Non-Analytic Behaviors of (SU($\mathcal{N}_c$)) Fermi Liquid

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Acknowledgment

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- Chi-Ho Cheng (NCUE, Taiwan)

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Outline

Some history

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- Past effort focused on specific heat or spin susceptibility
- Uncertainties in interaction parameters prevent precise comparison

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    - New perspective to an old problem
  - Possible SU(N) enhancement for large N
    - $N = 6$ for ^{173}Yb and $N = 10$ for ^{87}Sr

Outline

Introduction

What is not analytic?

Why is it not analytic?

Crossover Between T and H

SU($\mathcal{N}_c$)

Conclusion

Free energy at finite T

$$\frac{\Omega}{V} \sim E_F k_F^3 \left[\omega_0 + \left(\frac{T}{E_F} \right)^2 + \left(\frac{T}{E_F} \right)^4 + \ldots \right]$$

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$$\frac{\Omega}{V} \sim E_F k_F^3 \left[\omega_0 + \left(\frac{T}{E_F} \right)^2 + \left(\frac{T}{E_F} \right)^4 \log \left(\frac{T}{E_F} \right) \right]$$

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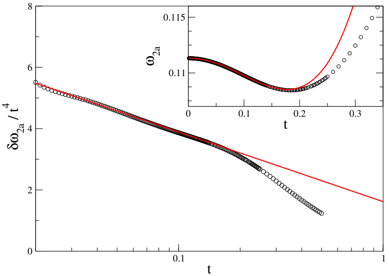
- Fourth order term is already beyond FL
  - needs properties of systems *away* from FS
- But FL with interaction universally yields a logarithmic term

Free energy at finite T (II)



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PRA **95**, 033619 (2017)

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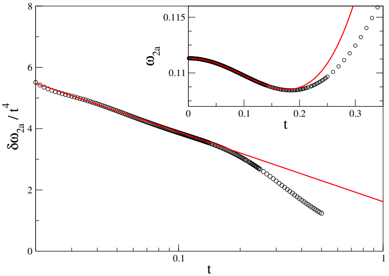

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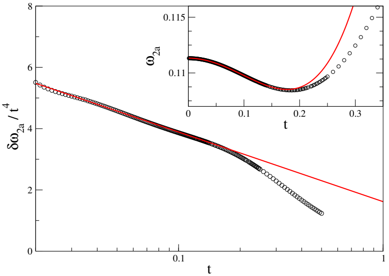

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- When $H \neq 0$: additional crossover!

When both T and H are non-zero

$$\frac{\Omega}{V} \sim E_F k_F^3 \left[\cdots - t^4 \log t + h^4 \log |h| + F_4(t,h) + \ldots\right]$$
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- $H \rightarrow$ Zeeman energy of spins
  - Shift in spin-dependent chemical potential
  - Tuned by density difference in quantum gas

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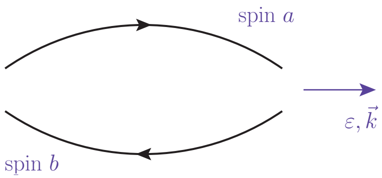
- These limits correspond to specific heat/susceptibility
- What if $t/h \sim 1$?

Outline

Particle-hole pair



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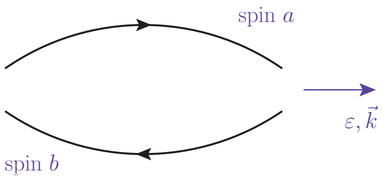

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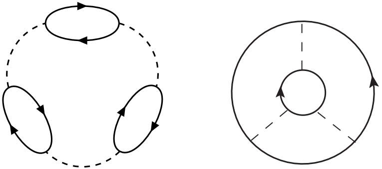
  - Peruturbation series has IR problem
  - Relevant IR scale acts as cutoff
- T smears out sharp Fermi surface: cutoff candidate
- If $a \neq b$: gapped by $H \neq 0$. Another scale!

Ring and ladder diagrams



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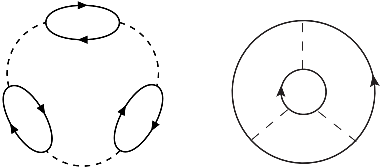

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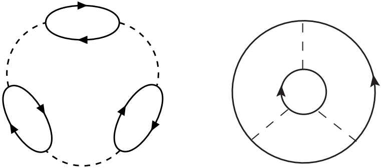

- Strings of particle-hole pairs. Left: ring; right: ladder
- Not full-fledged IR divergence; non-analytic at zero energy
- Ladder diagram is sensitive to both T and H
  - IR cutoff by $\max(T, H)$
  - Crossover

Outline

Our model

- Non-relativistic Fermi gas
- Contact interaction characterized by scattering length a
- Dilute limit: 2nd order perturbation theory

Thermodynamic potential

$$\frac{\Omega}{V} = \frac{k_F^5}{12\pi m}\Bigg\{ \dots + (k_F a)^2 \underbrace{\left[-\frac{\pi^2}{20} t^4 \log t + \frac{1}{32\pi^2} h^4 \log |h| + F_4(t,h) \right]}_{\text{ladder type non-analyticity}} + \dots \Bigg\}$$

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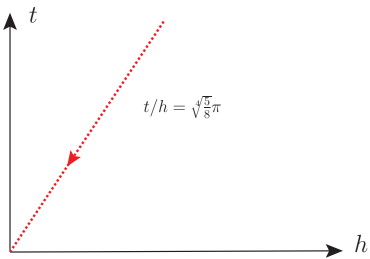
Closed form for the crossover function F_4:

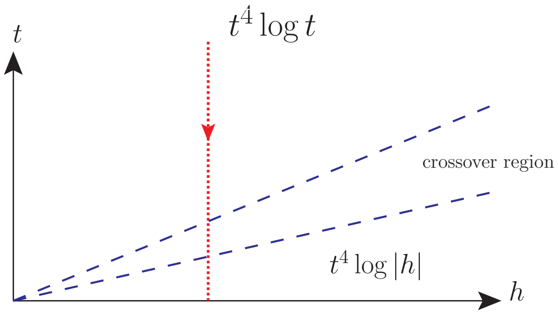
$$F_4(t,h) = \frac{3}{8\pi^2} \int_0^\infty \mathrm{d}x \, \frac{x^2}{e^{x/t}-1} (x+h) \log\left|\frac{x}{x+h}\right| + (h \to -h)$$

The analytic line



- $t^4 \log t$ and $h^4 \log h$ of opposite sign: "competing"
- Along the line $\frac{t}{h} = \sqrt[4]{\frac{5}{8}}\pi$, the thermodynamic potential is a completely *analytic* function

Approaching $T = 0$ at constant H



- Perhaps most relevant experimentally
- Competition of T and H as IR scale

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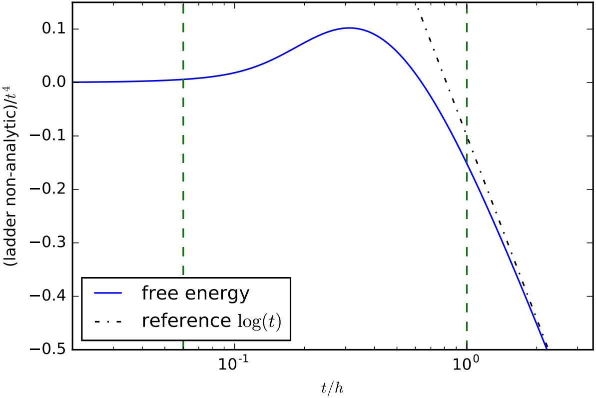

- straight line: $t^4 \log t$

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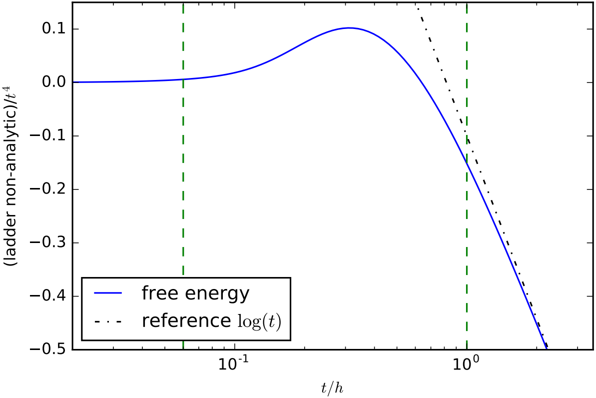
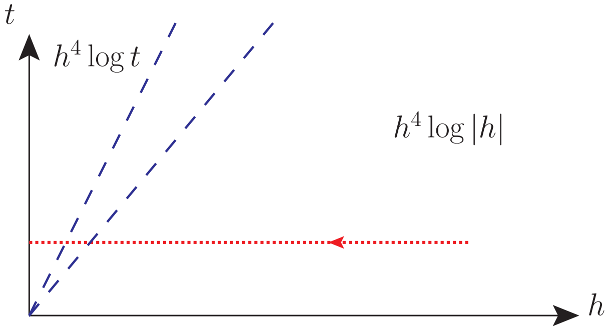
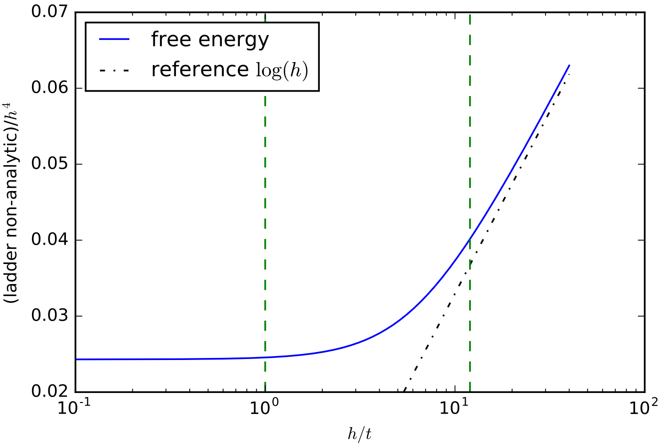


- straight line: $t^4 \log t$
- t gets smaller:
- crosses over to $t^4 \log h$

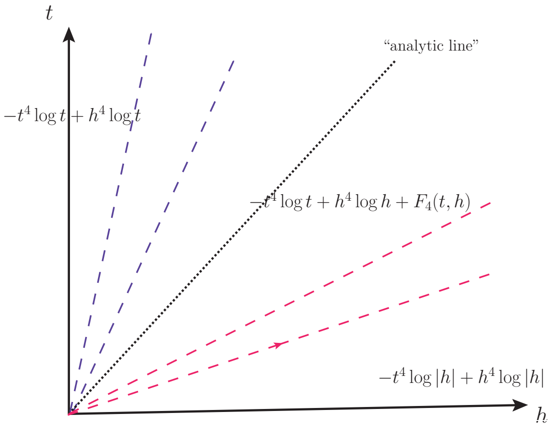
Approaching $H = 0$ at constant T



Approaching $H = 0$ at constant T II



The overall "phase diagram"



- Near axes: $\log t$ or $\log h$ dominates all
- middle: need F_4 in full
- two sets of crossover
- analytic line where the two non-analytic terms cancel

Outline

Simple generalization of SU(2) and Large-$\mathcal{N}_c$ enhancement

- Consider an even $\mathcal{N}_c$
- Spin $1, \ldots, \mathcal{N}_c/2$ has chemical potential μ_a
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  - $H \sim \mathrm{diag}(-(N-1), 1, 1, \dots)$ v.s.
    $H \sim \mathrm{diag}((N-1), -1, -1, \dots)$

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- Interesting crossover behaviors reflected in equation of state
  - accessible with cold quantum gas experiment
- SU($\mathcal{N}_c$) pseudo-spin may enhance the interaction effect
- But also adds new difficulty because $H \leftrightarrow -H$ is not a symmetry in general