Bose-Einstein condensation: an application to pions

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S. Carignano, A. Mammarella and M.M. Phys.Rev. D93 (2016) no.5, 051503
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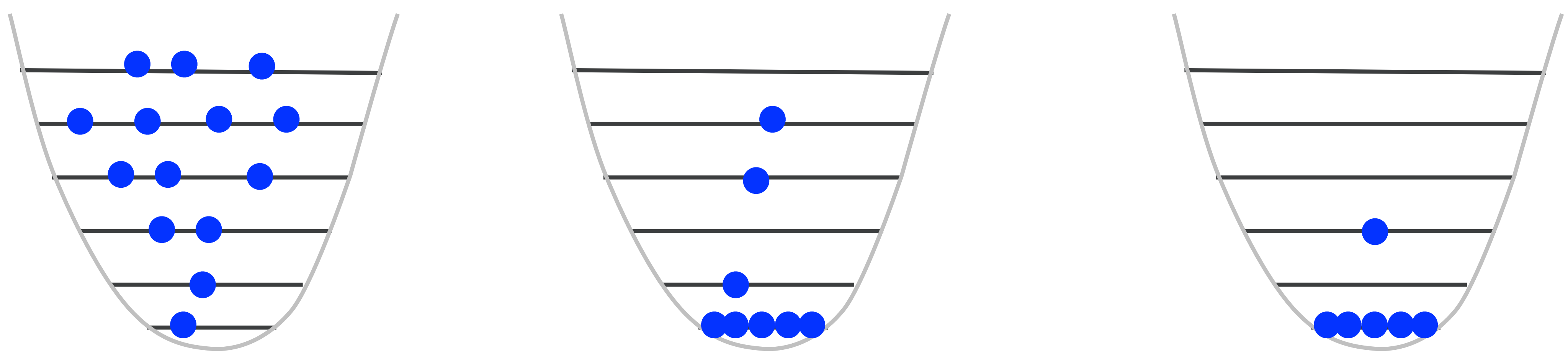
Macroscopic quantum phenomenon

The Bose-Einstein condensate (BEC) is a **coherent state of matter** characterised by a "thermodynamically" large number of particles

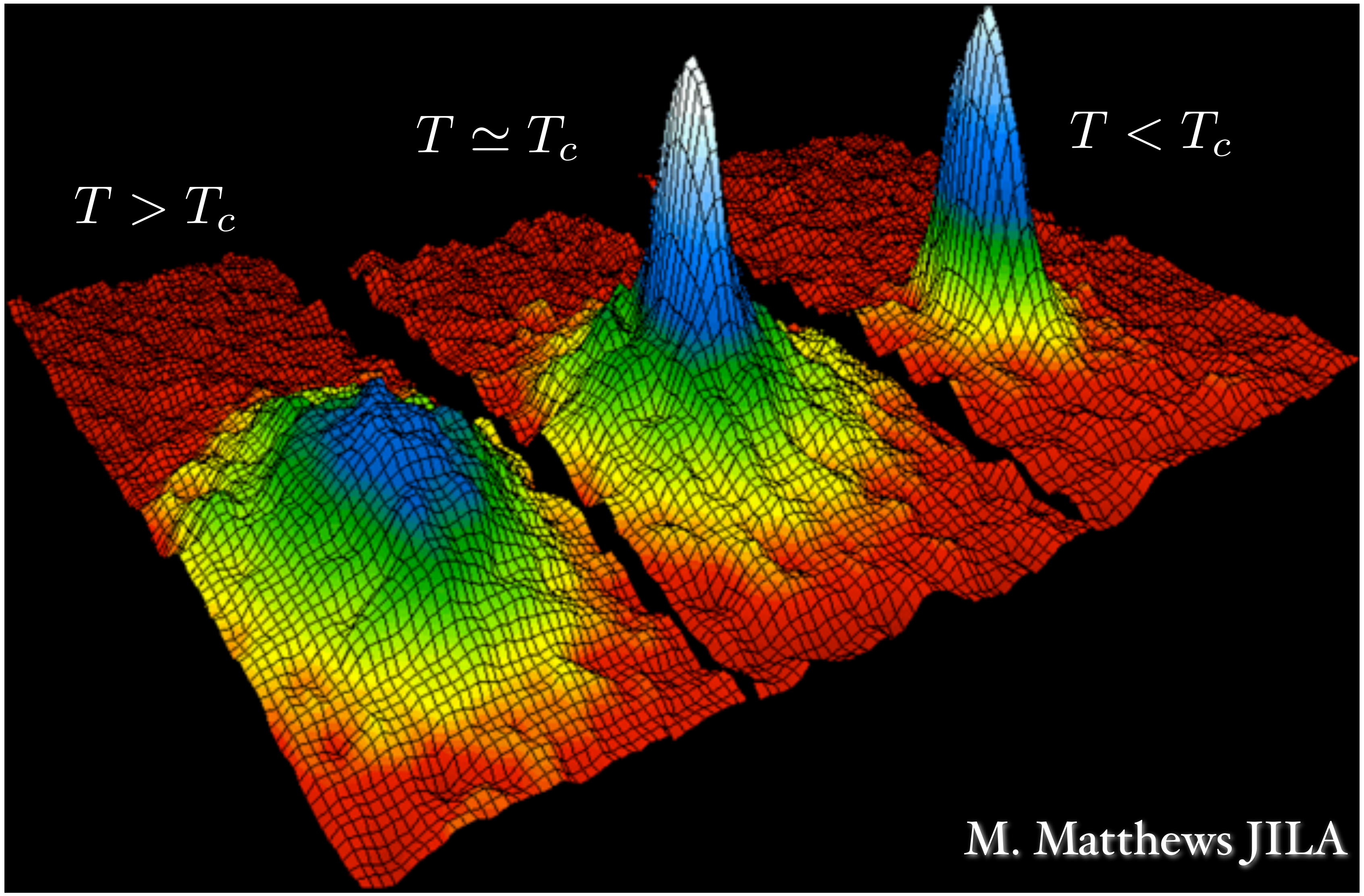
1. Particles must be **bosons** or must have a bosonic-like behaviour
2. **Cold system**: A fight between thermal disorder and quantum coherence
3. Particles must be **stable**

BOSONS@ low temperature in an harmonic potential



$T < T_c$ $T \simeq T_c$ $T < T_c$

Ultracold atoms in an optical trap



2001
Nobel prize
in Physics

Velocity distribution of ^{87}Rb atoms $T_c \simeq 200$ nK

1. ^{87}Rb is bosonic
2. can be cooled
3. has a lifetime of about 10^{10} years (the experiments lasts $\sim 10^3$s)

A BEC of pions?

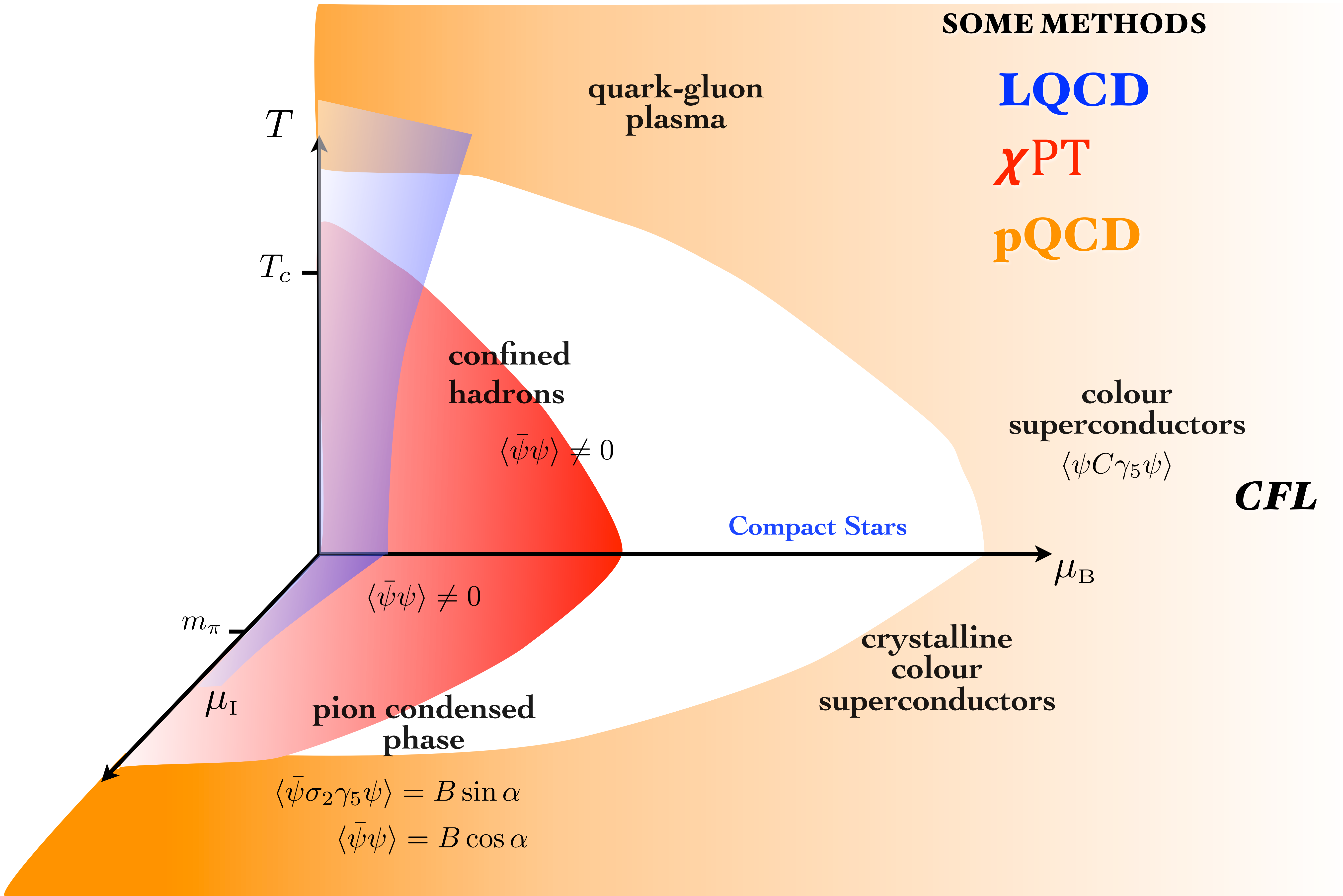
1. Pions are bosons
2. Can be produced at low temperature
3. $\pi^{\pm}$ has a lifetime of about 10^{-8} s

Assuming that it is possible to realise it, is it relevant?

What for?

- Very asymmetric nuclear matter
- Nuclear matter in compact stars
- **Understanding QCD** — In a regime in which different methods can be used

Quark matter phase diagram



Chiral perturbation theory (χPT)

χPT is a realisation of hadronic matter at soft energy scales

$$p \ll \Lambda \sim 1\,\mathrm{GeV}$$

Qualitative picture
We variationally derive the nonperturbative vacuum and we "expand" around that vacuum by low energy modes

Since we are expanding, we do have control parameters

We do not include baryons and vector mesons

$$|\mu_B| \lesssim 940\text{ MeV} \qquad |\mu_I| \lesssim 770\text{ MeV}$$

Leading order Lagrangian

The $\mathcal{O}(p^2)$Lorentz invariant Lagrangian density for pseudoscalar mesons

$$\mathcal{L} = \frac{F_0^2}{4}\mathrm{Tr}(D_\nu \Sigma D^\nu \Sigma^\dagger) + \frac{F_0^2 m_\pi^2}{2}\mathrm{Tr}(\Sigma)$$

Trick for introducing the isospin. We define the covariant derivative

$$D_\mu \Sigma = \partial_\mu \Sigma - \frac{i}{2}[v_\mu, \Sigma]$$

Gasser and Leutwyler, Annals Phys. 158, 142 (1984)

Formally preserving the Lorentz invariance

Then we take $v^\mu = \mu_I \, \sigma_3 \, \delta^{\mu 0}$

A general $\mathrm{SU}(2)$ static and homogeneous vev

$$\bar{\Sigma} = e^{i\boldsymbol{\alpha}\cdot\boldsymbol{\sigma}} = \cos\alpha + i\boldsymbol{n}\cdot\boldsymbol{\sigma}\sin\alpha$$

α and $\boldsymbol{n}$ variational parameters

Static Lagrangian

$$\mathcal{L}_0(\alpha, \mu_I, n_3) = F_0^2 m_\pi^2 \cos\alpha + \frac{F_0^2}{2}\mu_I^2 \sin^2\alpha(1-n_3^2)$$

Maximising the Lagrangian

for $\mu_I < m_\pi$	$\cos\alpha = 1$	$\mathcal{L}_0$ independent of $\boldsymbol{n}$
for $\mu_I > m_\pi$	$\cos\alpha_\pi = m_\pi^2/\mu_I^2$	$n_3 = 0$ residual $O(2)$ symmetry

The vacuum has been tilt in some direction in isospin space

Pion fluctuations

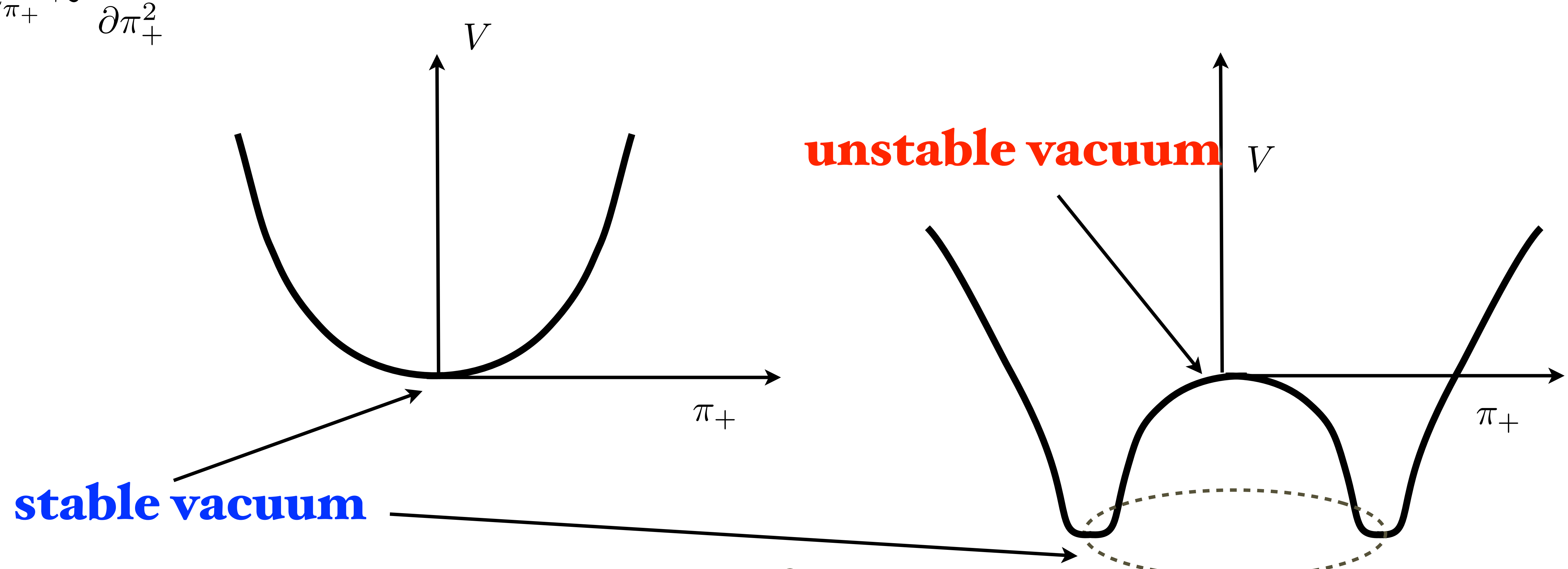
Mass splitting

proportional to the isospin charge

$$m_{\pi^0} = m_\pi$$
$$m_{\pi^-} = m_\pi + \mu_I$$
$$m_{\pi^+} = m_\pi - \mu_I$$

The meson mass vanishes at the phase transition

$$m^2_{\pi_+} \sim \frac{\partial^2 V}{\partial \pi_+^2}$$



BEC of pions!

Rotated condensates

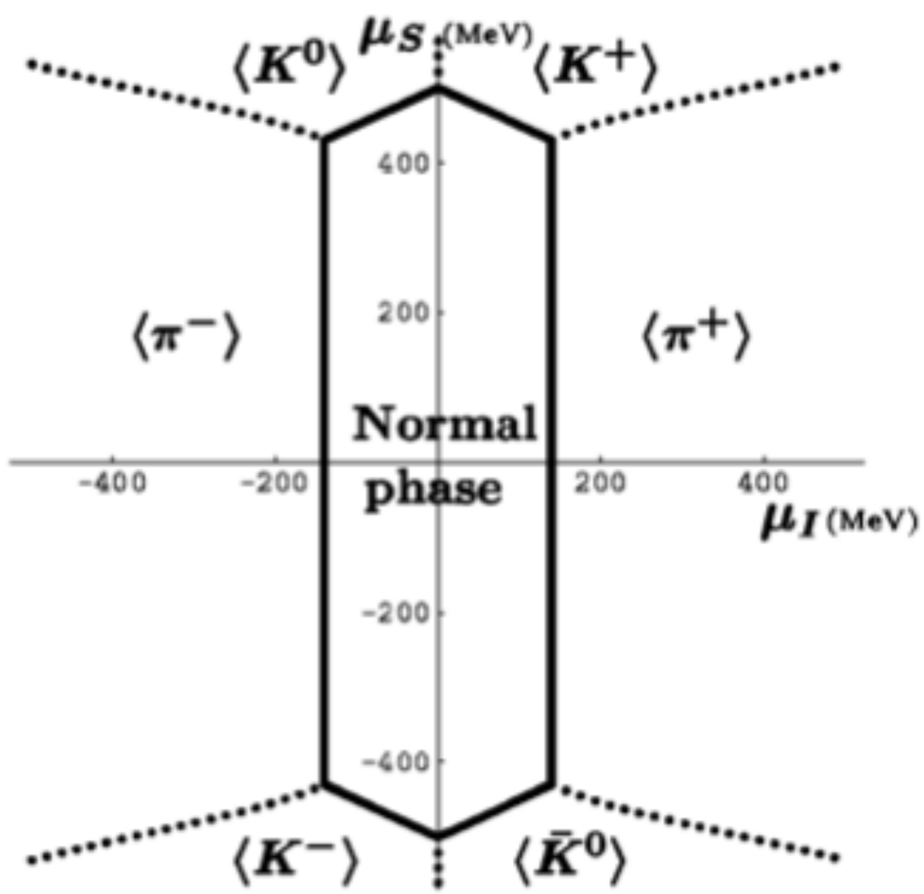
$$\langle \bar{u}u \rangle = \langle \bar{d}d \rangle \propto \cos\alpha$$
$$\langle \bar{d}\gamma_5 u + \text{h.c.} \rangle \propto \sin\alpha$$

Control parameter $\gamma = \frac{\mu_I}{m_\pi}$

Pressure $P = \frac{f_\pi^2 m_\pi^2}{2}\gamma^2 \left(1 - \frac{1}{\gamma^2}\right)^2$

Ground state occupation number $n_I = f_\pi^2 m_\pi \gamma \left(1 - \frac{1}{\gamma^4}\right)$

Phase diagram



solid line: second order

dotted line: first order

Kogut and Toublan PhysRevD.64.034007

In the condensed phases, a superfluid of charged bosons: a superconductor!

$$M_D^2 = M_M^2 = F_0^2 e^2 (\sin\alpha)^2$$

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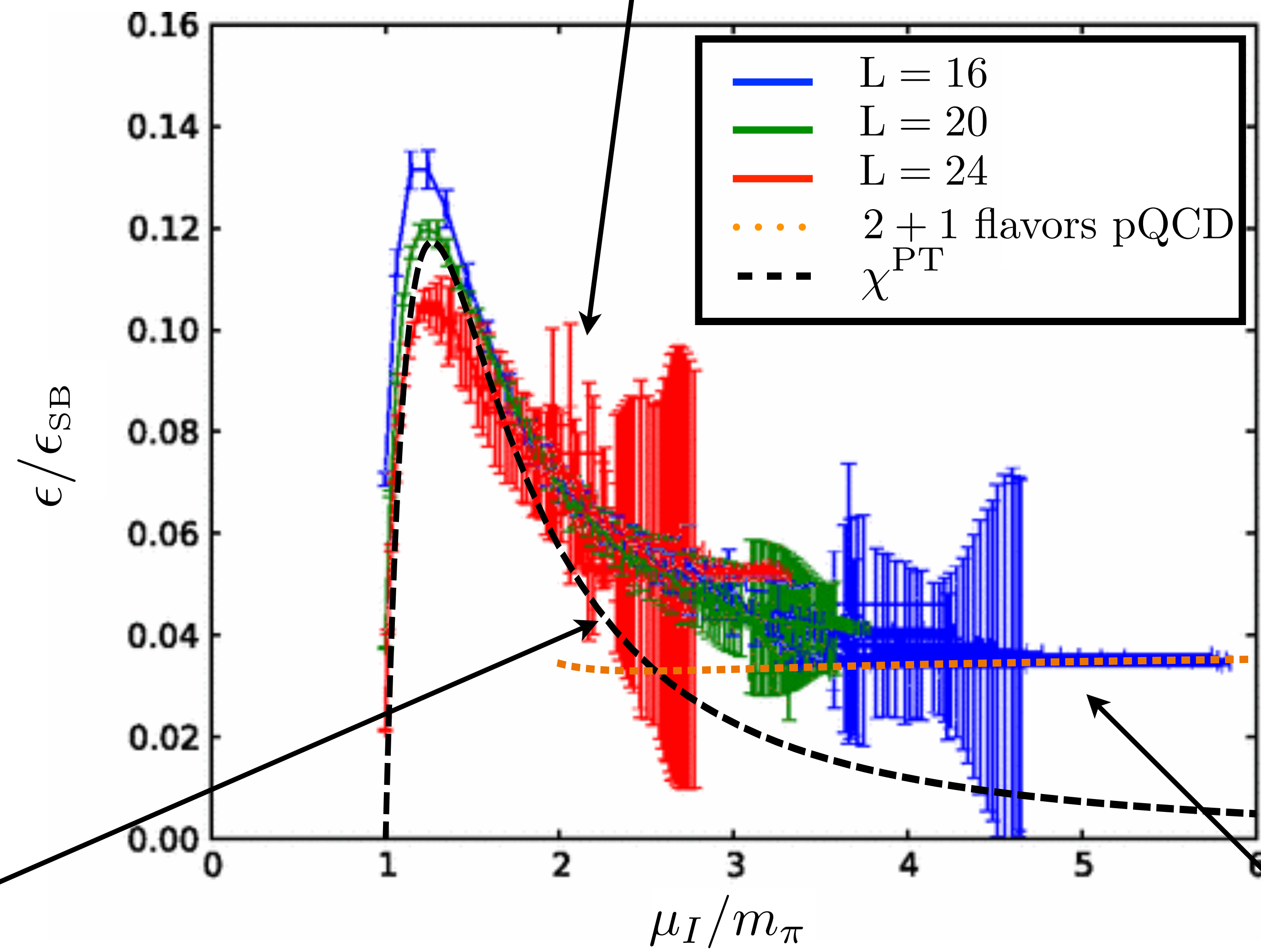
Comparison of the energy density

Microcanonical lattice QCD simulations

W. Detmold, K. Orginos, and Z. Shi, Phys. Rev. D86, 054507 (2012)



$$\epsilon_{SB} = \frac{N_c N_f}{4\pi^2}\mu_I^4$$

***χ*PT**

S. Carignano, A. Mammarella and M.M. Phys.Rev. D93 (2016) no.5, 051503

pQCD

T. Graf, et al. Phys. Rev. D 93, 085030 (2016)

Leading order ***χ***PT correctly reproduces the peak structure

$$\mu_{I,\rm LQCD}^{\rm peak} = \{1.20, 1.25, 1.275\}m_\pi \qquad \mu_{I,\chi\rm PT}^{\rm peak} = (\sqrt{13}-2\,)^{1/2}m_\pi \simeq 1.276\,m_\pi$$

Conclusions

- The BEC is **a macroscopic quantum state of** matter

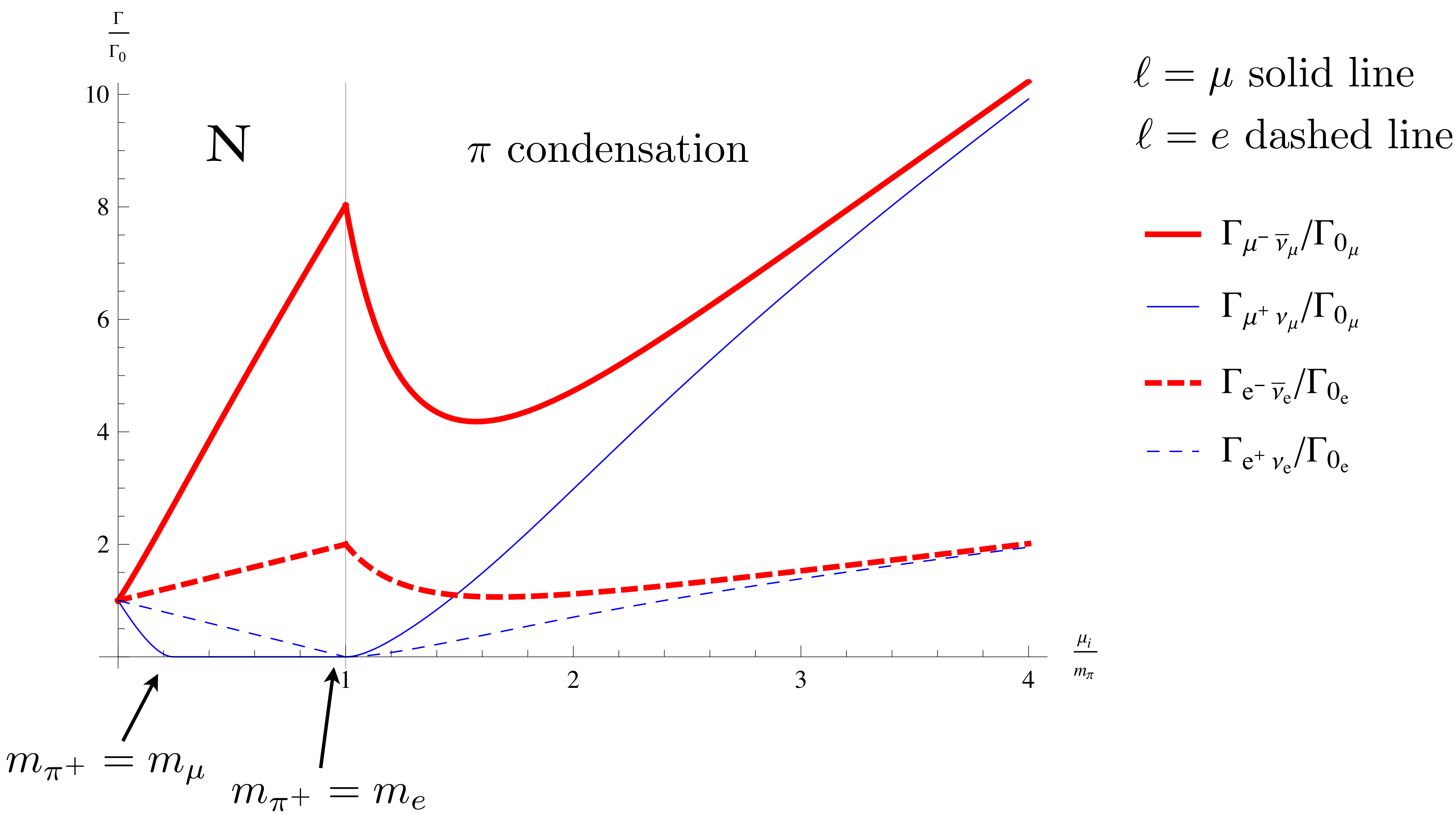
- **Pions** can condense if somehow they are stabilised

- Pion condensation open a paths **for understanding some properties of QCD**

- The comparison with lattice QCD simulation leads to impressive results *backup*

Leptonic decays

Processes $\tilde{\pi}_- \to \ell^\pm \nu_\ell$ and $\tilde{\pi}_+ \to \ell^\pm \nu_\ell$



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