The longitudinal or transverse spin transfer to Lambda and anti-Lambda hyperons in polarized proton-proton collisions is expected to be sensitive to the helicity or transversity distributions of strange and anti-strange quarks of the proton, and to the corresponding polarized fragmentation function. We report the first measurement of the transverse spin transfer to Lambda and anti-Lambda along the polarization direction of the fragmenting quark, $D_{TT}$, in transversely polarized proton-proton collisions at 200 GeV with the STAR detector at RHIC. The data correspond to an integrated luminosity of 18 pb$^{-1}$, and cover a kinematic range of $|\eta|<1.2$ and transverse momentum $p_T$ up to 8 GeV/c. We also report an improved measurement of the longitudinal spin transfer $D_{LL}$ to Lambda and anti-Lambda with $p_T$ up to 6 GeV/c, using data with about twelve times larger figure-of-merit than the previously published STAR results. The prospects of hyperon polarization measurements in the forward pseudo-rapidity region ($\eta$~3) in p+p collision in the year of 2021 and beyond will also be discussed, which is based on STAR forward detector upgrade plan including a forward tracking system and a forward calorimeter system.