QED radiative corrections for the P2 experiment

Spin 2018

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Overview

(1) Introduction — P2 project and the weak mixing angle

(2) How to measure the weak mixing angle?

(3) Electroweak radiative corrections to Q_W^p.

(4) The shift in Q^2 due to photon radiation.

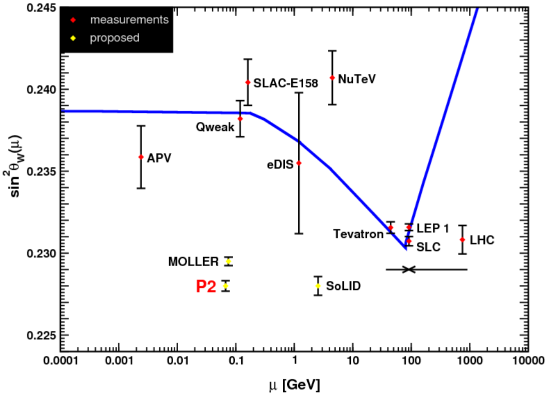
(5) $\mathcal{O}(\alpha^2)$ QED corrections

P2

- MESA =
  Mainz Energy-recovering Superconducting Accelerator
  A small superconducting accelerator for particle and nuclear physics
- Funded by PRISMA - Cluster of Excellence and
  Collaborative Research Center 1044
  German Science Foundation (DFG)
- P2 (Project 2):
  Parity-violating electron proton scattering
- Other Projects: Search for a dark photon,
  Nuclear physics program

$\sin^2\theta_W$ is scale dependent

$$\sin^2\hat{\theta}_W(\boldsymbol{Q})_{\overline{\mathrm{MS}}} = \kappa(\boldsymbol{Q})_{\overline{\mathrm{MS}}}\sin^2\theta_W(\boldsymbol{M_Z})_{\overline{\mathrm{MS}}}$$

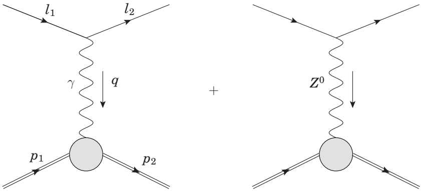


arXiv:1802.04759 (to appear in EPJA)

→ The future P2 experiment at low momentum transfer will complement other high-precision determinations and may thus help to resolve differences between previous measurements, or find interesting new effects.

How to measure $\sin^2\theta_W$?

$\rightarrow$ Extract Q_W^p (weak charge of the proton — the neutral equivalent of the proton's electric charge) in ep scattering with polarized e^- beam and unpolarized proton target. (P2 and Qweak approach)



$\rightarrow$ Measure the very small asymmetry $\approx 10^{-8}$ between cross sections for electrons with $+$ and $-$ helicities to filter out the weak interaction

$$A_{\text{PV}} = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}$$

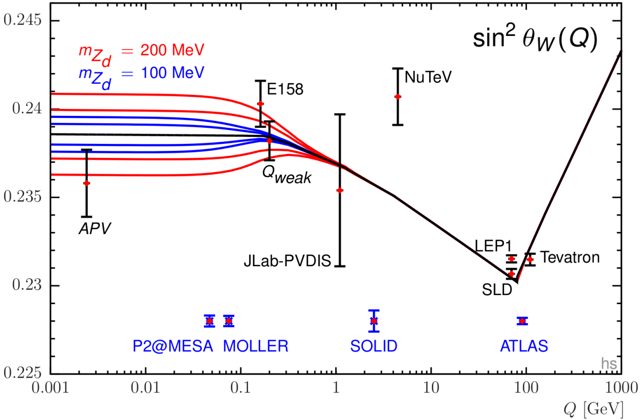
In the Born approximation

$$A_{PV} \xrightarrow{\text{low } Q^2} -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\left[Q_W^p - F(Q^2)\right].$$

SM at tree level $Q_W^{p,\text{tree}} \rightarrow 1 - 4\sin^2\theta_W^{\text{tree}} \approx 0.07$ (good canditate for New Physics search)

Why measure at low Q^2? $\rightarrow$ small contribution from $F(Q^2)$ (form factors) and $\gamma - Z$ box correction.
$\rightarrow$ Precision measurements at low-energies are sensitive to TeV-scale physics.

Dark Z boson



→ Models with dark photons predict a small shift of the running weak mixing angle at low mass scales, visible for P2, but not at higher energies. (Hooman Davoudiasl, Hye-Sung Lee, and William J. Marciano 2015)

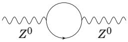
Radiative Corrections

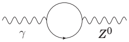
P2 accuracy: $\frac{\Delta A_{\text{PV}}}{A_{\text{PV}}} = 1.7\%$ $\rightarrow$ $\frac{\Delta \sin^2\theta_W}{\sin^2\theta_W} = 0.15\%$

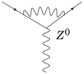
$\rightarrow$ Include full treatment of radiative corrections at $\mathcal{O}(\alpha^2)$ to match experimental precision.

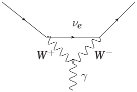
$$A_{PV} = -\frac{G_F Q'^2}{4\sqrt{2}\pi\alpha}\left[\widetilde{Q}^p_W - F(Q'^2)\right].$$

$$\widetilde{Q}^p_W = (\rho + \Delta_e)(1 - 4\kappa(Q'^2)\sin^2\hat{\theta}_W(0) + \Delta_{e'}) + \Box_{WW} + \Box_{ZZ} + \Box_{\gamma Z},$$

Universal corrections (loop diagrams) $\rightarrow$ $\rho = 1 + \Delta\rho$ 

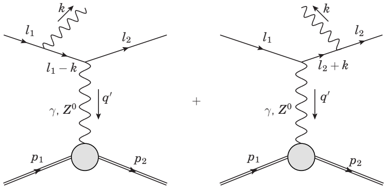
and $\kappa = 1 + \Delta\kappa$ 

Non-universal corrections (vertex corrections) $\rightarrow$ Δ_e 

and $\Delta_{e'}$ 

Form Factors $\rightarrow$ $F(Q'^2) = F_{EM}(Q'^2) + F_{axial}(Q'^2) + F_{strangeness}(Q'^2)$

Shift in momentum transfer due to photon radiation



Shifted kinematics:

$$Q^2 = -(l_1 - l_2)^2 \to Q'^2 = -(l_1 - l_2 - k)^2$$

$\to Q'^2$ can be on average much smaller than Q^2.

The average shift in momentum transfer squared due to hard-photon bremsstrahlung can be defined as

$$\langle \Delta Q^2 \rangle = \frac{1}{\sigma} \int \frac{\mathrm{d}^4\sigma^{1\gamma}}{\mathrm{d}E'\mathrm{d}\theta_l \mathrm{d}E_\gamma \mathrm{d}\theta_\gamma} \mathrm{d}E'\mathrm{d}\theta_l \mathrm{d}E_\gamma \mathrm{d}\theta_\gamma \Delta Q^2,$$

with

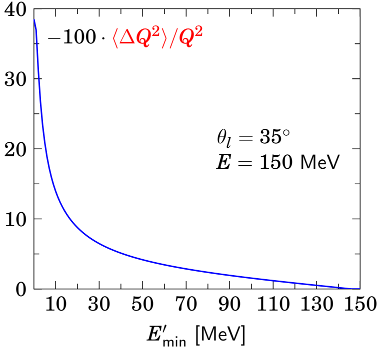
$$\Delta Q^2 = Q'^2 - Q^2,$$

$$\sigma = \sigma^{1\gamma}_{1-\text{loop}}\Big|_{E_\gamma < \Delta} + \sigma^{1\gamma}\Big|_{E_\gamma > \Delta}.$$

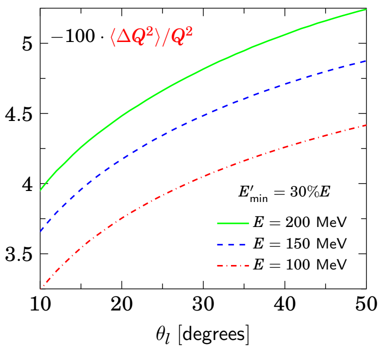
→ Strong dependence on experimental prescriptions for measuring kinematic variables

→ Need full Monte-Carlo treatment

→ Dependence on the detector acceptance

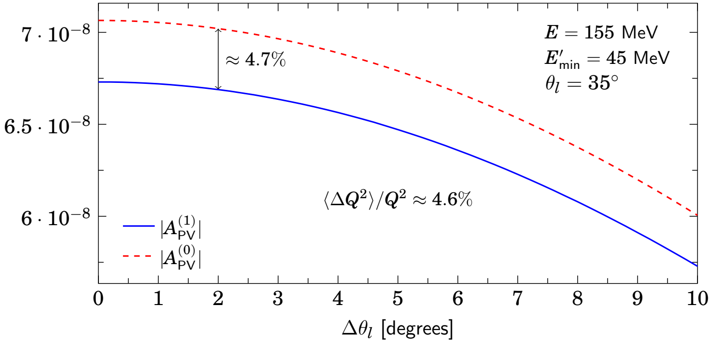


→ Dependence on the scattering angle



→ The shift in Q^2 due to photon bremsstrahlung induces a shift similar in size in the asymmetry.

→ A significant effect is also given by the bin size $\Delta\theta_l$ of the integration over the scattering angle.



The total asymmetry with first order QED corrections compared with the leading order asymmetry.

$\mathcal{O}(\alpha^2)$ QED corrections

- The cross-section with $\mathcal{O}(\alpha^2)$ QED corrections is given by

$$\begin{aligned} \mathbf{d}\sigma^{(2)} &= \mathbf{d}\sigma_{0\gamma} \\ &\times \left[1 + \delta^{(1)}_{1-\mathrm{loop}} + \delta^{(2)}_{2-\mathrm{loop}} + \delta^{(1)}_{1\gamma}(\Delta) + \delta^{(2)}_{2\gamma}(\Delta) + \delta^{(1)}_{1-\mathrm{loop}}\delta^{(1)}_{1\gamma}(\Delta)\right] \\ &+ \int_{E_\gamma > \Delta} \mathbf{d}^4\sigma_{1\gamma}\left[1 + \delta^{(2)}_{1-\mathrm{loop}} + \delta^{(2)}_{1\gamma}(\Delta)\right] + \int_{E_\gamma,\, E'_\gamma > \Delta} \mathbf{d}^7\sigma_{2\gamma}. \end{aligned}$$

  Δ is the cut-off energy that makes the separation between soft- and hard-photons.

- The second order soft real correction can be written as

$$\delta_{2\gamma}(\Delta) = \frac{1}{2}\left[\delta_{1\gamma}(\Delta)\right]^2$$

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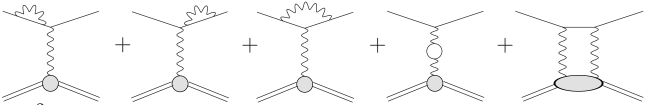
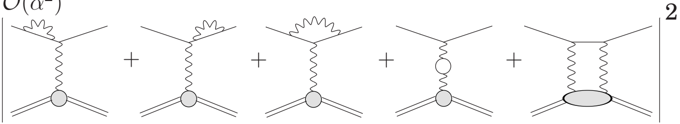
QED virtual corrections

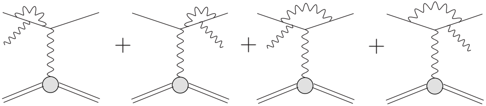
$\mathcal{O}(\alpha)$



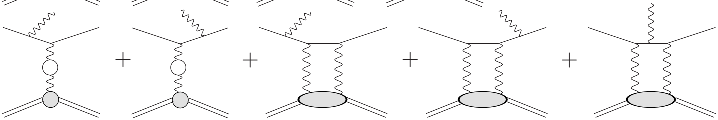
$\mathcal{O}(\alpha^2)$



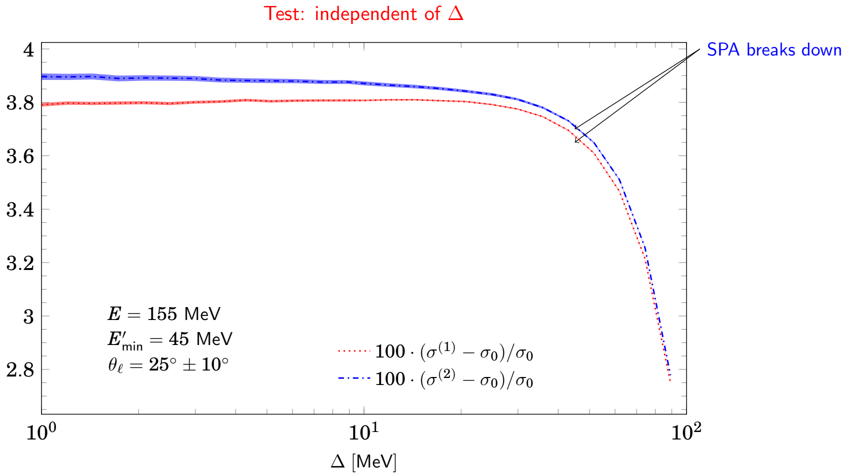
+ 2-loop



+ photon emitted from on-shell line



$\mathcal{O}(\alpha^2)$ QED corrections to the unpolarized cross section

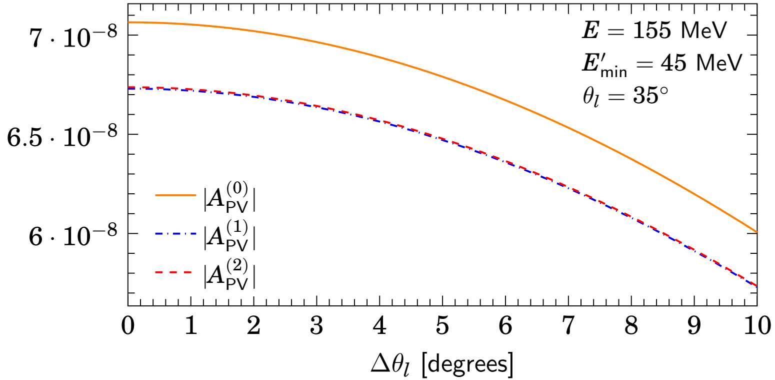


→ Can be used to decide which is the best value for Δ.

→ Soft photon approximation (SPA) breaks down if the cut-off Δ is too big and numerical uncertainties become too large if Δ is too small.

→ A nice plateau can be found between 1 and 10 MeV.

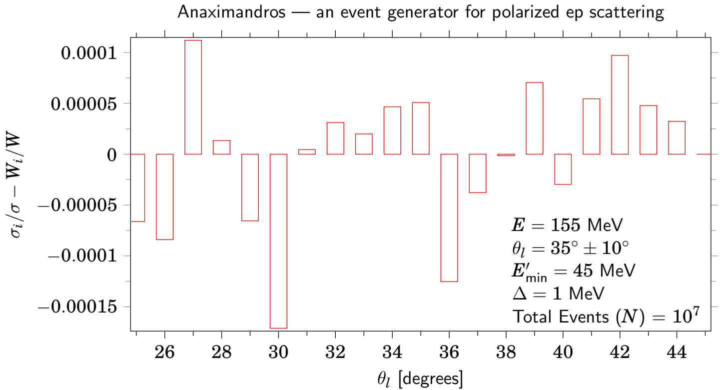
$\mathcal{O}(\alpha^2)$ QED corrections to the asymmetry (P2 kinematics)



The shift in Q^2 is a kinematical effect included in 1γ radiation $\rightarrow$ very small $\mathcal{O}(\alpha^2)$ corrections to the asymmetry.

Conclusion

It is important to include full treatment of radiative corrections at the level of the event generator.



→ A technical comparison between the numerical integration of the cross-section for each bin and the weights produced by the event generator.

Overview

- Significant $\mathcal{O}(\alpha)$ QED corrections to the asymmetry were found
  $\rightarrow$ need full Monte Carlo treatment

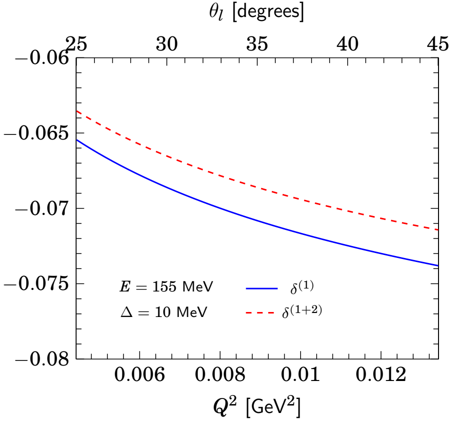
- Very small $\mathcal{O}(\alpha^{\mathbf{2}})$ to the asymmetry.

- A modern, flexible, easy to use event generator was developed that will include complete $\mathcal{O}(\alpha^{\mathbf{2}})$ electroweak corrections.

Thank you for your attention!

Extra Slides

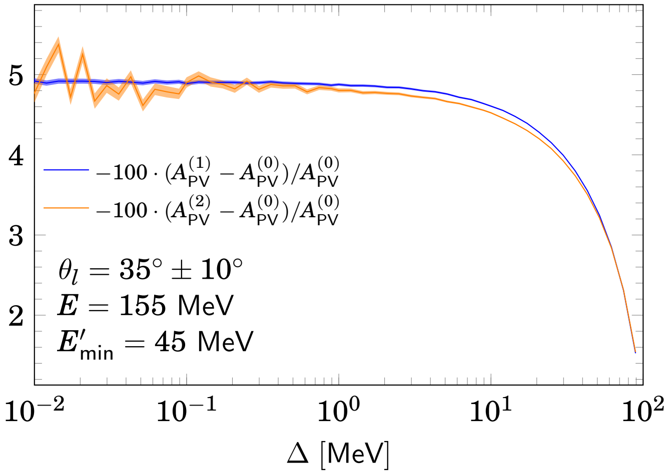
$\mathcal{O}(\alpha^2)$ QED loop corrections



$$\delta^{(1)} = \delta^{(1)}_{1-\text{loop}} + \delta^{(1)}_{1\gamma}$$
$$\delta^{(2)} = \delta^{(2)}_{2-\text{loop}} + \delta^{(2)}_{2\gamma} + \delta^{(1)}_{1-\text{loop}}\delta^{(1)}_{1\gamma}$$
$$\delta^{(1+2)} = \delta^{(1)} + \delta^{(2)}.$$

Richard J. Hill 2016

Total Asymmetry



Cross Section dependence on detector acceptance