# Process dependence of the gluon Sivers function in inclusive *pp* collisions: theory

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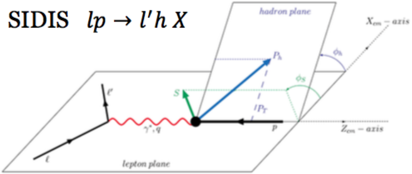
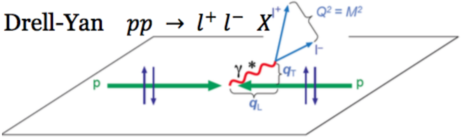
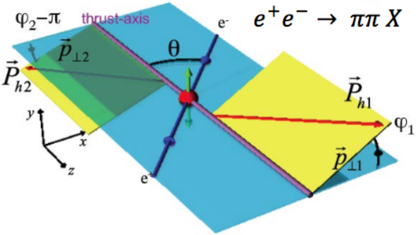
# TMD factorization and process dependence

Two scale processes $Q^2 \gg p_T^2$



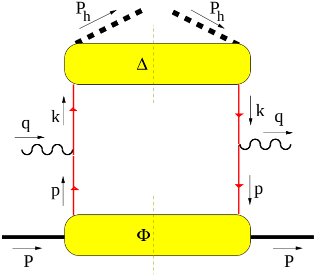




Factorization proven

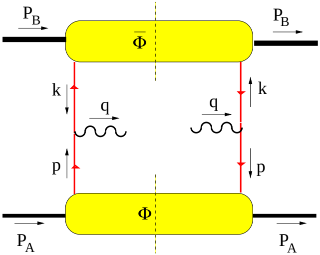
Hard partonic interactions can be separated from nonperturbative correlators

SIDIS



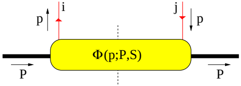

$\gamma^* p \to h X$

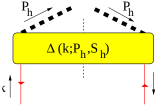
Drell-Yan




$p\, p \to \gamma^* X$

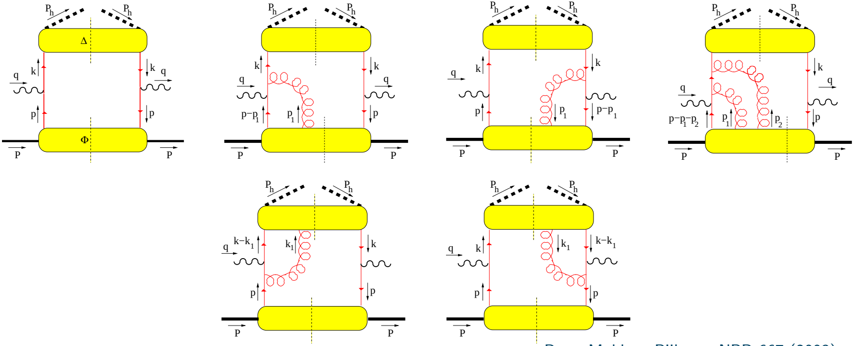
Parton correlators Φ and Δ describe the soft hadron ↔ parton transitions




Parametrized in terms of distribution and fragmentation functions

Resummation of all gluon exchanges leads to *gauge links* in the correlators $\Phi$, $\Delta$




Boer, Mulders, Pijlman, NPB 667 (2003)

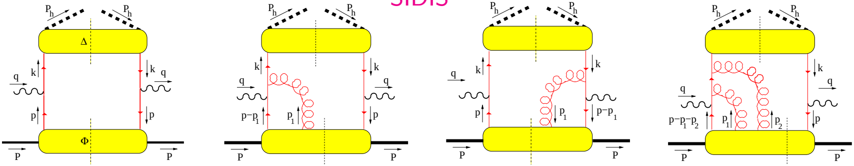
$$\mathcal{U}^{\mathcal{C}}_{[0,\xi]} = \mathcal{P}\exp\left(-ig\int_{\mathcal{C}[0,\xi]} \mathrm{d}s_\mu A^\mu(s)\right)$$

The path $\mathcal{C}$ depends on the color interactions, *i.e.* on the specific process

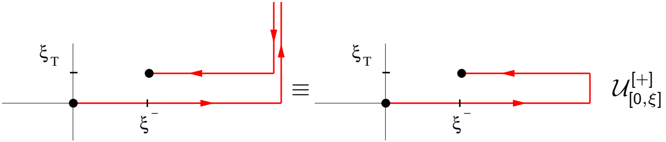
## Gauge invariant definition of Φ (not unique)

$$\Phi^{[\mathcal{U}]} \propto \left\langle P, S \left| \overline{\psi}(0)\, \mathcal{U}^{\mathcal{C}}_{[0,\xi]}\, \psi(\xi) \right| P, S \right\rangle$$

SIDIS



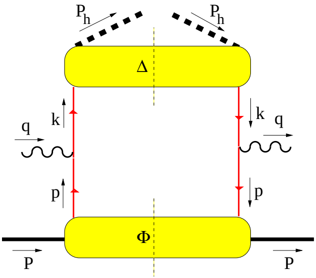

Belitsky, Ji, Yuan, NPB 656 (2003)
Boer, Mulders, Pijlman, NPB 667 (2003)



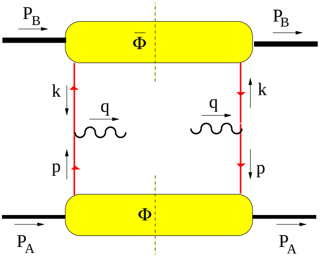

Possible effects in transverse momentum observables ($\xi_T$ is conjugate to $k_T$)

# TMD factorization
## Process dependence of gauge links

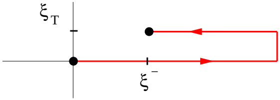
SIDIS

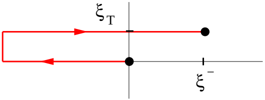



Drell-Yan




$\mathcal{U}^{[+]}_{[0,\xi]}$

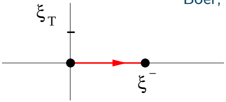



$\mathcal{U}^{[-]}_{[0,\xi]}$

Belitsky, Ji, Yuan, NPB 656 (2003)
Boer, Mulders, Pijlman, NPB 667 (2003)
Boer, talk at RBRC Synergies workshop (2017)

$\int \mathrm{d}k_T \longrightarrow \xi_T = 0 \longrightarrow$



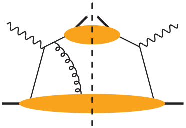

the same in both cases

# TMD factorization
## The quark Sivers function

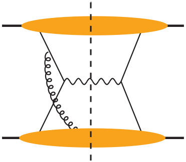
### Fundamental test of TMD theory

$$f_{1T}^{\perp\,[DY]}(x,\boldsymbol{k}_\perp^2) = -f_{1T}^{\perp\,[SIDIS]}(x,\boldsymbol{k}_\perp^2) \qquad h_1^{\perp\,[DY]}(x,\boldsymbol{k}_\perp^2) = -h_1^{\perp\,[SIDIS]}(x,\boldsymbol{k}_\perp^2)$$

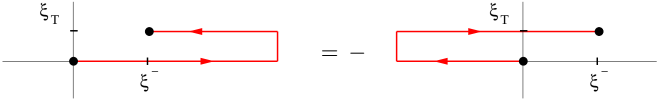
Collins, PLB 536 (2002)



FSI in SIDIS



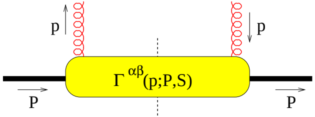
ISI in DY




ISI/FSI lead to process dependence of TMDs, could even break factorization

Collins, Qiu, PRD 75 (2007)
Collins, PRD 77 (2007)
Rogers, Mulders, PRD 81 (2010)

# Process dependence of gluon TMDs




### Gauge invariant definition of $\Gamma^{\mu\nu}$

$$\Gamma^{[\mathcal{U},\mathcal{U}']\mu\nu} \propto \langle P,S|\,\mathrm{Tr_c}\big[\,F^{+\nu}(0)\,\mathcal{U}^{\mathcal{C}}_{[0,\xi]}\,F^{+\mu}(\xi)\,\mathcal{U}^{\mathcal{C}'}_{[\xi,0]}\,\big]\,|P,S\rangle$$

Mulders, Rodrigues, PRD 63 (2001)
Buffing, Mukherjee, Mulders, PRD 88 (2013)
Boer, Cotogno, Van Daal, Mulders, Signori, Zhou, JHEP 1610 (2016)

The gluon correlator depends on two path-dependent gauge links

$ep \to e'\,Q\overline{Q}X$, $ep \to e'$ jet jet $X$ probe gluon TMDs with $[++]$ gauge links

$pp \to \gamma\gamma X$ (and/or other CS final state) probes gluon TMDs with $[--]$ gauge links

$pp \to \gamma\,\mathrm{jet}\,X$ probes an entirely independent gluon TMD: $[+-]$ links (dipole)

# The gluon Sivers functions
## Sign change test

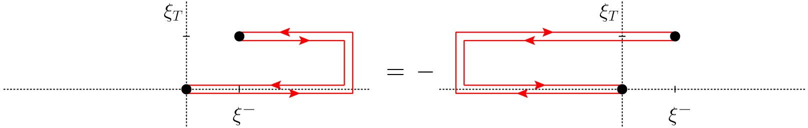
### Related Processes

$ep^\uparrow \to e' Q\overline{Q}X$, $ep^\uparrow \to e'\ \mathrm{jet\,jet}\,X$ probe GSF with $[++]$ gauge links (WW)

$p^\uparrow p \to \gamma\gamma X$ (and/or other CS final state) probe GSF with $[--]$ gauge links

**Analogue of the sign change of $f_{1T}^{\perp\,q}$ between SIDIS and DY (true also for $h_1^g$ and $h_{1T}^{\perp\,g}$)**

$$f_{1T}^{\perp\,g\,[e\,p^\uparrow \to e'\,Q\overline{Q}\,X]} = -f_{1T}^{\perp\,g\,[p^\uparrow\,p \to \gamma\,\gamma\,X]}$$



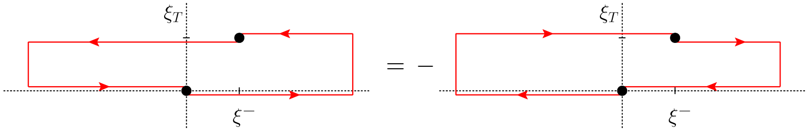

Boer, Mulders, CP, Zhou (2016)

Motivation to study gluon Sivers effects at both RHIC and the EIC

## Complementary Processes

$ep^{\uparrow} \to e' Q\overline{Q}X$ probes a GSF with $[++]$ gauge links (WW)

$p^{\uparrow}p \to \gamma\,\mathrm{jet}\,X$ $(gq \to \gamma q)$ probes a gluon TMD with : $[+-]$ links (DP)




At small-$x$ the WW Sivers function appears to be suppressed by a factor of $x$ compared to the unpolarized gluon function, unlike the dipole one

The DP gluon Sivers function at small-$x$ is the **spin dependent odderon** (single spin asymmetries from a single Wilson loop matrix element)

Boer, Echevarria, Mulders, Zhou, PRL 116 (2016)
Boer, Cotogno, Van Daal, Mulders, Signori, Zhou, JHEP 1610 (2016)

The first transverse moments of the WW and DP gluon Sivers functions

$$f_{1T}^{\perp(1)g\,(f/d)}(x) = \int \mathrm{d}^2\boldsymbol{k}_T \, \frac{k_T^2}{2M_p^2} \, f_{1T}^{\perp\, g\,(f/d)}(x, \boldsymbol{k}_T^2)$$

related to two different trigluon Qiu-Sterman functions $T_G^{(f/d)}$, involving the antisymmetric $f_{abc}$ and symmetric $d_{abc}$ color structures, respectively

Bomhof, Mulders, JHEP 0702 (2007)
Buffing, Mukherjee, Mulders, PRD 88 (2013)

The two distributions have a different behavior under charge conjugation

The Burkardt sum rule constraints only the $f$-type gluon Sivers function

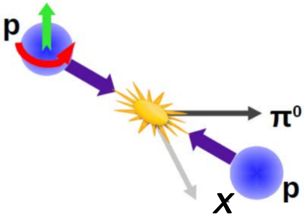
$$\sum_{a=q,\bar{q},g} \int \mathrm{d}x \, f_{1T}^{\perp(1)a}(x) = 0$$

Boer, Lorcé, CP, Zhou, AHEP 2015 (2015)

# The TMD Generalized Parton Model

# The Generalized Parton Model

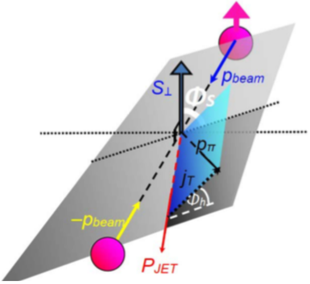
Phenomenological extension of the TMD formalism to processes like




$pp \to \pi X$

$(pp \to \text{jet}\, X,\ pp \to \gamma X)$

Single scale processes




$pp \to \text{jet}\, \pi X$

and more

Anselmino, Boglione, Murgia, PLB 362 (1995), ...
Aschenauer, D'Alesio, Murgia, EPJA52 (2016)

## Transverse Momentum Dependent – Generalized Parton Model (GPM)

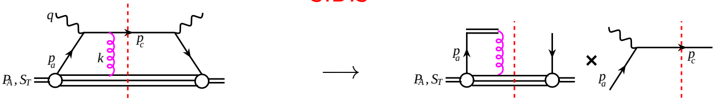
- Spin & $k_\perp$-dependent distribution and fragmentation functions as in TMD scheme
- $k_\perp$-dependence included in the hard scattering, unlike in the TMD formalism
- Universality and TMD factorization: assumption to be tested

The CGI-GPM takes into account the effects of initial and final state interactions

Gamberg, Kang, PLB 696 (2011)

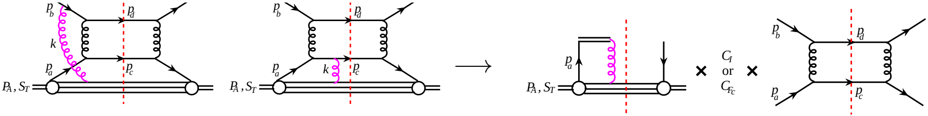
One-gluon exchange approx.: LO term of of the $\alpha_S$ expansion of the gauge link

SIDIS




$f_{1T}^{\perp q\,[\mathrm{SIDIS}]}$ × Hard part

$qq' \to qq'$



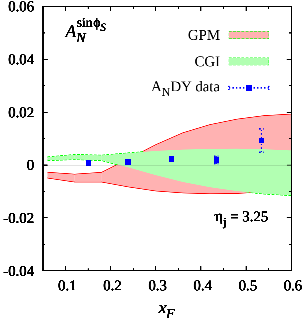

$f_{1T}^{\perp q\,[\mathrm{SIDIS}]}$ × (CF × Hard part)

$f_{1T}^{\perp q[\mathrm{SIDIS}]}$ is universal, process dependence absorbed in *modified* hard functions

The CGI-GPM recovers the relation $f_{1T}^{\perp\,[DY]} = -f_{1T}^{\perp\,[SIDIS]}$

In the CGI-GPM TMDs are process dependent, different predictions w.r.t. GPM

Gamberg, Kang, PLB 696 (2011)
D'Alesio, Gamberg, Kang, Murgia, CP, PLB 704 (2011)




$p^{\uparrow} p \to \text{jet}\, X$

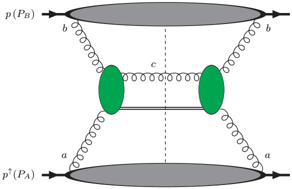
$(\sqrt{s} = 500$ GeV$)$

Extension of the CGI-GPM to the gluon Sivers function is now completed

D'Alesio, Murgia, CP, Taels, PRD 96 (2017)
D'Alesio, Flore, Murgia, CP, in preparation

Gluon Sivers function constrained from available data on $p^{\uparrow} p \to \pi^0 X$, $p^{\uparrow} p \to J/\psi\, X$, $p^{\uparrow} p \to DX$, predictions for $p^{\uparrow} p \to \gamma X$ at RHIC

In the Color Singlet Model, the dominant production channel is $gg \to J/\psi g$

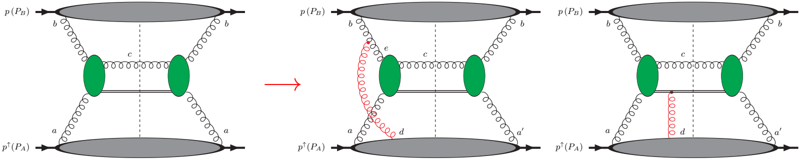



$$A_N \equiv \frac{\mathrm{d}\sigma^{\uparrow} - \mathrm{d}\sigma^{\downarrow}}{\mathrm{d}\sigma^{\uparrow} + \mathrm{d}\sigma^{\downarrow}} \equiv \frac{\mathrm{d}\Delta\sigma}{2\mathrm{d}\sigma}$$

$$\mathrm{d}\Delta\sigma^{\mathrm{GPM}} = \frac{2\alpha_s^3}{s} \int \frac{\mathrm{d}x_a}{x_a} \frac{\mathrm{d}x_b}{x_b} \, \mathrm{d}^2\boldsymbol{k}_{\perp a} \, \mathrm{d}^2\boldsymbol{k}_{\perp b} \; \delta(\hat{s} + \hat{t} + \hat{u} - M^2)$$
$$\times \left( -\frac{k_{\perp\, a}}{M_p} \right) f_{1T}^{\perp\, g}(x_a, k_{\perp a}) \cos\phi_a \, f_{g/p}(x_b, k_{\perp b}) \, H^U_{gg \to J/\psi g}(\hat{s}, \hat{t}, \hat{u})$$

$f_{1T}^{\perp\, g}$: Gluon Sivers function (one and process independent)

GPM CGI-GPM

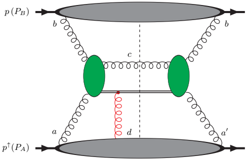



$C_U$ $C_I^{(f/d)}$ $C_{F_c}^{(f/d)}$ [Color Factors]

$$[\text{GPM}]\;\; f_{1T}^{\perp\, g}\, H^U_{gg\to J/\psi g} \longrightarrow f_{1T}^{\perp\, g\,(f)} H^{\text{Inc}\,(f)}_{gg\to J/\psi g} + f_{1T}^{\perp\, g\,(d)} H^{\text{Inc}\,(d)}_{gg\to J/\psi g} \;\; [\text{CGI}-\text{GPM}]$$

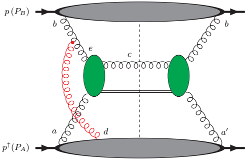
Two independent, universal $f_{1T}^{\perp}$'s, process dependence shifted into new hard parts

$$H^{\text{Inc}\,(f/d)}_{gg\to J/\psi g} \equiv \frac{C_I^{(f/d)} + C_{F_c}^{(f/d)}}{C_U}\, H^U_{gg\to J/\psi g}$$




$c\bar{c}$ pair in a color singlet state, no FSIs: $C_{F_c}^{(f)} = C_{F_d}^{(d)} = 0$

F. Yuan, PRD 78 (2003)



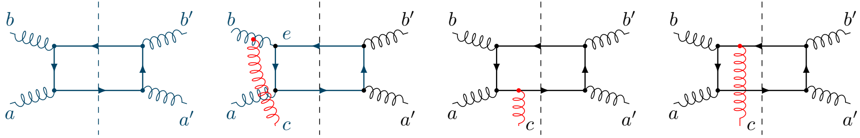

$$C_I^{(f)} = -\tfrac{1}{2}\, C_U \qquad C_I^{(d)} = 0$$

Only $f_{1T}^{\perp\, g\,(f)}$ contributes to $A_N$

$$\mathrm{d}\Delta\sigma^{\mathrm{CGI}} = \frac{2\alpha_s^3}{s} \int \frac{\mathrm{d}x_a}{x_a}\, \frac{\mathrm{d}x_b}{x_b}\, \mathrm{d}^2\boldsymbol{k}_{\perp a}\, \mathrm{d}^2\boldsymbol{k}_{\perp b}\, \delta(\hat{s}+\hat{t}+\hat{u}-M^2)$$
$$\times \left(-\frac{k_{\perp\, a}}{M_p}\right) f_{1T}^{\perp\, g\,(f)}(x_a, k_{\perp a}) \cos\phi_a\, f_{g/p}(x_b, k_{\perp b})\, H^{\mathrm{Inc}\,(f)}_{gg\to J/\psi g}(\hat{s}, \hat{t}, \hat{u})$$

$$H^{\mathrm{Inc}\,(f)}_{gg\to J/\psi g} = -\frac{1}{2}\, H^{U}_{gg\to J/\psi g}$$

LO channels are $gg \to c\bar{c}$ and $q\bar{q} \to c\bar{c}$. Color factors for $gg \to c\bar{c}$:




$C_U$ $\quad C_I^{(f/d)}$ $\quad C_{F_c}^{(f/d)}$ $\quad C_{F_d}^{(f/d)}$

Agreement with *gluonic pole strenghts* calculated for $p^\uparrow p \to h\, h\, X$

$$C_G^{(f/d)} \equiv \frac{C_I^{(f/d)} + C_{F_c}^{(f/d)} + C_{F_d}^{(f/d)}}{C_U}$$

Bomhof, Mulders, JHEP 0702 (2007)

Agreement with twist-three results for $p^\uparrow p \to D X$

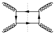
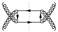
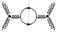
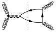
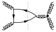
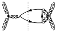
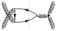
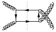
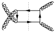
Kang, Qiu, Vogelsang, Yuan, PRD 78 (2008)

Both $f_{1T}^{\perp g\,(f)}$ and $f_{1T}^{\perp g\,(d)}$ contribute to $A_N(p^\uparrow p \to D X)$

# Gluon Sivers function in $p^{\uparrow} p \to D X$

## Color factors for $gg \to c\bar{c}$

$$C_I^{\mathrm{Inc}\,(f/d)} \equiv C_I^{(f/d)} + C_{F_c}^{(f/d)}$$

| $D$ | $C_U$ | $C_I^{(f)}$ | $C_{Fc}^{(f)}$ | $C_{F_d}^{(f)}$ | $C^{\mathrm{Inc}\,(f)}$ | $C_I^{(d)}$ | $C_{F_c}^{(d)}$ | $C_{F_d}^{(d)}$ | $C^{\mathrm{Inc}\,(d)}$ |
|---|---|---|---|---|---|---|---|---|---|
|  | $\frac{1}{4N_c}$ | $-\frac{N_c}{8(N_c^2-1)}$ | $\frac{1}{8N_c}$ | $-\frac{1}{8N_c(N_c^2-1)}$ | $-\frac{1}{8N_c(N_c^2-1)}$ | $\frac{N_c}{8(N_c^2-1)}$ | $\frac{1}{8N_c}$ | $\frac{1}{8N_c(N_c^2-1)}$ | $\frac{2N_c^2-1}{8N_c(N_c^2-1)}$ |
|  | $\frac{1}{4N_c}$ | $-\frac{N_c}{8(N_c^2-1)}$ | $-\frac{1}{8N_c(N_c^2-1)}$ | $\frac{1}{8N_c}$ | $-\frac{N_c^2+1}{8N_c(N_c^2-1)}$ | $-\frac{N_c}{8(N_c^2-1)}$ | $-\frac{1}{8N_c(N_c^2-1)}$ | $-\frac{1}{8N_c}$ | $-\frac{N_c^2+1}{8N_c(N_c^2-1)}$ |
|  | $\frac{N_c}{2(N_c^2-1)}$ | $-\frac{N_c}{4(N_c^2-1)}$ | $\frac{N_c}{8(N_c^2-1)}$ | $\frac{N_c}{8(N_c^2-1)}$ | $-\frac{N_c}{8(N_c^2-1)}$ | $0$ | $\frac{N_c}{8(N_c^2-1)}$ | $-\frac{N_c}{8(N_c^2-1)}$ | $\frac{N_c}{8(N_c^2-1)}$ |
|  | $\frac{N_c}{4(N_c^2-1)}$ | $-\frac{N_c}{8(N_c^2-1)}$ | $\frac{N_c}{8(N_c^2-1)}$ | $0$ | $0$ | $\frac{N_c}{8(N_c^2-1)}$ | $\frac{N_c}{8(N_c^2-1)}$ | $0$ | $\frac{N_c}{4(N_c^2-1)}$ |
|  | $\frac{N_c}{4(N_c^2-1)}$ | $-\frac{N_c}{8(N_c^2-1)}$ | $\frac{N_c}{8(N_c^2-1)}$ | $0$ | $0$ | $\frac{N_c}{8(N_c^2-1)}$ | $\frac{N_c}{8(N_c^2-1)}$ | $0$ | $\frac{N_c}{4(N_c^2-1)}$ |
|  | $-\frac{N_c}{4(N_c^2-1)}$ | $\frac{N_c}{8(N_c^2-1)}$ | $0$ | $-\frac{N_c}{8(N_c^2-1)}$ | $\frac{N_c}{8(N_c^2-1)}$ | $\frac{N_c}{8(N_c^2-1)}$ | $0$ | $\frac{N_c}{8(N_c^2-1)}$ | $\frac{N_c}{8(N_c^2-1)}$ |
|  | $-\frac{N_c}{4(N_c^2-1)}$ | $\frac{N_c}{8(N_c^2-1)}$ | $0$ | $-\frac{N_c}{8(N_c^2-1)}$ | $\frac{N_c}{8(N_c^2-1)}$ | $\frac{N_c}{8(N_c^2-1)}$ | $0$ | $\frac{N_c}{8(N_c^2-1)}$ | $\frac{N_c}{8(N_c^2-1)}$ |
|  | $-\frac{1}{4N_c(N_c^2-1)}$ | $0$ | $-\frac{1}{8N_c(N_c^2-1)}$ | $-\frac{1}{8N_c(N_c^2-1)}$ | $-\frac{1}{8N_c(N_c^2-1)}$ | $0$ | $-\frac{1}{8N_c(N_c^2-1)}$ | $\frac{1}{8N_c(N_c^2-1)}$ | $-\frac{1}{8N_c(N_c^2-1)}$ |
|  | $-\frac{1}{4N_c(N_c^2-1)}$ | $0$ | $-\frac{1}{8N_c(N_c^2-1)}$ | $-\frac{1}{8N_c(N_c^2-1)}$ | $-\frac{1}{8N_c(N_c^2-1)}$ | $0$ | $-\frac{1}{8N_c(N_c^2-1)}$ | $\frac{1}{8N_c(N_c^2-1)}$ | $-\frac{1}{8N_c(N_c^2-1)}$ |

D'Alesio, Murgia, Pisano, Taels, PRD 96 (2017)

Modified hard functions $H^{\mathrm{Inc}\,(f/d)}$ are not simply proportional to $H_U$

Similar tables and results for all the channels in $p^{\uparrow} p \to \pi X$

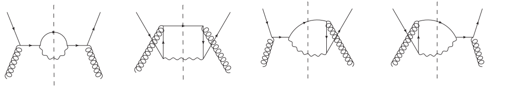
D'Alesio, Flore, Murgia, Pisano, Taels, in preparation

# Gluon Sivers function in $p^{\uparrow} p \to \gamma X$
## $A_N$ in the CGI-GPM

LO channels are $qg \to \gamma q, q\bar{q} \to \gamma g$ and $gq \to \gamma q$.

Gamberg, Kang, PLB 696 (2011)
D'Alesio, Flore, Murgia, Pisano, Taels, in preparation

Color factors for $gq \to \gamma q$:

| $D$ | $C_U$ | $C_I^{(f)}$ | $C_{F_d}^{(f)}$ | $C^{\mathrm{Inc}\,(f)}$ | $C_I^{(d)}$ | $C_{F_d}^{(d)}$ | $C^{\mathrm{Inc}\,(d)}$ |
|---|---|---|---|---|---|---|---|
|  | $\frac{1}{2N_c}$ | $-\frac{1}{4N_c}$ | $\frac{1}{4N_c}$ | $-\frac{1}{4N_c}$ | $\frac{1}{4N_c}$ | $\frac{1}{4N_c}$ | $\frac{1}{4N_c}$ |

Simple color structure, both $f_{1T}^{\perp\, g\,(f)}$ and $f_{1T}^{\perp\, g\,(d)}$ contribute to $A_N(p^{\uparrow} p \to \gamma X)$

$$H_{gq\to\gamma q}^{\mathrm{Inc}\,(f)} = H_{g\bar{q}\to\gamma\bar{q}}^{\mathrm{Inc}\,(f)} = -\frac{1}{2}\, H_{gq\to\gamma q}^{U} = -\frac{1}{2N_c}\left(-\frac{\hat{u}}{\hat{s}} - \frac{\hat{s}}{\hat{u}}\right)$$

$$H_{gq\to\gamma q}^{\mathrm{Inc}\,(d)} = -H_{g\bar{q}\to\gamma\bar{q}}^{\mathrm{Inc}\,(d)} = \frac{1}{2}\, H_{gq\to\gamma q}^{U} = \frac{1}{2N_c}\left(-\frac{\hat{u}}{\hat{s}} - \frac{\hat{s}}{\hat{u}}\right)$$

- Single spin asymmetries $A_N$ for $p^\uparrow p \to h(\gamma) X$ can be described within the GPM, which includes both spin and transverse momentum effects
- In the CGI-GPM, effects of ISI/FSI are taken into account in the one-gluon exchange approximation
- As a consequence, TMDs become process dependent
- In this framework, the gluon Sivers effect is given by the convolution of two independent gluon Sivers functions with *modified* hard functions
- These two distributions can be in principle singled out by looking at available data on $A_N$ in $p^\uparrow p \to \pi^0 X$, $p^\uparrow p \to J/\psi X$, $p^\uparrow p \to DX$, $p^\uparrow p \to \gamma X$ at RHIC

Talk by U. D'Alesio