From QCD to cosmology: θ dependence, axions and dark matter

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Based on work in collaboration with M. D'Elia, M. Mariti, G. Martinelli, M. Mesiti, F. Negro, F. Sanfilippo and G. Villadoro

Outline

1. θ dependence in QCD, strong CP and axions
2. How to constraint axion properties
3. Recent lattice works
4. Conclusions

What is θ dependence?

The terminology originated in QCD in late '70s, but more generally one has

θ term a total derivative term in the Lagrangian that has physical effects in quantum theory only (due to topology)

θ angle the θ term can be written as $\theta q(x,t)$, with $Q = \int q$ integer when the action is finite, and $\theta \in [0, 2\pi)$

θ dependence dependence of something (typically the effective potential or the free energy density) on the value of the θ angle.

Probably the simplest example is that of a quantum particle moving on a circumference of radius R:

$$L = \frac{1}{2} m R^2 \dot{\phi}^2 - V(\phi) - \theta \frac{\dot{\phi}}{2\pi}\ , \quad \int \frac{\dot{\phi}}{2\pi} \mathrm{d}t \in \mathbb{Z}\ , \quad \theta \sim \text{magnetic flux}$$

see e.g. Smilga QCD book & Bonati, D'Elia 1709.10034

Models with θ dependence: Schwinger model, CP^N models in 2d, non abelian Yang-Mills theories and in particular QCD, ...

θ term and the strong CP problem

The most general QCD lagrangian compatible with the fundamental symmetries of the Standard Model is:

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu} + \sum_f \bar{\psi}^a_f (i D^{ab}_\mu \gamma^\mu - m_f) \psi^b_f + \theta q(x)$$

$$q(x) = \frac{g^2}{64\pi^2} \epsilon_{\mu\nu\rho\sigma} G^{a\,\mu\nu} G^{a\,\rho\sigma} \qquad (\sim \text{“}\boldsymbol{E} \cdot \boldsymbol{B}\text{”})$$

where $\theta \in [0, 2\pi)$ is a dimensionless, RG invariant parameter.

For $\theta \neq 0, \pi$ the QCD lagrangian is not invariant under P and CP (and for $\theta = \pi$ spontaneous CP symmetry breaking is expected).

Signal of CP violation: electric dipole moment. From neutron EDM measures one gets $|\theta| \lesssim 10^{-10}$ (in going from $|d_n|$ to $|\theta|$ one need the $\theta-$polarizability, that is not well known, hence the $\lesssim$)

Why should $|\theta|$ be so small? Strong CP problem

General properties of θ dependence in QCD

Behaviour under $U(1)_A$: if $\psi_j \to e^{i\alpha\gamma_5}\psi_j$ and $\bar{\psi}_j \to \bar{\psi}_j e^{i\alpha\gamma_5}$ then $\theta \to \theta - 2\alpha N_f$ and $m_j \to m_j e^{2i\alpha}$ (if $m_j = 0$ no θ dependence).

Let $f(\theta, T)$ be the free energy density of QCD at temperature T, then

- $f(-\theta, T) = f(\theta, T)$
- $f(\theta = 0, T) \le f(\theta, T)$

An useful general parametrization is

$$f(\theta, T) - f(0, T) = \frac{1}{2}\chi(T)\theta^2\left[1 + b_2(T)\theta^2 + b_4(T)\theta^4 + \cdots\right]$$

$$Q = \int q(x)\mathrm{d}^4x\ , \quad \chi = \frac{1}{V_4}\langle Q^2\rangle_{\theta=0}\ , \quad b_2 = -\frac{\langle Q^4\rangle_{\theta=0} - 3\langle Q^2\rangle^2_{\theta=0}}{12\langle Q^2\rangle_{\theta=0}}$$

and χ is known as topological susceptibility.

Most famous example of why θ-dependence matters even if $\theta = 0$: $m^2_{\eta'} = \frac{2N_f}{f_\pi^2}\chi^{N_f=0}$ (Witten-Veneziano formula, $f_\pi \simeq 92\,\mathrm{MeV}$).

Possible solutions of the strong CP problem

1. $m_u = 0$ (in this case θ can be removed by a change of variables)
2. promote CP to be a fundamental symmetry of SM and explain all the CP violation in SM by spontaneous symmetry breaking
3. promote θ to be a dynamical variable

Realization of mechanism 3: add to SM a pseudoscalar field "a" with coupling $\frac{a}{f_a}q(x)$ and only derivative interactions. Since the energy has a minimum at $\theta = 0$, "a" will acquire a VEV such that $\theta + \frac{\langle a \rangle}{f_a} = 0$.

Goldstone bosons have only derivatives couplings, so the simplest possibility is to think of "a" as the GB of some $U(1)$ axial symmetry (Peccei-Quinn symmetry). The effective low-energy lagrangian is thus

$$\mathcal{L} = \mathcal{L}_{QCD}^{\theta=0} + \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a(x)}{f_a}q(x) + \frac{1}{f_a}\left({}^{\textit{model dependent}}_{\textit{terms}} \right)$$

Several models for UV completion exist and the most famous are the KSVZ and the DFSZ models.

Axion mass and quartic coupling

θ dependence of the QCD vacuum and axion properties are two names for the same thing $\theta \leftrightarrow a/f_a$. Classical results (Di Vecchia, Veneziano 1980) for the θ dependence obtained in LO chiral perturbation theory give ($f_\pi \simeq 92\,\mathrm{MeV}$)

$$m_a = \frac{\sqrt{m_u m_d}}{m_u + m_d}\frac{m_\pi f_\pi}{f_a}; \quad \lambda_a = -\left(\frac{m_a^2}{f_a^2}\right)\frac{m_u^2 - m_u m_d + m_d^2}{(m_u + m_d)^2}$$

For NLO corrections and the model dependent contributions see G. Grilli di Cortona, E. Hardy, J. Pardo Vega, G. Villadoro 1511.02867

$$m_a = 5.70(7)\mu\mathrm{eV}\left(\frac{10^{12}\,GeV}{f_a}\right) .$$

Since all the couplings are proportional to $1/f_a$, absence of direct detection gives lower bounds on the value of f_a, that turns out to be huge, typical values being $f_a \gtrsim 10^8 \div 10^9$ GeV.

Axions as dark matter

Since f_a is so large, axions are light and weakly interacting, natural candidates for dark matter

Cosmological sources of axions:

thermal production common to all particles, strongly dependent on dynamical details and on model dependent parameters

decay of topological objects both properties and existence of topological objects are model dependent (if $U(1)_{PQ} \to \varnothing$ no top. obj. at all)

misalignment mechanism typical of axions, almost (see later) model independent

Overclosure bound: axion density $\leq$ dark matter density

stronger form

axion density from misalignment $\leq$ dark matter density

Misalignment mechanism

The EoM of the axion is ($H = \dot{R}/R$ Hubble "constant")

$$\ddot{a}(t) + 3H(t)\dot{a}(t) + m_a^2(T)a(t) = 0$$

for $T \gg \Lambda_{QCD}$ the second term dominates and we have $a(t) \sim \mathrm{const}$ (assuming $\dot{a} \ll H$ initially);
when $m_a \sim 3H$ the field start oscillating around the minimum;
when $m_a \gg H$ a WKB-like approximation can be used

$$a(t) \sim A(t) \cos \int^t m_a(\tilde{t}) \mathrm{d}\tilde{t}; \qquad \frac{\mathrm{d}}{\mathrm{d}t}(m_a A^2) = -3H(t)(m_a A^2)$$

and thus the number of axions in the comoving frame $N_a = m_a A^2 / R^3$ is conserved in the late evolution (note that $N_a \propto f_a$ since $A \sim \theta_0 f_a$)

The temperature for which $m_a(T) = 3H(T)$ is called the oscillation temperature T_{osc}. All the nontrivial dynamics happens for $T \simeq T_{osc}$.

Misalignment mechanism

The nice feature of the overclosure bound for misalignment is that it provides an upper bound for f_a.

What we need to estimate this upper bound?

1. $H(t)$: we need to solve the Friedmann equations, thus we need the equation of state (Lattice QCD can help)
2. $m_a(T)$: we need the "temperature dependence" of the axion mass (Lattice QCD can help)
3. we need the initial value of the misalignment angle (nobody can help)

For the last point two possibilities exist depending on when the $U(1)_{PQ}$ symmetry breaking takes place

- before inflation: a single angle has to be used in the initial conditions and its value is a new parameter
- after inflation: we can expect the initial angle not to be the same in different regions and we average on it

Lattice studies of $\chi(T) = f_a^2 m_a^2(T)$

Recent studies aimed at phenomenology are:

- A. Trunin, F. Burger, E. M. Ilgenfritz, M. P. Lombardo and M. Müller-Preussker,
  J. Phys. Conf. Ser. **668**, 012123 (2016) [arXiv:1510.02265 [hep-lat]]
  (and more recent unpublished results, see QM2017 conference)
- C. Bonati, M. D'Elia, M. Mariti, G. Martinelli, M. Mesiti, F. Negro, F. Sanfilippo and G. Villadoro,
  JHEP **1603**, 155 (2016) [arXiv:1512.06746 [hep-lat]].
- P. Petreczky, H. P. Schadler and S. Sharma,
  Phys. Lett. B **762**, 498 (2016) [arXiv:1606.03145 [hep-lat]].
- Sz. Borsanyi, Z. Fodor, K. H. Kampert, S. D. Katz, T. Kawanai, T. G. Kovacs, S. W. Mages, A. Pasztor, F. Pittler, J. Redondo, A. Ringwald, K. K. Szabo,
  Nature **539** no.7627, 69 (2016) [arXiv:1606.07494 [hep-lat]].

Lattice results

All lattice studies agree on the following ($T_c \simeq 155\,\mathrm{MeV}$)

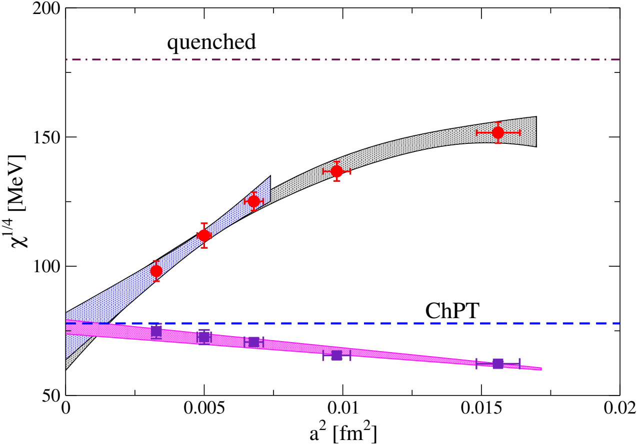
- for $T \lesssim T_c$ we have $\chi(T) \simeq \chi(T=0) = [75.5(5)\,\mathrm{MeV}]^4$
- for T slightly larger than T_c a transient behaviour is present
- for larger T the behaviour is well described by $\chi(T) = b/T^{\alpha}$

There is no agreement (so far) on the values of b, α. In particular α ranges from almost 3 to almost 8. This is not as tremendous as it could seem but still it introduces some $\mathcal{O}(10)$ discrepancies between the final results for f_a.

In Lattice QCD $\chi(T)$ is extracted by estimating $\langle Q^2\rangle/V_4$ by Markov chain Monte Carlo methods, where Q is the topological charge and V_4 the lattice volume. Several problems make a precise determination of $\chi(T)$ quite a difficult computational task, even for the standards of LQCD.

Problem 1

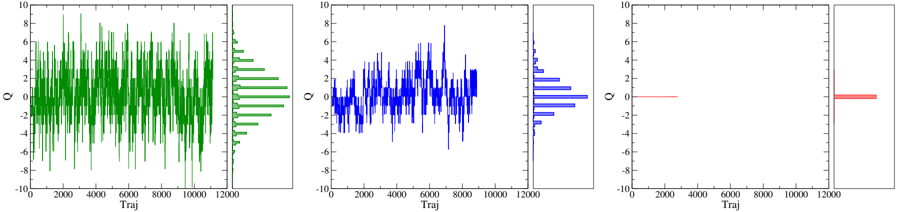
Example of (slow) convergence to the continuum for the $T = 0$ case (from Bonati et al. 1512.06746)



For $T = 0$ we have a good understanding of the physics from chiral perturbation theory and the convergence can be improved (violet points/strip).

Problem 2

Example of the "freezing" problem at $T = 0$ (from Bonati et al. 1512.06746)



$a \simeq 0.0824\,\mathrm{fm}$ $\quad$ $a \simeq 0.0572\,\mathrm{fm}$ $\quad$ $a \simeq 0.0397\,\mathrm{fm}$

From left to right the autocorrelation time of Q grows until the algorithm samples the distribution $P(Q)$ so badly that it is stuck in the $Q = 0$ sector. In this situation is clearly impossible to reliably estimate χ and we have a lower bound for the usable lattice spacings.

Problem 3

At high temperature states with $|Q| > 0$ are suppressed in a finite volume: $\lim_{T\to\infty}\chi(T) \to 0$ and the size of the typical fluctuation is $\delta \sim \sqrt{\chi V_4}$.

To estimate $\chi(T)$ very large lattices are needed and typically only the sectors with $|Q| \leq 1$ are explored.

We know that for very large temperatures the distribution of Q is well approximated by a Poisson distribution (i.e. we know everything once we know $P(Q=1)$), but we do not know the lowest temperature for which this approximation is quantitatively accurate.

It is difficult to understand if a distribution is a Poissonian or not by using only $Q=0$ and $Q=1$ data.

"Poissonian Q distribution" = "dilute instanton gas approxmation"

Exit strategies

Slow continuum convergence at finite temperature:

- normalize with the $T = 0$ result, Bonati et al. 1512.06746 (not enough, still problems).
- introduce *a posteriori* a corrective bias in the analysis Borsanyi et al. 1606.07494 (instanton gas inspired "eigenvalue reweighting". How to check its reliability?).

Freezing problem & high T finite size effects:

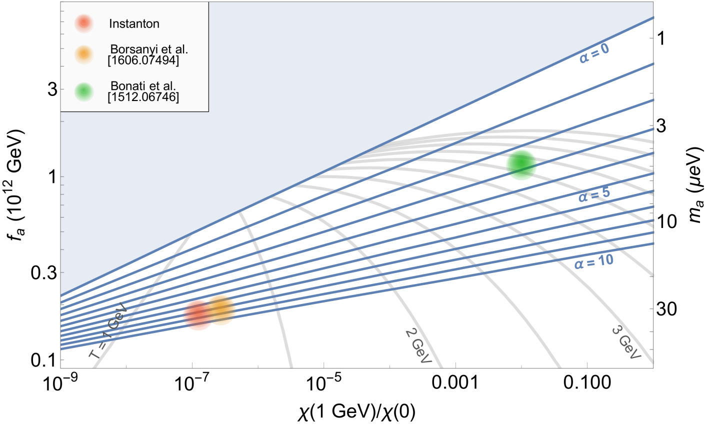
avoid it
- by using coarse enough lattice spacings and large enough lattice sizes
- by using different sampling strategies (open boundary conditions, metadynamics, sub-volume estimates, non-orientable manifolds...).

live with it "thermodynamical integration" Borsanyi et al. 1606.07494 (again a Poissonian instanton-like behaviour is assumed).

For a list of all the adopted/adoptable strategies see Bonati 1710.06410

The final results

upper bound for f_a lower bound for m_a



from Grilli di Cortona 2016 PhD thesis @ SISSA

Results by other groups come in between the ones shown in figure.

Conclusions

- "θ dependence" can be found in several QFT (or even QM) models and it is an intrinsically non-perturbative quantum phenomenon.
- For the case of QCD the study of θ dependence has both theoretical (e.g. understand the relation, if any, with confinement) and phenomenological aims.
- From the phenomenological point of view axions are becoming more and more popular as dark matter candidates and Lattice QCD can help in providing bounds for the axion mass and couplings (modulo cosmological assumption: e.g. if $U(1)_{PQ}$ breaks after inflation)
- Future lattice works will have to carefully asses the reliability of the procedures adopted so far and check for possible systematics, in order to provide a solid final result for $\chi(T)$.

Thank you for your attention!

Backup slides with something more

Model dependent interactions

$$\mathcal{L} = \mathcal{L}_{QCD}^{\theta=0} + \frac{1}{2}\partial_\mu a \partial^\mu a + \frac{a(x)}{f_a} q(x) +$$
$$+ \frac{1}{4} g^0_{a\gamma\gamma} a F_{\mu\nu}\tilde{F}^{\mu\nu} + \frac{1}{2f_a}\partial_\mu a \sum_{f=u,d\ldots} c^0_f \bar{\psi}_f \gamma^\mu \gamma_5 \psi_f$$

where

$$g^0_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\frac{E}{N} \qquad \frac{E}{N} = \frac{\text{em anomaly}}{\text{color anomaly}}$$

The coefficients E/N and c^0_f are all model dependent.

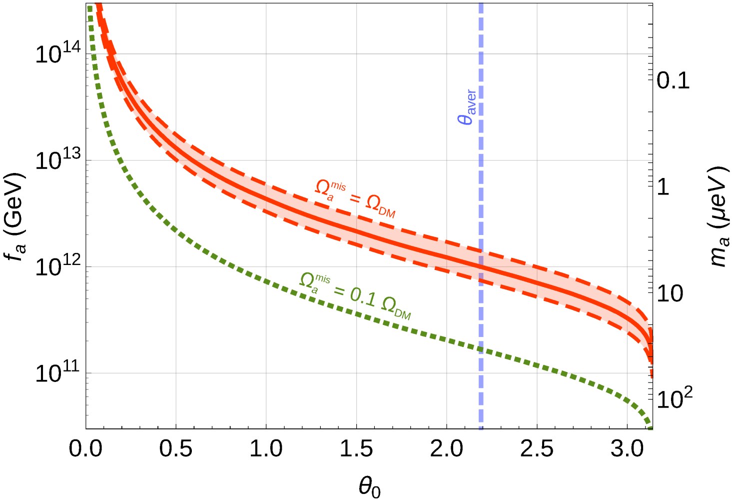
The low-energy effective $a\gamma\gamma$ interaction contains also a strongly induced model independent electromagnetic term and $g_{a\gamma\gamma}$ is given by

$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}\left(\frac{E}{N} - \frac{2}{3}\frac{4m_d + m_u}{m_d + m_u} + \cdots\right)$$

see e.g. G. Grilli di Cortona, E. Hardy, J. Pardo Vega, G. Villadoro 1511.02867 from Borsanyi et al. 1606.07494

All DM from misalignement	$m_a = 28(2)\mu eV$
50% from misalignement	$m_a = 40(4)\mu eV$
1% from misalignement	$m_a = 1500\mu eV$

upper bound for f_a lower bound for m_a



from Bonati et al. 1512.06746