Bound states and entanglement generation in waveguide quantum electrodynamics

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Bari Theory Xmas Workshop 2016

December 22, 2016

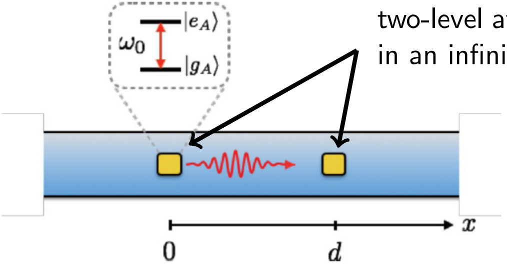
Rectangular Waveguide

A waveguide with a rectangular cross section, with sides $L_y < L_z$, has cut-off frequencies, in natural units:

$$M_{n,m} = \pi\sqrt{\frac{n^2}{L_y^2} + \frac{m^2}{L_z^2}}$$

with a dispersion relation: $k = \sqrt{\omega^2(k) - M_{n,m}^2}$.

Choosing to work with the lowest-cutoff-energy $TE_{1,0}$ mode, atomic transitions will couple only with photons belonging to this mode, created and annihilated by $b^\dagger(k)$ and $b(k)$.



two-level atoms A and B in an infinite waveguide

One-excitation sector

The dynamics is described, in dipolar and rotating wave approximations, by the Hamiltonian $H = H_0 + \lambda V$, where:

$$H_0 = \omega_0(|e_A\rangle\langle e_A| + |e_B\rangle\langle e_B|) + \int dk\ \omega(k) b^\dagger(k) b(k),$$

$$V = \int \frac{dk}{\omega(k)^{1/2}} [|e_A\rangle\langle g_A| b(k) + |g_A\rangle\langle e_A| b^\dagger(k)$$

$$+ |e_B\rangle\langle g_B| b(k) e^{ikd} + |g_B\rangle\langle e_B| b^\dagger(k) e^{-ikd}].$$

The Hamiltonian commutes with the excitation number,

$$\mathcal{N} = |e_A\rangle\langle e_A| + |e_B\rangle\langle e_B| + \int dk\ b^\dagger(k) b(k),$$

so the dynamics, in the $\mathcal{N} = 1$ sector, concerns the states:

$$|\psi\rangle = (c_A |e_A, g_B\rangle + c_B |g_A, e_B\rangle) \otimes |vac\rangle + |g_A, g_B\rangle \otimes |\varphi\rangle,$$

where $|\varphi\rangle = \int dk\ \varphi(k) b^\dagger(k) |vac\rangle$, with $|vac\rangle$ corresponding to the vacuum state of the electromagnetic field.

Resolvent Formalism

In the basis $\{|e_A, g_B\rangle, |g_A, e_B\rangle\}$:

$\lambda = 0$: free propagator $\mathcal{G}_0(z) = \dfrac{1}{z - \omega_0} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \longrightarrow$ pole $z = \omega_0$;

$\lambda > 0$: singularity splits into two simple poles,

$$\mathcal{G}(z) = [\mathcal{G}_0(z)^{-1} - \lambda^2 \Sigma(z)]^{-1},$$

where $\Sigma(z)$ is the self-energy.
The propagator is diagonal using the elements of the Bell basis $|\Psi^{\pm}\rangle = (|e_A, g_B\rangle \pm |g_A, e_B\rangle)/\sqrt{2}$:

$$\mathcal{G}(z) = \frac{|\Psi^+\rangle\langle\Psi^+|}{z - \omega_0 - \lambda^2 \Sigma_+(z)} + \frac{|\Psi^-\rangle\langle\Psi^-|}{z - \omega_0 - \lambda^2 \Sigma_-(z)},$$

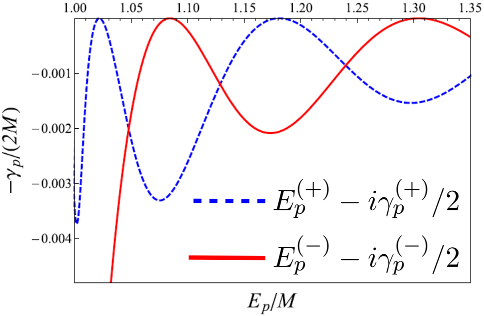
thus obtaining a description in the two parity operator sectors.

Entangled Bound State Stability

The propagator $\mathcal{G}(z)$ poles are:

$$z_p^{(\pm)} = E_p^{(\pm)} - i\gamma_p^{(\pm)}/2$$

respectively corresponding to $|\Psi^\pm\rangle$. Their trajectories are obtained in the variation of $\frac{\omega_0}{M} \in [0.95, 1.35]$, with $Md = 15, \lambda = 10^{-2}M$.



The non-perturbative analysis shows a cyclic behavior with respect to d, obtaining a long-lived pole falling on the real axis in the resonance condition:

$$c_A + c_B e^{\pm i\bar{k}d} = 0 \quad \Longrightarrow \quad \bar{k}d = n\pi$$

where $\bar{k} = \sqrt{E^2 - M^2}$ and $H\,|\psi\rangle = E\,|\psi\rangle$.
The optical path selects between the two sectors of the parity operator:

$c_A = (-1)^{n+1} c_B,$ giving rise to a triplet or singlet state.

Conclusions & Outlook

In conclusion the dynamics of a couple of two-level atoms in a one-dimensional waveguide shows the following characteristics:

- entangled bound state exists for discrete values of the interatomic distance, corresponding to the resonance condition;
- an initially factorized atomic state can spontaneously relax towards a long-lived entangled state, characterized by robustness to small variations in the model parameters.

Further devolpments may concern:

- the extension of the model for cylindrical waveguide, where the use of Bessel functions for the electromagnetic wavefront implies the employment of the Hankel transform in the analytical solution;
- the study of wavefronts deformation involved in the variation of boundary conditions, e.g. the Bessel function profile is modified through an external potential applied to the waveguide cross section perimeter.