Iso-vector Transversity Quark Distributions from Lattice QCD

Fernanda Steffens

University of Bonn

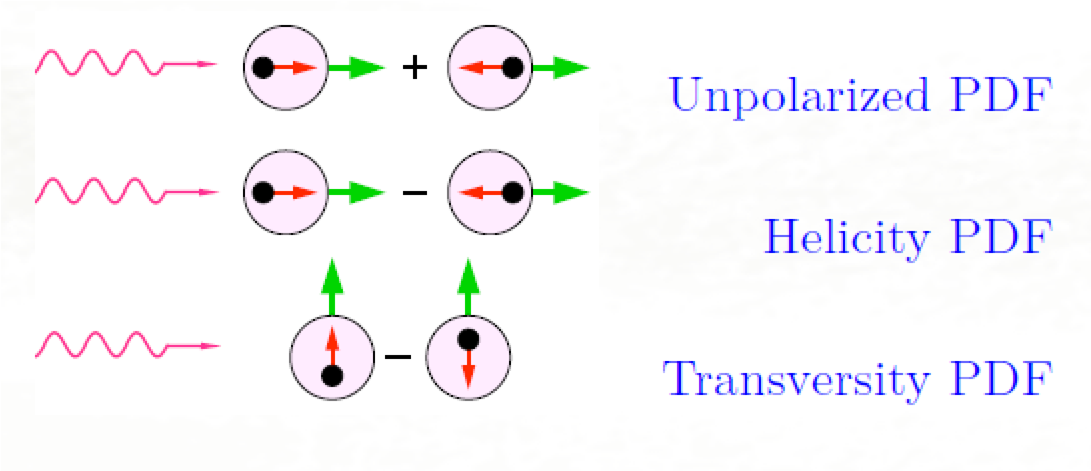
In collaboration with: Constantia Alexandrou (Univ. of Cyprus; Cyprus Institute),
Krzysztof Cichy (Adam Mickiewicz, Poland)
Martha Constantinou (Temple University)
Karl Jansen (DESY – Zeuthen)
Aurora Scapellato (HPC-LEAP; Uni. Of Cyprus; Uni. of Wuppertal)

Outline

- Introduction
- Quark distributions and quark quasi-distributions
- Extracting quark distributions from the quasi-distributions
- Computation of the matrix elements using lattice QCD
- Renormalization
- The x dependence of the iso-vector quark distributions
- Summary

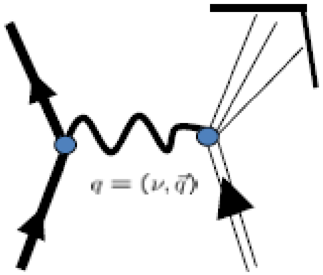
Introduction

Complete set of twist-2 parton distribution functions



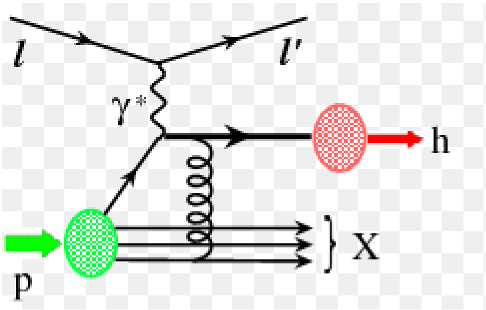
Cross sections are measured

Totally inclusive



Have access too to the chiral-even distributions $f_1(x)$ and $g_1(x)$

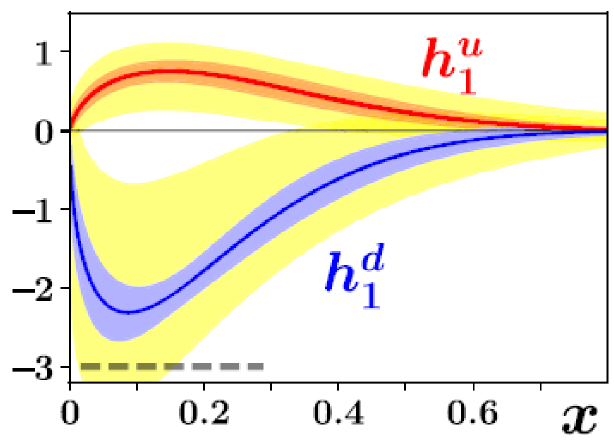
Semi-inclusive

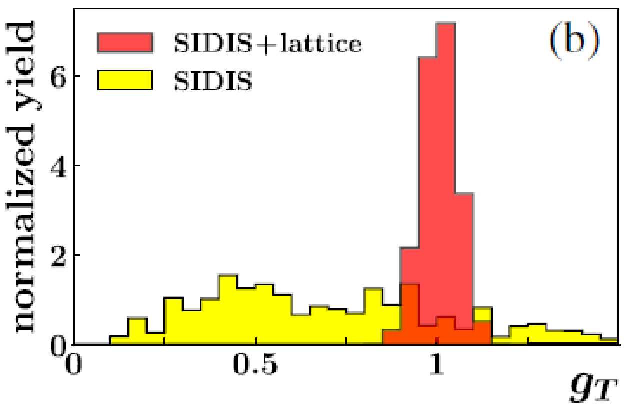


Have access to the chiral-odd distribution $h_1(x)$

It is naturally more difficult to have info on $h_1(x)$

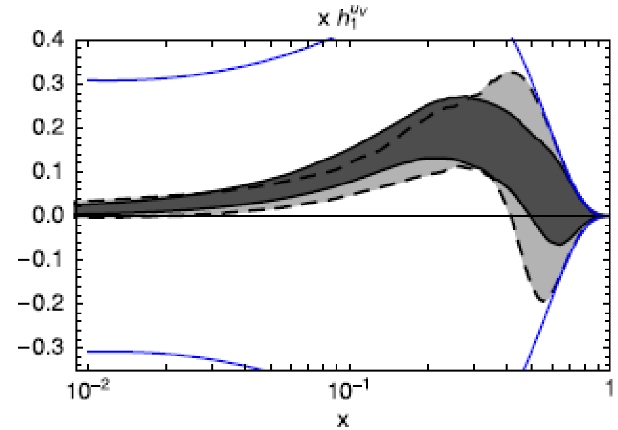
Two recent extractions

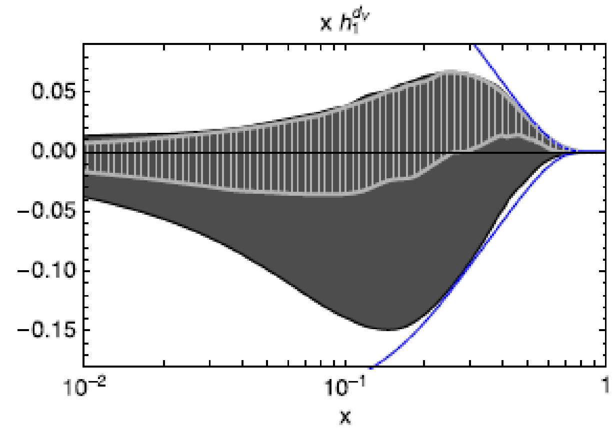




$$g_T = \int_0^1 dx\left(h_1^u(x) - h_1^d(x)\right) = 1.0(1)$$

H.-W. Lin PRL 120, 152502 (2018)





$$g_T = 0.53(25)$$

Radici and Bacchetta PRL 120, 192001 (2018)

Can we have a first principles calculation of $h_1^q(x)$?

Light-cone quark distributions

The most general form of the matrix element is:

$$\langle P|O^{\mu_1\mu_2\cdots\mu_n}|P\rangle = 2a_n^{(0)}\Pi^{\mu_1\mu_2\cdots\mu_n}$$

$$\Pi^{\mu_1\mu_2\cdots\mu_n} = \sum_{j=0}^{k}(-1)^j \frac{(2k-j)!}{2^j(2k)!}\{g\cdots gP\cdots P\}_{k,j}(P^2)^j$$

We use the following four-vectors

$$P = (P_0,0,0,P_3) \quad \lambda = (1,0,0,-1)/\sqrt{2} \quad \longrightarrow \quad \boxed{\lambda\cdot P = (P_0+P_3)/\sqrt{2} = P_+}$$

$$\lambda_{\mu_1}\lambda_{\mu_2}\left\langle P\middle|O^{\mu_1\,\mu_2}\middle|P\right\rangle = 2a_n^{(0)}\left(P^+P^+ - \lambda^2\frac{M^2}{4}\right) = 2a_n^{(0)}P^+P^+$$

In general, we have

$$\lambda_{\mu_1}\cdots\lambda_{\mu_n}\Pi^{\mu_1\cdots\mu_n} = (P^+)^n \quad \longrightarrow \quad \boxed{\langle P|O^{+\cdots+}|P\rangle = 2a_n^{(0)}(P^+)^n}$$

Matrix elements projected on the light-cone are protected from target mass corrections

Taking the inverse Mellin transform

$$a_n^{(0)} = \int dx\, x^{n-1} q(x) \qquad\qquad q(x) = \frac{1}{2\pi i}\int_{-i\infty}^{+i\infty} dn\, x^{-n} a_n^{(0)}$$

Using $a_n^{(0)} = \langle P|O^{+\cdots+}|P\rangle / 2(P^+)^n$

$$q(x) = \int_{-\infty}^{+\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle P|\bar{\psi}(\xi^-)\gamma^+ W(\xi^-,0)\psi(0)|P\rangle$$

$$W(\xi^-,0) = e^{-ig\int_0^{\xi^-} A^+(\eta^-)d\eta^-} \quad \text{(Wilson line)}$$

- Light cone correlations
- Equivalent to the distributions in the Infinite Momentum Frame
- Light cone dominated $\xi^2 = t^2 - z^2 \sim 0$
- Not calculable on Euclidian lattice $t^2 + z^2 \sim 0$

Quasi Distributions

X. Ji, "Parton Physics on a Euclidean Lattice," PRL 110 (2013) 262002.

Suppose we project outside the light-cone:

$$\lambda = (0,0,0,-1) \qquad P = (P_0, 0, 0, P_3) \qquad \lambda \cdot P = P_3$$

For example, for n=2

= -1

$$\langle P|O^{33}|P\rangle = 2\tilde{a}_n^{(0)}(P^3P^3 - \lambda^2 P^2/4) = 2\tilde{a}_n^{(0)}((P^3)^2 + P^2/4)$$

Mass terms contribute

After the inverse Mellin transform,

$$\tilde{q}(x, P_3) = \int_{-\infty}^{+\infty} \frac{dz}{4\pi} e^{-izxP_3} \langle P|\bar{\psi}(z)\gamma^3 W(z,0)\psi(0)|P\rangle + \mathcal{O}\left(\frac{M^2}{P_3^2}, \frac{\Lambda_{QCD}^2}{P_3^2}\right)$$

Higher twist

- Nucleon moving with finite momentum in the z direction
- Pure spatial correlation
- Can be simulated on a lattice

The light cone distributions:

$$x = \frac{k^+}{P^+}$$

$$0 \leq x \leq 1$$

Distributions can be defined in the infinite momentum frame: $P_3, P^+ \to \infty$

Quasi distributions:

P_3 large but finite

Usual partonic interpretation is lost

$x < 0$ or $x > 1$ is possible

But they can be related to each other!

Extracting quark distributions from quark quasi-distributions

Infrared region untouched when going from finite to infinite momentum

Infinite momentum: $p_3 \to \infty$ (before integrating over the quark transverse momentum k_T)

$$q(x,\mu) = q_{bare}(x)\left\{1 + \frac{\alpha_s}{2\pi} Z_F(\mu)\right\} + \frac{\alpha_s}{2\pi}\int_x^1 \Gamma\left(\frac{x}{y},\mu\right) q_{bare}(y)\frac{dy}{y} + \mathcal{O}(\alpha_s^2)$$

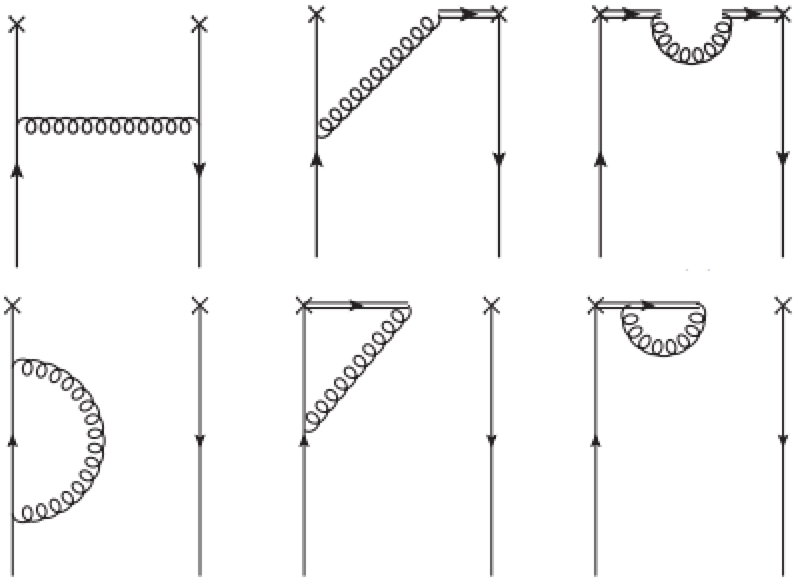
Finite momentum: p_3 fixed

$$\tilde{q}(x,P_3) = q_{bare}(x)\left\{1 + \frac{\alpha_s}{2\pi} \tilde{Z}_F\,(P_3)\right\} + \frac{\alpha_s}{2\pi}\int_{x/y_c}^1 \tilde{\Gamma}\left(\frac{x}{y},P_3\right) q_{bare}(y)\frac{dy}{y} + \mathcal{O}(\alpha_s^2)$$

$$\tilde{q}(\pm y_c) = 0$$

In principle, $y_c \to \infty$

One-loop correction using pQCD



Vertex: Γ or $\tilde{\Gamma}$

Self-energy: Z_F **or** $\tilde{Z}_F$

For the transversity operator:

X. Xiong, X. Ji, J. H. Zhang and Y. Zhao, PRD 90 014051 (2014), in a cut-off scheme

C. Alexandrou et al., 1807.00232, in the $\overline{MS}$ scheme

The two equations can be solved for the quark distributions, resulting in a matching equation:

$$q(x,\mu) = \tilde{q}(x,p_3) - \frac{\alpha_s}{2\pi}\tilde{q}(x,p_3)\delta Z_F\left(\frac{\mu}{p_3},x_c\right) - \frac{\alpha_s}{2\pi}\int_{-x_c}^{-|x|/y_c}\delta\Gamma\left(y,\frac{\mu}{p_3}\right)\tilde{q}\left(\frac{x}{y},p_3\right)\frac{dy}{|y|} - \frac{\alpha_s}{2\pi}\int_{+|x|/y_c}^{+x_c}\delta\Gamma\left(y,\frac{\mu}{p_3}\right)\tilde{q}\left(\frac{x}{y},p_3\right)\frac{dy}{|y|}$$

Matching equation

Where $\delta\Gamma = \tilde{\Gamma} - \Gamma$

$\delta Z_F = \tilde{Z}_F - Z_F$ are calculated using perturbation theory in the continuum

The integral in x in the quasi-quark self-energy, $\tilde{Z}_F$, is left unintegrated, hence the dependence on the limits of integration, $\pm x_c$. At the end, $x_c \to \infty$ in δZ_F.

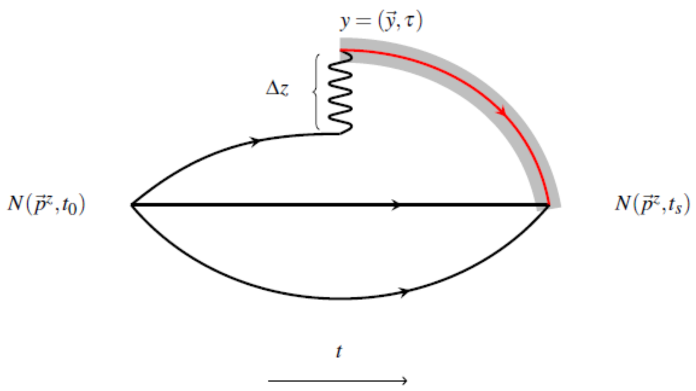
Because quasi-quark vertex correction, $\tilde{\Gamma}$, only vanishes at the infinity, the range of integration in the vertex also extends to zero in the convolution as $y_c \to \infty$

Main steps of the procedure:

1. Compute the matrix elements between proton states with finite P_3;
2. Non-perturbative renormalization of the matrix elements;
3. Fourier transform to obtain the quasi-PDF $\tilde{q}(x, P_3, \mu)$;
4. Matching procedure to obtain the light-cone PDF $q(x, \mu)$;
5. Apply Target Mass Corrections (TMCs) to correct for the powers of M^2/P_3^2 .

Computation of matrix elements using the lattice QCD

$$\frac{C^{3pt}(T_s,\tau,0;P_3)}{C^{2pt}(T_s,0;P_3)} \propto M_{h_1}(P_3,z), \qquad 0 \ll \tau \ll T_s$$



With the 3 point function given by:

$$C^{3pt}(t,\tau,0) = \langle N_\alpha(\vec{P},t)\mathcal{O}(\tau)\overline{N_\alpha}(\vec{P},0)\rangle$$

And

$$\mathcal{O}(z,\tau,Q^2=0) = \sum_{\vec{y}} \bar{\psi}(y+z)\,\sigma_{3i}\,W\,(y+z,y)\psi(y)$$

Where the matrix elements (ME) are: $M_{h_1}(P_3,z) = \langle P|\bar{\psi}(z)\sigma_{3i}\,W\,(z,0)\psi(0)|P\rangle$

Setup: $N_f = 2,\qquad \beta = \frac{6}{g_0^2} = 2.10,\qquad a = 0.0938(3)(2)\,fm$

$48^3 \times 96,\qquad L = 4.5fm,\qquad m_\pi = 0.1304(4)\ GeV,\qquad m_\pi L = 2.98(1)$

$$P_3 = \frac{6\pi}{L},\frac{8\pi}{L},\frac{10\pi}{L} = 0.84, 1.11, 1.38\ \text{GeV}$$

Computation made for the transversity (σ_{3i}) distributions

6 directions of Wilson line: $\pm x, \pm y, \pm z$

16 source positions

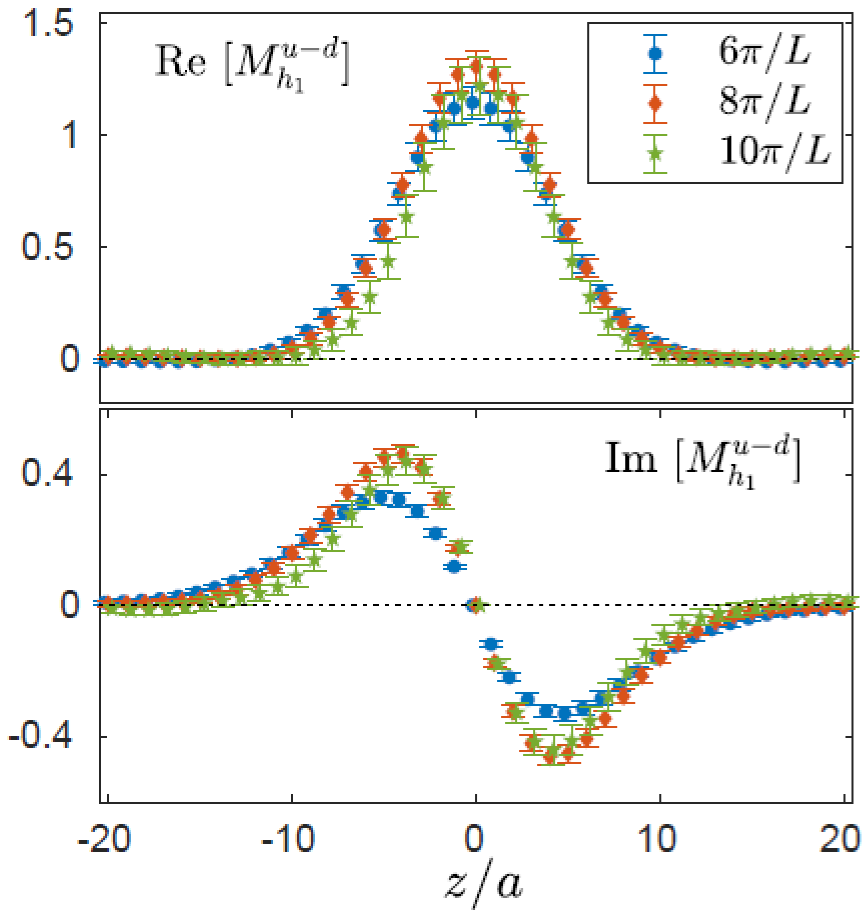
Separation $T_s \approx 1.1$ fm as the lowest safe choice

	$\frac{6\pi}{L}$ (0.83GeV)	$\frac{8\pi}{L}$ (1.11GeV)	$\frac{10\pi}{L}$ (1.38GeV)
N_{conf}	100	425	811
$N_{meas.}$	9600	38250	72990

Effectively, we compute the following matrix elements

$$M_{h_1}^{u-d} = \left\langle P \middle| \bar{\psi}(z) \frac{\sigma_{31} + \sigma_{32}}{2} W(z,0) \tau^3 \psi(0) \middle| P \right\rangle$$

C. Alexandrou et al., 1807.00232



Real part

Imaginary part

The bare matrix elements $\left\langle P \middle| \bar{\psi}(z) \frac{\sigma_{31}+\sigma_{32}}{2} W(z,0) \tau^3 \psi(0) \middle| P \right\rangle$, however, contain divergences:

Renormalization is necessary!

Renormalization

$$M_{h_1}^{R,u-d} = Z_{h_1} M_{h_1}^{u-d} = \left(Re\left[Z_{h_1}\right] + i\, Im\left[Z_{h_1}\right]\right)\left(Re\left[M_{h_1}^{u-d}\right] + i\, Im\left[M_{h_1}^{u-d}\right]\right)$$

Z_{h_1} renormalizes both the usual log divergence
and the linear divergence associated with the Wilson line

Nonperturbative renormalization using the RI'-MOM to remove both divergences

C. Alexandrou et al., NPB 923 (2017) 394 (Frontier Article)
J-W. Chen et al., PRD 97 014505 (2018)
C. Alexandrou et al., 1807.00232

Convert the ME from RI'-MOM to $\overline{MS}$ using 1-loop perturbation theory

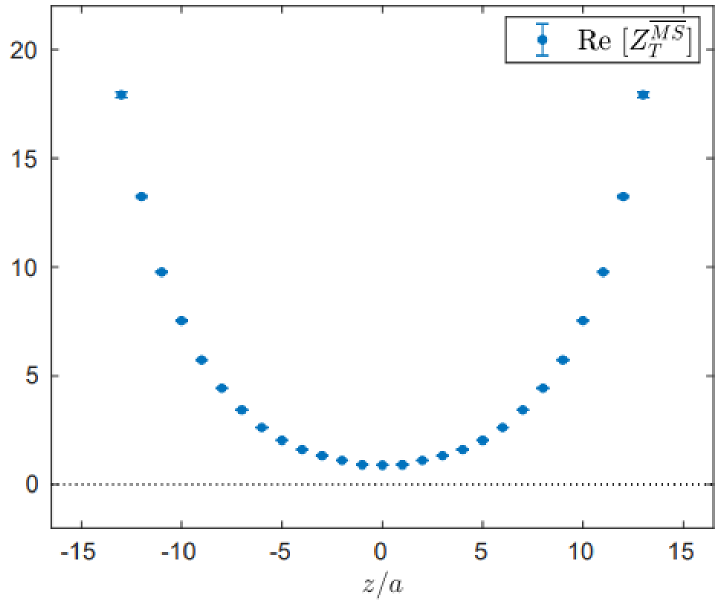
M. Constantinou, H. Panapaulos, PRD (2017)054506

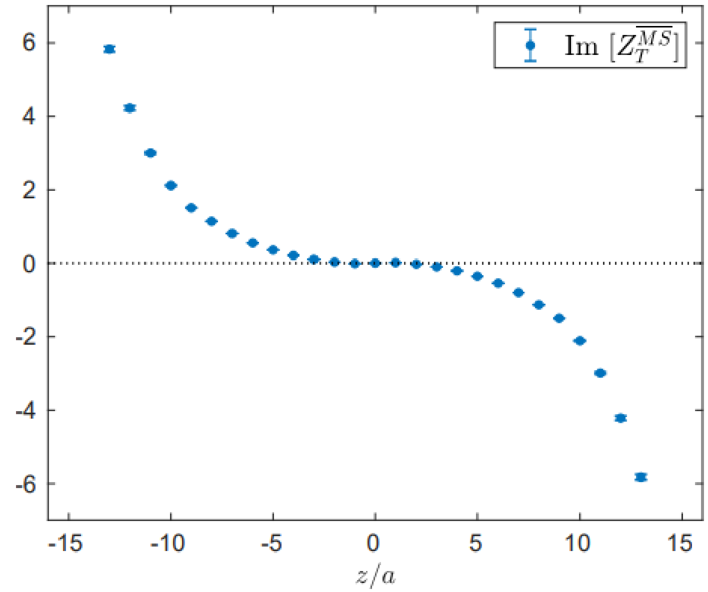
We present results for the $\overline{MS}$ scheme

Renormalization factor for transversity

RI'-MOM scheme at the scale $\bar{\mu}_0 = 3$ GeV

Perturbative conversion to $\overline{MS}$ scheme at the scale $\sqrt{2}$ GeV

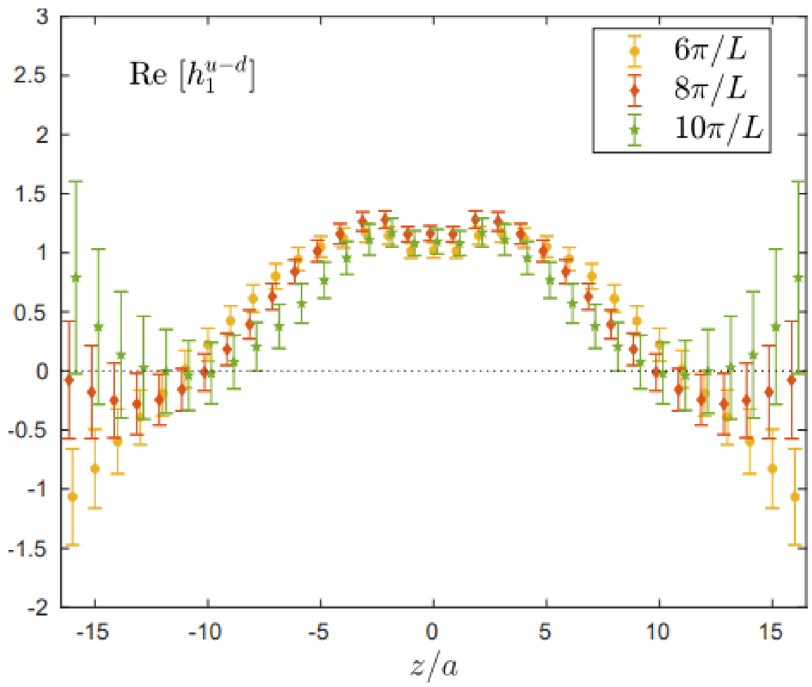




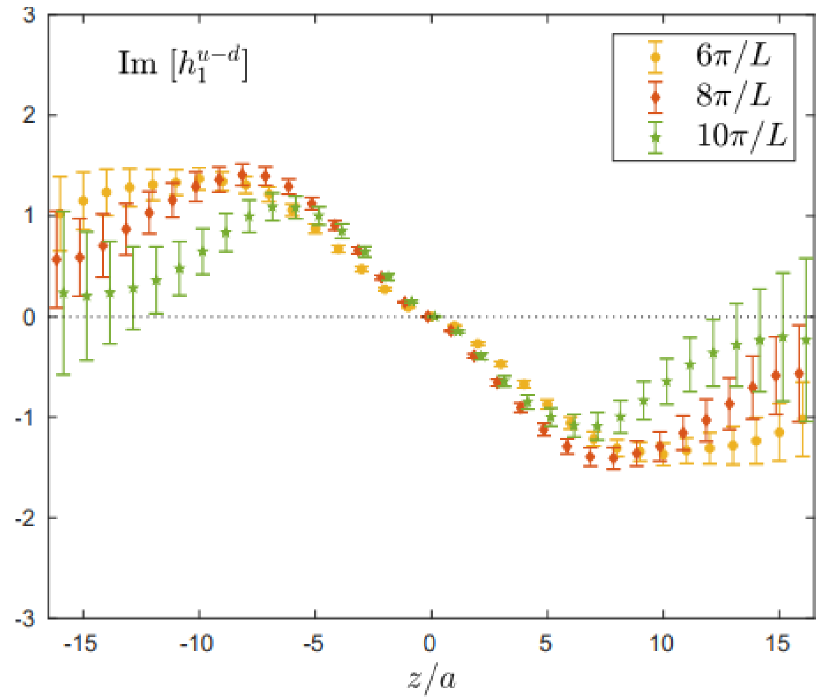
The linear divergence associated with the Wilson line makes Z_{h_1} to grow very fast for large z;

That makes the renormalized ME to have amplified errors at large z;

Renormalized ME for the transversity



$Re[M_{h_1}^{u-d}]\ Re[Z_{h_1}] - Im[M_{h_1}^{u-d}]\ Im[Z_{h_1}]$



$Re[M_{h_1}^{u-d}]\ Im[Z_{h_1}] + Im[M_{h_1}^{u-d}]\ Re[Z_{h_1}]$

The x dependence of the distributions

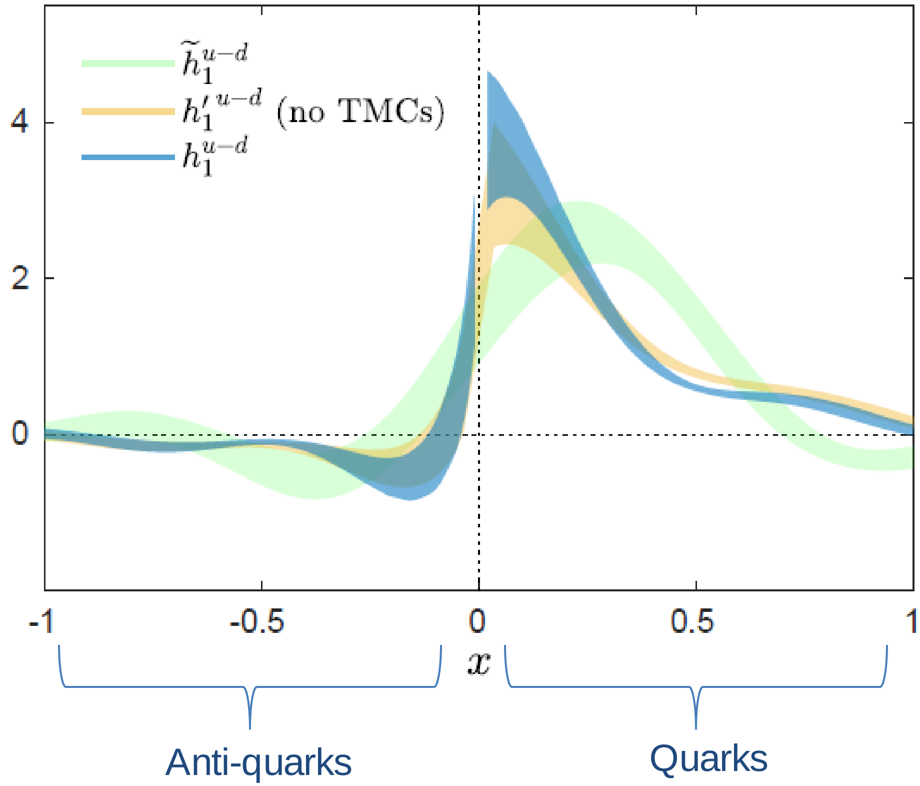
Once we have the ME, we compute the qPDF:

$$\tilde{h}_1(x,\mu^2,P_3) = \int \frac{dz}{4\pi} e^{-ixP_3 z}\langle P|\bar{\psi}(z)\sigma_{3i}W(z,0)\psi(0)|P\rangle$$

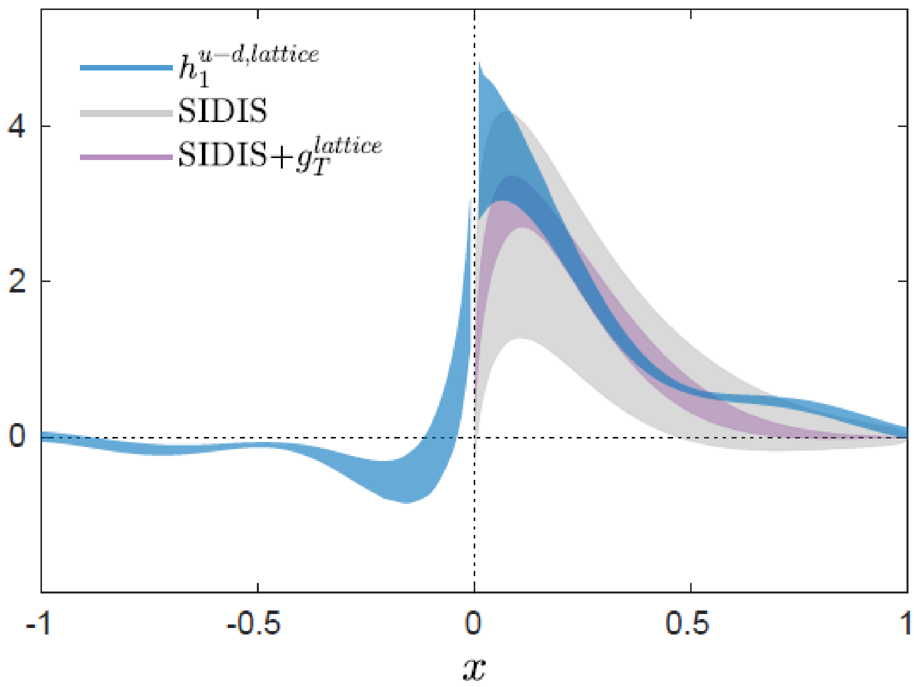
And then apply the matching plus target mass corrections to obtain the light-cone PDF:

$$h_1(x,\mu) = \int_{-\infty}^{+\infty} \frac{d\xi}{|\xi|}\delta C\left(\xi, \frac{\xi\mu}{xP_3}\right)\tilde{h}_1\left(\frac{x}{\xi},\mu,P_3\right)$$

C. Alexandrou et al., 1803.02685



Comparing with phenomenology:



$$g_T = \int_{-1}^{+1} dx h_1^{u-d} = 1.10(34)$$

This should be compared to: $g_T = 1.06(1)$ from dedicated lattice QCD calculation

C. Alexandrou et al., PRD95, 114514 (2017)

$g_T = 1.0(1)$ from Monte Carlo global analysis

H.-W. Lin PRL 120, 152502 (2018)

$g_T = 0.53(25)$ from global analysis of ep and pp data

Radici and Bacchetta PRL 120, 192001 (2018)

Summary

We have shown an *ab initio* computation of the x dependence of the iso-vector transversity PDF at the physical point;

Excited state contamination suppressed for the source-sink separation ($T_s \approx 1.12$ fm) and momentum $P_3 = 1.38$ GeV used here;

Enormous progress over the last couple of years:

- a complete non-perturbative prescription for the ME has emerged;
- a perturbative conversion from RI'-MOM and $\overline{MS}$ has been developed;
- the matching equations relating the quasi-PDFs to the light-cone PDFs for the transversity have been developed.

New computer architectures can lead to higher values of P_3;

Discretization effects need to be addressed: at least 3 different lattice spacings needed;

Volume effects also need to be addressed;

We have ahead of us the exciting possibility to calculate PDFs in general from first principles.

And the final matching can be written taking $x_c \to \infty$

$$\delta\Gamma^R\left(y,\frac{\mu}{p_3}\right) = \begin{cases} -\frac{2y}{1-y} ln\frac{y-1}{y} + \frac{2}{y} & y > 1 \\ -\frac{2y}{1-y} ln\frac{y^2\mu^2}{4p_3^2 y(1-y)} - \frac{2y}{1-y} & 0 < y < 1 \\ -\frac{2y}{1-y} ln\frac{y}{y-1} + \frac{2}{1-y} & y < 0 \end{cases}$$

$$\delta Z_F^R\left(\frac{\mu}{p_3}\right) = \begin{cases} \int_{-\infty}^{+\infty} d\eta \left(\frac{2\eta}{1-\eta} ln\frac{\eta-1}{\eta} - \frac{2}{\eta}\right) & \eta > 1 \\ \int_{-\infty}^{+\infty} d\eta \left(\frac{1+\eta^2}{1-\eta} ln\frac{y^2\mu^2}{4p_3^2 \eta(1-\eta)} + \frac{2\eta}{1-\eta}\right) & 0 < \eta < 1 \\ \int_{-\infty}^{+\infty} d\eta \left(\frac{2\eta}{1-\eta} ln\frac{\eta}{\eta-1} - \frac{2}{1-\eta}\right) & \eta < 0 \end{cases}$$

Regions outside the physical region, $0 < x < 1$, are not equal to zero!

Infrared divergences are the same in the quasi and light-cone PDFs: same splitting functions;

Automatically preserves quark number in all stages of the computation;