Reduced hadronic uncertainty in V_{ud} and CKM unitarity

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C-Y Seng, MG, H Patel, M J Ramsey-Musolf, arXiv: 1807.10197

Collaborators:
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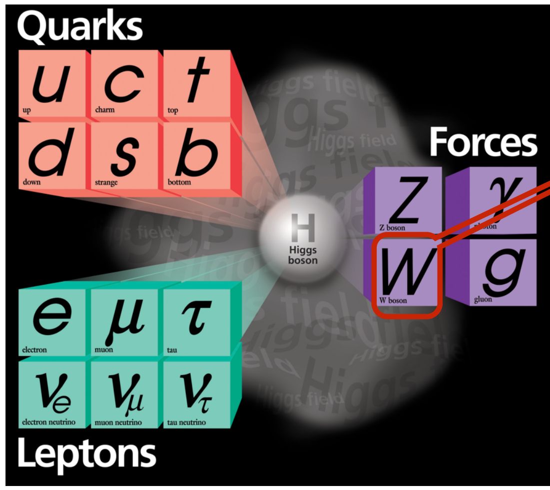
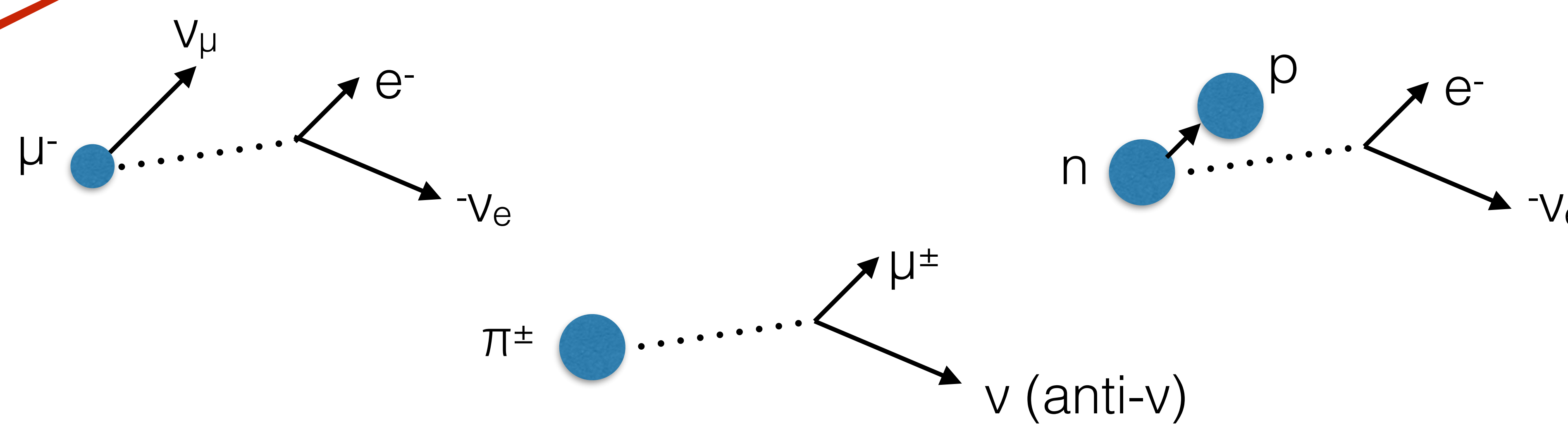
Precision measurements of V_{ud}



Charged current interaction - β-decay (μ, π±, n)



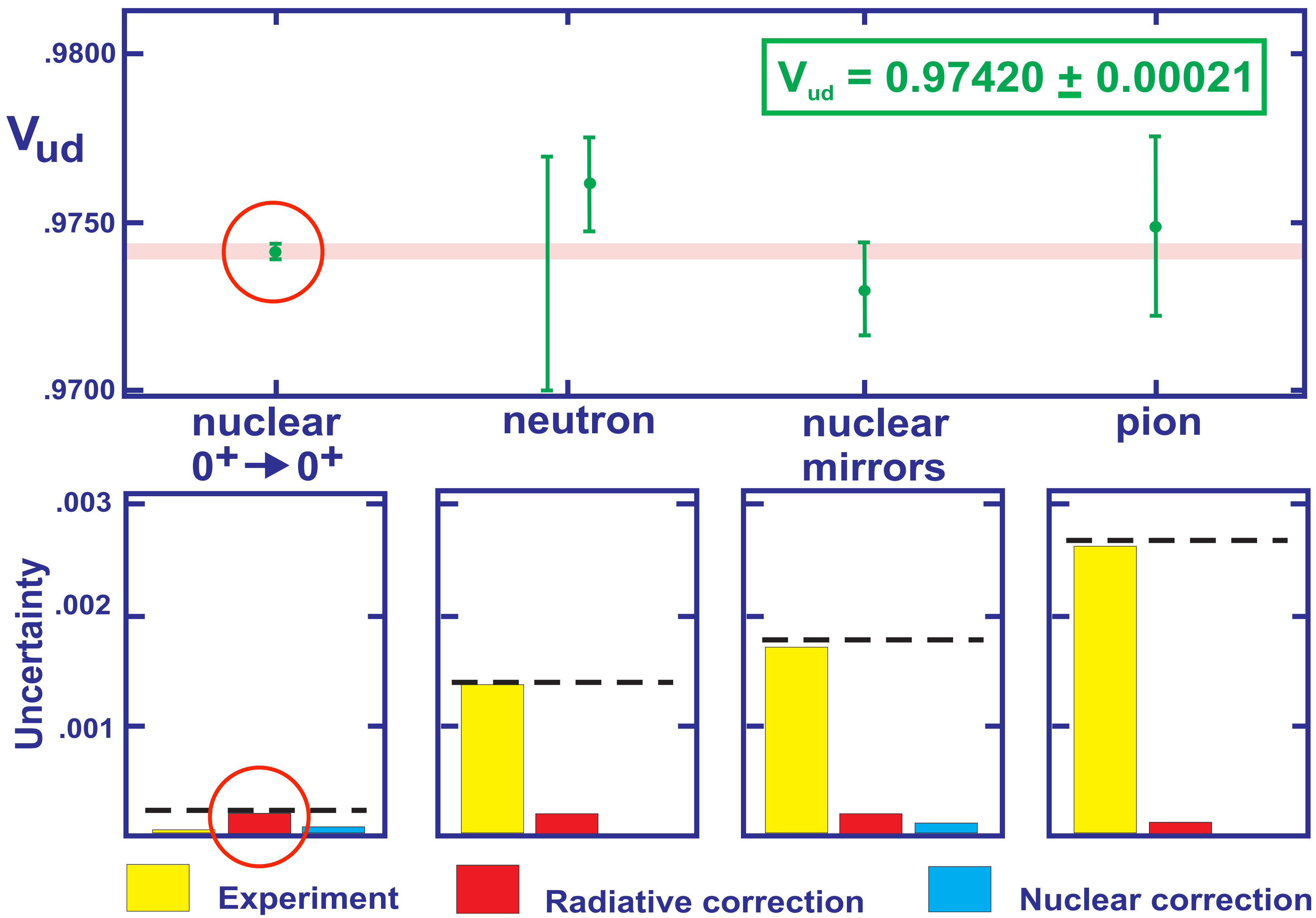
W coupling to leptons and hadrons very close but not exactly the same:
quark mixing - Cabbibo-Kabayashi-Maskawa matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

CKM - Determines the relative strength of the weak CC interaction of quarks vs. that of leptons

CKM unitarity - measure of completeness of the SM: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

Current status of V_{ud} and CKM unitarity



$$|V_{ud}|^2 + |V_{ud}|^2 + |V_{ud}|^2 = 0.9994 \pm 0.0005$$

0^+-0^+ nuclear decays $|V_{ud}|^2 = 0.94906 \pm 0.00041$

K decays $|V_{us}|^2 = 0.05031 \pm 0.00022$

B decays $|V_{ub}|^2 = 0.00002$

CKM unitarity: V_{ud} the main contributor to the sum and to the uncertainty

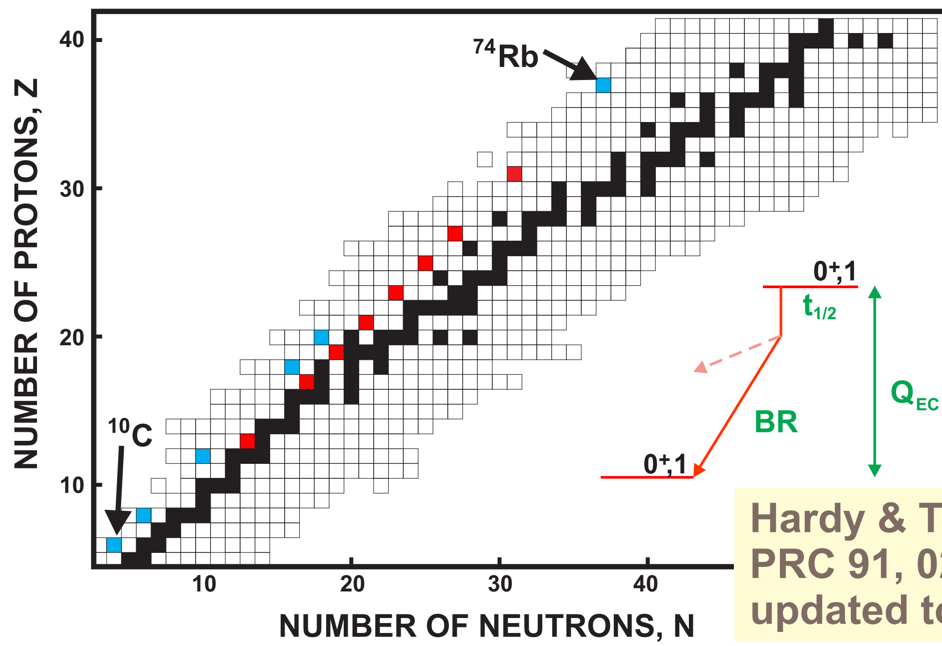
Current status of V_{ud} and CKM unitarity

Superallowed 0^+-0^+ nuclear decays - only conserved vector current; many decays

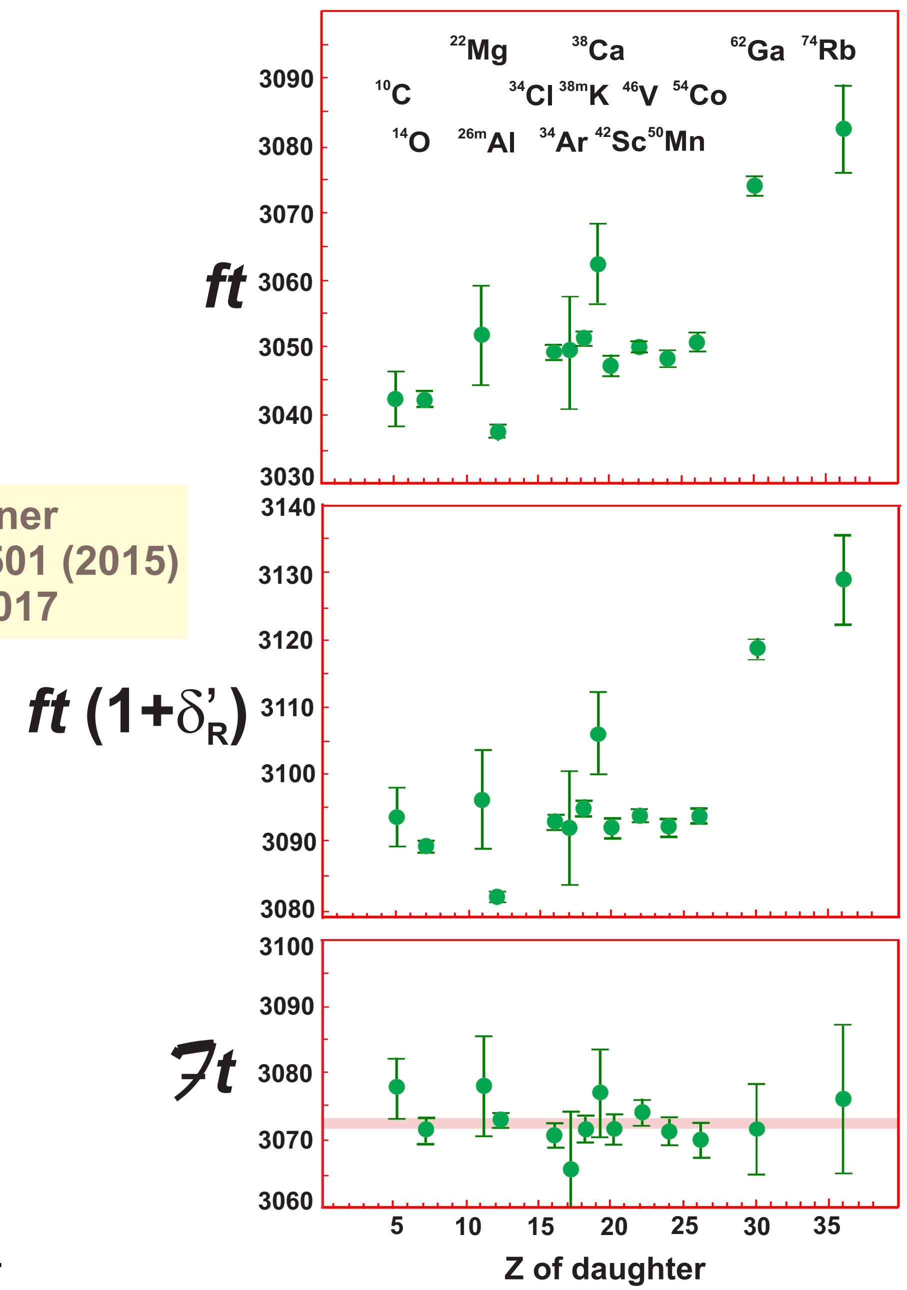
$$\mathcal{F}t = ft(1+\delta'_R)[1-(\delta_C-\delta_{NS})]$$

Experiment: f - phase space (Q_{EC})
t - partial half-life ($t_{1/2}$, BR)

Theory: δ'_R - Bremsstrahlung, IR-sensitive
δ_C - Isospin breaking (Coulomb, …)
δ_{NS} - Nuclear structure



Hardy & Towner
PRC 91, 025501 (2015)
updated to 2017



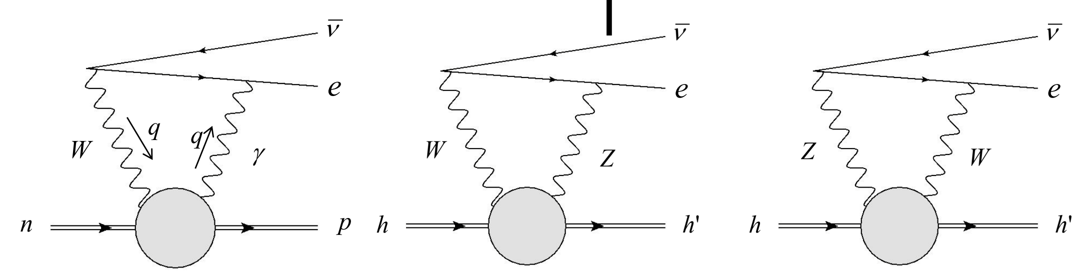
- 8 cases with *ft*-values measured to <0.05% precision; 6 more cases with 0.05-0.3% precision.
- ~220 individual measurements with compatible precision

Average $\overline{\mathcal{F}t} = 3072.1 \pm 0.7$

Current status of V_{ud} and CKM unitarity

Basis for precision V_{ud} extraction from superallowed decays:

$$|V_{ud}|^2 = \frac{2984.432(3)}{\mathcal{F}t(1+\Delta_R^V)}$$



Universal correction - γW-box + ZW-box
Marciano&Sirlin '87, '06

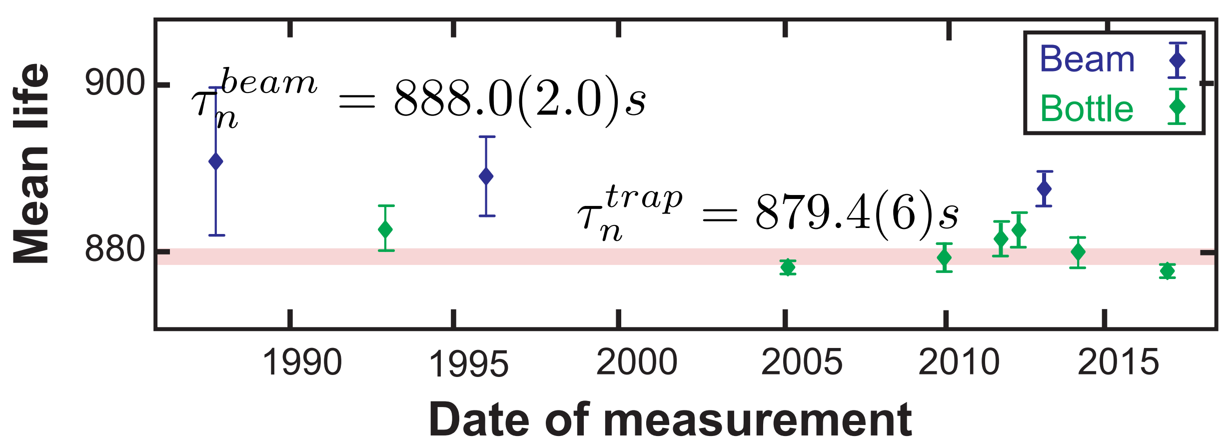
$$|V_{ud}^{0^+-0^+}| = 0.97420(21)$$

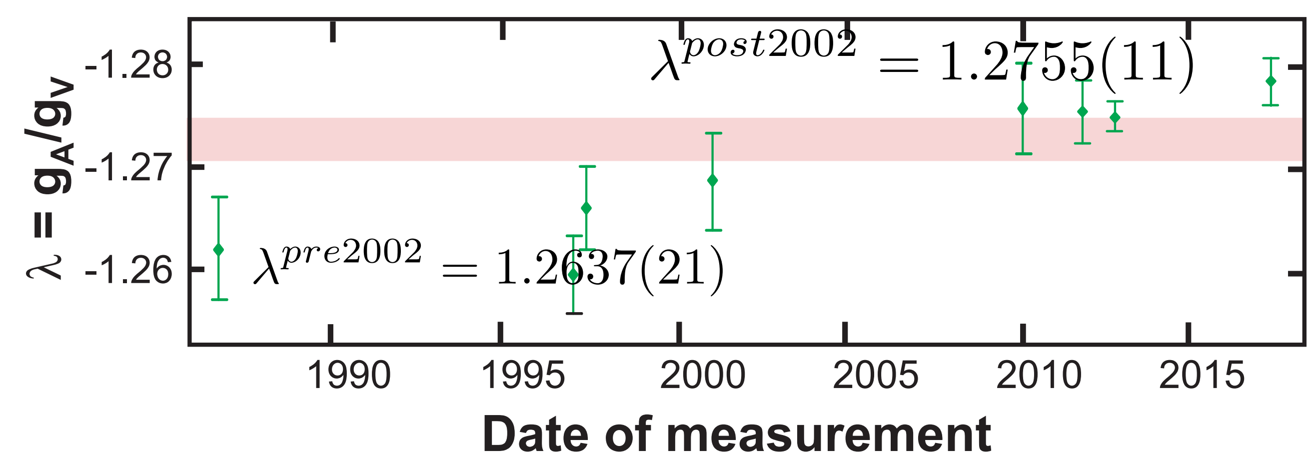
Free neutron decay: axial coupling
- requires additional measurements

$$|V_{ud}|^2 = \frac{5024.49(30)\ s}{\tau_n(1+3\lambda^2)(1+\Delta_R^V)}$$

$\lambda = g_A/g_V$

Unfortunate discrepancy between decay in flight vs. trapped UCN



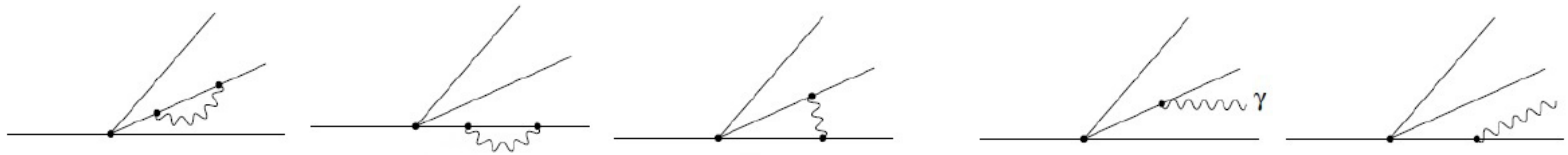


If using bottle τ_n + latest λ: consistent but 7 times less precise $\quad |V_{ud}^n| = 0.9743(15)$

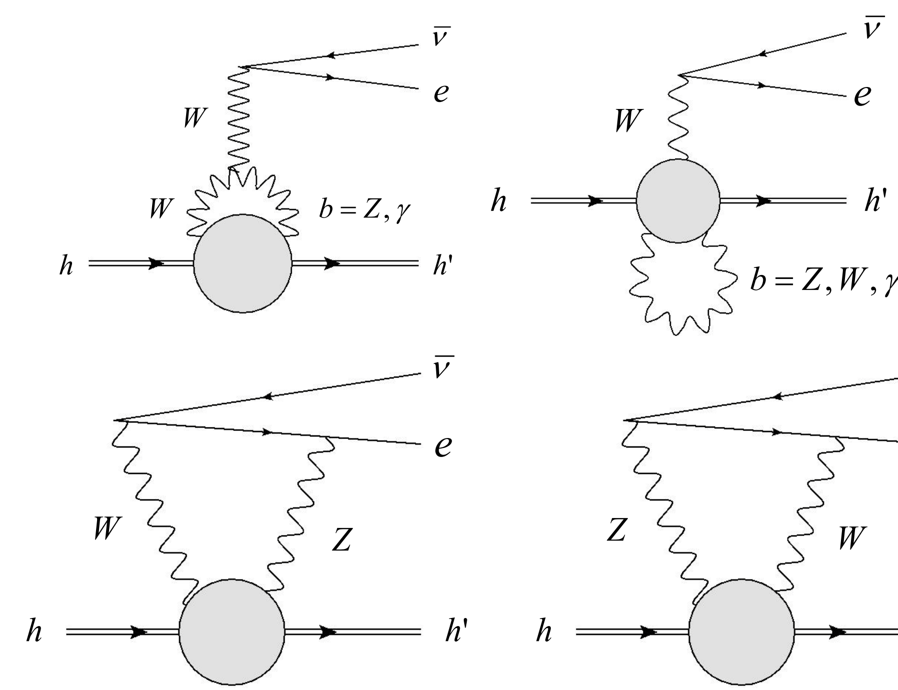
Same universal correction Δ_R^V - main limitation for V_{ud} extraction from superallowed decays

Radiative corrections - Inner & Outer

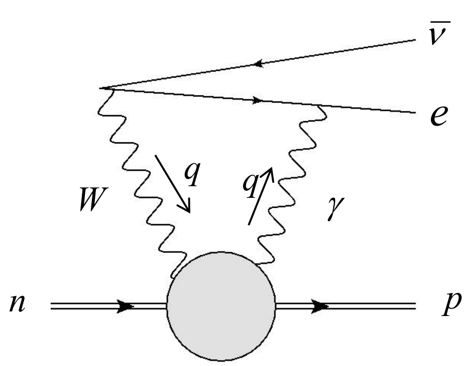
Outer (depend on e-energy) retain only IR divergent pieces



Inner (energy-independent)



W,Z-exchange: UV-sensitive, pQCD;
model-independent

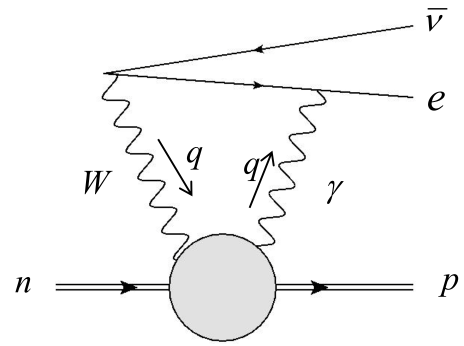


When γ involved:
sensitive to long range physics;
model-dependent!

Until recently: best determination
Marciano & Sirlin 2006

$$\Delta_R^V = 0.02361(38)$$

γW-box



Box at zero energy and momentum transfer
Corrections - ($\boldsymbol{\alpha}/2\boldsymbol{\pi}$) ($M_n$-$M_p$)/$m_\pi$ ~ 10^{-5}

$$T_{\gamma W} \;=\; \sqrt{2}e^2 G_F V_{ud}\int\frac{d^4q}{(2\pi)^4}\frac{\bar{u}_e\gamma^\mu(\not{k}-\not{q}+m_e)\gamma^\nu(1-\gamma_5)v_\nu}{q^2[(k-q)^2-m_e^2]}\frac{M_W^2}{q^2-M_W^2}T^{\gamma W}_{\mu\nu}$$

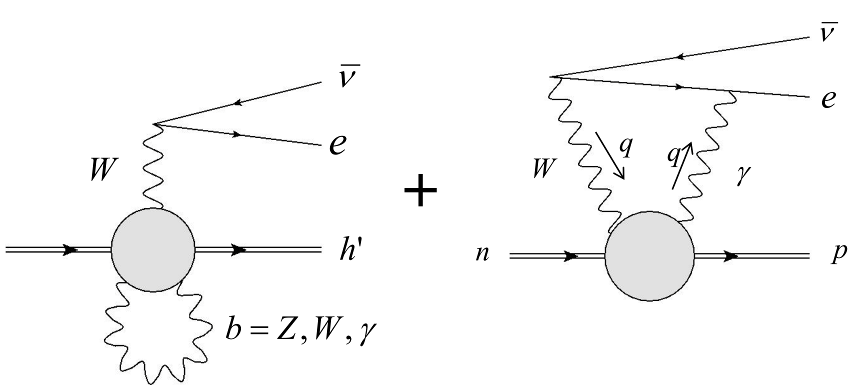
Hadronic tensor: two-current correlator

$$T^{\mu\nu}_{\gamma W}=\int dx e^{iqx}\langle p|T[J^\mu_{em}(x)J^\nu_W(0)]|n\rangle$$

General gauge-invariant decomposition (spin-independent)

$$T^{\mu\nu}_{\gamma W} \;=\; \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)T_1+\frac{1}{(p\cdot q)}\left(p-\frac{(p\cdot q)}{q^2}q\right)^\mu\left(p-\frac{(p\cdot q)}{q^2}q\right)^\nu T_2+\frac{i\epsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta}{2(p\cdot q)}T_3$$

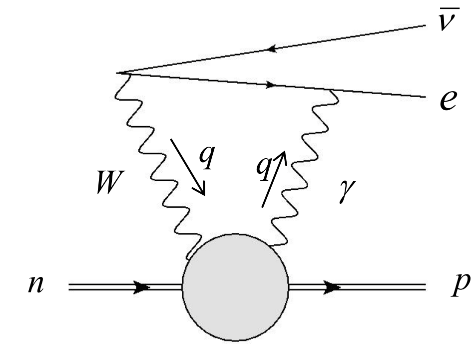
V-V correlator cancels 3-current correlator
Sirlin 1967 - current algebra
Reason: conserved vector-isovector current



Axial current not conserved -> A-V correlator model-dependent

γW-box in the pre-2018 era

$$\Box^{VA}_{\gamma W} = 4\pi\alpha \mathrm{Re} \int \frac{d^4q}{(2\pi)^4} \frac{M_W^2}{M_W^2+Q^2} \frac{Q^2+\nu^2}{Q^4} \frac{T_3(\nu,Q^2)}{M\nu}$$



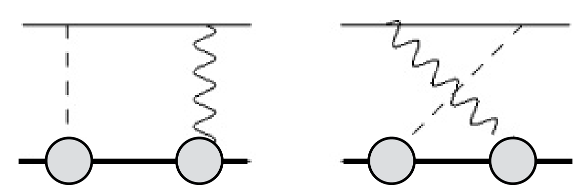
$Q^2 = -q^2$
$\nu = (pq)/M$

Marciano & Sirlin used loop techniques:
$$\Box^{VA}_{\gamma W} = \frac{\alpha}{8\pi} \int_0^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2} F(Q^2)$$

Short distance Q^2>> $F^{\mathrm{DIS}}(Q^2) = \frac{1}{Q^2}$
$$\Box^{\mathrm{DIS}}_{\gamma W} = \frac{\alpha}{8\pi} \int_{\Lambda^2}^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2} F^{\mathrm{DIS}}(Q^2) = \frac{\alpha}{4\pi} \ln \frac{M_W}{\Lambda}$$

Finite Q^2 -
pQCD corrections:
$$F^{\mathrm{DIS}} = \frac{1}{Q^2} \to \frac{1}{Q^2}\left[1 - \frac{\alpha_s^{\overline{MS}}}{\pi}\right]$$

Long distance Q^2<< - elastic box



MS 1987: asymptotic + pQCD + Born
$$\left[\Delta_R^V\right]^{\gamma W} = \frac{\alpha}{2\pi}\left[\ln\frac{M_W}{\Lambda} + A_g + 2C_B\right]$$

~4 -0.24 1.85

Problem: connecting short and long distances

γW-box in the pre-2018 era

MS 2006 update: a more sophisticated analysis to improve precision

Short distance: DIS to N[3]LO

$$F^{\rm DIS} = \frac{1}{Q^2} \to \frac{1}{Q^2}\left[1 - \frac{\alpha_s^{\overline{MS}}}{\pi} - C_2\left(\frac{\alpha_s^{\overline{MS}}}{\pi}\right)^2 - C_3\left(\frac{\alpha_s^{\overline{MS}}}{\pi}\right)^3\right]$$

GLS and Bjorken SR to N3LO
Larin, Vermaseren 1997

$$Q^2 > Q_2^2$$

Vector Dominance Model Ansatz

$$F^{\rm INT}(Q^2) = -\frac{1.490}{Q^2+m_\rho^2} + \frac{6.855}{Q^2+m_A^2} - \frac{4.414}{Q^2+m_{\rho'}^2}$$

$$F^{Int}(0) = 0$$

Matching conditions:

$$\int_{Q_2^2}^{\infty} \frac{dQ^2 M_W^2}{Q^2(M_W^2+Q^2)} F^{Int}(Q^2) = \int_{Q_2^2}^{\infty} \frac{dQ^2 M_W^2}{Q^2(M_W^2+Q^2)} F^{DIS}(Q^2)$$

$$F^{Int}(Q_1^2) = F^B(Q_1^2)$$

$$Q^2 < Q_1^2$$

Long distance: Born

$$F(Q^2) = F^B(Q^2)$$

$$\left[\Delta_R^V\right]^{\gamma W} = \frac{\alpha}{2\pi}\left[\underbrace{\ln\frac{M_W}{\Lambda}}_{\sim 3.86} + \underbrace{A_g + 2C_B}_{1.78}\right]$$

$$\left[\Delta_R^V\right]^{\gamma W} = \frac{\alpha}{2\pi}\left[\underbrace{\ln\frac{M_W}{\Lambda}}_{\sim 3.77} + \underbrace{A_g + C^{Int}}_{0.14(14)} + \underbrace{2C_B}_{1.66}\right]$$

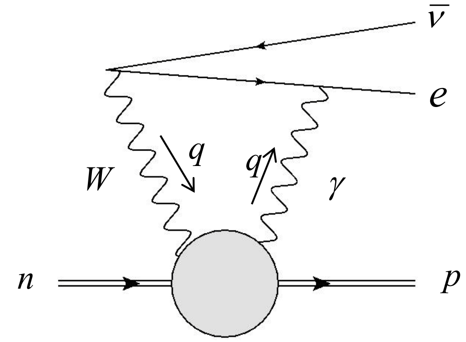
Uncertainty reduced by a factor ~2

γW-box from Dispersion Relations

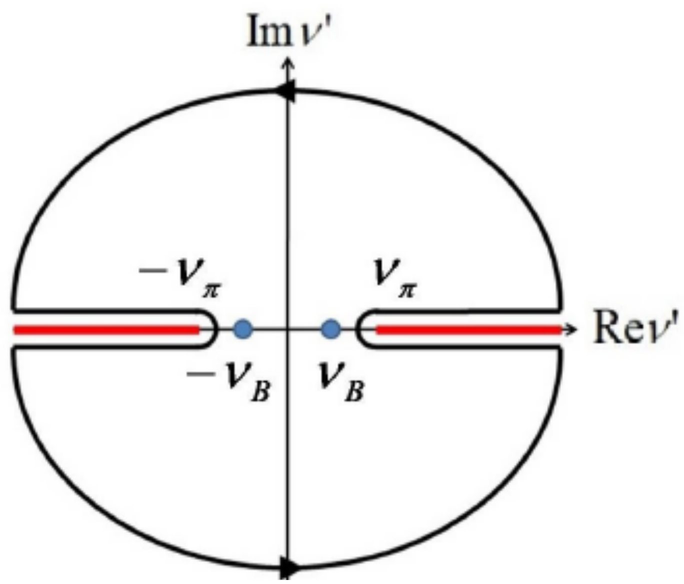
Seng, MG, Patel, Ramsey-Musolf, arXiv: 1807.10197

Check MS result + uncertainty independently

$$\Box^{VA}_{\gamma W} = 4\pi\alpha \mathrm{Re} \int \frac{d^4q}{(2\pi)^4} \frac{M_W^2}{M_W^2+Q^2} \frac{Q^2+\nu^2}{Q^4} \frac{T_3(\nu,Q^2)}{M\nu}$$



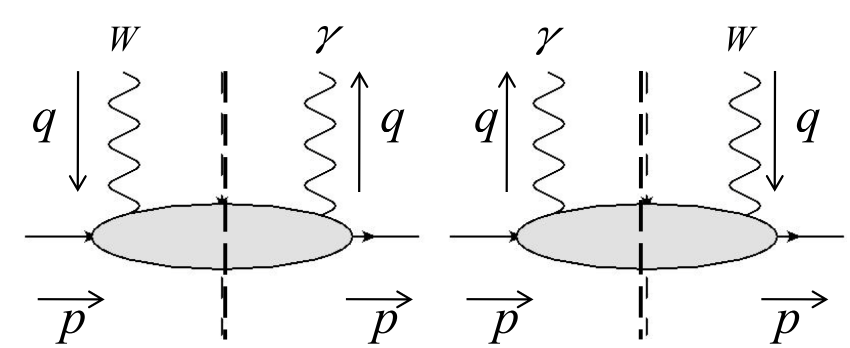
$Q^2 = -q^2$
$\nu = (pq)/M$



T_3 - analytic function inside the contour C in the complex ν-plane determined by its singularities on the real axis - poles + cuts

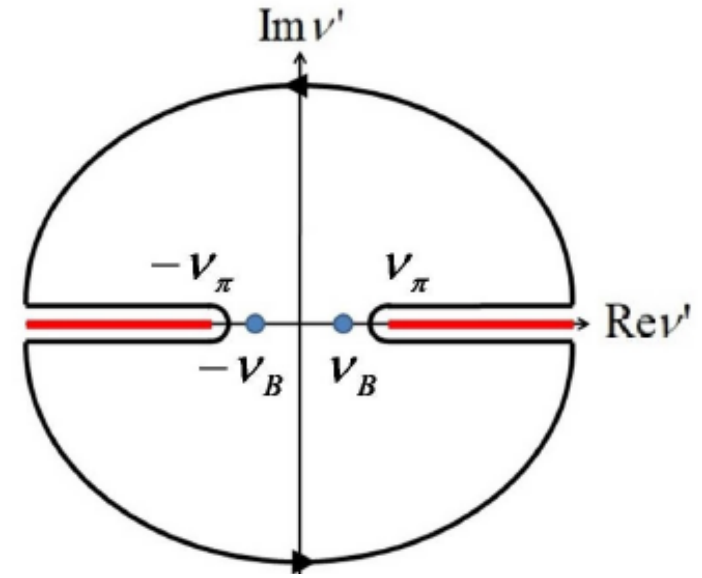
$$T_3(\nu,Q^2) = \frac{1}{2\pi i}\oint_C \frac{T_3(z,Q^2)dz}{z-\nu} \quad \nu \in C$$

Forward amplitude T_3 - unknown;
Its absorptive part can be related to production of on-shell intermediate states
a γW-analog of the SF F_3



$$\mathrm{Im}\, T_3^{\gamma W}(\nu,Q^2) = 2\pi F_3^{\gamma W}(\nu,Q^2)$$

γW-box from Dispersion Relations



Crossing behavior: photon is isoscalar or isovector

Different isospin channels behave differently

$$T_3^{\gamma W} = T_3^{(0)} + T_3^{(3)}$$

$$T_3^{(0)}(-\nu, Q^2) = -T_3^{(0)}(\nu, Q^2), \quad T_3^{(3)}(-\nu, Q^2) = +T_3^{(3)}(\nu, Q^2)$$

Dispersion representation of the γW-box correction at zero energy

$$\boxed{\begin{aligned} \Box_{\gamma W}^{VA\,(0)} &= \frac{3\alpha}{2\pi}\int_0^\infty \frac{dQ^2 M_W^2}{Q^2(M_W^2+Q^2)} M_3^{(0)}(1, Q^2) \\ \Box_{\gamma W}^{VA\,(3)} &= 0 \end{aligned}}$$

First Nachtmann moment of $F_3^{(0)}$

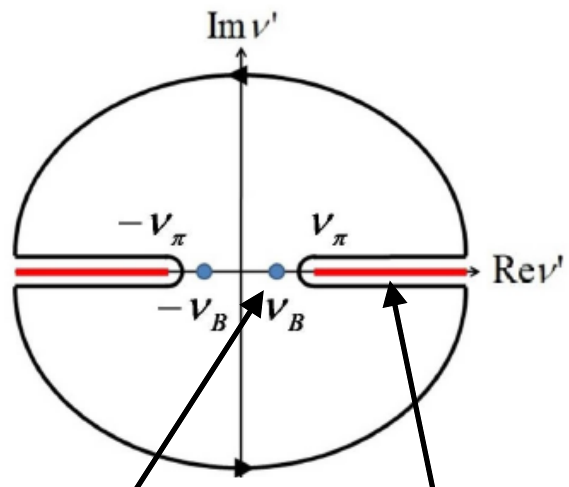
$$M_3^{(0)}(1, Q^2) = \frac{4}{3}\int_0^1 dx \frac{1+2r}{(1+r)^2} F_3^{(0)}(x, Q^2)$$

$$r = \sqrt{1 + 4M^2x^2/Q^2}$$

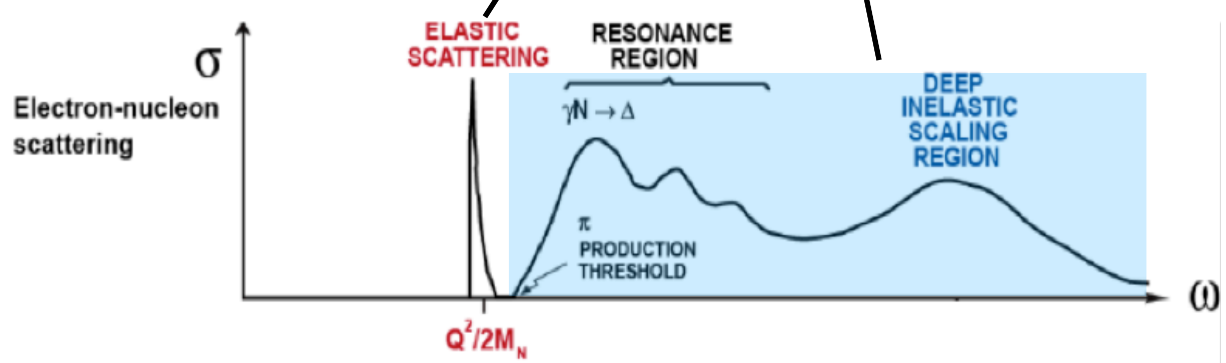
MS's loop function F in terms of M_3

$$F_{\mathrm{M.S.}}(Q^2) = \frac{12}{Q^2} M_3^{(0)}(1, Q^2)$$

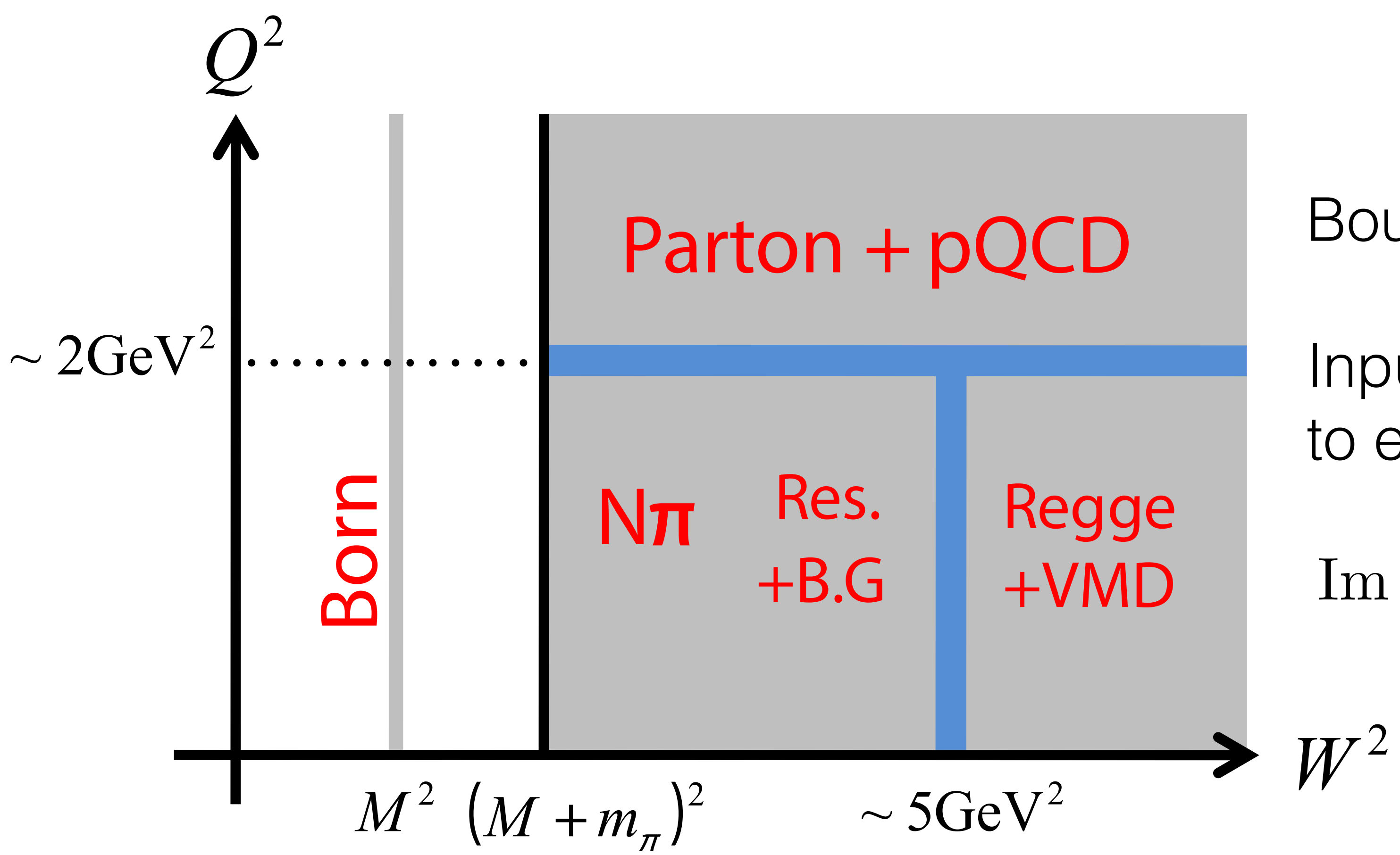
Input into dispersion integral



Dispersion in energy:
scanning hadronic intermediate states



Dispersion in Q²:
scanning dominant physics pictures

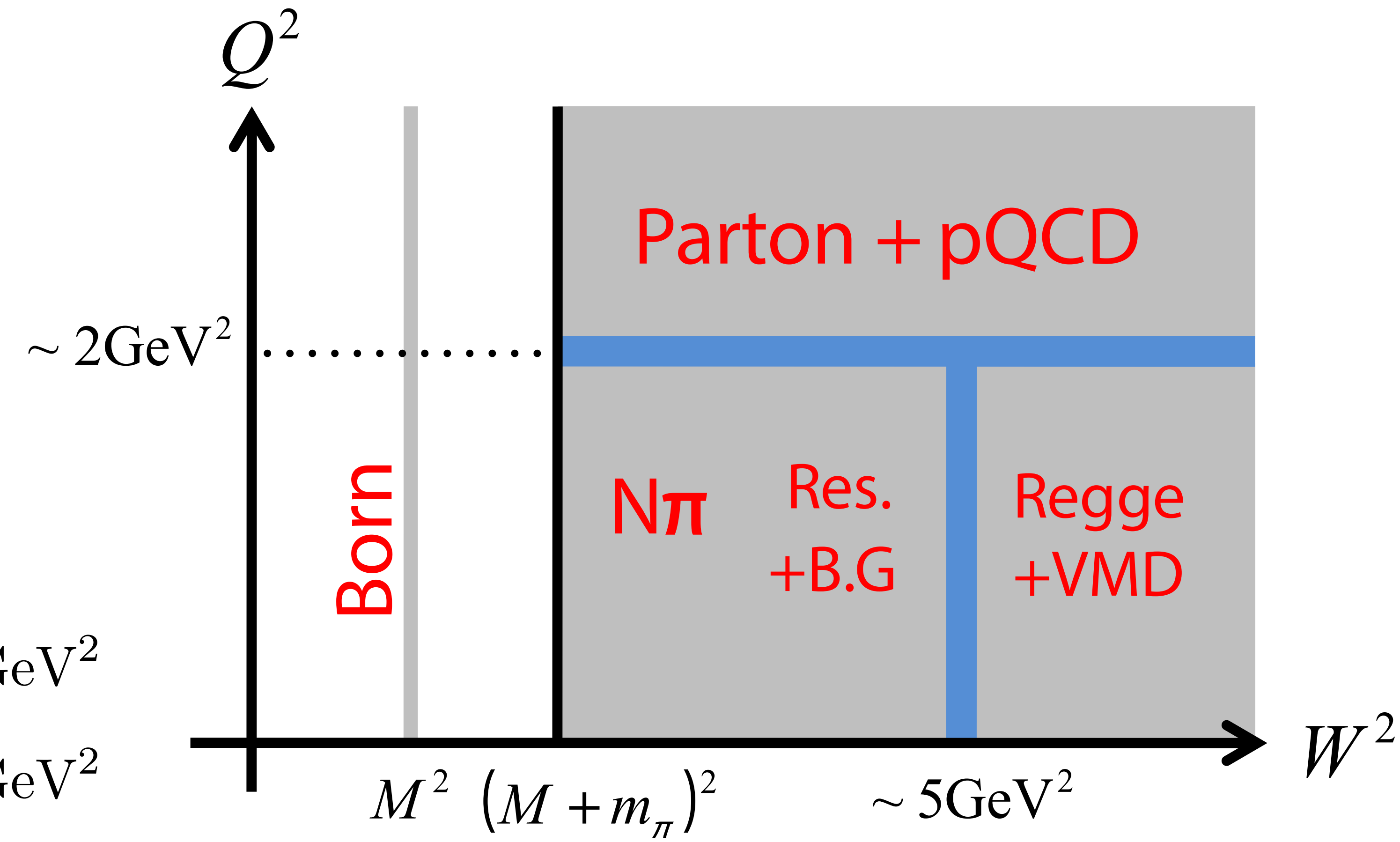


Boundaries between regions - approximate

Input in DR related (directly or indirectly) to experimentally accessible data

$$\mathrm{Im}\,[W^{+*} + n \to \gamma^* + p] \leftrightarrow \begin{cases} \sigma(\gamma^* + p \to X) \\ \sigma(W^{+*} + n \to X) \end{cases}$$

Input into dispersion integral



Our parametrization of the needed SF follows from this diagram

$$F_3^{(0)} = F_{\text{Born}} + \begin{cases} F_{\text{pQCD}}, & Q^2 \gtrsim 2\ \text{GeV}^2 \\ F_{\pi N} + F_{\text{res}} + F_{\mathbb{R}}, & Q^2 \lesssim 2\ \text{GeV}^2 \end{cases}$$

Born:

elastic FF from most recent electron and neutrino scattering data

πN:

relativistic ChPT calculation plus nucleon FF

Resonances:

axial excitation from PCAC (Lalakulich et al 2006) - neutrino scattering

isoscalar photo-excitation from MAID and PDG - electron and γ inelastic scattering

Above resonance region:

multiparticle continuum economically described by Regge exchanges

Input into dispersion integral

Unfortunately, no data can be obtained for $F_3^{\gamma W\,(0)}$

Use the data for the pure CC processes

$$\frac{d^2\sigma^{\nu(\bar\nu)}}{dxdy} = \frac{G_F^2 ME}{\pi}\left[xy^2 F_1 + \left(1 - y - \frac{Mxy}{2E}\right) F_2 \pm x\left(y - \frac{y^2}{2}\right) F_3\right]$$

$$\sigma^{\nu p} - \sigma^{\bar\nu p} \sim F_3^{\nu p} + F_3^{\bar\nu p} = u_v^p(x) + d_v^p(x)$$

Data used in the past to check Gross-Llewellyn-Smith sum rule $\int_0^1 dx(u_v^p(x) + d_v^p(x)) = 3$

The plan: validate the model for CC process; apply an isospin rotation to obtain γW

$$F^{\nu p+\bar\nu p}_{3,\,\mathrm{low}-Q^2} = F^{\nu p+\bar\nu p}_{3,\,el.} + F^{\nu p+\bar\nu p}_{3,\,\pi N} + F^{\nu p+\bar\nu p}_{3,\,R} + F^{\nu p+\bar\nu p}_{3,\,\mathrm{Regge}}$$

The same ingredients but in a different isospin channel

Low-W part of spectrum: neutrino data from MiniBooNE, Minerva, …

- fix (to some extent) the axial FF, resonance contributions, pi-N continuum
- High energies (low x): Regge behavior $F_3 \sim q^{\nu} \sim x^{-\alpha}$, $\alpha \sim 0.5$-0.7

Inelastic states - low Q^2, high W

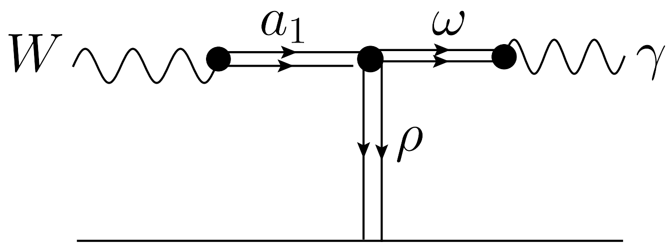
Scattering at high energy can be very effectively described by Regge exchanges

$$F_3^{(0),\mathrm{Regge}}(\nu, Q^2) = C_R(Q^2)\left(\frac{\nu}{\nu_0}\right)^{\alpha_\rho}$$

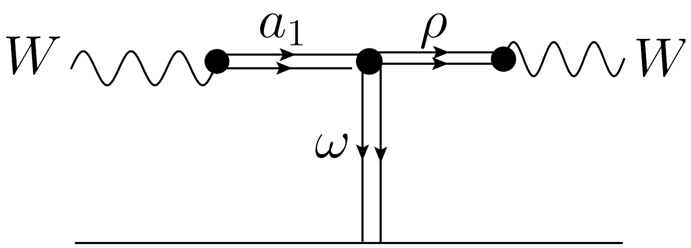
Content of the model (and parameters) - depend on the quantum numbers
Consider vector - axial vector interference relevant for F_3

Regge behavior in EW processes: hadron-like behavior of HE electroweak probes - Vector/Axial Vector Dominance is the proper language

γW-box: conversion of $W^\pm$ (charged, I=1, axial) to γ (neutral, vector, I=0)
requires charged vector exchange w. I=1 - $\rho^\pm$
effective a_1 - ρ - ω vertex



Inclusive ν scattering: conversion of $W^\pm$ (charged, I=1, axial) to $W^\pm$ (charged, I=1, axial)
requires neutral vector exchange w. I=0 - ω
effective a_1 - ω - ρ vertex



Minimal model for both reactions - check with data.

Parameters of the Regge model from neutrino scattering

$$F_3^{WW,\,\mathrm{Regge}}(\nu,Q^2) = C_R^{WW}(Q^2)\left(\frac{\nu}{\nu_0}\right)^{\alpha_\omega} \qquad F_3^{W\gamma(0),\,\mathrm{Regge}}(\nu,Q^2) = C_R^{\gamma W}(Q^2)\left(\frac{\nu}{\nu_0}\right)^{\alpha_\rho}$$

Vector/axial vector dominance: Stodolsky, Piketty '70

$$C_R^{WW}(Q^2) = C_R^{WW}(0)\frac{m_\rho^2}{m_\rho^2+Q^2}\frac{m_{a_1}^2}{m_{a_1}^2+Q^2}\times h^{WW}(Q^2) \qquad C_R^{\gamma W}(Q^2) = C_R^{\gamma W}(0)\frac{m_\omega^2}{m_\omega^2+Q^2}\frac{m_{a_1}^2}{m_{a_1}^2+Q^2}\times h^{\gamma W}(Q^2)$$

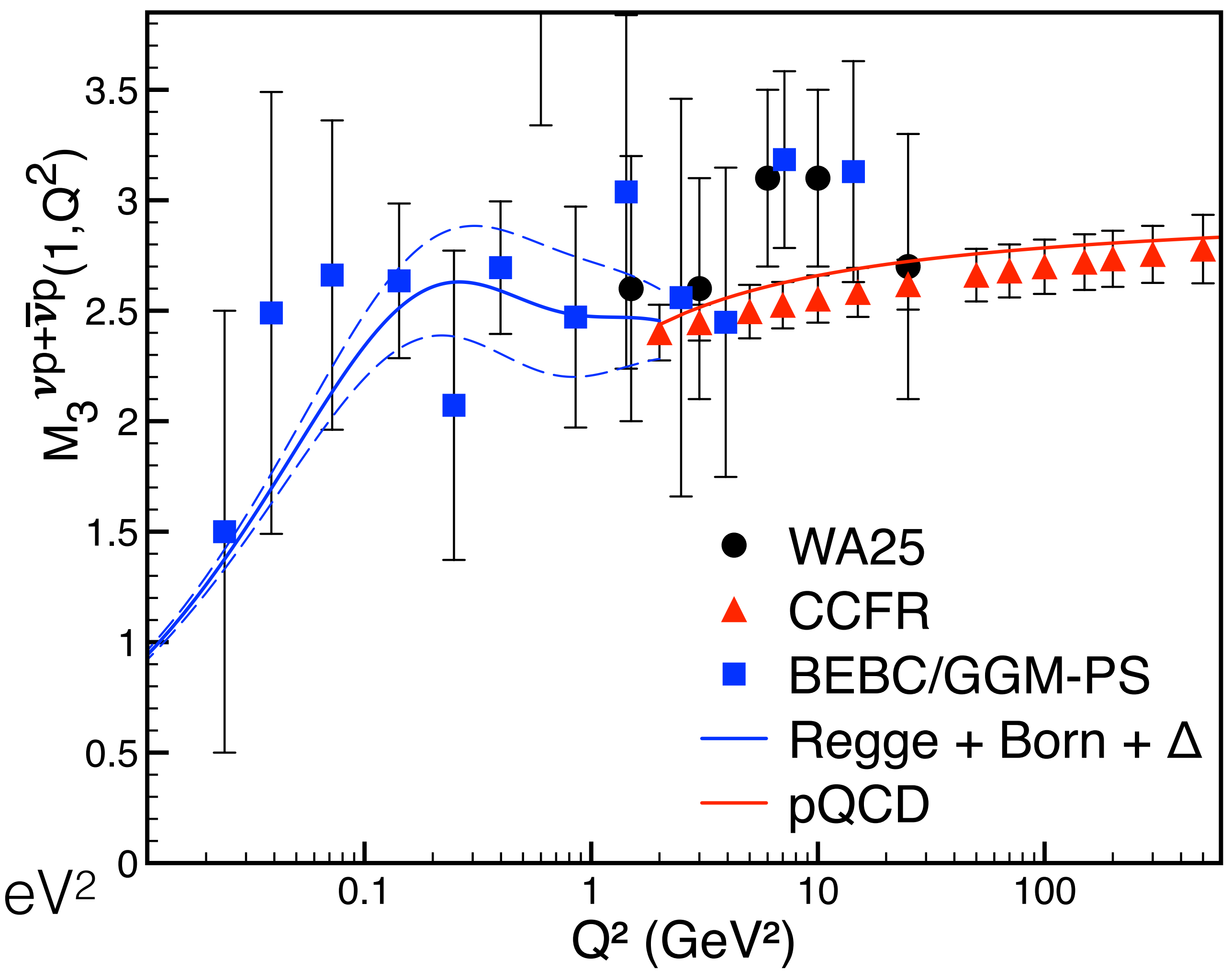
Pure VDM may not work at $Q^2 = 2$ GeV^2

$$h^{WW}(Q^2) = 1 + aQ^2$$

C_R^{WW}, h^{WW} and uncertainties
- from neutrino data;

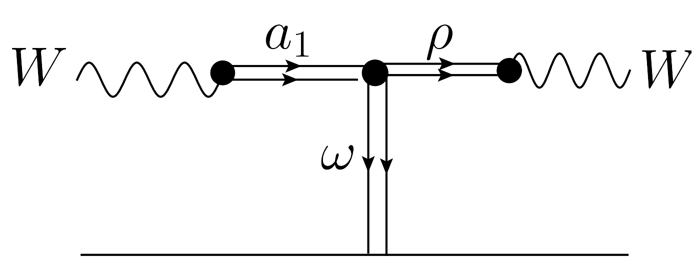
$$C_R^{WW} = 5.2 \pm 1.5 \qquad a^{WW} = 1.08^{+0.48}_{-0.28}\ \mathrm{GeV}^2$$

Uncertainties anti correlated

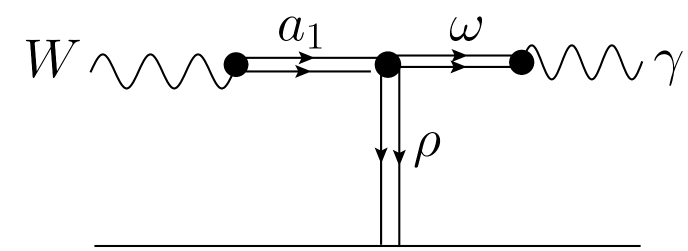


Low $Q^2 < 0.1$ GeV^2: Born + Δ(1232) dominate
Not fitted: modern data more precise but cover only limited energy range
Fit driven by 4 data points between 0.2 and 2 GeV^2

Parameters of the Regge model from neutrino scattering



—> Isospin rotation —>



$$C_R^{WW}(Q^2) = C_R^{WW}(0)\frac{m_\rho^2}{m_\rho^2+Q^2}\frac{m_{a_1}^2}{m_{a_1}^2+Q^2}\times h^{WW}(Q^2)$$

$$C_R^{\gamma W}(Q^2) = C_R^{\gamma W}(0)\frac{m_\omega^2}{m_\omega^2+Q^2}\frac{m_{a_1}^2}{m_{a_1}^2+Q^2}\times h^{\gamma W}(Q^2)$$

$C_R^{\gamma W}(0)$ - matching in Regge + VDM

$$C_R^{WW}(0) \sim \frac{e}{g_\rho}\times g_{\omega NN}\times \overset{\text{V*A+A*V}}{2}\times \underset{(\bar\nu p+\nu p)}{2}$$

$$C_R^{\gamma W}(0) \sim \frac{e}{g_\omega}\cdot g_{\rho NN} = \frac{1}{36}C_R^{WW}(0)$$

$$g_\omega \approx 3g_\rho, \quad g_{\omega NN}\approx 3g_{\rho NN}$$

$h^{\gamma W}$ - match at Q^2 = 2 GeV^2 (pQCD)

$$C_R^{WW}(Q^2)h^{WW}(Q^2)\big|_{2\,GeV^2} \propto \int_0^1 dx(F_3^{\nu p+\bar\nu p}) = 3$$

$$C_R^{\gamma W}(Q^2)h^{\gamma W}(Q^2)\Big|_{2\,GeV^2} \propto \int_0^1 dx F_3^{\gamma W\,(0)} = \frac{1}{12}$$

At both matching points the same relation holds, $C_R^{\gamma W}\, h^{\gamma W}$ = (1/36) $C_R^{WW}\, h^{WW}$

Only overall normalization C_R is changed! In the isospin-symmetry limit $h^{\gamma W}(Q^2) = h^{WW}(Q^2)$

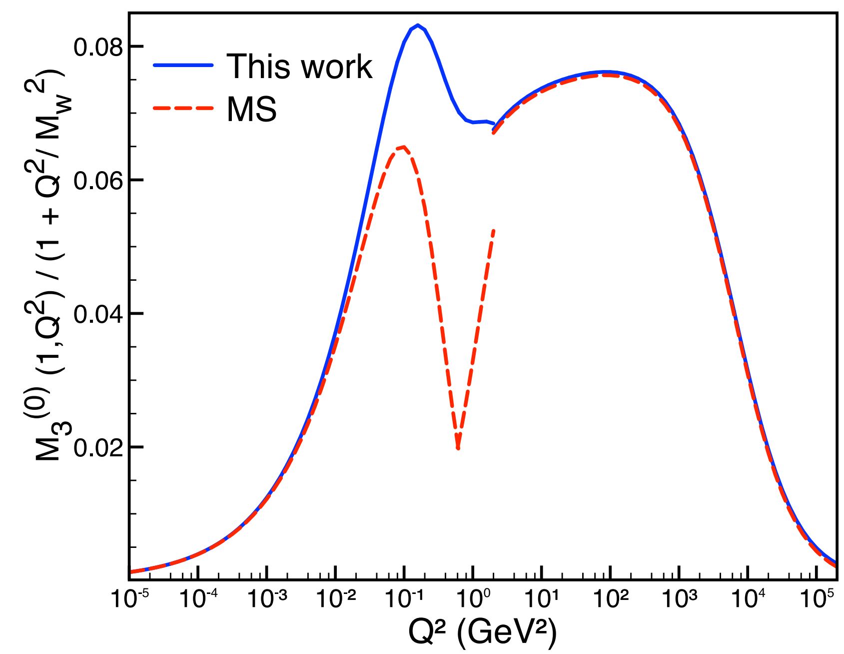
No additional uncertainty due to isospin rotation!

Input into dispersion integral

Model & Uncertainty fully specified
- compare M&S vs This work

Log scale for x-axis:
the integral = surface under the curve

M&S: integrand discontinuous at $Q^2 = 2.25\ \mathrm{GeV}^2$



$$^{MS}\Box_{\gamma W}^{VA\,(0)} = \frac{3\alpha}{2\pi}\left[\int_0^{(0.823\,GeV)^2}\frac{dQ^2}{Q^2}M_3^{Born} + \int_{(0.823\,GeV)^2}^{(1.5\,GeV)^2}\frac{dQ^2}{Q^2}M_3^{Int} + \int_{(1.5\,GeV)^2}^{\infty}\frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}M_3^{pQCD}\right]$$

(0.96±0.1)×10^-3 (0.14±0.14)×10^-3 2.14(0)×10^-3

$$^{New}\Box_{\gamma W}^{VA\,(0)} = \frac{3\alpha}{2\pi}\left[\int_0^{2\,GeV^2}\frac{dQ^2}{Q^2}\left(M_3^{Born} + M_3^{\pi N} + M_3^{Res} + M_3^{Regge}\right) + \int_{2\,GeV^2}^{\infty}\frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}M_3^{pQCD}\right]$$

1.06(6)×10^-3 <10^-5

0.051(5)×10^-3 0.51(8)×10^-3 2.17(0)×10^-3

MS Total (3.24±0.18)×10^-3

New Total (3.79±0.10)×10^-3

Uncertainty reduced by almost factor 2;
~ 3-5 sigma shift from the old value

Universal γW-box

$$|V_{ud}|^2 = \frac{2984.432(3)\,\mathrm{s}}{\mathcal{F}t(1+\Delta_R^V)}$$

Marciano & Sirlin 2006

$$\Delta_R^V = 0.02361(38)$$

$$|V_{ud}| = 0.97420(21)$$

Dispersion relations

$$\Delta_R^V = 0.02467(22)$$

$$|V_{ud}| = 0.97366(15)$$

DR treatment allowed to reduce the uncertainty in Δ_R^V by almost factor of 2
due to the use of neutrino data

But the shift is more significant than anticipated from the uncertainty estimate by MS

Raises tension with CKM unitarity

Shifts the emphasis on V_{us} and on the nuclear corrections entering Ft

Before

$$|V_{ud}|^2 + |V_{ud}|^2 + |V_{ud}|^2 = 0.9994 \pm 0.0005$$

After

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9983(4)$$

How to further test the model?

Neutrino data at low Q^2 are not precise:
upcoming DUNE experiment @ Fermilab may provide better data for the first Nachtmann moment of F_3

Isospin rotation needs to be tested separately!

Use the connection between the CC V-A interference and a similar V-A term in the PVES!

Assuming the axial Z-N coupling to be a pure isovector,
build an isovector combination to isolate the isoscalar part of the photon

$$4F_3^{(0)} \approx F_{3,\gamma Z}^{p} - F_{3,\gamma Z}^{n} \approx 2F_{3,\gamma Z}^{p} - F_{3,\gamma Z}^{d}$$

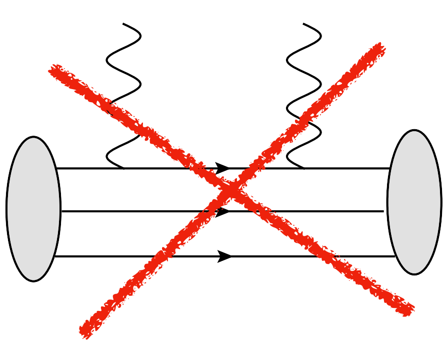
Need to measure F_3 in PVES at high energy (above resonance region) and low Q^2
- MOLLER (the background measurement on proton) - can use deuteron target?

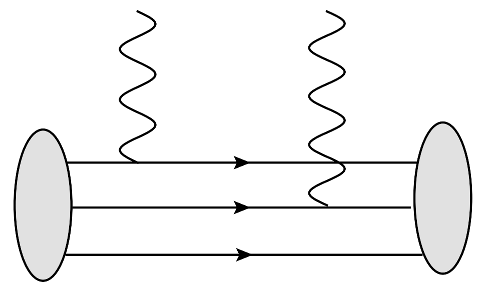
Summary & Outlook

- New dispersive representation of the γW-box:
  - data-driven uncertainties;
  - hadronic uncertainty almost halved;
  - raises tension with CKM unitarity
- Further data to better constrain the model: DUNE, MOLLER, … ?
- Nuclear effects? A shift in Ft value may partly cancel the shift in Δ_R^V
- Electroweak boxes interconnected: how large is the shift for γZ-box?
- Stay tuned!

Nuclear β-decay

General structure of RC for nuclear decay



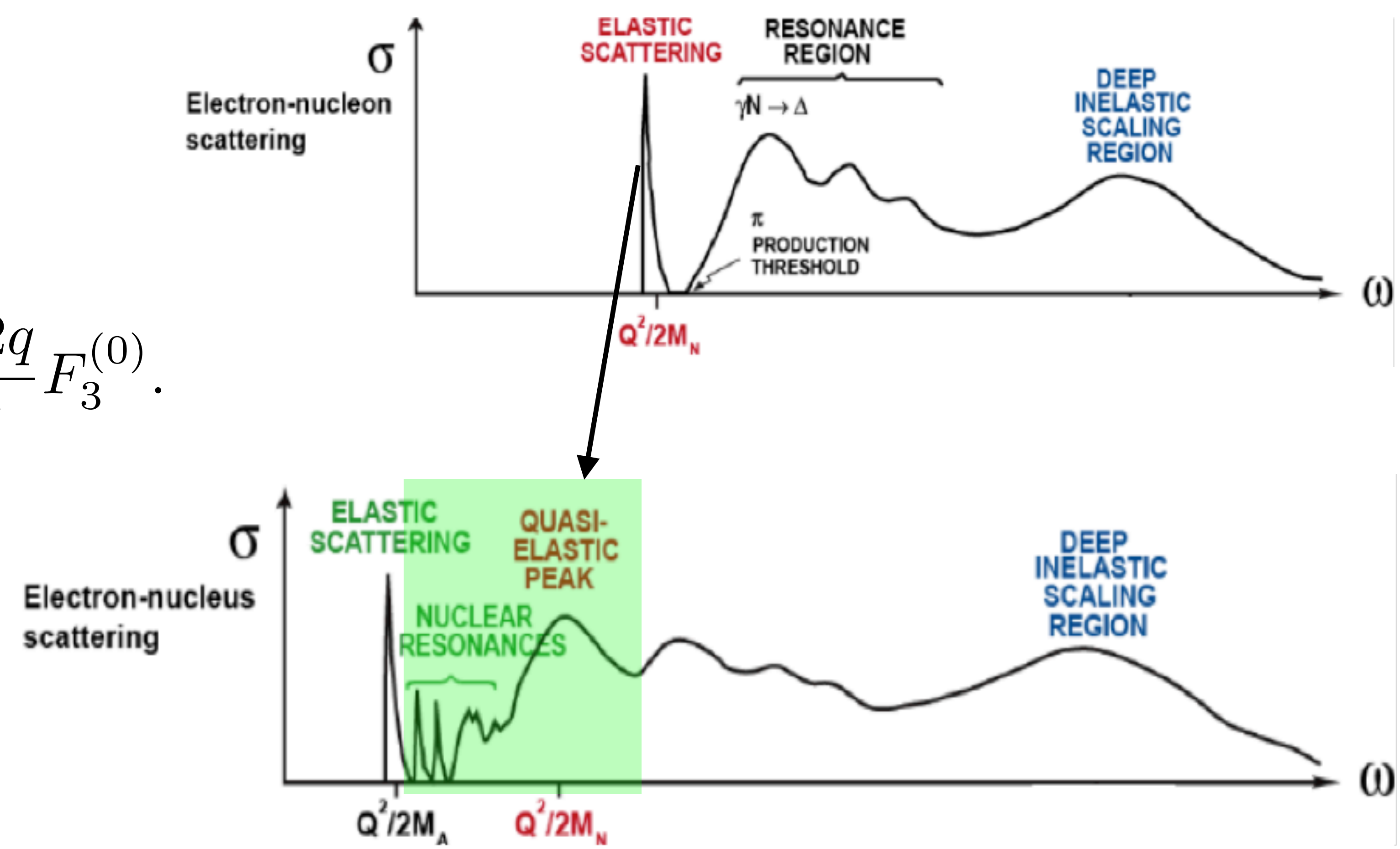


$$1 + \mathrm{RC} = (1 + \underbrace{\delta_R}_{\text{Nuclear structure (NS)}})(1 - \overbrace{\delta_C}^{\text{Isospin breaking}})(1 + \underbrace{\Delta}_{\text{Universal (no NS)}}).$$

Nuclear Green fn: only with 2 active N

But data tell us differently:
prominent broad QE peak -
mostly 1N knock-out

$$\Box^{\rm Nucl.}_{\gamma W} = \frac{\alpha}{\pi}\int_0^{\Lambda^2} dQ^2 \int_{\nu_N}^\infty \frac{d\nu}{(\nu+q)^2}\frac{\nu+2q}{M\nu}F_3^{(0)}.$$



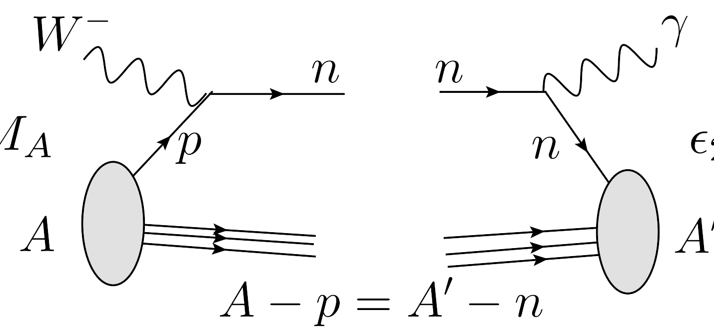
QE peak is common for all nuclei
Modify the universal correction
to account for bulk QE effect

QE contribution to γW-box

Bulk nuclear properties: Fermi momentum and break-up threshold

20 decays: $^{10}C \to {}^{10}B$ through $^{74}Rb \to {}^{74}Kr$ (Towner&Hardy '14 review)

$\epsilon_1 = M_{A-p} + M_n - M_A$



$\epsilon_2 = M_{A'-n} + M_n - M_A$

$$\bar{\epsilon} = \sqrt{\epsilon_1 \epsilon_2}$$

Decay	ϵ_2 (MeV)	ϵ_1 (MeV)	$\bar{\epsilon}$ (MeV)
$^{10}C \to {}^{10}B$	8.44	4.79	6.36
$^{14}O \to {}^{14}N$	10.55	5.41	7.55
$^{18}Ne \to {}^{18}F$	9.15	4.71	6.56
$^{22}Mg \to {}^{22}Na$	11.07	6.28	8.34
$^{26}Si \to {}^{26}Al$	11.36	6.30	8.46
$^{30}S \to {}^{30}P$	11.32	5.18	7.66
$^{34}Ar \to {}^{34}Cl$	11.51	5.44	7.91
$^{38}Ca \to {}^{38}K$	12.07	5.33	8.02
$^{42}Ti \to {}^{42}Sc$	11.55	4.55	7.25
$^{26m}Al \to {}^{26}Mg$	11.09	6.86	8.72
$^{34}Cl \to {}^{34}S$	11.42	5.92	8.22
$^{38m}K \to {}^{38}Ar$	11.84	5.79	8.28
$^{42}Sc \to {}^{42}Ca$	11.48	5.05	7.61
$^{46}Va \to {}^{46}Ti$	13.19	6.14	9.00
$^{50}Mn \to {}^{50}Cr$	13.00	5.37	8.35
$^{54}Co \to {}^{54}Fe$	13.38	5.13	8.28
$^{62}Ga \to {}^{62}Zn$	12.90	3.72	6.94
$^{66}As \to {}^{66}Ge$	13.29	3.16	6.48
$^{70}Br \to {}^{70}Se$	13.82	3.20	6.65
$^{74}Rb \to {}^{74}Kr$	13.85	3.44	6.90

Effective removal energies - all in a small range

$$\bar{\epsilon} = 7.68 \pm 1.32 \text{ MeV}$$

Fermi momentum also not too different for all A

$$k_F(A=10) = 228\,\text{MeV}, \quad k_F(A=74) = 245\,\text{MeV}$$

Can define a universal correction that correctly represents bulk nuclear effect!

Further ingredients:
Free Fermi gas model (or superscaling)
+ Pauli blocking

QE contribution to γW-box

Elastic γW-box for bound neutron: $\Box_{\gamma W}^{\rm free\, n} = \frac{\alpha}{2\pi} 0.91(5) \quad \to \quad \Box_{\gamma W}^{\rm QE} = \frac{\alpha}{2\pi} 0.44(4)$

Reduction due to finite breakup threshold and Fermi motion

Towner & Hardy 2014: quenching of spin operators in nuclei:
about half the effect we observe.
Indicates that Ft values may shift down and compensate
part of the shift in the universal correction

$$\Box_{\gamma W}^{B,\, Quenched} - \Box_{\gamma W}^{B,\, free\ n} \approx \frac{\alpha}{2\pi}(-0.25)$$

Turn “inner” correction inside-out?

γW-box correction at zero energy

$$\Box_{\gamma W}^{VA\,(0)} = \frac{\alpha}{\pi M}\int_0^\infty \frac{dQ^2 M_W^2}{M_W^2+Q^2}\int_0^\infty d\nu \frac{(\nu+2q)}{\nu(\nu+q)^2}F_3^{(0)}(\nu,Q^2),$$
$$\Box_{\gamma W}^{VA\,(3)} = 0,$$

γW-box correction with linear E-dependece

$$E_{min} = (\nu+\sqrt{\nu^2+Q^2})/2$$

$$\mathrm{Re}\,\overline{\Box_{\gamma W}^{even}} = \frac{\alpha_{em}}{\pi}\int\limits_{\nu_{thr}}^{\infty} d\nu \int\limits_0^\infty dQ^2 \frac{F_3^{(0)}}{2M\nu}\left(\frac{1}{E_{min}} - \frac{\nu}{4E_{min}^2}\right),$$

$$\mathrm{Re}\,\Box_{\gamma W}^{odd} = \frac{\alpha_{em}}{\pi}E\int\limits_{\nu_{thr}}^{\infty} d\nu \int\limits_0^\infty dQ^2 \left[\frac{F_1^{(0)}}{6ME_{min}^3} + \left(\frac{\sqrt{\nu^2+Q^2}}{2E_{min}^2\nu Q^2} - \frac{1}{12E_{min}^3\nu}\right)F_2^{(0)} + \frac{F_3^{(-)}}{2M\nu}\left(\frac{1}{2E_{min}^2} - \frac{\nu}{6E_{min}^3}\right)\right]$$

Common wisdom: E-dep. negligible because should come as $(\alpha/2\pi)\, E/m_\pi < 10^{-5}$

But nuclear excitations live at few MeV —> large nuclear polarizabilities

$$\alpha_E + \beta_M = \frac{2\alpha_{em}}{M}\int \frac{d\omega}{\omega^3}F_1(\omega,Q^2=0) = 2\alpha_{em}\int \frac{d\omega}{\omega^2}\left.\frac{F_2(\omega,Q^2)}{Q^2}\right|_{Q^2=0}$$

New energy scale: polarizability/radius^2

$$\mathrm{Re}\,\Box_{\gamma W}^{odd} \sim \frac{2}{\pi}E\frac{\alpha_E+\beta_M}{R_{Ch}^2}$$

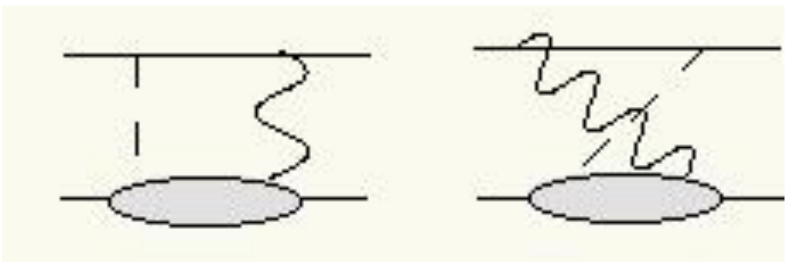
$$R_{\mathrm{Ch}} \sim 1.2\mathrm{fm}A^{1/3}$$

$$\alpha_E \sim (2.2\times10^{-3}\,\mathrm{fm})A^{5/3}$$

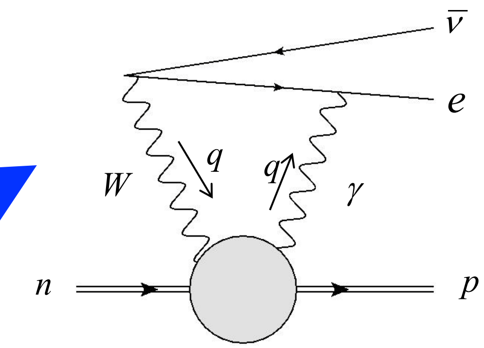
Expect $\mathrm{Re}\,\Box_{\gamma W}^{odd} \sim 1\times10^{-3}\left(\frac{E}{5\,\mathrm{MeV}}\right)\left(\frac{A}{30}\right)$

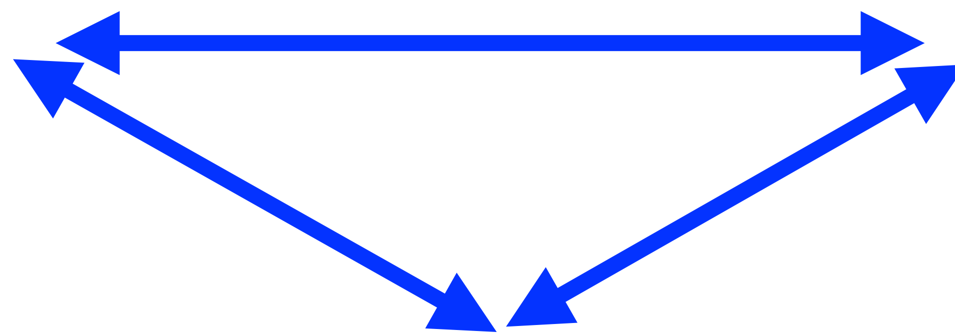
Connecting boxes: γW - γZ - WW

γZ-box: PV in atoms and e-scattering
MESA: P2 and C12 expo.

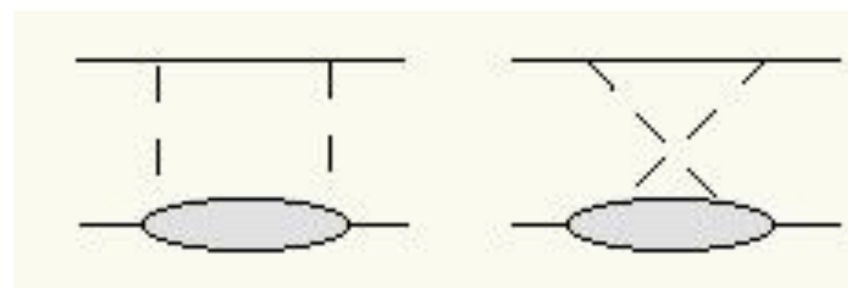


γW-box: beta decays





WW-box: neutrinoless double-beta decay



External particles, probes and kinematics
may be different

The input information is very much the same!