The generalized Lemaitre-Tolman-Bondi solutions with nonzero pressure in modeling the cosmological black holes.

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The plan of my talk is as follows:

1. Motivation
2. The method for obtaining the exact solutions in General Relativity
3. The solution for black hole inserted in dust
4. The solution describing the black hole in the universe filled with dust and radiation
5. The solution for the charged static black hole in dust
6. The generalized case for the charged black hole immersed in the universe filled with dust and radiation
7. Conclusions

Motivation

- the problem of the black hole horizon dynamics
- the problem of interplay between cosmological expansion and local gravity
- the problem of structure formation in the early universe and other

The method for obtaining the exact solutions in General Relativity

spherically symmetric interval

$$ds^2 = e^{\nu(R,t)}dt^2 - e^{\lambda(R,t)}dR^2 - r^2(R,t)d\sigma^2$$

definition of mass function

$$m(R,t) = r(R,t)(1 + e^{\nu(R,t)}\dot{r}^2 - e^{\lambda(R,t)}r'^2)$$

Einstein equation

$$\begin{aligned}
m' &= \epsilon r^2 r' \\
\dot{m} &= -p_{\|} r^2 \dot{r} \\
2\dot{r}' &= \nu' \dot{r} + \dot{\lambda} r' \\
2\dot{m}' &= m' \frac{\dot{r}}{r'}\nu' + \dot{m}\frac{r'}{\dot{r}}\dot{\lambda} - 4r\dot{r}r' p_{\perp}
\end{aligned}$$

first Einstein equation

$$m' = \epsilon r^2 r'$$

total mass

$$m(R,t) = \int_0^R \epsilon(R,t) r^2(R,t) \frac{\partial r}{\partial R} dR$$

and this equation shows the meaning of the mass function as total mass of the dust enclosed in the shell $R = const$ including gravitational energy

event horizon

$$m(R,t) = r(R,t)$$

Our calculations are from the view of the commoving observer and the metric describing the inhomogenous matter distribution on commoving coordinates

$$ds^2 = dt^2 - \frac{r'^2(R,t)}{f^2(R)}dR^2 - r^2(R,t)d\sigma^2$$

mass function for commovinng coordinate

$$m(R,t) = r(R,t)(1 + \dot{r}^2 - f^2(R))$$

mass function for flat case has form

$$m(R,t) = r(R,t)\dot{r}^2(R,t)$$

Solution for black hole inserted in dust

Schwarzschild metric

$$ds^2 = \left(1 - \frac{2MG}{r}\right)dt^2 - \left(1 - \frac{2MG}{r}\right)^{-1} dR^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi$$

mass function for Schwarzschild metric

$$m(R) = r_g$$

where r_g is Schwarzschild radius

- P. Jaluvkova, E. Kopteva, Z. Stuchlik, The model of the black hole enclosed in dust. The flat space case. Gen. relativity and cosmology(2016)

Friedman metric

$$ds^2 = dt^2 - a^2(t)(dR^2 + F^2(R)d\sigma^2)$$

mass function for dust

$$m(R) = a_0 R^3$$

mass function for LTB metric

$$m(R) = r_g + a_0 R^3$$

and solution is

$$r(R,t) = \left[\pm \frac{3}{2}\sqrt{r_g + a_0 R^3}(t - t_0(R)) \right]^{2/3}$$

The solution describing the black hole in the universe filled with dust and radiation

The mass function for universe with dust and radiation:

$$m(R,t) = a_{dust} R^3 + \frac{a_{rad}^2 R^4}{r(R,t)}$$

mass function for universe describing a black hole embedded on the background of dust and radiation

$$m(R,t) = r_g + a_{dust} R^3 + \frac{a_{rad}^2 R^4}{r(R,t)}$$

equation for calculate

$$\int dt = \int \frac{1}{\sqrt{\frac{r_g + a_{dust} R^3 + \frac{a_{rad}^2 R^4}{r(R,t)}}{r(R,t)}}} dr(R,t)$$

and finally solution

$$t(R) - t_0 = \; = \; \frac{2}{3} r^{3/2} \frac{\left(m_{SchwCH} - \frac{3 a_{rad}^2 R^4}{r(R,t)} \right) \sqrt{m_{SchwCH}}}{(a_{dust}^2 R^3 + r_g)^2}$$

The solution for the charged static black hole in dust

Reissner-Nordstram metric

$$ds^2 = \left(1 - \frac{2MG}{r} + \frac{q^2}{r}\right) dt^2 - \left(1 - \frac{2MG}{r} + \frac{q^2}{r}\right)^{-1} dR^2 - r^2 d\sigma^2$$

mass function for RN metric

$$m(R) = r_g - \frac{q^2}{r}$$

mass function for the charged black hole immersed in the universe filled with dust

$$m(R,t) = r_g + a_{dust} R^3 - \frac{q^2}{r}$$

equation for calculate

$$\int dt = \int \frac{1}{\frac{\sqrt{r_g + a_{dust} R^3 - \frac{q^2}{r}}}{r(R,t)}} dr(R,t)$$

and solution is

$$\frac{2r^{2/3}\sqrt{m(R,t)}\left(m(R,t) + 2\frac{q^2}{r}\right)}{3\left(a_{dust}R^3 + r_g\right)^2} \tag{1}$$

The generalized case for the charged black hole immersed in the universe filled with dust and radiation

mass function for charged static black hole

$$m(R) = r_g - \frac{q^2}{r}$$

mass function for the charged black hole immersed in the universe filled with dust and radiation

$$m(R,t) = r_g + a_{dust} R^3 + \frac{a_{rad}^2 R^4}{r(R,t)} - \frac{q^2}{r}$$

equation for calculate

$$\int dt = \int \frac{1}{\sqrt{\frac{r_g + a_{dust}R^3 + \frac{a_{rad}^2 R^4}{r(R,t)} - \frac{q^2}{r(R,t)}}{r(R,t)}}} dr(R,t)$$

finally solution

$$t(R) - t_0 = \frac{2r^{3/2}\sqrt{m(R,t)}\left(m(R,t) - \frac{3a_{rad}^2 R^4}{r} + \frac{3q^2}{r}\right)}{3\left(a_{dust}R^3 + r_g\right)^2}$$

Conclusions

In this work we have obtained the set of new exact solutions of the Einstein equations that generalize the known LTB solution for the particular case of nonzero pressure under zero spatial curvature. These solutions are pretending to describe the black hole immersed in nonstatic cosmological background and give a possibility to investigate the hot problems concerning the effects of the cosmological expansion in gravitationally bounded systems.

Exact solutions it is further used to calculate an event horizon, the energy density, determined on the R-T-region, to calculate the velocity of the particles and other properties of the particle of the dynamically inhomogeneous universe.

Thank you for your attention !