Extracting the scalar dynamical polarizabilities from real Compton scattering data

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SUMMARY

- Introduction
  - RCS: theoretical framework
  - Static and Dynamical polarizabilities
  - Dispersion Relations and Low Energy Expansion

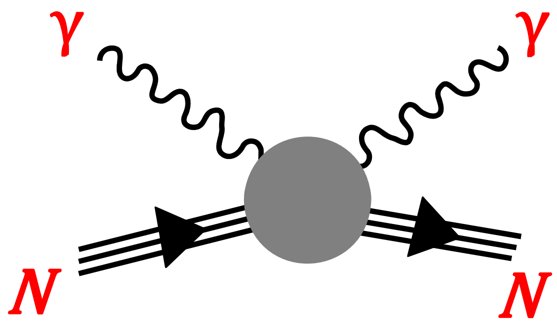
- First evaluation of the scalar proton dynamical polarizabilities
  - Problems in the existing data set
  - New fit approach: simplex + bootstrap
  - A cross check: static polarizabilities
  - Dynamical polarizabilities

- Conclusions and perspectives

B. Pasquini, P. Pedroni, S. Sconfietti,
Phys. Rev.C 98, 015204 (2018)

Real Compton Scattering

Expansion of the Hamiltonian in incident photon energy (ω)



0th order ⟹ **charge, mass**

1st order ⟹ **magnetic moment**

«point-like» nucleon (Born terms)

2nd order ⟹ **2 scalar polarizabilitites** (≈ known)

Baldin's sum rule: $(\alpha_{E1}+\beta_{M1}) \sim$ **known**

$$H^{(2)}_{eff} = -4\pi\left[\frac{1}{2}\alpha_{E1}\vec{E}^2 + \frac{1}{2}\beta_{M1}\vec{H}^2\right]$$

3rd order ⟹ **4 spin (vector) polarizabilitites** **only one direct measurement**

P. Martel et al, PRL 114, 112501 (2015)

$$H^{(3)}_{eff} = -4\pi\left[\begin{array}{c}\frac{1}{2}\gamma_{E1E1}\vec{\sigma}\cdot\left(\vec{E}\times\dot{\vec{E}}\right) + \frac{1}{2}\gamma_{M1M1}\vec{\sigma}\cdot\left(\vec{H}\times\dot{\vec{H}}\right) \\ -\gamma_{M1E2}E_{ij}\sigma_i H_j + \gamma_{E1M2}H_{ij}\sigma_i E_j\end{array}\right]$$

$$\boxed{\begin{array}{l} E_{ij} = \frac{1}{2}\left(\nabla_i E_j + \nabla_j E_i\right) \\ H_{ij} = \frac{1}{2}\left(\nabla_i H_j + \nabla_j H_i\right)\end{array}}$$

$$\boxed{\gamma_0} = -\gamma_{E1E1} - \gamma_{E1M2} - \gamma_{M1M1} - \gamma_{M1E2}$$

$$\boxed{\gamma_\pi} = -\gamma_{E1E1} - \gamma_{E1M2} + \gamma_{M1M1} + \gamma_{M1E2}$$

(≈ known)

Multipole Expansion for RCS

R.Hildebtrandt et al., EPJA 20, 293 (2004)

$$T_{fi} = \frac{4\pi W}{M} \sum_{i=1}^{6} \rho_i R_i(\omega, \cos\theta)$$

R_i $\Rightarrow$ 6 Independent amplitudes

$$R_1 = \sum_{l\geq 1} \{[(l+1) f_{EE}^{l+} + l f_{EE}^{l-}](l P_l' + P_{l-1}'') - [(l+1) f_{MM}^{l+} + l f_{MM}^{l-}] P_l''\}$$

$$R_2 = \sum_{l\geq 1} \left\{[(l+1) f_{MM}^{l+} + l f_{MM}^{l-}](l P_l' + P_{l-1}'') - [(l+1) f_{EE}^{l+} + l f_{EE}^{l-}] P_l''\right\}$$

2 spin-independent amplitudes

$f_{TT'}^{l\pm}$ Corresponds to the transition $Tl \rightarrow T'l'$ with $T, T' = E, M \quad ; \quad l' = l \pm \{0,1\}$

Multipole expansion + nucleon polarizabilities $\Rightarrow$ **Dynamical polarizabilities**

$$\alpha_{E1-DYN}(\omega) = \frac{2 f_{EE}^{1+}(\omega) + f_{EE}^{1-}(\omega)}{\omega^2} \quad ; \quad \beta_{M1-DYN}(\omega) = \frac{2 f_{MM}^{1+}(\omega) + f_{MM}^{1-}(\omega)}{\omega^2}$$

$$\alpha_{E1} = \lim_{\omega\to 0} \alpha_{E1-DYN}(\omega) \quad ; \quad \beta_{M1} = \lim_{\omega\to 0} \beta_{M1-DYN}(\omega)$$

How to extract dynamical polarizabilites from data ?

Our method $\Rightarrow$ Dispersion relations (DRs) + Low Energy Expansion (LEX) $(\omega < 140$ MeV)

B.Holstein et al., PRC61, 034316 (2000)

RCS differential cross section $\rightarrow$ 6 amplitudes $A_i(\nu,t)$ $\nu \rightarrow \omega + t/4M$

$t \rightarrow$ transferred momentum

$A_i(\nu,t)$ are connected to the multipoles $f^{l\pm}_{TT'}$

$\left[A_i(0,0) - A_i^B(0,0)\right]$ are connected to the 6 static polarizabilites

$$\mathrm{Re}\, A_i(\nu,t) = A_i^B(\nu,t) + \left[A_i(0,t) - A_i^B(0,t)\right] + \frac{2}{\pi}\nu^2 P\int_{\nu_{thr}}^{+\infty} d\nu' \frac{\mathrm{Im}_s A_i(\nu',t)}{\nu'(\nu'^2-\nu^2)}$$

Dispersion relations

Born terms (can be exactly calculated)

Can be evaluated from $\gamma\gamma \rightarrow \pi\pi, \pi\pi \rightarrow N\overline{N}$, $\gamma N \rightarrow N\pi(\pi)$ data

$$\alpha^{DR}_{E1-DYN}(\omega) = f_\alpha(\alpha_{E1}, \beta_{M1}, \alpha_{E1,\nu}, \beta_{M1,\nu}) + g_\alpha(\gamma_i) + h_\alpha(\text{any other term})$$

$$\beta^{DR}_{M1-DYN}(\omega) = f_\beta(\alpha_{E1}, \beta_{M1}, \alpha_{E1,\nu}, \beta_{M1,\nu}) + g_\beta(\gamma_i) + h_\beta(\text{any other term})$$

(up to ω^5)

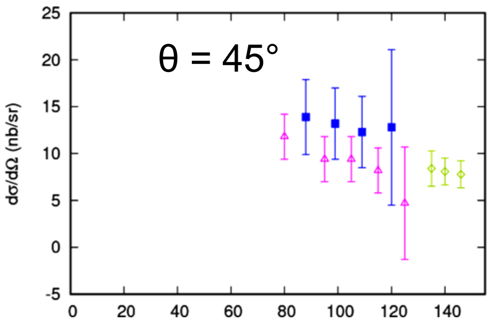
2 new additional parameters to be fitted

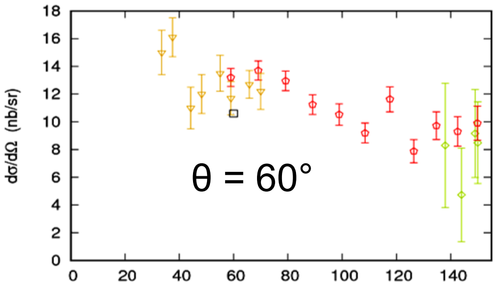
Calculated using measured γ_i values

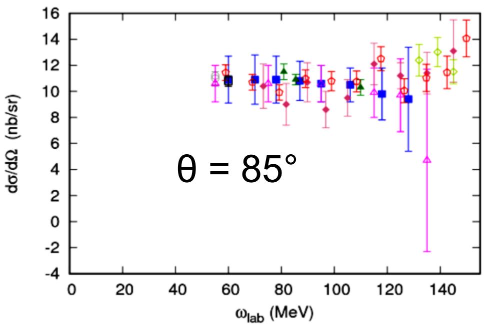
evaluated with DRs

The proton data base

($\omega < 140$ MeV)



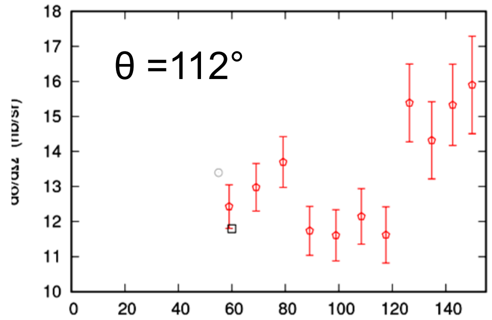


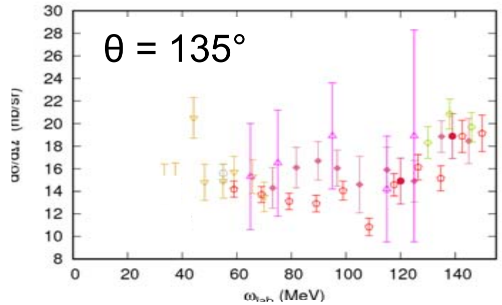


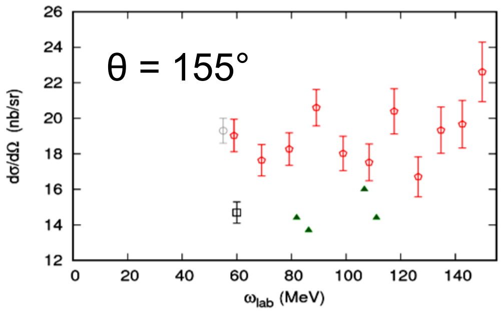
Symbol	Set	Ref
	GOL 60	Goldansky et al.
	OdL 01	Olmos de León
	HAL 93	Hallin et al.
	HYM 59	Hyman et al.
	PUG 67	Pugh et al.
	FED 91	Federspiel et al.
	BER 61	Bernardini et al.
	BAR 74	Baranov et al.
	OXL 58	Oxley
	MAC 95	MacGibbon et al.

(Only) 150 points

*(**Half of the Spartans** that King Leonidas led to the Battle of Thermopylae in 480 BC)*







$\Rightarrow$ **Poor quality of the data set**

Large statistic -and systematical- errors ; possible inconsistencies between subsets

Complications

3-parameter fit $(\alpha_{E1}-\beta_{M1})$; $\alpha_{E1,\nu}$; $\beta_{M1,\nu}$ $\rightarrow$ $(\alpha_{E1}+\beta_{M1})$ from Baldin's sum rule

Standard gradient (Newton) method to find the minimum of the "χ^2 function" using first and second derivatives

MINUIT WARNING IN HESSE
============== MATRIX FORCED POS-DEF BY ADDING 0.13727E-01 TO DIAGONAL.

Too high correlations between fitted parameters!

VERY low sensitivity of the data to dynamical polarizabilities

NO WAY to find the "right" minimum and to define "right" errors on fit parameters

Combination of **SIMPLEX** method and **BOOTSTRAP** technique
(purely geometrical search) (Monte Carlo)

Sampling using parametric bootstrap

$$S_{i,boot} \sim \mathrm{Gauss}(S_{i,\exp}, \sigma^2_{i,\exp}) \;\; ; \;\; i = 1, \ldots, N_{tot}$$

Gaussian distributed statistical errors

Differential cross section value of the i-th point

After each bootstap cycle

- **Simplex minimization is performed and fitted parameters are stored in histograms.**
- **Their probability distribution can then empirically found after a sufficiently large number of cycles**

How can systematical errors (common scale factor) be included ?

«standard method»

$$\chi^2_{mod} = \sum_{i=1}^{N_{tot}} \left[\frac{\mathcal{N} S_{i,exp} - S_{i,theory}}{\mathcal{N} \sigma_{i,exp}} \right]^2 + \left(\frac{\mathcal{N} - 1}{\sigma_{i,sys}} \right)^2$$

...one additional normalization factor per data subset is needed!

«bootstrap method»

$$S^{SYS}_{j,boot} = \xi^{Subset}_j \cdot S_{j,boot} \;\; ; \;\; j = 1, \ldots, N^{Subset}_{TOT}$$

$$\xi \sim P\left(1, \sigma^2_{SYS}\right)$$

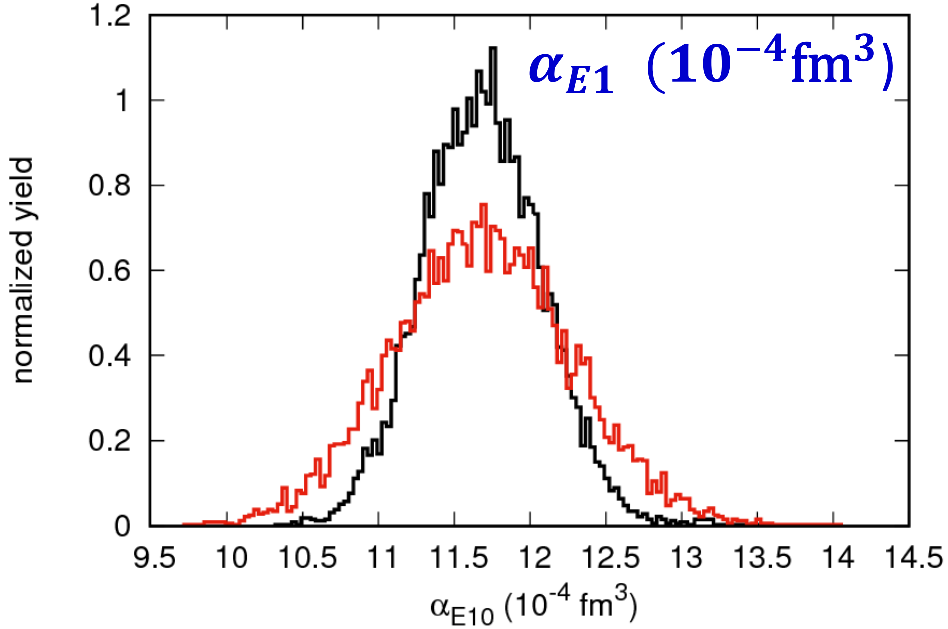
Often a uniform distribution

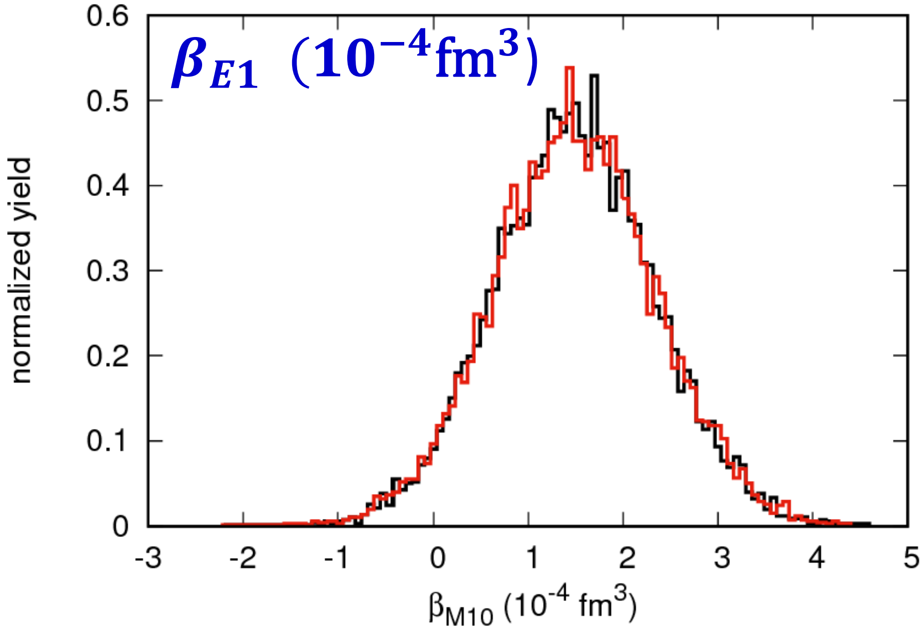
At every bootstrap cycle the systematical error is sampled independently for each subset

A check: fit only scalar static polarizabilites

2-parameter fit α_{E1} ; β_{M1}

10,000 replicas

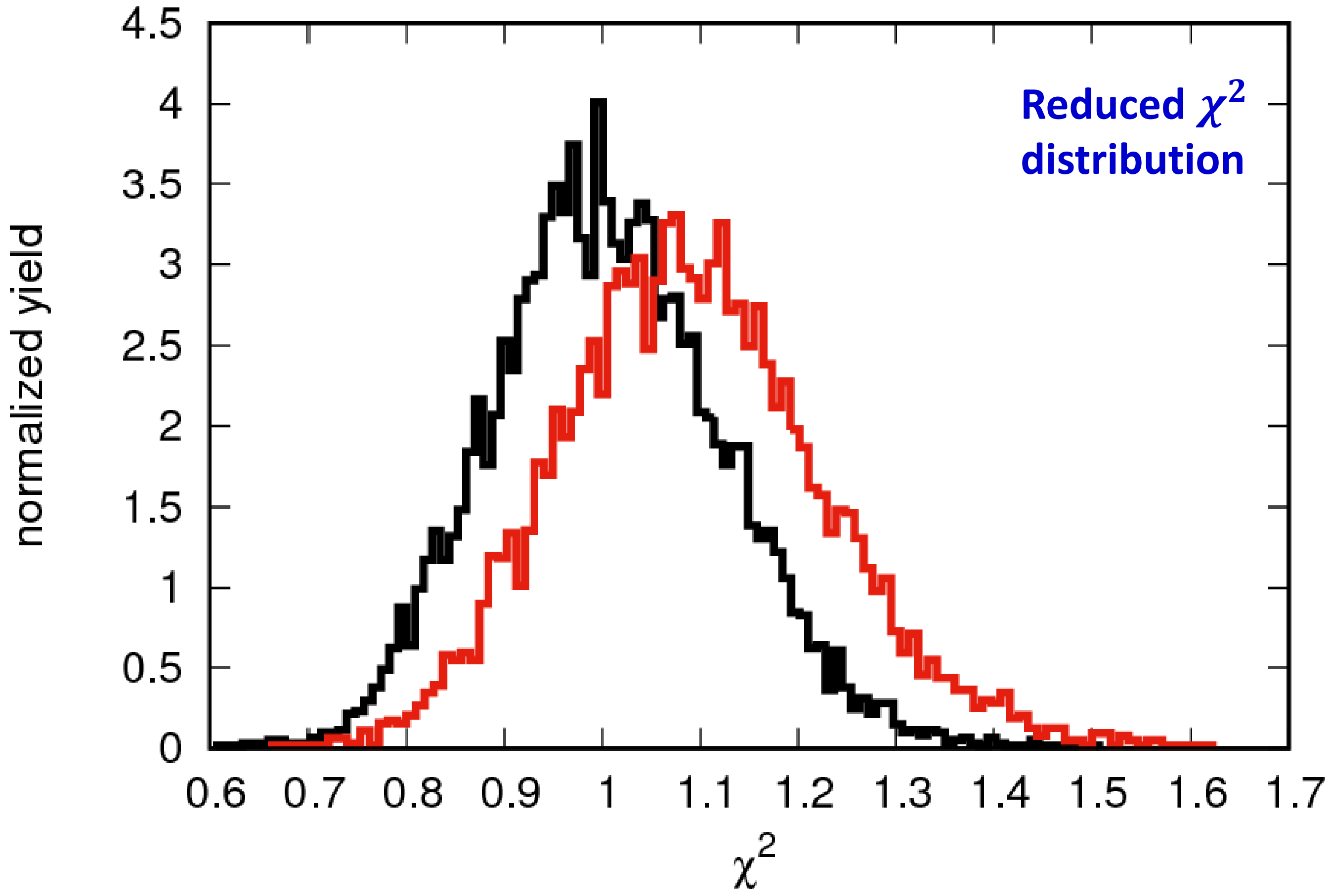




Expected Gaussian shape + broadening due to systematics

Systematics OFF **Systematics ON**

“equivalent χ^2” probability distribution from bootstrap



Static polarizabilities fit

Systematics OFF

Systematics ON

Not even a “true” χ^2 distribution (due to correlations among points given by systematics)

Comparison: bootstrap vs gradient

1-parameter fit $(\alpha_{E1}-\beta_{M1})$ $\rightarrow$ $(\alpha_{E1}+\beta_{M1})$ from Baldin's sum rule

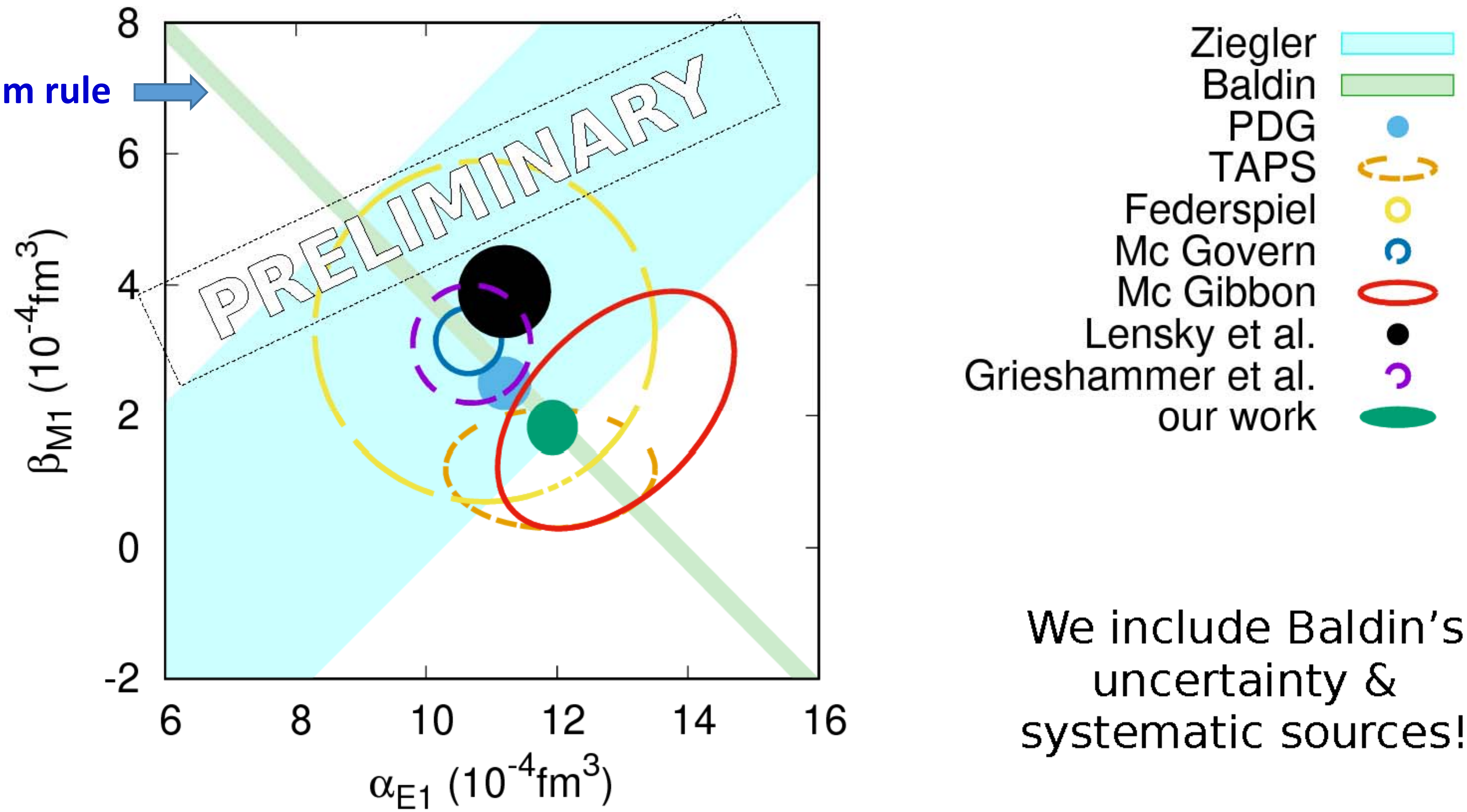
	α_{E1}	β_{M1}
GRADIENT SYS OFF	11.8 ± 0.2	2.0 ∓ 0.2
BOOTSTRAP SYS OFF	11.8 ± 0.2	2.0 ∓ 0.2
BOOTSTRAP SYS ON	11.8 ± 0.3	2.0 ∓ 0.3

Model: DRs+LEX

Systematical errors enlarge the error band of the polarizabilities

Summary plot

Baldin's sum rule



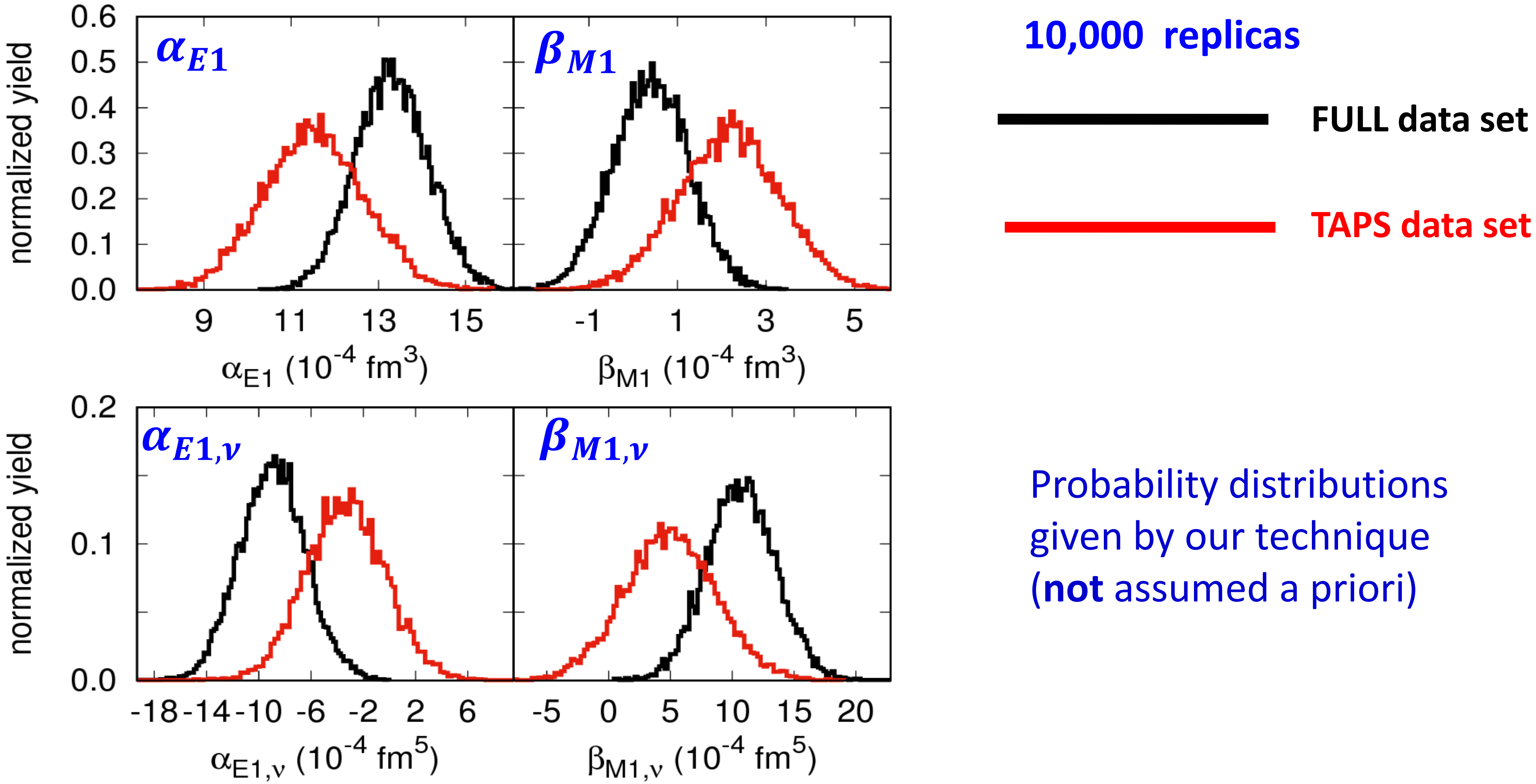
We include Baldin's uncertainty & systematic sources!

Bootstrap and dynamical polarizabilities

3-parameter fit $(\alpha_{E1} - \beta_{M1})$; $\alpha_{E1,\nu}$; $\beta_{M1,\nu}$

- ✓ Baldin's sum rule
- ✓ Systematical errors ON
- ✓ FULL data set (150 data)
- ✓ TAPS data set (55 data) → O. De Leon et al., EPJA 10, 207 (2001)
- ✓ Errors on Baldin's sum rule and γ_π included in the procedure

Dynamical polarizabilities: fit distributions

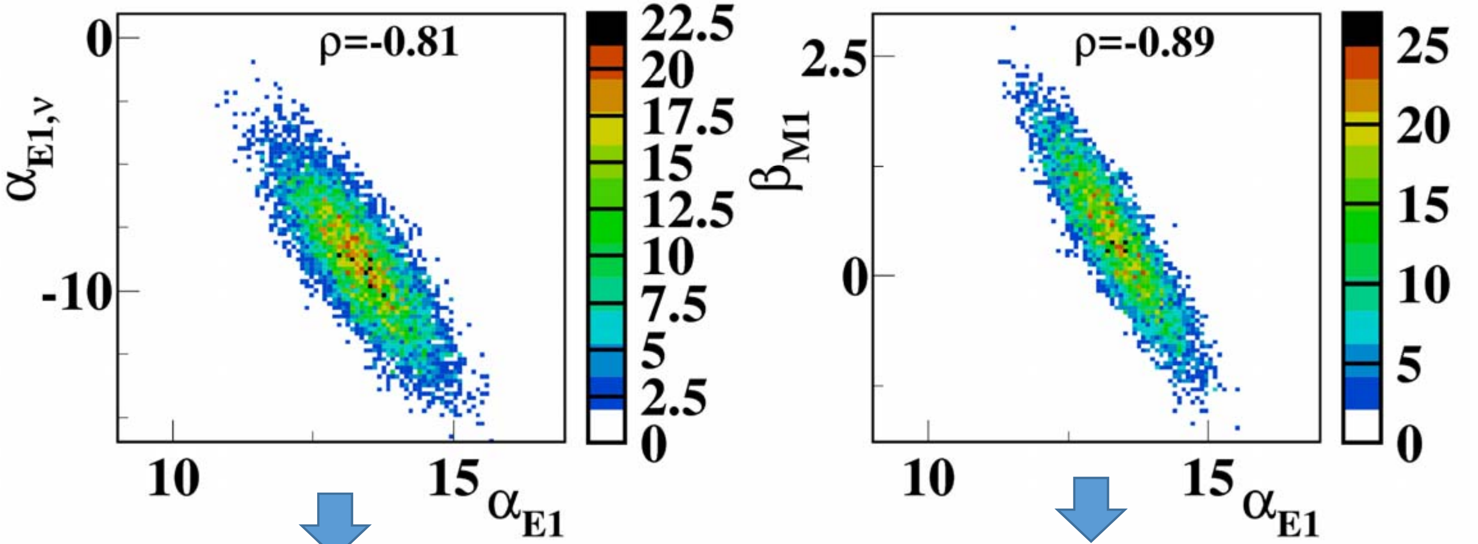


Probability distributions given by our technique (**not** assumed a priori)

Dynamical polarizabilities: fit results

		FULL	TAPS
α_{E1}	(10^{-4}fm^3)	13.3 ± 0.8	11.6 ± 1.1
$\alpha_{E1,\nu}$	(10^{-4}fm^5)	-8.8 ± 2.5	-3.2 ± 3.1
β_{M1}	(10^{-4}fm^3)	0.4 ∓ 0.9	2.2 ∓ 1.1
$\beta_{M1,\nu}$	(10^{-4}fm^5)	10.8 ± 2.8	5.1 ± 3.7

Quite strong dependence on data set (maybe due to different covered angular regions ?)

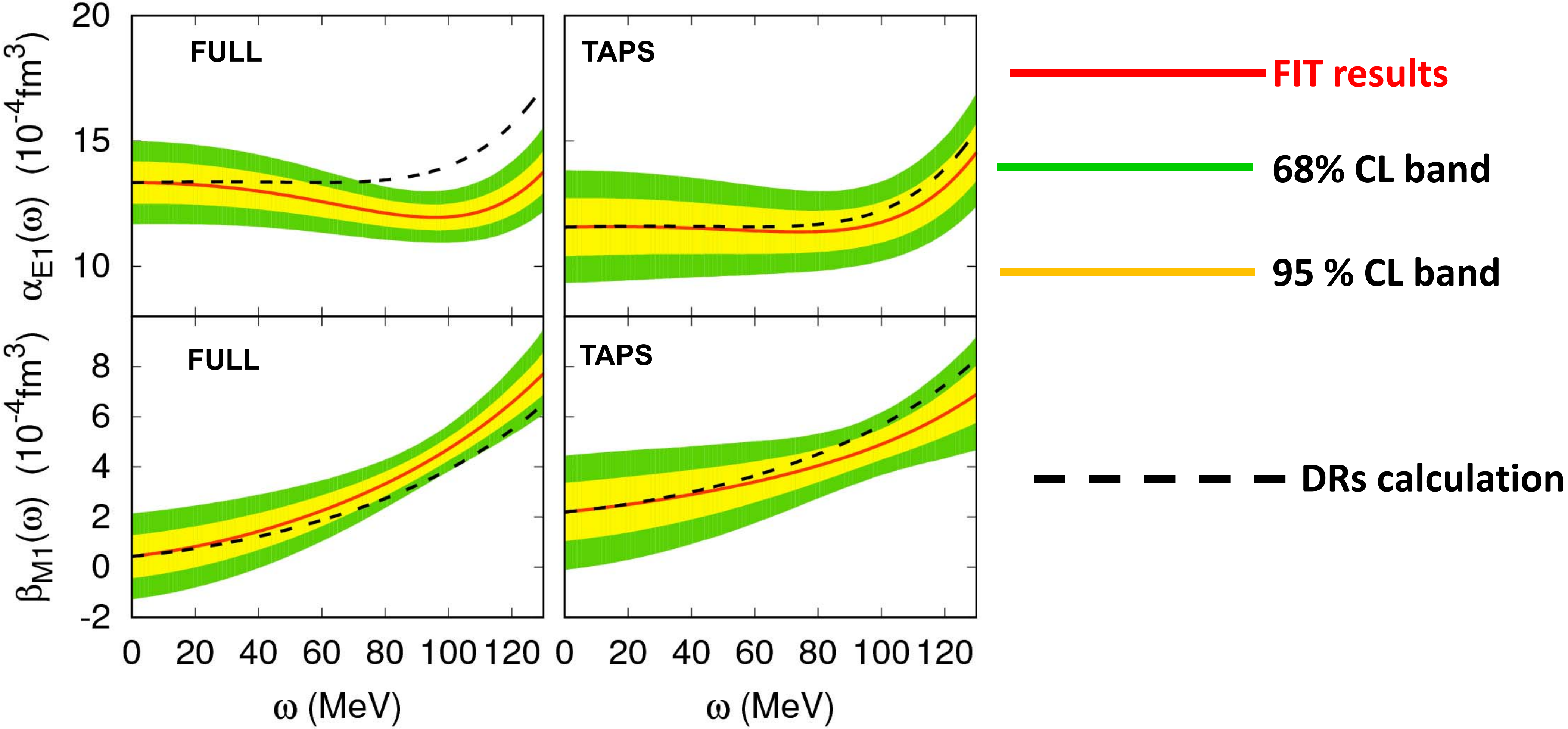


Very strong correlations between the fit parameters

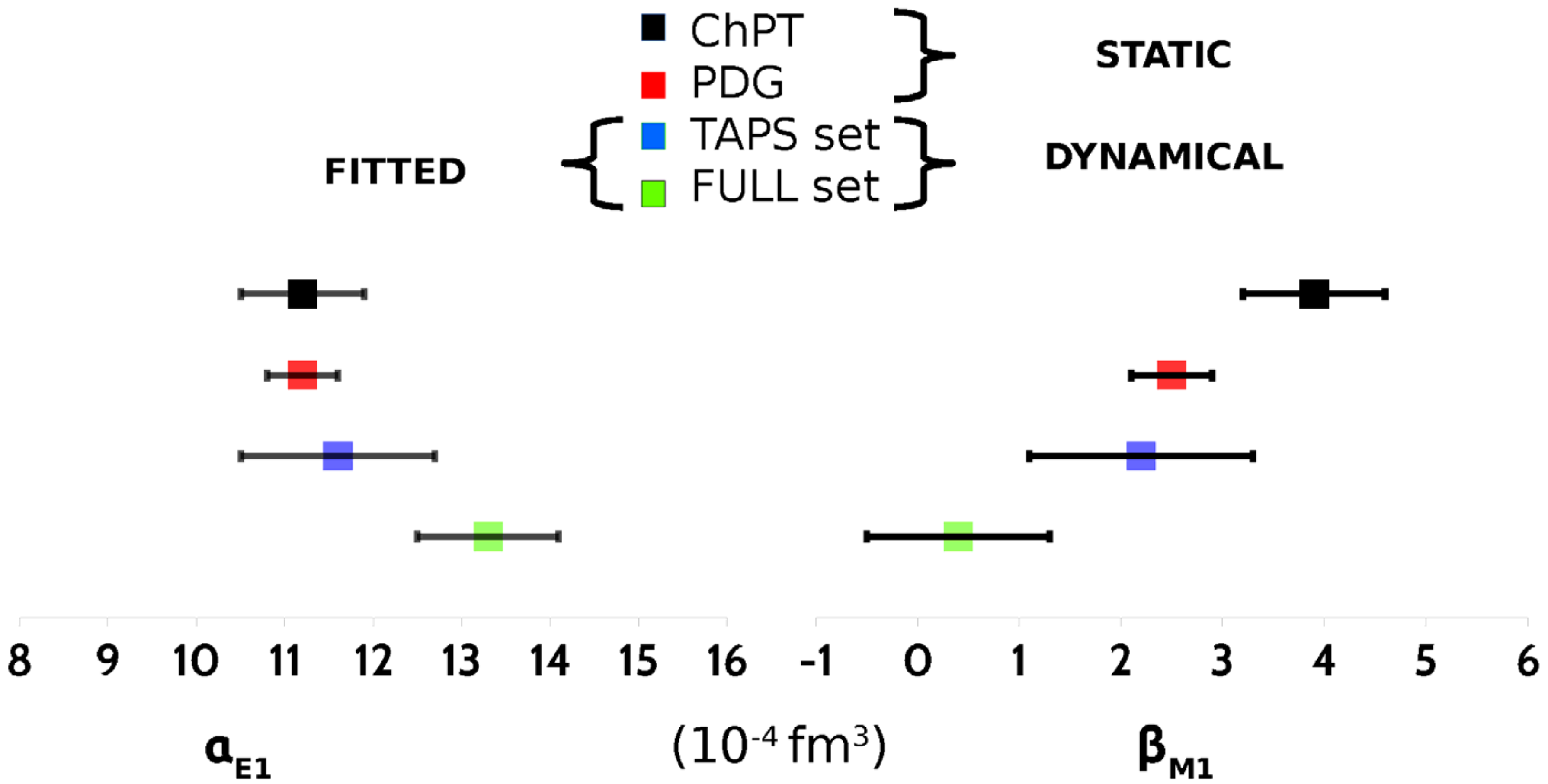
Very low sensitivity of the data to $\alpha_{E1,\nu}$

$(\alpha_{E1}+\beta_{M1})$ Constrained by the Baldin's rum rule value ($\rho \neq 1$ due to its uncertainty)

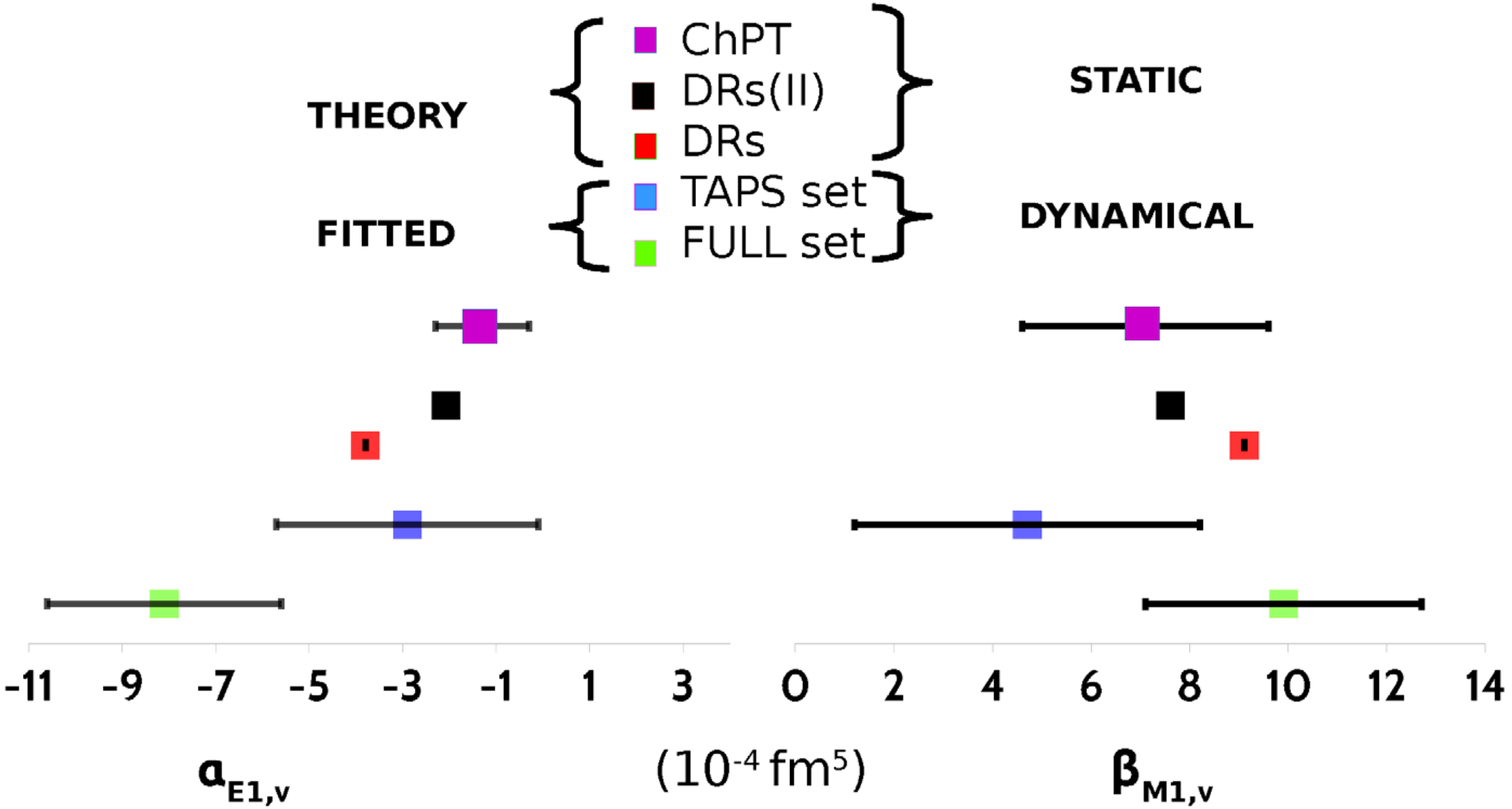
Dynamical polarizabilities: fit results



Dynamical polarizabilities: comparisons



Dynamical polarizabilities: comparisons



V. Lensky et al., EPJC 75, 604 (2015)

B. Pasquini et al., PRC 76, 015203 (2007)

B. Holstein et al., PRC 61, 034316 (2000)

Conclusions and perspectives

- **First evaluation of the scalar proton dynamical scalar polarizabilites**

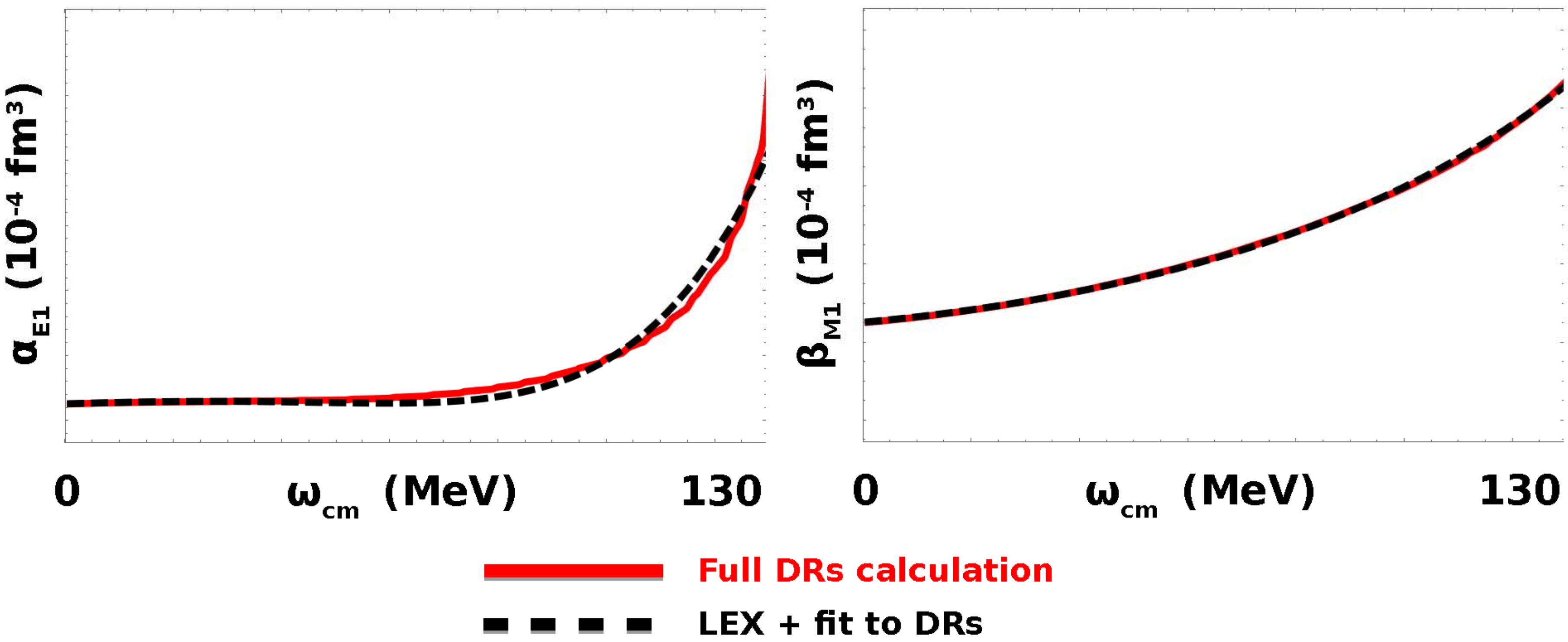
- **Bootstrap technique**
  - Very useful and versatile method for data fitting
  - No a priori assumption needed for the experimental errors
  - Effect of systematic uncertainties included in a straightforward way

- **The only way to get a more accurate and consistent determination of the values of all different polarizabilities is to improve bot the quantity and the quality of the present Compton data base**

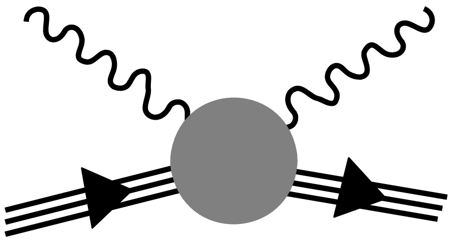
- Ongoing experiment at Mainz by the A2 collaboration
  E. J. Downie et al., Proposal MAMI-A2/04-16 (2016)

Backup

LEX + residual functions



Dispersion Relation formalism RCS ⇒ differential cross section → 6 amplitudes A_i



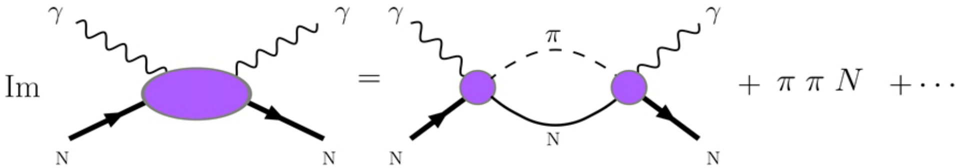
$A_i(0,0) = a_i$ ⟶ **Static polarizabilities**

$$A_i(\nu,t) = A_i^s(\nu,0) + A_i^t(0,t) + A_i(0,0)$$

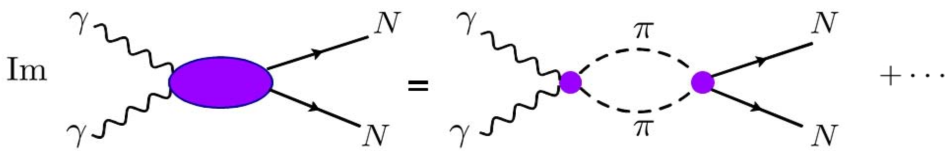
Subtracted Dispersion Relations (*s*-channel)

$$A_i^s(\nu,0) \;=\; \frac{2}{\pi}\,\nu^2\,\mathrm{P}\int_{\nu_{thr}}^{\infty} \mathrm{Im_s}\;A_i(\nu',t)\;\frac{d\nu'}{\nu'(\nu'^2-\nu^2)}$$

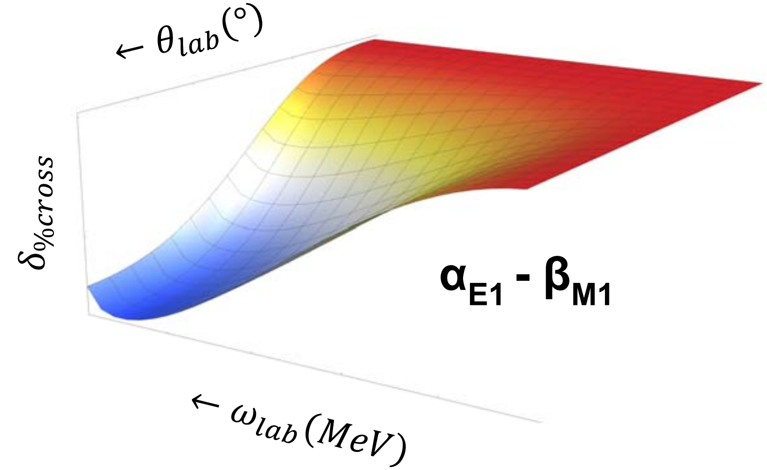
s-CHANNEL

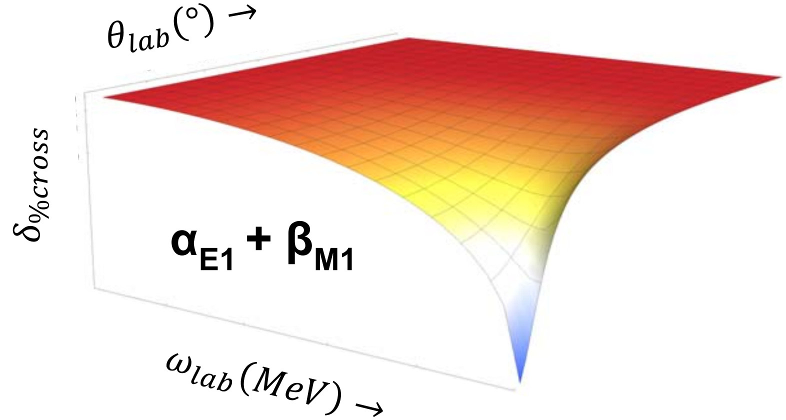


t-CHANNEL

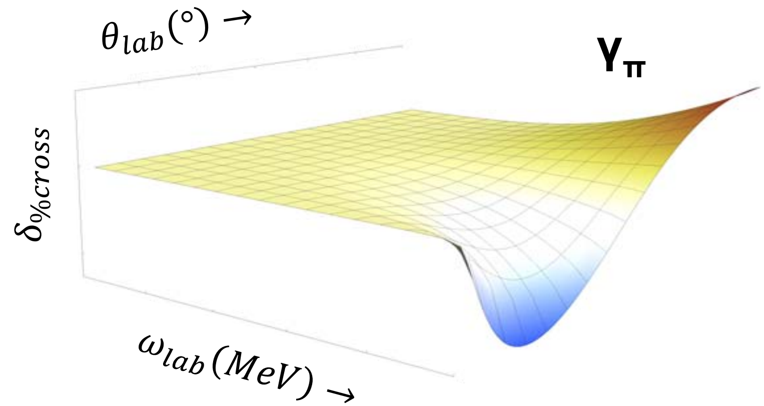


Unpolarized differential cross section: sensitivity plots

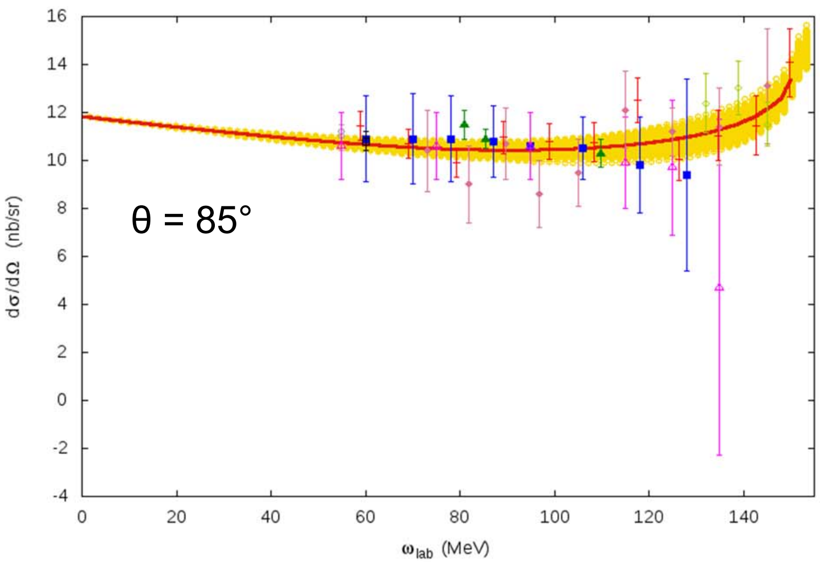


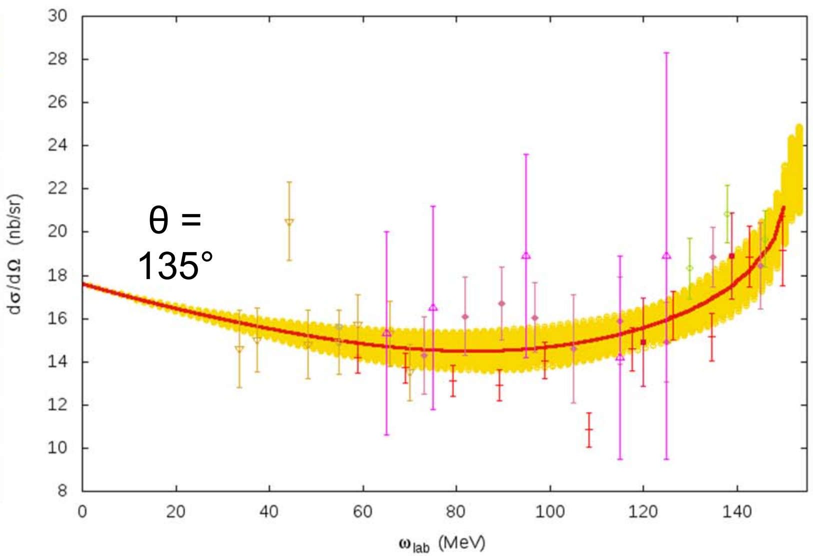


% variation of the observable when the particular polarizability is changed by a factor ± 10%



Differential cross section





dσ/dΩ VS lab energy

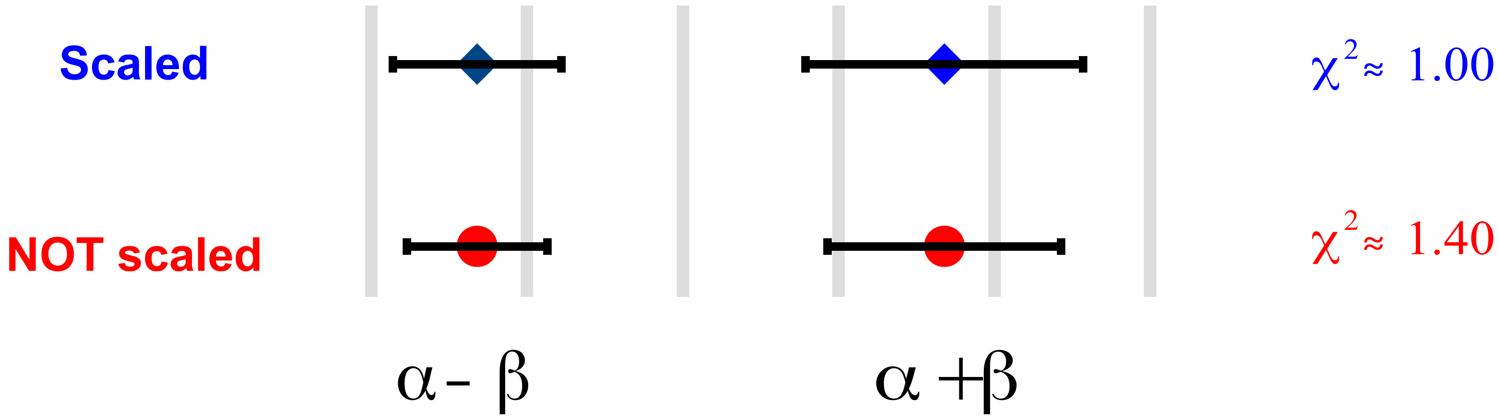
100% error band from the bootstrap fit

Problems with the data set

Very strong dependence of polarizabilities on the specific data set!

Outliers → rescaling of all the statistic uncertainties by a factor

$$\sqrt{\chi^2}$$



Effect: enlarging of errors on fitted parameters (~20%)