edge modes and crystalline phases in atomic synthetic la

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Sezione di Pisa

"Quantum gases, Fundamental interactions and Cosmology"

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Outlook

- **Ultracold atoms in optical lattices**
  - ladders & synthetic dimensions
  - a microscopic model
- **Periodic synthetic boundaries (cylinder)**
  - magnetic crystals
- **Open synthetic boundaries (stripe)**
  - chiral edge currents

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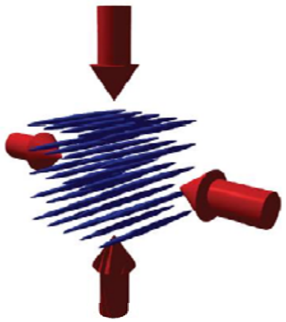
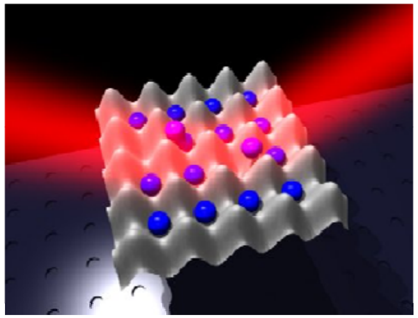
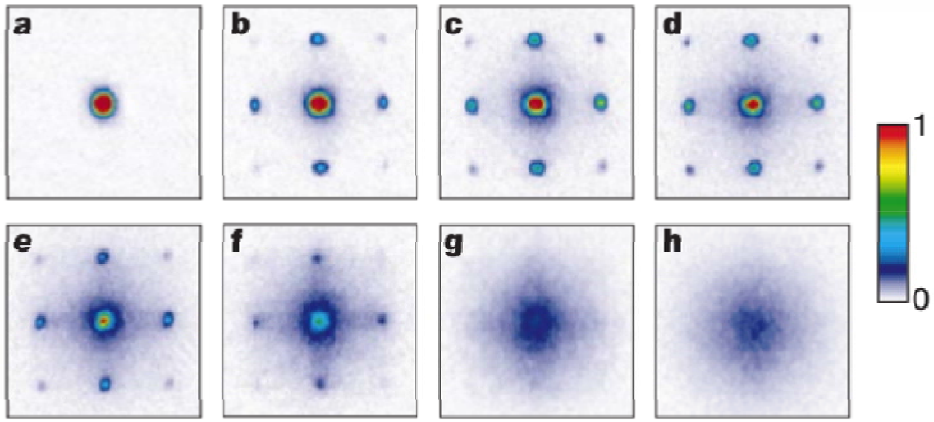




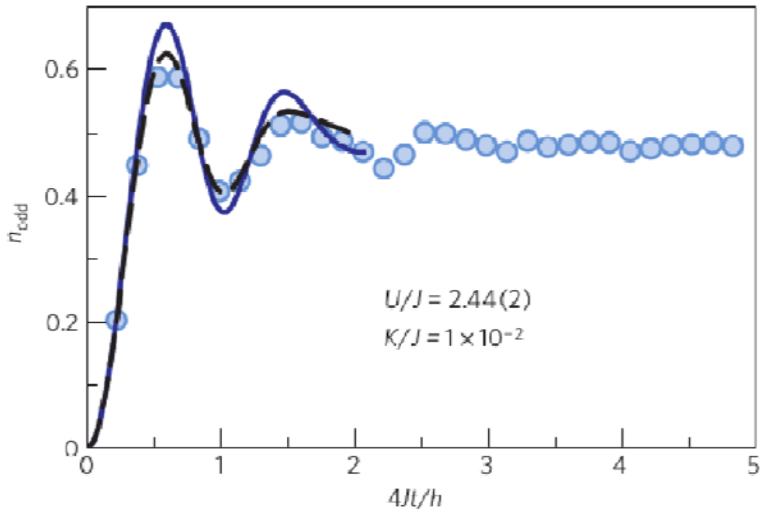
Ultracold atoms in optical lattices

Hubbard model (SF–MI phase transition …)



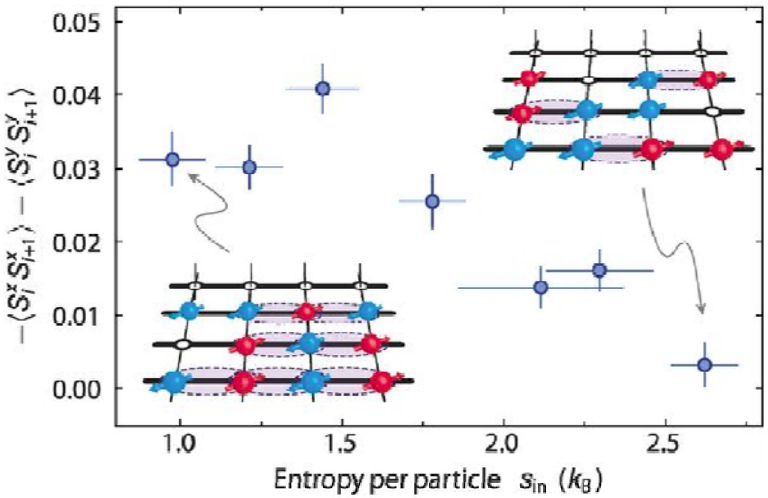
Greiner *et al.*, Nature **415**, 39 (2002)

Out-of-equilibrium dynamics
(thermalization, localization …)



S. Trotzky *et al.*, Nat. Phys. **8**, 325 (2012)

Strongly correlated states (spin liquids …)



D. Greif *et al.*, Science **340**, 1307 (2013)

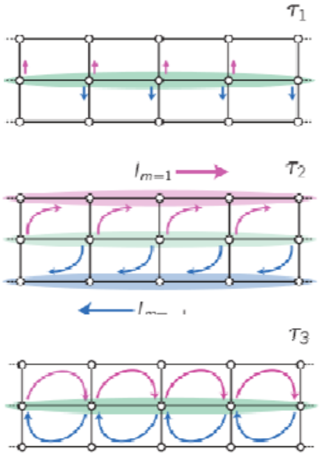
Hall physics

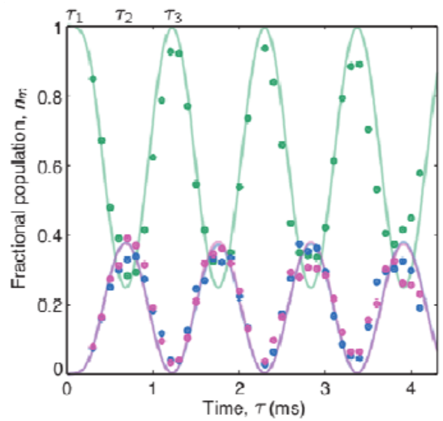
↓

gauge fields
synthetic dimensions

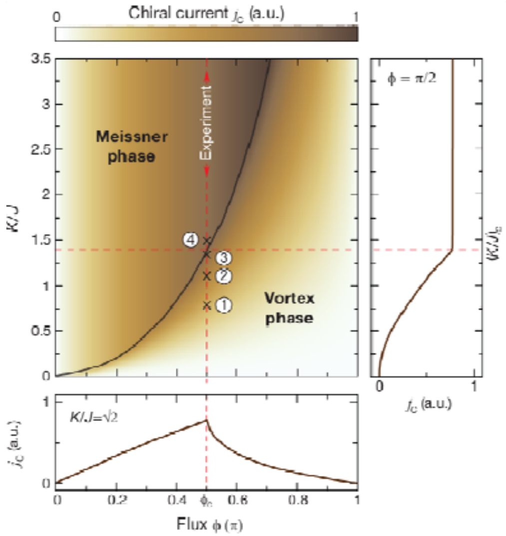
Hall physics with cold atoms

- Observation of chiral edges in *bosonic systems*



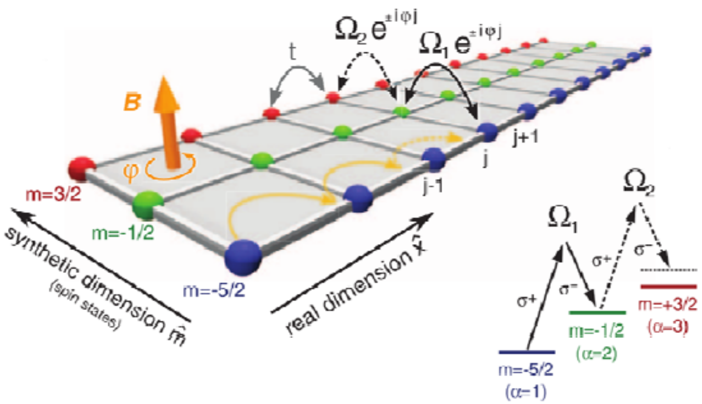


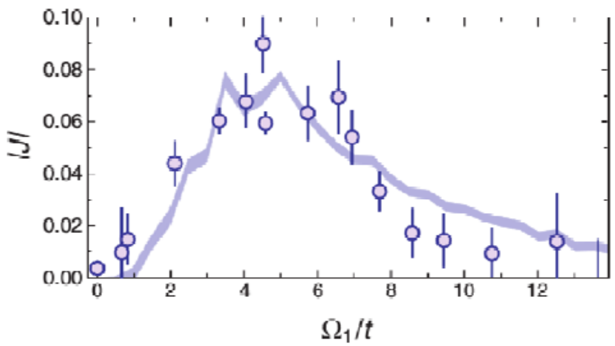
B.H. Stuhl *et al.*, Science **349**, 1514 (2015)



M. Atala *et al.*, Nat. Phys. **10**, 588 (2014)

- Observation of chiral edges in *fermionic systems*

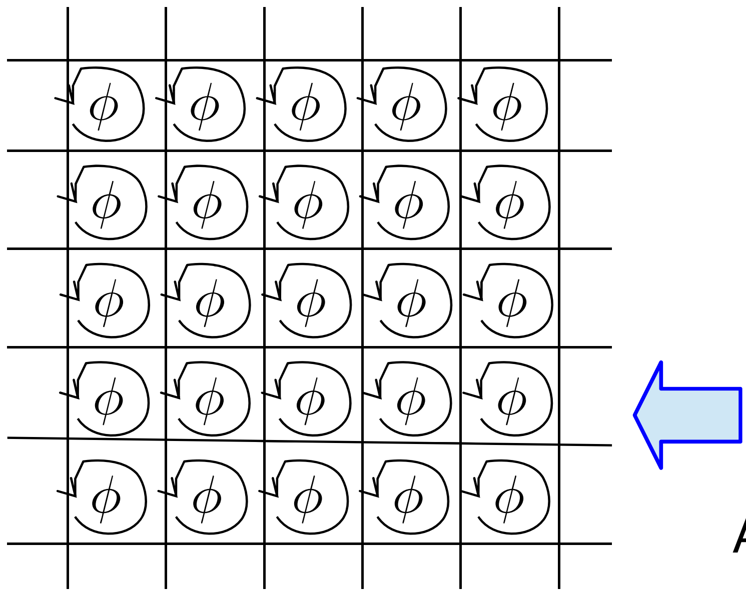




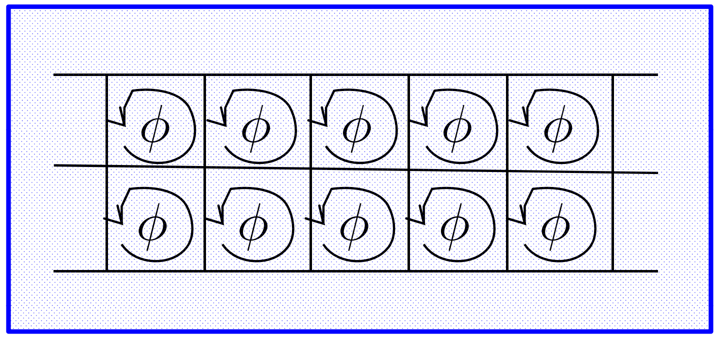
M. Mancini *et al.*, Science **349**, 1510 (2015)

Ladders & synthetic dimension

A minimal setup in order to mimic the Hall physics in quasi-1D systems



ladder system



A system of (interacting) particles, in presence of an ap

→ signatures of Quantum Hall States

One direction needs not to be a physical dimension:

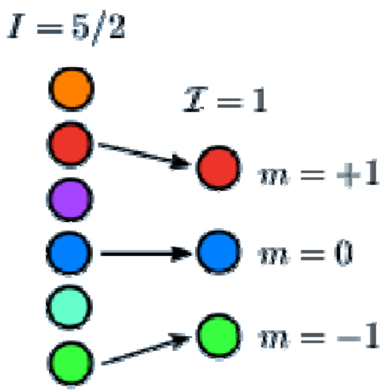
An **extra-dimension** can by synthetically engineered in a different way!

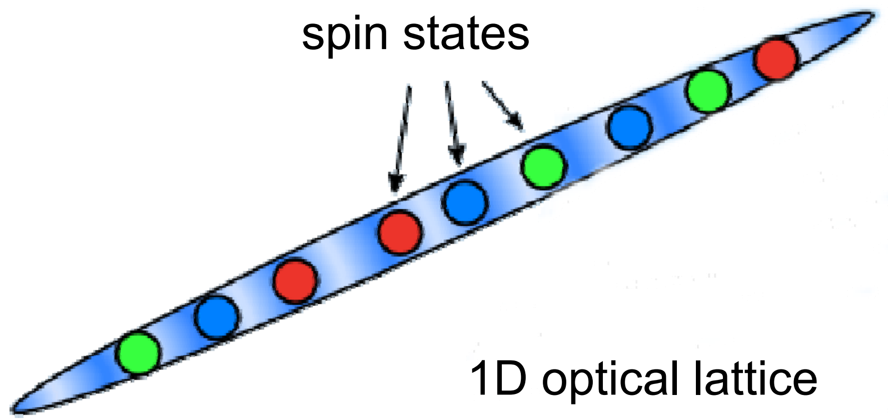
Synthetic dimension

IDEA:

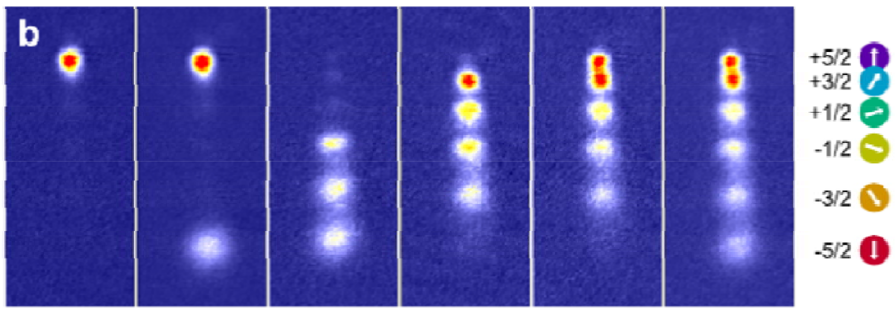
Use a system with *D spatial dimensions*

Encode the *(D+1)th dimension* in a *different degree of freedom* (e.g. the spin)





Large-*I* systems: Rubidium, Ytterbium, …



G. Pagano *et al.*, Nat. Phys. **10**, 198 (2014)

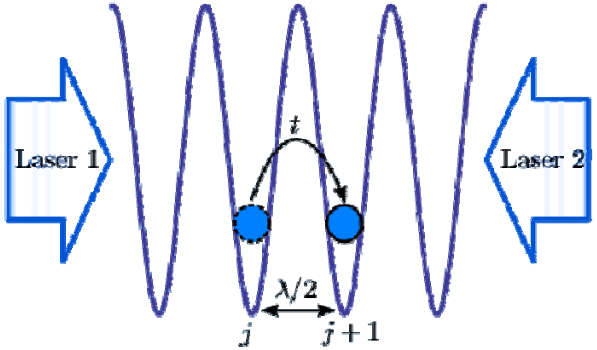
O. Boada *et al.,* PRL **108**, 133001 (2012)

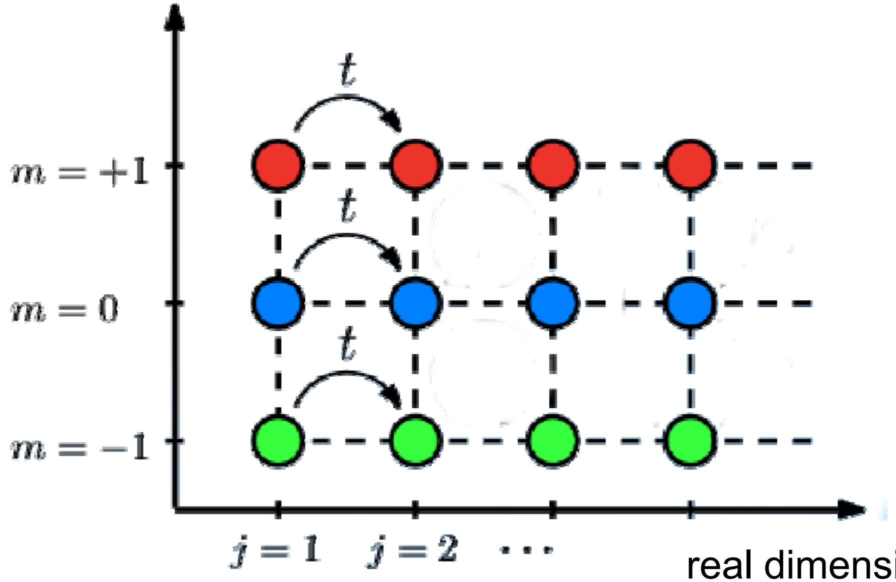
A. Celi *et al.*, PRL **112**, 043001 (2014)

The model

Tunneling in the real dimension (tight binding):

$$H_t = -t \sum_j \sum_m \left[c^\dagger_{j,m} c_{j+1,m} + \text{h.c.} \right]$$





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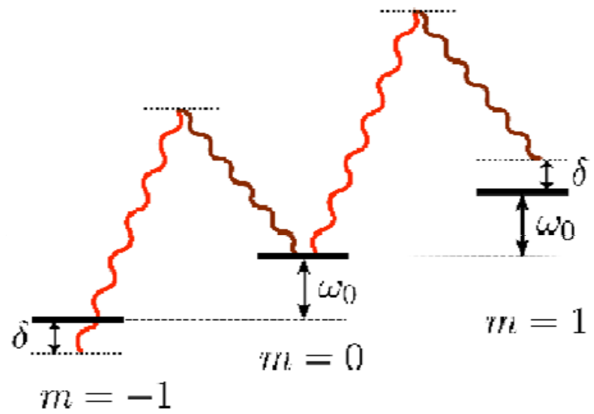
Spin states are coupled through *Raman transitions*:

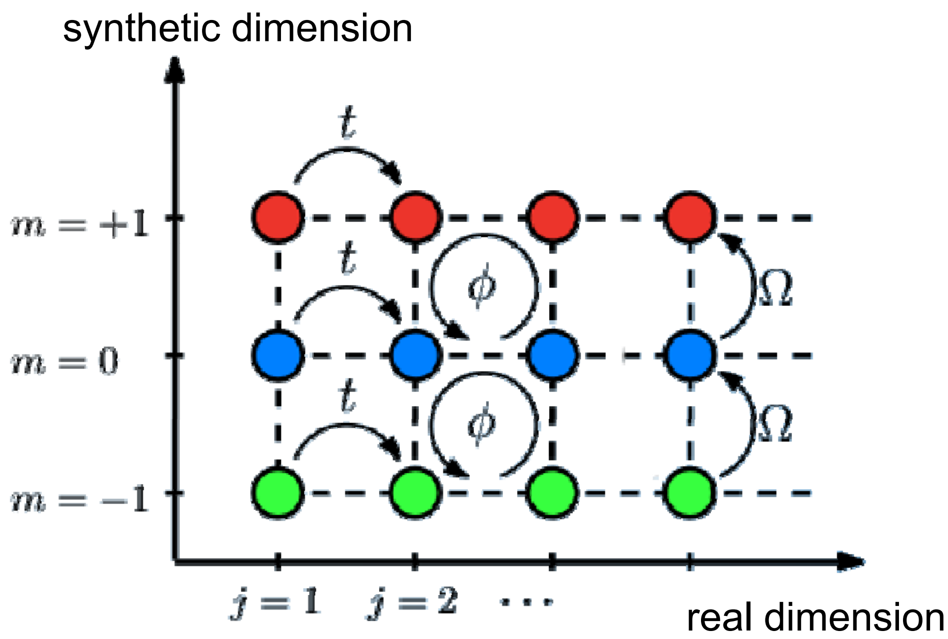


$$H_R = \Omega \sum_j \sum_m \left[e^{-i2\pi\phi j} c^\dagger_{j,m} c_{j,m+1} + \text{h.c.} \right]$$

- *tunneling in the synthetic dimension* (Ω)
- ***synthetic magnetic field*** (Φ)

A) Open boundary conditions in the synthetic dimension

→ plain ladder, edge currents



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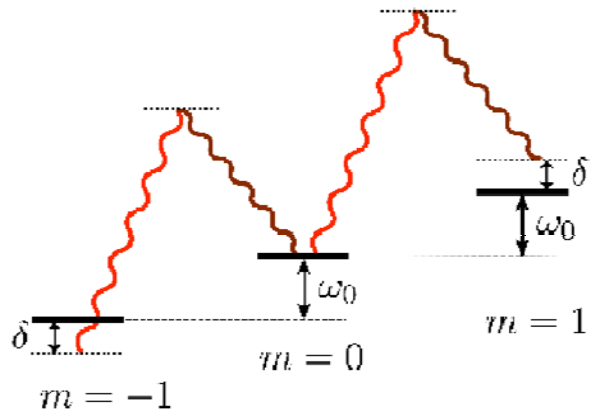

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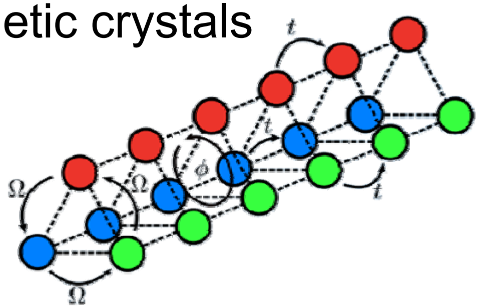
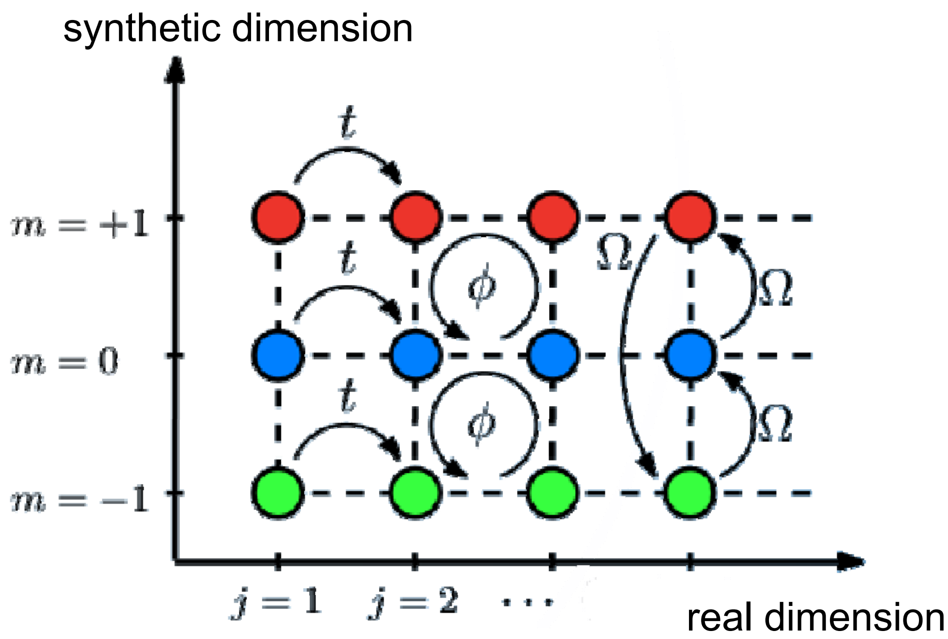
- *tunneling in the synthetic dimension* (Ω)
- ***synthetic magnetic field*** (Φ)

B) Periodic boundary conditions in the synthetic dimension

→ torus, magnetic crystals

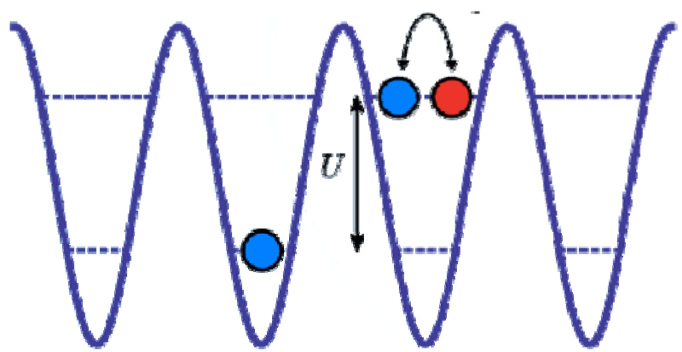




The model: two-body interactions

Interactions in earth-alkali atoms (Yb, Sr, …) are ***SU(N) invariant***:

$$H_{\text{int}} = U \sum_j \sum_m \sum_{m'<m} n_{j,m} n_{j,m'}$$



→ Repulsive ($U>0$) among two atoms in the same spatial site;

→ ***Highly anisotropic***: local in real space, infinite-range in synthetic space;

→ SU(N) invariant in spin space.

Here we consider *Fermionic* particles

The global Hamiltonian is:

$$H = H_t + H_R + H_{\text{int}}$$

$$= \sum_{jm} \left\{ \left[-t c^\dagger_{j,m} c_{j+1,m} + \Omega e^{-i2\pi\phi j} c^\dagger_{j,m} c_{j,m+1} + \text{h.c.} \right] + U \sum_{m'<m} n_{j,m} n_{j,m'} \right\}$$

Connection with Rashba SOC

$$H = H_t + H_R + H_{\text{int}}$$
$$= \sum_{jm} \left\{ \left[-tc^\dagger_{j,m} c_{j+1,m} + \Omega e^{-i2\pi\phi j} c^\dagger_{j,m} c_{j,m+1} + \text{h.c.} \right] + U \sum_{m'<m} n_{j,m} n_{j,m'} \right\}$$

Since the Hamiltonian is *not translationally invariant*, we perform the transformation:

$$d_{j,m} = U c_{j,m} U^\dagger = e^{-i2\pi\phi m j} c_{j,m} \qquad (\nu_{j,m} = d^\dagger_{j,m} d_{j,m})$$

$$\tilde{H} = \sum_{jm} \left\{ \left[-te^{i2\pi\phi m} d^\dagger_{j,m} d_{j+1,m} + \Omega d^\dagger_{j,m} d_{j,m+1} + \text{h.c.} \right] + U \sum_{m'<m} \nu_{j,m} \nu_{j,m'} \right\}$$

lattice Rashba SOC → $-2t \sum_{km} \cos(k - 2\pi m\phi) d^\dagger_{k,m} d_{k,m}$

magnetic field along x

The quadratic part is now readily diagonalizable in Fourier space…

A few words on the method:
tensor networks

Tensor-network approach

Use the variational class of ***matrix product states***, in order to address ground-state properties of (quasi) 1D lattice systems:

$$|\psi\rangle = \sum_{i_1, i_2, \dots, i_L} A^{[1], i_1} A^{[2], i_2} \dots A^{[L], i_L} |i_1\rangle_1 \otimes |i_2\rangle_2 \otimes \cdots \otimes |i_L\rangle_L$$

S. R. White, PRL **69**, 2863 (1992)
U. Schollwöck, Ann. Phys. **326**, 96 (2011)

The *density-matrix renormalization group*

Area-law entanglement ⟶ ***very efficient in 1D***

- For ladder systems, we group each physical site belonging to a given rung (and a di

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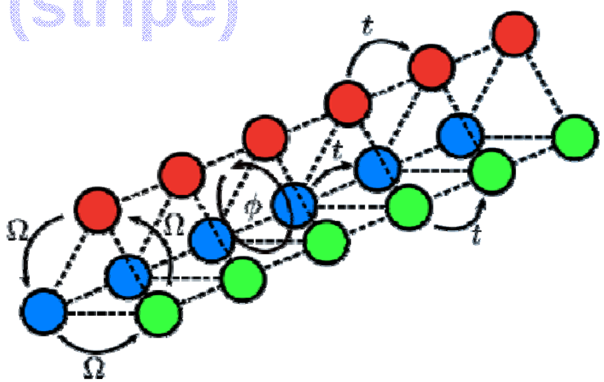
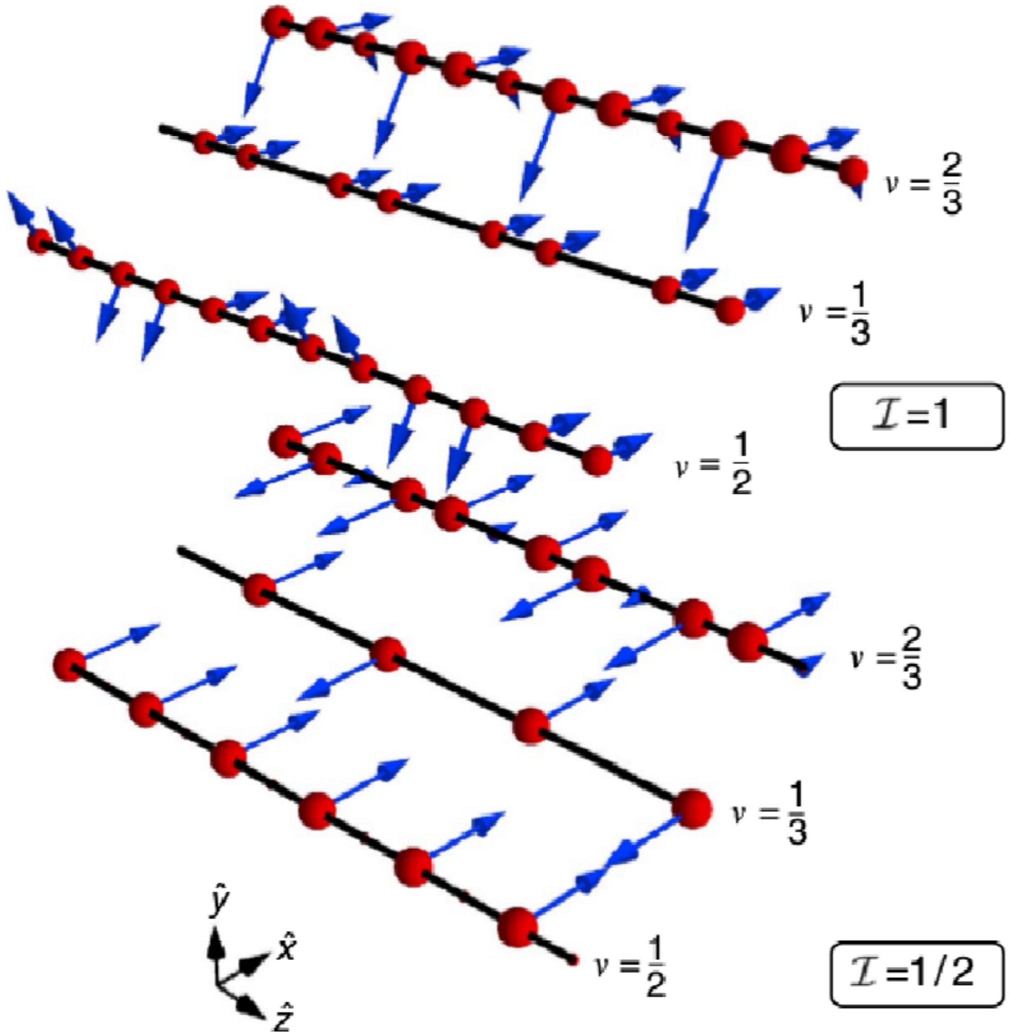


Magnetic crystals



S. Barbarino *et al.*, Nat. Commun. **6**, 8134 (2015)

L. Taddia *et al.*, PRL **118**, 230402 (2017)

Magnetic crystals

Emergence of gapped phases with regular pattern in *charge density* & *spin textures*

They occur at rational values of the **filling factor**: $\nu = p/q$

$$\nu = \frac{N}{N_\phi} = \frac{n}{\phi(2\mathcal{I}+1)}$$

$N = nL$ particle number

$N_\phi = \phi(2\mathcal{I}+1)L$ total magnetic flux

Integer filling

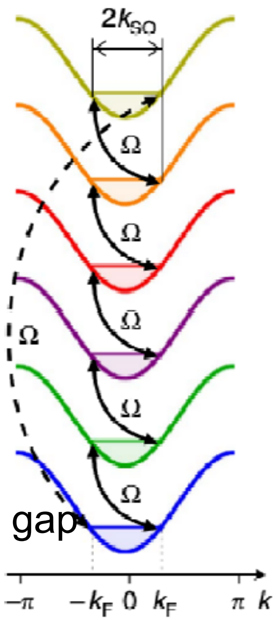
$(\nu = 1, 2, \ldots)$

⇩

non-interacting model

$$2k_F = 2\pi\phi = 2k_{\rm SO}$$

system develops a full gap



Fractional filling

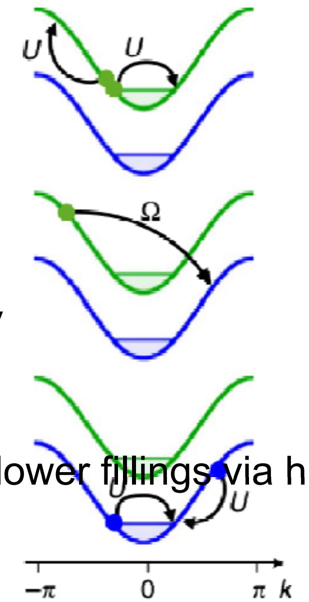
$(\nu = 1/2,\ 1/3,\ 2/3, \ldots)$

⇩

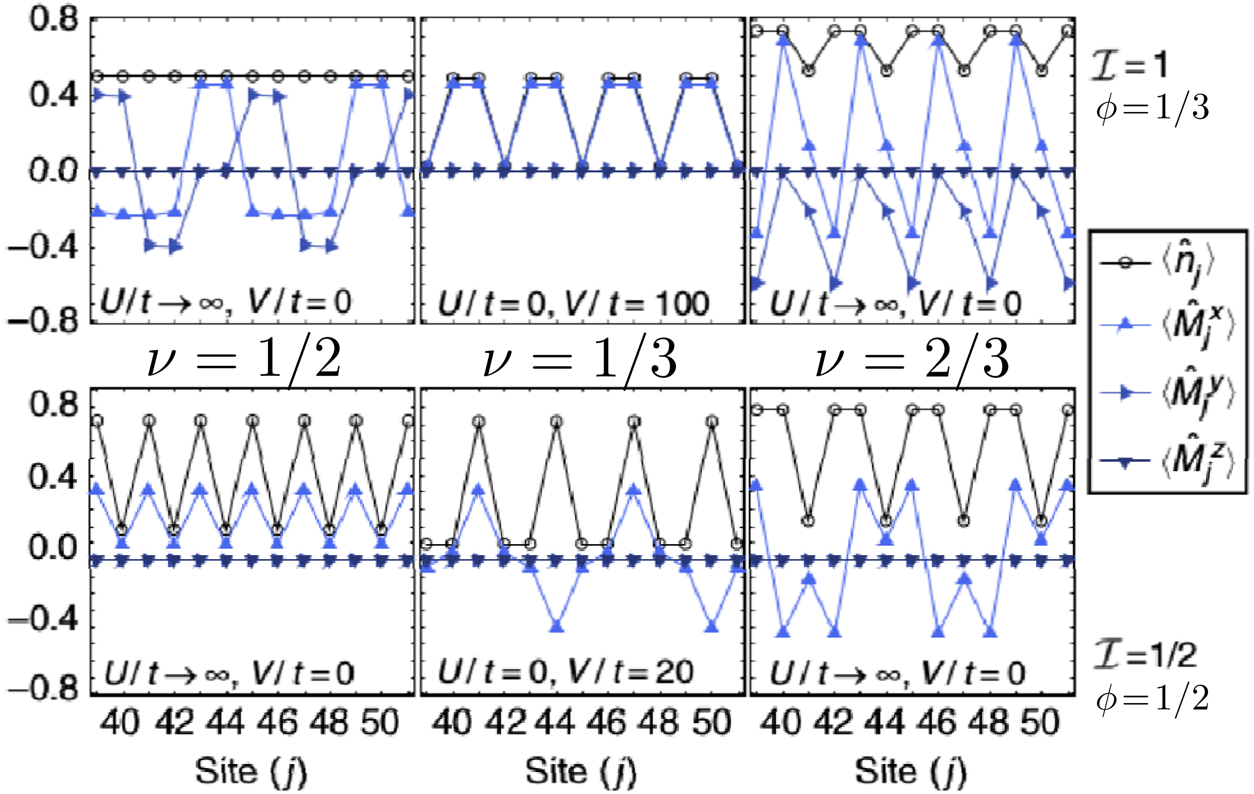
interactions are mandatory

⇩

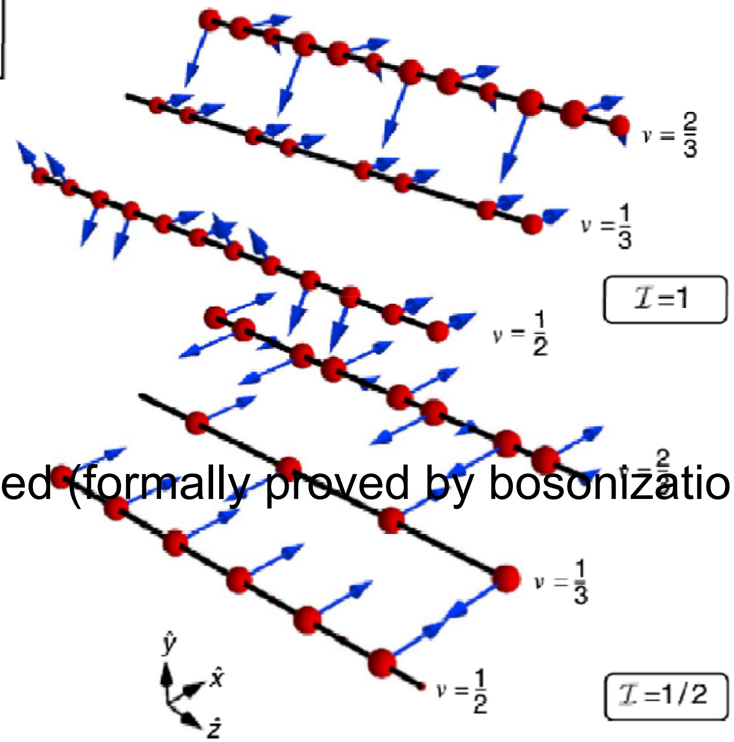
system can develop a gap for lower fillings via higher-orde



Magnetic crystals



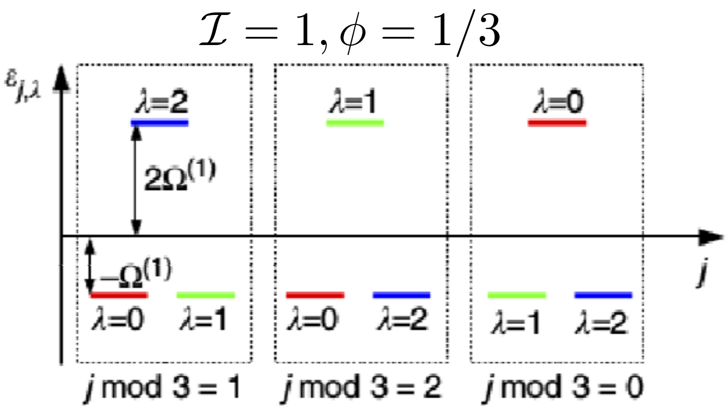
Crystalline order is stabilized for part

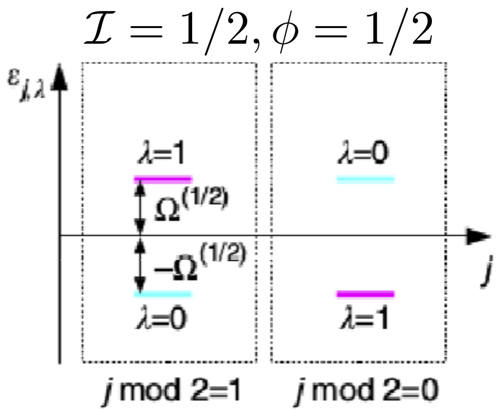


Decreasing the filling, <u>*finite-range interactions*</u> are needed (formally proved by bosonization)

for $q>1$ and odd

Magnetic crystals

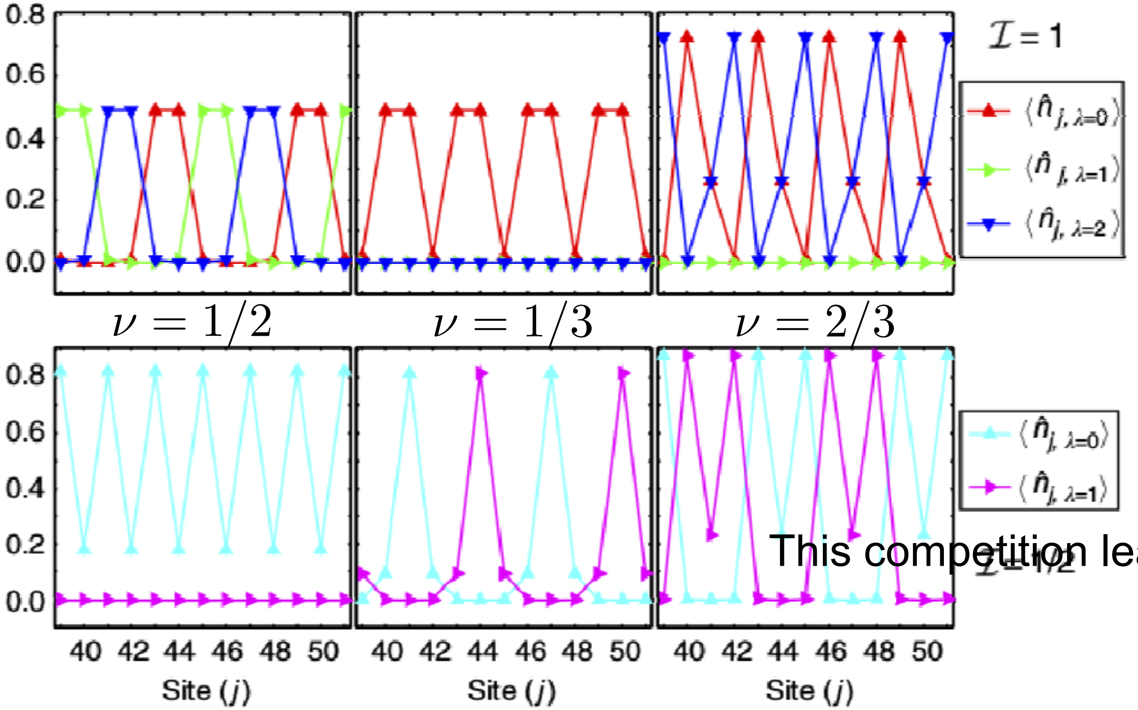




The structure of crystals is better unders

$$d_{j\lambda} = \frac{1}{\sqrt{2\mathcal{I}+1}} \sum_m e^{\left(\frac{2\pi i}{2\mathcal{I}+1}\right)\lambda m} c_{j,m}$$

$$\varepsilon_{j,\lambda} = 2\Omega \cos\left[\frac{2\pi\lambda}{2\mathcal{I}+1} + 2\pi\phi j\right]$$

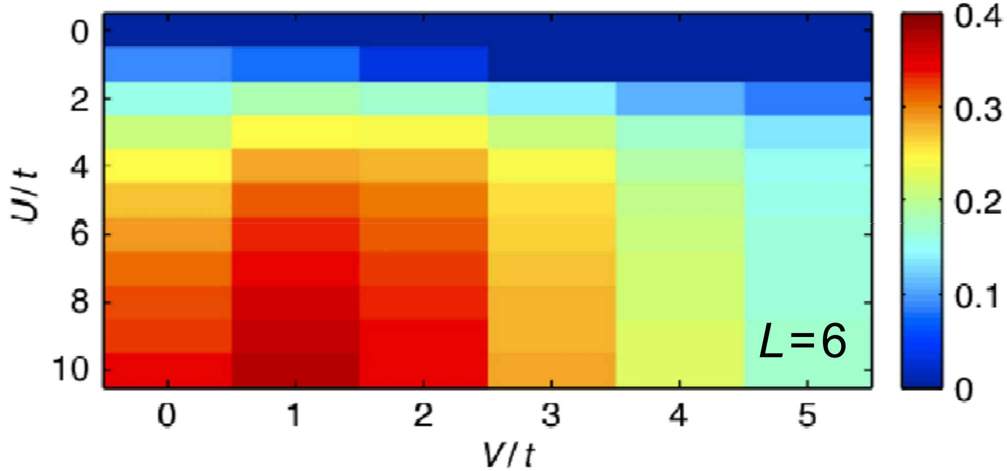


When the space periodicity matches int

- hopping → delocalization
- repulsion → localization

This competition leads to the formation of dimers locked together

Finite temperature & trapping effects



Energy gaps for $\mathcal{I} = 1,\ \nu = 1/2$

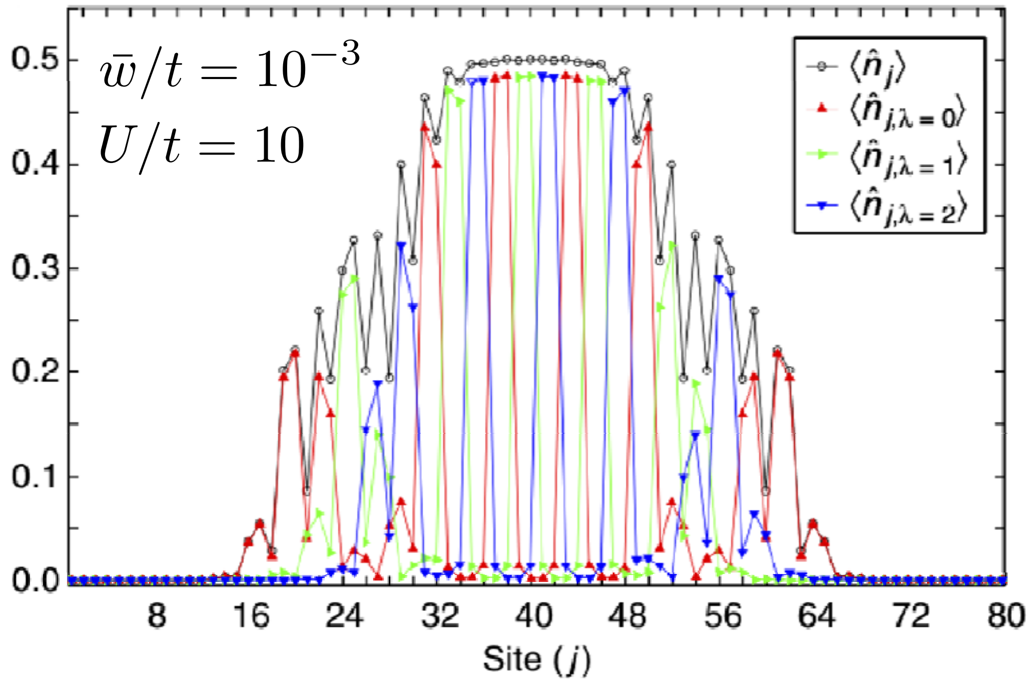
repulsive interactions enhance the gap

$$\Rightarrow \boxed{k_B T \lesssim \varepsilon_{\text{gap}} \sim t/2}$$

Harmonic confinement:

$$H_{\text{trap}} = \sum_{j,m} \bar{w}\left(j - \frac{L}{2}\right)^2 c^\dagger_{j,m} c_{j,m}$$

Robustness → crystals form in a definite region of the trap



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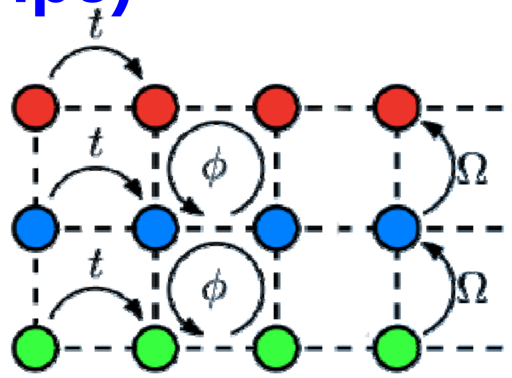


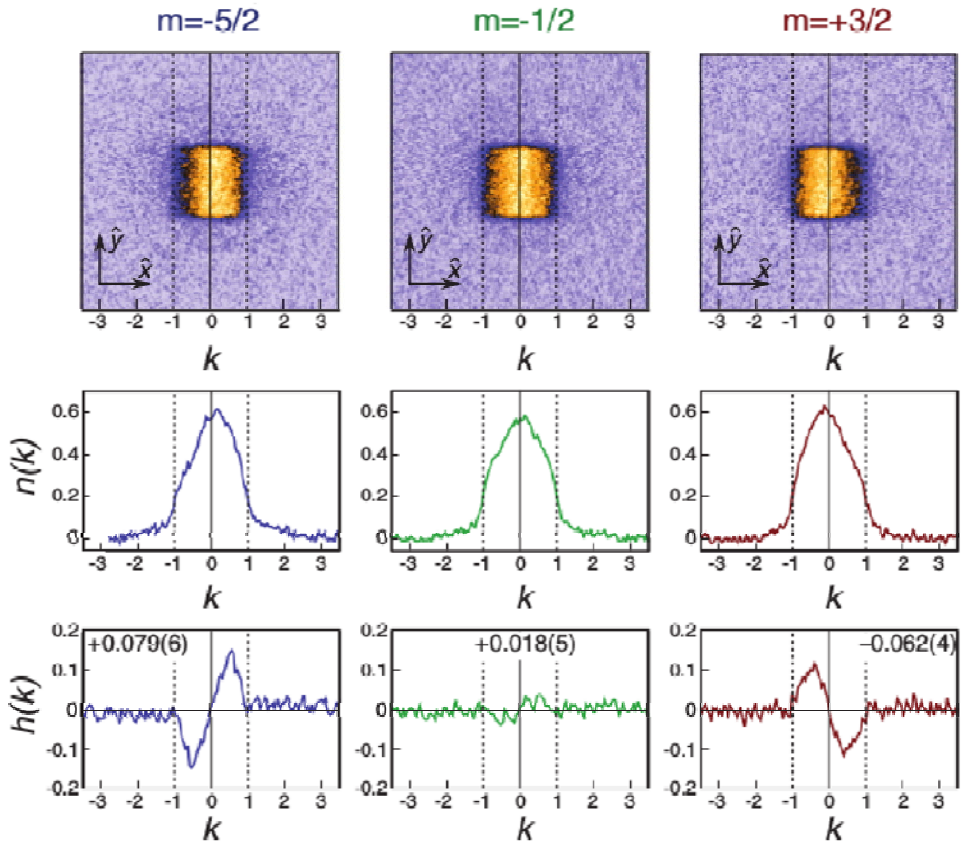
Edge-like modes: experimental facts

exp: → M. Mancini *et al.*, Science **349**, 1510 (2015)

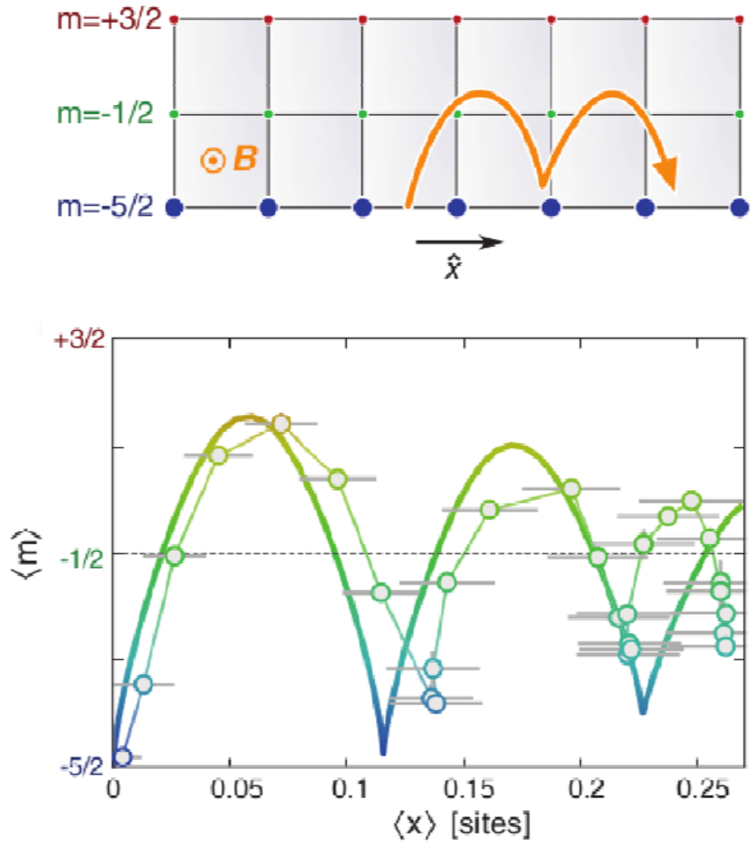
Momentum distribution: $n(k) = \sum_m n_m(k)$

Asymmetry: $h(k) = n(k) - n(-k)$

$$n_m(k) = \frac{1}{L}\sum_{j,l} e^{-ik(j-l)}\langle c^\dagger_{j,m} c_{l,m}\rangle$$



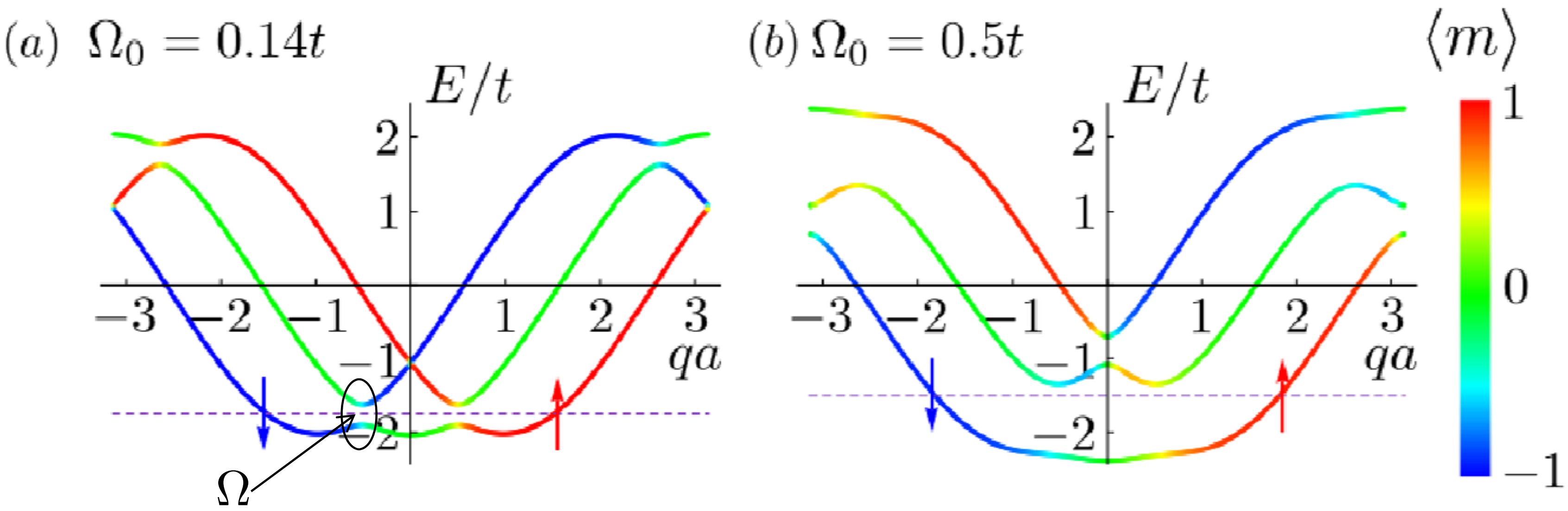
Edge-cyclotron skipping orbits



Single particle spectrum

The *non-interacting case* can be easily diagonalized in the Rashba basis.

Single-particle spectrum for three legs ($l=1$):



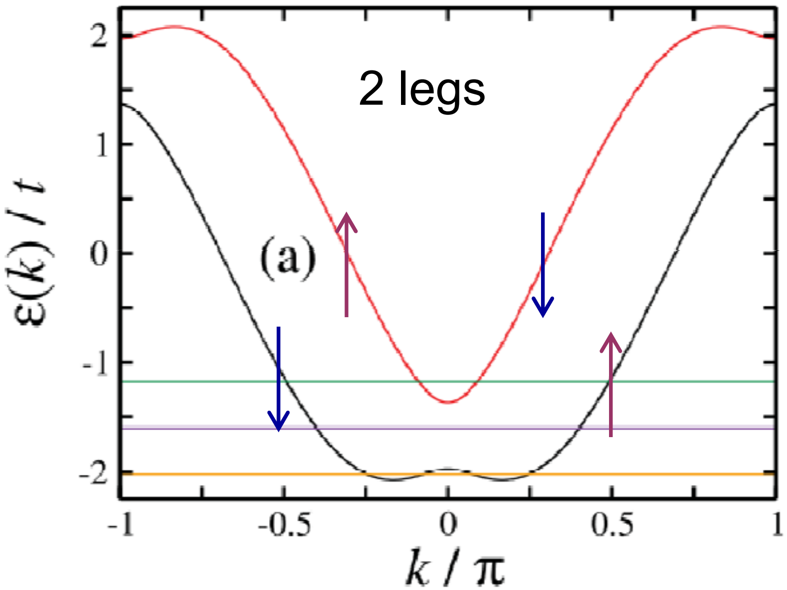
→ The ground-state branch displays spin-polarized edges

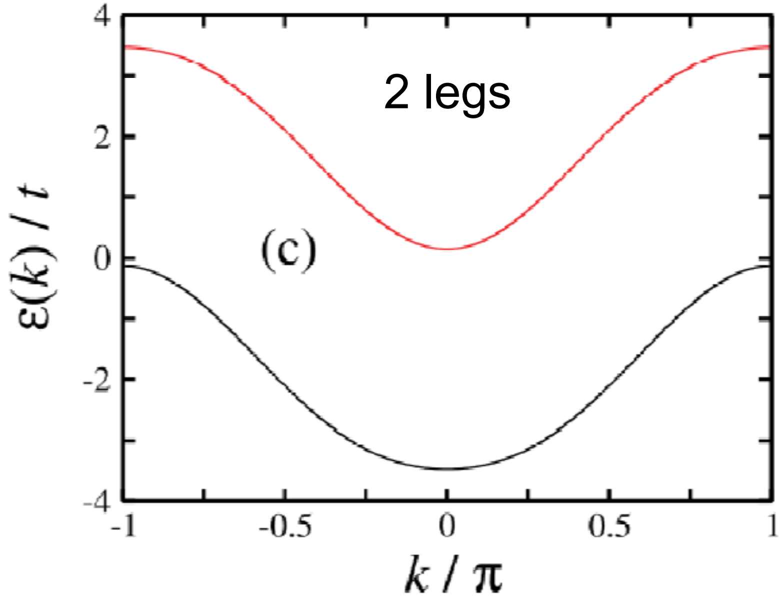
→ low-energy excitations are gapless and with defined momentum & spin

→ **chiral edge modes** on the synthetic lattice

A. Celi *et al.*, PRL **112**, 043001 (2014)

Different regimes





Weak Raman coupling $\Omega/t \lesssim 1$:

- **low** / **high** filling→4 low-energy excit.
- **intermediate** filling→helical liquid

Strong Raman coupling $\Omega/t \gtrsim 1$:

- quasi-spinless gas (x-polarized)
- interactions almost ineffective

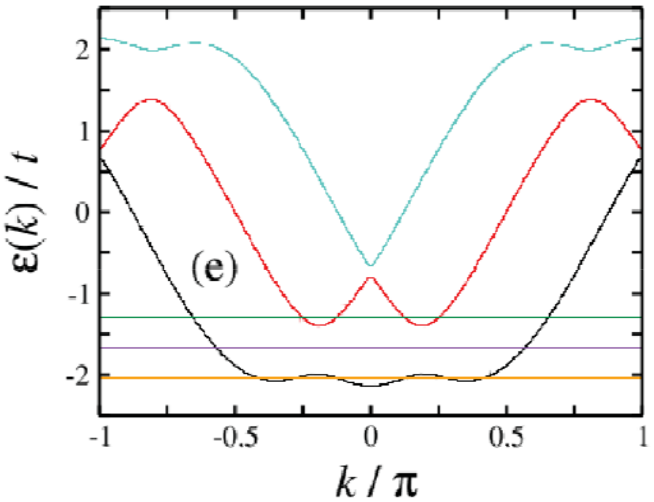
An interacting system is predicted to behave effectively as a free system with a renormali

B. Braunecker *et al.*, PRB **82**, 045127 (2010)

S. Barbarino *et al.*, New J. Phys. **18**, 035010 (2016)

M. Calvanese Strinati *et al.*, Phys. Rev. X **7**, 021033 (2017)

Role of interactions

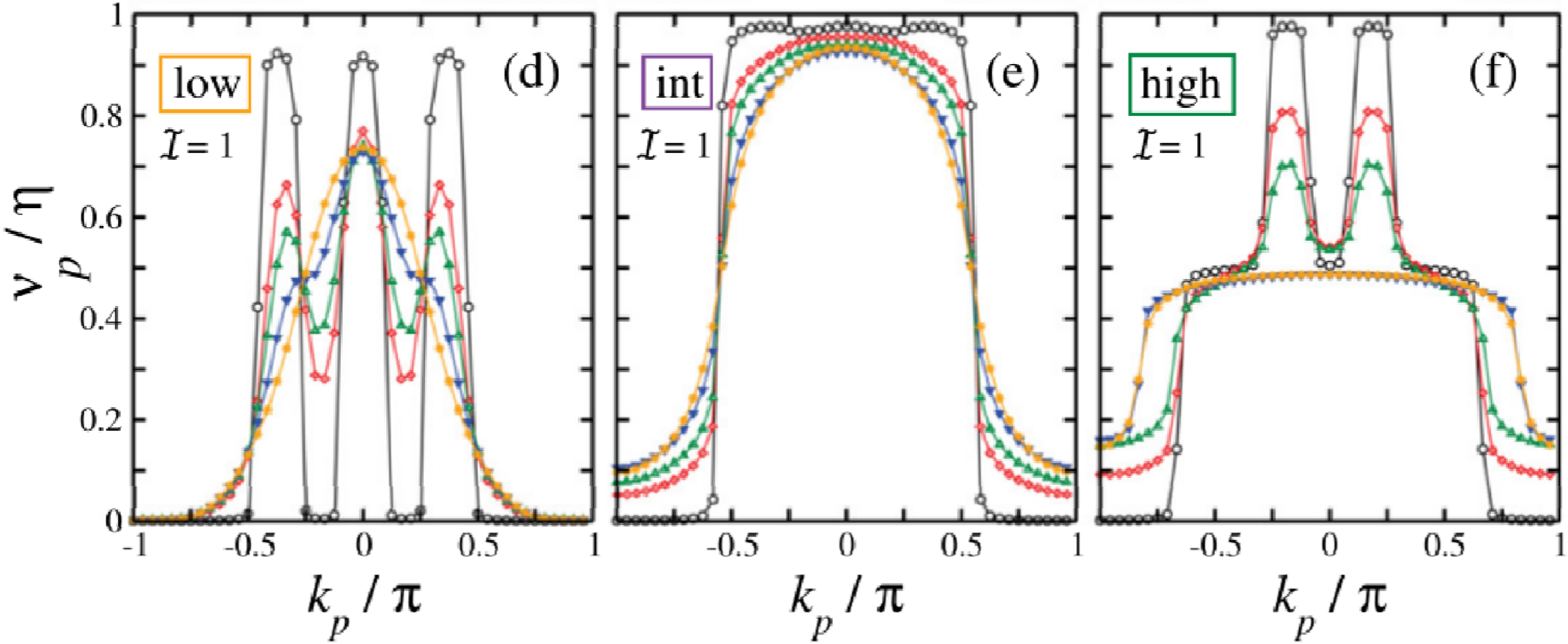


Total momentum distribution in *k*-space

$$\nu_p = \sum_m \nu_{pm} = \sum_m \langle d^\dagger_{p,m} d_{p,m} \rangle$$

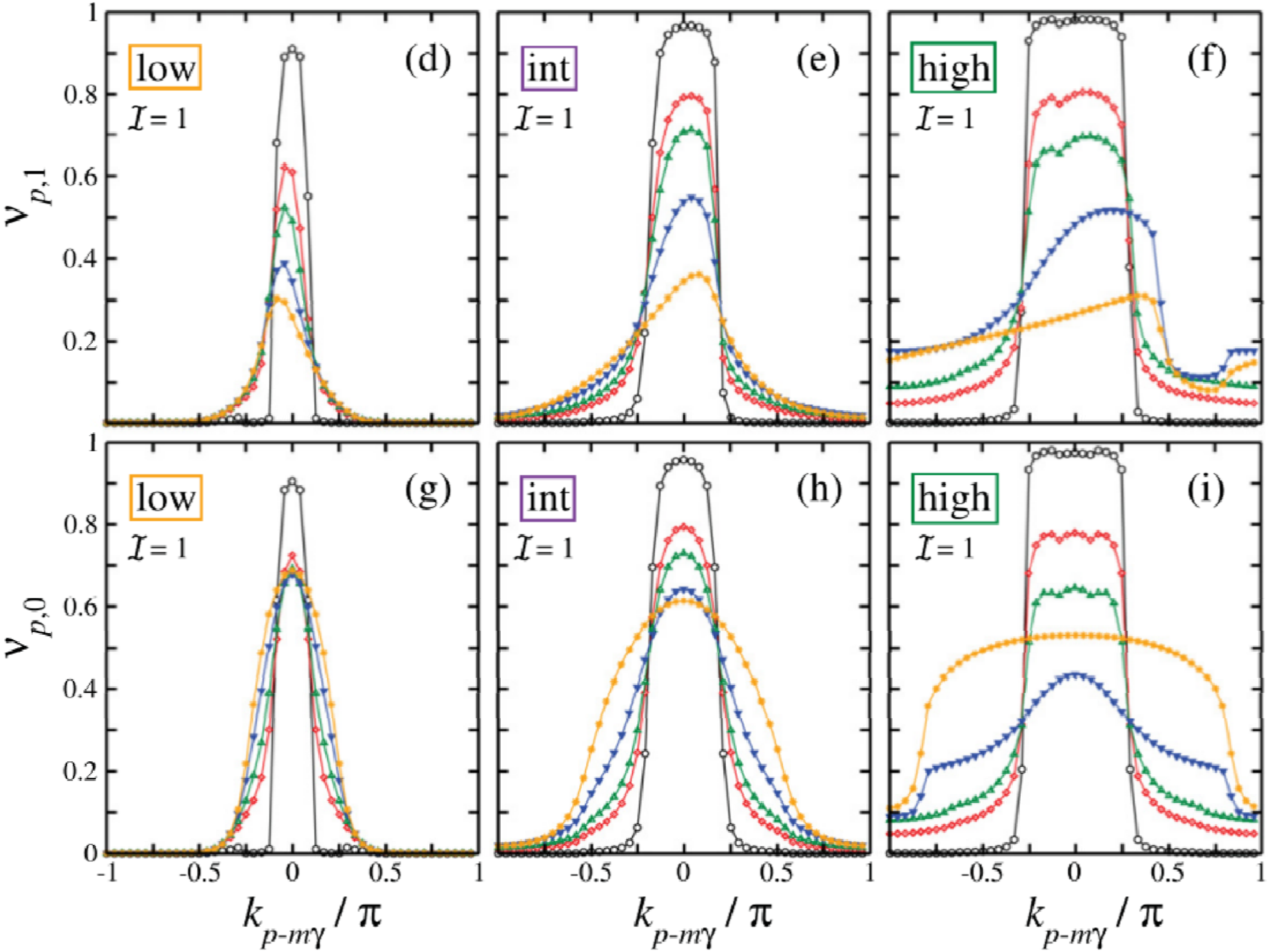
Increasing *U* results in *effectively increasing* Ω

$$\nu_{p,m} = n_{p-2\pi m\phi,m}$$



Role of interactions

Increasing U results in *effectively enhancing the asymmetry*

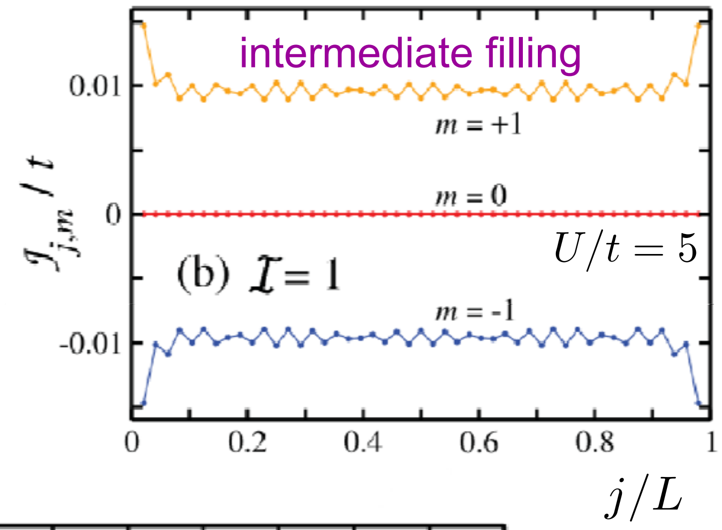


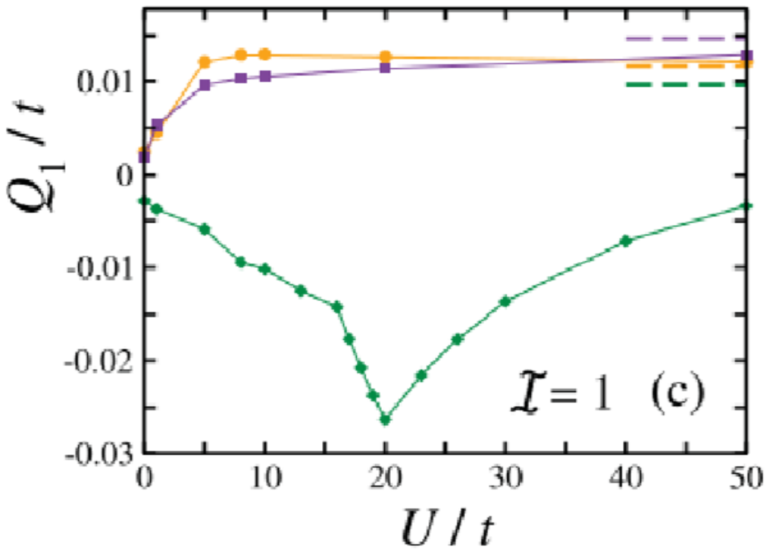
Chiral currents

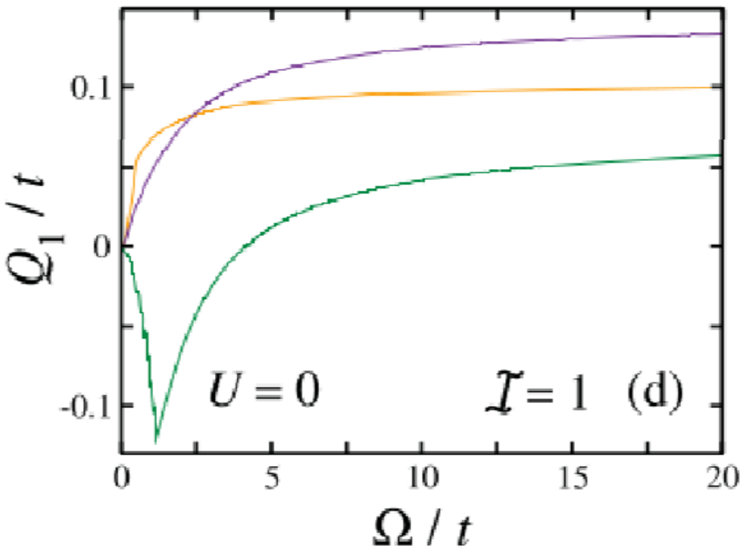
Chiral current: $\mathcal{J}_{j,m} = -it(c^\dagger_{j,m} c_{j+1,m} - \text{h.c.})$

$$Q_m = \frac{1}{L} \sum_j \langle \mathcal{J}_{j,m} \rangle$$

$$= -\frac{2t}{L} \sum_{p>0} \sin k_p (n_{p,m} - n_{-p,m})$$







→ Similar trend for the current vs. *U* and Ω [*U* effectively enhances Ω]

→ Strong non-monotonic behavior of the current with *U and Ω*

(a priori unexpected: classically the magnetic field determines the direction of current)

Conclusions

- Atomic simulator of quantum Hall-like effects
- Focus on the role of interaction
- Gapped phases at fractional fillings
- Interaction strongly affect chiral currents

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