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TMDs from Parton Branching

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- Introduction

- The Parton Branching (PB) method

- New results and applications
I. Introduction

TRANSVERSE MOMENTUM DEPENDENT (TMD) PARTON DISTRIBUTION FUNCTIONS

• Parton correlation functions at non-lightlike distances:

\[ f(y) = \langle P | \bar{\psi}(y) V_y^+(n) \gamma^+ V_0(n) \psi(0) | P \rangle , \quad y = (0, y^-, y_\perp) \]

\[ V_y(n) = \mathcal{P} \exp \left( i g_s \int_0^\infty d\tau \ n \cdot A(y + \tau \ n) \right) \]

• TMD pdfs:

\[ f(x, k_\perp) = \int \frac{dy^-}{2\pi} \frac{d^{d-2} y_\perp}{(2\pi)^{d-2}} e^{-ixp^+y^- + ik_\perp \cdot y_\perp} \tilde{f}(y) \]
Evolution equations for TMD parton distribution functions

\[ \text{low } q_T : q_T \ll Q \]

\[ \text{high } \sqrt{s} : \sqrt{s} \gg M \]

\[ \alpha_s^n \ln^m \frac{Q}{q_T} \]

CSS evolution equation

CCFM evolution equation

TMD distributions (unpolarized and polarized)

II. The Parton Branching (PB) approach

- Connected with DGLAP evolution of collinear parton distribution functions
- Applicable over broad kinematic range from low to high transverse momenta, for inclusive as well as non-inclusive observables
- Implementable in Monte Carlo event generators
Parton Branching (PB) method: collinear PDFs

QCD evolution and soft-gluon resolution scale

\[ \tilde{f}_a(x, \mu^2) = \Delta_a(\mu^2) \tilde{f}_a(x, \mu_0^2) + \sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'}{\mu'^2} \frac{\Delta_a(\mu'^2)}{\Delta_a(\mu^2)} \int_{x}^{z_M} dz \ P_{ab}^{(R)}(\alpha_s(\mu'^2), z) \tilde{f}_b(x/z, \mu'^2) \]

where \[ \Delta_a(z_M, \mu^2, \mu_0^2) = \exp \left( -\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'}{\mu'^2} \int_0^{z_M} dz \ z P_{ba}^{(R)}(\alpha_s(\mu'^2), z) \right) \]

\[ \text{\textgreater soft-gluon resolution parameter } z_M \text{ separates resolvable and nonresolvable branchings} \]
\[ \text{\textgreater no-branching probability } \Delta; \text{ real-emission probability } P^{(R)} \]

- Equivalent to DGLAP evolution equation for \( z_M \rightarrow 1 \)
Parton Branching (PB) method: TMD PDFs

\[
\tilde{\mathcal{A}}_a(x, k, \mu^2) = \Delta_a(\mu^2) \tilde{\mathcal{A}}_a(x, k, \mu_0^2) + \sum_b \int \frac{d^2 q'}{\pi q'^2} \frac{\Delta_a(\mu^2)}{\Delta_a(q'^2)} \Theta(\mu^2 - q'^2) \Theta(q'^2 - \mu_0^2) \\
\times \int_x^{z_M} dz \mathcal{P}^{(R)}_{ab}(\alpha_s(q'^2), z) \tilde{\mathcal{A}}_b(x/z, k + (1 - z)q', q'^2)
\]

Solve iteratively: \( \tilde{\mathcal{A}}^{(0)}_a(x, k, \mu^2) = \Delta_a(\mu^2) \tilde{\mathcal{A}}_a(x, k, \mu_0^2) \),

\[
\tilde{\mathcal{A}}^{(1)}_a(x, k, \mu^2) = \sum_b \int \frac{d^2 q'}{\pi q'^2} \Theta(\mu^2 - q'^2) \Theta(q'^2 - \mu_0^2) \\
\times \frac{\Delta_a(\mu^2)}{\Delta_a(q'^2)} \int_x^{z_M} dz \mathcal{P}^{(R)}_{ab}(\alpha_s(q'^2), z) \tilde{\mathcal{A}}_b(x/z, k + (1 - z)q', \mu_0^2) \Delta_b(q'^2)
\]

Jung, Lelek, Radescu, Zlebcik & H, JHEP 01 (2018) 070

NB: angular ordering

- A new evolution equation!

\[
\mu = |q_c|/(1 - z)
\]
Validation of the method with semi-analytic result from QCDNUM at LO

Agreement to better than 1 % over several orders of magnitude in x and mu
Validation of the method with semi-analytic result from QCDNUM at NLO.

Very good agreement at NLO over all x and mu.
NB: the same approach is designed to work at NNLO.
TMDs and soft gluon resolution effects

Well-defined TMDs require appropriate ordering condition
PB method in xFitter

Determine starting distribution

\[ x f_a(x, \mu^2) = x \int dx' \int dx'' A_{0,b}(x') \tilde{A}_a^b(x'', \mu^2) \delta(x' x'' - x) \]
\[ = \int dx' A_{0,b}(x') \cdot \frac{x}{x'} \tilde{A}_a^b \left( \frac{x}{x'}, \mu^2 \right) \]

- fit to HERA data (using xFitter) with \( Q^2 \geq 3.5 \text{ GeV}^2 \) gives \( \chi^2/ndf \sim 1.2 \)

A Bermudez et al, arXiv:1804.11152
A. Lelek et al REF 2016
TMD distributions from fits to precision HERA data

A Bermudez et al, arXiv:1804.11152

- NLO determination of TMDs with uncertainties
Where to find TMDs? TMDlib and TMDplotter

- TMDlib proposed in 2014 as part of the REF Workshop and developed since
- A library of parameterizations and fits of TMDs (LHAPDF-style)

http://tmdlib.hepforge.org
http://tmdplotter.desy.de

- Also contains collinear (integrated) pdfs
Next REF Workshop: Cracow, 19-22 November 2018

https://indico.cern.ch/event/696311
III. ONGOING WORK:
new results and applications

- Drell-Yan pT spectrum from convolution of two transverse momentum dependent distributions

- Comparison of parton branching results with analytic TMD resummation (Collins-Soper-Sterman method)

- First implementation for jets (using NLO matrix elements for color-charged final states)
Application of PB method to Z-boson transverse momentum spectrum in Drell-Yan production

- Parton branching TMD defined by using angular ordering
- Scale in running coupling also by angular ordering

\[ \alpha_s(\mu^2 (1-z)^2) \]

- Mu-dependent soft-gluon resolution scale parameter \( z_M \)

\[ z_M(\mu) = 1 - q_0/\mu \]

LHC Electroweak WG Meeting, CERN, June 2018
Z-boson transverse momentum spectrum: soft-gluon angular ordering effects


ATLAS data, EPJC 76 (2016) 291
Comparison with CSS (Collins-Soper-Sterman) resummation

◊ The resummed DY differential cross section is given by

\[
\frac{d\sigma}{d^2q dM^2 dy} = \sum_{q, \bar{q}} \frac{\sigma^{(0)}}{s} H(\alpha_s) \int \frac{d^2b}{(2\pi)^2} e^{iq \cdot b} A_q(x_1, b, M) A_{\bar{q}}(x_2, b, M) + O\left(\frac{|q|}{M}\right)
\]

where

\[
A_i(x, b, M) = \exp \left\{ \frac{1}{2} \int_{c_0/b^2}^{M^2} \frac{d\mu^{'2}}{\mu^{'2}} \left[ A_i(\alpha_s(\mu^{'2})) \ln \left( \frac{M^2}{\mu^{'2}} \right) + B_i(\alpha_s(\mu^{'2})) \right] \right\} \exp \left( \frac{-b^2}{2\lambda^2} \right)
\]

\[
\times \sum_j \int_{x}^{1} \frac{dz}{z} C_{ij}(z, \alpha_s \left( \frac{c_0}{b^2} \right)) f_j \left( \frac{x}{z}, \frac{c_0}{b^2} \right)
\]

and the coefficients $H$, $A$, $B$, $C$ have power series expansions in $\alpha_s$.

◊ The parton branching TMD is expressed in terms of real-emission $P^{(R)}$:

\[ \text{via momentum sum rules, use unitarity to relate } P^{(R)} \text{ to virtual emission} \]

\[ \text{identify the coefficients in the two formulations, order by order in } \alpha_s, \text{ at LL, NLL, ...} \]
Comparison with CSS (Collins-Soper-Sterman) resummation

More precisely:

- The parton branching TMD contains Sudakov form factor in terms of

\[ P_{ab}^{(R)}(\alpha_s, z) = K_{ab}(\alpha_s) \frac{1}{1 - z} + R_{ab}(\alpha_s, z) \text{ where} \]

\[ K_{ab}(\alpha_s) = \delta_{ab} k_a(\alpha_s), \quad k_a(\alpha_s) = \sum_{n=1}^{\infty} \left( \frac{\alpha_s}{2\pi} \right)^n k^{(n-1)}_a, \quad R_{ab}(\alpha_s, z) = \sum_{n=1}^{\infty} \left( \frac{\alpha_s}{2\pi} \right)^n R^{(n-1)}_{ab}(z) \]

- Via momentum sum rules, use unitarity to re-express this in terms of

\[ P^{(V)} = P - P^{(R)}, \text{ where} \]

\[ P_{ab}(\alpha_s, z) = D_{ab}(\alpha_s) \delta(1 - z) + K_{ab}(\alpha_s) \frac{1}{(1 - z)_+} + R_{ab}(\alpha_s, z) \]

is full splitting function (at LO, NLO, etc.)

with \[ D_{ab}(\alpha_s) = \delta_{ab} d_a(\alpha_s), \quad d_a(\alpha_s) = \sum_{n=1}^{\infty} \left( \frac{\alpha_s}{2\pi} \right)^n d^{(n-1)}_a \]

- Identify \( d_a(\alpha_s) \) and \( k_a(\alpha_s) \) with resummation formula coefficients (LL, NLL, . . .)
Comparison with CSS (Collins-Soper-Sterman) resummation

- $d_a(\alpha_s)$ and $k_a(\alpha_s)$ perturbative coefficients

one-loop:

$$d_q^{(0)} = \frac{3}{2} C_F, \quad k_q^{(0)} = 2 C_F$$

two-loop:

$$d_q^{(1)} = C_F^2 \left( \frac{3}{8} - \frac{\pi^2}{2} + 6 \zeta(3) \right) + C_F C_A \left( \frac{17}{24} + \frac{11\pi^2}{18} - 3 \zeta(3) \right) - C_F T_R N_f \left( \frac{1}{6} + \frac{2\pi^2}{9} \right),$$

$$k_q^{(1)} = 2 C_F \Gamma, \quad \text{where} \quad \Gamma = C_A \left( \frac{67}{18} - \frac{\pi^2}{6} \right) - T_R N_f \frac{10}{9}$$

- The $k$ and $d$ coefficients of the PB formalism match, order by order, the $A$ and $B$ coefficients of the CSS formalism
Di-jets from PB method: towards NLO-matched parton-shower Monte Carlo generators with TMDs

- Events by NLO POWHEG 2 jets
- Parton branching TMD (with angular ordering)
- TMD parton shower
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Conclusions

- method to take into account simultaneously soft-gluon emission at \( z \rightarrow 1 \) and transverse momentum \( q_T \) recoils in the parton branchings along the QCD cascade

- potentially relevant for calculations both in collinear factorization and in transverse momentum dependent factorization \( \rightarrow \) cf. parton shower calculations and analytic resummation

- terms in powers of \( \ln (1 - zM) \) can be related to large-\( x \) resummation \( \rightarrow \) relevant to near-threshold, rare processes to be investigated at high luminosity

- numerical examples in Drell-Yan production and jets \( \rightarrow \) systematic studies of ordering effects and color coherence
EXTRA SLIDES
Stability with respect to resolution scale $z_M$
PB method at NNLO

- In NNLO VFNS discontinuities both in $\alpha_s$ and PDFs
- These discontinuities ensure continuity of observables, e.g. $F_2$

Discontinuities in the quark and gluon Sudakov factors

R. Zlebcik, talk at REF 2017, November 2017

Workshop REF2017, Universidad Complutense Madrid, 13-16 November 2017
PB method at NNLO

The Monte Carlo solution vs QCDNUM

LO  NLO  NNLO

The Monte Carlo evolution implemented up to NNLO and cross-checked against the semi-analytical solution of DGLAP

The solution's uncertainties are mainly statistical
(~ number of generated MC evolutions)

R. Zlebcik, talk at REF 2017, November 2017

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Effects of coupling's scale and angular ordering in integrated parton distributions
Z-boson pT spectrum including TMD uncertainties

- Cf. predictions from fixed-order + resummed calculations
  Bizon et al., arXiv:1805.05916