EFT for Black Hole perturbations: testing extensions to GR with gravitational waves

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In progress
The CC problem

$$\langle T_{\mu\nu} \rangle = -\rho_{\text{vacuum}} g_{\mu\nu}$$

$$\rho_{\text{vacuum}} = \Lambda + \rho_{\text{vacuum}}^{SM}$$

$$\approx (10^{-3}\text{eV})^4$$
The CC problem

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\[ \rho_{\text{vacuum}} = \Lambda + \rho_{\text{SM}}^{\text{vacuum}} \approx (10^{-3} \text{eV})^4 \]

\[ \int d^4x \sqrt{-g} \left( M_{\text{Pl}}^2 R - \Lambda \right) \]
The CC problem

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\[ \int d^4x \sqrt{-g} \left( M_{Pl}^2 R - \Lambda \right) \]

\[ \rho_{\text{vacuum}} = \mathcal{O}(M_*) + \mathcal{O}(M_* m_e^2) + \mathcal{O}(m_e^4 \log M_*/m_e) \]
What is the theory of gravity?

GR is the **only** consistent Lorentz invariant theory of a massless spin 2 field at low energies.

If we want to modify the phenomenology at large distances, there must be extra *light degrees of freedom*.
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Test deviations from General Relativity:

With no single 'best-motivated' proposal at hand, useful to resort to a maximally model-independent EFT approach.

\[ \mathcal{L} = \sum_n c_n \frac{\Lambda^4}{g_*^2} \hat{\mathcal{L}} \left( \frac{\partial}{\Lambda}, \frac{g_* \phi}{\Lambda} \right) \]
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Assume the light degrees of freedom: graviton + a scalar field 3 DOF.

\[ \mathcal{L} = \mathcal{L}(g_{\mu\nu}, \phi) \]
EFT in Cosmology

“Cosmology is nothing but gravity at play with condensed matter”

On cosmological scales, FRW universes are characterized by a “medium” that gives a homogeneous and isotropic stress energy tensor.

This medium, at variance with a CC, breaks spontaneously Lorentz invariance.

The simplest example: in single field Inflation a scalar with a time-dependent expectation value breaks time translations and Lorentz boosts to $ISO(3)$.
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“covariant” EFT
(trivial background)

EFT for perturbations around the interesting solution
In the Standard Model

\[ L = m^2 h^2 + \lambda h^3 + \lambda_4 h^4 + \ldots \]

EFT before expanding

Full $SU(2) \times U(1)$ gauge invariance

\[ H(x) = \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \]

EFT for perturbations around

Only $U(1)_{EM}$ gauge invariance
In the Standard Model

\[ H(x) = \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \]

\[ \mathcal{L} = -\mu^2(H^\dagger H) + \lambda(H^\dagger H)^2 + c_6(H^\dagger H)^3 + \ldots \]

EFT before expanding

Full $SU(2) \times U(1)$ gauge invariance

\[ \mathcal{L} = m_h^2 h^2 + \lambda_3 h^3 + \lambda_4 h^4 + \ldots \]

EFT for perturbations around

Only $U(1)_{EM}$ gauge invariance

Resum the contribution of many operators if non-linearities are large
Quasi De sitter

\[ \sqrt{-g} \left( M_{Pl}^2 R - g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi - V(\phi) \right) \]

Solve the equation of motion to compute the background

\[ \phi(x) = \phi_0(t) \]

\[ ds^2 = -dt^2 + a(t)^2 dx^2 \quad H \equiv \frac{\dot{a}}{a} \]

Expand in small perturbations

\[ \mathcal{L} = (\partial \phi)^2 + (\partial h)^2 + \mathcal{O}(\phi^3, h^3, \ldots) \]
Quasi De sitter

\[ \sqrt{-g} \left( M_{Pl}^2 R - g_{\mu \nu} \partial^\mu \phi \partial^\nu \phi - V(\phi) + \sum_n a_n \frac{(\partial_\mu \phi \partial^\mu \phi)^n}{\Lambda^{4n-4}} \right) \]

Solve the equation of motion to compute the background

\[ \phi(x) = \phi_0(t) \]
\[ ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \quad H \equiv \dot{a} \quad a \]

Expand in small perturbations

\[ \mathcal{L} = (\partial \phi)^2 + (\partial h)^2 + \mathcal{O}(\phi^3, h^3, \ldots) \]
Quasi De sitter

\[ \sqrt{-g} \left( M_{P1}^2 R - g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi - V(\phi) + \sum_n a_n \frac{(\partial_\mu \phi \partial^\mu \phi)^n}{\Lambda^{4n-4}} + \sum_n b_n \frac{(\partial_\mu \phi \partial^\mu \phi)^n \Box \phi}{\Lambda^{4n-1}} + \ldots \right) \]

Solve the equation of motion to compute the background

\[ \phi(x) = \phi_0(t) \]

\[ ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \quad H \equiv \frac{\dot{a}}{a} \]

Expand in small perturbations

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Quasi De sitter

\[ \sqrt{-g} \left( M_{P1}^2 R - g_{\mu \nu} \partial^\mu \phi \partial^\nu \phi - V(\phi) + \sum_n a_n \frac{(\partial_\mu \phi \partial^\mu \phi)^n}{\Lambda^{4n-4}} + \sum_n b_n \frac{(\partial_\mu \phi \partial^\mu \phi)^n \Box \phi}{\Lambda^{4n-1}} + \ldots \right) \]

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Expand in small perturbations

\[ \mathcal{L} = (\partial \phi)^2 + (\partial h)^2 + \mathcal{O}(\varphi^3, h^3, \ldots) \]

\[ \partial \phi_0 \sim \Lambda^2 \quad \frac{\partial}{\Lambda} \ll 1 \]
Quasi De sitter

\[ \sqrt{-g}\left( M_{Pl}^2 R - g_{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) + \sum_n a_n \frac{(\partial_\mu \phi \partial_\mu \phi)^n}{\Lambda^{4n-4}} + \sum_n b_n \frac{(\partial_\mu \phi \partial_\mu \phi)^n \Box \phi}{\Lambda^{4n-1}} + \ldots \right) \]

Solve the equation of motion to compute the background

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Expand in small perturbations

\[ \mathcal{L} = (\partial \phi)^2 + (\partial h)^2 + \mathcal{O}(\phi^3, h^3, \ldots) \]

\[ \partial \phi_0 \sim \Lambda^2 \quad \frac{\partial}{\Lambda} \ll 1 \]

Instead...
Quasi De sitter

Choose a background solution \( ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \)

Construct an EFT for perturbations
Quasi De sitter

Choose a background solution \( ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \)

Construct an EFT for perturbations

Choose a foliation of spacetime (unitary gauge) such that \( \phi = \phi_0(t) \)

Use 3+1 ADM decomposition for the metric variables \( h_{ij}, N, N_i \)

Write down in a derivative expansions all the operators that are invariant under the residual symmetries (spatial diffs)

Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore ’07
Quasi De sitter

Choose a background solution \( ds^2 = -dt^2 + a(t)^2 d\bar{x}^2 \)

Construct an EFT for perturbations

The \textit{quadratic} action is already interesting

Lorentz is spontaneously broken: no a priori reason to expect luminal speed

\[
\int d^4x a^3 M_{P1}^2 \left[ (\dot{h}_{ij})^2 - c_T^2 (\partial_k h_{ij})^2 \right]
\]

\[
\int d^4x a^3 A^2 \left[ \dot{\zeta}^2 - c_S^2 (\partial_i \zeta)^2 \right]
\]
Quasi De sitter

Choose a background solution

\[ ds^2 = -dt^2 + a(t)^2 d\bar{x}^2 \]

Construct an EFT for perturbations

The **quadratic** action is already interesting

Lorentz is spontaneously broken: no a priori reason to expect luminal speed

\[
\int d^4x a^3 M_{Pl}^2 \left[ (h_{ij})^2 - c_T^2 (\partial_k h_{ij})^2 \right]
\]

\[
\int d^4x a^3 A^2 \left[ \zeta^2 - c_S^2 (\partial_i \zeta)^2 \right]
\]

\[ A^2 = \frac{-\dot{H} M_{Pl}^2}{H^2} \]

\[ <\zeta \zeta> \sim \frac{H^2}{A^2} \sim \frac{H^2}{\epsilon M_{Pl}^2} \]
Late time cosmology

Choose a background solution \( ds^2 = -dt^2 + a(t)^2 d\bar{x}^2 \)

Construct an EFT for perturbations

The quadratic action is already interesting

Lorentz is spontaneously broken: no a priori reason to expect luminal speed

\[
\int d^4x a^3 M_{P1}^2 \left[ (\dot{h}_{ij})^2 - c_T^2 (\partial_k h_{ij})^2 \right]
\]
Late time cosmology

Choose a background solution \( ds^2 = -dt^2 + a(t)^2 d\bar{x}^2 \)

Construct an EFT for perturbations

The *quadratic* action is already interesting

Lorentz is spontaneously broken: no a priori reason to expect luminal speed

\[
\int d^4 x \alpha^3 M_{Pl}^2 \left[ (\dot{h}_{ij})^2 - c_T^2 (\partial_k h_{ij})^2 \right]
\]

\( c_T^2 - 1 \lesssim 10^{-15} \)
Late time cosmology

Choose a background solution \[ ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \]

Construct an EFT for perturbations

\[ c_T^2 - 1 = -\frac{2m_4^2}{M^2} \]

\[ S = \int d^4x \sqrt{-g} \left[ \frac{M^2}{2} f^{(4)R} - \Lambda - c g^{00} + \frac{m_4^2}{2} (\delta g^{00})^2 \right. \]
\[ \left. - \frac{m_3^3}{2} \delta K \delta g^{00} - \frac{m_4^2}{2} \delta \mathcal{K}_2 + \frac{\tilde{m}_4^2}{2} \delta g^{00} R \right] \]

\[ \delta \mathcal{K}_2 \equiv \delta K^2 - \delta K_{\mu}^{\nu} \delta K_{\nu}^{\mu} \]

\[ m_4^2 \sim 0 \]

Creminelli, Vernizzi '17 + many others
Late time cosmology

Choose a background solution \[ ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \]

Construct an EFT for perturbations \[ c_T^2 - 1 = -2m_4^2/M^2 \]

\[
S = \int d^4x \sqrt{-g} \left[ \frac{M_*^2}{2} f^{(4)R} - \Lambda - c g^{00} + \frac{m_3^4}{2} (\delta g^{00})^2 - \frac{m_3^3}{2} \delta K \delta g^{00} - m_4^2 \delta \mathcal{K}_2 + \frac{\tilde{m}_4^2}{2} \delta g^{00} R \right]
\]

\[ \delta \mathcal{K}_2 \equiv \delta K^2 - \delta K^\mu \delta K^\mu \]

\[ m_4^2 \sim 0 \]

Depends on the background (dark matter abundance, ...): robustly set it to zero!

Creminelli, Vernizzi '17
+ many others
Late time cosmology

Choose a background solution  

\[ ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \]

Construct an EFT for perturbations

\[ c_T^2 - 1 = -2m_4^2/M^2 \]

\[
S = \int d^4x\sqrt{-g}\left[ \frac{M^2}{2} f^{(4)}R - \Lambda - c_{g^{00}} + \frac{m^4_2}{2}(\delta g^{00})^2 \\
- \frac{m_3^2}{2} \delta K \delta g^{00} - \tilde{m}_4^2 \delta \mathcal{H}_2 + \tilde{m}_4^2 \delta g^{00}R - \frac{m_5^2}{2} \delta g^{00} \delta \mathcal{H}_2 \\
- \frac{m_6^2}{3} \delta \mathcal{H}_3 - \tilde{m}_6 \delta g^{00} \delta \mathcal{H}_2 - \frac{m_7^2}{3} \delta g^{00} \delta \mathcal{H}_3 \right]
\]

\[
\delta \mathcal{H}_2 \equiv \delta K^2 - \delta K^\mu \delta K_\mu \\
\delta \mathcal{H}_2 \equiv \delta K^\nu R_\nu - \delta K R/2 \\
\delta \mathcal{H}_3 \equiv \delta K^3 - 3\delta K \delta K^\mu \delta K_\mu + 2\delta K^\nu \delta K_\mu \delta K_\nu
\]

\[
m_4^2 \sim 0 \\
\tilde{m}_4^2 = m_5^2 \\
m_6 = \tilde{m}_6 = m_7 = 0
\]

Depends on the background (dark matter abundance,…): robustly set it to zero!

Creminelli, Vernizzi ‘17
+ many others
Quadratic action of gravity in FRW
Quadratic action of gravity in FRW

Can we study it in some other background?
Perturbations around Black Holes

Inspiral

Merger

Ringdown

Ligo & Virgo ‘17
Perturbations around Black Holes

\[ g_{\mu\nu} = g_{\mu\nu}^{\text{BH}}(r) + h_{\mu\nu} \quad \text{Schwarzschild: static, spherically symmetric background} \]

\[ h(t, r, \theta, \phi) = \sum_{lm} h_{lm}(r)Y_{lm}(\theta, \phi)e^{i\omega t} \]

Classified accordingly to the behavior under parity \((\theta, \phi) \rightarrow (\pi - \theta, \phi + \pi)\)
Perturbations around Black Holes

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Axial (odd) perturbations \quad \text{Regge Wheeler '57}

Polar (even) perturbations \quad \text{Zerilli '70}

\[
\left[ \frac{d^2}{dr^2} + \omega^2 \right] h(r) = V^{(-)}(r) h(r)
\]

\[ V^{(-)}(r) = \frac{l(l+1)}{r^2} \left( 1 - \frac{r_s}{r} \right) - 3 \frac{r_s}{r^3} \left( 1 - \frac{r_s}{r} \right) \]
Perturbations around Black Holes

Exponentially damped sinusoidal waves: Quasi Normal Modes (QNM)

Spectrum of characteristic (complex) frequencies $\omega_{nlm}$
Perturbations around Black Holes

Exponentially damped sinusoidal waves: Quasi Normal Modes (QNM)

Spectrum of characteristic (complex) frequencies $\omega_{nlm}$

In GR black holes are characterized only by 3 parameters: M, J, Q

No-hair hypothesis
Perturbations around Black Holes

Exponentially damped sinusoidal waves: Quasi Normal Modes (QNM)

Spectrum of characteristic (complex) frequencies $\omega_{nlm}$

In GR black holes are characterized only by 3 parameters: $M$, $J$, $Q$

No-hair hypothesis

<table>
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<tr>
<th>$n$</th>
<th>$2M_*\omega (L = 2)$</th>
<th>$2M_*\omega (L = 3)$</th>
<th>$2M_*\omega (L = 4)$</th>
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<td>0</td>
<td>0.747 343 + 0.177925i</td>
<td>1.198 887 + 0.185406i</td>
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<tr>
<td>3</td>
<td>0.503 010 + 1.410296i</td>
<td>1.023 924 + 1.380674i</td>
<td>1.479 68 + 1.36784i</td>
</tr>
</tbody>
</table>

Nollert '99
EFT for perturbations around BH

\[
\left[ \frac{d^2}{dr^2} + \omega^2 \right] h(r) = V^{(-)}(r) h(r) \quad \text{QNM spectrum } \omega_{n \ell m}
\]

The linearized equations of motion are modified in extended theories of gravity
EFT for perturbations around BH

\[ \left[ \frac{d^2}{dr^2} + \omega^2 \right] h(r) = V^{(-)}(r)h(r) \quad \text{QNM spectrum} \quad \omega_{nlm} \]

The linearized equations of motion are modified in extended theories of gravity

Construct the EFT around static and spherically symmetric backgrounds up to quadratic order in perturbations

\[
S_{EFT}^2 = \int d^4x \sqrt{-G} \left[ \frac{M_{Pl}^2}{2} f(r)R - \Lambda(r) - c(r)G^{rr} - \alpha(r)K_{\mu\nu}K^{\mu\nu} + \right. \\
+ G(r)(\delta G^{rr})^2 + B(r)\delta G^{rr} \delta K + B_{\mu\nu} \delta G^{rr} \delta K^{\mu\nu} + \\
+ G^{(1)}(r)(\partial_r \delta G^{rr})^2 + B^{(1)}(r)(\partial_r \delta G^{rr}) \delta K + B^{(1)}_{\mu\nu}(\partial_r \delta G^{rr}) \delta K^{\mu\nu} + \\
+ F_0(r) \delta K \delta K + F_1(r) \delta K_{\mu\nu} \delta K^{\mu\nu} + F_{2\mu\nu} \delta K^{\mu\nu} \delta K + F_{3\mu\nu} \delta K^{\mu\nu} \delta K^\rho + \\
\left. + C(r)\delta G^{rr} \delta^{(3)} R + C_{\mu\nu} \delta G^{rr} \delta^{(3)} R^{\mu\nu} + ... \right]
\]

to appear...
Conclusions

Gravitational waves observation allows to study the quadratic action of gravity around non-trivial backgrounds (FRW, BH)

Dramatic improvement in our understanding of dark energy/modified gravity
The CC problem

\[ \langle T_{\mu\nu} \rangle = -\rho_{\text{vacuum}} g_{\mu\nu} \]

\[ \rho_{\text{vacuum}} = \Lambda + \rho_{\text{vacuum}}^{SM} \approx (10^{-3}\text{eV})^4 \]

\[ \int d^4x \sqrt{-g} \left( M_{\text{Pl}}^2 R - \Lambda \right) \]

\[ \rho_{\text{vacuum}} = \mathcal{O}(M^4) + \mathcal{O}(M^2 m_e^2) + \mathcal{O}(m_e^4 \log M_*/m_e) \]
The CC problem

\[ \langle T_{\mu\nu} \rangle = -\rho_{\text{vacuum}} g_{\mu\nu} \]

\[ \rho_{\text{vacuum}} = \Lambda + \rho_{\text{vacuum}}^{SM} + \rho_{\text{vacuum}}^{NP} \simeq (10^{-3}\text{eV})^4 \]

\[ \int d^4 x \sqrt{-g} \left( M_{Pl}^2 R - \Lambda \right) \]

\[ \rho_{\text{vacuum}} = \mathcal{O}(M_*^4) + \mathcal{O}(M_*^2 m_e^2) + \mathcal{O}(m_e^4 \log M_*/m_e) \]
Relaxing the CC

\[ H(t) \]

- Relaxation
- Phase transition
- NEC violation
- Inflation and reheating

Null Energy Condition
\[ T_{\mu\nu}n^\mu n^\nu \geq 0 \]

\[ \rho + p \geq 0 \]

\[ \dot{H} = -4\pi G(\rho + p) \]
\[ \dot{\rho} = -3H(\rho + p) \]

- \( \phi_1 \) dominates
- Phase transition: \( \phi_2 \simeq M_2 \)
- \( \phi_2 \) violates the NEC
- Energy density stored in \( \chi \)

Alberte, Creminelli, Khmelnitsky, Pirtskhalava, ET ‘17
Constraints on the Covariant Theory

\[ \mathcal{L} = \mathcal{L}(g_{\mu\nu}, \phi) \]

\[
L_2 \equiv G_2(\phi, X), \quad L_3 \equiv G_3(\phi, X) \Box \phi, \\
L_4 \equiv G_4(\phi, X) (^{(4)}R - 2G_{4,X}(\phi, X)(\Box^2 - \phi^{\mu\nu} \phi_{\mu\nu}) + F_4(\phi, X) e^{\mu\nu\rho\sigma} e^{\mu'\nu'\rho'\sigma'} \phi_{\mu\phi_{\mu'}} \phi_{\nu\phi_{\nu'}} \phi_{\rho\phi_{\rho'}} , \\
L_5 \equiv G_5(\phi, X) (^{(4)}G_{\mu\nu} \phi^{\mu\nu} \\
+ \frac{1}{3} G_{5,X}(\phi, X)(\Box^3 - 3 \Box \phi \phi_{\mu\nu} \phi^{\mu\nu} + 2 \phi_{\mu\nu} \phi^{\mu\sigma} \phi^{\nu}_{\sigma}) \\
+ F_5(\phi, X) e^{\mu\nu\rho\sigma} e^{\mu'\nu'\rho'\sigma'} \phi_{\mu\phi_{\mu'}} \phi_{\nu\phi_{\nu'}} \phi_{\rho\phi_{\rho'}} \phi_{\sigma\phi_{\sigma'}} ,
\]

\[
G_{5,X} = 0, \quad F_5 = 0, \quad 2G_{4,X} - XF_4 + G_{5,\phi} = 0
\]

\[
X \equiv g_{\mu\nu} \nabla^\mu \phi \nabla^\nu \phi \\
\phi_{\mu} \equiv \nabla_{\mu} \phi \\
\phi_{\mu\nu} \equiv \nabla_{\mu} \nabla_{\nu} \phi \\
G_i(\phi, X) = \sum_{nm} c_{nm} \phi^n X^m
\]
Constraints on the Covariant Theory

\[ \mathcal{L} = \mathcal{L}(g_{\mu\nu}, \phi) \]

\[ L_2 \equiv G_2(\phi, X) , \quad L_3 \equiv G_3(\phi, X) \square \phi , \]
\[ L_4 \equiv G_4(\phi, X) (4)^R - 2G_{4,X}(\phi, X)(\square^2 \phi - \phi^{\mu\nu} \phi_{\mu\nu}) \]
\[ + F_4(\phi, X) \varepsilon^{\mu\nu\rho\sigma} \varepsilon^{\mu'\nu'\rho'\sigma'} \phi_{\mu\mu'} \phi_{\nu\nu'} \phi_{\rho\rho'} , \]
\[ L_5 \equiv G_5(\phi, X) (4)G_{\mu\nu} \phi^{\mu\nu} \]
\[ + \frac{1}{3} G_{5,X}(\phi, X)(\square^3 \phi - 3 \square \phi \phi^{\mu\nu} \phi_{\mu\nu} + 2 \phi^{\mu\nu} \phi^{\mu\sigma} \phi_{\sigma} + \phi_{\mu\nu} \phi_{\nu} \phi_{\mu} \phi_{\nu} \phi_{\rho}\phi_{\rho} \phi_{\sigma} \phi_{\sigma'}) \]

\[ G_{5,X} = 0 , \quad F_5 = 0 , \quad 2G_{4,X} - XF_4 + G_{5,\phi} = 0 \]

\[ X \equiv g_{\mu\nu} \nabla^\mu \phi \nabla^\nu \phi \]
\[ \phi_\mu \equiv \nabla_\mu \phi \]
\[ \phi_{\mu\nu} \equiv \nabla_\mu \nabla_\nu \phi \]
\[ G_i(\phi, X) = \sum_{nm} c_{nm} \phi^n X^m \]

\[ L_{CT} = G_2(\phi, X) + G_3(\phi, X) \square \phi + B_4(\phi, X) (4)^R \]
\[ - \frac{4}{X} B_{4,X}(\phi, X)(\phi^{\mu} \phi^{\nu} \phi_{\mu\nu} \square \phi - \phi^{\mu} \phi_{\mu\nu} \phi_{\lambda} \phi^{\lambda\nu}) \]

\[ B_4 \equiv G_4 + XG_{5,\phi}/2. \]
Higher number of events + higher sensitivity

Possibility to measure multiple frequencies

Signal to noise ratio (SNR) to detect multiple modes

*Non spinning binary BH merger with masses ratio q (fundamental mode l=m=2)

\[
(SNR)^2 = 4 \int_0^\infty \frac{\tilde{h}^*(f) \tilde{h}(f)}{S_h(f)} df
\]