Understanding SIDIS data from QCD

J. Osvaldo Gonzalez-Hernandez

University of Turin
Outline

• Comments on high precision TMD Extractions:
  • Theoretical limitations
  • Data precision

• Sivers function in Generalized Parton Picture
  (new COMPASS analysis of Sivers asymmetry)
Sivers asymmetry in SIDIS

\[ A_{UT}^{\sin(\phi_h - \phi_S)} = 2 \frac{\int d\phi_S d\phi_h \left[ d\sigma^\uparrow - d\sigma^\downarrow \right] \sin(\phi_h - \phi_S)}{\int d\phi_S d\phi_h \left[ d\sigma^\uparrow + d\sigma^\downarrow \right]} = \frac{F_{UT}^{\sin(\phi_h - \phi_S)}}{F_{UU}} \]

Anselmino, et al. JHEP 1704 (2017) 046 (sign change)

Sivers asymmetry in SIDIS

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- New COMPASS analysis: \( Q^2 \) binning of azimuthal asymmetries.


- Results on new phenomenological analysis by Torino-Cagliari collaboration:

M. Boglione, U. D'Alesio, C. Flore, JOGH

JHEP 1807 (2018) 148
Sivers asymmetry in SIDIS

\[ A_{UT}^{\sin(\phi_h - \phi_S)} = 2 \frac{\int d\phi_S d\phi_h \left[d\sigma^\uparrow - d\sigma^\downarrow\right] \sin(\phi_h - \phi_S)}{\int d\phi_S d\phi_h \left[d\sigma^\uparrow + d\sigma^\downarrow\right]} = \frac{F_{UT}^{\sin(\phi_h - \phi_S)}}{F_{UU}} \]

Generalized Parton Picture (no evolution)

\[ \Delta^N f_{q/p^\uparrow}(x, k_\perp) = 4N_q x^{\alpha_q} (1 - x)^{\beta_q} M_p \frac{k_\perp}{\langle k_\perp^2 \rangle_S} \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle_S}}{\pi \langle k_\perp^2 \rangle_S} \]
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\Delta^N f_{q/p^\uparrow}(x, k_\perp) = 4N_c x^{\alpha_q} (1-x)^{\beta_q} \frac{M_p}{\langle k^2 \rangle_S} \frac{k_\perp}{\pi \langle k^2 \rangle_S} e^{-k_\perp^2/\langle k^2 \rangle_S}
\]

TMD evolution (CSS)

\[
\tilde{F}_{1T}^{\perp}(x, b_T; \mu, \zeta_F) = \tilde{F}_{1T}^{\perp}(x, b_T; \mu_0, Q_0^2) \exp \left\{ \ln \frac{\sqrt{\zeta_F}}{Q_0} \tilde{K}(b_s; \mu_b) + \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left[ \gamma_F(g(\mu'); 1) - \ln \frac{\sqrt{\zeta_F}}{\mu'} \gamma_K(g(\mu')) \right] \
+ \int_{\mu_0}^{\mu_b} \frac{d\mu'}{\mu'} \ln \frac{\sqrt{\zeta_F}}{Q_0} \gamma_K(g(\mu')) - g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{Q_0} \right\}.
\]

Sivers asymmetry in SIDIS

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Generalized Parton Picture (no evolution)

\[ \Delta^N f_{q/p^\uparrow} (x, k_\perp) = 4N_q x^\alpha_q (1 - x)^{\beta_q} \]

No constraint from collinear PDF

Gaussian ansatz

\[ M_p \frac{k_\perp}{\langle k_\perp^2 \rangle_S} \frac{e^{-k_\perp^2 / \langle k_\perp^2 \rangle_S}}{\pi \langle k_\perp^2 \rangle_S} \]
Sivers asymmetry in SIDIS

\[
A_{UT}^{\sin(\phi_h-\phi_S)} = 2 \frac{\int d\phi_S d\phi_h \left[ d\sigma^\uparrow - d\sigma^\downarrow \right] \sin(\phi_h - \phi_S)}{\int d\phi_S d\phi_h \left[ d\sigma^\uparrow + d\sigma^\downarrow \right]} = \frac{F_{UT}^{\sin(\phi_h-\phi_S)}}{F_{UU}}
\]

\[
F_{UT}^{\sin(\phi_h-\phi_S)} \sim \Delta^N f_{q/p^\uparrow}(x, k_\perp, Q^2) \otimes D(z, p_\perp, Q^2)
\]

\[
F_{UU} \sim F(x, k_\perp, Q^2) \otimes D(z, p_\perp, Q^2)
\]

Using results from Anselmino, et al. JHEP 04, 005 (2014), 1312.6261
Combined fit of HERMES COMPASS (reanalysis) and Jlab data (excluding K\(^-\)).

\[
\Delta^N_{f_q/p^+}(x, k_\perp) = 4N_q x^{\alpha_q} (1 - x)^{\beta_q} \frac{M_p}{\langle k_\perp^2 \rangle_s} \frac{k_\perp}{\pi \langle k_\perp^2 \rangle_s} e^{-k_\perp^2/\langle k_\perp^2 \rangle_s}
\]

<table>
<thead>
<tr>
<th>n. of data points = 220</th>
</tr>
</thead>
<tbody>
<tr>
<td>One flavour fits (3 parameters)</td>
</tr>
<tr>
<td>( \chi^2_{\text{tot}} )</td>
</tr>
<tr>
<td>( u )</td>
</tr>
<tr>
<td>( d )</td>
</tr>
<tr>
<td>Two flavour fits (5 parameters)</td>
</tr>
<tr>
<td>( \chi^2_{\text{tot}} )</td>
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<tr>
<td>( u, \bar{u} )</td>
</tr>
<tr>
<td>( u, \bar{d} )</td>
</tr>
<tr>
<td>( u, d )</td>
</tr>
</tbody>
</table>

\( \alpha = 0 \)

What flavor information can we extract?

Reference fit
\[ \Delta^N f_{q/p^+(x, k_\perp)} = 4N_q x^{\alpha_q} (1 - x)^{\beta_q} \frac{M_p}{\langle k_\perp^2 \rangle_S} \frac{k_\perp e^{-k_\perp^2/\langle k_\perp^2 \rangle_S}}{\pi \langle k_\perp^2 \rangle_S} \quad \alpha = 0 \]

<table>
<thead>
<tr>
<th>Reference fit - no evolution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \chi^2_{\text{tot}} = 212.8 ) n. of points = 220</td>
</tr>
<tr>
<td>( \chi^2_{\text{dof}} = 0.99 ) n. of free parameters = 5</td>
</tr>
<tr>
<td>( \Delta \chi^2 = 11.3 )</td>
</tr>
<tr>
<td>HERMES ( \langle k_\perp^2 \rangle = 0.57 \text{ GeV}^2 ) ( \langle p_\perp^2 \rangle = 0.12 \text{ GeV}^2 )</td>
</tr>
<tr>
<td>COMPASS ( \langle k_\perp^2 \rangle = 0.60 \text{ GeV}^2 ) ( \langle p_\perp^2 \rangle = 0.20 \text{ GeV}^2 )</td>
</tr>
<tr>
<td>( N_u = 0.40 \pm 0.09 ) ( \beta_u = 5.43 \pm 1.59 )</td>
</tr>
<tr>
<td>( N_d = -0.63 \pm 0.23 ) ( \beta_d = 6.45 \pm 3.64 )</td>
</tr>
<tr>
<td>( \langle k_\perp^2 \rangle_S = 0.30 \pm 0.15 \text{ GeV}^2 )</td>
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$$\Delta^N f_{q/p^+}(x, k_\perp) = 4N_qx^{\alpha_q}(1-x)^{\beta_q} \frac{M_p}{\langle k_\perp^2 \rangle_S} k_\perp \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle_S}}{\pi \langle k_\perp^2 \rangle_S}$$

Varying $\alpha$

\[\begin{array}{l}
\chi^2_{\text{tot}} = 211.5 \\
\chi^2_{\text{dof}} = 0.99 \\
\Delta \chi^2 = 14.3 \\
\text{HERMES} \quad \langle k^2_\perp \rangle = 0.57 \text{ GeV}^2 \\
\text{COMPASS} \quad \langle k^2_\perp \rangle = 0.60 \text{ GeV}^2 \\
N_u = 0.40 \pm 0.09 \quad \beta_u = 5.93 \pm 3.86 \quad \alpha_u = 0.073 \pm 0.46 \\
N_d = -0.63 \pm 0.23 \quad \beta_d = 5.71 \pm 7.43 \quad \alpha_d = -0.075 \pm 0.83 \\
\langle k^2_\perp \rangle_S = 0.30 \pm 0.15 \text{ GeV}^2
\end{array}\]
First moment

Sivers TMD at fixed value of $x$
Better assessment of errors at low-x using alpha
Signals of scale dependence (first moment)

\[
\frac{\partial T_{q,F}(x, x, \mu)}{\partial \ln \mu^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \left\{ \frac{P_{qg}(z)}{2} T_{q,F}(\xi, x, \mu) + \frac{N_c}{2} \left[ \frac{1 + z^2}{1 - z} (T_{q,F}(\xi, x, \mu) - T_{q,F}(\xi, \xi, \mu)) + z T_{q,F}(\xi, x, \mu) + T_{\Delta q,F}(x, \xi, \mu) \right] \rightarrow \right.

\left. - N_c \delta(1 - z) T_{q,F}(x, x, \mu) + \frac{1}{2N_c} [(1 - 2z) T_{q,F}(x, x - \xi, \mu) + T_{\Delta q,F}(x, x - \xi, \mu)] \right\}
\]

Signals of scale dependence

\[ \langle k^2 \rangle_s = g_1 + g_2 \ln \frac{Q^2}{Q_0^2} \]

\( g_2 \) here to “mimic” TMD evolution
Studies on asymmetries strongly depend on our Knowledge of the unpolarized TMDs.
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Different assumptions about the unpolarized functions Have dramatic effects on other extractions
To proceed to a high precision stage
we first need reliable extractions for unpolarized functions.
The more constraints from first principles the better.
Ultimately, we want to extract TMDs (in all their glory)

Fourier Transform of:

\[
\tilde{F}_j(x, b_T, Q, \zeta_F) = \left( \frac{\sqrt{\zeta_F}}{\mu_b} \right)^{\tilde{K}(b_*, \mu_b)} \sum_j \int_x^1 \frac{d\hat{x}}{\hat{x}} \tilde{C}_{ji}^{\text{in}}(x/\hat{x}, b_*, \mu_\beta, \mu_b^2) f_i(\hat{x}, \mu_b)
\]
\[
\times \exp \left\{ \int_{\mu_b}^Q \frac{d\mu}{\mu} \left( \gamma_F(\mu; 1) - \ln \left( \frac{\sqrt{\zeta_F}}{\mu} \right) \gamma_K(\mu) \right) \right\}
\]
\[
\times \exp \left\{ -g_P(x, b_T) - g_K(b_T) \ln \left( \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F0}}} \right) \right\} ,
\]

Unpolarized TMD PDF

\[
\tilde{D}_j(z, b_T, Q, \zeta_D) = \left( \frac{\sqrt{\zeta_D}}{\mu_b} \right)^{\tilde{K}(b_*, \mu_b)} \sum_k \int_z^1 \frac{d\hat{z}}{\hat{z}^3} \tilde{C}_{kj}^{\text{out}}(z/\hat{z}, b_*, \mu_\beta, \mu_b^2) D_j(\hat{z}, \mu_b)
\]
\[
\times \exp \left\{ \int_{\mu_b}^Q \frac{d\mu}{\mu} \left( \gamma_D(\mu; 1) - \ln \left( \frac{\sqrt{\zeta_D}}{\mu} \right) \gamma_K(\mu) \right) \right\}
\]
\[
\times \exp \left\{ -g_H(z, b_T) - g_K(b_T) \ln \left( \frac{\sqrt{\zeta_D}}{\sqrt{\zeta_{D0}}} \right) \right\} .
\]

Unpolarized TMD FF
Drell Yan

High precision phenomenology:
LO, NLO, NNLO analysis
Ignazio Scimemi, Alexey Vladimirov

Still Some trouble at low energies
Global Fits? First attempts

Drell Yan

LO analysis
A. Bacchetta, F. Delcarro, C. Pisano, M. Radici, A. Signori
JHEP 1706 (2017) 081

PDFs

Fragmentation Functions

SIDIS
SIDIS variables: \(Q^2, x_{Bj}, z_h, P_T\)

Often useful to consider: \(q_T = P_T/z_h, \quad y_h = \frac{1}{2} \log \left( \frac{P_{h^+}}{P_{h^-}} \right)\)
Hard gluon radiation

Non-perturbative Transverse Momentum

Matching region

Hard gluon radiation
Small transverse momentum:

Harder to reproduce data as compared to simple gaussian model
$W$ at $\mathcal{O}(\alpha_s)$ (no $Y$ term)
Large transverse momentum:

One would expect to be able to describe data with F.O.

At LO, F.O. is roughly an order of magnitude smaller than data

Try NLO ...
COMPASS 17 $h^+$

\[
\frac{\frac{d\sigma}{dx_{bj}dQ^2dzdP_T^2}}{\frac{d\sigma}{dx_{bj}dQ^2}} (\text{GeV}^{-2}) \ vs. \ q_T (\text{GeV})
\]
This is probably preventing us from successfully match TMD and large qT regions.

We Need to understand the different regions in order to trust TMD extraction.
We need more studies on these to move to the next Stage (low energy SIDIS and Drell-Yan).

\[ F_{UT}^{\text{sin} (\phi_n - \phi_S)} \sim \Delta^N f_{q/p^\uparrow} (x, k_\perp, Q^2) \otimes D(z, p_\perp, Q^2) \]

\[ F_{UU} \sim F(x, k_\perp, Q^2) \otimes D(z, p_\perp, Q^2) \]

And come back to this
And this!
Conclusions.

New analysis on Sivers asymmetries was presented.

Sivers function in u/d channels compatible with previous extractions

Need more data to constraint d (sea)

TMD evolution effects are hard to observe in the data

Scale dependence due to Twist-3 related may be visible from data

Need to continue improving on unpolarized function (low energies):
  
    Understand large qT behavior
    Successful matching to TMD region

Good progress, hopefully more studies will come in the near future that allow to move from exploratory to high precision (low energy)
Thank you.
Back up
\[ f_{q/p}(x, k_{\perp}) = f_{q/p}(x) \frac{e^{-k_{\perp}^2/\langle k_{\perp}^2 \rangle}}{\pi \langle k_{\perp}^2 \rangle} \]

\[ D_{h/q}(z, p_{\perp}) = D_{h/q}(z) \frac{e^{-p_{\perp}^2/\langle p_{\perp}^2 \rangle}}{\pi \langle p_{\perp}^2 \rangle}. \]

Different widths
Matching is crucial, cannot afford to miss any constraint.
Historical note

\[ y_h = \frac{1}{2} \ln \left( \frac{E_h + p_{h,L}}{E_h - p_{h,L}} \right), \]

where \( E_h, p_{h,L} \) are the energy and longitudinal component of momentum of hadron \( h \). (Longitudinal is defined by the direction of the momentum \( q \).) The full range of \( y_h \) allowed kinematically is \( Y = \ln W_X^2 = \ln(Q^2(1-x)/x) \); \( W_X \) is the invariant mass of the system \( X \) in the fully inclusive \( eA \rightarrow e'X \).

It has been established\(^{13}\) experimentally that the typical hadronic correlation length in rapidity is \( \Delta y_h \approx 2 \). Therefore, if the dynamics of quark fragmentation is to be studied independently of "contamination" from target fragmentation, it is necessary that \( Y \gtrsim 4 \), or, equivalently, that

\[ W_X = \left[ \frac{Q^2(1-x)}{x} \right]^{1/3} \gtrsim 7.4 \text{GeV}. \]  

(17)
Historical note

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(17)

If the inequality Eq. (17) is satisfied, it should be possible to measure fragmentation functions \( D(z, Q^2) \) over essentially the full range of \( z \), \( 0 < z < 1 \). Somewhat smaller values of \( W_X \) may be adequate if attention is restricted to the large \( z \) region. As \( Y \) is increased
More recently

![Diagram](image)

FIGURE 3. Relation between $z$ - *values* in fragmentation and CM rapidity for $W = 20$ GeV.

A real-life example

Rapidity distributions can be extremely useful


Fig. 23a–c. Normalized cms-rapidity distribution of positive hadrons in $\mu$Xe scattering, in three bins of $W$, for three variants of the particle identification procedure (see Sect. 3.1): assignment of the proton mass if $x_F (m_\pi)$ is $<-0.15$ (stars), or if $x_F (m_\pi)$ is $<-0.20$ (circles), or if $x_F (m_\pi)$ is $<-0.25$ (triangles)