The Non-existence of the Proton Spin Crisis

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The controversy began some decades ago when the European Muon Collaboration published results of their experiment on polarized fully inclusive DIS, which suggested that the constituents of the proton could not provide enough angular momentum to explain the fact that the proton had spin 1/2.

30 years ago Mauro Anselmino and E.L. published a paper in response to results of the European Muon Collaboration (EMC)

A crisis in the parton model: Where, oh where is the proton's spin?

Z.Phys. C41 (1988) 239

However, it has long been understood that **there is no such crisis**.

Nonetheless papers keep appearing referring to **"The proton spin crisis"**

Why??????

Because people have forgotten that

the belief in a spin-crisis emerged from an **over naive** interpretation of the EMC experiment.

For a modern summary of the situation and access to the literature, see the review

Kuhn, S. E., Chen, J. -P. and Leader, E.: Prog. Part. Nucl. Phys., Vol 63, (2009) 1-50

The naive interpretation of the EMC results

Quark model of the nucleon

Successful description of nucleon and baryon resonances.

Nucleon = bound state of 3 massive quarks (Q) ($M_Q \approx M_N/3$)

The nucleon corresponds to the ground state, and for any reasonably behaved potential this will be an s-state.

Simplest non-relativistic case: in s-state the constituent quarks have no OAM (orbital angular momentum)

Hence, for Nucleon at rest, say polarized in the positive Z-direction

$$1/2 = S_z^N = \sum_Q S_z^Q.$$

Theoretical interpretation of the EMC polarized DIS experimental results

The quantity of interest is the first moment of the spin-dependent structure function $g_1(x, Q^2)$

$$\Gamma_1^p \equiv \int_0^1 g_1^p(x)dx$$

In NLO the expression for Γ_1^p is renormalization scheme dependent. In the $\overline{MS}$ scheme, at leading twist, valid for $Q^2 \gg M^2$,

$$\Gamma_1^p(Q^2) = \frac{1}{12}\Big[\Big(a_3 + \frac{1}{3}\,a_8)\,\Delta C_{NS}^{\overline{MS}} + \frac{4}{3}\,a_0(Q^2)\,\Delta C_S^{\overline{MS}}\Big],$$

where the ΔC are the known singlet and non-singlet Wilson coefficients.

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$a_{3,8}$ are matrix elements of non-singlet SU(3) currents and a_0 0f the singlet SU(3) current.

Now the values of $a_{3,8}$ can be obtained from neutron and hyperon β-decay.
Hence a mesurement of Γ_1^p *at some value of* Q^2 *is effectively a measure of* $a_0(Q^2)$, where the Q^2 dependence of a_0 arises because the singlet current has to be renormalized, and it is customary and convenient to choose Q^2 as the renormalization scale.

Now

$$
\begin{aligned}
a_0 &= \Delta\Sigma \equiv (\Delta u + \Delta\overline{u}) + (\Delta d + \Delta\overline{d}) + (\Delta s + \Delta\overline{s}) \\
&= 2\left[\sum_q \langle S_z^q \rangle + \sum_{\bar{q}} \langle S_z^{\bar{q}} \rangle\right]
\end{aligned}
$$

Now

$$a_0 = \Delta\Sigma \equiv (\Delta u + \Delta\overline{u}) + (\Delta d + \Delta\overline{d}) + (\Delta s + \Delta\overline{s}) \quad (3)$$

$$= 2\left[\sum_q \langle S_z^q\rangle + \sum_{\bar{q}} \langle S_z^{\bar{q}}\rangle\right]$$

IF there is no other source of angular momentum one expects

$$\left[\sum_q \langle S_z^q\rangle + \sum_{\bar{q}} \langle S_z^{\bar{q}}\rangle\right] = S_z^{\text{proton}} = 1/2$$

implying, naively,

$$a_0 = 1.$$

The EMC experiment gave $a_0 \approx 0$ and later experiments confirmed that $a_0 \ll 1$, giving rise to the spin crisis in the (*naive*) parton model.

However,

$$a_0 = 1.$$

cannot possibly be true because the right hand side is a fixed number, whereas the left hand side is, *beyond the naive level*, equal to $a_0(Q^2)$, i.e. a function of Q^2! Thus failure of this to hold cannot, in principle, be used to infer that there is a spin crisis

Non-naive interpretation

Clearly a correct relation between the spin of a nucleon and the angular momentum of its constituents should include their OAM and a contribution from the gluons.

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Clearly a correct relation between the spin of a nucleon and the angular momentum of its constituents should include their OAM and a contribution from the gluons.

Unfortunately this is much more complicated than it sounds, because there is some **controversy** as to which operators should be used to represent the **angular momentum**, especially in the case of the massless gluon. (See Phys.Rev. D83, (2011) 096012; Phys.Rep. 541 (2013) 163)

Expression based on the canonical (can) version of the angular momentum

$$\frac{1}{2} = \langle\langle \widehat{S}_z^q \rangle\rangle + \langle\langle \widehat{L}_z^q \rangle\rangle + \langle\langle \widehat{S}_z^G \rangle\rangle + \langle\langle \widehat{L}_z^G \rangle\rangle$$

This looks totally intuitive; can't be incorrect!

Usually written in the Jaffe-Manohar form :

$$\frac{1}{2} = \frac{1}{2}a_0 + \Delta G + \langle\langle \hat{L}_z^q \rangle\rangle + \langle\langle \hat{L}_z^G \rangle\rangle$$

but more correctly it should read :

$$\frac{1}{2} = \frac{1}{2}a_0 + \Delta G + \langle\langle \hat{L}_{can,z}^q \rangle\rangle + \langle\langle \hat{L}_{can,z}^G \rangle\rangle.$$

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But is the Jaffe-Manohar form really identical to the original canonical form???

DANGER!

ΔG **is** a gauge invariant quantity but $\langle\langle S^{G}_{\mathrm{can},z}\rangle\rangle$ **is not.** However one can show that

$$\Delta G = \langle\langle \widehat{S}^{G}_{\mathrm{can},z}\rangle\rangle\big|_{\mathrm{Gauge}\,A^0=0},$$

or, as the nucleon momentum $P \to \infty$

$$\Delta G = \langle\langle \widehat{S}^{G}_{\mathrm{can},z}\rangle\rangle\big|_{\mathrm{Gauge}\,A^+=0}.$$

Moreover the operators $\widehat{L}^{q,G}$ are also not gauge invariant. Thus all the gauge non-invariant operators appearing in the Jaffe-Manohar sum rule should be evaluated in the gauge $A^0 = 0$ or $A^+ = 0$.

Rigorous statement of AM sum rule

Finally for a fast moving proton with helicity $+1/2$ the angular momentum sum rule becomes, in contrast to the naive result

$$\frac{1}{2} = \frac{1}{2}a_0 + \Delta G + \langle\langle \widehat{L}^{q}_{can,z} \rangle\rangle|_{A^+=0} + \langle\langle \widehat{L}^{G}_{can,z} \rangle\rangle|_{A^+=0}.$$

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It should not be forgotten that each individual term in is actually a function of Q^2, but that the sum is not.

ΔG can be measured and seems to be relatively small, but not negligible, typically $\Delta G \approx 0.29 \pm 0.32$ for $Q^2 \approx 10 GeV^2$.

The real challenge is to find a way to measure the orbital terms and thus to check whether the sum rule holds. Only then could one claim that there is or is not a spin crisis.

Conclusions

- What appeared to be a spin crisis in the parton model, 30 years ago, was a consequence of a mis-interpretation of the results of the European Muon Collaboration experiment on polarized deep inelastic scattering.

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- What appeared to be a spin crisis in the parton model, 30 years ago, was a consequence of a misinterpretation of the results of the European Muon Collaboration experiment on polarized deep inelastic scattering.

- This was caused by a failure to distinguish adequately between constituent and partonic quarks. In constituent quark models the spin of the nucleon is built up largely from the spins of its constituents.

Conclusions

- This is not true for partonic quarks, and the smallness of the spin contribution of the partonic quarks is perfectly reasonable, given that they certainly possess orbital angular momentum as well, and that the gluons, too, carry some angular momentum.

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- This is not true for partonic quarks, and the smallness of the spin contribution of the partonic quarks is perfectly reasonable, given that they certainly possess orbital angular momentum as well, and that the gluons, too, carry some spin.

- So, at present, there is **absolutely no basis for the assertion that there is a nucleon spin crisis**.