# SPIN DEPENDENT GLUON DISTRIBUTIONS: THEIR MEASUREMENT IN HEAVY QUARK PRODUCTION PROCESSES

**Gary R. Goldstein**

**Tufts Univeristy**

**SPIN 2018 - Ferrara**

- **ABSTRACT**
- Gluon pdf's, GPD's and TMD's play a significant role in an array of scattering processes, including SIDIS, DVCS, exclusive meson electroproduction and p p scattering.
- Spin dependent gluon distributions can lead to distinctive features in the angular dependences and asymmetries of the scattering processes. Of particular interest are heavy quark production processes, wherein spin observables of the heavy quarks adumbrate the underlying gluon spin dependences. Top pair production at LHC is a prime example that proceeds primarily via gluon fusion. Decays of polarized top pairs through various channels produce a variety of correlations among the decay products - particles and jets. Combinations of the gluon distributions, either polarized or unpolarized, can be accessed experimentally through angular dependences of decay products, as will be shown, along with predictions from a "flexible" spectator model of gluon distributions

Gluon Distributions

Transversity

Top quarks

Gluon Transversity → Top Pair Spin Correlations

# OUTLINE – top quarks to gluons

- GPD's & TMD's in Models– e.g. Reggeized spectator "flexible parameterization"
- electroproduction
    - Gluon GPDs Polarized Gluons?
      Transversity NOW Seen!
- t+t-bar production & decay to measure Gluon polarization in p+p @ LHC. Inclusive → TMDs
- Top spin correlations & Observable quantities

# Gluon GPDs

$$\frac{1}{\bar{P}^+}\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-} \langle P',\Lambda'|G^{+i}(-\frac{1}{2}z)G^{+i}(\frac{1}{2}z)|P,\Lambda\rangle\Big|_{z^+=0,\vec{z}_T=0} =$$
$$\frac{1}{2\bar{P}^+}\bar{U}(P',\Lambda')[H^g(x,\xi,t)\gamma^+ + E^g(x,\xi,t)\frac{i\sigma^{+\alpha}(-\Delta_\alpha)}{2M}]U(P,\Lambda)$$

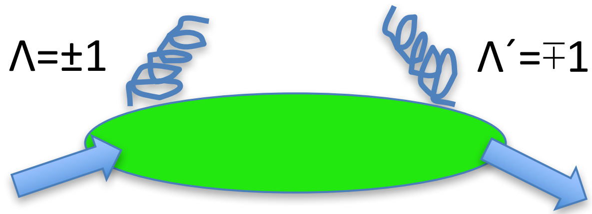
Even t-channel parity & Gluon helicity conserving

$$\frac{-i}{\bar{P}^+}\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+z^-} \langle P',\Lambda'|G^{+i}(-\frac{1}{2}z)\tilde{G}^{+i}(\frac{1}{2}z)|P,\Lambda\rangle\Big|_{z^+=0,\vec{z}_T=0} =$$
$$\frac{1}{2\bar{P}^+}\bar{U}(P',\Lambda')[\tilde{H}^g(x,\xi,t)\gamma^+\gamma_5 + \tilde{E}^g(x,\xi,t)\frac{\gamma_5(-\Delta^+)}{2M}]U(P,\Lambda)$$

Odd t-channel parity & Gluon helicity conserving

Must have 4 more Gluon helicity NONconserving

# Extension to Gluon "Transversity"




$$
\begin{aligned}
-\frac{1}{P^+}\int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle p',\lambda'|\, \mathbf{S} F^{+i}(-\tfrac{1}{2}z)\, F^{+j}(\tfrac{1}{2}z)\, |p,\lambda\rangle\Big|_{z^+=0,\,\mathbf{z}_T=0} \\
= \mathbf{S}\, \frac{1}{2P^+}\, \frac{P^+\Delta^j - \Delta^+ P^j}{2mP^+} \\
\times \bar{u}(p',\lambda') \Big[ H_T^g\, i\sigma^{+i} + \tilde{H}_T^g\, \frac{P^+\Delta^i - \Delta^+ P^i}{m^2} \\
+ E_T^g\, \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \tilde{E}_T^g\, \frac{\gamma^+ P^i - P^+\gamma^i}{m} \Big] u(p,\lambda).
\end{aligned}
$$

4 GPDs: see M.Diehl, EPJC19, 485 (2001)

4 Gluon helicity NONconserving Double flip

# Gluon "transversity" Double helicity flip *does not mix* with quark distributions

**Transversity for on-shell gluons or photons : no |0> helicity**

$$|+1)_{trans} = \{|+1> + |-1>\}/2 = |-1)_{trans}$$

$$|0\ )_{trans} = \{|+1> - |-1>\}/\sqrt{2}$$

$$\text{helicity } |\pm 1> = \{-/+\hat{x} - i\hat{y}\}/\sqrt{2}$$

$$\hat{x} = -|0)_{trans} = P_{parallel}$$ **Linear polarization in the plane**

$$\hat{y} = i\sqrt{2}\,|+1)_{trans} = P_{normal}$$ **Linear polarization normal to the plane**

GG&M.J.Moravcsik, Ann.Phys.195,213(1989).

Using Reggeized Spectators Model
Many other models & recently

How to Measure? What Processes? Long standing question.

M. Diehl, T. Gousset, B. Pire, and J. P. Ralston, Phys. Lett. B411, 193 (1997).
X. Ji and J. Osborne, UMD PP#98-074, hep-ph/9801260.
P. Kroll, M. Schurmann, and P. A. M. Guichon, Nucl. Phys. A598, 435 (1996).
P. Hoodbhoy & X. Ji, PRD58, 054006 (1998).

TMDs

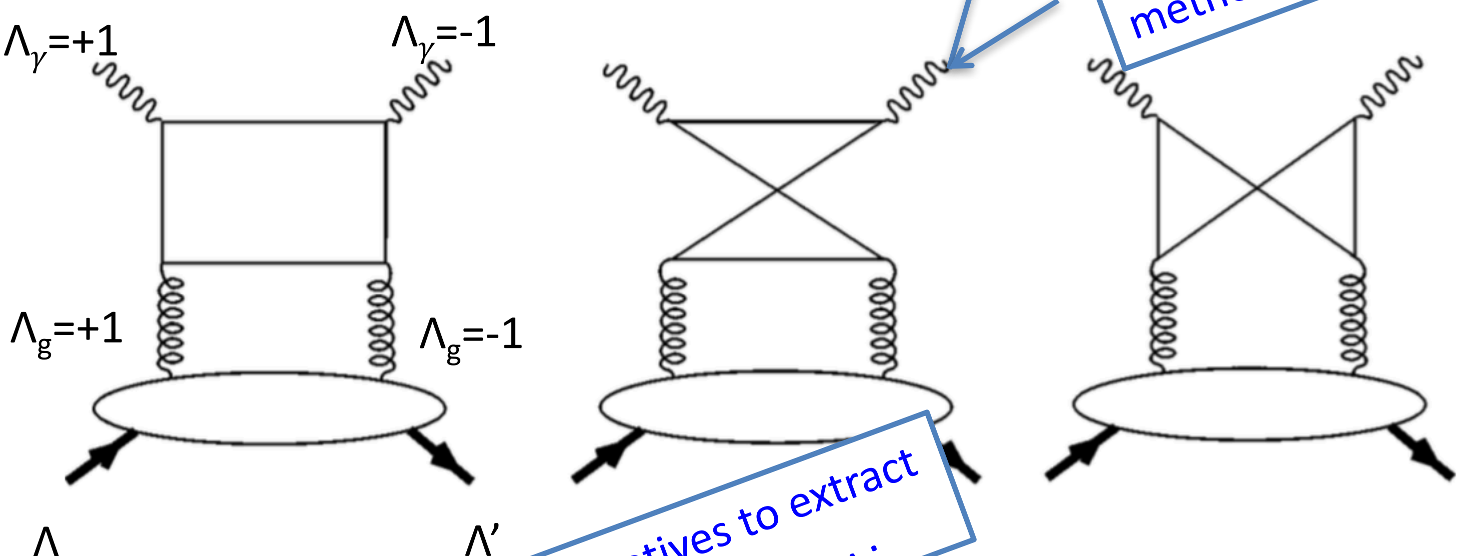
P. Mulders, J. Rodrigues,PRD 63, 094021 (2001).
D. Boer, Few-Body Syst. (2017); C. Pisano, et al., JHEP 10, 024 (2013);
D. Boer, et al., PRL 106, 132001 (2011); . . . .

# *Helicity flip $A_{\Lambda',-1;\Lambda,+1}$* contributes to DVCS ~ $\alpha_S$

$$M_{\Lambda',\Lambda'\gamma=-1;\Lambda,\Lambda\gamma=+1} = -\frac{\alpha s}{2\pi}\sum_q e_q^2 \int_{-1}^{+1} dx \frac{A_{\Lambda',\Lambda' g=-1;\Lambda,\Lambda g=+1}(x,\xi,t)}{(\xi - x - i\varepsilon)(\xi + x - i\varepsilon)} \; C'(x,\xi,Q^2)$$

Interference with Bethe-Heitler contains $cos 3\varphi$ modulation
to distinguish from (leading twist) quark contribution

$\mu^- \mu^+$ alternative method to extract




other alternatives to extract
J/ψ production; jets . . . .

See Hoodbhoy & Ji, PRD58, 054006 (1998)

# Gluon GPDs from DVCS – Hall A

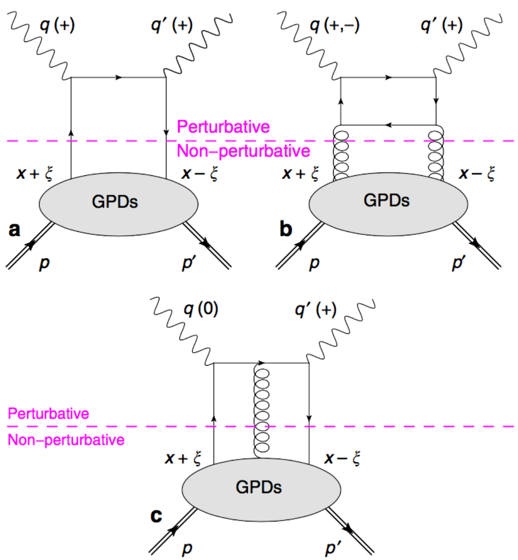
**Polarized** & unpolarized beam measurements

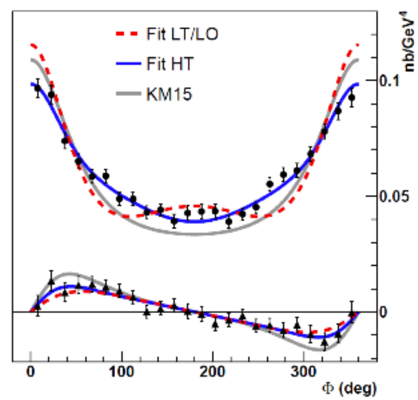
$$\frac{d^4\sigma(h)}{dQ^2 dx_B dt d\phi} = \frac{d^2\sigma_0}{dQ^2 dx_B} \times \left[ \left|\mathcal{T}^{BH}\right|^2 + \left|\mathcal{T}^{DVCS}(h)\right|^2 - \mathcal{I}(h) \right]$$

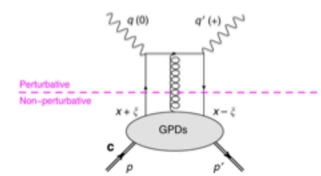
## Evidence of gluon transversity

Fitting ϕ distribution requires $F_{++}$ and both $F_{+-}$ gluon transversity and $F_{0+}$ higher twist

**A glimpse of gluons** through deeply virtual compton scattering on the proton, published in *Nature Communications* 8, 1408 (2017).
doi:10.1038/s41467-017-01819-3




*Analysis of 6 GeV Hall A DVCS data on the proton.*

Jefferson Lab

See Latifa Elouadrhiri talk at QCD Evolution 2018

LHC – many opportunities for studying gluons

p+p unpolarized → jets, hadrons, leptons

Interactions via g+g→ Q+Qbar +X

gluon TMDs in some kinematics

Extension to Gluon "Transversity"

c.f. TMDs $h_1^{\perp g}(x, p_T^2)$ Mulders & Rodrigues (2001), Gluon Boer-Mulders function
see D. Boer, Frascati talk (Nov.2016) & many references for measurements at EIC, RHIC, LHC

# Gluon TMDs

$$\begin{pmatrix} G+\Delta G_L & \frac{|\boldsymbol{k}_T|e^{-i\phi}}{M}[\Delta G_T - iG_T] & -e^{-2i\phi}[H^{\perp(1)}+i\Delta H_L^{\perp(1)}] & -i\frac{|\boldsymbol{k}_T|e^{-3i\phi}}{M}\boxed{\Delta H_T^{\perp(1)}} \\ \frac{|\boldsymbol{k}_T|e^{i\phi}}{M}[\Delta G_T + iG_T] & G-\Delta G_L & -i\frac{|\boldsymbol{k}_T|e^{-i\phi}}{M}\Delta H_T & -e^{-2i\phi}[H^{\perp(1)}-i\Delta H_L^{\perp(1)}] \\ -e^{2i\phi}[H^{\perp(1)}-i\Delta H_L^{\perp(1)}] & i\frac{|\boldsymbol{k}_T|e^{i\phi}}{M}\Delta H_T & G-\Delta G_L & -\frac{|\boldsymbol{k}_T|e^{-i\phi}}{M}[\Delta G_T + iG_T] \\ i\frac{|\boldsymbol{k}_T|e^{3i\phi}}{M}\boxed{\Delta H_T^{\perp(1)}} & -e^{2i\phi}[H^{\perp(1)}+i\Delta H_L^{\perp(1)}] & -\frac{|\boldsymbol{k}_T|e^{i\phi}}{M}[\Delta G_T - iG_T] & G+\Delta G_L \end{pmatrix}$$

Mulders & Rodrigues, PRD63, 94021 (2001)

The matrix representation is also convenient to find the physical meaning of the distributions. Well known is *G* which measures the number of gluons with momentum ($x$,$\boldsymbol{k}T$) in a hadron. The functions *GL* ( *GT*) represents the difference of the numbers of gluons with opposite circular polarizations in a longitudinally transversely polarized nucleon. The off- diagonal function *H* also is a difference of densities, but in this case of linearly polarized gluons in an unpolarized hadron. Using the circular polarizations, *H* flips the polarization.

Corresponding GTMDs generalize GPDs & TMDs.

Unintegrated **models** connect all

Other notation $\Delta H_T^{\perp(1)}$, $h_1^{g\perp}$ **gluon Boer-Mulders function**

Unpolarized Nucleon → polarized gluon | factorization & evolution

# Gluon TMDs

## TMD Color gauge invariance

*Small x gluons, Kharzeev, Kovchegov, Tuchin, saturation,*

*2 gluon distributions: WW vs. DP,*

*Saturation issues: McLerran-Venugopalan model, ColorGlassCondensate, . .?*

*See Mulders, et al.: small x DP is pure gauge link*

$$\Gamma^{\mu\nu\,[\mathcal{U},\mathcal{U}']}(x,\boldsymbol{k}_T) \equiv \int \frac{d(\xi\cdot P)\,d^2\xi_T}{(P\cdot n)^2(2\pi)^3}\,e^{i(xP+k_T)\cdot\xi}\left\langle P\left|\mathrm{Tr}_c\left[F^{n\nu}(0)\,\mathcal{U}_{[0,\xi]}\,F^{n\mu}(\xi)\,\mathcal{U}'_{[\xi,0]}\right]\right|P\right\rangle\Big|_{\xi\cdot n=0}.$$

Gauge link

$\xi=[0^+, \xi^-, \xi_T]$

$$\mathcal{U}_{\mathcal{C}}[0,\xi] = \mathcal{P}\exp\left(-ig\int_{\mathcal{C}[0,\xi]} ds_\mu\, A^\mu(s)\right)$$

Can be forward light front pointing link $+\infty$ FSI. Weizsacker-Williams

Or mixed light front pointing link $-\infty$ ISI Dipole

**For *U* and *U'* *have* [+ +] or [+ - ] (& parity opposites)**

## Consider TMD & GPD vs. data at intermediate x

The Model for valence quarks– Reggeized Diquarks

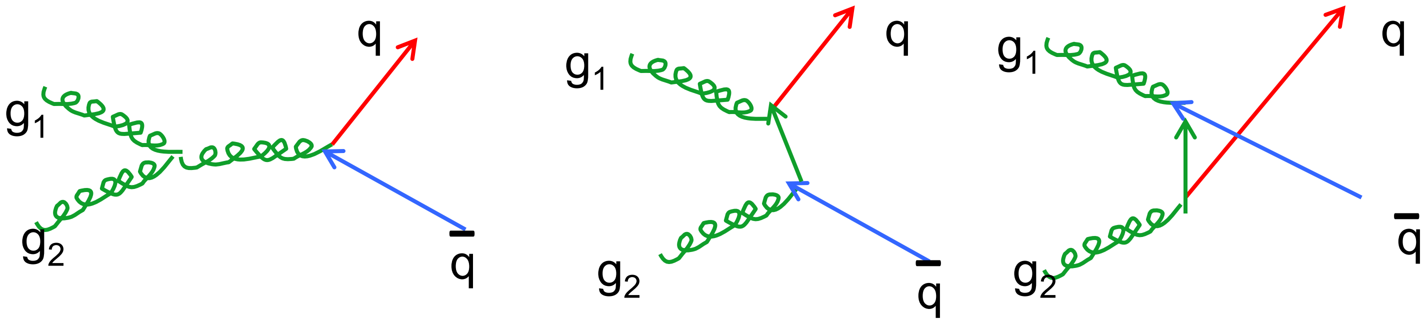
↓

The Model – first for Chiral Even – then Odd
Reggeized Diquark Spectator
Diquark: Color anti-3, scalar & axial vector

↓

Gluon GPDs, TMDs, GTMDs

# From p+p to gluon TMDs to quark pairs



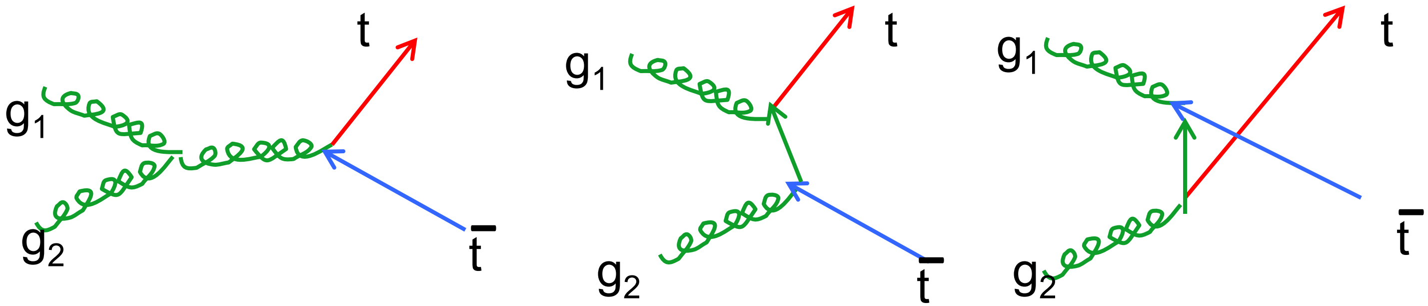

Form quarkonia & different possibilities for gg

Complications from f.s.i. & jets – hadronization

See Boer, Brodsky, Pisano, et al., . . .

Factorization and evolution

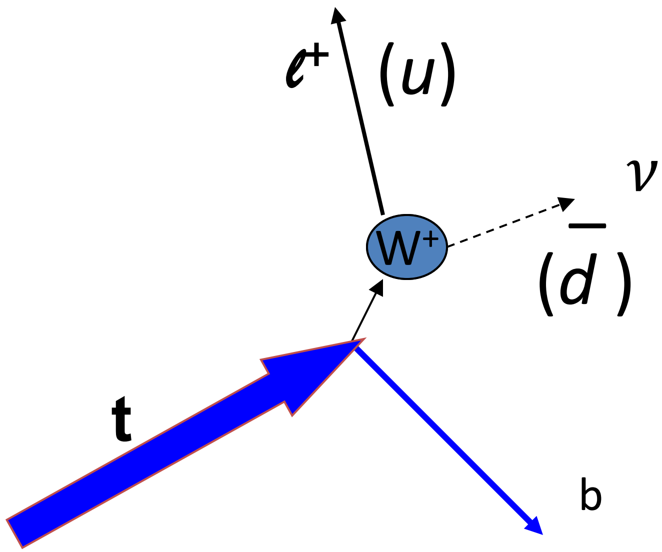
# For Gluon fusion top production at LHC




- $g_1$ & $g_2$ carry helicity $\Lambda_1\ \Lambda_2 = \pm 1$ & color 1, 8. . . & C=+ or -
- *t* & *t*-bar carry helicity $\lambda_t$ , $\lambda_{tbar} = \pm$ ½ & color 1 or 8
- *t* & *tbar decay before hadronizing* => no toponia & large scale

# How is top polarization determined?

## Its decay is good analyzer for transverse polarization.




$$U_{\lambda_t,\lambda_t'} = \sum_{\lambda_b} B^*_{\lambda_b,\lambda_t'} B_{\lambda_b,\lambda_t'}$$

$$\propto (I + \vec{p}_{\bar{l}} \cdot \vec{\sigma}_t / p_{\bar{l}})_{\lambda_t,\lambda_t'} (p_b \cdot p_\nu)$$

Calculated in top rest frame

OR

$$U = (p_t - m_t S_t) \cdot p_{\bar{l}} \, (p_b \cdot p_\nu)$$

$$S_t = \left[ \frac{\vec{p} \cdot \vec{P}_t}{m_t}, \vec{P}_t + \frac{(\vec{p} \cdot \vec{P}_t)\vec{P}_t}{m_t(E_t + m_t)} \right]$$

Covariant form in any frame

$P_t$ = strength of top polarization

Dalitz & GRG, PLB287,225(1992); PRD45, 1531(1992)

$(I + \vec{p}_{\bar{l}} \cdot \vec{\sigma}_t / p_{\bar{l}})$ lepton or u-quark moves parallel to transverse polarization

# What is known production of polarized tops?

## Top Single Spin Asymmetry and Double Spin Correlations – Measurements

**ATLAS** PRD93, 012002 (2016) & ref. PRL114, 142001 (2015)

** SSA: $B_1$ or $A_P$ =-0.035 +\- 0.040. (syst & stat)
*** Double: $C_{helicity}$ =0.315 +\- 0.07 vs. NLO QCD =0.31
(Bernreuther, et al., PRL 87,242002 (2001) QCD corrections but unpolarized gluons)

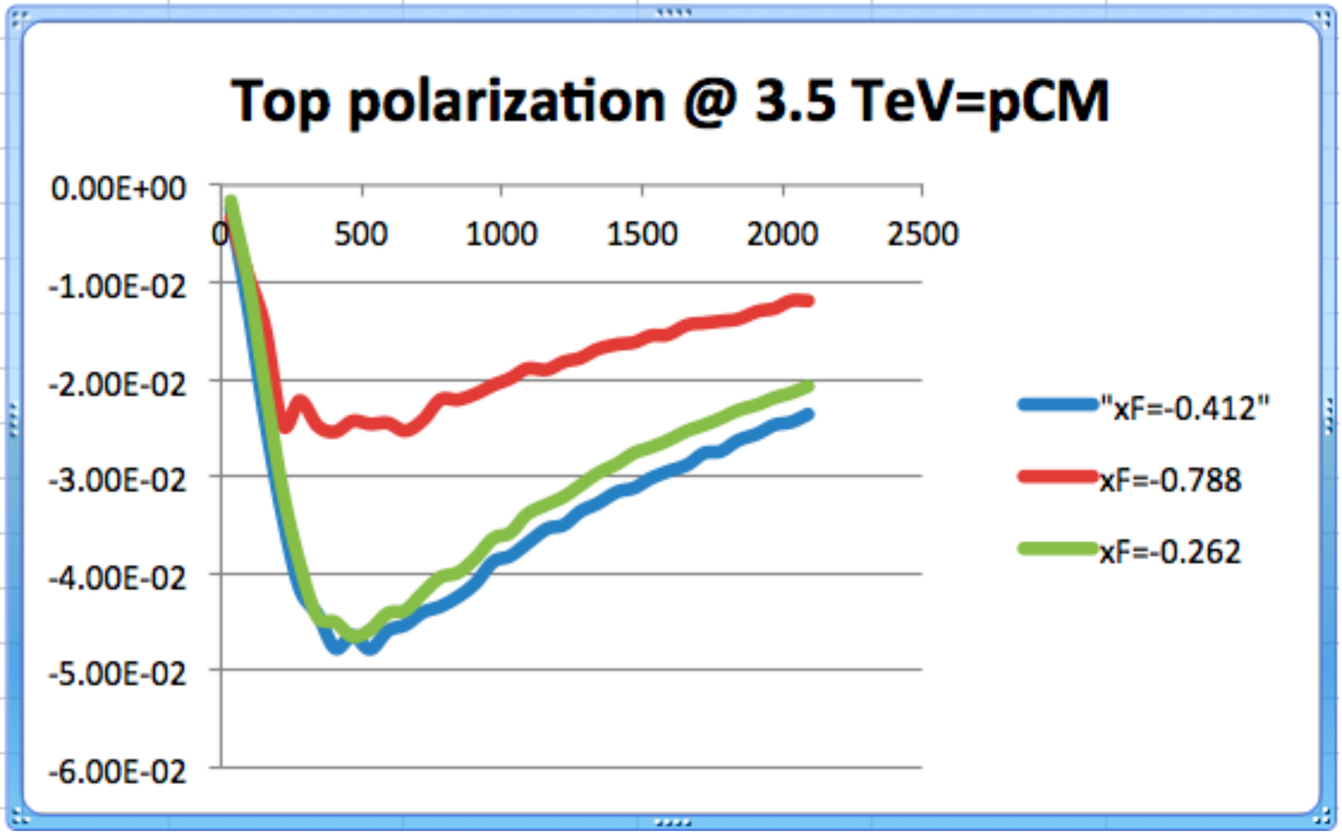
**CMS** PRL112, 182001 (2014): Different kinematics & selection criteria

** SSA: $A_P$=0.005+\- 0.01.
*** Double: $A_{\Delta\phi}=0.113$ +\- 0.01. vs. 0.110 +\- 0.001 (MC & QCD)
$A_{c1c2}$ = -0.021 +\- 0.03 vs -0.078 +\- 0.001

$$\frac{1}{\sigma}\frac{d^2\sigma}{d\cos\theta_1 d\cos\theta_2} = \frac{1}{4}(1 + B_1\cos\theta_1 + B_2\cos\theta_2 - C_{\mathrm{helicity}}\cos\theta_1\cdot\cos\theta_2)$$

$\theta_1$ $\theta_2$ decay product angles w.r.t. t+tbar CM

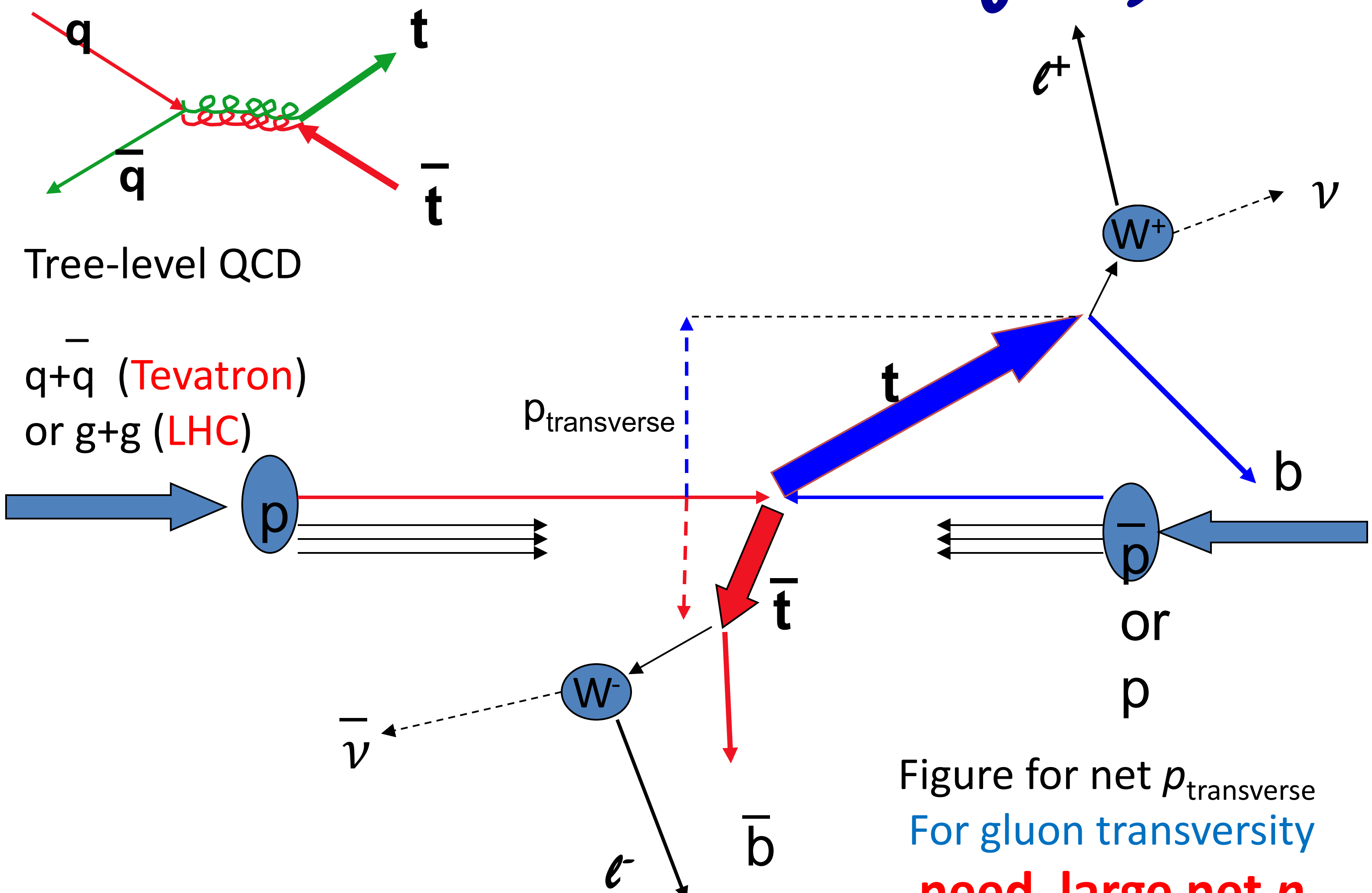
# Direct measure of hard process- top polarization
# Top decays weakly before hadronizing
# ⇒ decay "self-analyzing"




Analyze t--> $W^+$ b

Contributions to order $\alpha_S$
Imaginary Part (Dharmaratna & GRG 1990,1996)

# *Dilepton events or lepton+hadron jets or all Hadron jets)*




Tree-level QCD

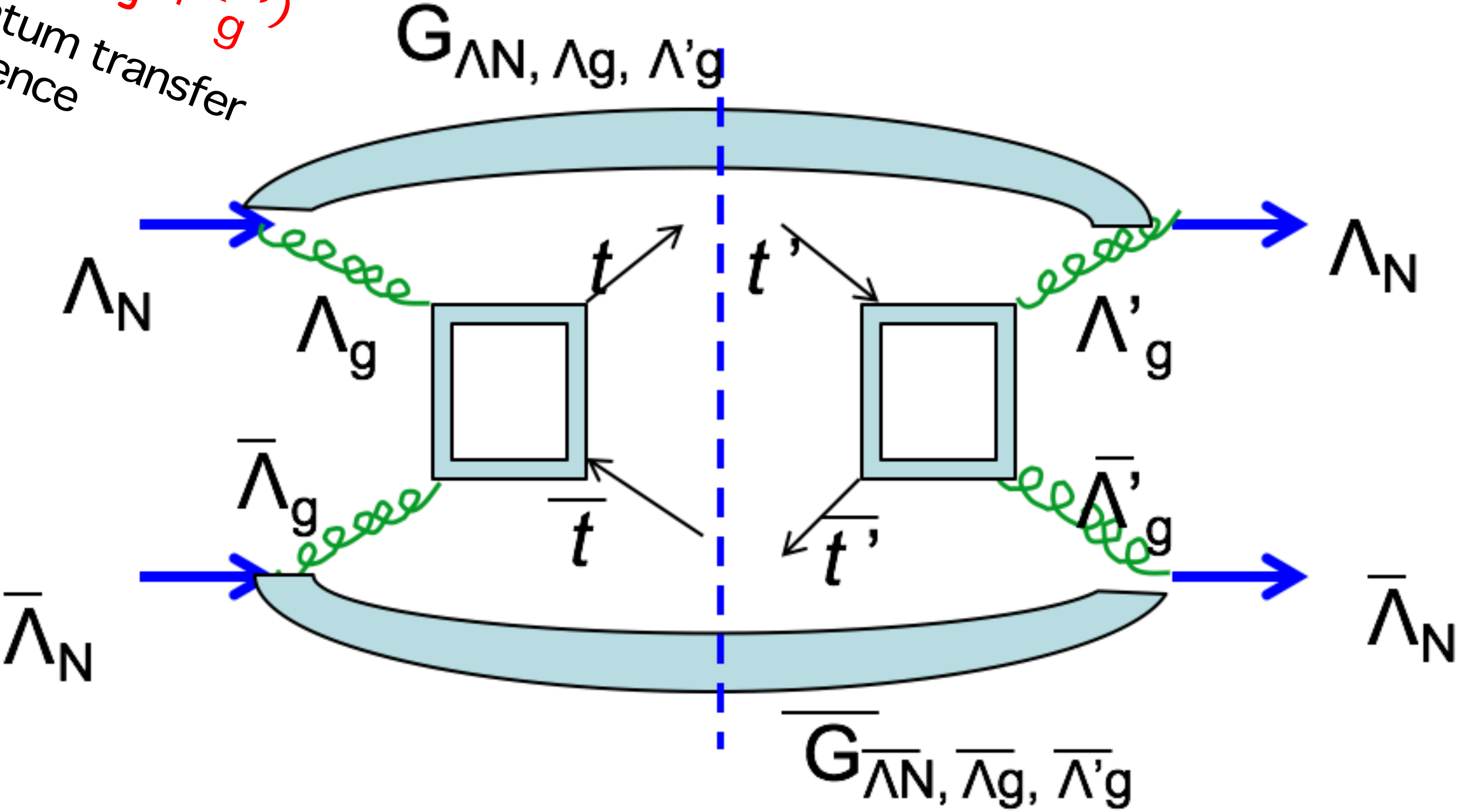
$q+\bar{q}$ (Tevatron)
or g+g (LHC)

Figure for net $p_{transverse}$
For gluon transversity
**need large net $p_{transverse}$ to access transversity**

# For inclusive p+p→t+tbar+X

$\Delta(\Lambda_N - \Lambda_{N'} - \Lambda_g + \Lambda_g')$

Momentum transfer dependence



# Gluon linear polarization with like and unlike t-tbar helicities

(work in progress S.Liuti, GRG, Gonzalez-Hernandez, Poage (thesis) )

## $F \sim G_{XX} + G_{YY}$ , $H \sim G_{XX} - G_{YY}$ or linear polarization

| $\rho_{t',\bar{t}';t,\bar{t}}$ | $\bar{F}\,F$ | $\bar{H}\,H$ | $\bar{F}\,H$ | $\bar{H}\,F$ |
|---|---|---|---|---|
| ++;++ | $\gamma^{-2}(1+\beta^2(1+\sin^4\theta))$ | $\gamma^{-2}(-1+\beta^2(1+\sin^4\theta))$ | $-2\dfrac{\beta^2}{\gamma^2}\sin^2\theta$ | $-2\dfrac{\beta^2}{\gamma^2}\sin^2\theta$ |
| +-;+- | $\beta^2\sin^2\theta(2-\sin^2\theta))$ | $-\beta^2\sin^4\theta$ | $0$ | $0$ |

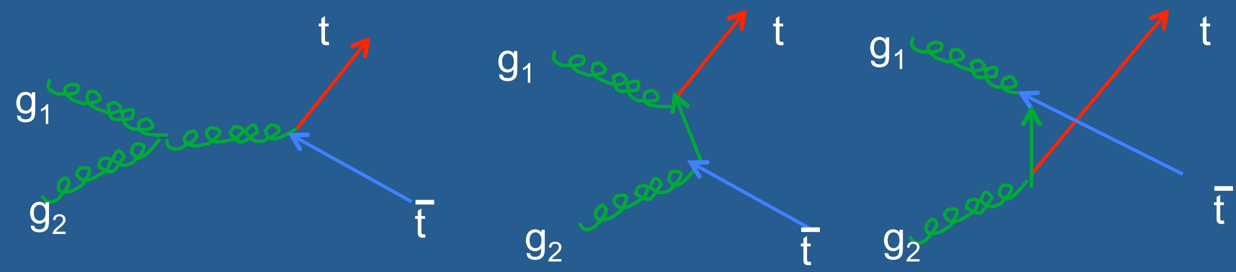
$$\rho_{t',\bar{t}';t,\bar{t}} \propto \sum_{all-helicities-not-tops} \bar{G}_{\bar{\Lambda}_N \bar{\Lambda}_g \bar{\Lambda}'_g} A^{*}_{\Lambda'_g \bar{\Lambda}'_g; t',\bar{t}'} A_{\Lambda_g \bar{\Lambda}_g; t,\bar{t}} G_{\Lambda_N \Lambda_g \Lambda'_g}$$

- The gluon spin correlations are transmitted to (determine the spin of) the decay products.
- The correlations between the lepton directions and the parent top spin (in the top rest frame) produce correlations between the lepton directions.
- The gluon fusion mechanism gives rise to a higher order (wrt quark antiquark) angular distribution due to the combination of two spin 1 gluons.

G.R.Goldstein,``Spin Correlations in Top Quark Production and the Top Quark Mass” in Proc. 12th Intl Symp. High Energy Spin Physics, Amsterdam, ed.C.W. deJager, et al., World Sci., Singapore (1997) p. 328

# At LHC:

Gluon fusion tree level mechanism
(Color gauge invariance)




$g_1$, $g_2$ carry helicity $\Lambda_1\ \Lambda_2 = \pm 1$ **OR transversity 1 or 0**

*t*, *t*-bar carry helicity $\lambda_t\ \lambda_{tbar} = \pm ½$ OR transversity $\pm 1/2$

Introduced in:

G.R.Goldstein,``Spin Correlations in Top Quark Production and the Top Quark Mass” in Proc. 12th Intl Symp. High Energy Spin Physics, Amsterdam, ed.C.W. deJager, et al., World Sci., Singapore (1997) p. 328.

R.H. Dalitz, G.R. Goldstein and R. Marshall, “Heavy Quark Spin Correlations in e+e- annihilations”, Phys. Lett. B215, 783 (1988);

R.H. Dalitz, G.R. Goldstein and R. Marshall, “On the Helicity of Charm Jets”, Zeits.f. Phys. C42, 441 (1989).

# $q$+$q$-bar → $t$ + $t$-bar
# Dilepton channel

- The light quark-antiquark annihilation mechanism gives rise to the **angular distribution between opposite charge lepton pairs**, more information than $C_{helicity}$ or $A_{c1\ c2}$

$$
\begin{aligned}
W(\theta,p,p_{\bar{l}},p_l) = \frac{1}{4}\{1 &+ [\sin^2\theta([p^2+m^2](\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{x}} + [p^2-m^2](\hat{p}_{\bar{l}})_y(\hat{p}_l)_{\bar{y}}) \\
&- 2mp\cos\theta\sin\theta((\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}} + (\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{x}}) + ([p^2-m^2] \\
&+[p^2+m^2]\cos^2\theta)(\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{z}}]/[(p^2+m^2)+(p^2-m^2)cos^2\theta]\} \\
= \frac{1}{4} + \frac{1}{4}\{&(2-\beta^2)\sin^2\theta(\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{x}} + \beta^2(\hat{p}_{\bar{l}})_y(\hat{p}_l)_{\bar{y}} \\
&+[\beta^2+(2-\beta^2)\cos^2\theta](\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{z}} \\
&-\frac{2}{\gamma}\cos\theta\sin\theta((\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}} + (\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{x}})\Big\}/[(2-\beta^2)+\beta^2\cos^2\theta]
\end{aligned}
$$

$m$ =top mass, $\theta$ = t production angle in q+q-bar CM
$p$= light quark 3-momentum in CM
Unit vectors $p$-hat are anti-lepton$^+$ and lepton$^-$ 3-momenta directions in the top and anti-top rest frames.

See G.R.Goldstein,``Spin Correlations in Top Quark Production and the Top Quark Mass'' in Proc. 12th Intl Symp. High Energy Spin Physics, Amsterdam, ed.C.W. deJager, et al., World Sci., Singapore (1997) p. 328.

# $g_1+g_2 \rightarrow t + t\text{-bar}$
# Spin correlations – dilepton channel

Correlations expressed as a weighting factor first ***for unpolarized gluons***.

- The gluon fusion mechanism gives rise to a higher order angular distribution ($\sin^4\theta$) due to the combination of two spin 1 gluons.

$$
\begin{aligned}
W(\theta, p, p_{\bar{l}}, p_l) = \frac{1}{4} - \frac{1}{4}\{&[p^4\sin^4\theta + m^4](\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{x}} + [p^2(p^2-2m^2)\sin^4\theta - m^4](\hat{p}_{\bar{l}})_y(\hat{p}_l)_{\bar{y}} \\
&+ [p^4\sin^4\theta - 2p^2(p^2-m^2)\sin^2\theta + m^2(2p^2-m^2)](\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{z}} \\
&+ 2mp^2\sqrt{p^2-m^2}\cos\theta\sin^3\theta[(\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}} - (\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{x}}]\} \\
&/ \left[p^2(2m^2-p^2)\sin^4\theta + 2p^2(p^2-m^2)\sin^2\theta + m^2(2p^2-m^2)\right] \qquad (20)
\end{aligned}
$$

$$
\begin{aligned}
= \frac{1}{4} - \frac{1}{4}\{&[(1-\beta^2)^2 + \sin^4\theta)](\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{x}} \\
&+[-(1-\beta^2)^2 - (1-2\beta^2)\sin^4\theta](\hat{p}_{\bar{l}})_y(\hat{p}_l)_{\bar{y}} \\
&+[(1-\beta^4) - 2\beta^2\sin^2\theta + \sin^4\theta](\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{z}} \\
&+2\frac{\beta}{\gamma}\sin^3\theta\cos\theta[(\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}} - (\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{x}}]\Big\} \\
&/ \left[(1-\beta^4) + 2\beta^2\sin^2\theta + (1-2\beta^2)\sin^4\theta\right] \qquad (21)
\end{aligned}
$$

*m* =top mass, $\theta$ = t production angle in g+g CM; *p*= gluon 3-momentum in CM

*p*-hat's are lepton$^-$ 3-momenta directions in the top and anti-top rest frames.

**Use these to test SM vs. BSM – Integrated version agrees –**

**with big errors -- GRG *in process* – see also Mahlon & Parke**

**See GG& Liuti,** 1710.01683; 2024742 (APS-DPF 2017)

# $g_1+g_2 \rightarrow t + t\text{-}bar$
# Spin correlations

Correlations expressed as a weighting factor first ***for polarized gluons***.

- The gluon fusion mechanism gives rise to a higher order angular distribution ($\sin^4\theta$) due to the combination of two spin 1 gluons.

$$
\begin{aligned}
W^{(LP,LP)}(\theta,p,p_{\bar{l}},p_l) = -\frac{1}{4} + \frac{1}{4}\Big\{ & [(1-\beta^4)+\beta^2\sin^2\theta(-2+(2-\beta^2)\sin^2\theta)](\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{x}} \\
& +[(1-\beta^4)+\beta^2\sin^2\theta(2-\beta^2\sin^2\theta)](\hat{p}_{\bar{l}})_y(\hat{p}_l)_{\bar{y}} \\
& +[-(1-\beta^2)^2+\beta^2(2-\beta^2)\sin^4\theta](\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{z}} \\
& -4\frac{\beta^2}{\gamma}\sin^3\theta\cos\theta[(\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}}-(\hat{p}_{\bar{l}})_z(\hat{p}_l)_{\bar{x}}]\Big\} \\
& /\left[(1-\beta^2)^2+\beta^4\sin^4\theta\right]
\end{aligned}
$$

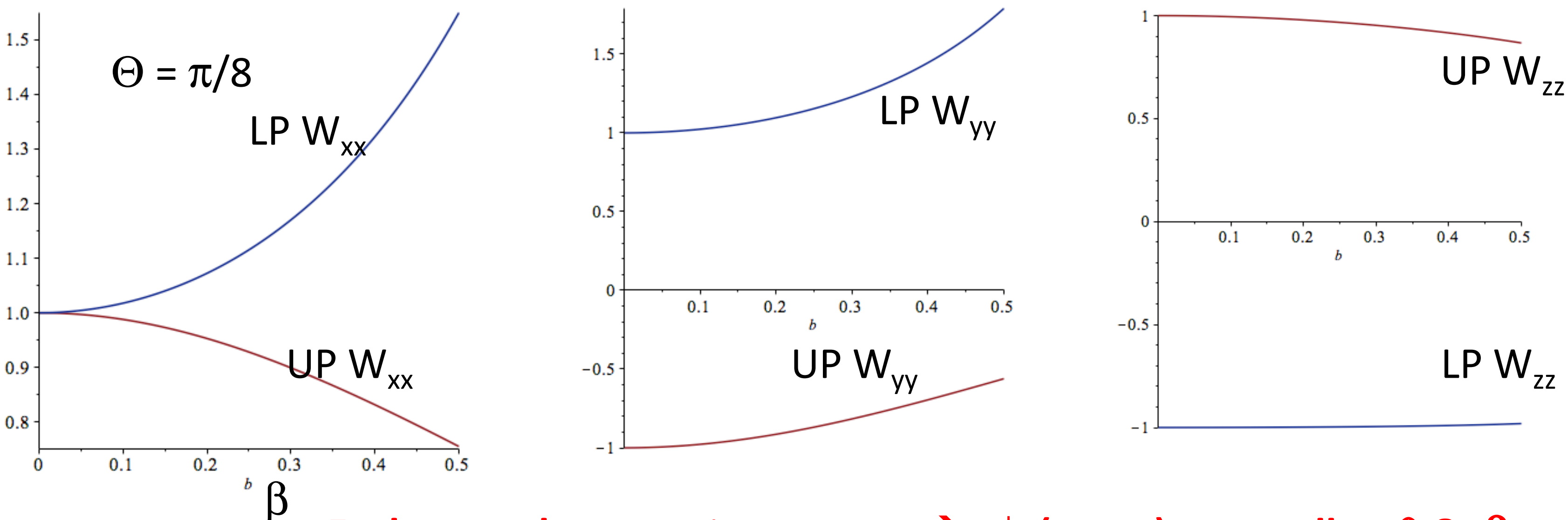
**Crucial measurements** $(\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{x}} = W_{xx}$, $(\hat{p}_{\bar{l}})_x(\hat{p}_l)_{\bar{z}} = W_{xz}$, . . . . **Weighting tensor**

- **Use these to compare with unpolarized to extract the Gluon transversity**
- **or linear polarizations $G_{xx}$-$G_{yy}$**
- **Careful about Frames:**
- **Collider LAB, $t + \bar{t}$ pair CM, separate *t* & *t*-bar rest, $W^{+/-}$ rest frames**

# Comparing lepton directional correlations

**Weighting tensor** for lepton$^+$ lepton$^-$ when $\theta=\pi/8$

*or* lepton$^+$ d-quark *or* u-quark lepton$^-$




**Each event has $\mu^-$ $\mu^+$ momenta → $p^{\pm}$ (x, y, z) as well as $\theta$ & $\beta$**

**Probability for given event configuration is given by**

**G(UP) W($\theta$,p,p¯l,pl) +G(LP) $W^{LP}$ ($\theta$,p,p¯l,pl)**

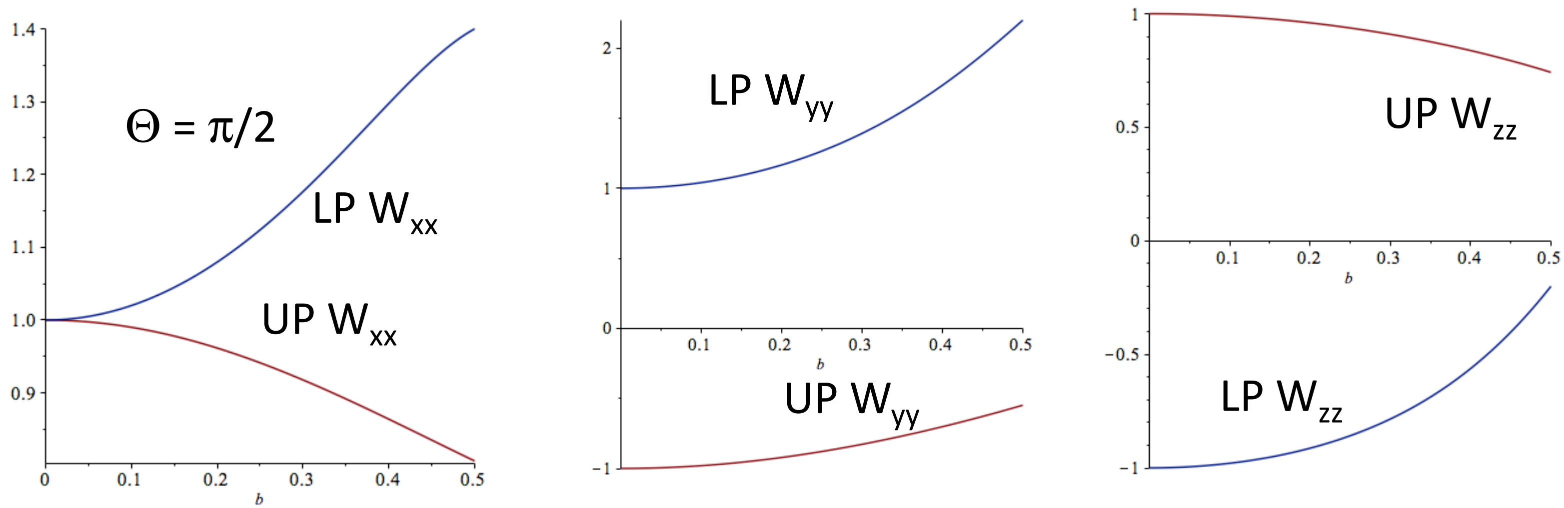
**Quite distinct! x & y components are aligned for LP, anti-aligned for UP**

**Can Diagonalize (with $W_{XY,\ YX}$) to obtain positive ellipsoidal weighting**

# Comparing lepton directional correlations

Weighting factors for lepton$^+$ lepton$^-$ when $\theta=\pi/2$

$W_{xz}=0$ for the off-diagonal




$\beta$

**Each event has $\mu^-$ $\mu^+$ momenta → $p^\pm$ (x, y, z) as well as $\theta$ & $\beta$**

**Probability for given event configuration is given by**

**$G(UP)\ W(\theta,p,\bar{p}l,pl)\ +G(LP)\ W^{LP}(\theta,p,\bar{p}l,pl)$**

**Quite distinct! x & y components are aligned for LP, anti-aligned for UP**

**Diagonalize (with $W_{XY,\ YX}$) to obtain positive ellipsoidal weighting**

# Separating polarized gluons

* Each event has $\mu^-$ $\mu^+$ momenta → $p^{\pm}$ (x, y, z)
in t & tbar rest frame

* t+tbar CM determines $\theta$ direction as well as $\beta$ for t & tbar

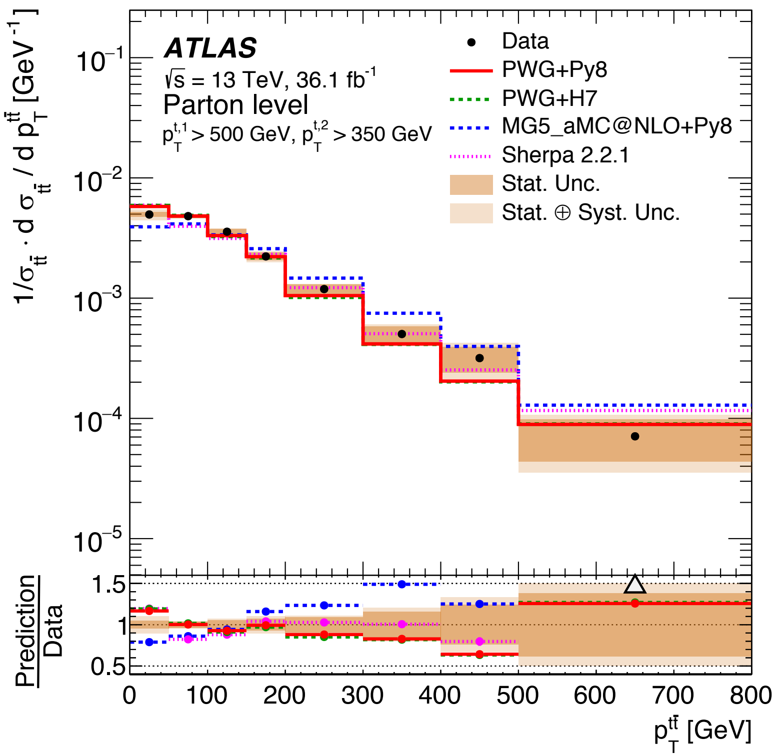
* Probability for given event configuration is given by

G(UP) W($\theta$,p,$\bar{p}$ l,pl) +G(LP) $W^{LP}$ ($\theta$,p,$\bar{p}$ l,pl) (ignoring light quarks)

- Quite distinct! x & y components are
- aligned for LP, anti-aligned for UP
- G's convoluted with W's all gluon $k_T$ & $\bar{k}_T$ satisfying
- measured $p_t + p_{anti\text{-}t}$ <-> large transverse momenta : transversity

# Large transverse momentum

- t-tbar inclusive at 13 TeV



# How are top pair polarizations measured at LHC?

- Purely hadronic events ⊃ 6 particles/jets (b,u,d +b̄,ū,d). Combinatorics!
- Dilepton events leave unknown $\nu$ & $\bar{\nu}$ momenta (b,$e^+$ $\nu$ +b̄,ū,d). Clean, lower dσ
- Single lepton events ⊃ one $\nu$ missing
  – Most promising: with H.Beauchemin(ATLAS), M. Yampolskaya, T. Lachance
- What is t or t̄ momentum?
- Measuring e or μ and b-jet fixes t to an ellipse

# Finding top momentum

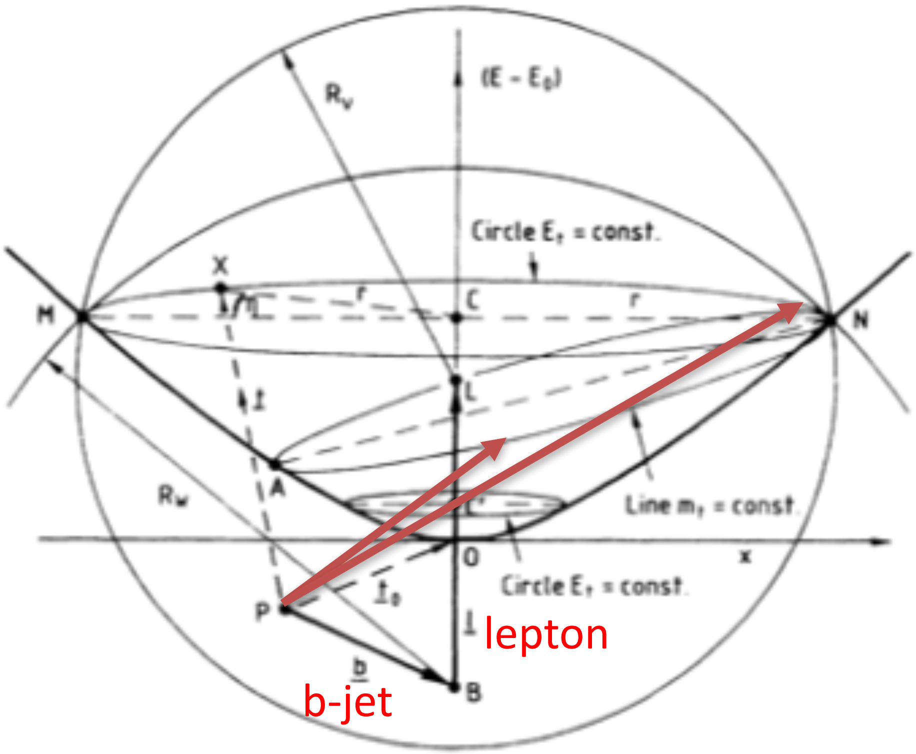



FIG. 5. Momentum vectors **b** and $\bar{l}$ observed in the laboratory frame for bottom quark and lepton, and the construction for locating all top-quark momenta **t** such that these three vectors can correspond to the decay sequence $t \to bW^+$, $W^+ \to \bar{l}^+ \nu_l$ for a given top-quark mass $m_t$.

- top leptonic decay in lab
- → momenta in lab
- b-jet & lepton measured ⇒
- Ellipse of t-vectors determined
- Boost to rest frame
- helicity preserved
- Lepton direction correlated

Dalitz & GG, PRD45,1531(1992)

# Summary

- Gluon GPDs & TMDs (from spectator & Regge R✖Dq )
- *Helicity* conserving & Helicity flip →gluon ***Transversity***
- Electroproduction & DVCS → gluon transversity GPDs
- pp→gluons→t+tbar + X
- Measurements? Single Top polarization
- **t+tbar spin correlations** **via lepton decays or hadron jets**

  – To Do List
- **More phenomenology to come**
- **Parton showers & jets**
- **Care about evolution, factorization, power counting, . . .**


## Collaborators: Gluons

Simonetta Liuti[2], Osvaldo Gonzalez Hernandez[3], Jon Poage[1]

- GRG, Gonzalez, Liuti, PRD91, 114013 (2015)
- GRG, Gonzalez Hernandez, Liuti, J. Phys. G: Nucl. Part. Phys. **39** 115001 (2012)
- GRG, Liuti, IJMP: Conf. 37, 1560038 (2015); arXiv: 1710.01683 [hep-ph]
- J.Poage, Tufts U. dissertation (2016)
- GRG & Liuti, Hernandez, PoS QCDEV2017, 037 (2017)

## Collaborators: Tops

Richard Dalitz,

Discussions: Krzysztof Sliwa, Hugo Beauchemin Tufts and Atlas

- Dalitz, R.H., and GRG, Phys. Rev. D45, 1531 (1992); Phys.Lett.B287, 225 (1992);
- GRG, Sliwa, K., Dalitz, R.H., Phys. Rev. D47, 967 (1993).

## Collaborators: Transversity

Micheal J. Moravcsik,

- GRG & M.J. Moravcsik, Ann. Phys. 98, 128 (1976); ibid. 142, 219 (1982);
- Ibid. 195, 213 (1989).

See also K. Chen, GRG, R.L. Jaffe, X.-D. Ji, Nucl Phys B 445 (1995) 380-396.


Thank you!

# Backup Slides

# Construct helicity flip amps Spectator Model, then GPDs

$$A_{++,+-} = \sqrt{1-\xi^2}\,\frac{t_0-t}{4M^2}\Big(\tilde{H}_T^g + (1-\xi)\frac{E_T^g+\tilde{E}_T^g}{2}\Big)$$

$$A_{-+,--} = \sqrt{1-\xi^2}\,\frac{t_0-t}{4M^2}\Big(\tilde{H}_T^g + (1+\xi)\frac{E_T^g-\tilde{E}_T^g}{2}\Big)$$

$$A_{++,--} = +e^{-i\phi}(1-\xi^2)\frac{\sqrt{t_0-t}}{2M}\Big(H_T^g + \frac{t_0-t}{M^2}\tilde{H}_T^g - \frac{\xi^2}{1-\xi^2}E_T^g + \frac{\xi}{1-\xi^2}\tilde{E}_T^g\Big)$$

$$A_{-+,+-} = -e^{i\phi}(1-\xi^2)\frac{\sqrt{t_0-t}^3}{8M^3}\tilde{H}_T^g,$$

Compare to spectator model results

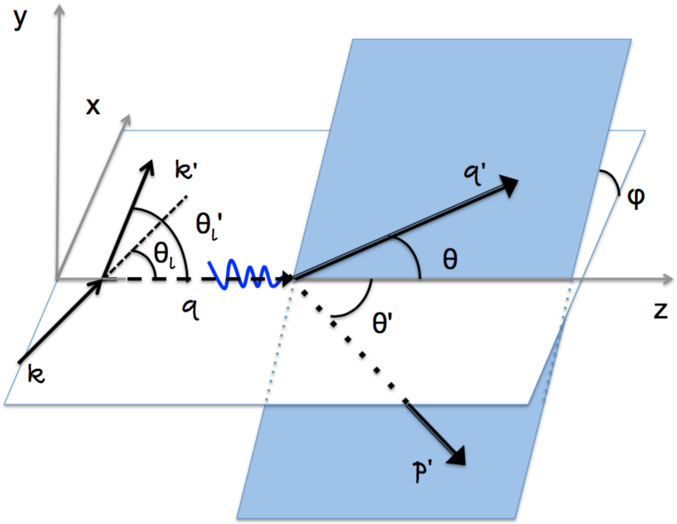
$$\tilde{H}_T^g = 0$$

$$(1-X)A^0_{-+.--} = (1-X')A^0_{++,+-}$$

$$\tilde{E}_T^g = 0.$$

As in Hoodbhoy & Ji, PRD58, 054006 (1998)

# Measuring Gluon GPDs in Nucleons




**DVCS**

$$\frac{d^5\sigma}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}}|T|^2$$

$$T(k,p,k',q',p') = T_{DVCS}(k,p,k',q',p') + T_{BH}(k,p,k',q',p'),$$

$$|T|^2 = |T_{\rm BH} + T_{\rm DVCS}|^2 = |T_{\rm BH}|^2 + |T_{\rm DVCS}|^2 + \mathcal{I}\,.$$

$$\mathcal{I} = T^*_{BH}T_{DVCS} + T^*_{DVCS}T_{BH}.$$

**For unpolarized e+p→e'+$\gamma$+p' cross section depends on azimuthal angle $\phi$.**
**cos3$\phi$ modulation in interference d$\sigma$ measures gluon transversity GPDs (CFF's)**

$$\frac{\sqrt{t_0-t}^3}{8M^3}\left[H_T^g F_2 - E_T^g F_1 - 2\tilde{H}_T^g\left(F_1 + \frac{t}{4M^2}F_2\right)\right]\cos 3\phi$$

$\mathcal{H}_T^g \sim \int dx\, H^g_T/(x-\xi)(x+\xi)$ CFF's

**But $\mathcal{H}_T^g \sim$ may need EIC**

See Diehl, *et al*. PLB411, 193 (1997);
Diehl, EPJC25, 223 (2002);
Belitsky, Mueller, PLB486, 369 (2000).

A glimpse of gluons through deeply virtual compton scattering on the proton, published in *Nature Communications* 8, 1408 (2017).
doi:10.1038/s41467-017-01819-3

# Evidence of gluon transversity

Fitting ϕ distribution requires

$F_{++}$ and both $F_{+-}$ gluon transversity

and $F_{0+}$ higher twist

**Table 2 Results of the cross-section fits**

| Fit description | LO/LT | Higher twist | NLO |
|---|---|---|---|
| **Helicity states** | **++** | **++/0+** | **++/−+** |
| $t=-0.18\ \text{GeV}^2$ | 250 | 204 | 206 |
| $t=-0.24\ \text{GeV}^2$ | 367 | 206 | 208 |
| $t=-0.30\ \text{GeV}^2$ | 415 | 189 | 190 |

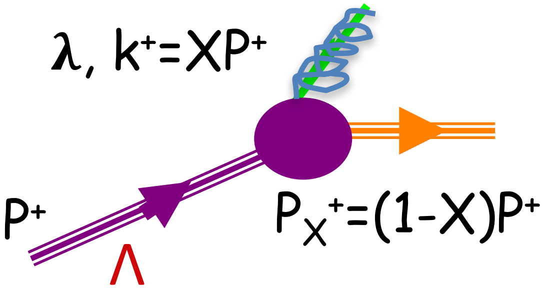
Values of $\chi^2$ (ndf = 208) obtained in the leading-order, leading-twist (++); higher-twist (++/0 +); and next-to-leading-order (++/−+) scenarios. The fit is not performed at the highest value of $-t$ because of the lack of full acceptance in $\phi$, resulting in a large statistical uncertainty. The fits include statistical and point-to-point systematic uncertainties

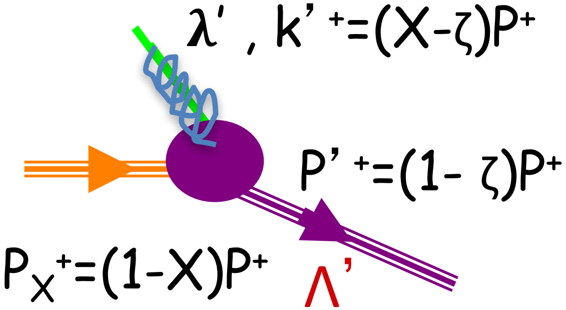
# Gluon Double Flip Amps

$$
\begin{aligned}
A_{++,+-} &= \frac{\Delta_\perp^2}{4M^2(1-\zeta)(1-\frac{\zeta}{2})}\Big(\tilde{H}_T^g + \frac{1-\zeta}{1-\frac{\zeta}{2}}\frac{E_T^g+\tilde{E}_T^g}{2}\Big)\\
A_{-+,--} &= \frac{\Delta_\perp^2}{4M^2(1-\zeta)(1-\frac{\zeta}{2})}\Big(\tilde{H}_T^g + \frac{1}{1-\frac{\zeta}{2}}\frac{E_T^g-\tilde{E}_T^g}{2}\Big)\\
A_{++,--} &= \frac{\Delta_1 - i\Delta_2}{2M\sqrt{1-\zeta}(1-\frac{\zeta}{2})}\Big(\frac{1-\zeta}{1-\frac{\zeta}{2}}H_T^g + \frac{\Delta_\perp^2}{M^2(1-\frac{\zeta}{2})}\tilde{H}_T^g - \frac{\zeta}{2}\Big[\frac{\zeta}{1-\frac{\zeta}{2}}E_T^g - \tilde{E}_T^g\Big]\Big)\\
A_{-+,+-} &= -\frac{\Delta_1 + i\Delta_2}{2M\sqrt{1-\zeta}(1-\frac{\zeta}{2})}\frac{\Delta_\perp^2}{4M^2(1-\frac{\zeta}{2})}\tilde{H}_T^g,
\end{aligned}
$$

**All will involve $\Delta$ powers for each net helicity flip**
**So need t and tbar 3-momenta with $\Delta$ non-zero!**

# Constructing gluon GPDs
## Gluon 'vertex functions' $G_{\Lambda x\ ;\ \Lambda g,\ \Lambda}$



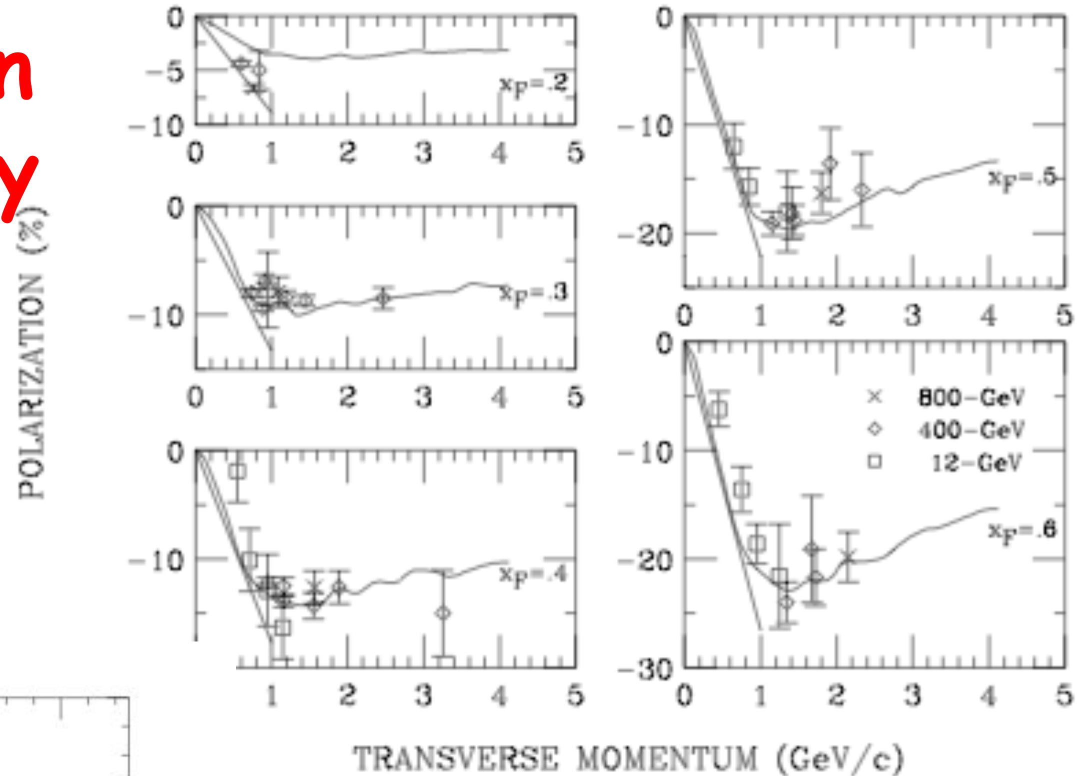

| | |
|---|---|
| $\mathcal{G}_{+++}(x,\vec{k}_T^2)$ | $-\frac{2}{\sqrt{2(1-X)}}\frac{(k_x-ik_y)}{X}$ |
| $\mathcal{G}_{-++}(x,\vec{k}_T^2)$ | $-\frac{2}{\sqrt{2(1-X)}}(M(1-X)-M_x)$ |
| $\mathcal{G}_{++-}(x,\vec{k}_T^2)$ | $0$ |
| $\mathcal{G}_{-+-}(x,\vec{k}_T^2)$ | $-\frac{2}{\sqrt{2(1-X)}}(1-X)\frac{(k_x-ik_y)}{X}$ |
| $\mathcal{G}^*_{+++}(x,\vec{k}_T'^2)$ | $-\frac{2}{\sqrt{2(1-X')}}\frac{(\tilde{k}_x+i\tilde{k}_y)}{X'}$ |
| $\mathcal{G}^*_{-++}(x,\vec{k}_T'^2)$ | $-\frac{2}{\sqrt{2(1-X')}}(M(1-X')-M_x)$ |
| $\mathcal{G}^*_{++-}(x,\vec{k}_T'^2)$ | $0$ |
| $\mathcal{G}^*_{-+-}(x,\vec{k}_T'^2))$ | $-\frac{2}{\sqrt{2(1-X')}}(1-X')\frac{(\tilde{k}_x+i\tilde{k}_y))}{X'}$ |

$$X' = \frac{X-\zeta}{1-\zeta},\ \tilde{k}_{i=1,2} = k_i - \frac{1-X}{1-\zeta}\Delta_i$$

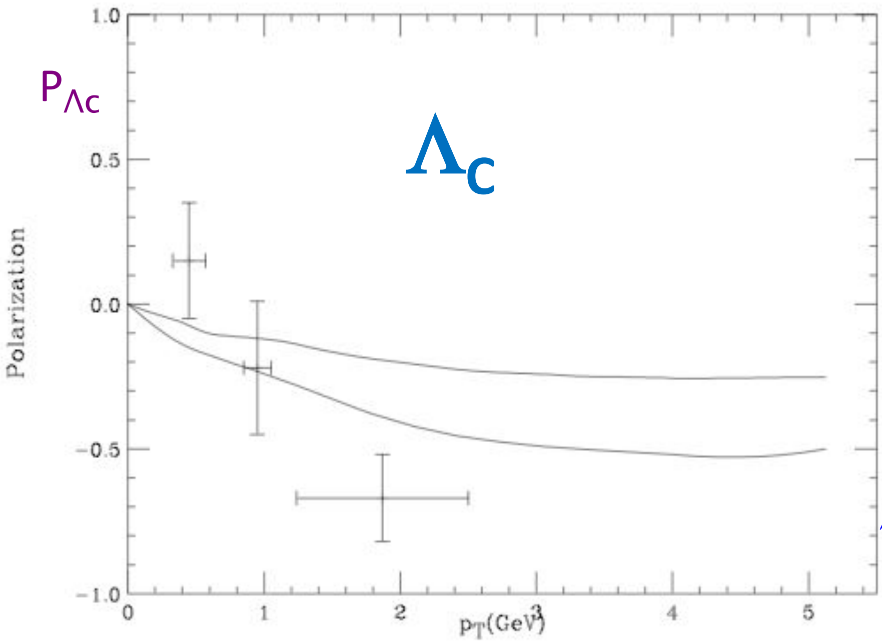
GRG & S. Liuti, QCD Evolution 2014, IJMP: Conf. 37, 1560038 (2015); arXiv: 1710.01683 [hep-ph]
GRG, Gonzalez Hernandez, Liuti, Poage, in progress

# Single Spin Asymmetry

$$pp \rightarrow \Lambda^{\uparrow}(\Lambda_c^{\uparrow})X$$

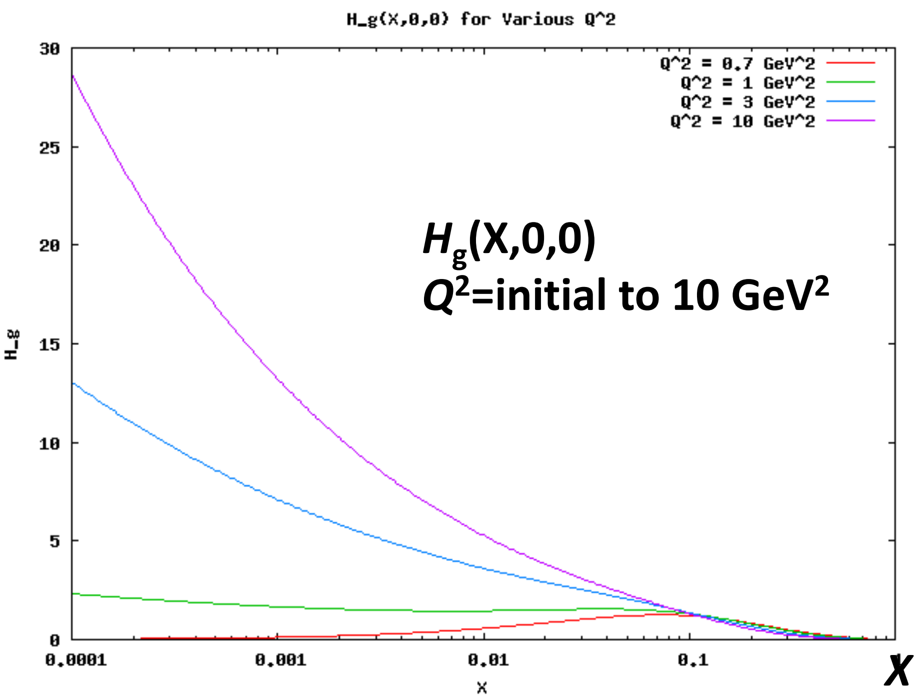


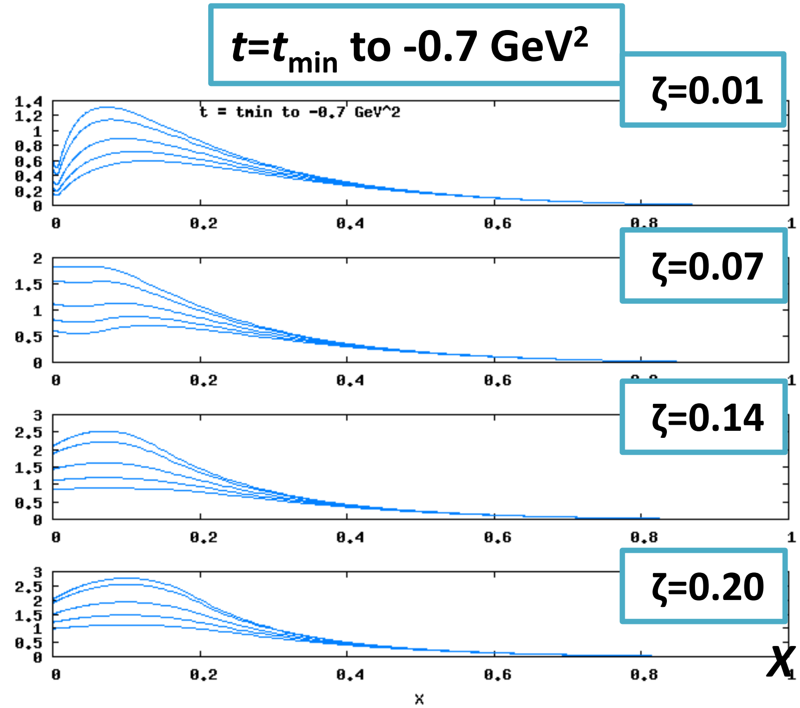

K. Heller, PRD1997
curves from model of
Dharmaratna & GRG PRD '90 & '97




E791, PLB 471, 449 (2000)
$\pi^- + p \rightarrow \Lambda_C + X$
curves from GRG hep-ph/9907573

# After pdf's vs. $Q^2$ → fix x dependence
# Regge behavior determines $t$ dependence
# Spectator determines $\zeta$ dependence




from J. Poage

# Gluon & Sea quark distributions Spectator Model

- N→ ***g*** + "color octet N" spectator ($8\otimes 8 \supset 1$)

  (could be spin ½ or 3/2)
- (N→ ***anti-u*** + color 3 "tetraquark"uuud)

- How to normalize?

$$H_g(x,\xi,t)_{Q^2} \rightarrow H_g(x,0,0)_{Q^2} = xG(x)_{Q^2}$$

Evolution & small x phenomenology

- Sea quark distributions $H_{anti\text{-}u}(x,0,0)$ . . .

- Use pdf's to fix x dependence

- Small x ~ Pomeron
- Model generalizes to GTMDs > TMDs . . .

# Top spin correlations & gluon polarizations

| $\rho_{t',\bar{t}';t,\bar{t}}$ | UP,UP | LP,LP | UP,LP + LP,UP |
|---|---|---|---|
| $++,++$ | $\gamma^{-2}(1+\beta^2(1+sin^4\theta))$ | $\gamma^{-2}(-1+\beta^2(1+sin^4\theta))$ | $-4\gamma^{-2}\beta^2 sin^2\theta$ |
| $+-,+-$ | $\beta^2 sin^2\theta(2-sin^2\theta))$ | $-\beta^2 sin^4\theta$ | $0$ |
| $++,--$ | $\gamma^{-2}(-1+\beta^2(1+sin^4\theta))$ | $\gamma^{-2}(+1+\beta^2(1+sin^4\theta))$ | $+4\gamma^{-2}\beta^2 sin^2\theta$ |
| $+-,-+$ | $\beta^2 sin^4\theta$ | $-\beta^2 sin^2\theta(2-sin^2\theta))$ | $0$ |
| $++,+-$ | $-2\gamma^{-1}\beta^2 sin^3\theta cos\theta$ | $-2\gamma^{-1}\beta^2 sin^3\theta cos\theta$ | $-4\gamma^{-1}\beta^2 sin\theta cos\theta$ |
| $++,-+$ | $2\gamma^{-1}\beta^2 sin^3\theta cos\theta$ | $2\gamma^{-1}\beta^2 sin^3\theta cos\theta$ | $4\gamma^{-1}\beta^2 sin\theta cos\theta$ |

11 using values of

$$G^{(1)}_{\Lambda_{N1},R,R} + G^{(1)}_{\Lambda_{N1},L,L} = G^{(1)}_{\Lambda_{N1},XX} + G^{(1)}_{\Lambda_{N1},YY} = G^{(1)}_{\Lambda_{N1},UP}$$

$$G^{(1)}_{\Lambda_{N1},R,L} + G^{(1)}_{\Lambda_{N1},L,R} = G^{(1)}_{\Lambda_{N1},YY} - G^{(1)}_{\Lambda_{N1},XX} = G^{(1)}_{\Lambda_{N1},LP}$$

UP = unpolarized, LP = Linearly polarized gluon distributions
assuming g+g→ t + t-bar in single plane CM
$\gamma$ & $\beta$ for top & antitop in CM.
$\theta$ = top production angle in CM relative to (t+tbar) momentum direction in lab
Taking X-Z plane for p+p→$(t+tbar)_{CM}$+X gives $\phi$ dependence to
t+tbar plane for opposite helicities: Re($e^{\pm(1or2)i\phi} \cdot e^{\pm(-i(1or2)\phi)}$)
leading to cos2$\phi$ for UP,LP and LP,UP and cos4$\phi$ modulations
for LP,LP.