Bari Theory Xmas Workshop 2016

Bari, 22 Dicembre 2016

Correlation Plenoptic Imaging

Francesco Pepe

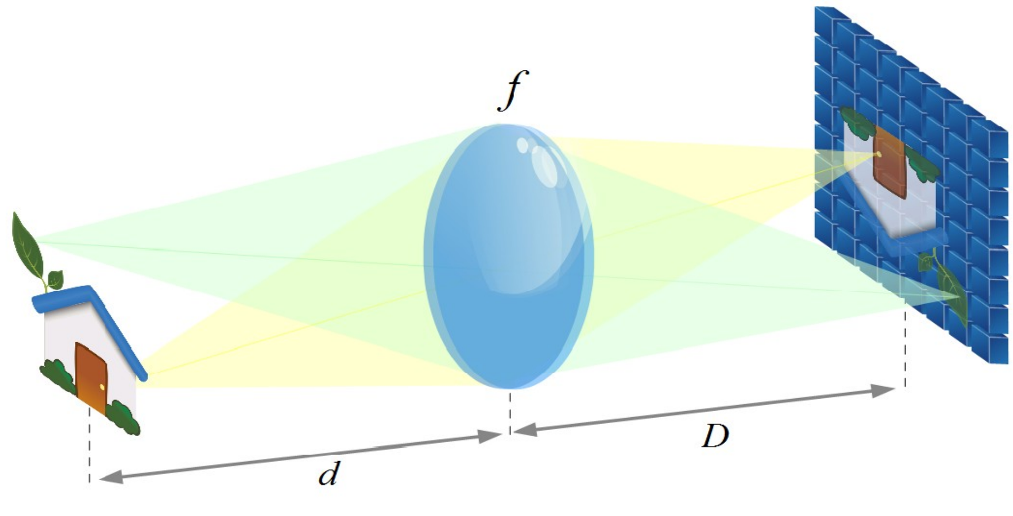
Centro Fermi, @UniBa Dipartimento di Fisica

INFN – Sezione di Bari

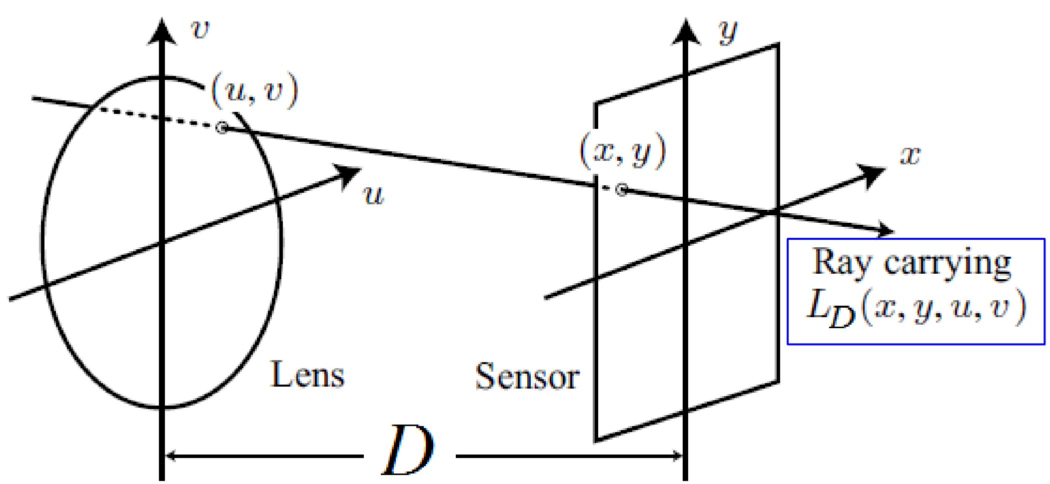
Standard (classical) imaging

In **standard imaging**, a thin lens induces a (quasi) one-to-one correspondence between points on the sensor and points of a *scene*

$$\frac{1}{d}+\frac{1}{D}=\frac{1}{f}$$



The signal at a point $\boldsymbol{x}$ on the sensor is proportional to the **irradiance**



$$I_D(\boldsymbol{x}) \propto \int_{\text{lens}} L_D(\boldsymbol{x}, \boldsymbol{u})\, d^2 u$$

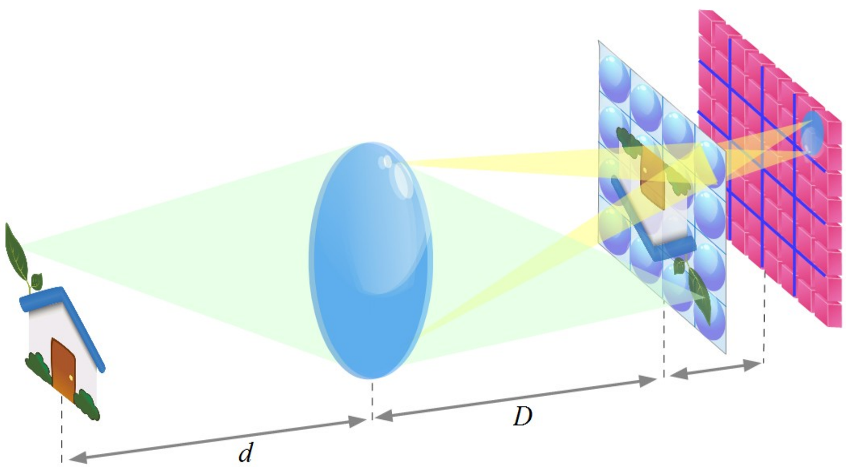
LIGHT FIELD

Radiance propagated from point $\boldsymbol{u}$ on the lens to point $\boldsymbol{x}$ on the sensor

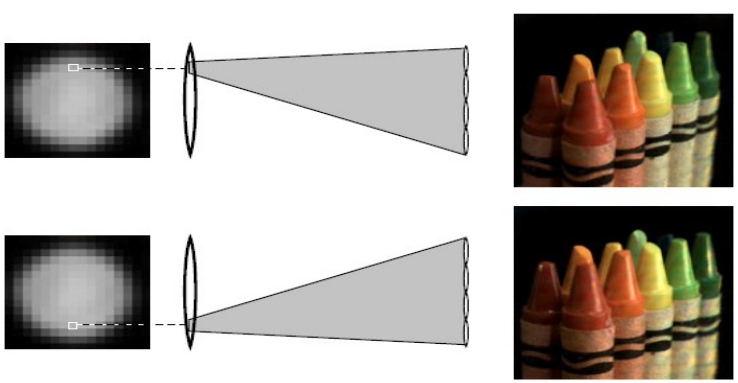
Plenoptic imaging

Plenoptic imaging aims at reconstructing not only the irrradiance at a fixed distance D, but the whole **3-dimensional light field**

REFOCUSING planes beyond the depth of field



- The *image of the object* is focused on the microlens array by the main lens
- Multiple *images of the main lens* are focused on the sensor by the microlenses



From Ren Ng *et al.* (2005)

Inverse proportionality between spatial and angular resolution

Spatial resolution larger than diffraction limit

Properties of chaotic light

$$\left\langle a^{\dagger}_{\boldsymbol{k}_1} a_{\boldsymbol{k}_2} \right\rangle \propto \delta\left(\boldsymbol{k}_1 - \boldsymbol{k}_2\right)$$ → Absence of first-order interference

$$\left\langle a^{\dagger}_{\boldsymbol{k}_1} a^{\dagger}_{\boldsymbol{k}_2} a_{\boldsymbol{k}_3} a_{\boldsymbol{k}_4} \right\rangle \propto \delta\left(\boldsymbol{k}_1 - \boldsymbol{k}_3\right)\delta\left(\boldsymbol{k}_2 - \boldsymbol{k}_4\right) + \delta\left(\boldsymbol{k}_1 - \boldsymbol{k}_4\right)\delta\left(\boldsymbol{k}_3 - \boldsymbol{k}_4\right)$$

⇓

Nontrivial correlations emerge from *photon indistinguishability*

Intensity correlations are related with the second-order correlation function

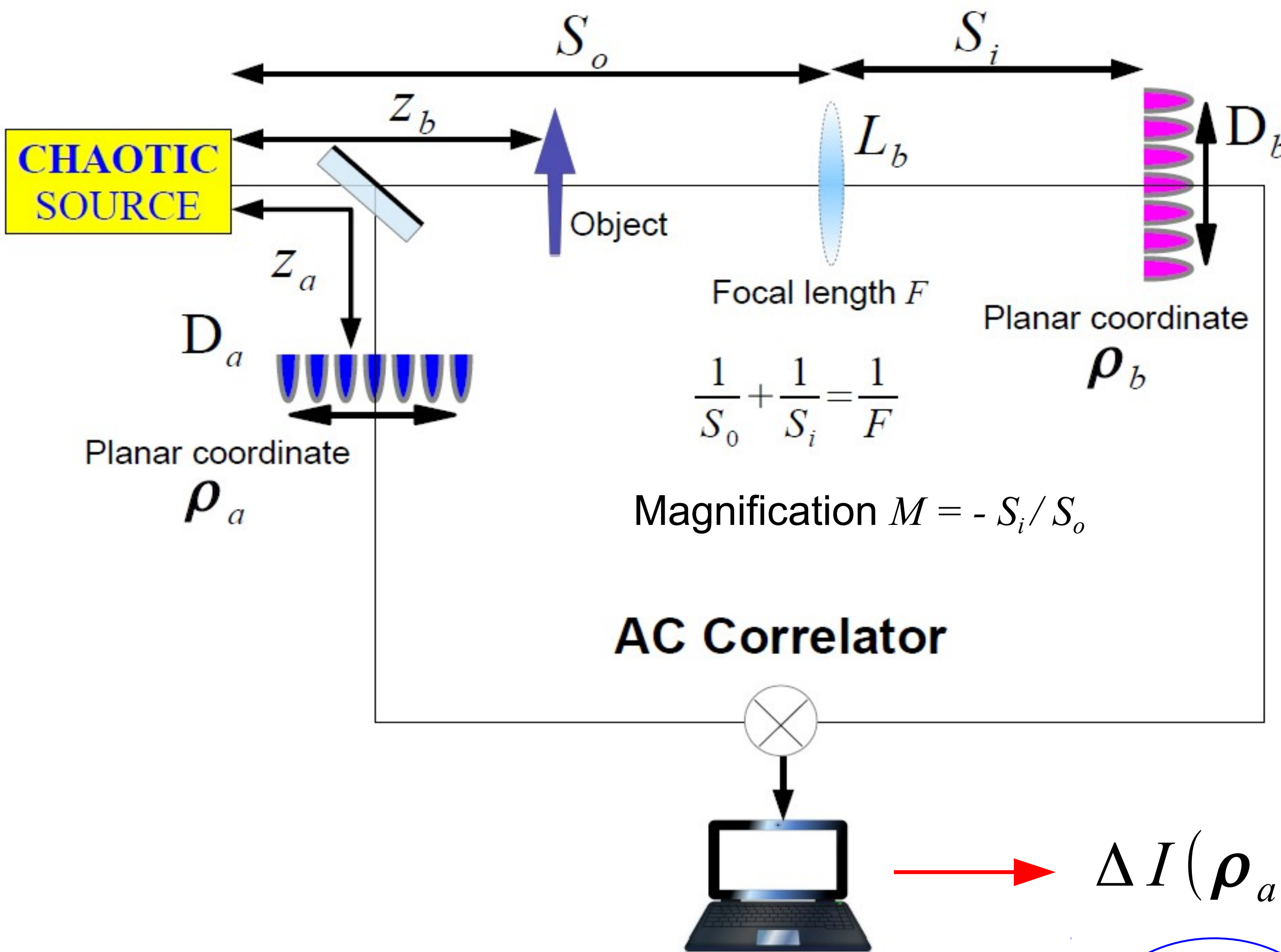
$$G^{(2)}\left(\boldsymbol{\rho}_a, \boldsymbol{\rho}_b\right) = \left\langle E_a^{(-)}\left(\boldsymbol{\rho}_a\right) E_b^{(-)}\left(\boldsymbol{\rho}_b\right) E_b^{(+)}\left(\boldsymbol{\rho}_b\right) E_a^{(+)}\left(\boldsymbol{\rho}_a\right) \right\rangle$$

$$= I_a\left(\boldsymbol{\rho}_a\right) I_b\left(\boldsymbol{\rho}_b\right) + \Gamma\left(\boldsymbol{\rho}_a, \boldsymbol{\rho}_b\right)$$

Directly retrieved by measuring correlations of intensity fluctuations

$$\left| \int d\boldsymbol{k}_{\perp}\, g_a\left(\boldsymbol{\rho}_a, \boldsymbol{k}\right)^* g_b\left(\boldsymbol{\rho}_b, \boldsymbol{k}\right) \right|^2$$

Correlation plenoptic imaging



The lens L_b focuses a real image of the source on the sensor D_b

$$\Delta I(\boldsymbol{\rho}_a)\Delta I(\boldsymbol{\rho}_b)=\Gamma(\boldsymbol{\rho}_a,\boldsymbol{\rho}_b)$$

$$\Gamma_{(z_a,z_b)}(\boldsymbol{\rho}_a,\boldsymbol{\rho}_b)=\left|\int d\boldsymbol{\rho}_o\int d\boldsymbol{\rho}_s A(\boldsymbol{\rho}_o)F(\boldsymbol{\rho}_s)\mathrm{e}^{\frac{i\omega}{2c}\left(\frac{1}{z_b}-\frac{1}{z_a}\right)\boldsymbol{\rho}_s^2}\mathrm{e}^{\frac{i\omega}{cz_b}\left[\left(\boldsymbol{\rho}_o-\frac{z_b}{z_a}\boldsymbol{\rho}_a\right)\cdot\boldsymbol{\rho}_s+\frac{1}{M}\boldsymbol{\rho}_o\cdot\boldsymbol{\rho}_b\right]}\right|^2$$

The structure of correlation imaging setup weakens the tradeoff between spatial and angular res.

Light-field imaging and refocusing

In the geometrical optics limit, a stationary-phase approximation to the double integral inside $\Gamma(\boldsymbol{\rho}_a, \boldsymbol{\rho}_b)$ yields

$$\Gamma(\boldsymbol{\rho}_a, \boldsymbol{\rho}_b) \sim F\left(-\frac{\boldsymbol{\rho}_b}{M}\right)^2 \left| A\left[\frac{z_b}{z_a}\boldsymbol{\rho}_a - \frac{\boldsymbol{\rho}_b}{M}\left(1-\frac{z_b}{z_a}\right)\right]\right|^2$$

Integration over $\boldsymbol{\rho}_b$ erases information on the object

However, object and source can be decoupled by properly scaling the first argument of the retrieved correlation function:

$$\Gamma\left(\frac{z_a}{z_b}\boldsymbol{\rho}_a - \frac{\boldsymbol{\rho}_b}{M}\left(1-\frac{z_a}{z_b}\right), \boldsymbol{\rho}_b\right) \sim F\left(-\frac{\boldsymbol{\rho}_b}{M}\right)^2 \left|A(\boldsymbol{\rho}_a)\right|^2$$