QED radiative corrections for the P2 experiment

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Overview

(1) Introduction — P2 project and the weak mixing angle
(2) How to measure the weak mixing angle?
(3) Electroweak radiative corrections to $Q_W^p$.
(4) The shift in $Q^2$ due to photon radiation.
(5) $O(\alpha^2)$ QED corrections
• **MESA =**
  Mainz Energy-recovering Superconducting Accelerator
  A small superconducting accelerator for particle and nuclear physics

• Funded by PRISMA - Cluster of Excellence and Collaborative Research Center 1044 German Science Foundation (DFG)

• **P2 (Project 2):**
  Parity-violating electron proton scattering

• Other Projects: Search for a dark photon,
  Nuclear physics program
\( \sin^2 \theta_W \) is scale dependent

\[
\sin^2 \hat{\theta}_W(Q)_{\overline{MS}} = \kappa(Q)_{\overline{MS}} \sin^2 \theta_W(M_Z)_{\overline{MS}}
\]

→ The future P2 experiment at low momentum transfer will complement other high-precision determinations and may thus help to resolve differences between previous measurements, or find interesting new effects.

arXiv:1802.04759 (to appear in EPJA)
How to measure $\sin^2 \theta_W$?

→ Extract $Q_W^p$ (weak charge of the proton — the neutral equivalent of the proton's electric charge) in $ep$ scattering with polarized $e^-$ beam and unpolarized proton target. (P2 and Qweak approach)

→ Measure the very small asymmetry $\approx 10^{-8}$ between cross sections for electrons with $+$ and $-$ helicities to filter out the weak interaction

$$A_{PV} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}$$

In the Born approximation

$$A_{PV} \stackrel{\text{low } Q^2}{\longrightarrow} - \frac{G_F Q^2}{4\sqrt{2}\pi\alpha} \left[ Q_W^p - F(Q^2) \right].$$

SM at tree level $Q_{W}^{p,\text{tree}} \rightarrow 1 - 4 \sin^2 \theta_W^{\text{tree}} \approx 0.07$ (good candidate for New Physics search)

Why measure at low $Q^2$? → small contribution from $F(Q^2)$ (form factors) and $\gamma - Z$ box correction. → Precision measurements at low-energies are sensitive to TeV-scale physics.
Models with dark photons predict a small shift of the running weak mixing angle at low mass scales, visible for P2, but not at higher energies. (Hooman Davoudiasl, Hye-Sung Lee, and William J. Marciano 2015)
Radiative Corrections

P2 accuracy: \( \frac{\Delta A_{PV}}{A_{PV}} = 1.7\% \rightarrow \frac{\Delta \sin^2 \theta_W}{\sin^2 \theta_W} = 0.15\% \)

→ Include full treatment of radiative corrections at \( \mathcal{O}(\alpha^2) \) to match experimental precision.

\[
A_{PV} = - \frac{G_F Q'^2}{4 \sqrt{2} \pi \alpha} \left[ \tilde{Q}_W^p - F(Q'^2) \right].
\]

\[
\tilde{Q}_W^p = (\rho + \Delta_e) \left( 1 - 4\kappa(Q'^2) \sin^2 \hat{\theta}_W(0) + \Delta_e' \right) + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z},
\]

Universal corrections (loop diagrams) → \( \rho = 1 + \Delta \rho \) and \( \kappa = 1 + \Delta \kappa \)

Non-universal corrections (vertex corrections) → \( \Delta_e \) and \( \Delta_e' \)

Form Factors → \( F(Q'^2) = F_{EM}(Q'^2) + F_{axial}(Q'^2) + F_{strangeness}(Q'^2) \)
Shift in momentum transfer due to photon radiation

\[
\begin{align*}
  l_1 - k & \rightarrow q' \\
  l_2 & \rightarrow l_2 + k
\end{align*}
\]

Shifted kinematics:

\[
Q^2 = -(l_1 - l_2)^2 \rightarrow Q'^2 = -(l_1 - l_2 - k)^2
\]

\( Q'^2 \) can be on average much smaller than \( Q^2 \).

The average shift in momentum transfer squared due to hard-photon bremsstrahlung can be defined as

\[
\langle \Delta Q^2 \rangle = \frac{1}{\sigma} \int \frac{d^4 \sigma_1}{dE'd\theta'dE_\gamma d\theta_\gamma} dE'd\theta'dE_\gamma d\theta_\gamma \Delta Q^2,
\]

with

\[
\Delta Q^2 = Q'^2 - Q^2,
\]

\[
\sigma = \sigma_{1-loop}^{1\gamma} \big|_{E_\gamma < \Delta} + \sigma_{1\gamma}^{1\gamma} \big|_{E_\gamma > \Delta}.
\]
→ Strong dependence on experimental prescriptions for measuring kinematic variables

→ Need full Monte-Carlo treatment

→ Dependence on the detector acceptance

→ Dependence on the scattering angle
The shift in $Q^2$ due to photon bremsstrahlung induces a shift similar in size in the asymmetry.

A significant effect is also given by the bin size $\Delta \theta_l$ of the integration over the scattering angle.

The total asymmetry with first order QED corrections compared with the leading order asymmetry.
\[ \mathcal{O}(\alpha^2) \text{ QED corrections} \]

- The cross-section with \( \mathcal{O}(\alpha^2) \) QED corrections is given by

\[
d\sigma^{(2)} = d\sigma_{0\gamma} \\
\times \left[ 1 + \delta_{1\text{-loop}}^{(1)} + \delta_{2\text{-loop}}^{(2)} + \delta_{1\gamma}^{(1)}(\Delta) + \delta_{2\gamma}^{(2)}(\Delta) + \delta_{1\text{-loop}}^{(1)} \delta_{1\gamma}^{(1)}(\Delta) \right] \\
+ \int_{E\gamma > \Delta} d^4\sigma_{1\gamma} \left[ 1 + \delta_{1\text{-loop}}^{(2)} + \delta_{1\gamma}^{(2)}(\Delta) \right] + \int_{E\gamma, E'\gamma > \Delta} d^7\sigma_{2\gamma}.
\]

\( \Delta \) is the cut-off energy that makes the separation between soft- and hard-photons.

- The second order soft real correction can be written as

\[
\delta_{2\gamma}(\Delta) = \frac{1}{2} \left[ \delta_{1\gamma}(\Delta) \right]^2
\]
\( \mathcal{O}(\alpha^2) \) QED corrections

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$$+ \int_{E_{\gamma}>\Delta} \text{d}^4\sigma_{1\gamma} \left[ 1 + \delta_{1\text{-loop}}^{(2)} + \delta_{1\gamma}^{(2)}(\Delta) \right] + \int_{E_{\gamma}, E'_{\gamma}>\Delta} \text{d}^7\sigma_{2\gamma}.$$ 

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+ \int_{E_\gamma > \Delta} d^4\sigma_{1\gamma} \left[ 1 + \delta^{(2)}_{1\text{-loop}} + \delta^{(2)}_{1\gamma}(\Delta) \right] + \int_{E_\gamma, E'_\gamma > \Delta} d^7\sigma_{2\gamma}.
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QED virtual corrections

\[ O(\alpha) \]

\[ O(\alpha^2) \]

\[ \frac{2}{16} \]

+ photon emitted from on-shell line

+ 2-loop
\( \mathcal{O}(\alpha^2) \) QED corrections to the unpolarized cross section

\[ E = 155 \text{ MeV} \]
\[ E'_{\text{min}} = 45 \text{ MeV} \]
\[ \theta_\ell = 25^\circ \pm 10^\circ \]

→ Can be used to decide which is the best value for \( \Delta \).
→ Soft photon approximation (SPA) breaks down if the cut-off \( \Delta \) is too big and numerical uncertainties become too large if \( \Delta \) is too small.
→ A nice plateau can be found between 1 and 10 MeV.
$O(\alpha^2)$ QED corrections to the asymmetry (P2 kinematics)

The shift in $Q^2$ is a kinematical effect included in $1\gamma$ radiation → very small $O(\alpha^2)$ corrections to the asymmetry.
Conclusion

It is important to include full treatment of radiative corrections at the level of the event generator.

$E = 155$ MeV
$\theta_l = 35^\circ \pm 10^\circ$
$E_{\text{min}}' = 45$ MeV
$\Delta = 1$ MeV
Total Events ($N$) = $10^7$

→ A technical comparison between the numerical integration of the cross-section for each bin and the weights produced by the event generator.
Overview

• Significant $\mathcal{O}(\alpha)$ QED corrections to the asymmetry were found → need full Monte Carlo treatment

• Very small $\mathcal{O}(\alpha^2)$ to the asymmetry.

• A modern, flexible, easy to use event generator was developed that will include complete $\mathcal{O}(\alpha^2)$ electroweak corrections.

Thank you for your attention!
Extra Slides
$O(\alpha^2)$ QED loop corrections

\[ \begin{align*}
\theta_l \text{ [degrees]} & \\
\begin{array}{c}
\text{\blue} \delta^{(1)} \\
\text{\red} \delta^{(1+2)}
\end{array} & \\
\begin{array}{c}
E = 155 \text{ MeV} \\
\Delta = 10 \text{ MeV}
\end{array}
\end{align*} \]

\[ \begin{align*}
\delta^{(1)} &= \delta^{(1)}_{1\text{-loop}} + \delta^{(1)}_{1\gamma} \\
\delta^{(2)} &= \delta^{(2)}_{2\text{-loop}} + \delta^{(2)}_{2\gamma} + \delta^{(1)}_{1\text{-loop}} \delta^{(1)}_{1\gamma} \\
\delta^{(1+2)} &= \delta^{(1)} + \delta^{(2)}.
\end{align*} \]

Richard J. Hill 2016
$\theta_l = 35^\circ \pm 10^\circ$

$E = 155$ MeV

$E_{\text{min}}' = 45$ MeV
Cross Section dependence on detector acceptance

\[ \frac{100 \cdot (\sigma^{(1)} - \sigma_0)}{\sigma_0} \]

\[ \theta_l = 20^\circ \pm 5^\circ \]

\[ E = 150 \text{ MeV} \]

\[ E'_{\text{min}} \text{ [MeV]} \]

\[ E'_{\gamma}^{\text{max}} \text{ [MeV]} \]