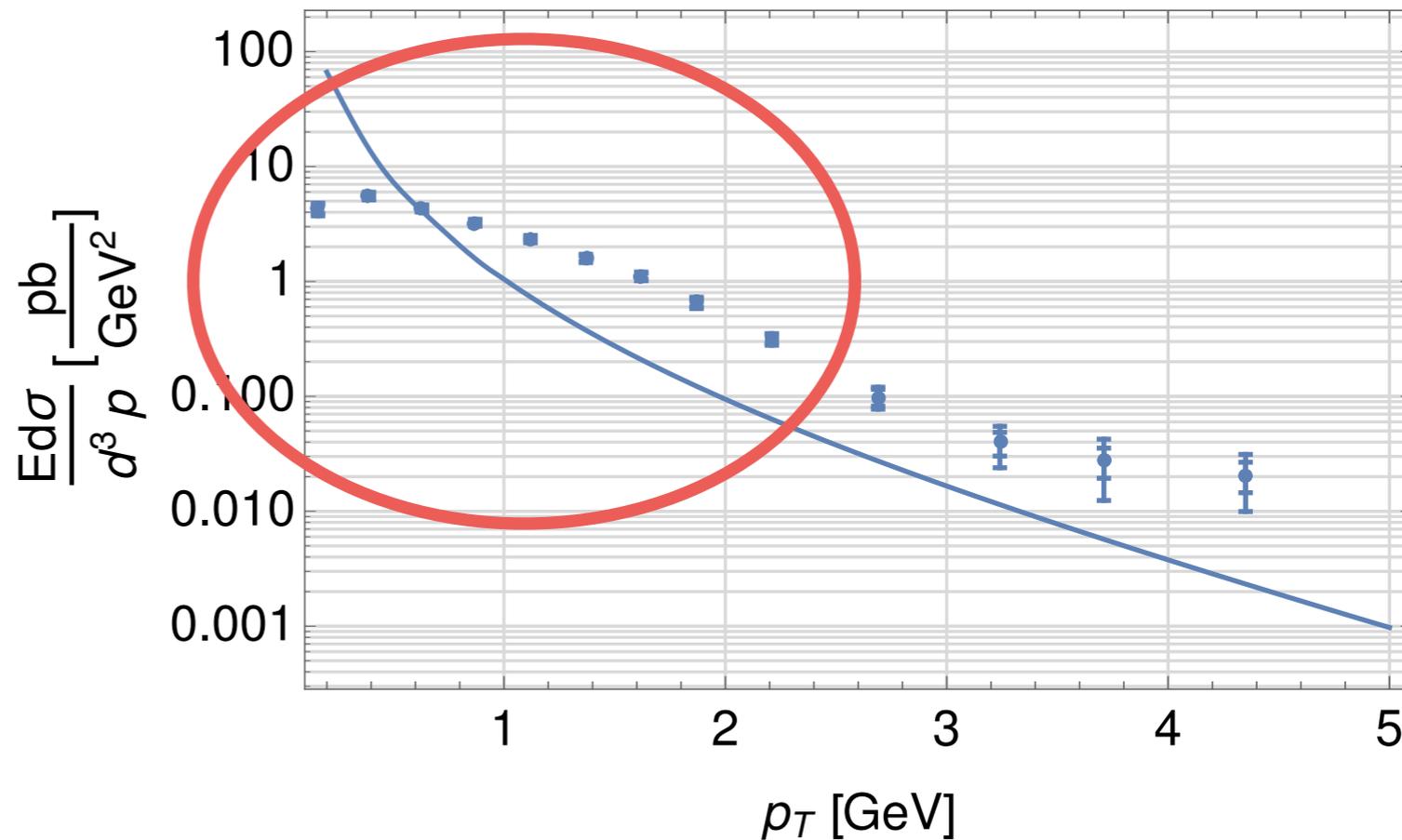


TMD
parametrizations

Q=5.2–6.2 GeV, $x_F=0.15-0.35$

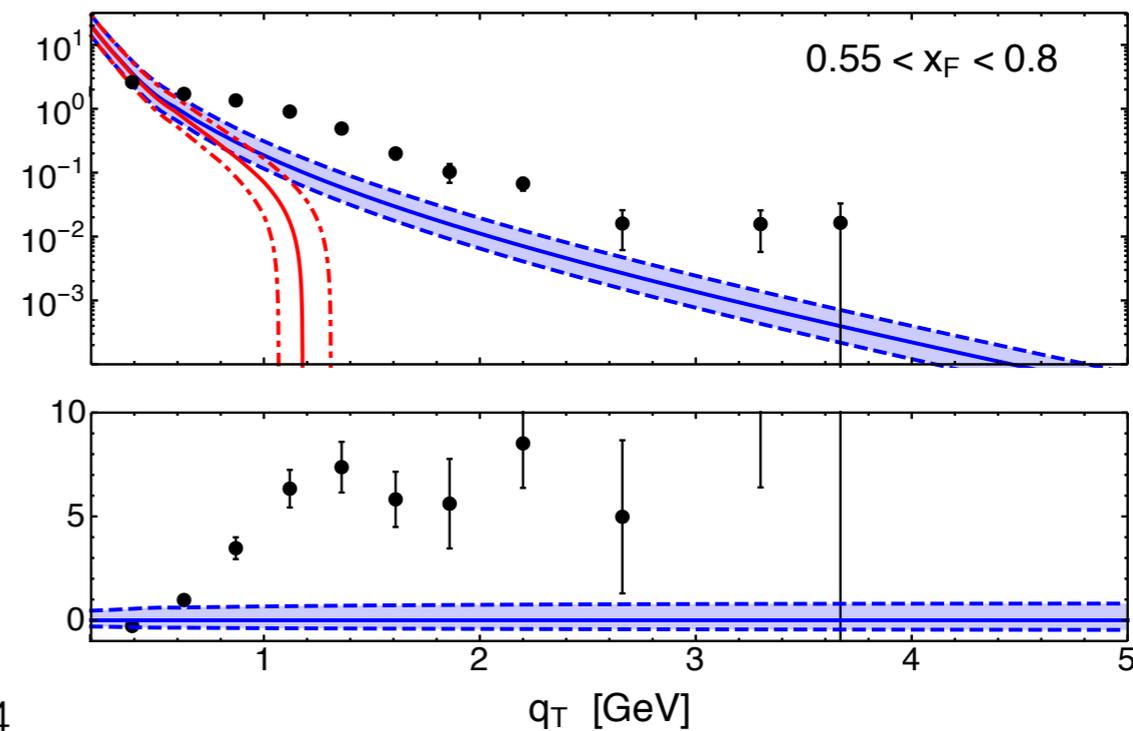
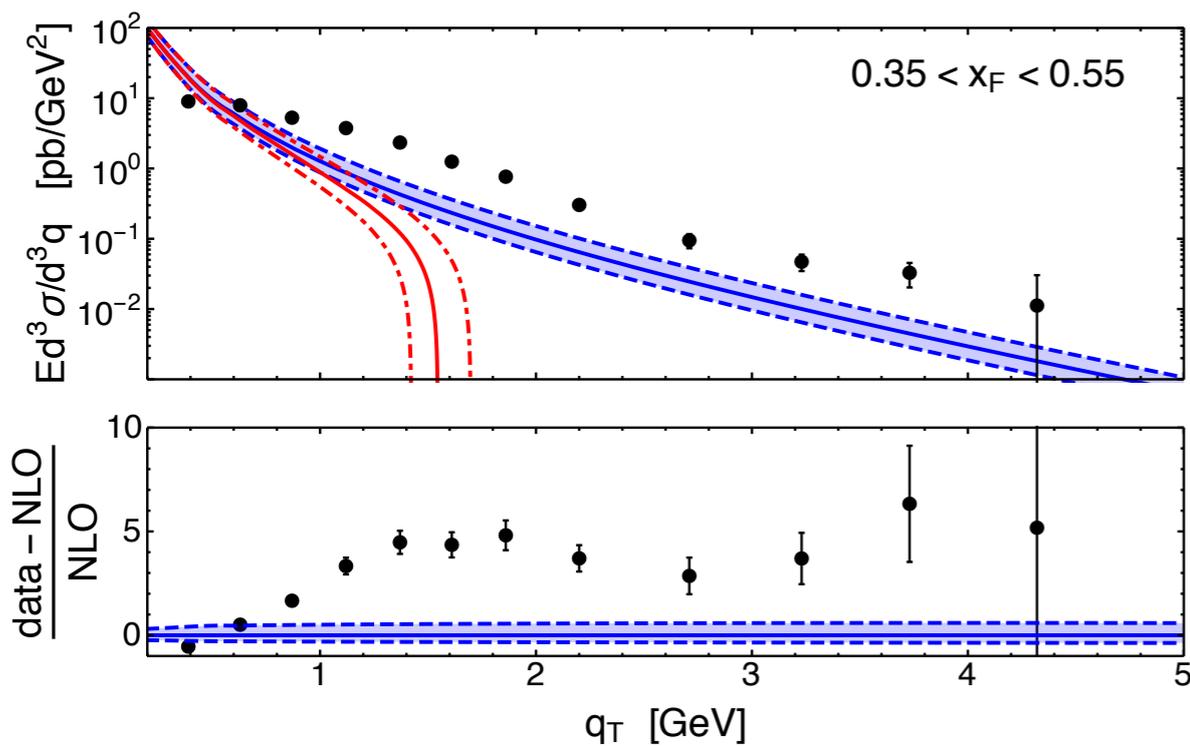
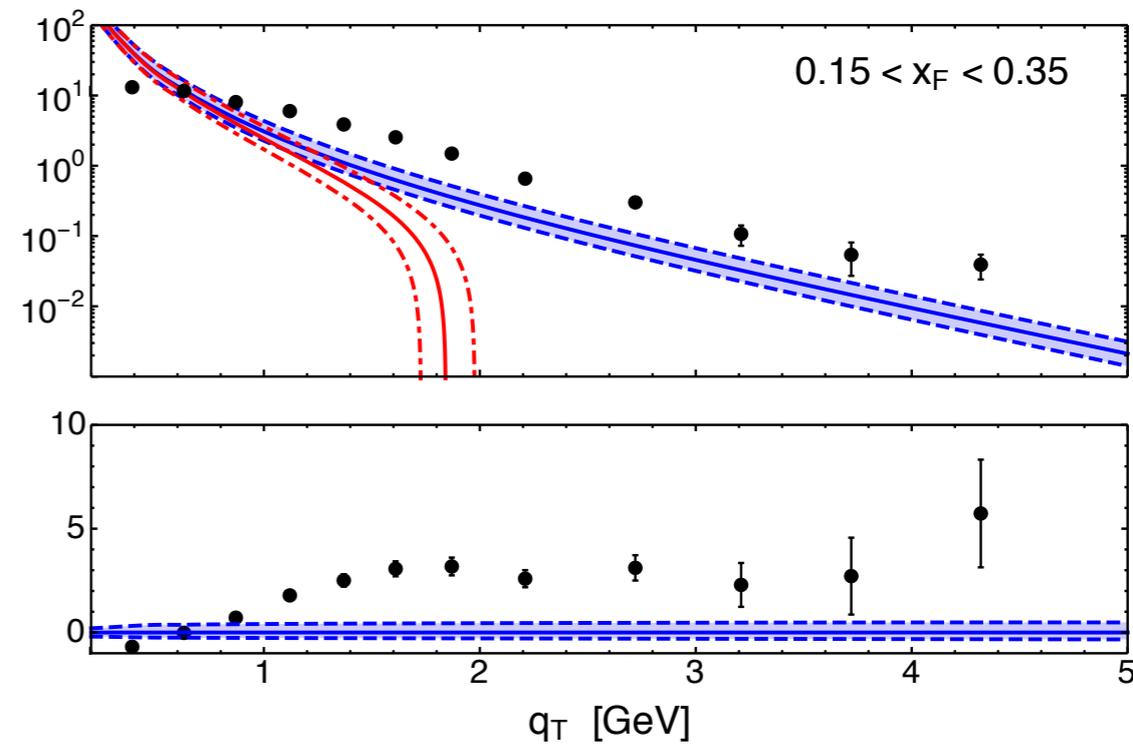
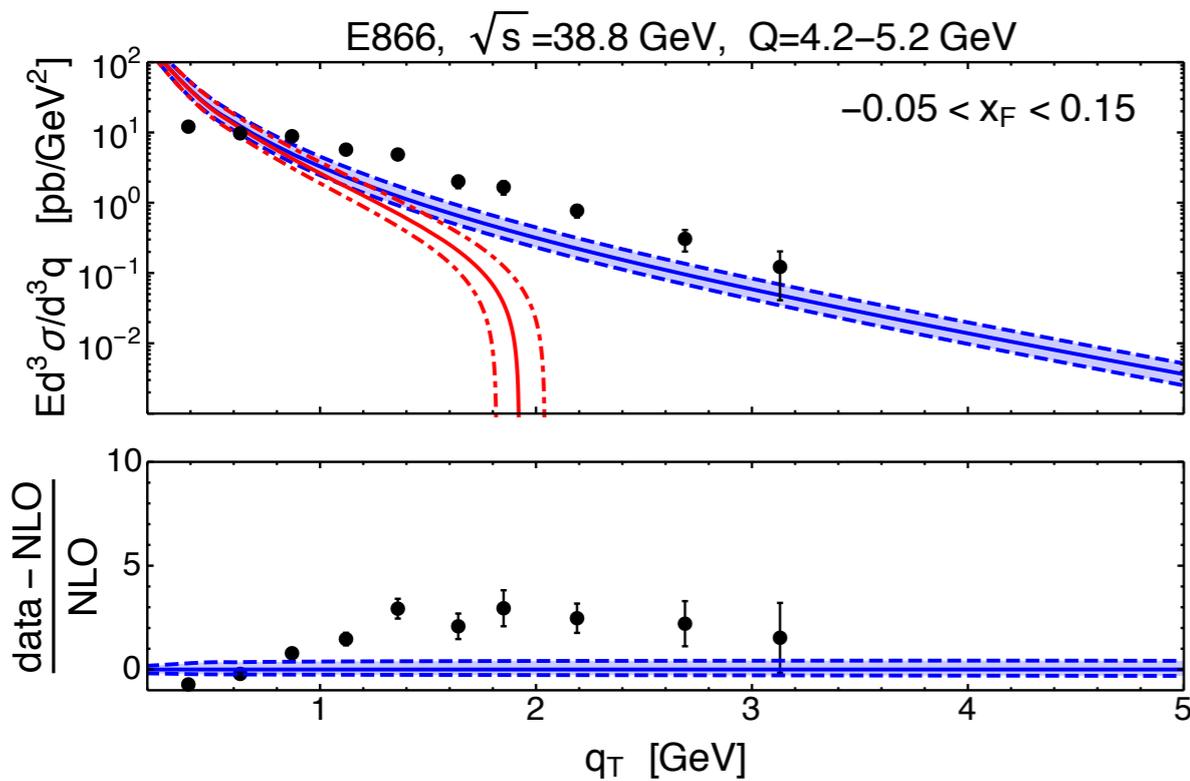
E866/NuSea

$pp \rightarrow \mu^+ \mu^- X$
 $\sqrt{s} = 38.8 \text{ GeV}$



E866/NuSea

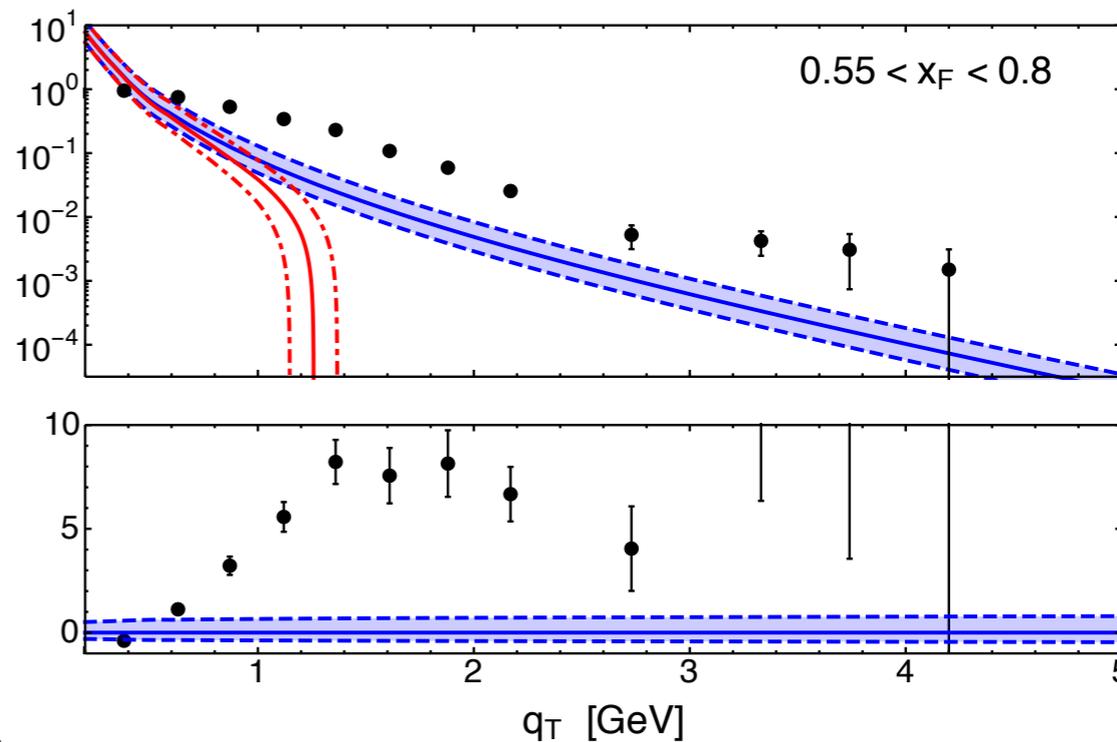
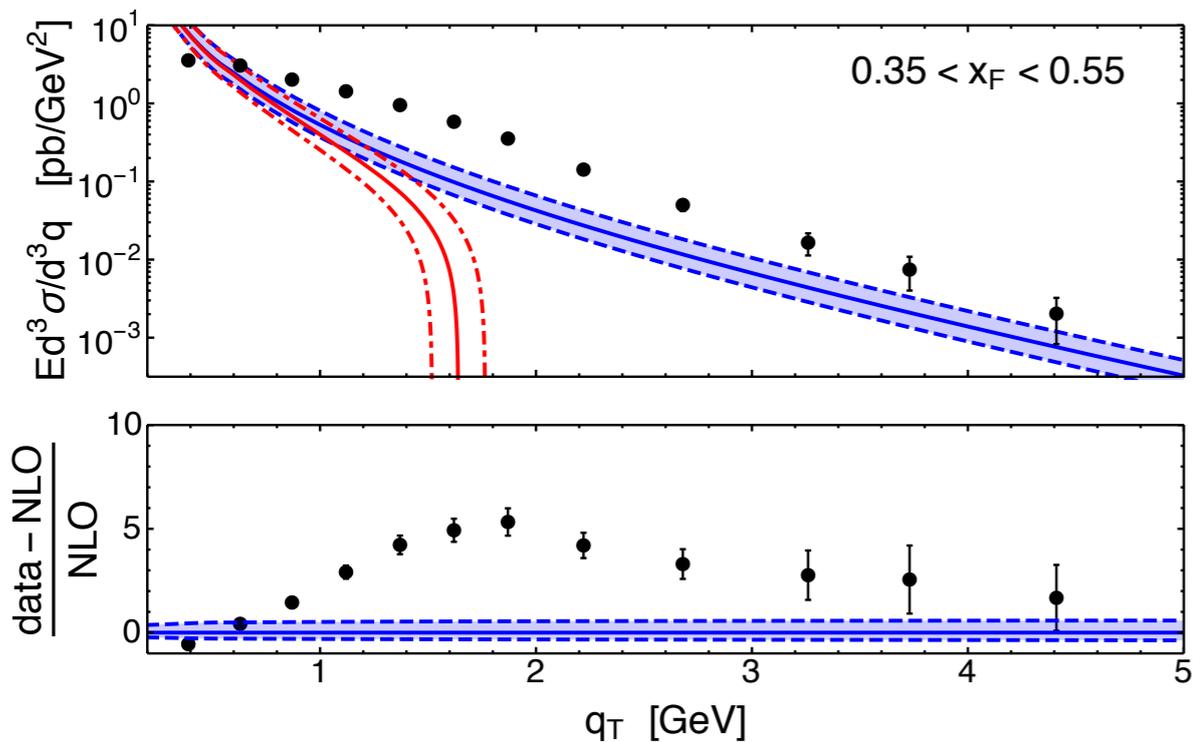
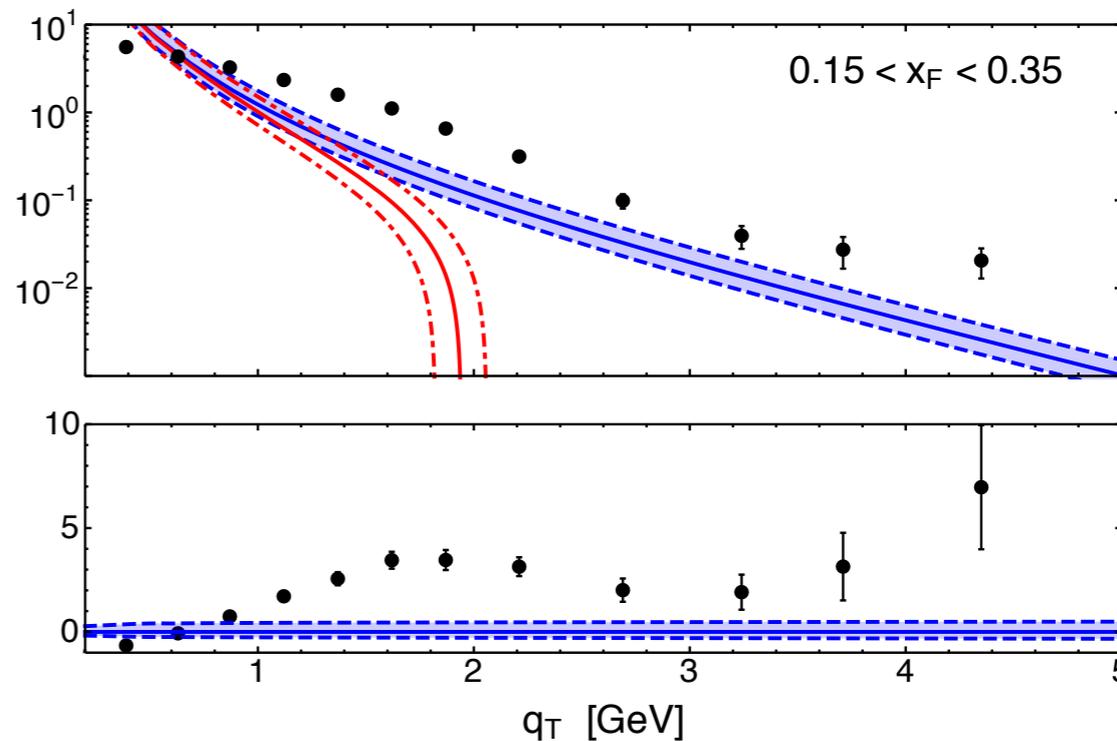
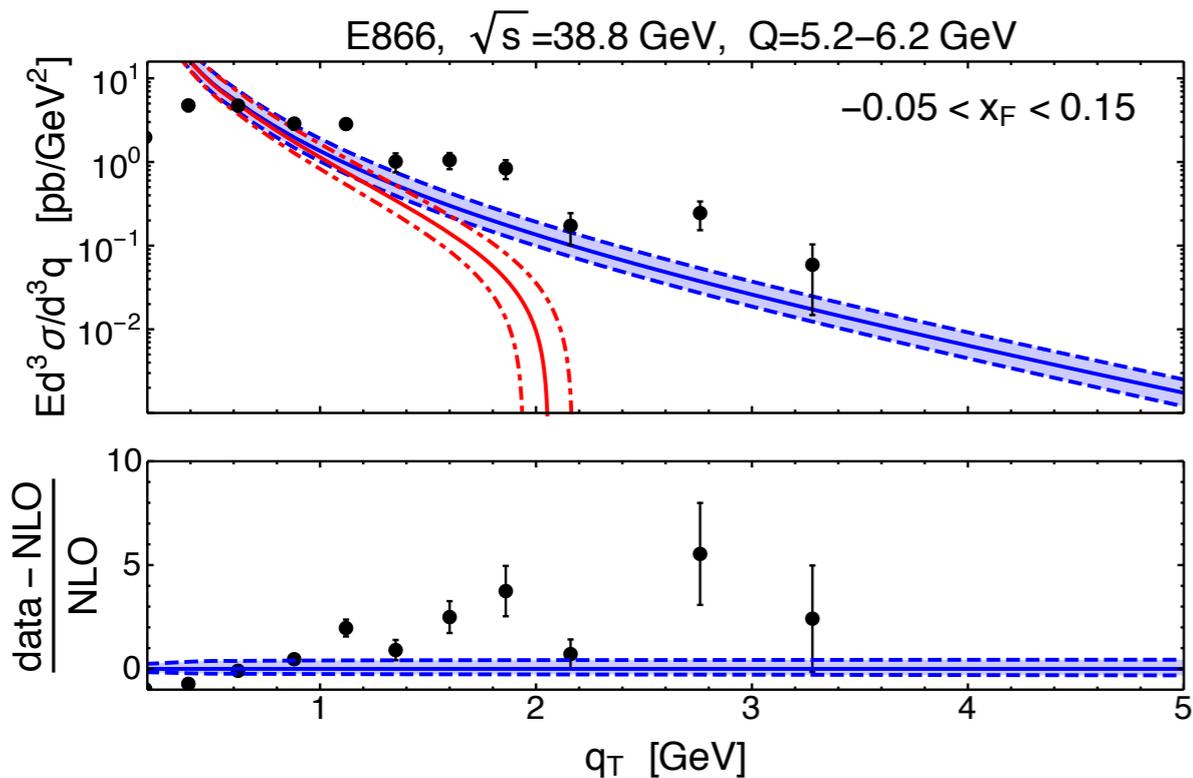
$$pp \rightarrow \mu^+ \mu^- X \quad \sqrt{s} = 38.8 \text{ GeV}$$



--- NLO

E866/NuSea

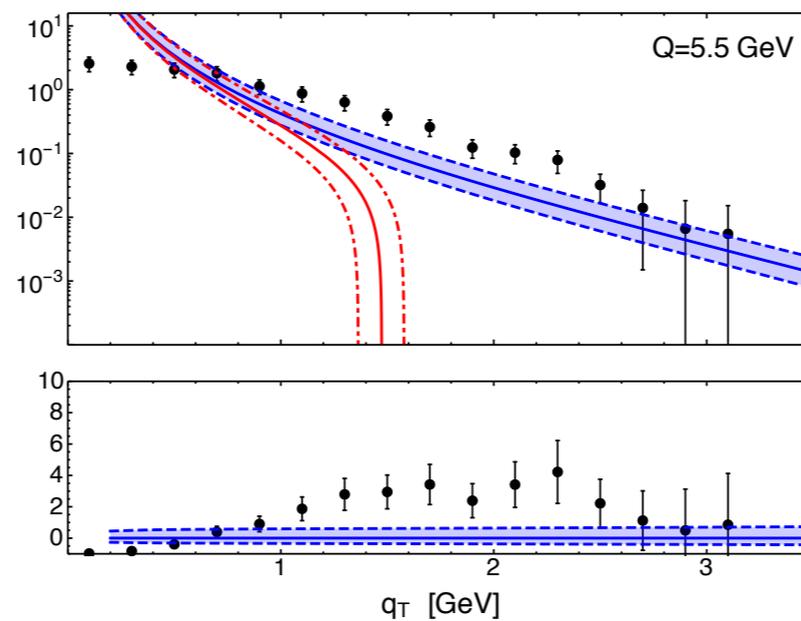
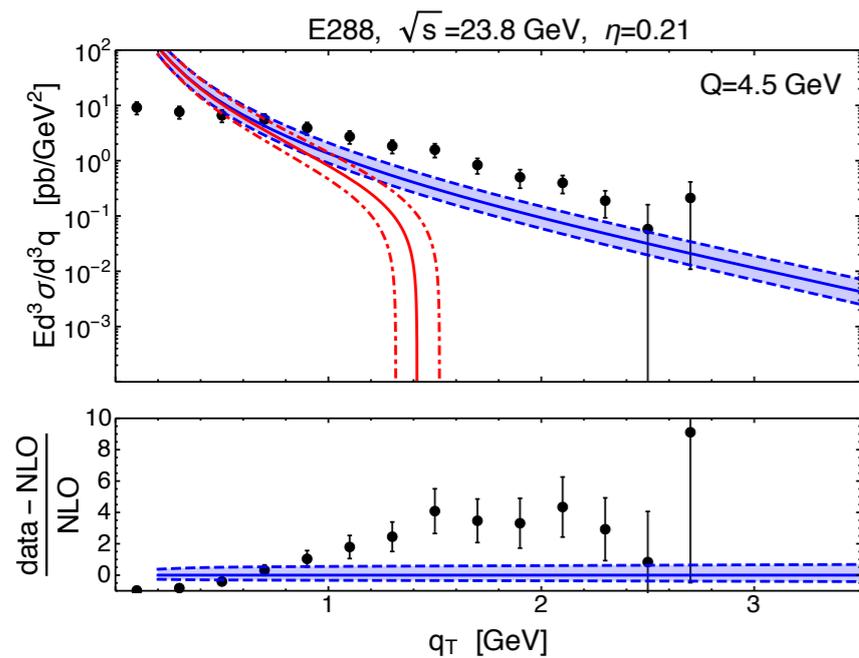
$$pp \rightarrow \mu^+ \mu^- X \quad \sqrt{s} = 38.8 \text{ GeV}$$



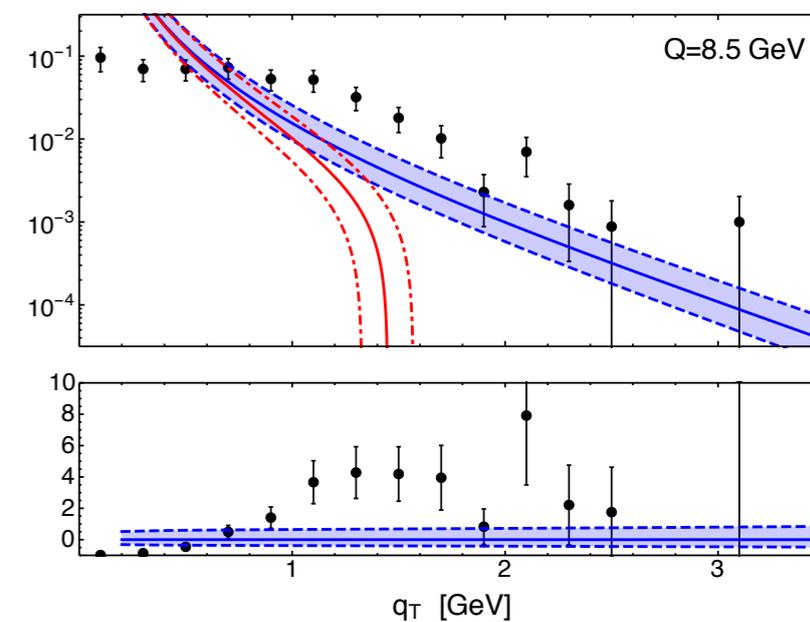
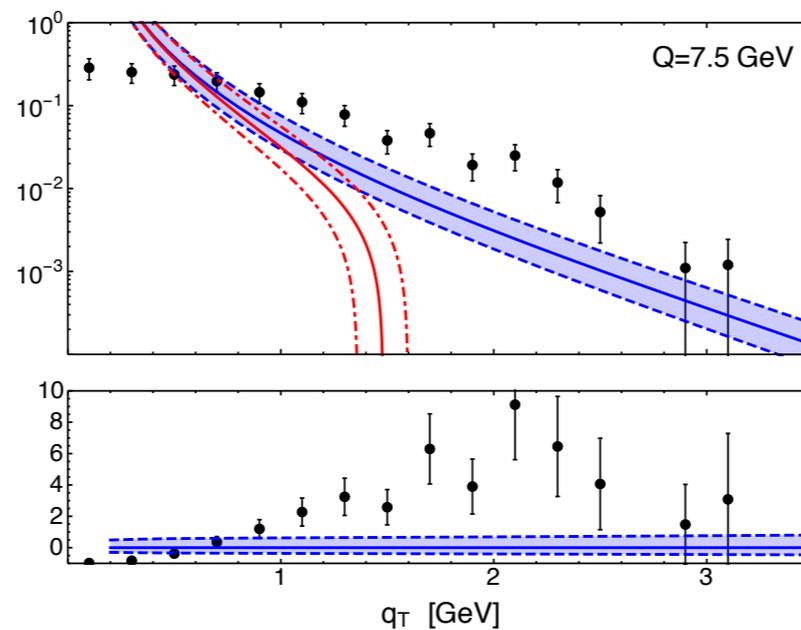
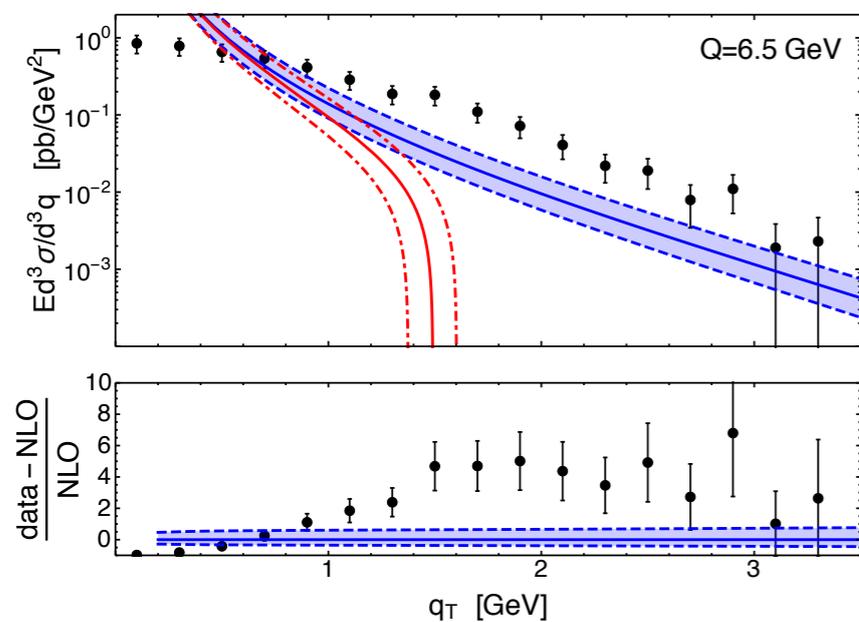
--- NLO

E288

$$p \text{ Cu}, p \text{ Pt} \rightarrow \mu^+ \mu^- X$$

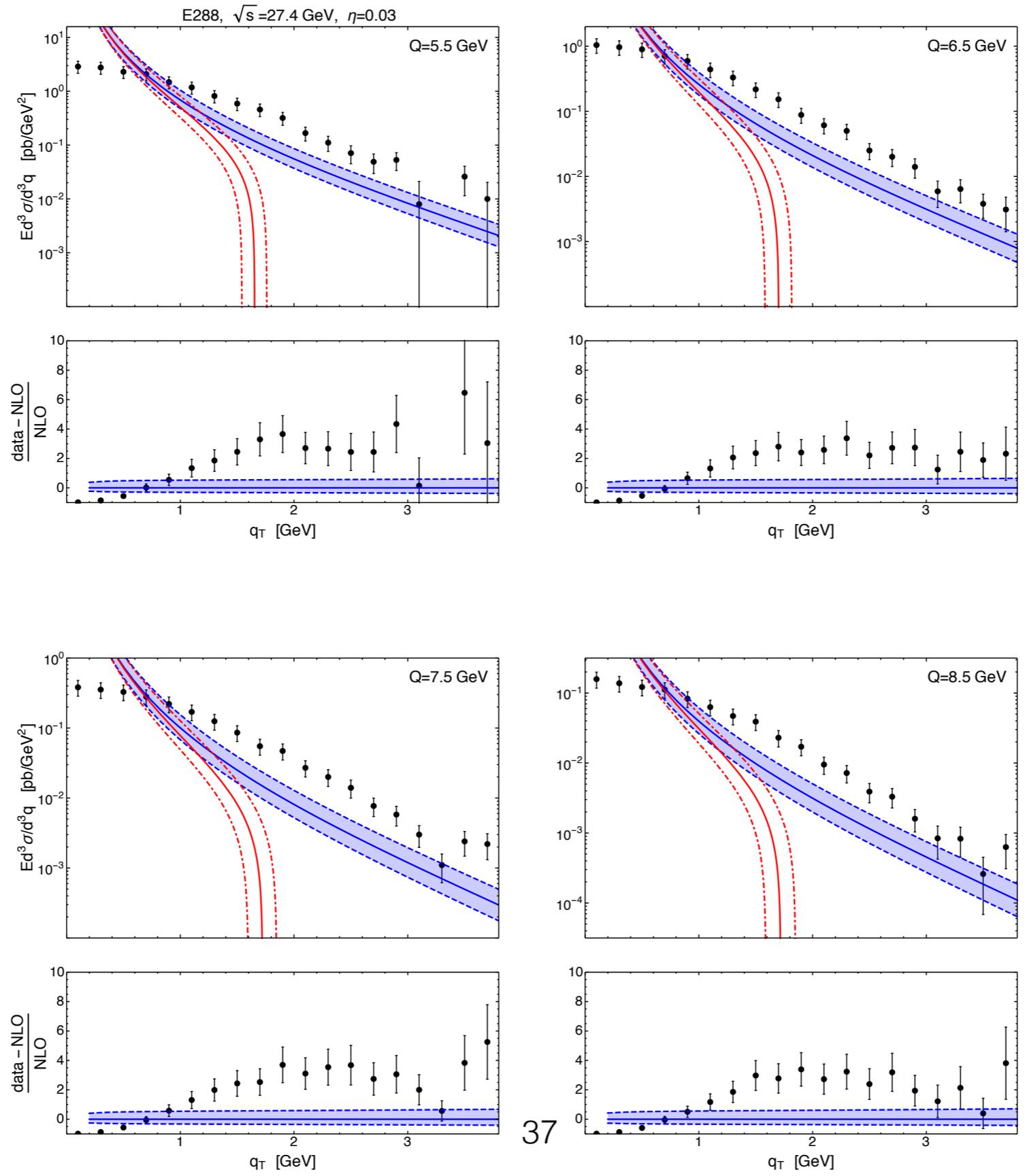


--- NLO
--- asymptotic



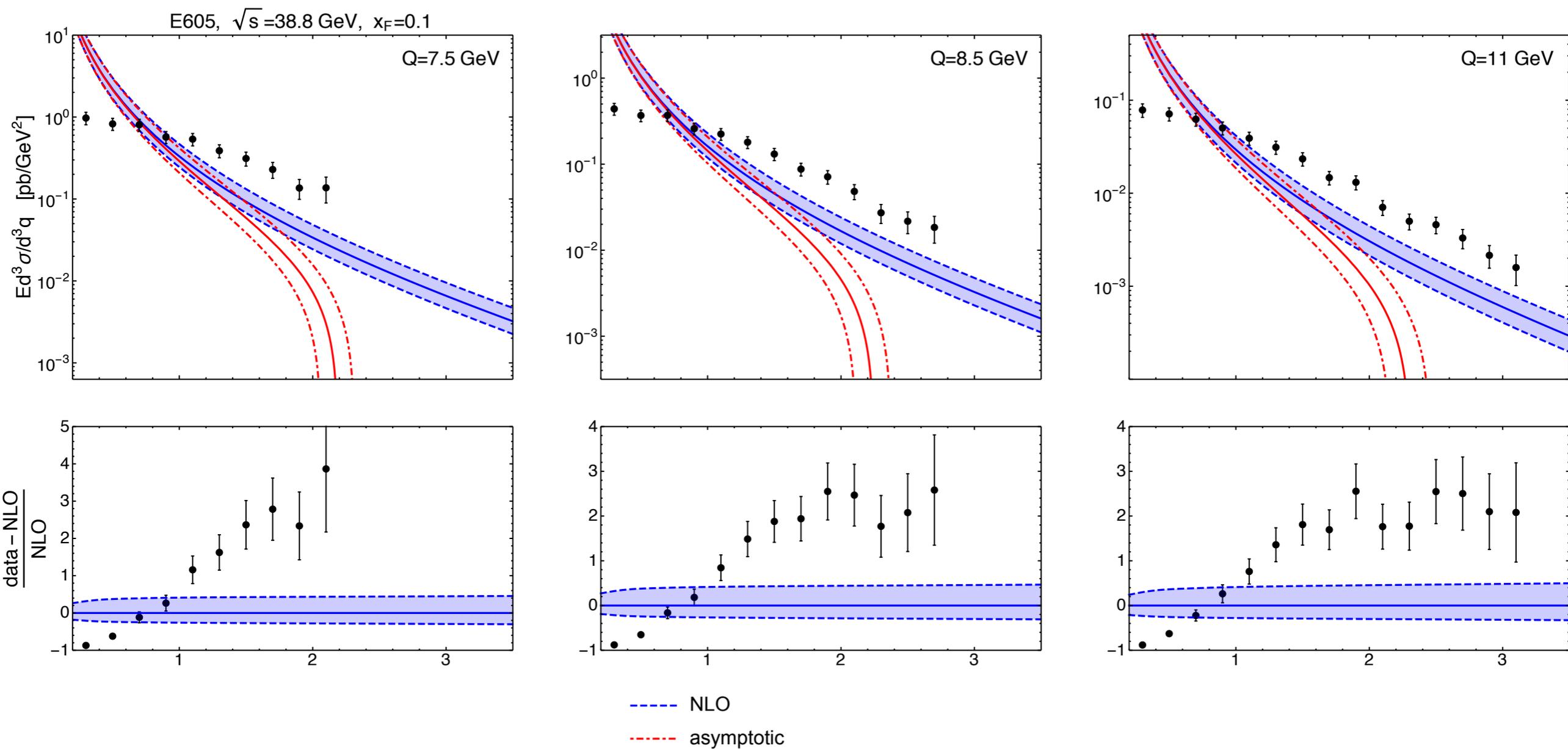
E288

$$p\text{Cu}, p\text{Pt} \rightarrow \mu^+ \mu^- X$$

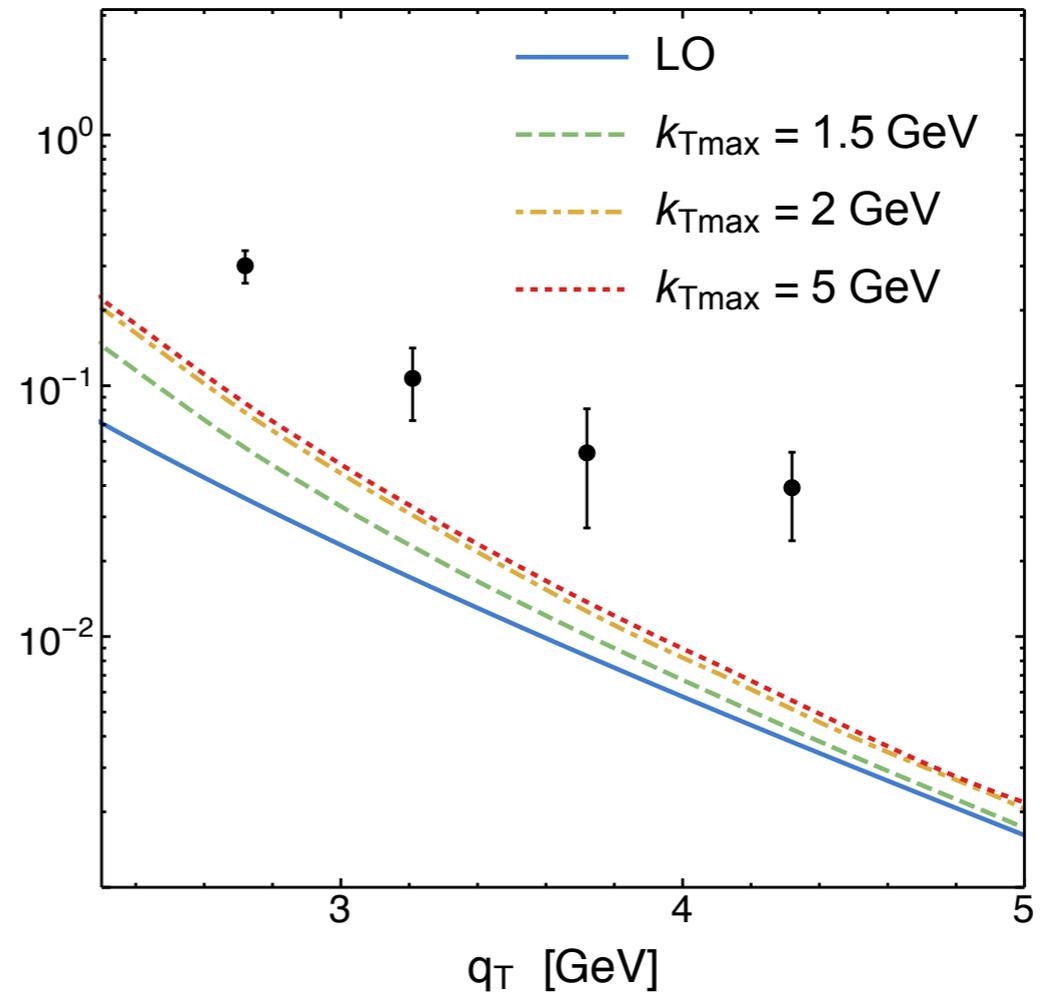
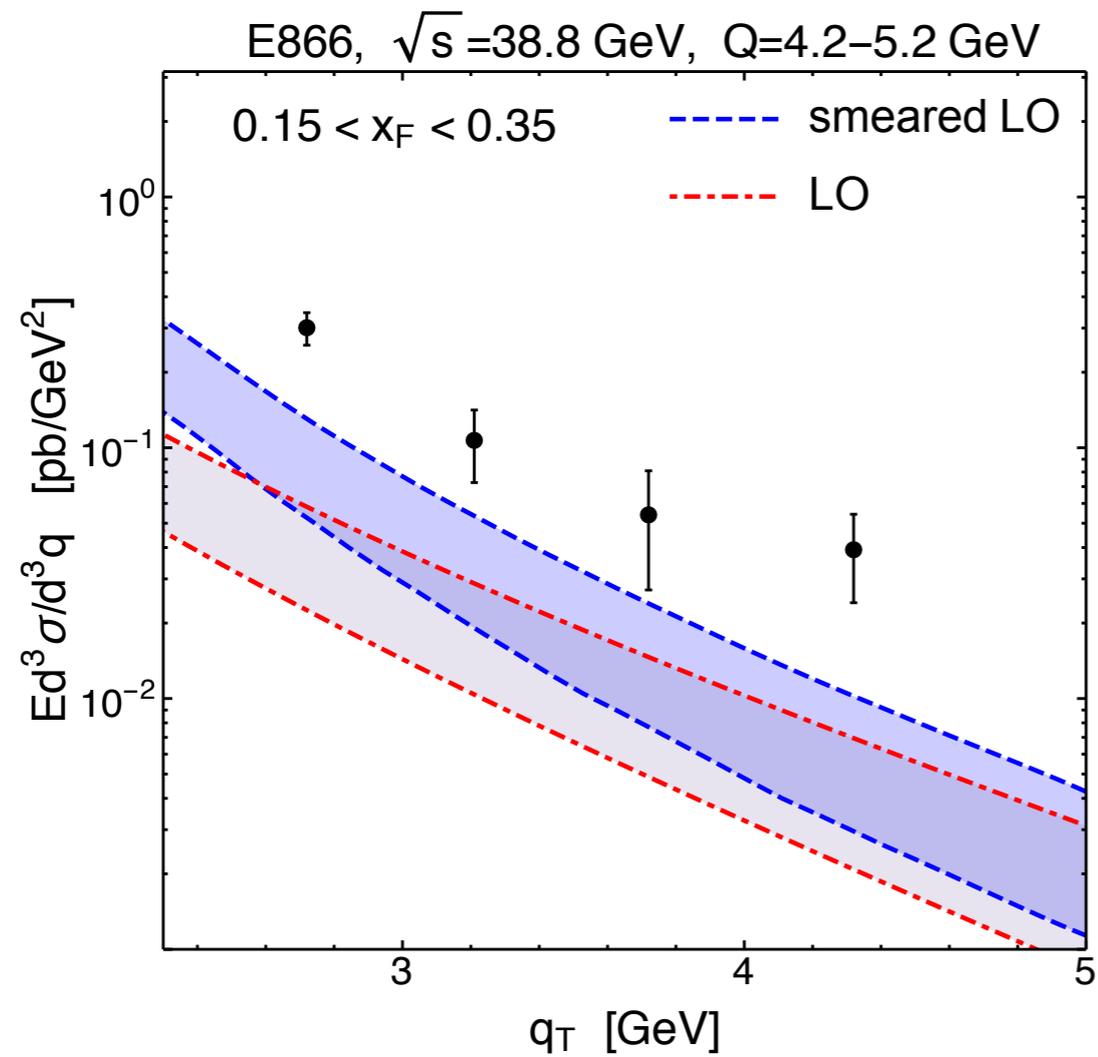


E605

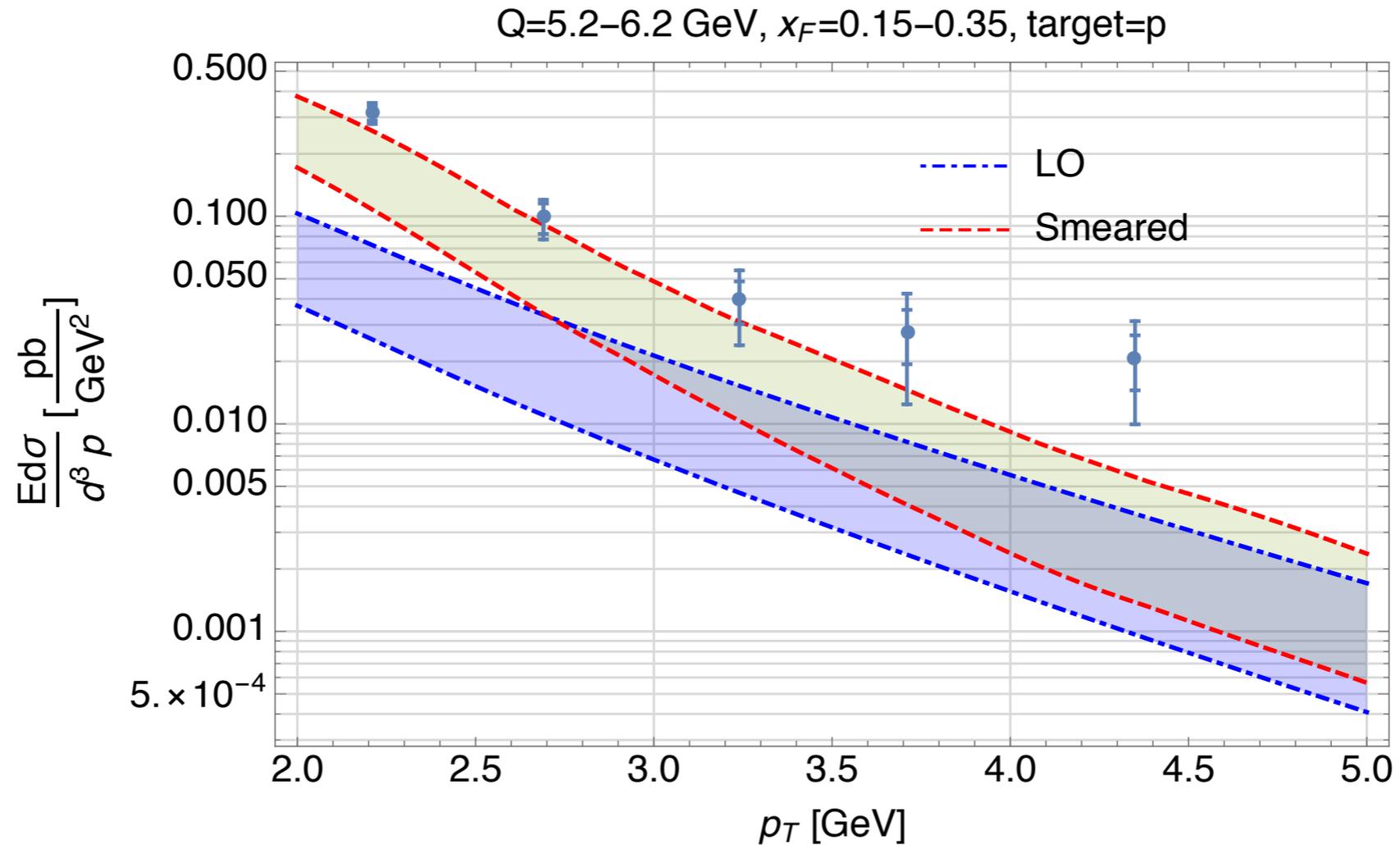
$$p Cu \rightarrow \mu^+ \mu^- X$$



intrinsic k_T smearing



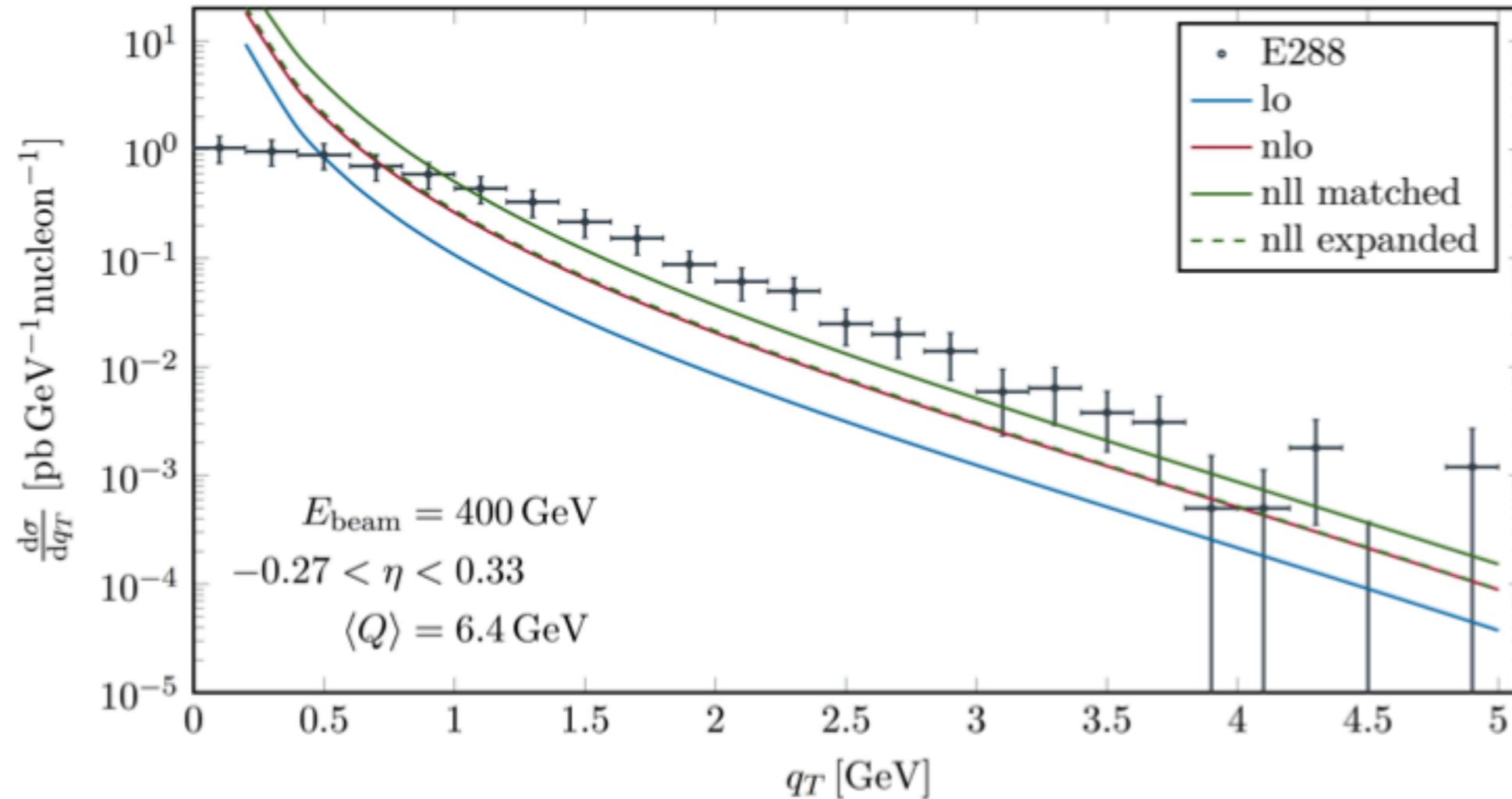
intrinsic k_T smearing



$$f_q(x, \mathbf{k}_T) = f(x) \frac{1}{\pi \langle k_T^2 \rangle} e^{-\frac{k_T^2}{\langle k_T^2 \rangle}}$$

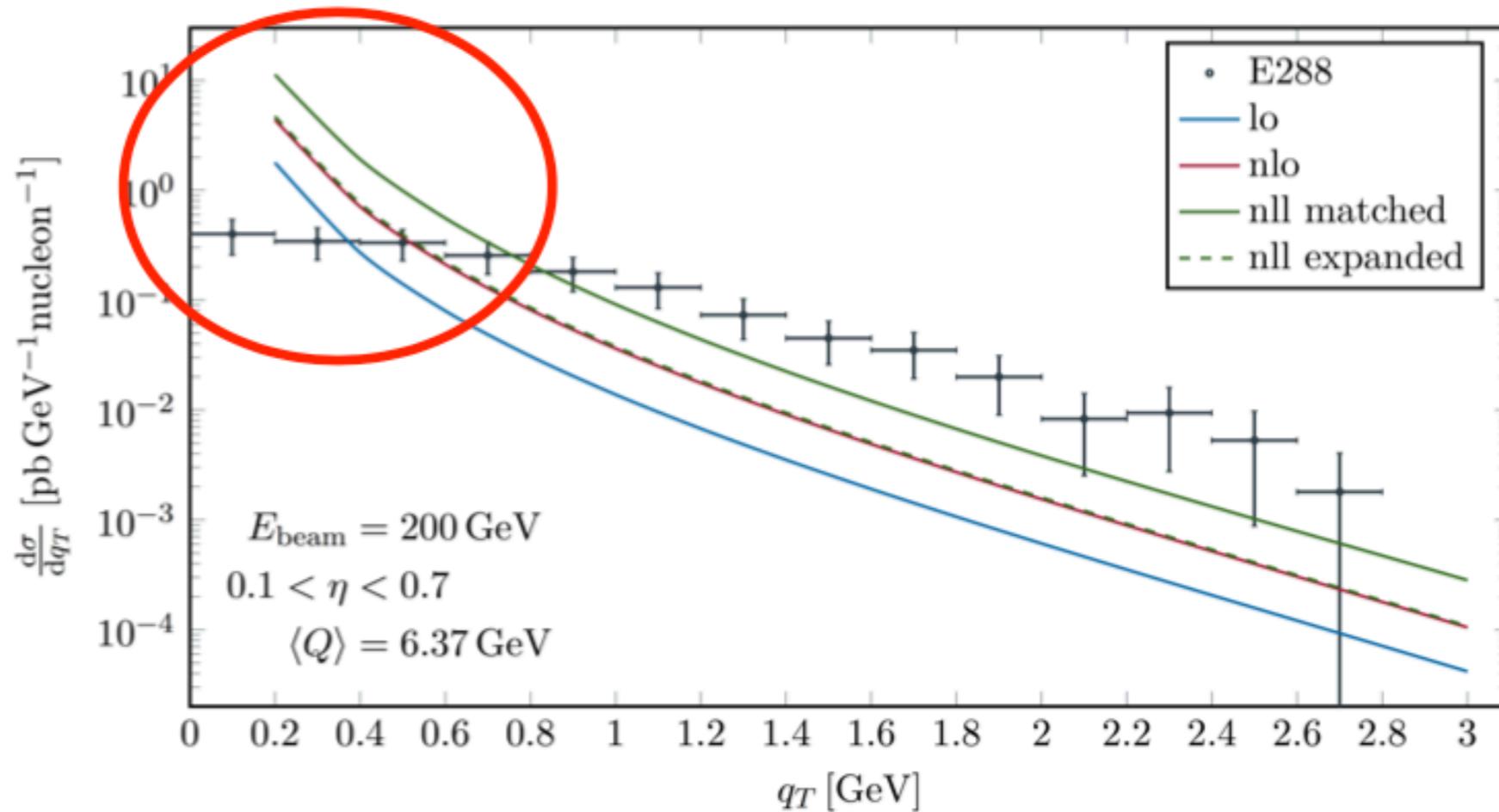
$$\langle k_T^2 \rangle = 1 \text{ GeV}^2$$

threshold resummation



W. Vogelsang @Transversity 2017

threshold resummation



Lambertsen,
Steiglechner,
WV

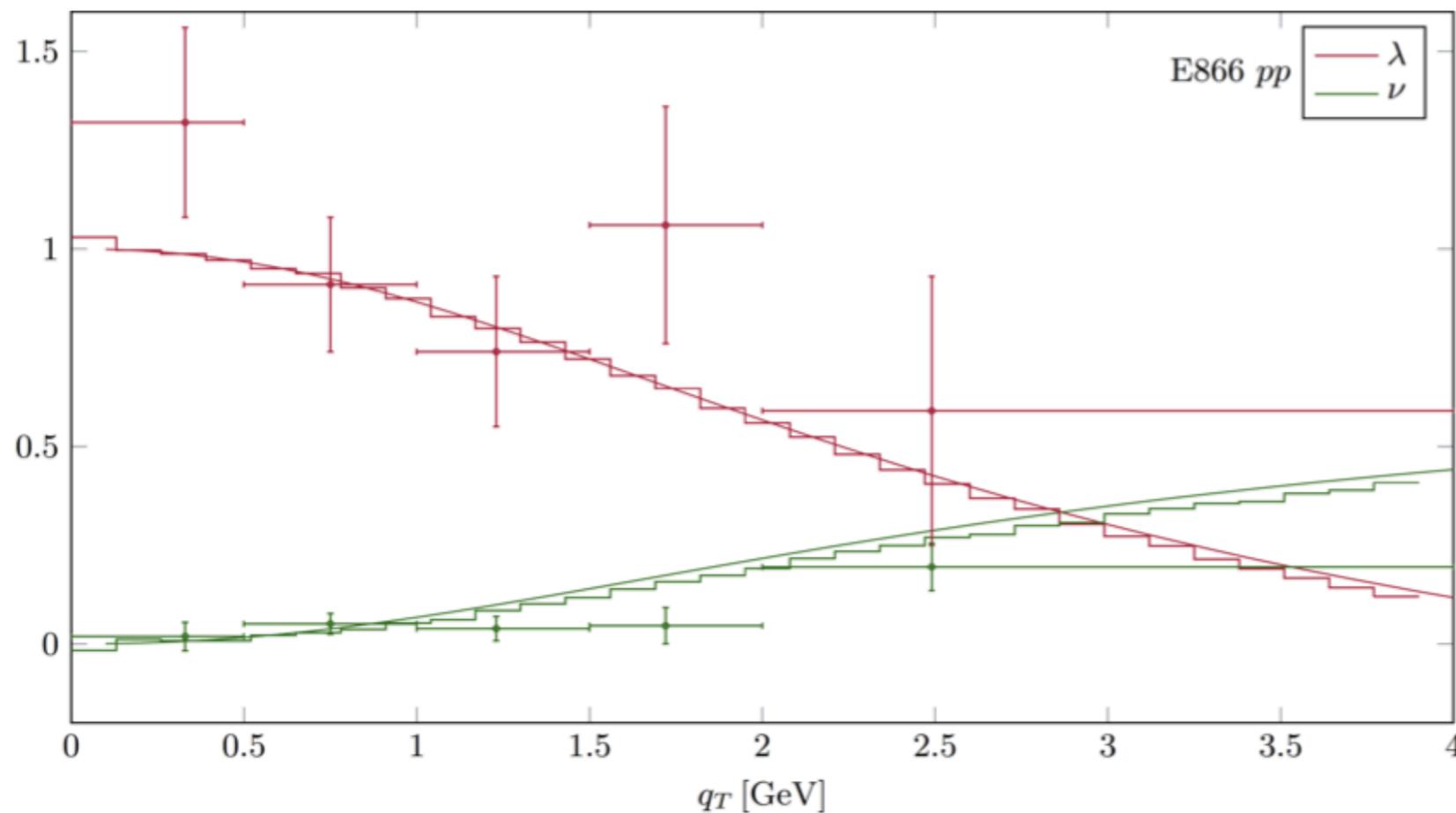
W. Vogelsang @Transversity 2017

angular coefficients

Lambertsen, WV '16

pp , $E = 800$ GeV

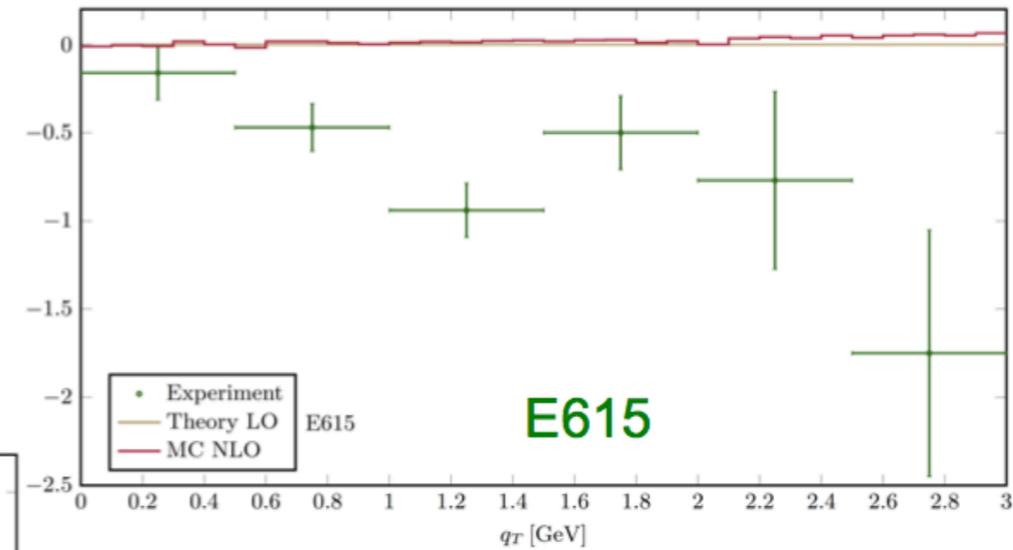
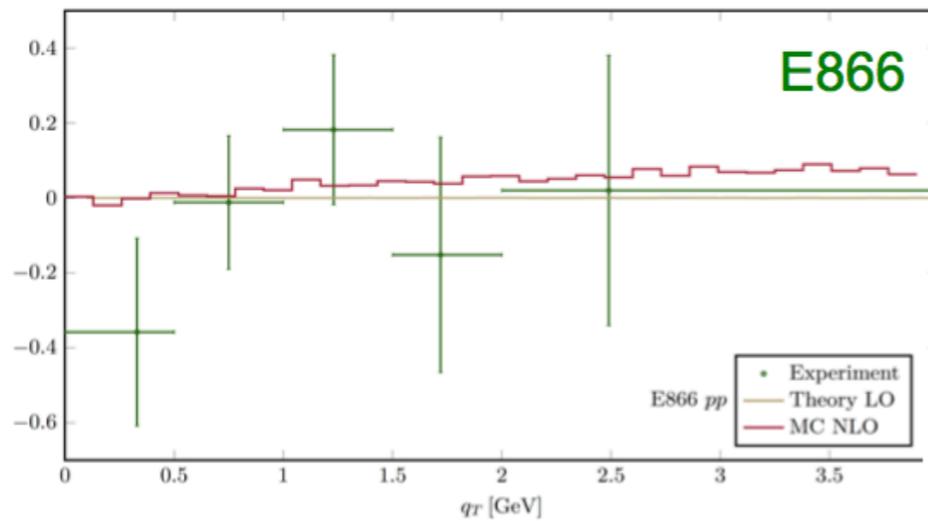
E866



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angular coefficients

Lam-Tung $1 - \lambda - 2\nu$



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