edge modes and crystalline phases in atomic synthetic la

Davide Rossini
University of Pisa & INFN, Pisa (Italy)

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Outlook

- Ultracold atoms in optical lattices
  - ladders & synthetic dimensions
  - a microscopic model

- Periodic synthetic boundaries (cylinder)
  - magnetic crystals

- Open synthetic boundaries (stripe)
  - chiral edge currents
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Ultracold atoms in optical lattices

Hubbard model (SF–MI phase transition …)

Out-of-equilibrium dynamics (thermalization, localization …)


Strongly correlated states (spin liquids …)

S. Trotzky et al., Nat. Phys. 8, 325 (2012)


Hall physics
gauge fields
 synthetic dimensions
Hall physics with cold atoms

• Observation of chiral edges in *bosonic systems*


• Observation of chiral edges in *fermionic systems*

Ladders & synthetic dimension

A minimal setup in order to mimic the Hall physics in quasi-1D systems

A system of (interacting) particles, in presence of an extra-dimension, can synthetically engineer in a different way!

One direction needs not to be a physical dimension:

An extra-dimension can be synthetically engineered in a different way!
**Synthetic dimension**

**IDEA:**
Use a system with $D$ spatial dimensions. Encode the $(D+1)$th dimension in a different degree of freedom (e.g. the spin).

Large-$I$ systems: Rubidium, Ytterbium, …

1D optical lattice


A. Celi *et al.*, PRL **112**, 043001 (2014)
The model

*Tunneling in the real dimension* (tight binding):

\[ H_t = -t \sum_j \sum_m \left[ c_{j,m}^\dagger c_{j+1,m} + h.c. \right] \]
The model

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**Spin states** are coupled through **Raman transitions**:

\[ H_R = \Omega \sum_j \sum_m \left[ e^{-i2\pi\phi_j} c_{j,m}^\dagger c_{j,m+1} + h.c. \right] \]

- **tunneling in the synthetic dimension** (\( \Omega \))
- **synthetic magnetic field** (\( \Phi \))

A) Open boundary conditions in the synthetic dimension
   → plain ladder, edge currents
The model

*Tunneling in the real dimension* (tight binding):

\[ H_t = -t \sum_j \sum_m \left[ c^\dagger_{j,m} c_{j+1,m} + h.c. \right] \]

Spin states are coupled through *Raman transitions*:

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- *tunneling in the synthetic dimension* ($\Omega$)
- *synthetic magnetic field* ($\Phi$)

B) Periodic boundary conditions in the synthetic dimension
   \[ \rightarrow \text{torus, magnetic crystals} \]
The model: two-body interactions

Interactions in earth-alkali atoms (Yb, Sr, ...) are SU(N) invariant:

\[ H_{\text{int}} = U \sum_j \sum_m \sum_{m'<m} n_{j,m} n_{j,m'} \]

→ Repulsive \((U>0)\) among two atoms in the same spatial site;
→ **Highly anisotropic**: local in real space, infinite-range in synthetic space;
→ SU(N) invariant in spin space.

The global Hamiltonian is:

\[
H = H_t + H_R + H_{\text{int}}
\]

\[
= \sum_{j,m} \left\{ \left[ -t c_{j,m}^\dagger c_{j+1,m} + \Omega e^{-i2\pi\phi j} c_{j,m}^\dagger c_{j,m+1} + \text{h.c.} \right] + U \sum_{m'<m} n_{j,m} n_{j,m'} \right\}
\]
Connection with Rashba SOC

\[ H = H_t + H_R + H_{\text{int}} \]

\[ = \sum_{j m} \left\{ -t c_{j,m}^\dagger c_{j+1,m} + \Omega e^{-i2\pi\phi} c_{j,m}^\dagger c_{j,m+1} + \text{h.c.} \right\} + U \sum_{m'<m} n_{j,m} n_{j,m'} \]

Since the Hamiltonian is not translationally invariant, we perform the transformation:

\[ d_{j,m} = U c_{j,m} U^\dagger = e^{-i2\pi\phi m j} c_{j,m} \quad (\nu_{j,m} = d_{j,m}^\dagger d_{j,m}) \]

\[ \tilde{H} = \sum_{j m} \left\{ -t e^{i2\pi\phi m} d_{j,m}^\dagger d_{j+1,m} + \Omega d_{j,m}^\dagger d_{j,m+1} + \text{h.c.} \right\} + U \sum_{m'<m} \nu_{j,m} \nu_{j,m'} \]

\[ \Downarrow \]

\[ -2t \sum_{k m} \cos(k - 2\pi m \phi) d_{k,m}^\dagger d_{k,m} \]

The quadratic part is now readily diagonalizable in Fourier space…
A few words on the method: tensor networks
Tensor-network approach

Use the variational class of **matrix product states**, in order to address ground-state properties of (quasi) 1D lattice systems:

$$|\psi\rangle = \sum_{i_1, i_2, \ldots, i_L} A^{[1],i_1} A^{[2],i_2} \ldots A^{[L],i_L} |i_1\rangle_1 \otimes |i_2\rangle_2 \otimes \cdots \otimes |i_L\rangle_L$$

The **density-matrix renormalization group**

Area-law entanglement $\rightarrow$ **very efficient in 1D**

- For ladder systems, we group each physical site belonging to a given rung (and a dii
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Magnetic crystals

S. Barbarino et al., Nat. Commun. 6, 8134 (2015)
L. Taddia et al., PRL 118, 230402 (2017)
Magnetic crystals

Emergence of gapped phases with regular pattern in charge density & spin textures

They occur at rational values of the filling factor: \( \nu = p/q \)

\[
\nu = \frac{N}{N_\phi} = \frac{n}{\phi(2I + 1)}
\]

- **Integer filling**  
  \( \nu = 1, 2, \ldots \)
  - non-interacting model
  - \( 2k_F = 2\pi\phi = 2k_{SO} \)
  - system develops a full gap

- **Fractional filling**  
  \( \nu = 1/2, 1/3, 2/3, \ldots \)
  - interactions are mandatory
  - system can develop a gap for lower fillings via higher-order interactions

\( N = nL \)  \( N_\phi = \phi(2I + 1)L \)  \( \)  \( \)  \( \)  \( \)  \( \)  \( \)
Magnetic crystals

Crystalline order is stabilized for parl

Decreasing the filling, finite-range interactions are needed (formally proved by bosonization)

for $q > 1$ and odd
Magnetic crystals

The structure of crystals is better understood when the space periodicity matches integer values. This competition leads to the formation of dimers locked together.

\[
d_{j,\lambda} = \frac{1}{\sqrt{2I + 1}} \sum_{m} e^{(\frac{2\pi i}{2I + 1})\lambda m} c_{j,m}
\]

\[
\varepsilon_{j,\lambda} = 2\Omega \cos \left[ \frac{2\pi \lambda}{2I + 1} + 2\pi \phi j \right]
\]

- hopping → delocalization
- repulsion → localization

When the space periodicity matches integers...
Finite temperature & trapping effects

Energy gaps for $I = 1, \nu = 1/2$

repulsive interactions enhance the gap

$\Rightarrow k_B T \lesssim \varepsilon_{\text{gap}} \sim t/2$

Harmonic confinement:

$$H_{\text{trap}} = \sum_{j,m} \bar{w} \left( j - \frac{L}{2} \right)^2 c^\dagger_{j,m} c_{j,m}$$

Robustness $\rightarrow$ crystals form in a definite region of the trap

$\bar{w}/t = 10^{-3}$

$U/t = 10$
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Edge-like modes: experimental facts

exp: → M. Mancini et al., Science 349, 1510 (2015)

Momentum distribution: $n(k) = \sum m n_m(k)$

$$n_m(k) = \frac{1}{L} \sum_{j,l} e^{-ik(j-l)} \langle c_j^\dagger, m c_l, m \rangle$$

Asymmetry: $h(k) = n(k) - n(-k)$

Edge-cyclotron skipping orbits
The non-interacting case can be easily diagonalized in the Rashba basis. Single-particle spectrum for three legs ($l=1$):

$\Omega_0 = 0.14t$

$E/t$ vs $q/a$ for $\Omega_0 = 0.14t$

$E/t$ vs $q/a$ for $\Omega_0 = 0.5t$

$\langle m \rangle$

The ground-state branch displays spin-polarized edges

low-energy excitations are gapless and with defined momentum & spin

chiral edge modes on the synthetic lattice

A. Celi et al., PRL 112, 043001 (2014)
Different regimes

Weak Raman coupling $\Omega/t \lesssim 1$:
- low/high filling $\rightarrow$ 4 low-energy excit.
- intermediate filling $\rightarrow$ helical liquid

Strong Raman coupling $\Omega/t \gtrsim 1$:
- quasi-spinless gas (x-polarized)
- interactions almost ineffective

An interacting system is predicted to behave effectively as a free system with a renormalization.

B. Braunecker et al., PRB 82, 045127 (2010)

Role of interactions

Total momentum distribution in $k$-space

\[ \nu_p = \sum_m \nu_{pm} = \sum_m \langle d_{p,m}^\dagger d_{p,m} \rangle \]

Increasing $U$ results in effectively increasing $\Omega$

\[ \nu_{p,m} = n_{p-2\pi m \phi,m} \]
Role of interactions

Increasing $U$ results in effectively enhancing the asymmetry.
Chiral currents

Chiral current: \( \mathcal{J}_{j,m} = -it(c_{j,m}^\dagger c_{j+1,m} - \text{h.c.}) \)

\[ Q_m = \frac{1}{L} \sum_j \langle \mathcal{J}_{j,m} \rangle \]

\[ = -\frac{2t}{L} \sum_{p>0} \sin k_p (n_{p,m} - n_{-p,m}) \]

\[ \rightarrow \text{Similar trend for the current vs. } U \text{ and } \Omega \quad [U \text{ effectively enhances } \Omega] \]

\[ \rightarrow \text{Strong non-monotonic behavior of the current with } U \text{ and } \Omega \]

(a priori unexpected: classically the magnetic field determines the direction of current)
Conclusions

➢ Atomic simulator of quantum Hall-like effects

➢ Focus on the role of interaction

➢ Gapped phases at fractional fillings

➢ Interaction strongly affect chiral currents
Collaborators

Simone Barbarino (@ SISSA, Trieste)

Marcello Calvanese Strinati (@ SNS, Pisa & ENS, Paris)

Luca Taddia (formerly @ SNS, Pisa)

Eyal Cornfeld (@ Tel Aviv Univ.)

Leonardo Mazza (@ ENS, Paris)

Marcello Dalmonte (@ ICTP, Trieste)

Rosario Fazio