Role of transverse momentum dependent TMD unpolarised PDFs and FFs in azimuthal and transverse single spin asymmetries

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The Message [A warning]

- **At present most of information on the unpolarized TMDs, the Sivers and Collins functions and the related spin and azimuthal asymmetries comes from (un)polarized data for semi-inclusive DIS processes, $\ell p^{(\uparrow)} \to \ell' h\, X$, partly from e^+e^- annihilations and even less from Drell-Yan processes**

- **For observables measured in SIDIS processes (multiplicities, Sivers and Collins asymmetries, etc) the transverse momentum dependences of the TMD PDFs and FFs are strongly correlated, since the transverse momentum of the observed hadron, P_T, is kinematically given by a combination of those in the PDF, $k_\perp$, and in the FF, $p_\perp$, sectors. At leading order in a $k_\perp/Q$ power expansion**

$$P_T \simeq p_\perp + z k_\perp$$

- **Due to this strong correlation, comparably good fits of SIDIS data can be obtained with (even very) different combinations of $< {k_\perp}^2 >$ and $< {p_\perp}^2 >$**

- **As a consequence, these comparable fits to SIDIS data might give even sensibly different predictions for Drell-Yan ($k_\perp$) and e^+e^- ($p_\perp$) processes**

The goal

- To investigate in a general way the effects of the strong correlation between $< k_\perp{}^2 >$ and $< p_\perp{}^2 >$, coming mainly from unpolarised SIDIS data (Cahn effect, multiplicities), on phenomenological predictions for:

- The Sivers function and the Sivers asymmetry in Drell-Yan processes

- The transversity distribution and the Collins function and the Collins azimuthal correlations in $e^+e^- \to h_1 h_2 \, X$ processes

- To point out, at the present stage of knowledge of TMDs, the relevance of these effects and the need of carefully considering them when studying important properties of TMDs like their universality and process dependence and their (TMD) evolution properties, and when comparing predictions coming from different extractions of TMDs

Work tools

In order to catch the main qualitative results avoiding unessential complications we work in a simplified scheme:

- **TMD factorisation approach at leading order and leading twist**
- **Factorized longitudinal and transverse momentum dependences in (un)polarized TMDs;**
- **Gaussian (Gaussian-like) and flavour-independent functional forms for the transverse momentum dependent components of TMDs**
- **(un)polarized cross sections integrated over the transverse momentum of the observed hadron(s) [SIDIS and e^+e^-] or lepton pair [DY] in the range of validity of the TMD approach, $P_T, q_T \leq 1 - 2 \text{ GeV} \ll Q$**
- **In this scheme, cross sections and asymmetries factorize into a collinear and a simple transverse-momentum integrated term**

Unpolarised TMD-PDFs and TMD-FFs

$$f_{q/p}(x,k_\perp) = f_{q/p}(x)\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle} \qquad D_{h/q}(z,p_\perp) = D_{h/q}(z)\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}$$

TMD transversity distribution

$$h_1^q(x,k_\perp) = h_1^q(x)\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_T}}{\pi\langle k_\perp^2\rangle_T}$$

Sivers distribution function

$$\Delta^N f_{q/p^\uparrow}(x,k_\perp) = \Delta^N f_{q/p^\uparrow}(x)\,\sqrt{2e}\,\frac{k_\perp}{M_S}\,e^{-k_\perp^2/M_S^2}\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle} \equiv \Delta^N f_{q/p^\uparrow}(x)\,\sqrt{2e}\,\frac{k_\perp}{M_S}\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_S}}{\pi\langle k_\perp^2\rangle}$$

Collins fragmentation function

$$\Delta^N D_{h/q^\uparrow}(z,p_\perp) = \Delta^N D_{h/q^\uparrow}(z)\,\sqrt{2e}\,\frac{p_\perp}{M_C}\,e^{-p_\perp^2/M_C^2}\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle} \equiv \Delta^N D_{h/q^\uparrow}(z)\,\sqrt{2e}\,\frac{p_\perp}{M_C}\,\frac{e^{-p_\perp^2/\langle p_\perp^2\rangle_C}}{\pi\langle p_\perp^2\rangle}$$

$$\langle k_\perp^2\rangle_S = \frac{\langle k_\perp^2\rangle\,M_S^2}{\langle k_\perp^2\rangle + M_S^2} \qquad \langle p_\perp^2\rangle_C = \frac{\langle p_\perp^2\rangle\,M_C^2}{\langle p_\perp^2\rangle + M_C^2}$$

Sivers azimuthal asymmetry for SIDIS processes - 1

$$A_{UT}^{\sin(\phi_h-\phi_S)}(x,z,P_T) = A^S_{\rm DIS}(x,z)\,F^S_{\rm DIS}(z,P_T)$$

$$A^S_{\rm DIS}(x,z) \;=\; \frac{\sum_q e_q^2\,\Delta^N f_{q/p^\uparrow}(x)\,D_{h/q}(z)}{2\sum_q e_q^2\,f_{q/p}(x)\,D_{h/q}(z)}$$

$$F^S_{\rm DIS}(z,P_T) \;=\; \frac{\sqrt{2e}\,\dfrac{P_T}{M_S}\,\dfrac{z\,\langle k_\perp^2\rangle_S^2\;\exp[-P_T^2/\langle P_T^2\rangle_S]}{\pi\,\langle k_\perp^2\rangle\,\langle P_T^2\rangle_S^2}}{\dfrac{\exp[-P_T^2/\langle P_T^2\rangle]}{\pi\,\langle P_T^2\rangle}}$$

$$\langle P_T^2\rangle = \langle p_\perp^2\rangle + z^2\,\langle k_\perp^2\rangle \qquad\qquad \langle P_T^2\rangle_S = \langle p_\perp^2\rangle + z^2\langle k_\perp^2\rangle_S$$

Sivers azimuthal asymmetry for SIDIS processes - 2

Integrating separately the numerator and the denominator of the P_T −dependent component of the asymmetry over the modulus of the transverse momentum of the observed hadron, $P_T dP_T$, in the full P_T range [0,+∞], one obtains:

$$A_{UT}^{\sin(\phi_h-\phi_S)}(x,z) = A_{\rm DIS}^{S}(x,z)\,\mathcal{F}_{\rm DIS}^{S}(z,\xi_1,\rho_S)$$

$$\mathcal{F}_{\rm DIS}^{S}(z,\xi_1,\rho_S) = \sqrt{\frac{e\pi}{2}}\left[\frac{\rho_S^3(1-\rho_S)}{\rho_S+\xi_1/z^2}\right]^{1/2}$$

$$\xi_1 = \frac{\langle p_\perp^2\rangle}{\langle k_\perp^2\rangle} \qquad\qquad \rho_S = \frac{\langle k_\perp^2\rangle_S}{\langle k_\perp^2\rangle} = \frac{M_S^2}{M_S^2+\langle k_\perp^2\rangle}$$

Notice: due to the Gaussian(like) shape of the TMDs, the dominant contribution to these integrals comes from the region $P_T \ll Q$, the TMD factorization regime

Collins azimuthal asymmetry for SIDIS processes

$$A_{UT}^{\sin(\phi_h+\phi_S)}(x,y,z) = A_{\rm DIS}^{C}(x,y,z)\,\mathcal{F}_{\rm DIS}^{C}(z,\rho_C,\xi_1/\xi_T)$$

$$A_{\rm DIS}^{C}(x,y,z) \;=\; \frac{1-y}{1+(1-y)^2}\,\frac{\sum_q e_q^2\, h_1^q(x)\,\Delta^N D_{h/q^\uparrow}(z)}{\sum_q e_q^2\, f_{q/p}(x)\, D_{h/q}(z)}$$

$$\mathcal{F}_{\rm DIS}^{C}(z,\rho_C,\xi_1/\xi_T) = \sqrt{\frac{e\pi}{2}}\left[\frac{\rho_C^3(1-\rho_C)}{\rho_C + z^2(\xi_T/\xi_1)}\right]^{1/2}$$

$$\xi_T = \frac{\langle k_\perp^2\rangle_T}{\langle k_\perp^2\rangle} \qquad\qquad \rho_C = \frac{\langle p_\perp^2\rangle_C}{\langle p_\perp^2\rangle} = \frac{M_C^2}{M_C^2+\langle p_\perp^2\rangle}$$

Sivers single spin asymmetry for Drell-Yan processes

$$A_N^{\rm DY}(y, M) = A_{\rm DY}^S(x_1, x_2)\,\mathcal{F}_{\rm DY}^S(\rho_S, \xi_{21})$$

$$A_{\rm DY}^S(x_1, x_2) \;\equiv\; A_{\rm DY}^S(y, M) = \frac{\sum_q e_q^2\,\Delta^N f_{q/h_1^\uparrow}(x_1) f_{\bar{q}/h_2}(x_2)}{2\sum_q e_q^2\, f_{q/h_1}(x_1) f_{\bar{q}/h_2}(x_2)}$$

$$\mathcal{F}_{\rm DY}^S(\rho_S, \xi_{21}) = \sqrt{\frac{e\pi}{2}}\left[\frac{\rho_S^3(1-\rho_S)}{\rho_S + \xi_{21}}\right]^{1/2}$$

$$\xi_{21} = \frac{\langle k_{\perp 2}^2\rangle}{\langle k_{\perp 1}^2\rangle} \qquad\qquad \rho_S = \frac{\langle k_\perp^2\rangle_S}{\langle k_{\perp 1}^2\rangle} = \frac{M_S^2}{M_S^2 + \langle k_{\perp 1}^2\rangle}$$

Collins azimuthal asymmetry for $e^+e^- \to h_1 h_2\, X$ processes (hadronic-plane frame)

$$A_0^{UL(C)} \simeq P_0^U - P_0^{L(C)}$$

$$P_0^{h_1 h_2}(z_1, z_2; \theta) = A_{\rm ee}^{h_1 h_2}(z_1, z_2; \theta)\, \mathcal{F}_{\rm ee}^C$$

$$A_{\rm ee}^{h_1 h_2}(z_1, z_2; \theta) \;=\; \frac{1}{4}\, \frac{\sin^2\theta}{1+\cos^2\theta}\, \frac{z_1 z_2}{z_1^2 + z_2^2}\, \frac{\sum_q e_q^2\, \Delta^N D_{h_1/q^\uparrow}(z_1)\, \Delta^N D_{h_2/\bar{q}^\uparrow}(z_2)}{\sum_q e_q^2\, D_{h_1/q}(z_1)\, D_{h_2/\bar{q}}(z_2)}$$

$$\mathcal{F}_{\rm ee}^C(\rho_C) = 2\, e\, \rho_C^2 (1 - \rho_C)$$

Note 1: assuming that $h_{1,2}$ are both either pions or kaons
Cases like πK pairs would in general require two different values of $< {p_\perp}^2 >$

Note 2: Similar results can be obtained adopting the thrust-axis frame

Remarks

Additional simplifying assumptions:

- **For the transversity distribution, we will assume the same (flavour-independent) Gaussian functional dependence on transverse momentum as for the unpolarized TMD PDFs**

$$\xi_T = \frac{\langle k_\perp^2 \rangle_T}{\langle k_\perp^2 \rangle} \to 1$$

- **We only consider Drell-Yan processes in pp collisions; the πp COMPASS case would require two independent values for the average square transverse momentum in the initial proton and pion beams**

$$\xi_{21} = \frac{\langle k_{\perp\,2}^2 \rangle}{\langle k_{\perp\,1}^2 \rangle} \to 1 \qquad \text{i.e.} \qquad \langle k_{\perp\,1}^2 \rangle = \langle k_{\perp\,2}^2 \rangle = \langle k_\perp^2 \rangle$$

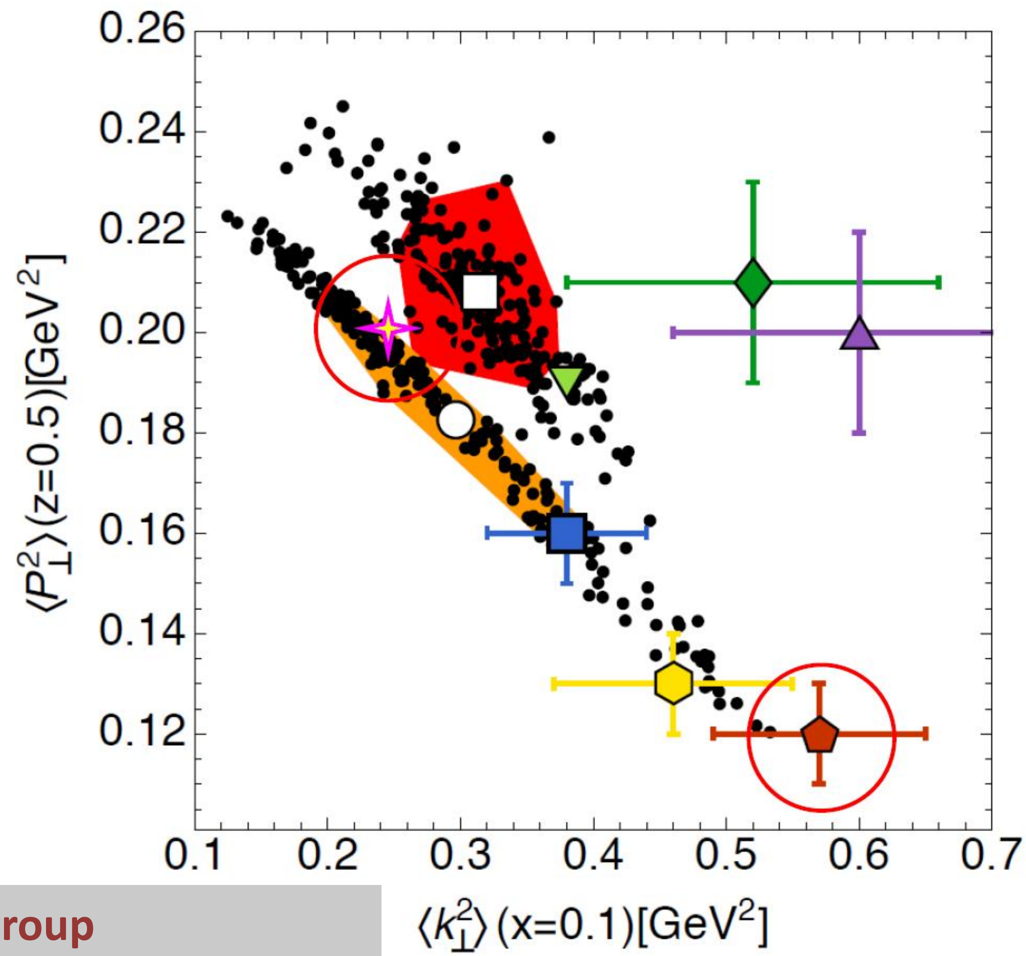
- **In this simplified (but realistic) scheme, the transverse momentum integrated components of the asymmetries are functions only of $\xi_1 = < p_\perp{}^2 > / < k_\perp{}^2 >$, which is fixed by fitting unpolarised observables (Cahn effect, multiplicities), and $\rho_{S(C)}$, that are fixed, using ξ_1, by fitting data on the asymmetries [and z in the SIDIS case]**

More on the parameter $\xi_1 = < p_\perp{}^2 > / < k_\perp{}^2 >$

Mathematically, $0 < \xi_1 < +\infty$; zero would correspond to a completely collinear configuration in the fragmentation sector, + ∞ to full collinearity in the distribution sector

Physically, these limiting values are quite extreme and phenomenologically unrealistic. A plausible range of values is

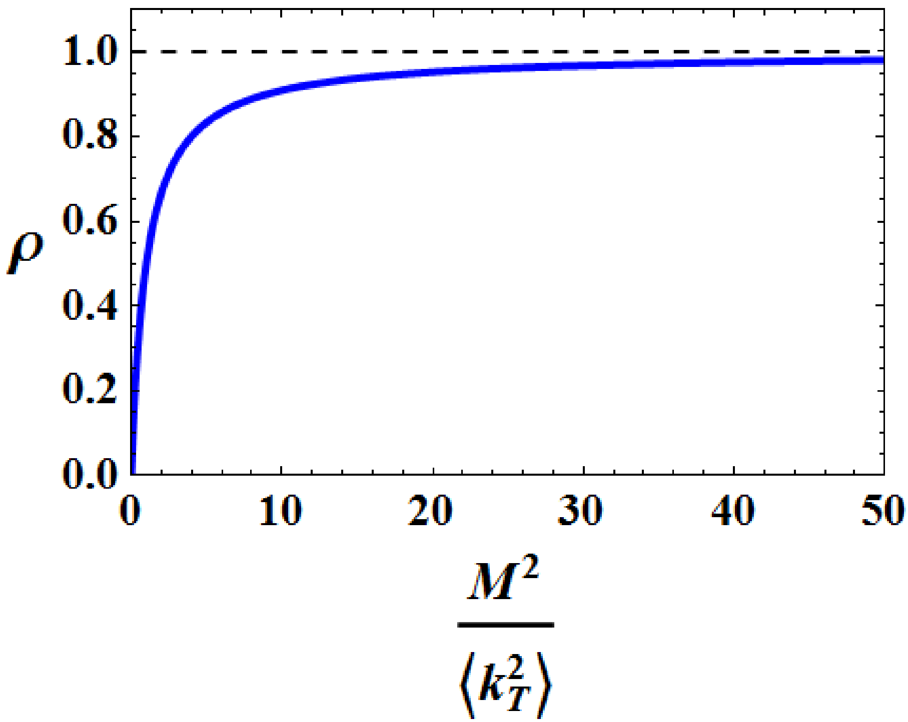
$$0.15 < \xi_1 < 2.2$$

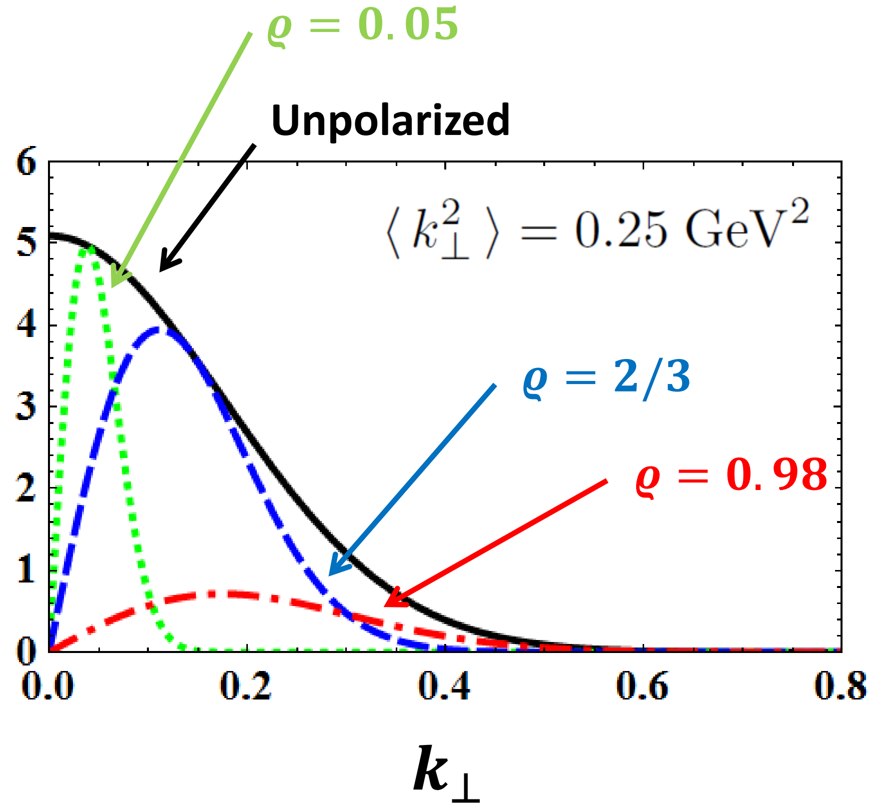


Pavia group
A. Bacchetta et al., JHEP 06 (2017) 081

More on the parameters $\rho_{S(C)}$

Mathematically $0 < \rho_{S(C)} < 1$;





$$\rho_S = \frac{M_S^2}{M_S^2 + \langle k_\perp^2 \rangle} = \frac{M_S^2/\langle k_\perp^2 \rangle}{1 + M_S^2/\langle k_\perp^2 \rangle}$$

How do our predictions for the Sivers and Collins asymmetries depend on the choice of ξ_1 and $\rho_{S(C)}$?

Study how the transverse components of the asymmetries, $\mathcal{F}^{S(C)}$, change when moving along a generic trajectory in the (ξ_1, ρ) parameter space, starting from some fixed point $(\hat{\xi}_1, \hat{\rho})$ corresponding to a phenomenological reference fit [FIT09 or FIT16].

To be definite, starting from the Sivers case, we consider two different and comparably good fits of the quark Sivers functions from the Cagliari-Torino group:

- **The fit of Ref. EPJ A39, 89 (2009), referred to as FIT09**

$$\langle k_\perp^2 \rangle = 0.25\,\mathrm{GeV}^2, \qquad \langle p_\perp^2 \rangle = 0.20\,\mathrm{GeV}^2, \qquad M_S^2 = 0.34\,\mathrm{GeV}^2$$
$$\hat{\xi}_1^{(09)} = 0.80, \qquad \hat{\rho}_S^{(09)} = 0.58$$

- **The fit of Ref. JHEP 04, 046 (2016), referred to as FIT16**

$$\langle k_\perp^2 \rangle = 0.57\,\mathrm{GeV}^2, \qquad \langle p_\perp^2 \rangle = 0.12\,\mathrm{GeV}^2, \qquad M_S^2 = 0.80\,\mathrm{GeV}^2$$
$$\hat{\xi}_1^{(16)} = 0.21, \qquad \hat{\rho}_S^{(16)} = 0.58$$

See the quoted references for details on the fitting procedure and the parameter extraction

Notice: We can always refer the generic expressions of $\mathcal{F}^{S(C)}$ (ξ_1, ρ) to that corresponding to a given fixed point, $\hat{\mathcal{F}}^{S(C)}(\hat{\xi}_1, \hat{\rho})$ in the 2D parameter space (ξ_1, ρ).

$$\mathcal{F}^S_{\rm DIS}(z, \xi_1, \rho_S) = R^S_{\rm DIS}\, \hat{\mathcal{F}}^S_{\rm DIS}(z, \hat{\xi}_1, \hat{\rho}_S)\,, \qquad R^S_{\rm DIS} = \left[\frac{\rho_S^3(1-\rho_S)}{\rho_S + \xi_1/z^2}\, \frac{\hat{\rho}_S + \hat{\xi}_1/z^2}{\hat{\rho}_S^3(1-\hat{\rho}_S)} \right]^{1/2}$$

$$\mathcal{F}^S_{\rm DY}(\xi_{21} = 1, \rho_S) = R^S_{\rm DY}\, \hat{\mathcal{F}}^S_{\rm DY}(\hat{\rho}_S)\,, \qquad R^S_{\rm DY} = \left[\frac{\rho_S^3(1-\rho_S)}{\rho_S + 1}\, \frac{\hat{\rho}_S + 1}{\hat{\rho}_S^3(1-\hat{\rho}_S)} \right]^{1/2}$$

$$\mathcal{F}^C_{\rm DIS}(z, \xi_1, \rho_C) = R^C_{\rm DIS}\, \hat{\mathcal{F}}^C_{\rm DIS}(z, \hat{\xi}_1, \hat{\rho}_C)\,, \qquad R^C_{\rm DIS} = \left[\frac{\rho_C^3(1-\rho_C)}{\rho_C + z^2/\xi_1}\, \frac{\hat{\rho}_C + z^2/\hat{\xi}_1}{\hat{\rho}_C^3(1-\hat{\rho}_C)} \right]^{1/2}$$

$$\mathcal{F}^C_{\rm ee}(\rho_C) = R^C_{\rm ee}\, \hat{\mathcal{F}}^C_{\rm ee}(\hat{\rho}_C)\,, \qquad R^C_{\rm ee} = \frac{\rho_C^2(1-\rho_C)}{\hat{\rho}_C^2(1-\hat{\rho}_C)}$$

Sivers case, the strategy

Start from a given reference fit, FIT09 or FIT16, corresponding to the point $(\hat{\xi}_1, \hat{\rho})$ in the parameter space

Look for different points (ξ_1, ρ) that still allow to fit reasonably well the SIDIS Sivers asymmetry

$$A^S_{\rm DIS}(x,z)\,\mathcal{F}^S_{\rm DIS}(z,\rho_S,\xi_1) \simeq \hat{A}^S_{\rm DIS}(x,z)\,\hat{\mathcal{F}}^S_{\rm DIS}(z,\hat{\rho}_S,\hat{\xi}_1)$$

$$\mathcal{F}^S_{\rm DIS} = R^S_{\rm DIS}\,\hat{\mathcal{F}}^S_{\rm DIS} \qquad A^S_{\rm DIS} \simeq \frac{1}{R^S_{\rm DIS}}\,\hat{A}^S_{\rm DIS} \qquad \text{with} \qquad R^S_{\rm DIS} = \left[\frac{\rho_S^3(1-\rho_S)}{\rho_S+\xi_1/z^2}\,\frac{\hat{\rho}_S+\hat{\xi}_1/z^2}{\hat{\rho}_S^3(1-\hat{\rho}_S)}\right]^{1/2}$$

Since very few low-statistics data are presently available for the Drell-Yan Sivers asymmetry, they are not used in the fit of the Sivers function.

However, since both the SIDIS and Drell-Yan Sivers asymmetries are linear in the Sivers function, we may reasonably assume that

$$\frac{A^S_{\rm DY}}{\hat{A}^S_{\rm DY}} \simeq \frac{A^S_{\rm DIS}}{\hat{A}^S_{\rm DIS}} \quad \Rightarrow A^S_{\rm DY} \simeq \left(\frac{A^S_{\rm DIS}}{\hat{A}^S_{\rm DIS}}\right)\hat{A}^S_{\rm DY} \equiv \frac{1}{R^S_{\rm DIS}}\,\hat{A}^S_{\rm DY}$$

Moreover, $\mathcal{F}^S_{\rm DY} = R^S_{\rm DY}\,\hat{\mathcal{F}}^S_{\rm DY}$

$$A_N^{\rm DY} = A^S_{\rm DY}\mathcal{F}^S_{\rm DY} \simeq \left(\frac{R^S_{\rm DY}}{R^S_{\rm DIS}}\right)\hat{A}^S_{\rm DY}\hat{\mathcal{F}}^S_{\rm DY} = R^N_{\rm DY}\hat{A}_N^{\rm DY} \qquad \text{with} \qquad R^N_{\rm DY} = \left[\frac{\rho_S+\xi_1/z^2}{\hat{\rho}_S+\hat{\xi}_1/z^2}\,\frac{\hat{\rho}_S+1}{\rho_S+1}\right]^{1/2}$$

Sivers case, comparison between the two reference sets FIT09 e FIT16

$$\hat{\xi}_1^{(09)} = 0.80, \qquad \hat{\rho}_S^{(09)} = 0.58\,; \qquad\qquad \hat{\xi}_1^{(16)} = 0.21, \qquad \hat{\rho}_S^{(16)} = 0.58$$

Both sets reproduce comparably well the SIDIS Sivers asymmetries; notice that

$$\hat{\rho}_S^{(09)} \cong \hat{\rho}_S^{(16)} = \hat{\rho}_S$$

When going from the FIT09 to the FIT16 set, however, due to the huge difference (uncertainty) on the ξ_1 parameter, the estimates for the corresponding Drell-Yan Sivers asymmetry are rescaled as follows:

$$A_N^{\rm DY}(\hat{\rho}_S, \hat{\xi}_1^{(16)}) = \left[\frac{\hat{\rho}_S + \hat{\xi}_1^{(16)}/z^2}{\hat{\rho}_S + \hat{\xi}_1^{(09)}/z^2} \right]^{1/2} \hat{A}_N^{\rm DY}(\hat{\rho}_S, \hat{\xi}_1^{(09)})$$

Using the values given above for the $\hat{\rho}_S$, $\hat{\xi}_1$ parameters, this rescaling factor varies from about 0.52 to 0.68 for 0.1 < z < 0.7. Since small z values dominate the SIDIS data, we find that

$$A_N^{\rm DY}(\hat{\rho}_S, \hat{\xi}_1^{(16)}) \simeq \frac{1}{2} \hat{A}_N^{\rm DY}(\hat{\rho}_S, \hat{\xi}_1^{(09)})$$

Sivers effect, scenario 2 - Fixing separately $A^S_{\rm DIS}$ and $\mathcal{F}^S_{\rm DIS}$

Consider a more general situation in which, moving from one of the two reference sets, FIT09 or FIT16, along a generic curve in the (ξ_1, ρ) parameter space, we want not only keep fixed the total SIDIS Sivers asymmetry, but also separately its collinear and transverse momentum integrated components

$$A^S_{\rm DIS}(x,z)\,\mathcal{F}^S_{\rm DIS}(z,\rho_S,\xi_1) \simeq \hat{A}^S_{\rm DIS}(x,z)\,\hat{\mathcal{F}}^S_{\rm DIS}(z,\hat{\rho}_S,\hat{\xi}_1)$$

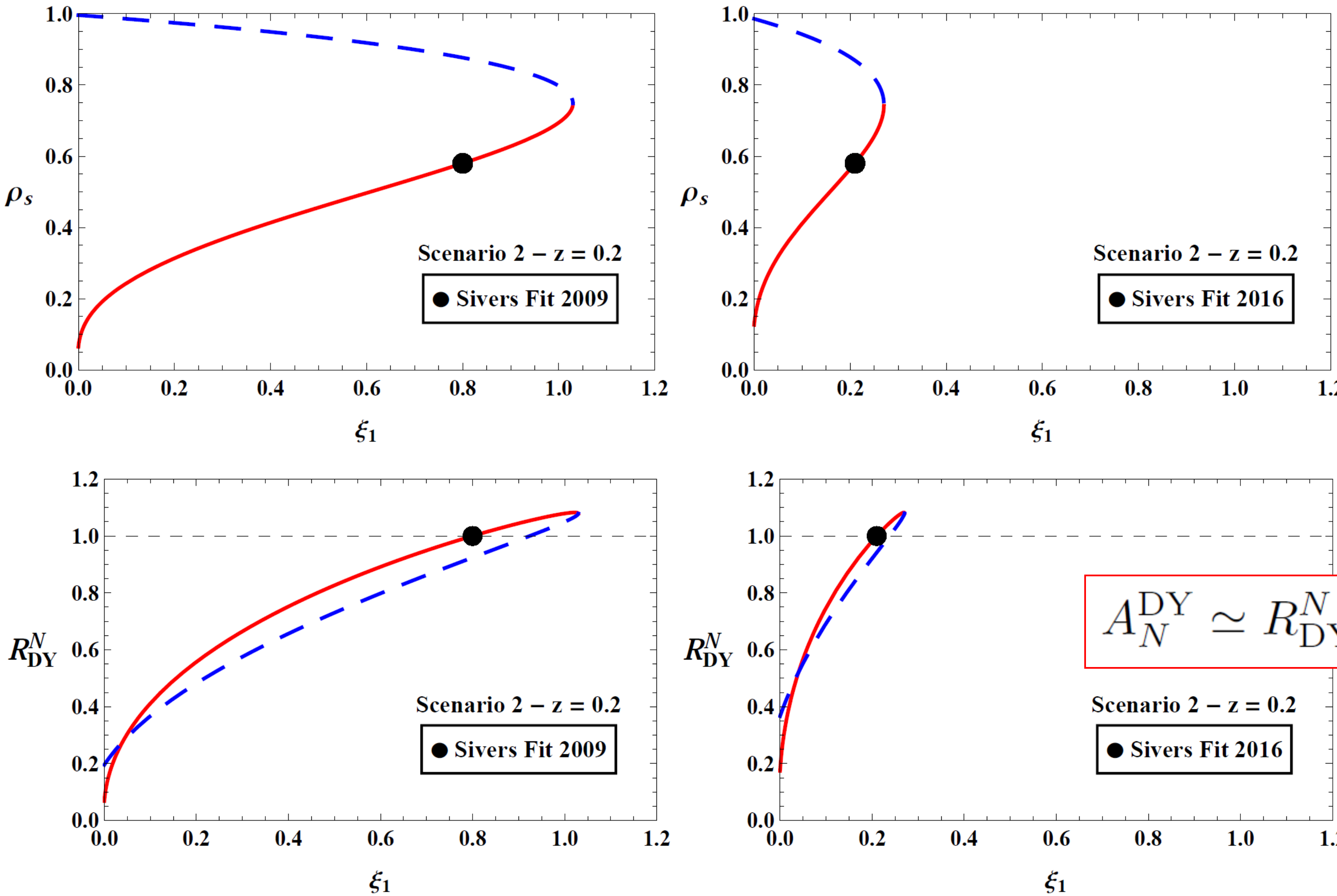
$$\mathcal{F}^S_{\rm DIS} = R^S_{\rm DIS}\,\hat{\mathcal{F}}^S_{\rm DIS} \qquad A^S_{\rm DIS} \simeq \frac{1}{R^S_{\rm DIS}}\,\hat{A}^S_{\rm DIS} \qquad \text{with} \qquad R^S_{\rm DIS} = \left[\frac{\rho_S^3(1-\rho_S)}{\rho_S+\xi_1/z^2}\,\frac{\hat{\rho}_S+\hat{\xi}_1/z^2}{\hat{\rho}_S^3(1-\hat{\rho}_S)}\right]^{1/2}$$

$$\Rightarrow\; R^S_{\rm DIS} = \left[\frac{\rho_S^3(1-\rho_S)}{\rho_S+\xi_1/z^2}\,\frac{\hat{\rho}_S+\hat{\xi}_1/z^2}{\hat{\rho}_S^3(1-\hat{\rho}_S)}\right]^{1/2} = 1$$

At fixed ξ_1 and z this constraint correspond to a 4th order algebraic equation for ρ_S

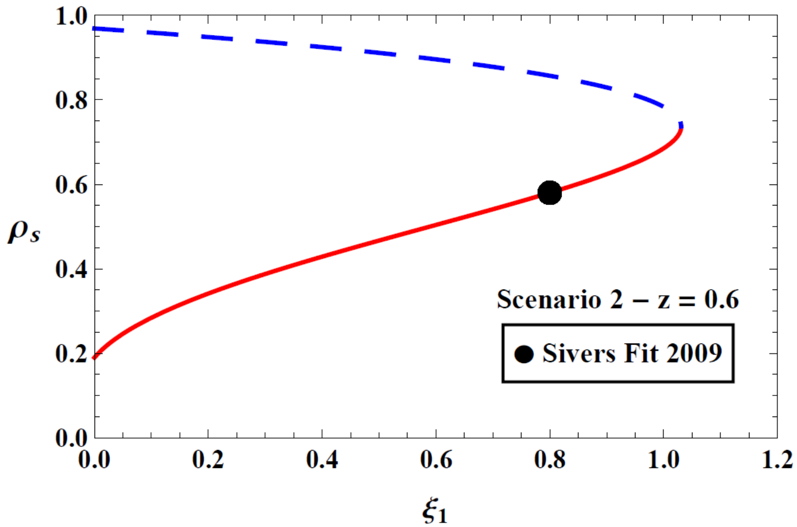
$$\rho_S^4 - \rho_S^3 + \hat{a}(z)\rho_S + \hat{a}(z)\frac{z^2}{\xi_1} = 0 \qquad \text{with} \qquad \hat{a}(z) = \frac{\hat{\rho}_S^3(1-\hat{\rho}_S)}{\hat{\rho}_S+\hat{\xi}_1/z^2}$$

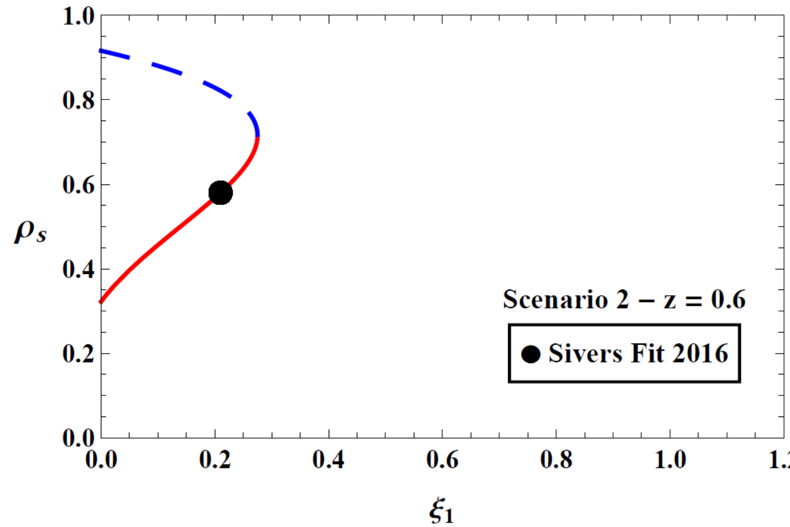
Sivers effect – scenario 2: Results

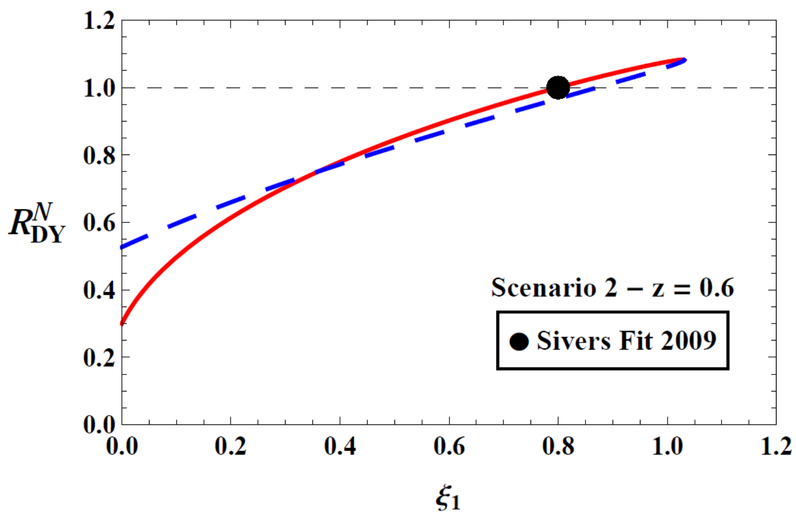


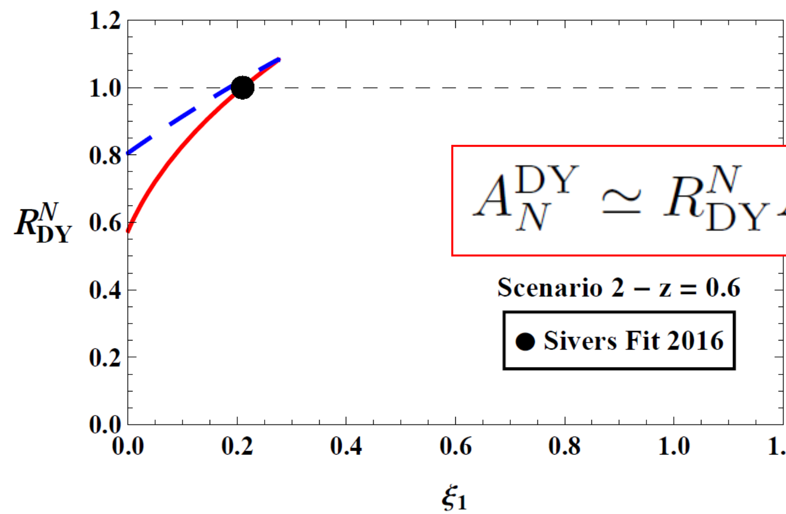
$$A_N^{\rm DY} \simeq R^N_{\rm DY}\,\hat{A}_N^{\rm DY}$$

Sivers effect – scenario 2: dependence on *z*









$$A_N^{\rm DY} \simeq R_{\rm DY}^N \hat{A}_N^{\rm DY}$$

Sivers effect – Scenario 3: fixing also the Drell-Yan asymmetry

If in the future enough data will be available for the total q_T-integrated Drell-Yan Sivers asymmetry, then we can look for trajectories in the (ξ_1, ρ_S) plane moving from a reference fit point $(\hat{\xi}_1, \hat{\rho}_S)$ and obeying the additional constraint

$$A_N^{\rm DY} \simeq \hat{A}_N^{\rm DY} \qquad \Rightarrow \qquad R_{\rm DY}^N = \frac{R_{\rm DY}^S}{R_{\rm DIS}^S} = \left[\frac{\rho_S + \xi_1/z^2}{\hat{\rho}_S + \hat{\xi}_1/z^2} \, \frac{\hat{\rho}_S + 1}{\rho_S + 1} \right]^{1/2} = 1 \quad \text{or} \quad R_{\rm DY}^S = R_{\rm DIS}^S$$

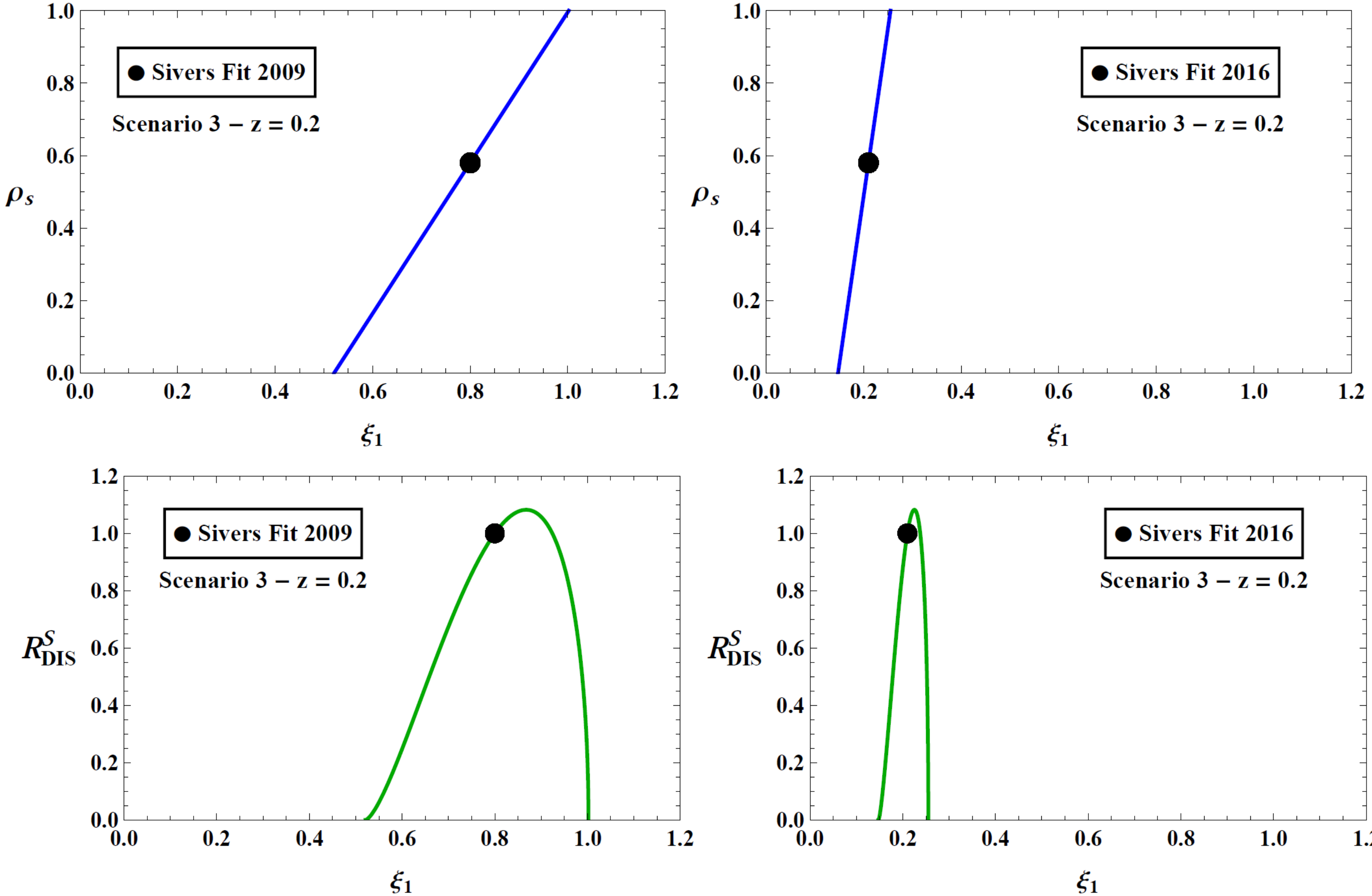
$$A_{\rm DIS}^S \simeq \frac{1}{R_{\rm DIS}^S} \, \hat{A}_{\rm DIS}^S \,, \quad \mathcal{F}_{\rm DIS}^S = R_{\rm DIS}^S \, \hat{\mathcal{F}}_{\rm DIS}^S \qquad \text{and} \qquad A_{\rm DY}^S \simeq \frac{1}{R_{\rm DIS}^S} \, \hat{A}_{\rm DY}^S \,, \quad \mathcal{F}_{\rm DY}^S = R_{\rm DIS}^S \, \hat{\mathcal{F}}_{\rm DY}^S$$

$$R_{\rm DY}^N = 1 \qquad \Rightarrow \qquad \rho_S = \frac{\hat{b}(z)\, \xi_1/z^2 - 1}{1 - \hat{b}(z)} \quad \text{with} \quad \hat{b}(z) = \frac{\hat{\rho}_S + 1}{\hat{\rho}_S + \hat{\xi}_1/z^2}$$

$\hat{b}$(z) is a rapidly increasing function of z

$$\hat{b}^{(09)}(z = 0.2) \simeq 0.077, \qquad \hat{b}^{(16)}(z = 0.6) \simeq 1.36$$

Sivers effect – scenario 3: Results



The Collins case for SIDIS and e^+e^- processes

More complicated than the Sivers case;
The Collins FF enters linearly in the SIDIS Collins asymmetry, convoluted with the transversity distribution, while in the e^+e^- case it appears *quadractically.*
Changes in ξ_1 can affect only the transversity distribution, or only the Collins FF, or both of them simultaneously. More scenarios are possible.
Sufficient experimental information is already available for both the SIDIS and e^+e^- Collins asymmetries
Again, we consider two different and comparably good fits of the transversity distribution and of the Collins FF from the Cagliari-Torino group:

The fit of Ref. PRD 75, 054032 (2007), referred to as FIT07

$$\langle k_\perp^2 \rangle = 0.25\,\mathrm{GeV}^2, \qquad \langle p_\perp^2 \rangle = 0.20\,\mathrm{GeV}^2, \qquad M_C^2 = 0.88\,\mathrm{GeV}^2$$
$$\hat{\xi}_1^{(07)} = 0.80, \qquad \hat{\rho}_C^{(07)} = 0.81$$

The fit of Ref. PRD 92, 114023 (2015), referred to as FIT15

$$\langle k_\perp^2 \rangle = 0.57\,\mathrm{GeV}^2, \qquad \langle p_\perp^2 \rangle = 0.12\,\mathrm{GeV}^2, \qquad M_C^2 = 0.28\,\mathrm{GeV}^2$$
$$\hat{\xi}_1^{(15)} = 0.21, \qquad \hat{\rho}_C^{(15)} = 0.70$$

The Collins case – basic constraints

$$
\begin{aligned}
A^C_{\rm DIS}\mathcal{F}^C_{\rm DIS}(\rho_C,\xi_1) &\simeq \hat{A}^C_{\rm DIS}\hat{\mathcal{F}}^C_{\rm DIS}(\hat{\rho}_C,\hat{\xi}_1)\,,\\
A^C_{\rm ee}\mathcal{F}^C_{\rm ee}(\rho_C) &\simeq \hat{A}^C_{\rm ee}\hat{\mathcal{F}}^C_{\rm ee}(\hat{\rho}_C)\,.
\end{aligned}
$$

$$
\mathcal{F}^C_{\rm DIS} = R^C_{\rm DIS}\,\hat{\mathcal{F}}^C_{\rm DIS} \qquad A^C_{\rm DIS} \simeq \frac{1}{R^C_{\rm DIS}}\,\hat{A}^C_{\rm DIS} \qquad \text{with} \qquad R^C_{\rm DIS} = \left[\frac{\rho_C^3(1-\rho_C)}{\rho_C + z^2/\xi_1}\,\frac{\hat{\rho}_C + z^2/\hat{\xi}_1}{\hat{\rho}_C^3(1-\hat{\rho}_C)}\right]^{1/2}
$$

$$
\mathcal{F}^C_{\rm ee} = R^C_{\rm ee}\,\hat{\mathcal{F}}^C_{\rm ee} \qquad A^C_{\rm ee} \simeq \frac{1}{R^C_{\rm ee}}\,\hat{A}^C_{\rm ee} \qquad \text{with} \qquad R^C_{\rm ee} = \frac{\rho_C^2(1-\rho_C)}{\hat{\rho}_C^2(1-\hat{\rho}_C)}
$$

The Collins case- scenario 1

We impose as further constraint that also the P_T-integrated SIDIS Collins factor is fixed

$$\mathcal{F}^C_{\rm DIS}(\rho_C, \xi_1) = \hat{\mathcal{F}}^C_{\rm DIS}(\hat{\rho}_C, \hat{\xi}_1)$$

Which is fulfilled if

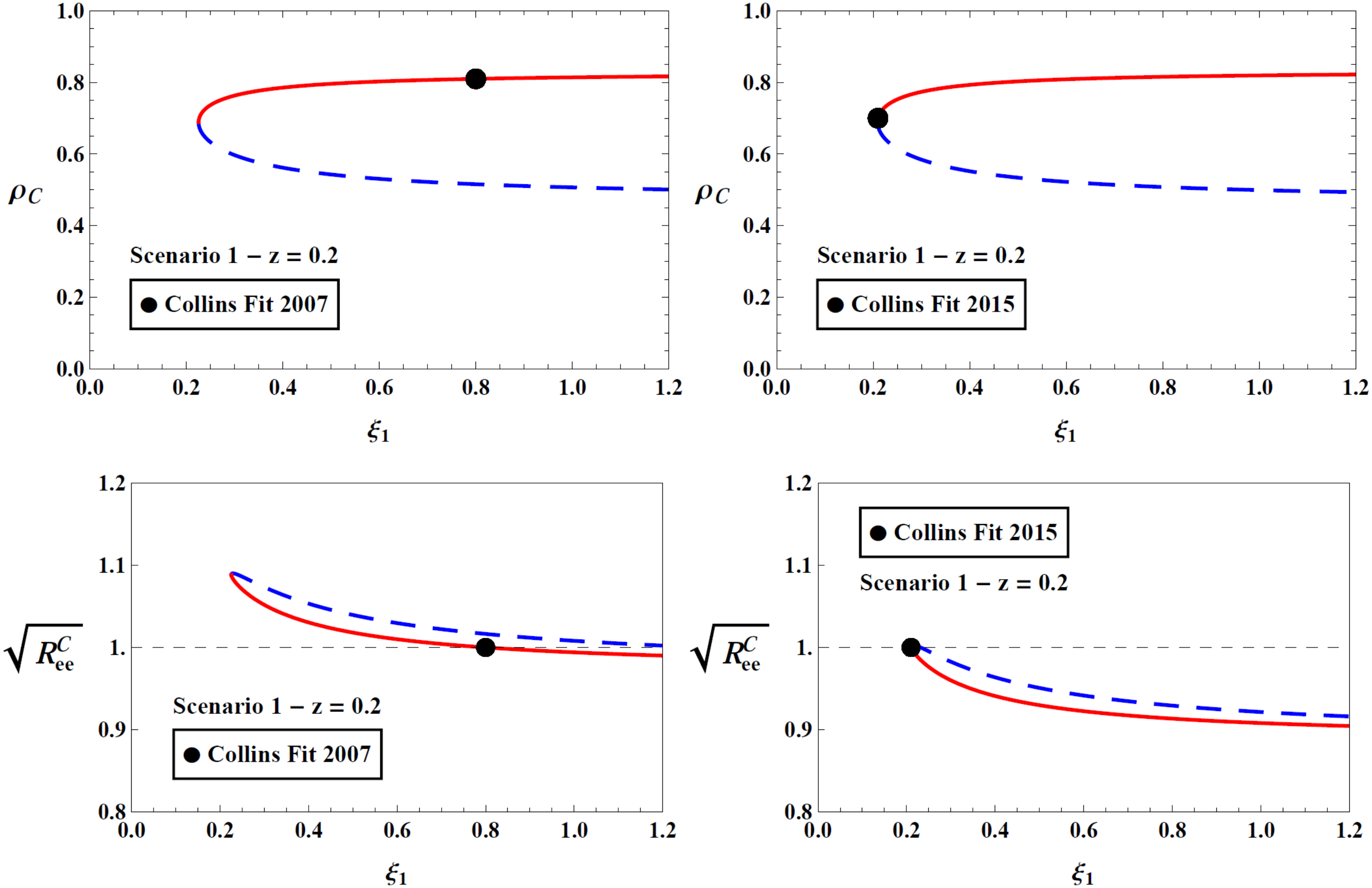
$$R^C_{\rm DIS} = \left[\frac{\rho_C^3(1-\rho_C)}{\rho_C + z^2/\xi_1} \, \frac{\hat{\rho}_C + z^2/\hat{\xi}_1}{\hat{\rho}_C^3(1-\hat{\rho}_C)} \right]^{1/2} = 1$$

As a consequence

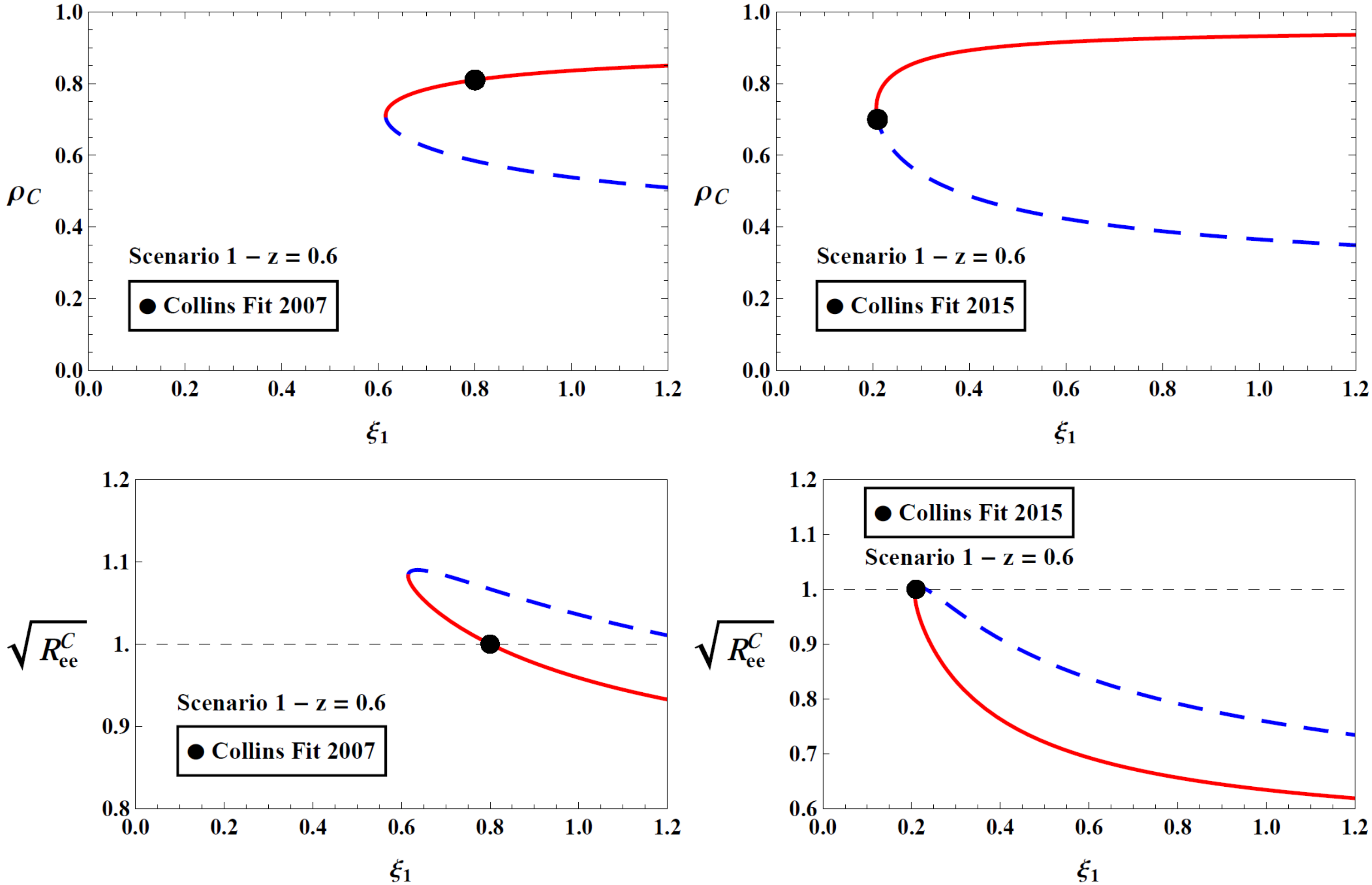
$$A^C_{\rm DIS} = \hat{A}^C_{\rm DIS} \qquad A^C_{\rm ee} = \frac{1}{R^C_{\rm ee}} \, \hat{A}^C_{\rm ee}$$

Since $A^C_{\rm DIS}$ is linear in both the transversity distribution and the Collins FF, while $A^C_{\rm ee}$ is quadratic in the Collins FF, the two relations above can be fulfilled if the Collins FF rescales as $1/\sqrt{R^C_{\rm ee}}$, while the transversity distributions rescales at the same time as $\sqrt{R^C_{\rm ee}}$

The Collins case – scenario 1: Results



The Collins case – scenario 1: dependence on z



Conclusions

- **All present parameterisations of the most studied TMDs, the transversity and the Sivers distributions and the Collins FF, mainly originate from SIDIS data, to some extent from e^+e^- data and marginally from Drell-Yan results**

- **We have investigated, in a simple but general approach, to what extent the unavoidable strong correlation between the average transverse momenta for the TMD PDFs and FFs extracted from unpolarised SIDIS observables may in turn affect the extraction of the collinear part of the TMDs and, ultimately, the estimates (predictions) of azimuthal and SSAs for Drell-Yan and $e^+e^- \rightarrow h_1 h_2 X$ processes, and even other pp processes [factorization issues]**

- **We have shown that since comparably good fits of the Sivers and Collins azimuthal asymmetries can be obtained with (even very) different values of $\xi_1 = \langle p_\perp^2 \rangle / \langle k_\perp^2 \rangle$, the corresponding estimates for the Sivers asymmetry in Drell-Yan processes may vary by a factor of up to 2**

- **Concerning the extraction of the Collins FF and the transversity distribution from SIDIS and e^+e^- data, the uncertainty on ξ_1 seems to have milder (but not negligible) effects, except for some marginal cases**

- **A more precise determination of $\langle k_\perp^2 \rangle$, $\langle p_\perp^2 \rangle$ and ξ_1 is therefore crucial while entering a new stage in the exploration of the 3D structure of the nucleon, aiming at a more precise determination of TMD PDFs and FFs and the understanding of their process dependence and a full implementation of TMD evolution [new RHIC, Jlab, COMPASS, EIC results will be crucial]**

Thanks for your attention!