Inflation models in the LHC era

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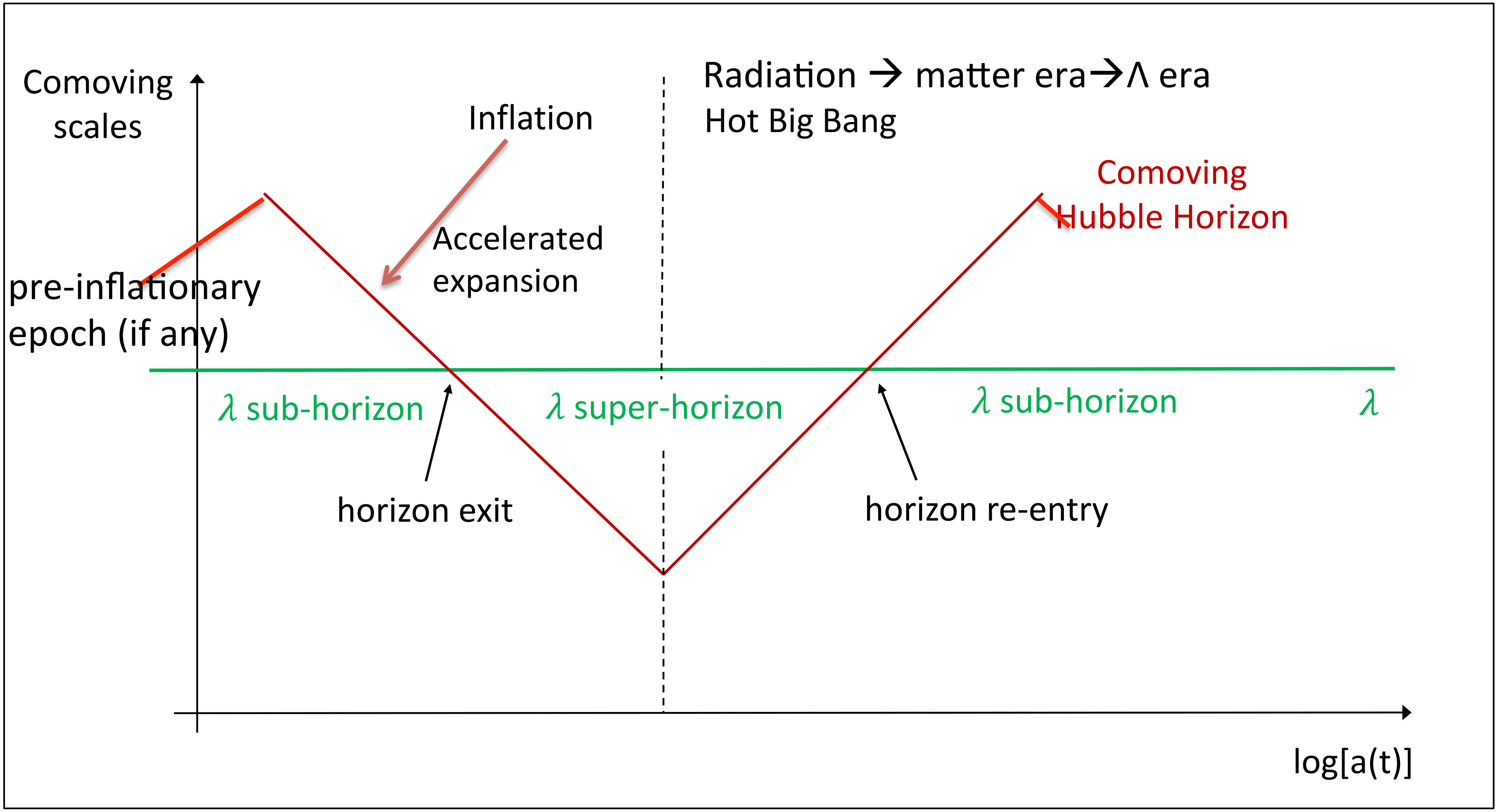
Outline

- A short introduction
- Current observational status
- Why and how inflation is sensitive to UV fundamental physics?
- Future prospects

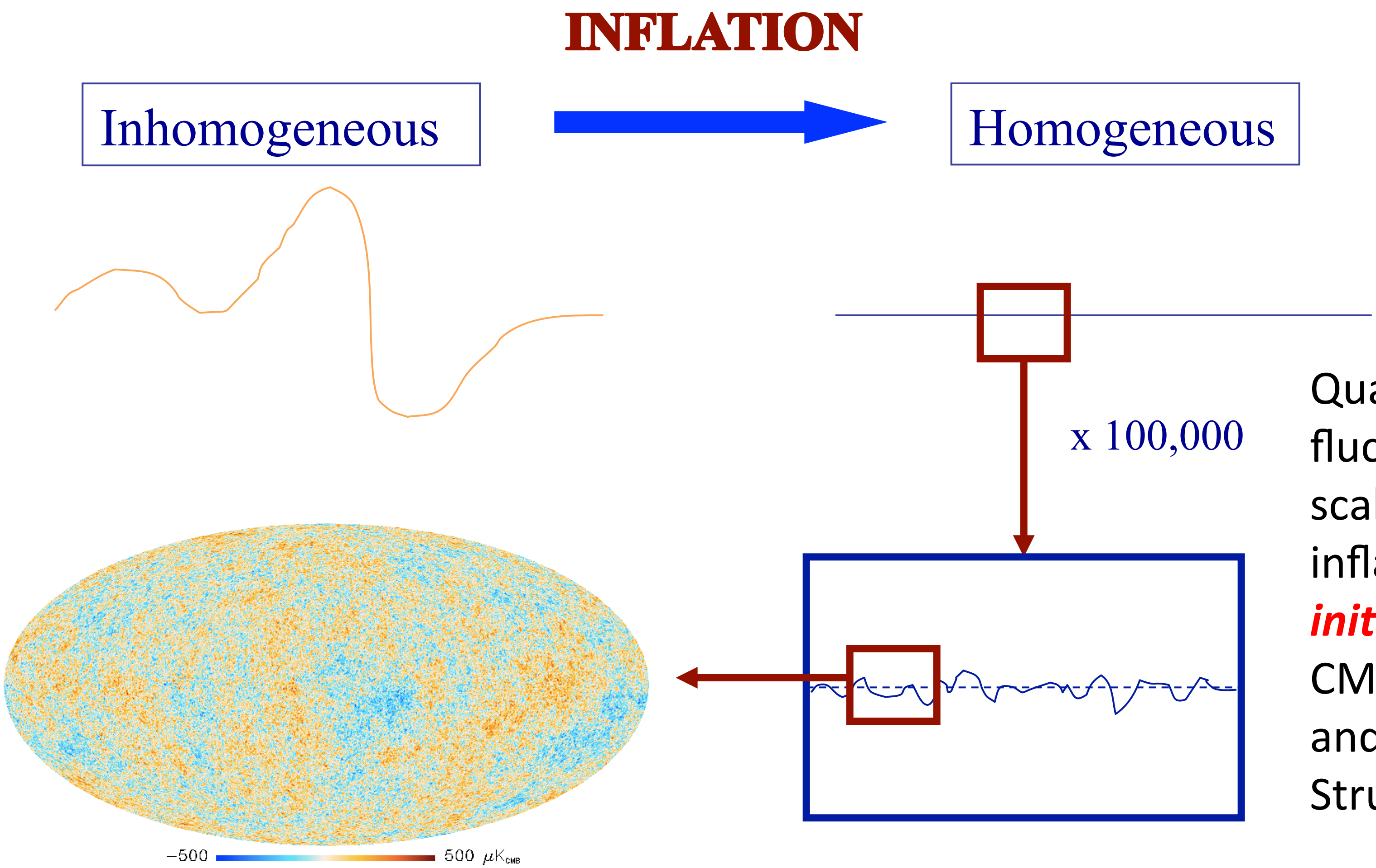
4 FACTS INFLATION CAN EXPLAIN

- The Universe is old
- The Universe is homogeneous and isotropic (on large scales)
- The Universe today is very close to be spatially flat
- Structures grew out of tiny, *almost* scale invariant perturbations

The rise and fall … of the comoving Hubble horizon

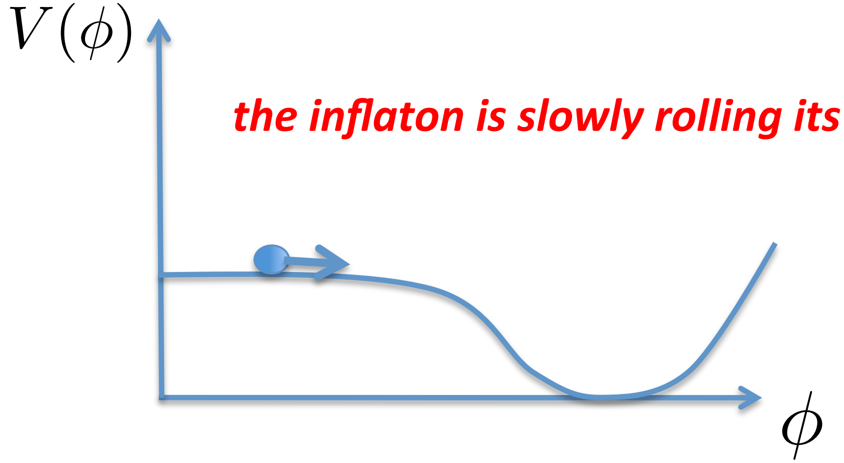


Initial conditions



Quantum fluctuations of a scalar field, the inflaton, set the ***initial conditions*** for CMB anisotropies and Large-Scale Structure formation

Inflation



✓ $V(\phi) >> \frac{1}{2}\dot{\phi}^2 \longrightarrow H^2 = \frac{8\pi G}{3} V(\phi) \simeq const. \longrightarrow a(t) \simeq e^{Ht} \quad \left(H(t) = \frac{\dot{a}}{a} \right)$

accelerated expansion in the early universe

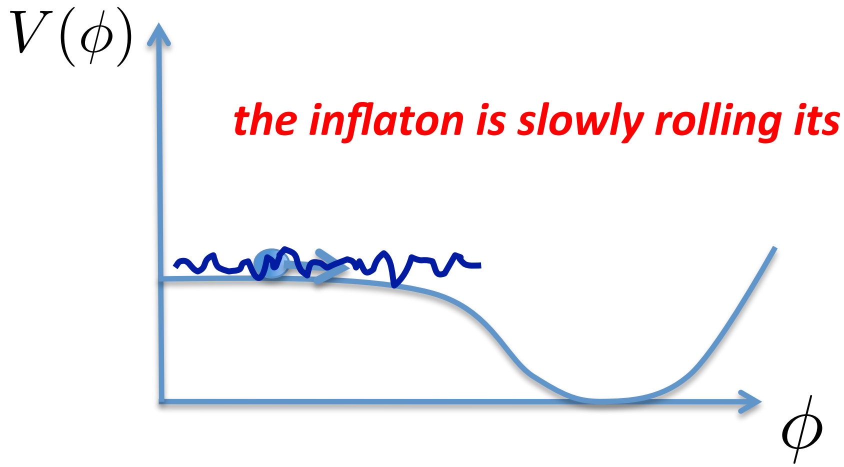
✓ To induce acceleration the potential must be flat

$$\varepsilon = \frac{M_{Pl}^2}{2}\left(\frac{V_\phi}{V}\right)^2 << 1$$

✓ To have long enough inflation, V(ϕ) must be flat for long enough

$$\eta = M_{Pl}^2 \frac{V_{\phi\phi}}{V} << 1$$

Inflation



Fluctuations in the inflaton produce fluctuations in the universe expansion from place to place, so that each region in the universe goes through the same expansion history but at slightly different times:

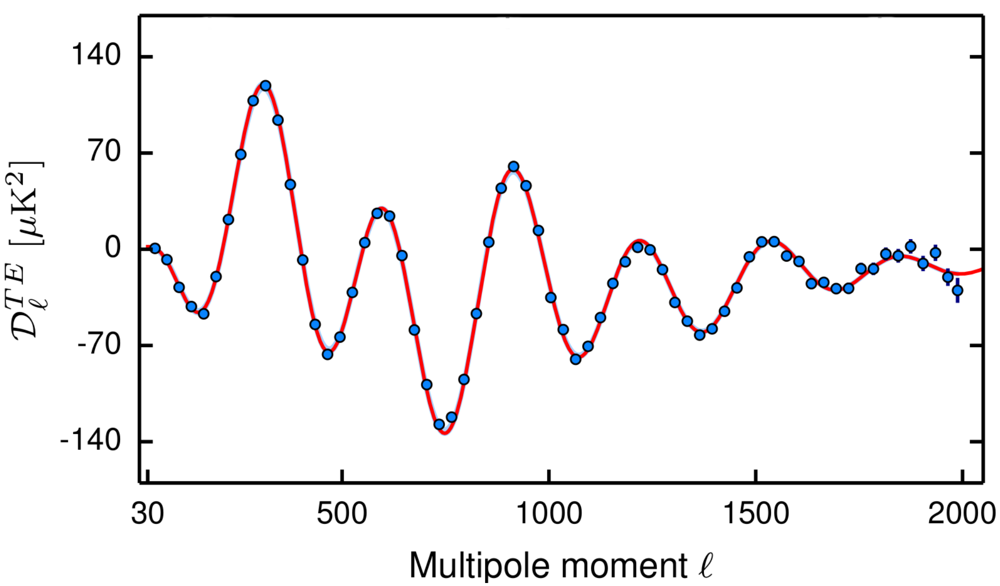
$$\zeta \sim \frac{\Delta T}{T} \sim \frac{\delta\rho}{\rho} \qquad \zeta \simeq \frac{H\delta\phi}{\dot{\phi}}$$

Cross-correlation T-E

You expect a cross-correlation because both T and E-modes are sourced by density perturbations

SUPERHORIZON CORRELATIONS

The ***anticorrelation between T and E for 50 <l <200*** is a distinctive signature of adiabatic superhorizon fluctuation at last scattering, which is a ***distinctive signature of inflation:***
Inflation produces fluctuations which are coherent on superhorizon scales at last scattering



Observational predictions

- Primordial density (scalar) perturbations

$$\mathcal{P}_\zeta(k) = \frac{16}{9}\frac{V^2}{M_{\rm Pl}^4\dot{\phi}^2}\left(\frac{k}{k_0}\right)^{n-1}$$

amplitude

spectral index: $n - 1 = 2\eta - 6\epsilon$

describes deviations from scale invariance

$$\epsilon = \frac{M_{\rm Pl}^2}{16\pi}\left(\frac{V'}{V}\right)^2 \ll 1; \;\; \eta = \frac{M_{\rm Pl}^2}{8\pi}\left(\frac{V''}{V}\right) \ll 1$$

- Primordial (tensor) gravitational waves

$$\mathcal{P}_{\rm T}(k) = \frac{128}{3}\frac{V}{M_{\rm Pl}^4}\left(\frac{k}{k_0}\right)^{n_{\rm T}}$$

Tensor spectral index: $n_{\rm T} = -2\epsilon$

- Tensor-to-scalar perturbation ratio***: parametrizes strength of primordial GW signal***

$$r = \frac{\mathcal{P}_{\rm T}}{\mathcal{P}_\zeta} = 16\epsilon$$

- **Consistency relation** (valid for *all* single field models of slow-roll inflation):

$$r = -8n_T$$

INFLATIONARY CONSISTENCY RELATION

single-field slow-roll inflation

(vacuum fluctuations)

$$r = -8n_T$$

test
for single-field
slow-roll inflation

Other inflationary models beyond the standard ones→ violation?

From Guzzetti, M, N.B., M. Liguori, S. Matarrese, ``Gravitational waves from Inflation'', arXiv:1605.01615

	Model	Tensor power-spectrum	Tensor spectral index		Consistency relation
Background	Standard infl.	$P_{\rm T} = \frac{8}{M_{\rm pl}^2}\left(\frac{H}{2\pi}\right)^2$	$n_{\rm T} = -2\epsilon$	red	$r = -8n_{\rm T}$
	EFT inflation[(a)]	$P_{\rm T} = \frac{8}{c_{\rm T} M_{\rm pl}^2}\left(\frac{H}{2\pi}\right)^2$	$n_{\rm T} = -2\epsilon + \frac{2}{3}\frac{m_{\rm T}^2}{\alpha H^2}\left(1 + \frac{4}{3}\epsilon\right)$	r/b	-
	EFT inflation[(b)]	$P_{\rm T} = \frac{8}{c_{\rm T} M_{\rm pl}^2}\frac{2^{\frac{-p}{1+p}}}{\pi}\Gamma^2\left(\frac{1}{2(1+p)}\right)\left(\frac{H}{2\pi}\right)^2$	$n_{\rm T} = \frac{p}{1+p}$	blue	violation
	Gen. G-Infl.	$P_{\rm T} = \frac{8}{M_{\rm pl}^2}\gamma_{\rm T}\frac{\mathscr{G}_{\rm T}^{1/2}}{\mathscr{F}_{\rm T}^{3/2}}\left(\frac{H}{2\pi}\right)^2$	$n_{\rm T} = 3 - 2\nu_{\rm T}$	r/b	-
	Pot.-driv. G-Infl.	$P_{\rm T} = \frac{8}{M_{\rm pl}^2}\left(\frac{H}{2\pi}\right)^2$	$n_{\rm T} = -2\epsilon$	r/b	$r \simeq -\frac{32\sqrt{6}}{9}n_{\rm T}$
Extra background	Particle prod.	$P_{\rm T}^{+} = 8.6 \times 10^{-7}\frac{4H^2}{M_{\rm pl}^2}\left(\frac{H}{2\pi}\right)^2\frac{e^{4\pi\xi}}{\xi^6}$	-	blue	violation
	Spectator field	$P_{\rm T} \simeq 3\frac{H^4}{c_{\rm S}^{18/5}M_{\rm pl}^4}$	$n_{\rm T} \simeq 2\left(\frac{2m^2}{3H^2} - 2\epsilon\right) - \frac{18}{5}\frac{\dot{c}_{\rm S}}{Hc_{\rm S}}$	r/b	violation

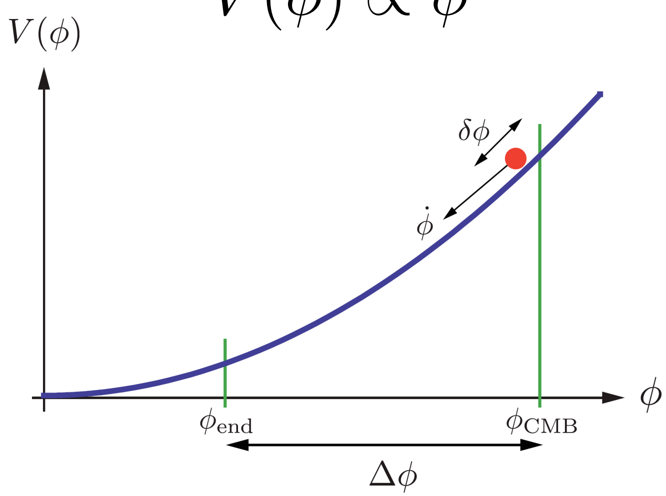
Courtesy of Maria Chiara Guzzetti

Inflaton dynamics and the level of gravity waves

Roughly speaking: ``Large field'' models can produce a high level of gravity waves;
``small field'' models produce a low level of gravity waves

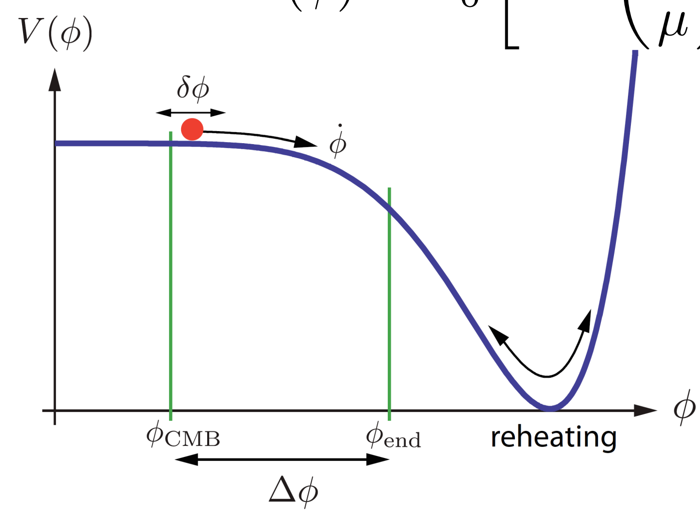
``Large field'' like potential

$$V(\phi) \propto \phi^{\alpha}$$



``Small field'' like potential

$$V(\phi) = V_0\left[1 - \left(\frac{\phi}{\mu}\right)^p\right]$$



$$\frac{\Delta\phi}{m_{\rm Pl}} \simeq \left(\frac{N}{30}\right) \times \left(\frac{r}{0.01}\right)^{1/2}$$

$30 \lesssim N \lesssim 60$

Current observational status

Planck parameters measurements

Parameter	TT+lowP 68 % limits	TT+lowP+lensing 68 % limits	TT+lowP+lensing+ext 68 % limits	TT,TE,EE+lowP 68 % limits	TT,TE,EE+lowP+lensing 68 % limits	TT,TE,EE+lowP+lensing+ext 68 % limits
$\Omega_b h^2$	0.02222 ± 0.00023	0.02226 ± 0.00023	0.02227 ± 0.00020	0.02225 ± 0.00016	0.02226 ± 0.00016	0.02230 ± 0.00014
$\Omega_c h^2$	0.1197 ± 0.0022	0.1186 ± 0.0020	0.1184 ± 0.0012	0.1198 ± 0.0015	0.1193 ± 0.0014	0.1188 ± 0.0010
$100\theta_{MC}$	1.04085 ± 0.00047	1.04103 ± 0.00046	1.04106 ± 0.00041	1.04077 ± 0.00032	1.04087 ± 0.00032	1.04093 ± 0.00030
τ	0.078 ± 0.019	0.066 ± 0.016	0.067 ± 0.013	0.079 ± 0.017	0.063 ± 0.014	0.066 ± 0.012
$\ln(10^{10} A_s)$	3.089 ± 0.036	3.062 ± 0.029	3.064 ± 0.024	3.094 ± 0.034	3.059 ± 0.025	3.064 ± 0.023
n_s	0.9655 ± 0.0062	0.9677 ± 0.0060	0.9681 ± 0.0044	0.9645 ± 0.0049	0.9653 ± 0.0048	0.9667 ± 0.0040
H_0	67.31 ± 0.96	67.81 ± 0.92	67.90 ± 0.55	67.27 ± 0.66	67.51 ± 0.64	67.74 ± 0.46
Ω_Λ	0.685 ± 0.013	0.692 ± 0.012	0.6935 ± 0.0072	0.6844 ± 0.0091	0.6879 ± 0.0087	0.6911 ± 0.0062
Ω_m	0.315 ± 0.013	0.308 ± 0.012	[illegible]	[illegible]	[illegible]	0.3089 ± 0.0062
$\Omega_m h^2$	0.1426 ± 0.0020	0.1415 ± 0.0019	[illegible]	[illegible]	[illegible]	0.14170 ± 0.00097
$\Omega_m h^3$	0.09597 ± 0.00045	0.09591 ± 0.00045	[illegible]	[illegible]	[illegible]	0.09598 ± 0.00029
σ_8	0.829 ± 0.014	0.8149 ± 0.0093	0.8154 ± 0.0090	0.831 ± 0.013	0.8150 ± 0.0087	0.8159 ± 0.0086
$\sigma_8 \Omega_m^{0.5}$	0.466 ± 0.013	0.4521 ± 0.0088	0.4514 ± 0.0066	0.4668 ± 0.0098	0.4553 ± 0.0068	0.4535 ± 0.0059
$\sigma_8 \Omega_m^{0.25}$	0.621 ± 0.013	0.6069 ± 0.0076	0.6066 ± 0.0070	0.623 ± 0.011	0.6091 ± 0.0067	0.6083 ± 0.0066
z_{re}	$9.9^{+1.8}_{-1.6}$	$8.8^{+1.7}_{-1.4}$	$8.9^{+1.3}_{-1.2}$	$10.0^{+1.7}_{-1.5}$	$8.5^{+1.4}_{-1.2}$	$8.8^{+1.2}_{-1.1}$
$10^9 A_s$	$2.198^{+0.076}_{-0.085}$	2.139 ± 0.063	2.143 ± 0.051	2.207 ± 0.074	2.130 ± 0.053	2.142 ± 0.049
$10^9 A_s e^{-2\tau}$	1.880 ± 0.014	1.874 ± 0.013	1.873 ± 0.011	1.882 ± 0.012	1.878 ± 0.011	1.876 ± 0.011
Age/Gyr	13.813 ± 0.038	13.799 ± 0.038	13.796 ± 0.029	13.813 ± 0.026	13.807 ± 0.026	13.799 ± 0.021
z_*	1090.09 ± 0.42	1089.94 ± 0.42	1089.90 ± 0.30	1090.06 ± 0.30	1090.00 ± 0.29	1089.90 ± 0.23
r_*	144.61 ± 0.49	144.89 ± 0.44	144.93 ± 0.30	144.57 ± 0.32	144.71 ± 0.31	144.81 ± 0.24
$100\theta_*$	1.04105 ± 0.00046	1.04122 ± 0.00045	1.04126 ± 0.00041	1.04096 ± 0.00032	1.04106 ± 0.00031	1.04112 ± 0.00029
z_{drag}	1059.57 ± 0.46	1059.57 ± 0.47	1059.60 ± 0.44	1059.65 ± 0.31	1059.62 ± 0.31	1059.68 ± 0.29
r_{drag}	147.33 ± 0.49	147.60 ± 0.43	147.63 ± 0.32	147.27 ± 0.31	147.41 ± 0.30	147.50 ± 0.24
k_D	0.14050 ± 0.00052	0.14024 ± 0.00047	0.14022 ± 0.00042	0.14059 ± 0.00032	0.14044 ± 0.00032	0.14038 ± 0.00029
z_{eq}	3393 ± 49	3365 ± 44	3361 ± 27	3395 ± 33	3382 ± 32	3371 ± 23
k_{eq}	0.01035 ± 0.00015	0.01027 ± 0.00014	0.010258 ± 0.000083	0.01036 ± 0.00010	0.010322 ± 0.000096	0.010288 ± 0.000071
$100\theta_{s,eq}$	0.4502 ± 0.0047	0.4529 ± 0.0044	0.4533 ± 0.0026	0.4499 ± 0.0032	0.4512 ± 0.0031	0.4523 ± 0.0023

n=1 excluded at 5.6 sigma!!

Observational constraints: *Planck*

Amplitude of primordial density (scalar) perturbations

$$\ln(10^{10} A_s) = 3.062 \pm 0.029 \quad (68\%\,\mathrm{CL})$$

Spectral index of primordial density (scalar) perturbations

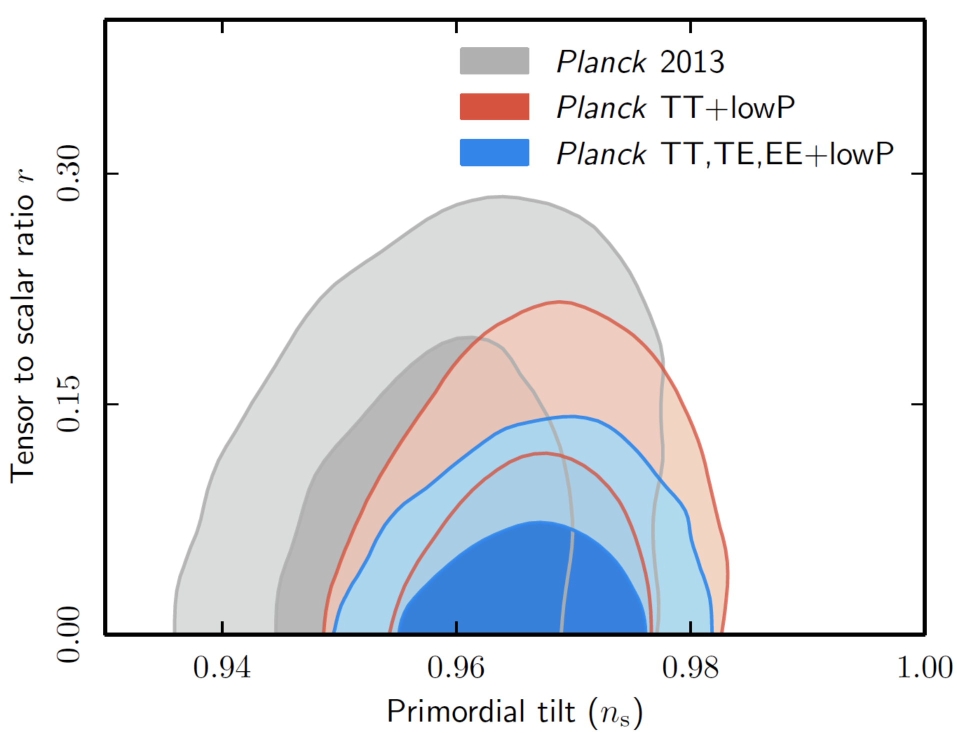
$$n_s = 0.9677 \pm 0.0060 \quad (68\%\,\mathrm{CL})$$

n=1 (Harrison Zeld' ovich spectrum) excluded at than 5.6 sigmas!

Two fundamental observational constants of cosmology in addition to three very well known (Ω_b, Ω_{cdm}, Ω_Λ).

Constraints on tensor modes

Model	Parameter	*Planck* TT+lowP	*Planck* TT+lowP+lensing	*Planck* TT+lowP+BAO	*Planck* TT,TE,EE+lowP
ΛCDM+r	n_s	0.9666 ± 0.0062	0.9688 ± 0.0061	0.9680 ± 0.0045	0.9652 ± 0.0047
	$r_{0.002}$	< 0.103	< 0.114	< 0.113	< 0.099
	$-2\Delta \ln \mathcal{L}_{max}$	0	0	0	0
ΛCDM+r +$dn_s/d\ln k$	n_s	0.9667 ± 0.0066	0.9690 ± 0.0063	0.9673 ± 0.0043	0.9644 ± 0.0049
	$r_{0.002}$	< 0.180	< 0.186	< 0.176	< 0.152
	r	< 0.168	< 0.176	< 0.166	< 0.149
	$dn_s/d\ln k$	$-0.0126^{+0.0098}_{-0.0087}$	$-0.0076^{+0.0092}_{-0.0080}$	-0.0125 ± 0.0091	-0.0085 ± 0.0076
	$-2\Delta \ln \mathcal{L}_{max}$	-0.81	-0.08	-0.87	-0.38

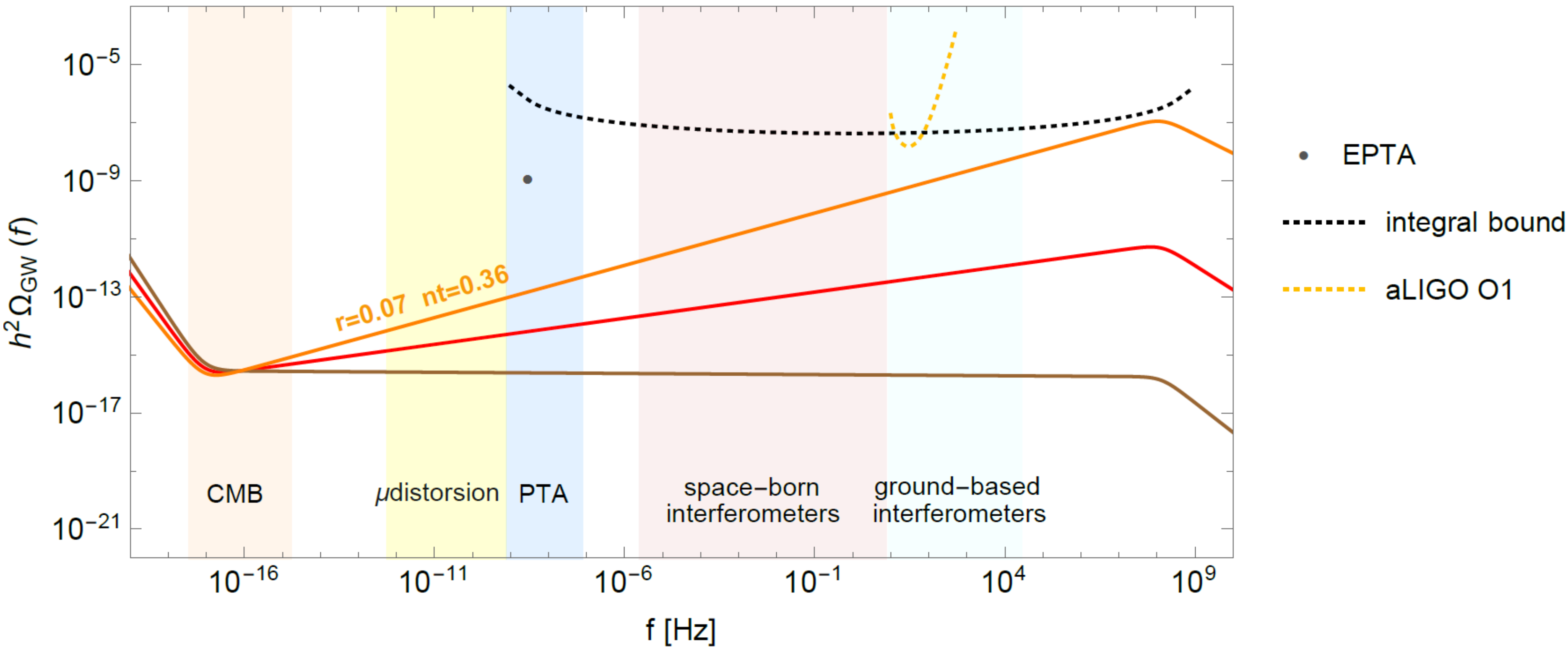


Current tightest constraint from a combination of Planck, BICEP2 and Keck Array data

$r<0.07$ (@ 95% C.L).

BICEP2, Keck Array, P.A.R. Ade et al., Phys. Rev. Lett. 116 (2016) 031302

CURRENT BOUNDS



primordial → **present time**

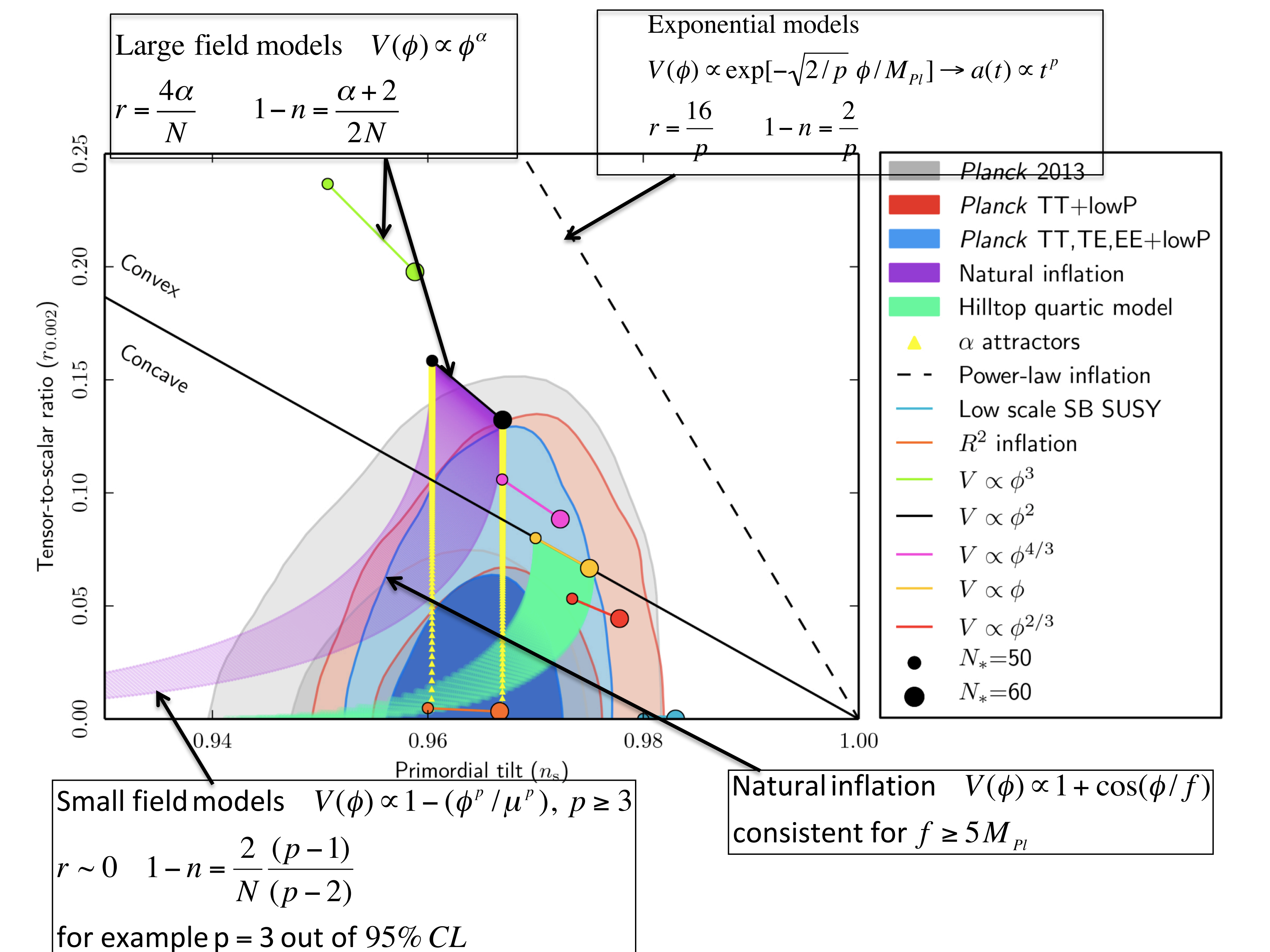
$$P_T(k) = A_T(k_*)\left(\frac{k}{k_*}\right)^{n_T}$$

present time
gw spectral energy density

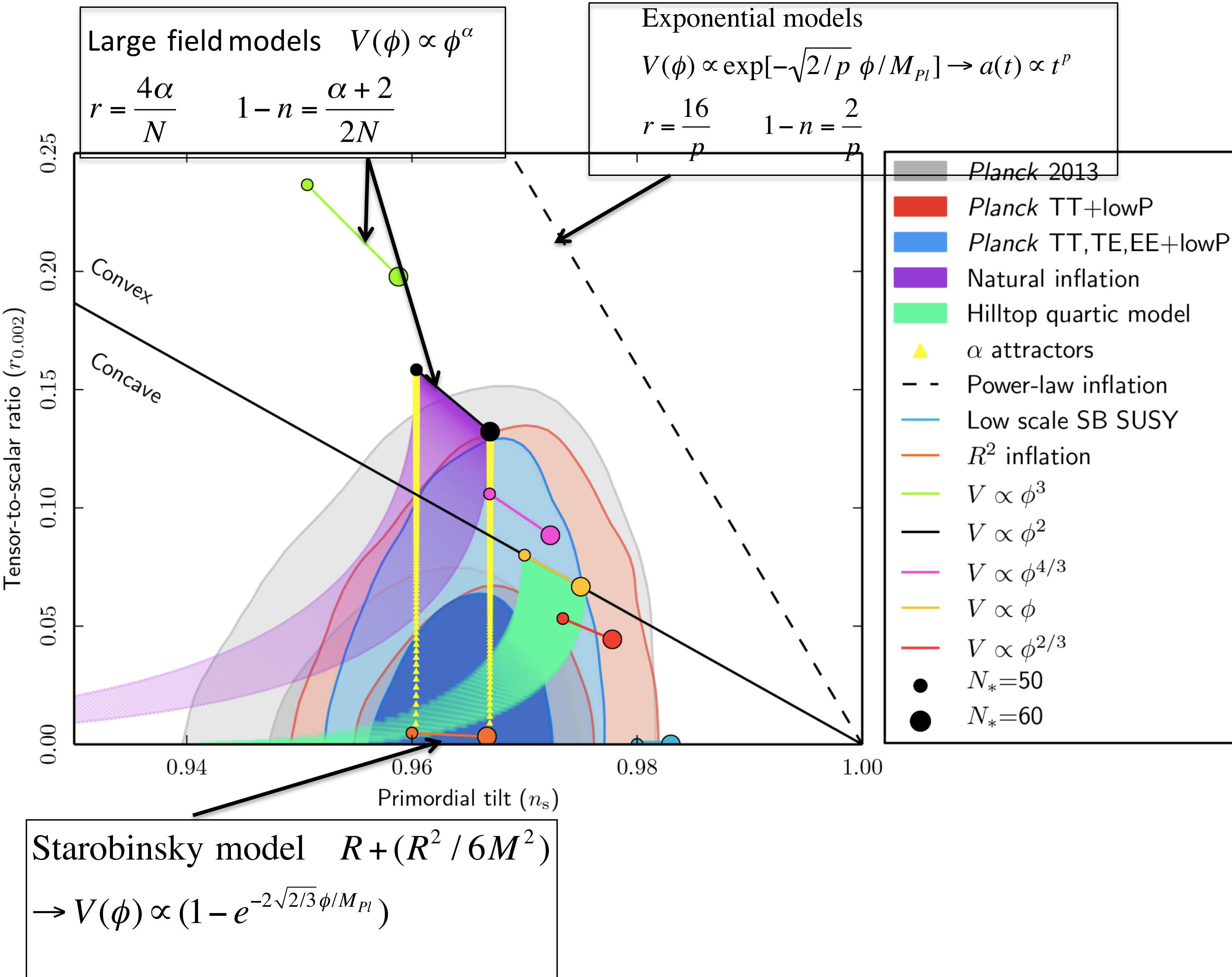
$$\Omega_{\rm GW}(k,\tau_0) \equiv \frac{1}{\rho_c}\frac{d\rho_{\rm gw}}{d\ln k} = \frac{1}{12}\left(\frac{k}{aH}\right)^2 T(k)\, P_{\rm T}(k)$$

Courtesy of Maria Chiara Guzzetti

What are the implications for inflationary models ?



Large field models $V(\phi) \propto \phi^{\alpha}$
$r = \frac{4\alpha}{N} \qquad 1-n = \frac{\alpha+2}{2N}$
Exponential models
$V(\phi) \propto \exp[-\sqrt{2/p}\,\phi/M_{Pl}] \rightarrow a(t) \propto t^{p}$
$r = \frac{16}{p} \qquad 1-n = \frac{2}{p}$
Convex
Concave
Tensor-to-scalar ratio ($r_{0.002}$)
0.00
0.05
0.10
0.15
0.20
0.25
0.94
0.96
0.98
1.00
Primordial tilt (n_s)
Planck 2013
Planck TT+lowP
Planck TT,TE,EE+lowP
Natural inflation
Hilltop quartic model
α attractors
Power-law inflation
Low scale SB SUSY
R^2 inflation
$V \propto \phi^3$
$V \propto \phi^2$
$V \propto \phi^{4/3}$
$V \propto \phi$
$V \propto \phi^{2/3}$
N_*=50
N_*=60
Small field models $V(\phi) \propto 1-(\phi^{p}/\mu^{p}),\ p \geq 3$
$r \sim 0 \quad 1-n = \frac{2}{N}\frac{(p-1)}{(p-2)}$
for example p = 3 out of 95% CL
Natural inflation $V(\phi) \propto 1+\cos(\phi/f)$
consistent for $f \geq 5M_{Pl}$



Large field models $V(\phi) \propto \phi^{\alpha}$
$r = \frac{4\alpha}{N}$ $1 - n = \frac{\alpha + 2}{2N}$
Exponential models
$V(\phi) \propto \exp[-\sqrt{2/p}\ \phi / M_{Pl}] \to a(t) \propto t^{p}$
$r = \frac{16}{p}$ $1 - n = \frac{2}{p}$
Starobinsky model $R + (R^2 / 6M^2)$
$\to V(\phi) \propto (1 - e^{-2\sqrt{2/3}\phi/M_{Pl}})$
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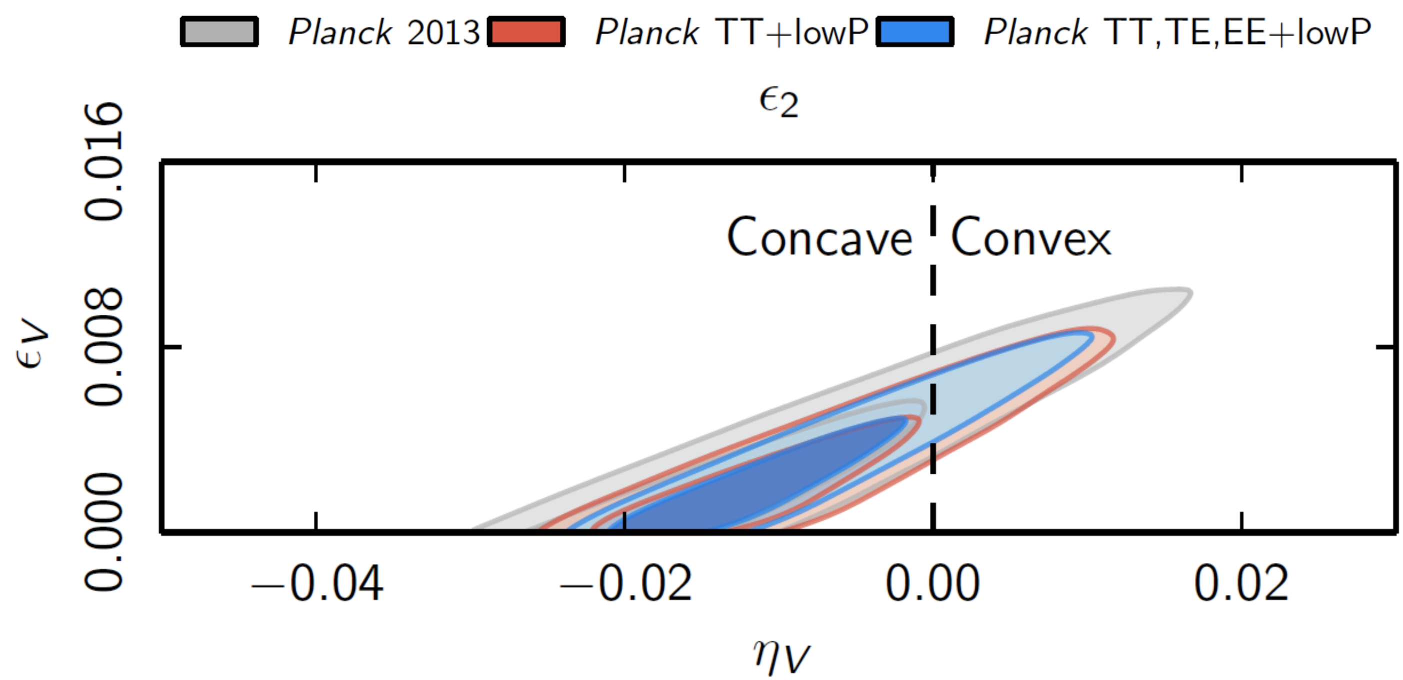
Constraints on slow-roll parameters



$$\epsilon_V < 0.012 \qquad (95\ \%\ \text{CL, } \textit{Planck}\ \text{TT+lowP})$$

$$\eta_V = -0.0080^{+0.0088}_{-0.0146} \qquad (68\ \%\ \text{CL, } \textit{Planck}\ \text{TT+lowP})$$

$$\xi_V = 0.0070^{+0.0045}_{-0.0069} \qquad (68\ \%\ \text{CL, } \textit{Planck}\ \text{TT+lowP})$$

Why Inflation is sensitive to high-energy fundamental physics?

At least two (main) avenues:

- ***gravitational waves***
- ***primordial non-Gaussianity***

Gravity waves from inflation

- A ***smoking gun*** of a period of inflation in the early universe: a stochastic background of gravitational waves is predicted by inflation independently of the specific inflationary model

- The amplitude of the inflationary gravity waves probes the ***energy scale of inflation***

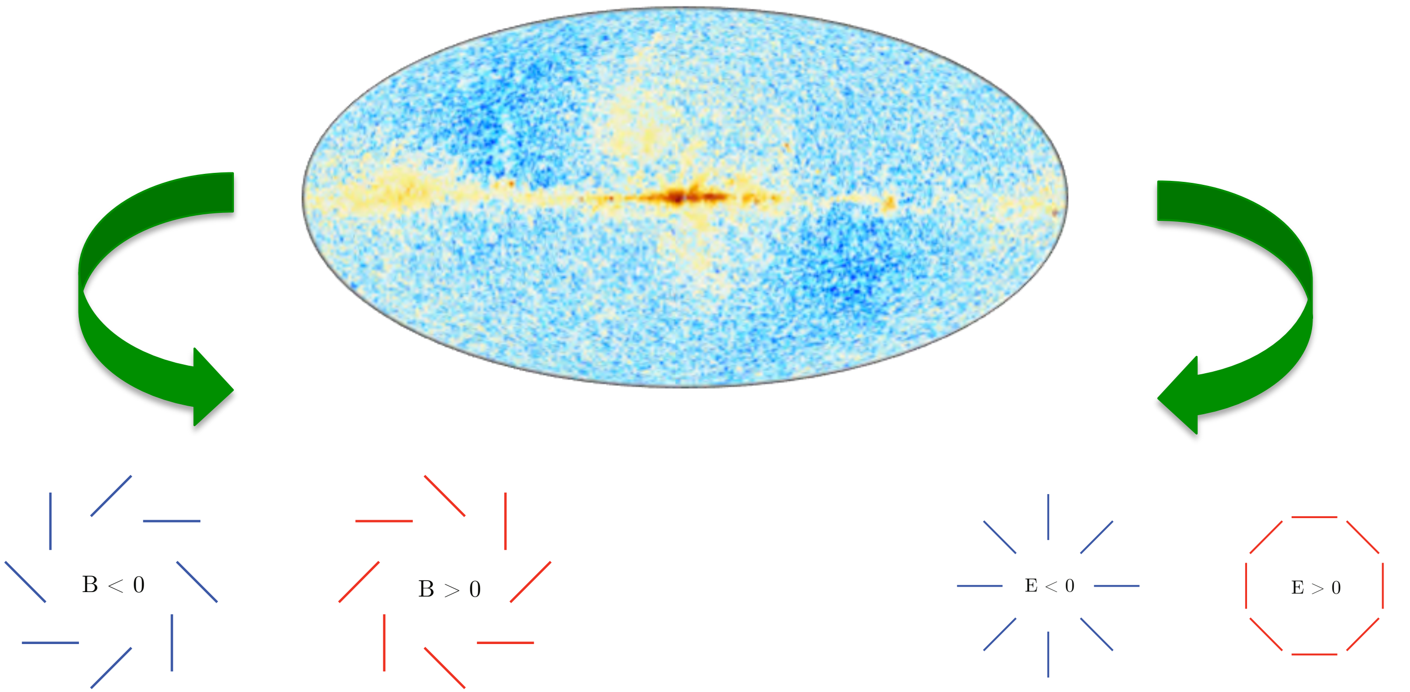
$$V^{1/4} = 1.06 \times 10^{16}\, GeV \left(\frac{r}{0.01} \right)^{1/4}$$

GUT SCALE

- ***a detection would provide a firm observational link to physics of the early universe, characterized by energies never achievable in labs***

- inflationary gravity waves generate a unique imprint into the CMB polarization pattern ***(the so called B-modes of polarization)***

Looking for gravitational waves via CMB polarization



Sourced by tensor (and vector) perturbations

$$P_{\mathrm{T}} \sim \left(\frac{V}{M_{\mathrm{Pl}}}\right)^4$$

Sourced by scalar and tensor (and vector) pertuabtions

Primary goal for future CMB experiments

Sensitivity of Inflation to fundamental physics and symmetries

A worked example

take $V(\phi)_{\text{slow-roll}}$

operators like $\phi^2 V(\phi)_{\text{slow-roll}} / \Lambda^2$

induce $\eta = M^2_{PL} (V''/V) = (M_{Pl}/\Lambda) \sim 1$!!

whatever physics there is around the Planck scale, it must ensure these terms are not induced (largely suppresses them)

→ Ultraviolet sensitivity

The issue can be solved by a shift symmetry $\phi \rightarrow \phi + \text{const}$

Sensitivity of Inflation to fundamental physics

✧ Case A: no shift symmetry; just $\phi \rightarrow -\phi$

Cutoff $\Lambda \sim M_{Pl}$

$$\mathcal{L}_{\rm eff}(\phi) = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4}\lambda\phi^4 - \sum_{p=1}^{\infty}\left[\lambda_p\phi^4 + \nu_p(\partial\phi)^2\right]\left(\frac{g\,\phi}{\Lambda}\right)^{2p} + \ldots$$

the general expectations is λ_p and $\nu_p \sim 1$, and the inflaton potential can get important correction for inflaton field excursion $\sim M_{Pl}$

→ need ***$\Delta\varphi \ll M_{Pl}$: small field models of inflation***

Sensitivity of Inflation to fundamental physics

✧ Case B: approximate shift symmetry $\phi \rightarrow \phi + \text{const}$

Cutoff $\Lambda \sim M_{Pl}$

$$\mathcal{L}_{\text{eff}}(\phi) = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4}\lambda\phi^4 - \sum_{p=1}^{\infty}\left[\lambda_p\phi^4 + \nu_p(\partial\phi)^2\right]\left(\frac{g\,\phi}{\Lambda}\right)^{2p} + \ldots$$

flatness of the inflaton potential is guaranteed because the ***symmetry of the UV theory*** forbids coefficients λ_p and ν_p~1.

Example: $V(\phi)=\mu^{4-p}\,\phi^p$,
with $\mu << M_{Pl}$ from scalar power spectrum

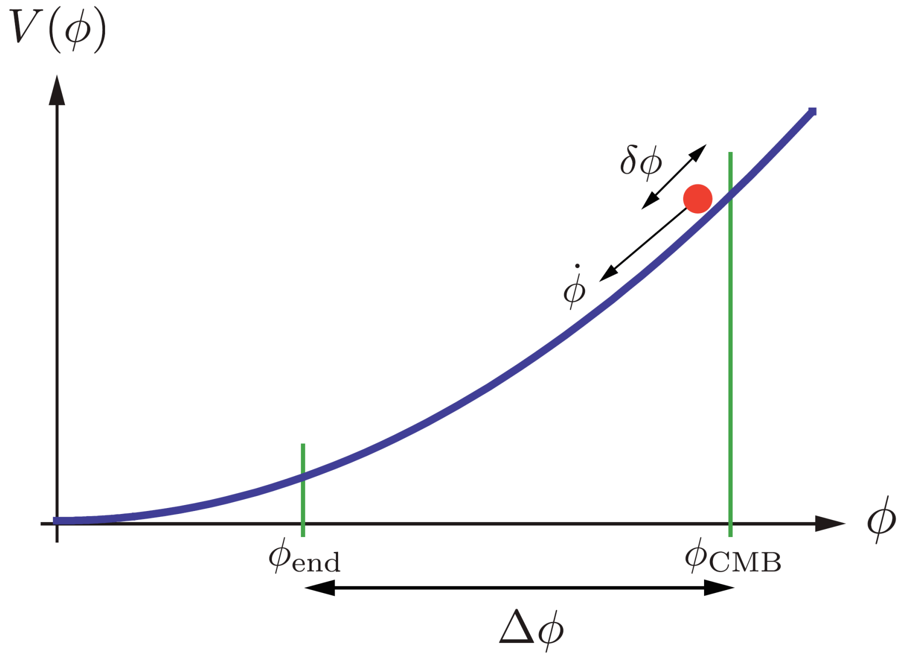
Such Lagrangians support large field models of inflation ($\Delta\varphi >> M_{Pl}$)

A couple of examples

Chaotic inflation like potentials

Case B.

$V(\varphi)=\mu^{4-p}\,\varphi^{p}$, with $\mu \ll M_{Pl}$ from scalar power spectrum



Axion inflation

$$\mathcal{L} = -\frac{1}{2}(\partial\varphi)^2 - \mu^{4-p}\varphi^p - \Lambda^4 \cos\left(\frac{\varphi}{f}\right) - \frac{\alpha}{4f}\varphi F_{\mu\nu}\tilde{F}^{\mu\nu} - \frac{F^2}{4}$$

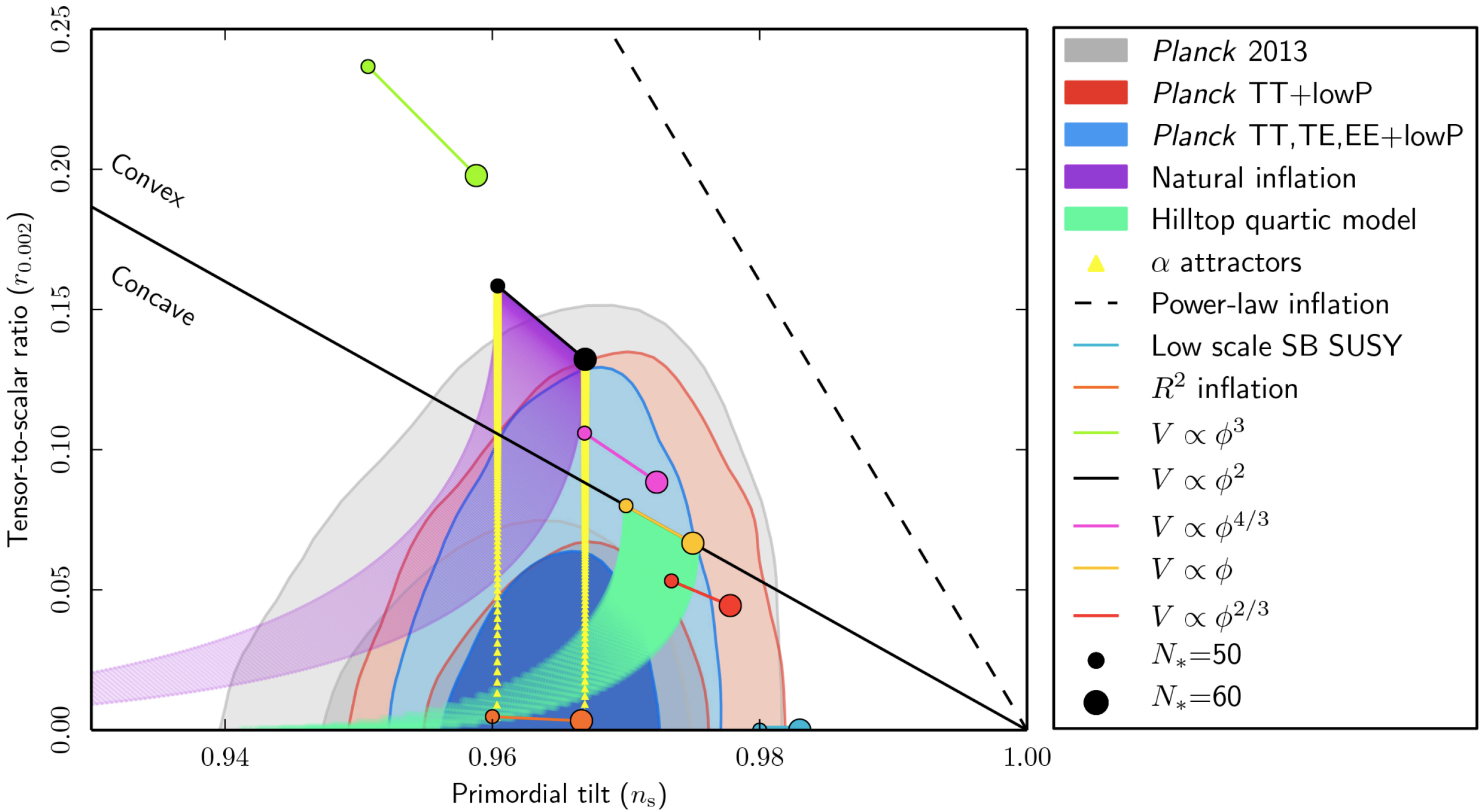
e.g.: axion monodromy

Oscillatory contribution

Coupling to gauge field

- Based on (slightly broken) shift symmetry that forbids corrections like $\varphi^2 V_{sr}/M_{\rm PL}^2$ which would spoil inflation
- From an effective field theory point of view the coupling to the gauge field should be included
- The coupling to the gauge field has a very rich phenomenology, both for primordial NG and for gravitational waves

(e.g. Barnaby & Peloso 2011; Barnaby, Pajer, Peloso 2011; Meerburg and Pajer 2012; Linde et al. 2013)



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$V \propto \phi^2$
$V \propto \phi^{4/3}$
$V \propto \phi$
$V \propto \phi^{2/3}$
N_*=50
N_*=60

Higgs inflation
(a short discussion and a few examples)

Higgs inflation

✓ potential of the Higgs field at large-field values $V(h) = \frac{\lambda}{4} h^4$
does not work: $\lambda \sim 10^{-13}$ would be required
to have enough inflation and to generate the right amplitude of primordial density perturbations

✓ Introduce a minimal modification

$$S = \int \mathrm{d}x^4 \sqrt{-g} \left[-\frac{1}{2} M_{Pl}^2 R - \frac{1}{2} \xi h^2 R + \frac{1}{2} \partial_\mu h \partial^\mu h - V(h) \right]$$

See, e.g., Bezrukov and Shaposhnikov (2008), or the review Bezrukov (2014)

✓ Via a Weyl transformation (+ a redefinition of the field to have a canonical kinetic term)

$$g_{\mu\nu} \longrightarrow \Omega^2 g_{\mu\nu} \qquad \Omega^2 = 1 + \frac{\xi h^2}{M_{Pl}^2} \qquad h = \frac{M_{\rm Pl}}{\sqrt{\xi}} \exp\left(\frac{\chi}{\sqrt{6} M_{\rm Pl}}\right)$$

$$S = \int \mathrm{d}x^2 \sqrt{-g} \left[-\frac{1}{2} M_{Pl}^2 R + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - U(\chi) \right]$$

$$U(\chi) \simeq \frac{\lambda M_{Pl}^2}{4\xi^2} \left(1 - \mathrm{e}^{-\frac{2\chi}{\sqrt{6} M_{Pl}}}\right)^2 \simeq \frac{\lambda h^4}{4\left(1 + \frac{\xi h^2}{M_{\rm Pl}}\right)^2}$$

(for large field values > $M_{Pl}/\xi^{1/2}$)

✓ to match the observed amplitude of primordial density perturbations

$$\xi \simeq 48000\sqrt{\lambda} \quad \rightarrow \quad n_s - 1 = -\frac{2}{N} \simeq 0.967 \qquad r = \frac{12}{N^2} \simeq 0.0031$$

✓ Interesting, but not without some issues

- Intrinsic theoretical uncertainty in computing the quantum corrections

- For $h >> M_{Pl}/\xi$ perturbative unitarity is violated (e.g. Burgess, Lee, Trott 2010)

- Stability of the Higgs potential up to $M_{Pl}/\xi^{1/2}$ is required.

- finally (but this is maybe a matter of taste……): why the Higgs and the scalar field driving inflation shoud be the same?

The Higgs as the inflaton

- An attempt to have Higgs as the inflaton without introducing nothing beyond SM
- exploits the fact that at high energies a plateau develops in the Higgs potential for a narrow range of Higgs and top masses.

First proposed in Isidori, Rychkov, Strumia, Tetradis, 2008.

The Higgs as the inflaton

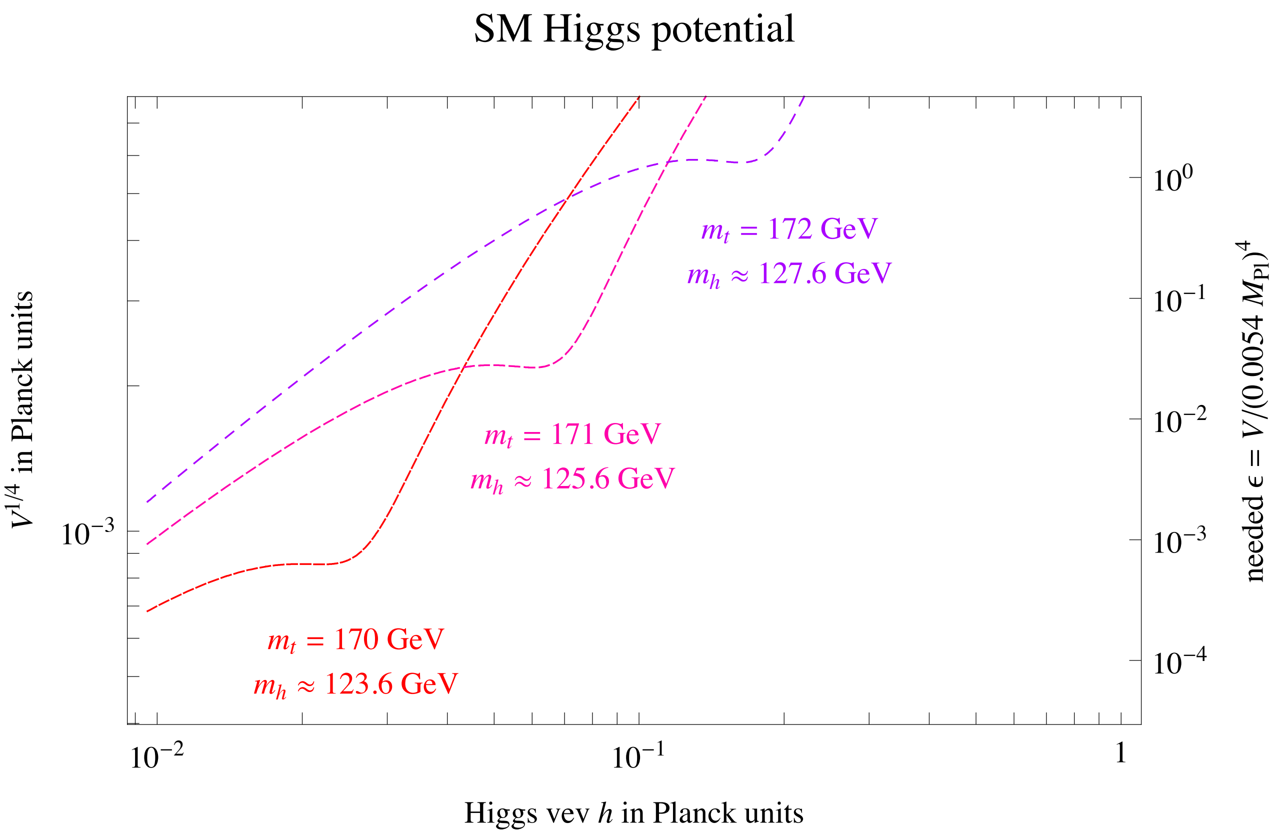


Figure 3: *Examples of fine-tuned SM potentials that might allow inflation. The right handed axis shows the value of the slow-roll parameter ε that would give the observed amount of anisotropies.*

The Higgs as the inflaton

- however:

$$\frac{\delta\rho}{\rho} \simeq 10^{-5} \quad \longleftrightarrow \quad \frac{V}{\varepsilon} \approx (0.0054 M_{\rm Pl})^4$$

and at the same time you must require

$$N_{\rm CMB} \simeq 60 \simeq \left(\frac{\Delta\phi}{M_{\rm Pl}}\right)\frac{1}{\epsilon}$$

The point is that the height of the potental in the flat region is fixed.

- Also: the potential is obtained for fined-tune values of the Higgs and top masses.

Higgs false vacuum inflation

- A second attempt to have the Higgs as the inflaton without introducing nothing beyond SM (e.g., Masina, Notari 2012)
- exploits the peculiarity of the SM to develop a second-minimum at high energies for a narrow band of the Higgs and top masses

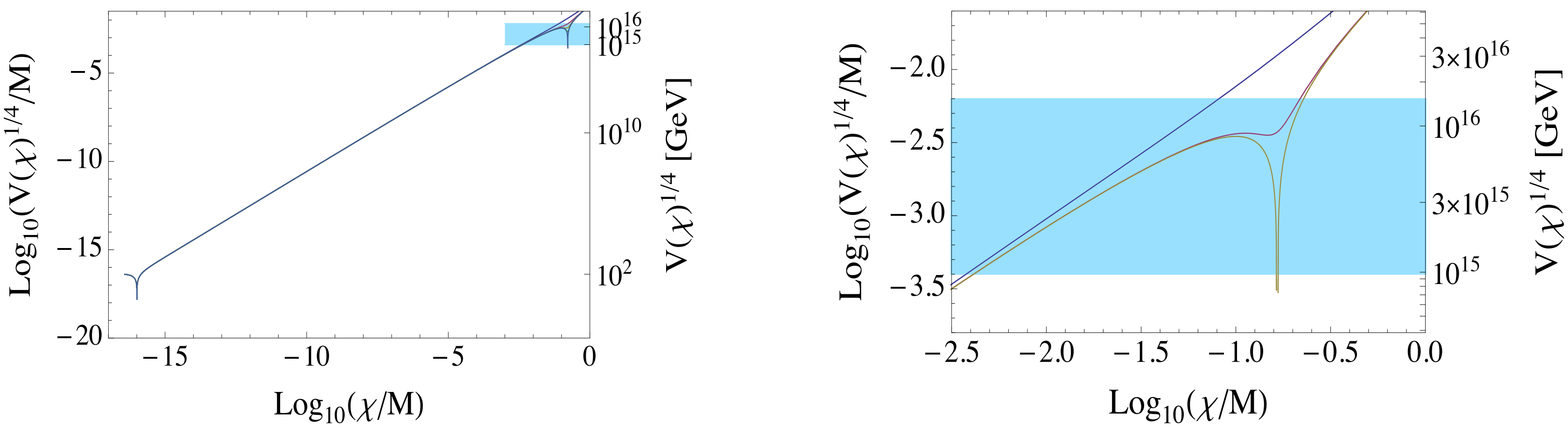


Figure 3: Higgs potential as a function of the Higgs field value χ. We fixed $m_t = 171.8$ GeV and, from top to bottom, $m_H = 125.2, 125.158, 125.157663$ GeV. We also fixed $\alpha_3(m_Z) = 0.1184$. The shaded region is the range selected by our inflationary model: $10^{-3.4} \leq V(\chi_0)^{1/4}/M \leq 10^{-2.2}$. The right panel is a magnification of the false vacuum region.

Higgs false vacuum inflation

- However: this is nothing but the old inflation scenario (Guth ’81), and as such it faces the same old issue: the graceful exit problem.
- To solve this one is forced to introduce in any case physics beyond the SM (non minimal coupling with gravity of a second-scalar field (as in the old days of extended inflation) or a second-scalar field to have hybrid inflation.
- moreover the issue of fine tuning remain.

Starobinsky inflation

- Actually first model of inflation (Starobinsky 1980)

$$S = \int d^4x \sqrt{-g} \frac{M_{\rm pl}^2}{2} \left(R + \frac{R^2}{6M^2} \right)$$

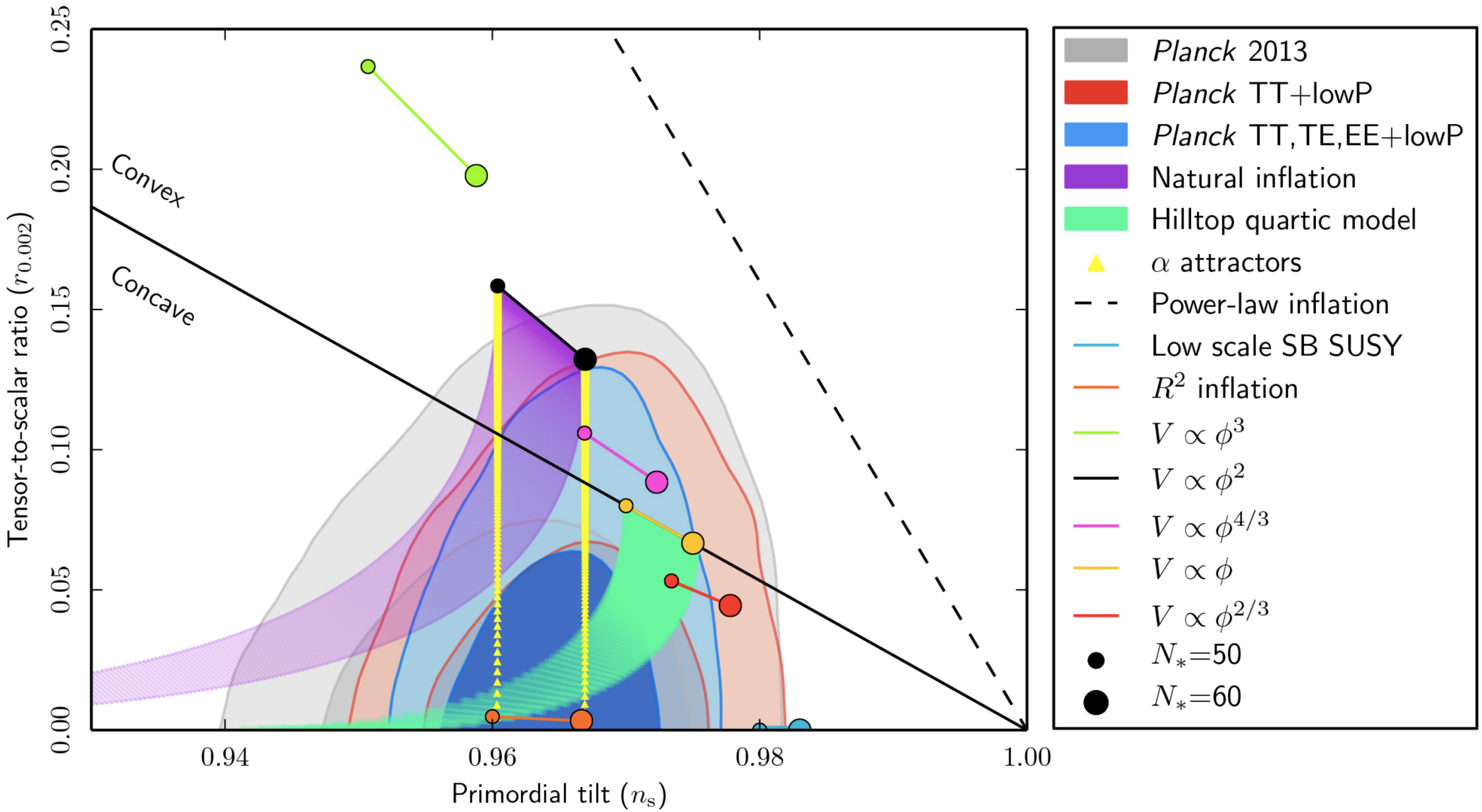
↓ Weyl transormation + field redifinition

$$V(\phi) = \frac{3}{4} M^2 M_{\rm Pl}^4 \left(1 - e^{-\sqrt{2/3}\phi/M_{\rm Pl}} \right)^2$$

(so for large field values it converges to Higgs-inflation)

``Extensions'' of Starobinksy models: the alpha-attractors

Building a bridge between the ``small'' and the ``large''



see,e.g, Kallosh & Linde arXiv:1306.5220, 1306.3214, arXiv:1309.2015; Ferarra, Linde, Porrati arXiv:1307.7696, Kallosh, Linde & Roest, arXiv:1310.3950

A simple toy model

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{\chi^2}{12}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{\phi^2}{12}R - \frac{F(\phi/\chi)}{36}\left(\phi^2 - \chi^2\right)^2\right]$$

Local conformal invariance under

$$\tilde{g}_{\mu\nu} = e^{-2\sigma(x)} g_{\mu\nu}$$

$$\tilde{\chi} = e^{\sigma(x)}\chi,$$

$$\tilde{\phi} = e^{\sigma(x)}\phi.$$

→ Choose a gauge

$$\chi = \sqrt{6}\cosh\frac{\varphi}{\sqrt{6}},$$

$$\phi = \sqrt{6}\sinh\frac{\varphi}{\sqrt{6}},$$

$$\mathcal{L} = \sqrt{-g}\left[\frac{R}{2} - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - F\left(\tanh\frac{\varphi}{\sqrt{6\alpha}}\right)\right],$$

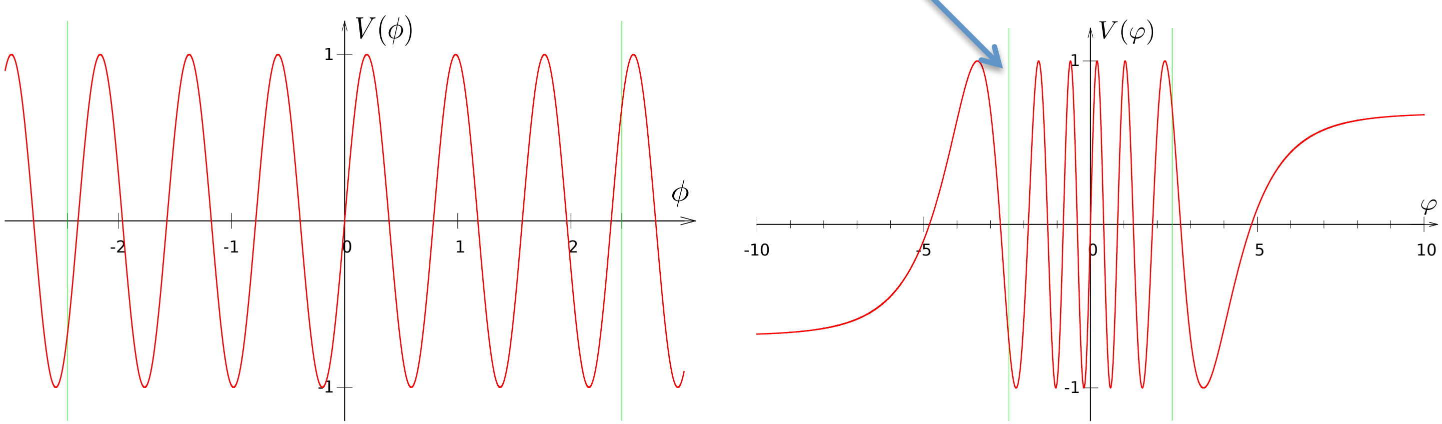
it flattens for large field values, exploiting a conformal stretching $\phi \rightarrow \sqrt{6}\tanh(\varphi/\sqrt{6})$

→ universality of predictions

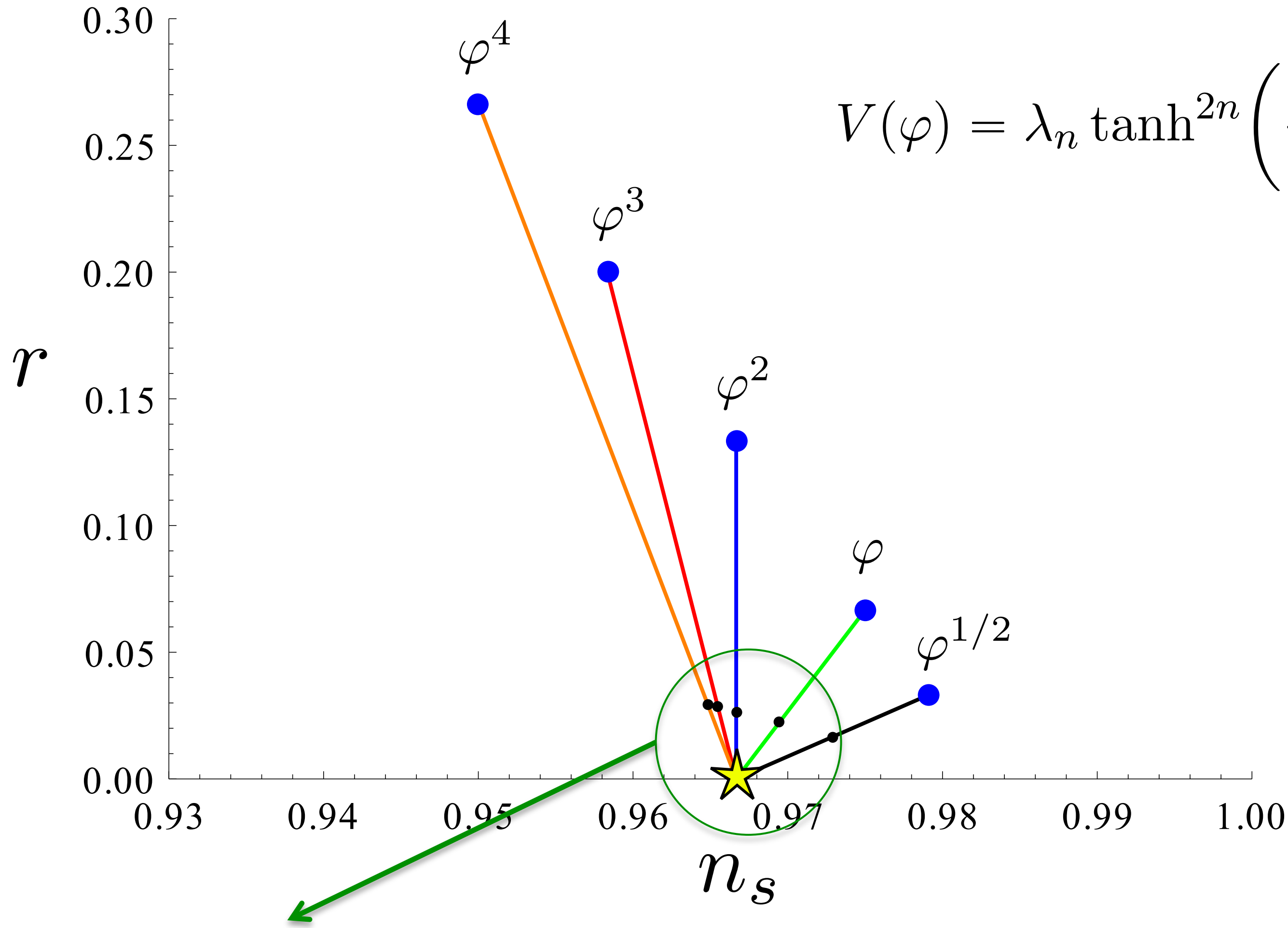
- F is an arbitray function (it quantifies the deviation from a pure cosmological constant): so one can obtain a generic potential starting from a conformal theory which is spontaneously broken.

- $F\left(\tanh\frac{\varphi}{\sqrt{6\alpha}}\right)$: it flattens for large field values: universality of predictions

- exploiting a conformal stretching $\phi \rightarrow \sqrt{6}\tanh(\varphi/\sqrt{6})$



Observational predictions (T-models)



$$V(\varphi) = \lambda_n \tanh^{2n}\left(\frac{\varphi}{\sqrt{6\alpha}}\right),$$

Attractor behaviour
for small α
α=1 corresponds to Starobinsky (Higgs) inflation

$$n_s - 1 \simeq -\frac{2}{N},$$

$$r \simeq \frac{12\alpha}{N^2},$$

Primordial non-Gaussianity

Primordial NG

$\zeta(\mathbf{x})$: primordial perturbations

If the fluctuations are Gaussian distributed then their statistical properties are completely characterized by the two-point correlation function, $\langle\zeta(\mathbf{x}_1)\zeta(\mathbf{x}_2)\rangle$ or its Fourier transform, the power-spectrum.

Thus a non-vanishing ***three point function***, or its Fourier transform, the ***bispectrum is an indicator of non-Gaussianity***

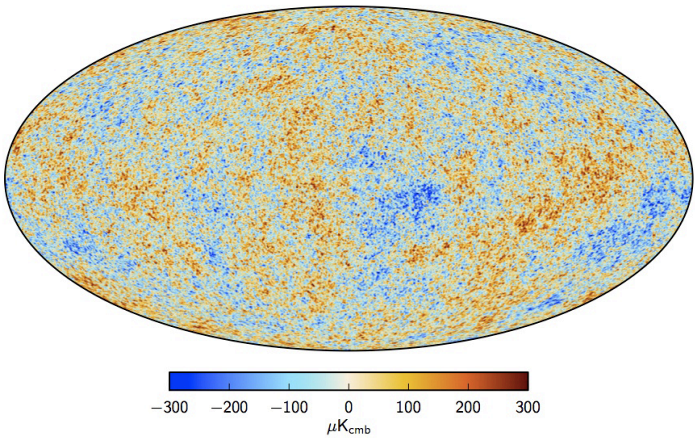
$$\left\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3) \right\rangle = (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) f_{NL} F(k_1, k_2, k_3)$$

Amplitude (f_{NL})

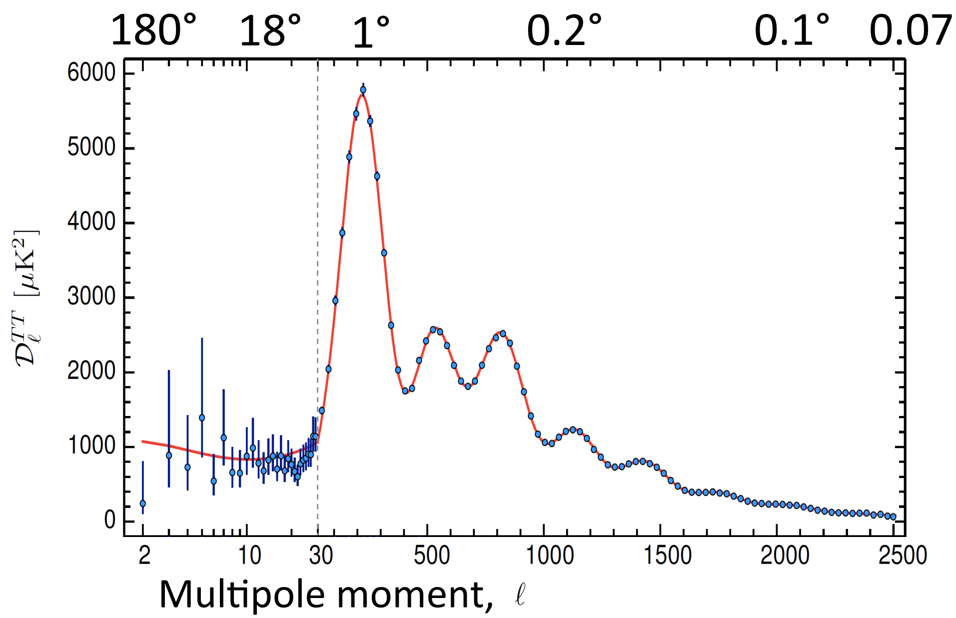
Shape ($F(k_1, k_2, k_3)$)

$$\rightarrow \quad \left\langle \frac{\Delta T}{T}(n_1) \frac{\Delta T}{T}(n_2) \frac{\Delta T}{T}(n_3) \right\rangle$$

Bispectrum vs power spectrum information



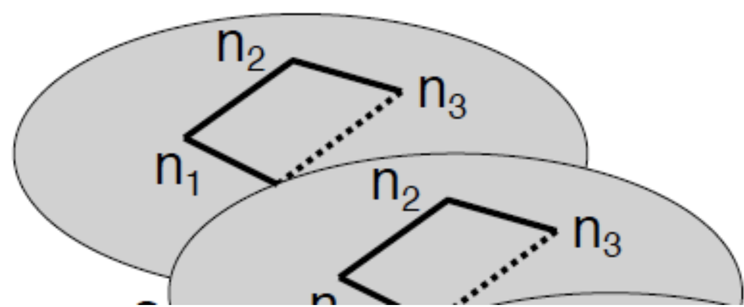
Planck 2015 Results. I. Overview of products and scientific results



$5{\times}10^6$ pixels compressed into ~2500 numbers: O.K. only if gaussian

If not we could miss precious information

Measure 3 point-function and higher-order



Primordial NG

Gaussian ⟺ free (i.e. non-interacting) field, linear theory

Collection of independent harmonic oscillators
(no mode-mode coupling)

Physical origin of primordial NG:
self-interactions of the inflaton field, e.g. $\lambda\,\phi^3$,
interactions between different fields,
non-linear evolution of the fields during inflation,
gravity itself is non linear.....

Why primordial NG is important?

One (among many) good reason:

f_{NL} and shape are model dependent:

e.g.: standard single-field models of slow-roll inflation predict

$$f_{NL} \sim O(\varepsilon,\eta) \ll 1$$

(Acquaviva, Bartolo, Riotto, Matarrese 2002; Maldacena 2002)

A detection of a primordial $|f_{NL}| \sim 1$ would rule out *all* standard single-field models of slow-roll inflation

A second good reason

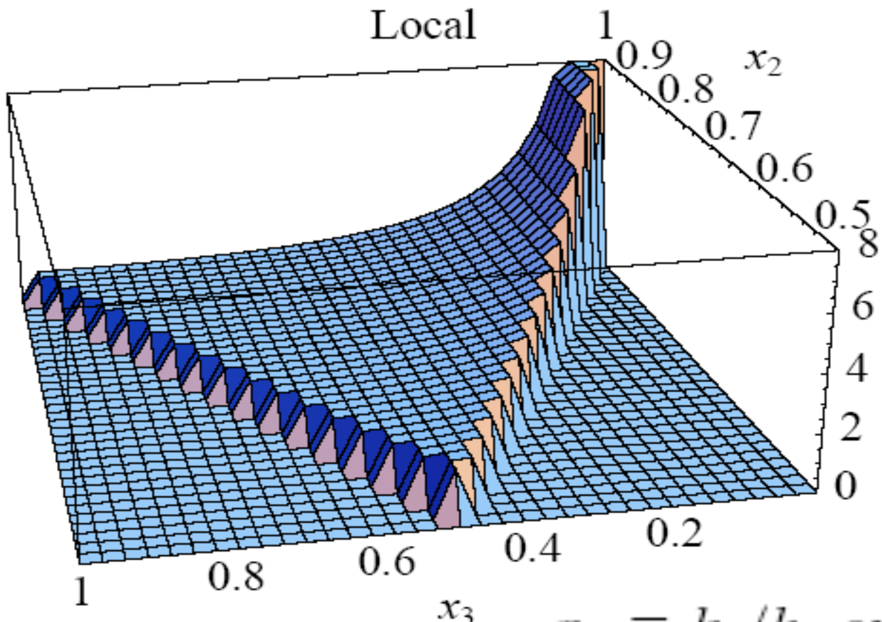
In the last years there has been an explosion of a ***new wave of physically well motivated inflationary models beyond the simplest*** ones capable of generating a large and detectable amount of NG (spurred by the present and future high precision data)

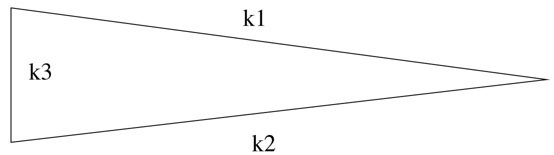
$$|f_{NL}| \gg 1$$

See N. Bartolo, E. Komatsu, S. Matarrese, A. Riotto, astro-ph/0406398
X. Chen, arXiv:1002.1416
N. Bartolo, S. Matarrese, A.Riotto arXiv:1001.3957

SHAPES OF NG: LOCAL NG

Bispectrum peaks for squeezed triangles $k_1 \ll k_2 \sim k_3$





$x_3 \equiv k_3/k_1$ and $x_2 \equiv k_2/k_1$

Babich et al. astro-ph/0405356

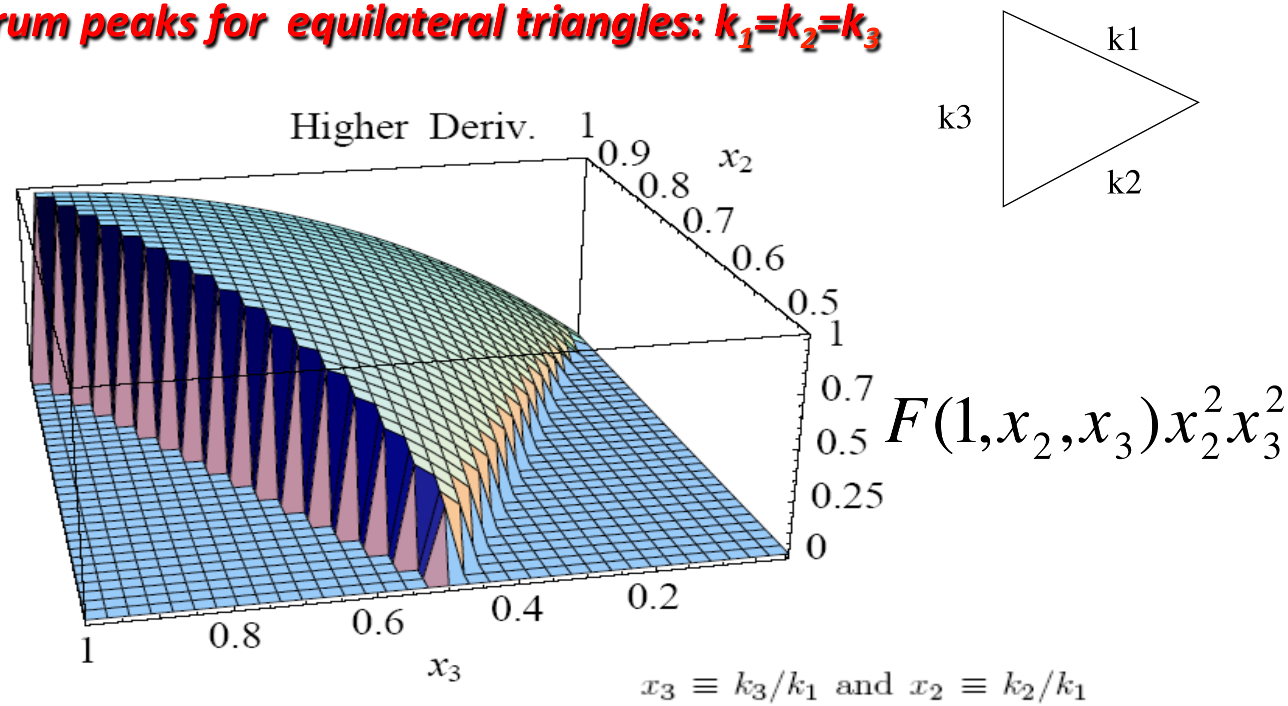
$$\zeta(\mathbf{x}) = \zeta^{\mathrm{G}}(\mathbf{x}) + \frac{3}{5} f_{\mathrm{NL}} \left(\zeta^{\mathrm{G}}(\mathbf{x})\right)^2$$

Non-linearities develop outside the horizon during or immediately after inflation

(e.g. ***multifield models of inflation***)

EQUILATERAL NG

Bispectrum peaks for equilateral triangles: $k_1=k_2=k_3$



Babich et al. (2004)

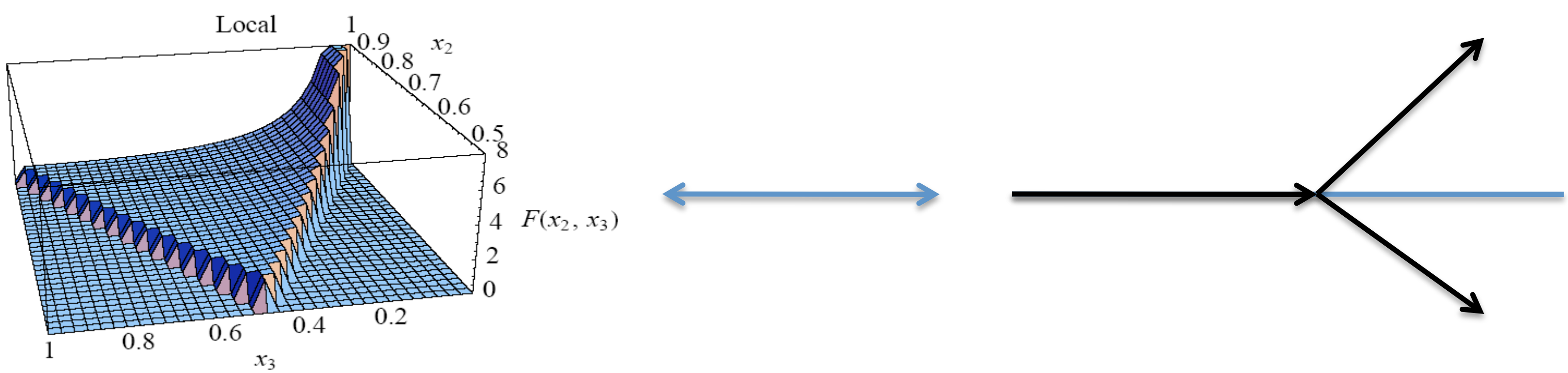
Babich et al. astro-ph/0405356

Single field models of inflation with non-canonical kinetic term L=P(φ, X) where X=(∂ φ)^2 (DBI or K-inflation) where NG comes from higher derivative interactions of the inflaton field

Example: $\delta\dot{\phi}(\nabla\delta\phi)^2$

LESSON: NG…IT'S NOT JUST A NUMBER

Measuring the amplitude and shape of non-Gaussianities, with their huge amount of information associated to triangular configurations is analogous to measuring a cross section as a function of the angle of the outgoing particles in particle and collider physics



Constraints on f_{NL} translates into constraints of the coefficients of the interactions of the inflaton Lagrangian)

Limits set by Planck

See Planck 2015 results. XVII. Constraints on primordial non-Gaussianity

Observational limits set by Planck

	f_{NL}(KSW)	
Shape and method	Independent	ISW-lensing subtracted
SMICA (T)		
Local	10.2 ± 5.7	**2.5 ± 5.7**
Equilateral	−13 ± 70	**−16 ± 70**
Orthogonal	−56 ± 33	**−34 ± 33**
SMICA (T+E)		
Local	6.5 ± 5.0	**0.8 ± 5.0**
Equilateral	3 ± 43	**−4 ± 43**
Orthogonal	−36 ± 21	**−26 ± 21**

e.g. models with non-standard kinetic terms

e.g. multi-field models of inflation

Planck 2015 results. XVII. Constraints on primordial non-Gaussianity.

Implications for inflation models

- The standard models of single-field slow-roll inflation has survived the most stringent tests of Gaussianity to-date: ***deviations from primordial Gaussianity are less than 0.01% level. This is a fantastic achievement, one of the most precise measurements in cosmology!***

$$\Phi(\mathbf{x}) = \Phi^{(1)}(\mathbf{x}) + f_{\mathrm{NL}} \left(\Phi^{(1)}(\mathbf{x}) \right)^2 + \ldots\ldots$$

$\sim 10^{-5}$ $\quad$ ~few $\quad$ $\sim 10^{-10}$

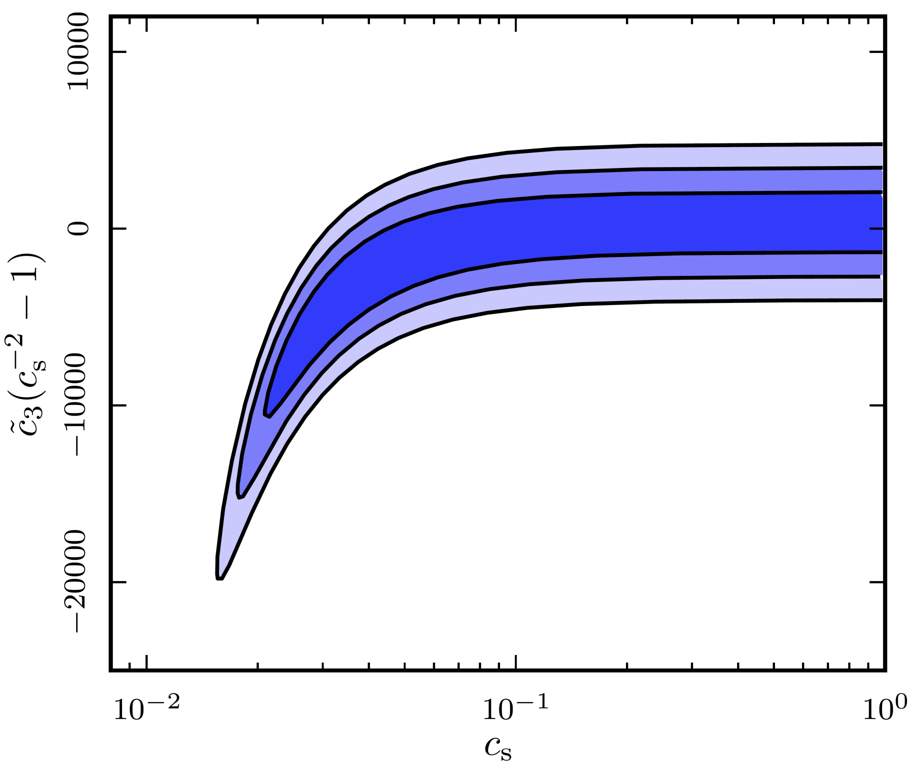
- ***The NG constraints*** on different primordial bispectrum shapes ***severly limit/rule out specific key (inflationary) mechanisms alternative to the standard models of inflation***

General single-field models of inflation: Implications for Effective Field Theory of Inflation

$$S = \int d^4x \sqrt{-g}\Big[-\frac{M_{\rm Pl}^2 \dot{H}}{c_s^2}\Big(\dot{\pi}^2 - c_s^2\frac{(\partial_i\pi)^2}{a^2}\Big) - M_{\rm Pl}^2\dot{H}(1-c_s^{-2})\dot{\pi}\frac{(\partial_i\pi)^2}{a^2} + \Big(M_{\rm Pl}^2\dot{H}(1-c_s^{-2}) - \frac{4}{3}M_3^4\Big)\dot{\pi}^3\Big]$$

$$f_{\rm NL} \propto \frac{1}{c_s^2}$$

(Cheung et al. 08; Weinberg 08)
for extensions see also N.B., Fasiello, Matarrese, Riotto 10)



Constraints obtained from

$$f_{\rm NL}^{\rm equil} = -16 \pm 70 \quad (68\%\ {\rm CL})$$

$$f_{\rm NL}^{\rm ortho} = -34 \pm 33 \quad (68\%\ {\rm CL})$$

$$c_s \geq 0.02 \ \text{ at } 95\%\ {\rm CL}$$

String inspired models of inflation

DBI (Dirac-Born-Infield) ***models*** (brane/string inspired models)
Alishahiha, Silverstein, Tong 04; Chen 05;07

$$\mathcal{L}(\phi, X) = -f(\phi)^{-1}\sqrt{1-2f(\phi)X} + f(\phi)^{-1} - V(\phi)$$

$$X = -\frac{1}{2}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\ , \quad f(\phi) = \lambda/\phi^4$$

Infrared DBI

$$V(\phi) = V_0 - \frac{1}{2}\beta H^2\phi^2$$

$0.1 < \beta < 10^9$ allowed

$$f_{\rm NL}^{\rm DBI} = -(35/108)\left(\frac{1}{c_s^2} - 1\right)\ , \quad c_s \simeq 3/(\beta N)$$

$$n_s - 1 = -4/N$$

Combining the NG constraint $f_{\rm NL}^{\rm DBI} = 11 \pm 69$ (68% C.L.)

with the spectral index $n_s - 1 = 0.9603 \pm 0.0073$

we get $\beta \leq 0.7 \quad (95\%\ \mathrm{CL})$.

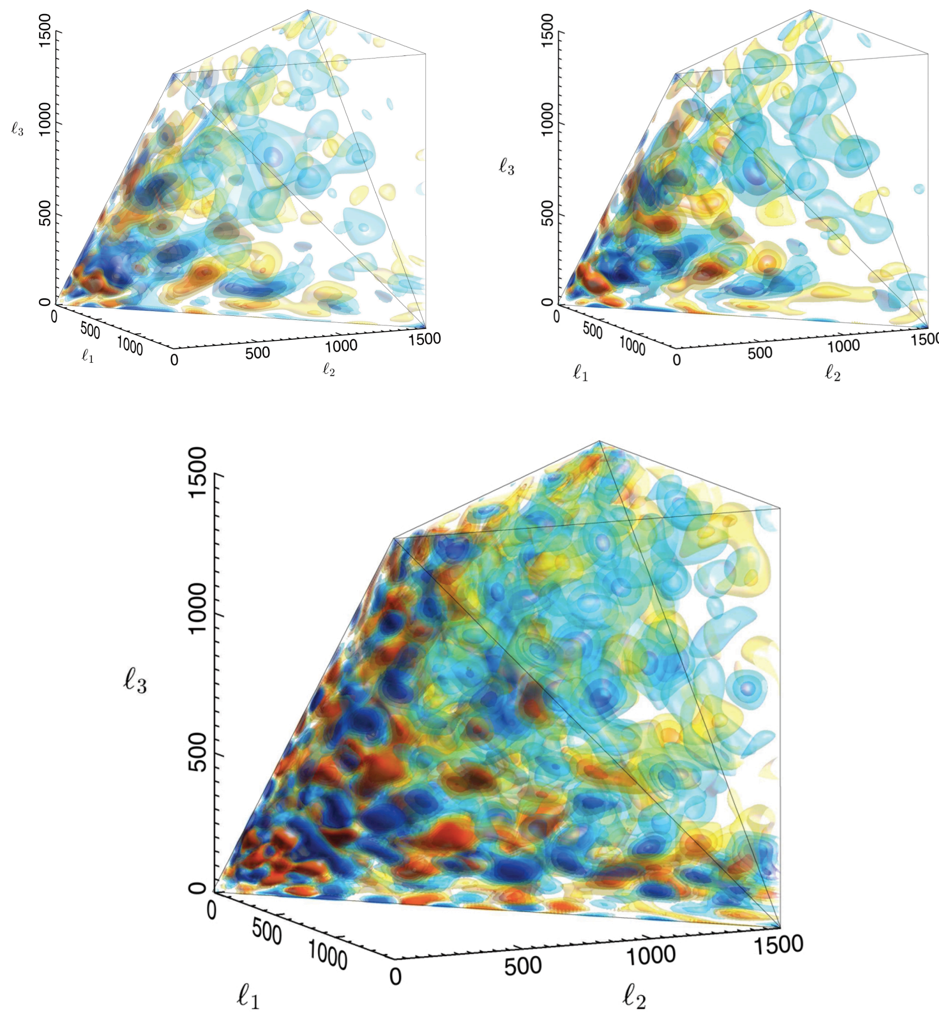
Paramater space of the model dramatically restricted

The CMB bispectrum as seen by Planck

$$\frac{\Delta T}{T}(\vartheta,\phi)=\sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} a_{\ell m}Y_{\ell m}(\vartheta,\phi)$$

$$B_{\ell_1\ell_2\ell_3}=\sum_m \begin{pmatrix}\ell_1 & \ell_2 & \ell_3\\ m_1 & m_2 & m_3\end{pmatrix}\langle a^{m_1}_{\ell_1}a^{m_2}_{\ell_2}a^{m_3}_{\ell_3}\rangle;$$

$$B_{\ell_1\ell_2\ell_3}=h_{\ell_1\ell_2\ell_3}b_{\ell_1\ell_2\ell_3}$$



Future prospects

Significant thresholds for r (Gravitational waves)

- When considering future sensitivity on r, it is important to have in mind some motivated theoretical thresholds

- One reasonable threshold is $r \sim 2 \times 10^{-3}$

  - It (approximately) corresponds to both the prediction of inflation models that become flat as $\exp\left(-\phi/M_{\rm P}\right)$
  (e.g. Higgs-inflation or Starobinsky-like inflation)

  - it corresponds to the threshold $\Delta\phi = M_{\rm P}$

(see discussions, e.g., in Creminelli et al. arXiv:1502.01983, Dodelson arXiv:1403.6310, Kamionkowski, Kovetz 1510.06042, Guzzetti et al. 1605.01615 or, recently Linde in 1612.00020).

Gravitational waves from inflation: CMB B-modes

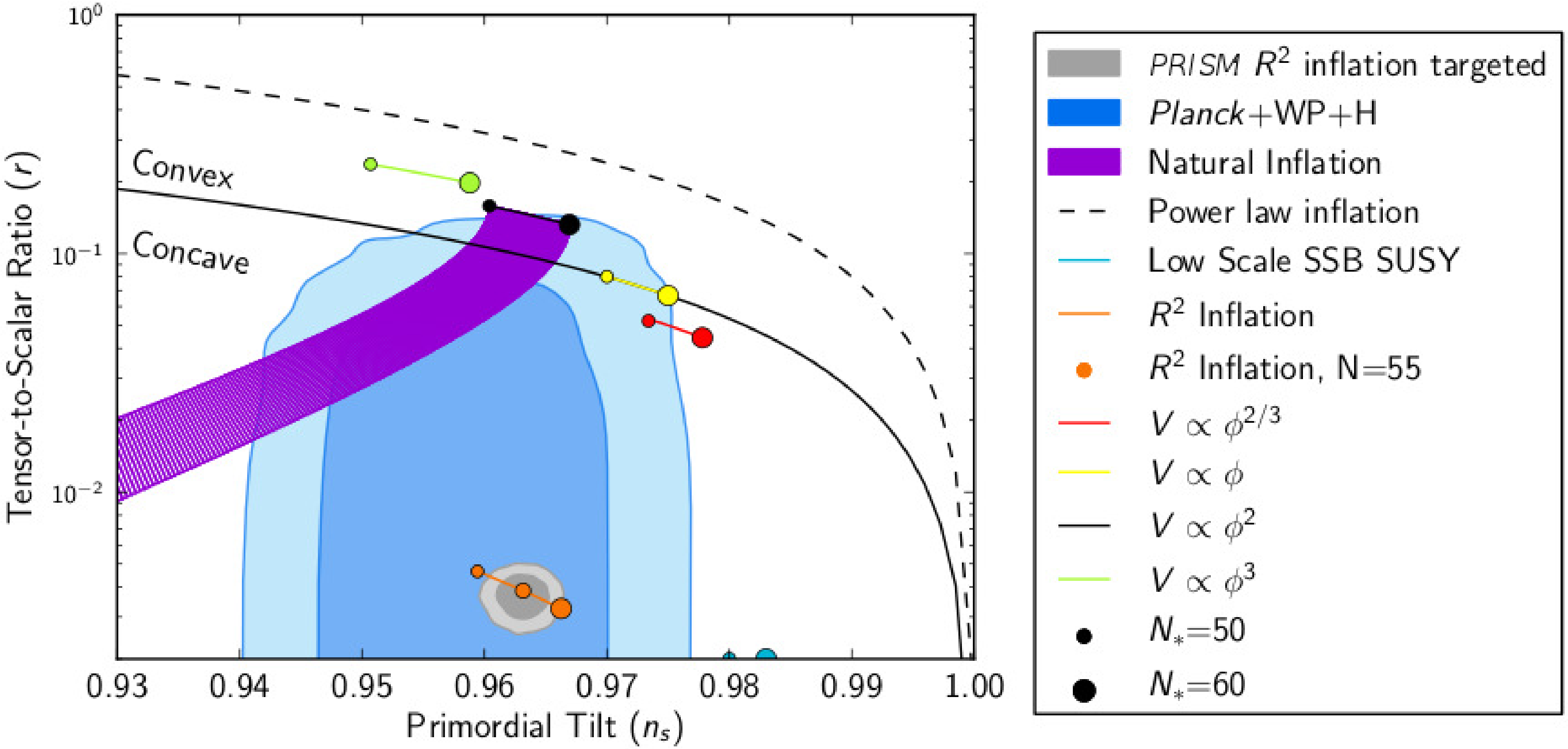
- The search for B-modes will be the main target for most future CMB surveys.

- **Current** constraints **$r < 0.07$,** (95% C.L.)**,** Planck + BICEP2 + Keck Array.

- From the **ground, claim: $\Delta r \sim 0.01$** maybe achievable.
- **Main obstacles**: **astrophysical foreground, B-mode lensing signal. Best remedies**: full-sky, wide multi-frequency coverage => **SPACE**

- Next generation of **space missions aiming for $\Delta r \sim 0.001$**

n_s-r plane: expected improvements



Vast improvement achievable from future polarization data (TE, EE, BB)

GRAVITATIONAL WAVES FROM INFLATION

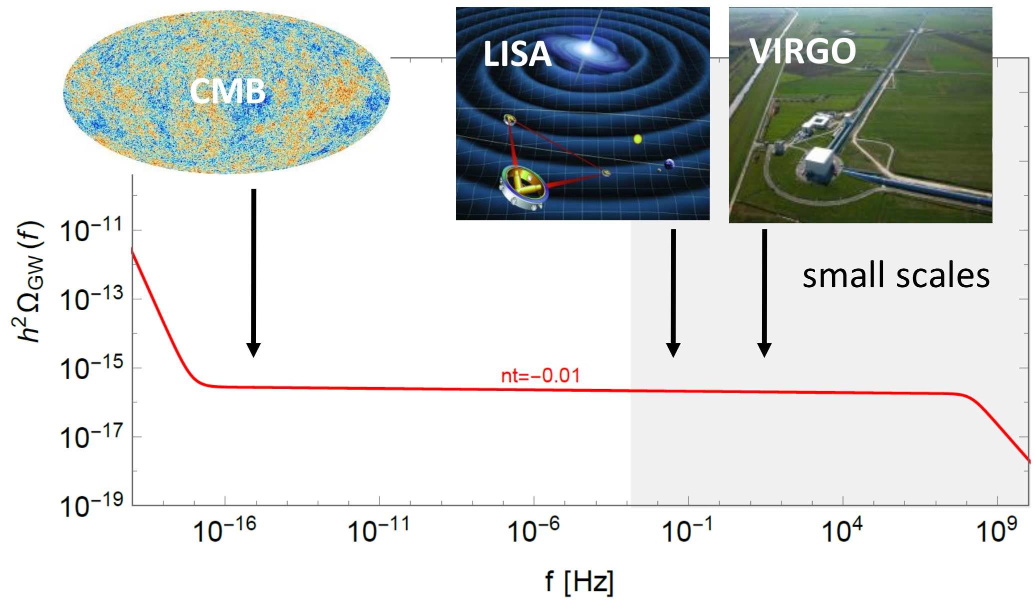
primordial → **present time**

$$P_T(k) = A_T(k_*)\left(\frac{k}{k_*}\right)^{n_T}$$

present time
gw spectral energy density

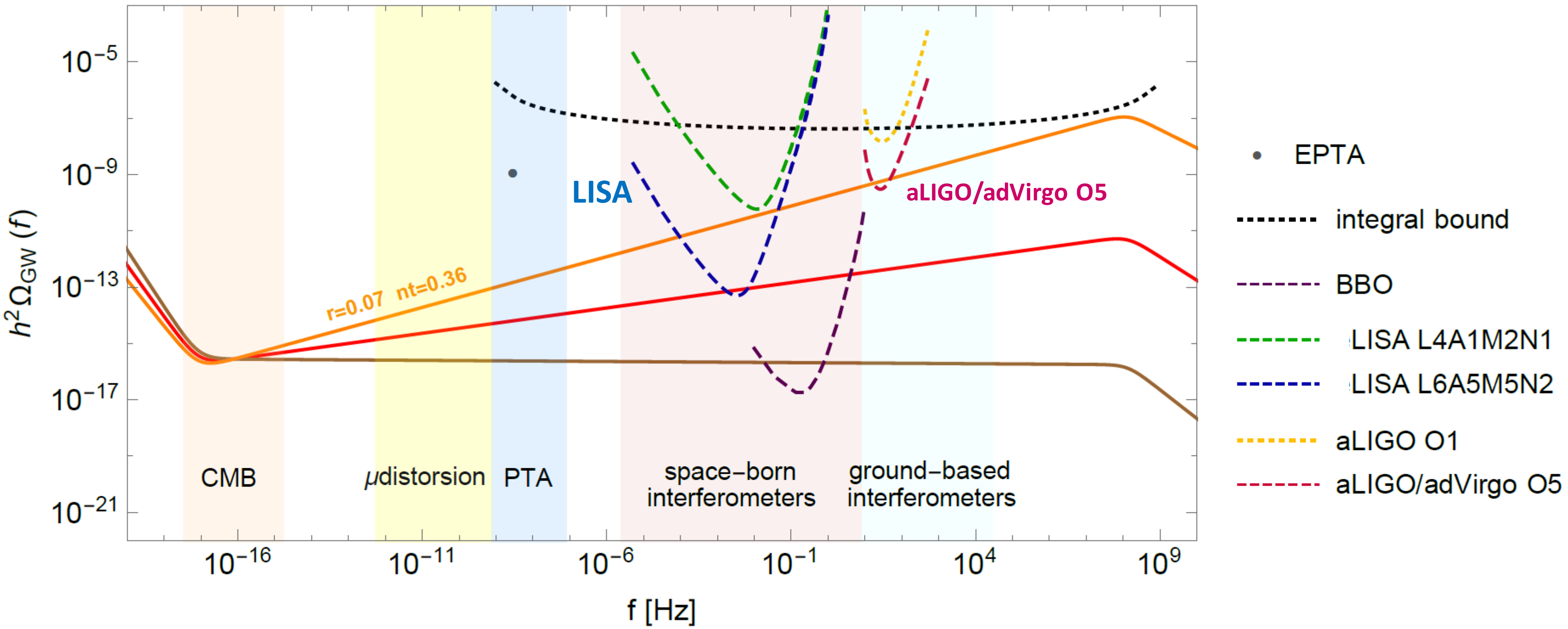
$$\Omega_{\mathrm{GW}}(k, \tau_0) \equiv \frac{1}{\rho_c}\frac{\mathrm{d}\rho_{\mathrm{gw}}}{\mathrm{d}\ln k} = \frac{1}{12}\left(\frac{k}{aH}\right)^2 T(k) P_{\mathrm{T}}(k)$$

stochastic gravitational wave **background**



Courtesy of Maria Chiara Guzzetti

CURRENT BOUNDS AND OBSERVATIONAL PROSPECTS



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INFLATIONARY CONSISTENCY RELATION

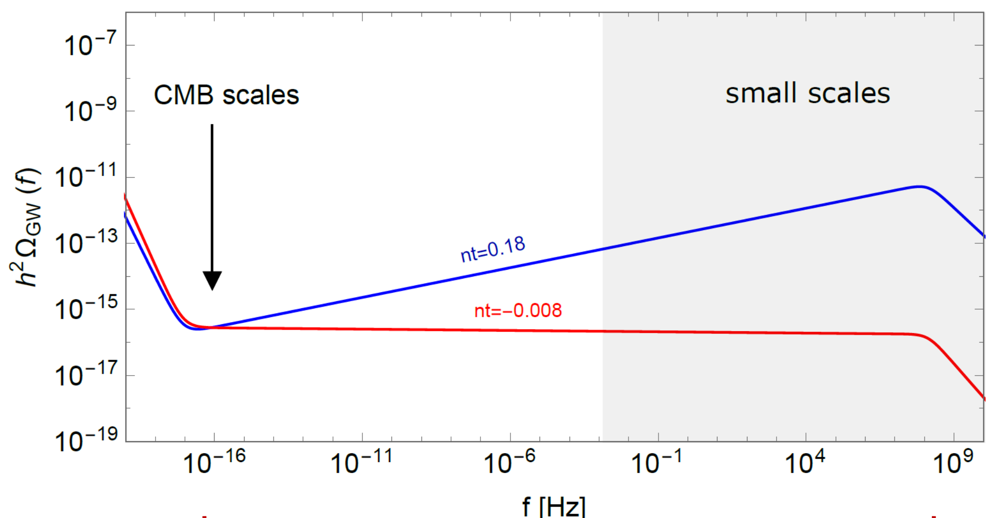
single-field slow-roll inflation
(vacuum fluctuations)

$$r = -8n_T$$

test
for single-field
slow-roll inflation

experiments at **small scales**
are crucial in order to exploit
the **long lever arm** between CMB scales and
laser interferometers scales

experiments at small scales improve the capabilities of testing the single-field slow-roll inflationary model



large range of scales

Courtesy of Maria Chiara Guzzetti

FURTHER MECHANISMS OF GW PRODUCTION

ANY inflationary model ⟶ quantum fluctuations of the gravitational field

POSSIBLE EXTRA PRODUCTION

due to further fields besides the gravitational one

CLASSICAL gw production

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = \frac{2}{M_{\rm pl}^2}\hat{\Pi}_{ij}^{lm}T_{lm}$$

SOURCE TERM

$$\Rightarrow \quad \Omega_{\rm gw} = \Omega_{\rm gw}^{vacuum} + \Omega_{\rm gw}^{sourced}$$

Courtesy of Maria Chiara Guzzetti

CONSTRAINING SPECIFIC INFLATIONARY MODELS, AN EXAMPLE

INFLATION WITH SPECTATOR FIELD

inflaton + spectator field σ

$$\Omega_{\rm gw} = \Omega_{\rm gw}^{vacuum} + \Omega_{\rm gw}^{sourced}$$

blue ?

c_s

σ speed of sound → gw **amplitude**

$$s = \dot{c}_s / H c_s$$

speed of sound evolution → gw **spectral index**

Courtesy of Maria Chiara Guzzetti

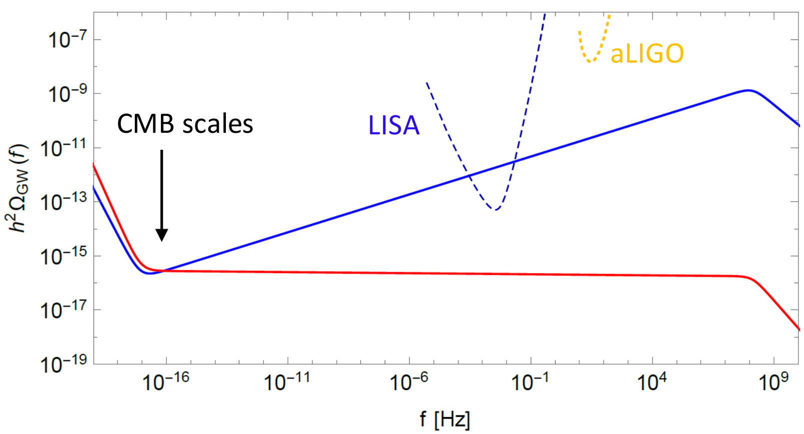
CONSTRAINING SPECIFIC INFLATIONARY MODELS, AN EXAMPLE

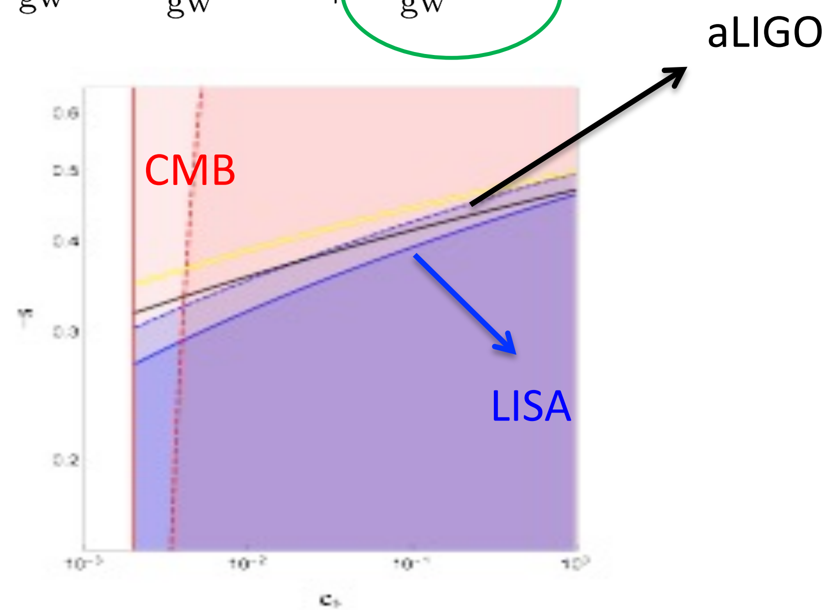
INFLATION WITH SPECTATOR FIELD

inflaton + spectator field σ

$$\Omega_{\rm gw} = \Omega_{\rm gw}^{vacuum} + \Omega_{\rm gw}^{sourced}$$

blue ?





experiments at small scales improve constraints on specific inflationary models, even in case of a non-detection

From N.B. et al., arXiv:1610.06481, ``Science with the space-based interferometer LISA. IV: Probing inflation with gravitational waves''

Courtesy of Maria Chiara Guzzetti

Primordial non-Gaussianity: expected improvements

CMB is a priviliged laboratory for cosmic inflation.

Improvements are possible thanks to CMB polarization.

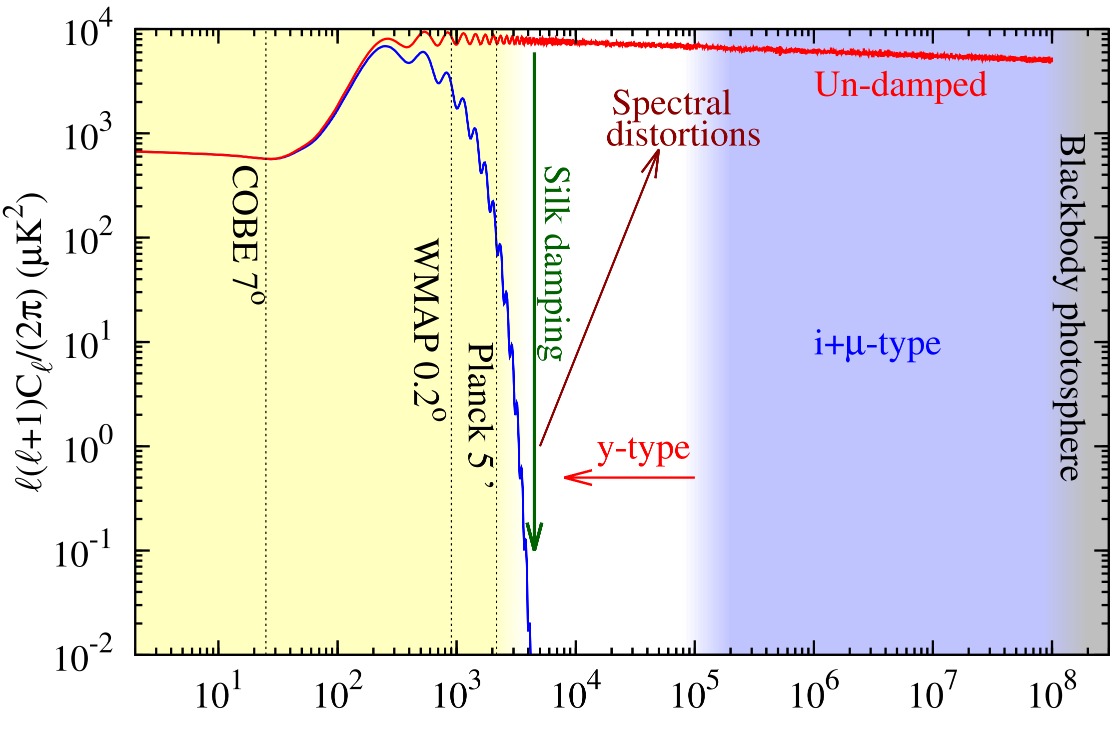
An experiment like PRISM or CMBpol, cosmic variance dominated in E-mode up to to l_max ~ 3000 can improve by a factor of 3 the error bars on f_NL for ***all shapes (no other observable can do that sxcept futristic 21-cm experiments).***

New observational strategies

CMB is a privileged laboratory for cosmic inflation. However different observables can be competitive, and in the future, have a better sensitivity to, e.g., primordial non-Gaussianity

- Large-Scale-Structure Surveys → f_{NL} ~1 or less.
- CMB spectral distortions → f_{NL}~0.001 (cosmic variance limited exp.)
- Future high-redshift large radio surveys → f_NL~ 1 or less.
- High-redshift 21cm fluctuations → f_{NL}~0.01 (cosmic variance limited experiment)

CMB spectral distortions: a new window



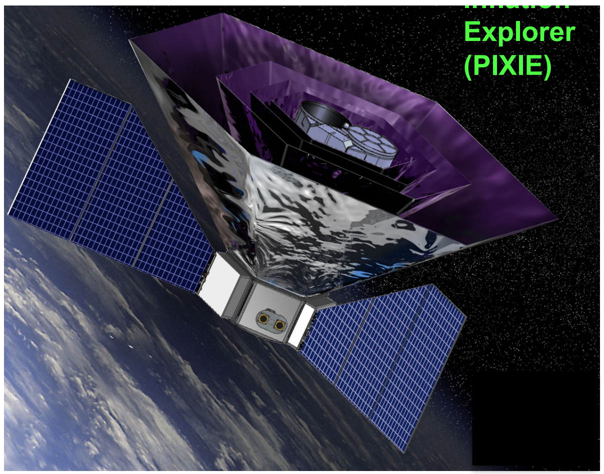
CMB spectral distortions from acoustic waves dissipation probe a large range of scales much smaller than CMB/LSS .
Many additional modes

AN ALMOST UNEXPLOITED OBSERVATIONAL WINDOW

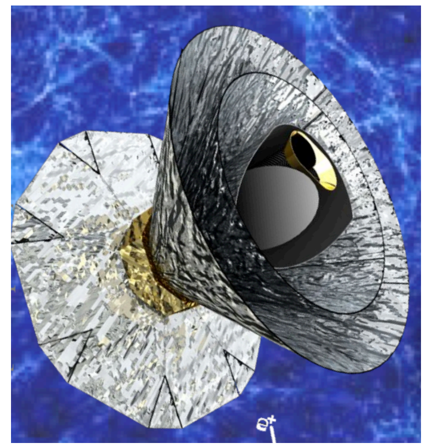
- If <u>μ anisotropies</u> are measured
  - Tμ cross-correlation: primordial local f_NL (Pajer & Zaldarriaga 2013) can in principle reach f_NL~10^{-2}-10^{-3}
  - μμ correlation: primordial 4-point function (amplitude τ_NL)
  - TTμ: primordial 4-point function g_NL: can in priciple improve by 4 orders of magnitude (N.B., Liguori, Shiraishi, 2016)

CMB spectral distortions

- Various planned and proposed satellite missions can achieve the required sensitivity to measure the primordial μ and y spectral distortions: these are predicted to be $<\mu>\approx 1.9\times 10^{-9}$ and $<y>\approx 4.2\times 10^{-8}$



Sensitive to a minimum $<\mu>_{min}\approx 10^{-9}$



PRISM

Sensitive to a minimum $<\mu>_{min}\approx 10^{-8}$

- Besides being a probe of the standard ΛCDM model (including inflation) it can unveil new physics, e.g. about
  - decaying and annihilating dark matter particles
  - black holes and cosmic strings

  and it can allow to measure a whole series of signals like y-distortions from re-ionized gas

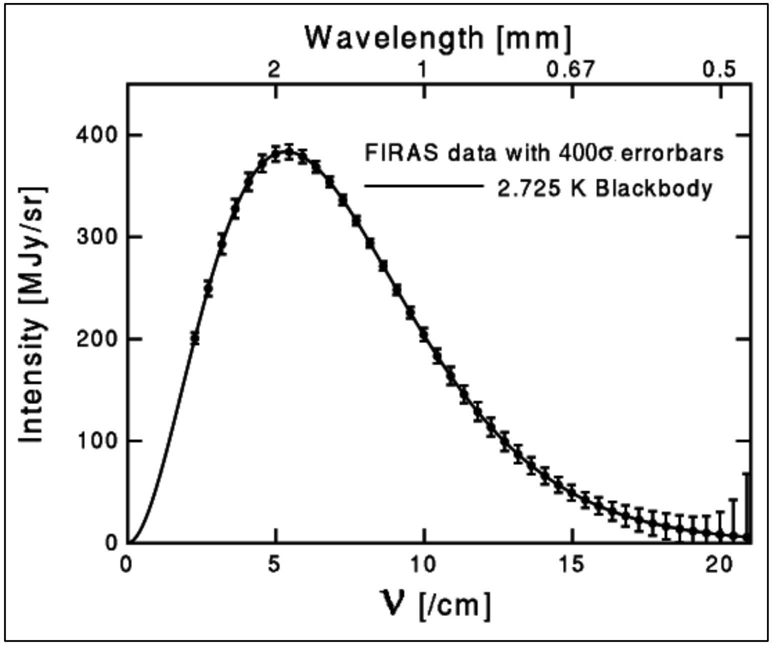
CMB spectral distortions

- We know there must be tiny deviations from a perfect black body of the CMB spectrum in the frequency domain

- Not detected yet (apart y-distortions from Sunyaev-Zel'dovich effect)

- $\frac{\Delta I_\nu}{I_\nu} < 10^{-4}$ $\quad \mu < 9 \times 10^{-5}$ $\quad y < 1.5 \times 10^{-5}$ $\quad$ (95% C.L)

FROM COBE/FIRAS



CMB spectral distortions and NG

- Pajer & Zaldarriaga (2012) and Ganc & Komatsu (2012) pointed out that the cross-correlation between CMB μ-distortion and CMB temperature fluctuations can be a diagnostic very sensitive to local-type bispectra peaking in the squeezed configuration: a cosmic variance limited experiment can achieve f_{NL}~0.01-0.001

Local primordial non-Gaussianity correlates short- with long-mode perturbations, so it induces a correlation between the dissipation process on small scales

$$\mu \sim \delta_\gamma^2 \sim \zeta_{\mathbf{k_1}} \zeta_{\mathbf{k_2}}$$

and the long-mode fluctuations in the CMB

$$\delta T/T \sim \zeta_{\mathbf{k}}$$

$$\downarrow$$

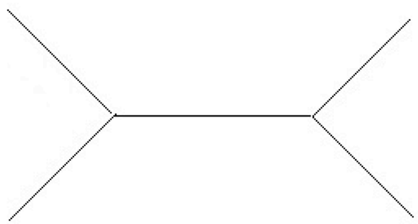
$$C_\ell^{\mu T} \sim \langle \zeta_{\mathbf{k_1}} \zeta_{\mathbf{k_2}} \zeta_{\mathbf{k_3}} \rangle$$

Looking at the inflationary trispectra
(4-point correlation functions)

Looking at the inflationary trispectra

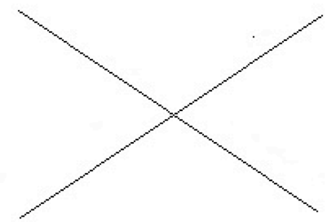
$$\langle \hat{\zeta}_{\vec{k}_1} \hat{\zeta}_{\vec{k}_2} \hat{\zeta}_{\vec{k}_3} \hat{\zeta}_{\vec{k}_4} \rangle = (2\pi)^3 \delta^{(3)}(\vec{k}_1 + \vec{k}_2 + \vec{k}_3 + \vec{k}_4) T_\zeta(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4)$$

Scalar exchange:
comes from terms in the 3-oder action, e.g. $(\delta\phi)^3$



$\tau_{NL} \propto f^2_{NL}$

Contact interaction: e.g. $\lambda\,(\delta\phi)^4$ (intrinsic contributions from the 4-th order action)



g_{NL}

Planck 2015 constraints

$$\tau_{\rm NL}^{\rm loc} < 2800 \quad (95\%\ {\rm CL})$$

$$\begin{aligned} g_{\rm NL}^{\rm local} &= (-9.0 \pm 7.7) \times 10^4; \\ g_{\rm NL}^{\dot{\sigma}^4} &= (-0.2 \pm 1.7) \times 10^6; \\ g_{\rm NL}^{(\partial\sigma)^4} &= (-0.1 \pm 3.8) \times 10^5. \qquad (68\%\ {\rm CL}) \end{aligned}$$

A warning

- ***Tμ (and μμ) cross-correlation is not able to determine the g_{NL} parameter***

- the TTμ *bispectrum* is a potential powerful way to measure g_{NL}
- An ideal, cosmic variance dominated experiment can reach g_{NL}~0.1

(N.B., Liguori and Shiraishi 2015)

Conclusions

- Cosmology has seen a tremendous progress in the last years, and more is expected in the near/long term, thanks to new high-precision data from a variety of new CMB and LSS surveys.

- Inflation is no exception: a large portion of the model parameter space has been ruled out, and many non-standard models of inflation have been tightly constrained (e.g via primordial non-Gaussiantiy)

- A crucial measurement will be the amplitude of the gravitational waves from inflation since this is directly proportional to the energy scale of inflation, and will allow to fully exploit the high sensitivity of inflation to high-scale physics.