EFT for Black Hole perturbations: testing extensions to GR with gravitational waves

Enrico Trincherini
(Scuola Normale Superiore & INFN)

with G. Franciolini, L. Hui, R. Penco & L. Santoni
In progress

The CC problem

$$\langle T_{\mu\nu} \rangle = -\rho_{\rm vacuum} g_{\mu\nu}$$

$$\rho_{\rm vacuum} = \Lambda + \rho^{SM}_{\rm vacuum} \qquad \simeq (10^{-3}{\rm eV})^4$$

The CC problem

$$\langle T_{\mu\nu} \rangle = -\rho_{\rm vacuum} g_{\mu\nu}$$

$$\rho_{\rm vacuum} = \Lambda + \rho^{SM}_{\rm vacuum} \qquad \simeq (10^{-3}{\rm eV})^4$$

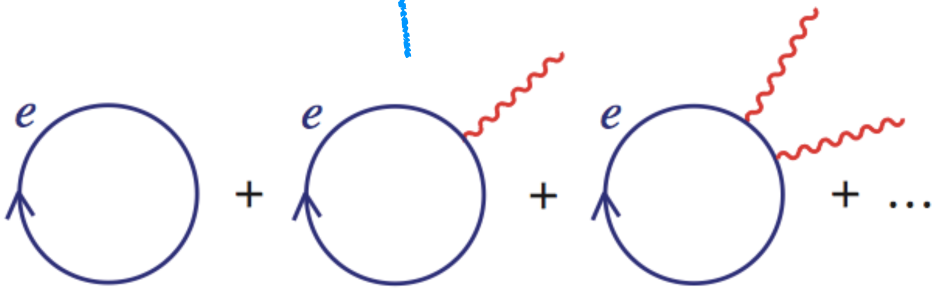
$$\int d^4x \sqrt{-g} \Big(M^2_{\rm Pl} R - \Lambda \Big)$$

The CC problem

$$\langle T_{\mu\nu} \rangle = -\rho_{\rm vacuum} g_{\mu\nu}$$

$$\rho_{\rm vacuum} = \Lambda + \rho^{SM}_{\rm vacuum} \qquad \simeq (10^{-3}{\rm eV})^4$$

$$\int d^4x \sqrt{-g}\Big(M^2_{\rm Pl} R - \Lambda\Big)$$



$$\rho_{\rm vacuum} = \mathcal{O}(M_*^4) + \mathcal{O}(M_*^2 m_{\rm e}^2) + \mathcal{O}(m_{\rm e}^4 \log M_*/m_{\rm e})$$

What is the theory of gravity?

GR is the only consistent Lorentz invariant theory of a massless spin 2 field at low energies

If we want to modify the phenomenology at large distances

there must be extra light degrees of freedom

What is the theory of gravity?

GR is the only consistent Lorentz invariant theory of a massless spin 2 field at low energies

If we want to modify the phenomenology at large distances

there must be extra light degrees of freedom

Today's perspective: put aside our prejudices in favor of empirical verification

What is the theory of gravity?

GR is the only consistent Lorentz invariant theory of a massless spin 2 field at low energies

If we want to modify the phenomenology at large distances
there must be extra light degrees of freedom

Today's perspective: put aside our prejudices in favor of empirical verification

Test deviations from General Relativity

With no single 'best- motivated' proposal at hand, useful to resort to a maximally model-independent EFT approach

$$\mathcal{L} = \sum_n c_n \frac{\Lambda^4}{g_*^2} \hat{\mathcal{L}}\Big(\frac{\partial}{\Lambda}, \frac{g_* \phi}{\Lambda}\Big)$$

What is the theory of gravity?

GR is the only consistent Lorentz invariant theory of a massless spin 2 field at low energies

If we want to modify the phenomenology at large distances

there must be extra light degrees of freedom

Today's perspective: put aside our prejudices in favor of empirical verification

Test deviations from General Relativity

With no single 'best- motivated' proposal at hand, useful to resort to a maximally model-independent EFT approach

$$\mathcal{L} = \sum_n c_n \frac{\Lambda^4}{g_*^2} \hat{\mathcal{L}}\Big(\frac{\partial}{\Lambda}, \frac{g_* \phi}{\Lambda}\Big)$$

Assume the light degrees of freedom: graviton + a scalar field 3 DOF

$$\mathcal{L} = \mathcal{L}(g_{\mu\nu}, \phi)$$

EFT in Cosmology

"Cosmology is nothing but gravity at play with condensed matter"

On cosmological scales, FRW universes are characterized by a "medium" that gives a homogeneous and isotropic stress energy tensor.

This medium, at variance with a CC, breaks spontaneously Lorentz invariance

The simplest example: in single field Inflation a scalar with a time-dependent expectation value breaks time translations and Lorentz boosts to *ISO(3)*

EFT in Cosmology

"Cosmology is nothing but gravity at play with condensed matter"

On cosmological scales, FRW universes are characterized by a "medium" that gives a homogeneous and isotropic stress energy tensor.

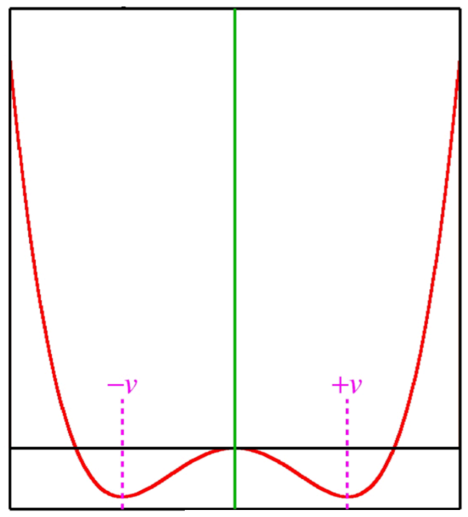
This medium, at variance with a CC, breaks spontaneously Lorentz invariance

The simplest example: in single field Inflation a scalar with a time-dependent expectation value breaks time translations and Lorentz boosts to *ISO(3)*

"covariant" EFT
(trivial background)

EFT for perturbations around
the interesting solution

In the Standard Model



$$H(x) = \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

$$\mathcal{L} = -\mu^2(H^\dagger H) + \lambda(H^\dagger H)^2 + c_6(H^\dagger H)^3 + \ldots$$

EFT before expanding

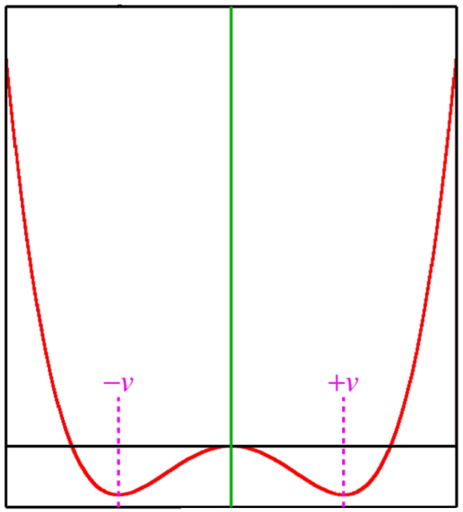
Full $SU(2) \times U(1)$ gauge invariance

$$\mathcal{L} = m_h^2 h^2 + \lambda_3 h^3 + \lambda_4 h^4 + \ldots$$

EFT for perturbations around

Only $U(1)_{\mathrm{EM}}$ gauge invariance

In the Standard Model



$$H(x) = \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

$$\mathcal{L} = -\mu^2 (H^\dagger H) + \lambda (H^\dagger H)^2 + c_6 (H^\dagger H)^3 + \ldots$$

EFT before expanding

Full $SU(2) \times U(1)$ gauge invariance

$$\mathcal{L} = m_h^2 h^2 + \lambda_3 h^3 + \lambda_4 h^4 + \ldots$$

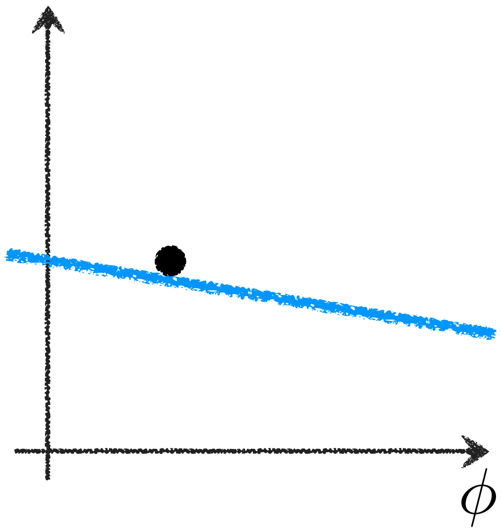
EFT for perturbations around

Only $U(1)_{\mathrm{EM}}$ gauge invariance

Resum the contribution of many operators if non-linearities are large

Quasi De sitter

$$\sqrt{-g}\Big(M_{\rm Pl}^2 R - g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - V(\phi)\Big)$$



Solve the equation of motion to compute the background

$$\phi(x) = \phi_0(t)$$

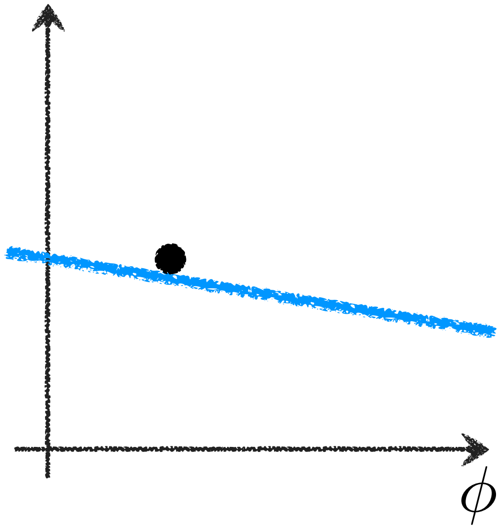
$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \qquad H \equiv \frac{\dot{a}}{a}$$

Expand in small perturbations

$$\mathcal{L} = (\partial\varphi)^2 + (\partial h)^2 + \mathcal{O}(\varphi^3, h^3, \ldots)$$

Quasi De sitter

$$\sqrt{-g}\Big(M_{\rm Pl}^2 R - g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - V(\phi) + \sum_n a_n \frac{(\partial_\mu\phi\partial^\mu\phi)^n}{\Lambda^{4n-4}}\ \Big)$$



Solve the equation of motion to compute the background

$$\phi(x) = \phi_0(t)$$

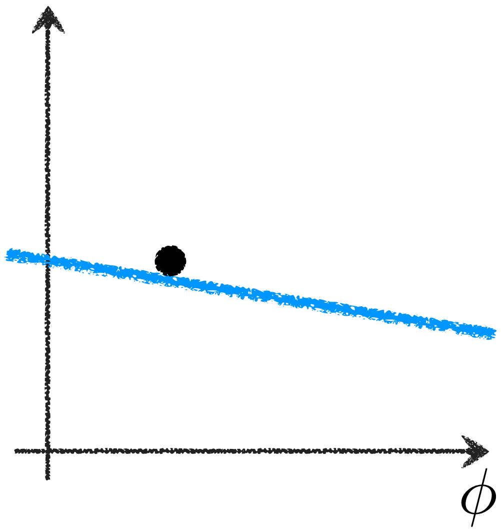
$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \qquad H \equiv \frac{\dot{a}}{a}$$

Expand in small perturbations

$$\mathcal{L} = (\partial\varphi)^2 + (\partial h)^2 + \mathcal{O}(\varphi^3, h^3, \dots)$$

Quasi De sitter

$$\sqrt{-g}\Big(M_{\rm Pl}^2 R - g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - V(\phi) + \sum_n a_n \frac{(\partial_\mu\phi\partial^\mu\phi)^n}{\Lambda^{4n-4}} + \sum_n b_n \frac{(\partial_\mu\phi\partial^\mu\phi)^n \,\Box\phi}{\Lambda^{4n-1}} + \dots \Big)$$



Solve the equation of motion to compute the background

$$\phi(x) = \phi_0(t)$$

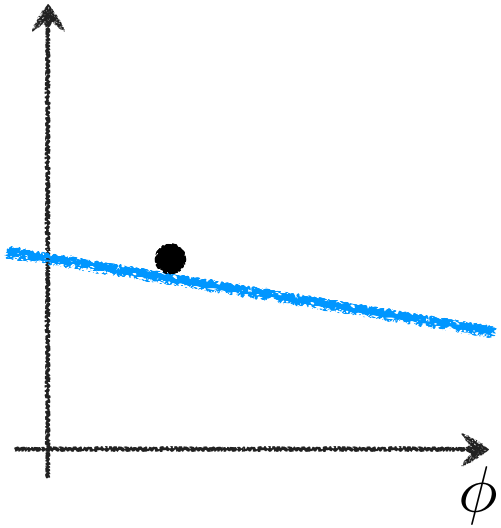
$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \qquad H \equiv \frac{\dot{a}}{a}$$

Expand in small perturbations

$$\mathcal{L} = (\partial\varphi)^2 + (\partial h)^2 + \mathcal{O}(\varphi^3, h^3, \dots)$$

Quasi De sitter

$$\sqrt{-g}\Big(M_{\rm Pl}^2 R - g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - V(\phi) + \sum_n a_n \frac{(\partial_\mu\phi\partial^\mu\phi)^n}{\Lambda^{4n-4}} + \sum_n b_n \frac{(\partial_\mu\phi\partial^\mu\phi)^n\,\Box\phi}{\Lambda^{4n-1}} + \dots\Big)$$



Solve the equation of motion to compute the background

$$\phi(x) = \phi_0(t)$$

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \qquad H \equiv \frac{\dot a}{a}$$

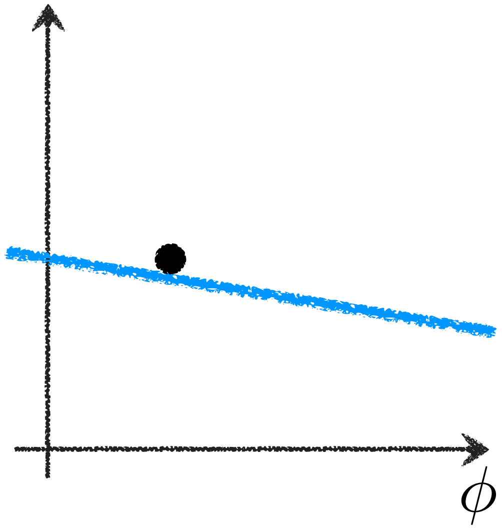
Expand in small perturbations

$$\mathcal{L} = (\partial\varphi)^2 + (\partial h)^2 + \mathcal{O}(\varphi^3, h^3, \dots)$$

$$\partial\phi_0 \sim \Lambda^2 \qquad \frac{\partial}{\Lambda} \ll 1$$

Quasi De sitter

$$\sqrt{-g}\left(M_{\rm Pl}^2 R - g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi - V(\phi) + \sum_n a_n \frac{(\partial_\mu\phi\partial^\mu\phi)^n}{\Lambda^{4n-4}} + \sum_n b_n \frac{(\partial_\mu\phi\partial^\mu\phi)^n\,\Box\phi}{\Lambda^{4n-1}} + \dots\right)$$



Solve the equation of motion to compute the background

$$\phi(x) = \phi_0(t)$$

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \qquad H \equiv \frac{\dot{a}}{a}$$

Expand in small perturbations

$$\mathcal{L} = (\partial\varphi)^2 + (\partial h)^2 + \mathcal{O}(\varphi^3, h^3, \dots)$$

$$\partial\phi_0 \sim \Lambda^2 \qquad \frac{\partial}{\Lambda} \ll 1$$

Instead…

Quasi De sitter

Choose a background solution $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$

Construct an EFT for perturbations

Quasi De sitter

Choose a background solution $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$

Construct an EFT for perturbations

Choose a foliation of spacetime (unitary gauge) such that $\phi = \phi_0(t)$

Use 3+1 ADM decomposition for the metric variables h_{ij}, N, N_i

Write down in a derivative expansions all the operators that are invariant under the residual symmetries (spatial diffs)

Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore '07

Quasi De sitter

Choose a background solution $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$

Construct an EFT for perturbations

The quadratic action is already interesting

Lorentz is spontaneously broken: no a priori reason to expect luminal speed

$$\int d^4x a^3 \, M_{\rm Pl}^2 \left[(\dot{h}_{ij})^2 - c_T^2 (\partial_k h_{ij})^2\right]$$

$$\int d^4x a^3 \, A^2 \left[\dot{\zeta}^2 - c_S^2 (\partial_i \zeta)^2\right]$$

Quasi De sitter

Choose a background solution $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$

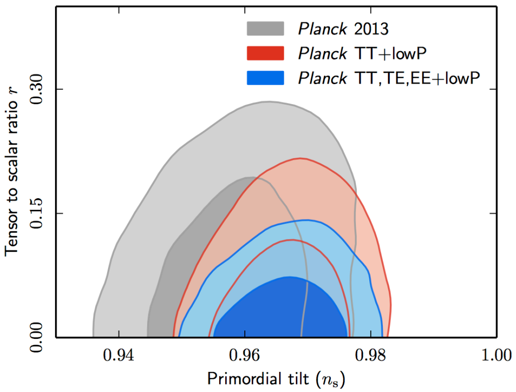
Construct an EFT for perturbations

The quadratic action is already interesting

Lorentz is spontaneously broken: no a priori reason to expect luminal speed

$$\int d^4x a^3 M_{\rm Pl}^2 \left[(\dot{h}_{ij})^2 - c_T^2(\partial_k h_{ij})^2\right]$$

$$\int d^4x a^3 A^2 \left[\dot{\zeta}^2 - c_S^2(\partial_i \zeta)^2\right]$$

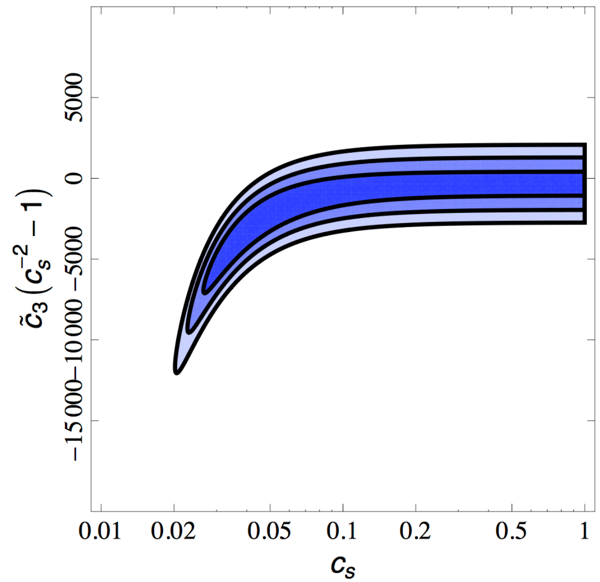


Planck 2015 results

$$< hh > \sim \frac{H^2}{M_{\rm Pl}^2}$$

$$A^2 = \frac{-\dot{H} M_{\rm Pl}^2}{H^2}$$

$$< \zeta\zeta > \sim \frac{H^2}{A^2} \sim \frac{H^2}{\epsilon M_{\rm Pl}^2}$$



Planck 2015 results

Late time cosmology

Choose a background solution $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$

Construct an EFT for perturbations

The quadratic action is already interesting

Lorentz is spontaneously broken: no a priori reason to expect luminal speed

$$\int d^4x a^3 \, M_{\rm Pl}^2 \left[(\dot{h}_{ij})^2 - c_T^2 (\partial_k h_{ij})^2 \right]$$

Late time cosmology

Choose a background solution $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$

Construct an EFT for perturbations

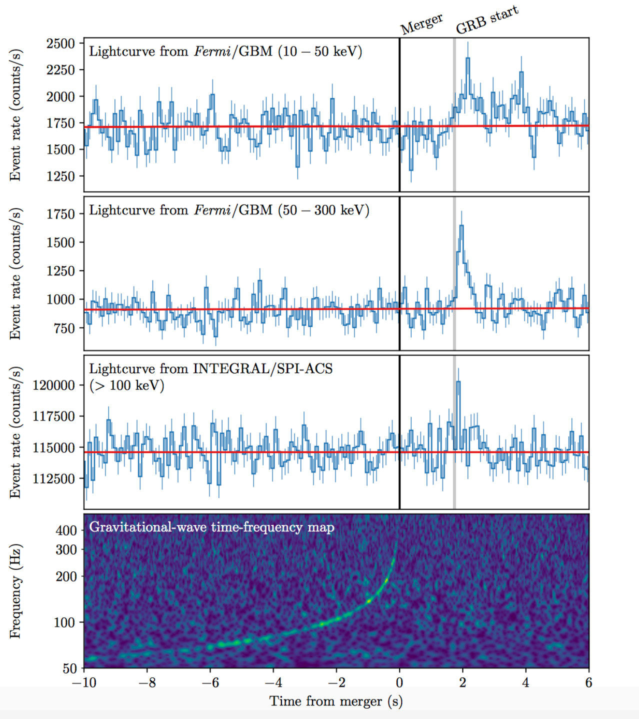
The quadratic action is already interesting

Lorentz is spontaneously broken: no a priori reason to expect luminal speed

$$\int d^4x a^3 M_{\rm Pl}^2 \left[(\dot{h}_{ij})^2 - c_T^2 (\partial_k h_{ij})^2 \right]$$

Abbott *et al.* '17

$$c_T^2 - 1 \lesssim 10^{-15}$$



Late time cosmology

Choose a background solution $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$

Construct an EFT for perturbations $c_T^2 - 1 = -2m_4^2/M^2$

$$S = \int d^4x\sqrt{-g}\left[\frac{M_*^2}{2}f^{(4)}R - \Lambda - cg^{00} + \frac{m_2^4}{2}(\delta g^{00})^2 - \frac{m_3^3}{2}\delta K \delta g^{00} - m_4^2 \delta\mathcal{K}_2 + \frac{\tilde{m}_4^2}{2}\delta g^{00} R\right.$$

$$\delta\mathcal{K}_2 \equiv \delta K^2 - \delta K_\mu^\nu \delta K_\nu^\mu$$

$$m_4^2 \sim 0$$

Creminelli, Vernizzi '17
+ many others

Late time cosmology

Choose a background solution $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$

Construct an EFT for perturbations $c_T^2 - 1 = -2m_4^2/M^2$

$$S = \int d^4x\sqrt{-g}\left[\frac{M_*^2}{2}f^{(4)}R - \Lambda - cg^{00} + \frac{m_2^4}{2}(\delta g^{00})^2 - \frac{m_3^3}{2}\delta K \delta g^{00} - m_4^2 \delta \mathcal{K}_2 + \frac{\tilde{m}_4^2}{2}\delta g^{00} R\right.$$

$$\delta \mathcal{K}_2 \equiv \delta K^2 - \delta K_\mu^\nu \delta K_\nu^\mu$$

$$m_4^2 \sim 0$$

Depends on the background (dark matter abundance,…): robustly set it to zero!

Creminelli, Vernizzi '17
+ many others

Late time cosmology

Choose a background solution $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$

Construct an EFT for perturbations $c_T^2 - 1 = -2m_4^2/M^2$

$$S = \int d^4x\sqrt{-g}\left[\frac{M_*^2}{2}f^{(4)}R - \Lambda - cg^{00} + \frac{m_2^4}{2}(\delta g^{00})^2 - \frac{m_3^3}{2}\delta K \delta g^{00} - m_4^2 \delta \mathcal{K}_2 + \frac{\tilde{m}_4^2}{2}\delta g^{00} R - \frac{m_5^2}{2}\delta g^{00}\delta \mathcal{K}_2 - \frac{m_6}{3}\delta \mathcal{K}_3 - \tilde{m}_6 \delta g^{00}\delta \mathcal{G}_2 - \frac{m_7}{3}\delta g^{00}\delta \mathcal{K}_3\right]$$

$$\delta \mathcal{K}_2 \equiv \delta K^2 - \delta K_\mu^\nu \delta K_\nu^\mu \qquad \delta \mathcal{G}_2 \equiv \delta K_\mu^\nu R_\nu^\mu - \delta K R/2$$

$$\delta \mathcal{K}_3 \equiv \delta K^3 - 3\delta K \delta K_\mu^\nu \delta K_\nu^\mu + 2\delta K_\mu^\nu \delta K_\rho^\mu \delta K_\nu^\rho$$

$$m_4^2 \sim 0$$
$$\tilde{m}_4^2 = m_5^2$$
$$m_6 = \tilde{m}_6 = m_7 = 0$$

Depends on the background (dark matter abundance,…): robustly set it to zero!

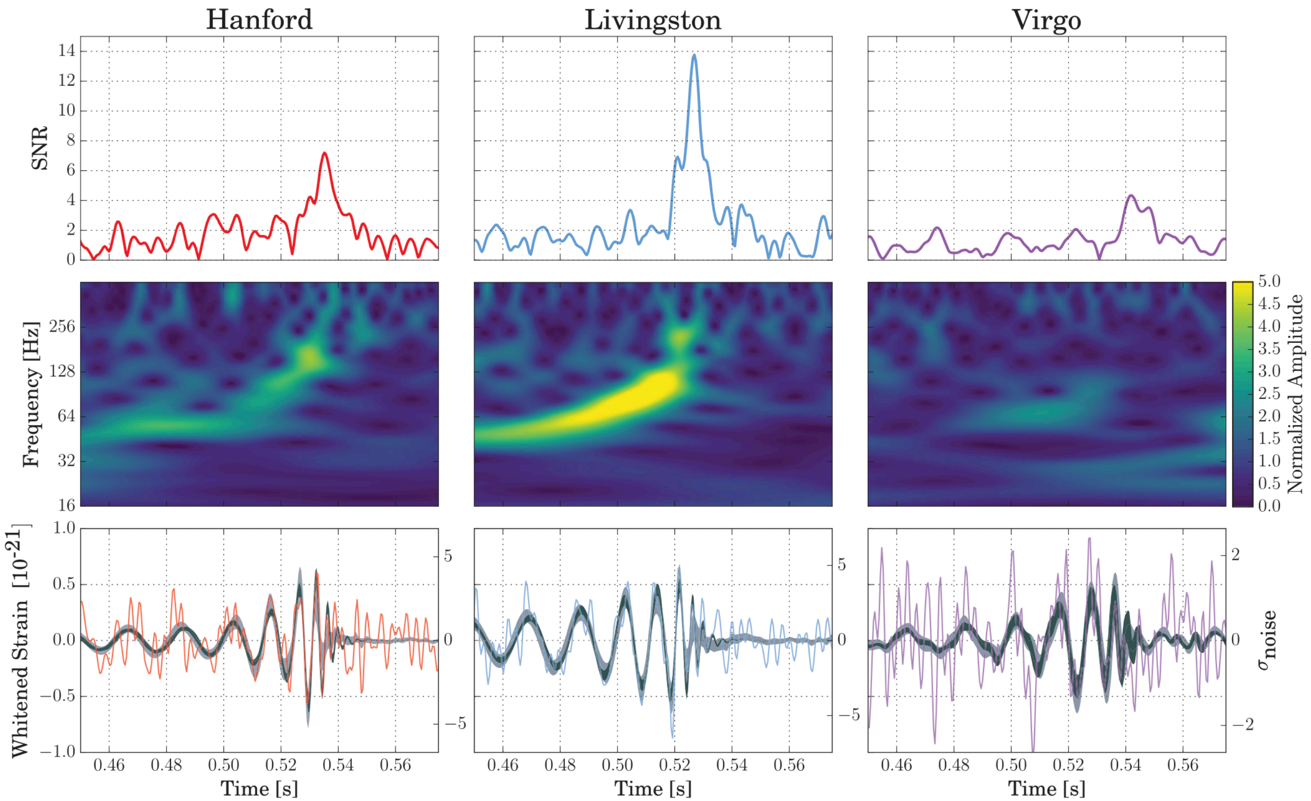
Creminelli, Vernizzi '17
+ many others

Quadratic action of gravity in FRW

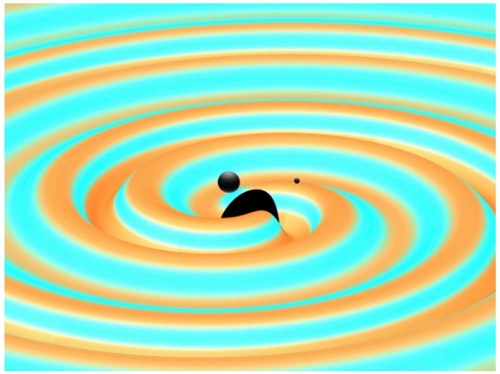
Quadratic action of gravity in FRW

Can we study it in some other background?

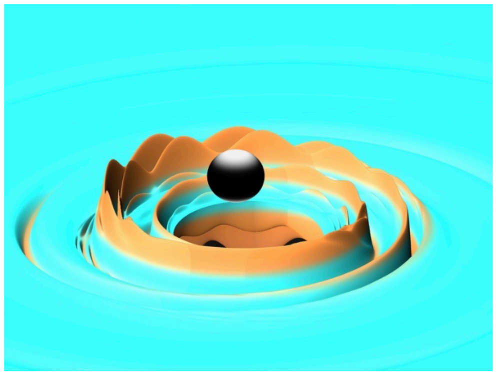
Perturbations around Black Holes



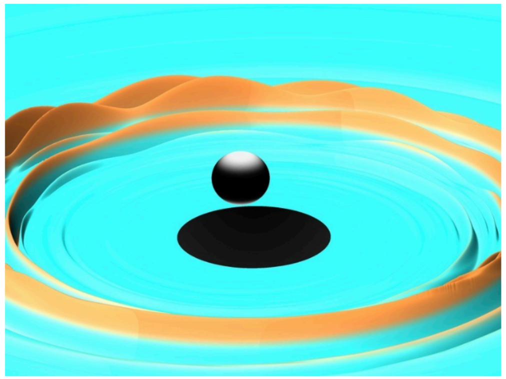
Ligo & Virgo '17



Inspiral



Merger



Ringdown

Perturbations around Black Holes

$g_{\mu\nu} = g^{\mathrm{BH}}_{\mu\nu}(r) + h_{\mu\nu}$ Schwarzschild: static, spherically symmetric background

$$h(t,r,\theta,\phi) = \sum_{lm} h_{lm}(r) Y_{lm}(\theta,\phi) e^{i\omega t}$$

Classified accordingly to the behavior under parity $(\theta,\phi) \to (\pi-\theta, \phi+\pi)$

Perturbations around Black Holes

$g_{\mu\nu} = g_{\mu\nu}^{\mathrm{BH}}(r) + h_{\mu\nu}$ Schwarzschild: static, spherically symmetric background

$$h(t,r,\theta,\phi) = \sum_{lm} h_{lm}(r) Y_{lm}(\theta,\phi) e^{i\omega t}$$

Classified accordingly to the behavior under parity $(\theta,\phi) \to (\pi-\theta, \phi+\pi)$

Axial (odd) perturbations Regge Wheeler '57

Polar (even) perturbations Zerilli '70

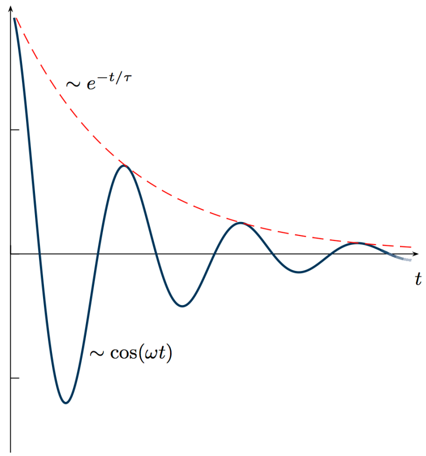
$$\left[\frac{d^2}{dr^2} + \omega^2\right] h(r) = V^{(-)}(r) h(r)$$

$$V^{(-)}(r) = \frac{l(l+1)}{r^2}\left(1 - \frac{r_S}{r}\right) - 3\frac{r_S}{r^3}\left(1 - \frac{r_S}{r}\right)$$

Perturbations around Black Holes

Exponentially damped sinusoidal waves: Quasi Normal Modes (QNM)

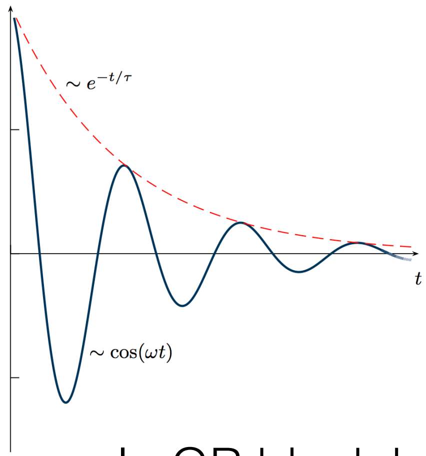
Spectrum of characteristic (complex) frequencies ω_{nlm}



Perturbations around Black Holes

Exponentially damped sinusoidal waves: Quasi Normal Modes (QNM)

Spectrum of characteristic (complex) frequencies ω_{nlm}



Nollert '99

n	$2M_{\bullet}\omega$ $(L=2)$	$2M_{\bullet}\omega$ $(L=3)$	$2M_{\bullet}\omega$ $(L=4)$
0	0.747 343 + 0.177 925i	1.198 887 + 0.185 406i	1.618 36 + 0.188 32i
1	0.693 422 + 0.547 830i	1.165 288 + 0.562 596i	1.593 26 + 0.568 86i
2	0.602 107 + 0.956 554i	1.103 370 + 0.958 186i	1.545 42 + 0.959 82i
3	0.503 010 + 1.410 296i	1.023 924 + 1.380 674i	1.479 68 + 1.367 84i

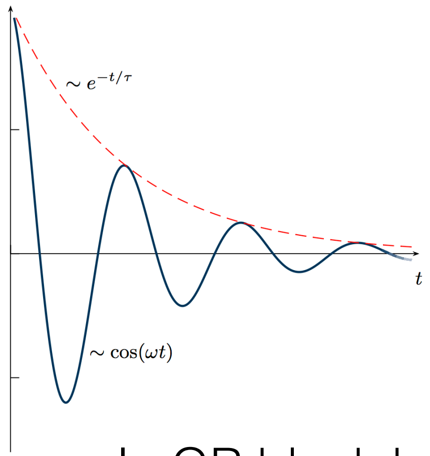
In GR black holes are characterized only by 3 parameters: M, J, Q

No-hair hypothesis

Perturbations around Black Holes

Exponentially damped sinusoidal waves: Quasi Normal Modes (QNM)

Spectrum of characteristic (complex) frequencies ω_{nlm}

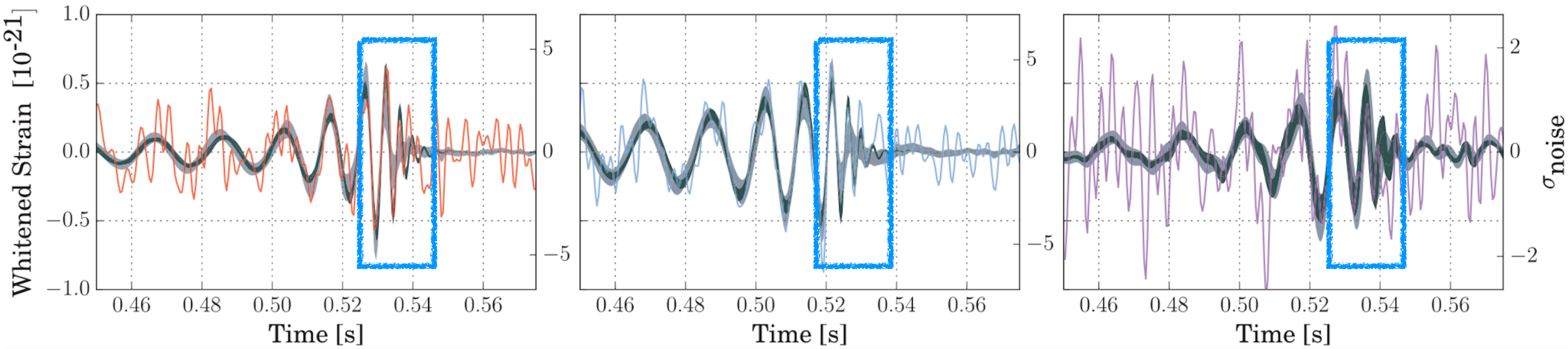


Nollert '99

n	$2M_\bullet\omega\ (L=2)$	$2M_\bullet\omega\ (L=3)$	$2M_\bullet\omega\ (L=4)$
0	0.747 343 + 0.177 925i	1.198 887 + 0.185 406i	1.618 36 + 0.188 32i
1	0.693 422 + 0.547 830i	1.165 288 + 0.562 596i	1.593 26 + 0.568 86i
2	0.602 107 + 0.956 554i	1.103 370 + 0.958 186i	1.545 42 + 0.959 82i
3	0.503 010 + 1.410 296i	1.023 924 + 1.380 674i	1.479 68 + 1.367 84i

In GR black holes are characterized only by 3 parameters: M, J, Q

No-hair hypothesis



EFT for perturbations around BH

$$\left[\frac{d^2}{dr^2} + \omega^2\right] h(r) = V^{(-)}(r) h(r)$$

QNM spectrum ω_{nlm}

The linearized equations of motion are modified in extended theories of gravity

EFT for perturbations around BH

$$\left[\frac{d^2}{dr^2}+\omega^2\right]h(r)=V^{(-)}(r)h(r)$$

QNM spectrum ω_{nlm}

The linearized equations of motion are modified in extended theories of gravity

Construct the EFT around static and spherically symmetric backgrounds
up to quadratic order in perturbations

$$\begin{aligned}\mathcal{S}^2_{EFT}=\int d^4x\sqrt{-G}\,\Bigg[&\frac{M_{Pl}^2}{2}f(r)R-\Lambda(r)-c(r)G^{rr}-\alpha(r)K_{\mu\nu}K^{\mu\nu}+\\&+\mathcal{G}(r)(\delta G^{rr})^2+\mathcal{B}(r)\delta G^{rr}\delta K+\mathcal{B}_{\mu\nu}\delta G^{rr}\delta K^{\mu\nu}+\\&+\mathcal{G}^{(1)}(r)(\partial_r\delta G^{rr})^2+\mathcal{B}^{(1)}(r)(\partial_r\delta G^{rr})\delta K+\mathcal{B}^{(1)}_{\mu\nu}(\partial_r\delta G^{rr})\delta K^{\mu\nu}+\\&+\mathcal{F}_0(r)\delta K\delta K+\mathcal{F}_1(r)\delta K_{\mu\nu}\delta K^{\mu\nu}+\mathcal{F}_{2\mu\nu}\delta K^{\mu\nu}\delta K+\mathcal{F}_{3\mu\nu}\delta K^{\mu\rho}\delta K^{\nu}_{\rho}+\\&+\mathcal{C}(r)\delta G^{rr}\delta^{(3)}R+\mathcal{C}_{\mu\nu}\delta G^{rr}\delta^{(3)}R^{\mu\nu}+\ldots\Bigg]\end{aligned}$$

to appear…

Conclusions

Gravitational waves observation allows to study
the quadratic action of gravity around non-trivial backgrounds (FRW, BH)

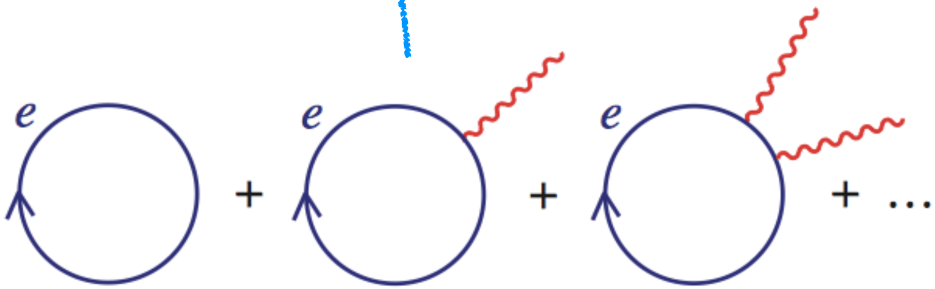
Dramatic improvement in our understanding of dark energy/modified gravity

The CC problem

$$\langle T_{\mu\nu} \rangle = -\rho_{\rm vacuum} g_{\mu\nu}$$

$$\rho_{\rm vacuum} = \Lambda + \rho^{SM}_{\rm vacuum} \qquad \simeq (10^{-3}{\rm eV})^4$$

$$\int d^4x \sqrt{-g} \Big(M_{\rm Pl}^2 R - \Lambda \Big)$$



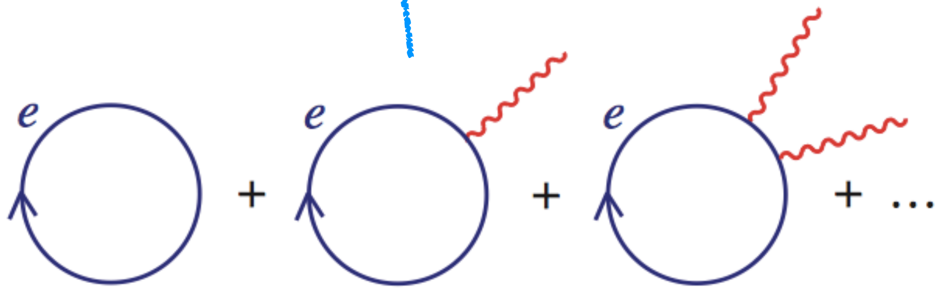
$$\rho_{\rm vacuum} = \mathcal{O}(M_*^4) + \mathcal{O}(M_*^2 m_{\rm e}^2) + \mathcal{O}(m_{\rm e}^4 \log M_*/m_{\rm e})$$

The CC problem

$$\langle T_{\mu\nu} \rangle = -\rho_{\text{vacuum}} g_{\mu\nu}$$

$$\rho_{\text{vacuum}} = \Lambda + \rho^{SM}_{\text{vacuum}} + \rho^{NP}_{\text{vacuum}} \simeq (10^{-3}\text{eV})^4$$

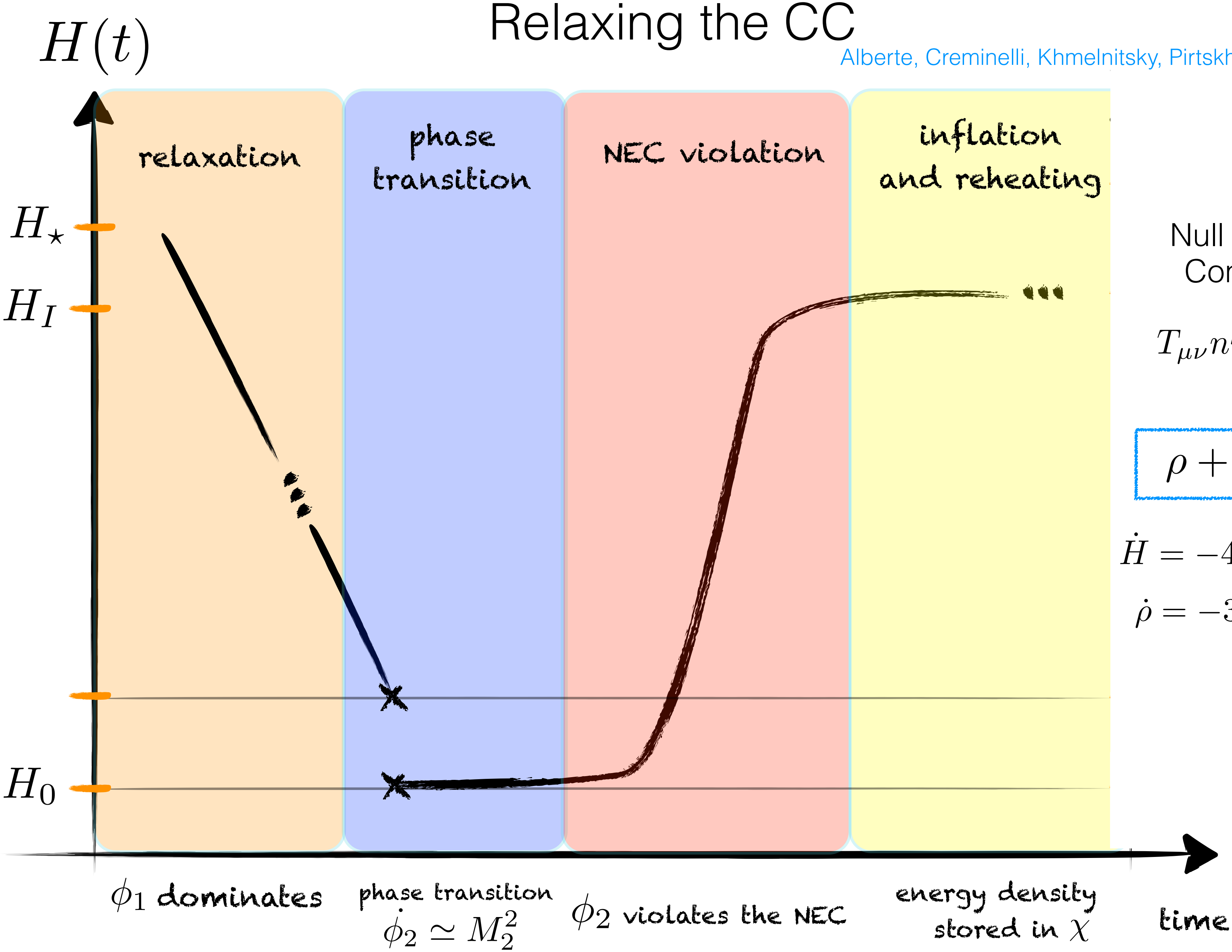
$$\int d^4x \sqrt{-g}\left(M_{\text{Pl}}^2 R - \Lambda\right)$$



$$\rho_{\text{vacuum}} = \mathcal{O}(M_*^4) + \mathcal{O}(M_*^2 m_{\text{e}}^2) + \mathcal{O}(m_{\text{e}}^4 \log M_*/m_{\text{e}})$$

Relaxing the CC

Alberte, Creminelli, Khmelnitsky, Pirtskhalava, ET '17



Null Energy Condition

$$T_{\mu\nu} n^\mu n^\nu \geq 0$$

$$\rho + p \geq 0$$

$$\dot{H} = -4\pi G(\rho + p)$$

$$\dot{\rho} = -3H(\rho + p)$$

Constraints on the Covariant Theory

$$\mathcal{L} = \mathcal{L}(g_{\mu\nu}, \phi)$$

$$\begin{aligned}
L_2 &\equiv G_2(\phi, X)\,, \qquad L_3 \equiv G_3(\phi, X)\Box\phi\,,\\
L_4 &\equiv G_4(\phi, X)\,{}^{(4)}R - 2G_{4,X}(\phi, X)(\Box\phi^2 - \phi^{\mu\nu}\phi_{\mu\nu})\\
&\quad + F_4(\phi, X)\varepsilon^{\mu\nu\rho}{}_{\sigma}\,\varepsilon^{\mu'\nu'\rho'\sigma}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}\,,\\
L_5 &\equiv G_5(\phi, X)\,{}^{(4)}G_{\mu\nu}\phi^{\mu\nu}\\
&\quad + \frac{1}{3}G_{5,X}(\phi, X)(\Box\phi^3 - 3\Box\phi\,\phi_{\mu\nu}\phi^{\mu\nu} + 2\,\phi_{\mu\nu}\phi^{\mu\sigma}\phi^{\nu}{}_{\sigma})\\
&\quad + F_5(\phi, X)\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\mu'\nu'\rho'\sigma'}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}\phi_{\sigma\sigma'}\,,
\end{aligned}$$

$$G_{5,X} = 0\,, \qquad F_5 = 0\,, \qquad 2G_{4,X} - XF_4 + G_{5,\phi} = 0$$

$$\begin{aligned}
X &\equiv g_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi\\
\phi_\mu &\equiv \nabla_\mu\phi\\
\phi_{\mu\nu} &\equiv \nabla_\mu\nabla_\nu\phi\\
G_i(\phi, X) &= \sum_{nm} c_{nm}\,\phi^n X^m
\end{aligned}$$

Constraints on the Covariant Theory

$$\mathcal{L} = \mathcal{L}(g_{\mu\nu}, \phi)$$

$$L_2 \equiv G_2(\phi, X)\,, \qquad L_3 \equiv G_3(\phi, X)\Box\phi\,,$$

$$L_4 \equiv G_4(\phi, X)\,{}^{(4)}R - 2G_{4,X}(\phi, X)(\Box\phi^2 - \phi^{\mu\nu}\phi_{\mu\nu})$$

$$+ F_4(\phi, X)\varepsilon^{\mu\nu\rho}{}_{\sigma}\,\varepsilon^{\mu'\nu'\rho'\sigma}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}\,,$$

$$L_5 \equiv G_5(\phi, X)\,{}^{(4)}G_{\mu\nu}\phi^{\mu\nu}$$

$$+ \frac{1}{3}G_{5,X}(\phi, X)(\Box\phi^3 - 3\Box\phi\,\phi_{\mu\nu}\phi^{\mu\nu} + 2\,\phi_{\mu\nu}\phi^{\mu\sigma}\phi^{\nu}{}_{\sigma})$$

$$+ F_5(\phi, X)\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\mu'\nu'\rho'\sigma'}\phi_\mu\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}\phi_{\sigma\sigma'}\,,$$

$$G_{5,X} = 0\,, \qquad F_5 = 0\,, \qquad 2G_{4,X} - XF_4 + G_{5,\phi} = 0$$

$$X \equiv g_{\mu\nu}\nabla^\mu\phi\nabla^\nu\phi$$

$$\phi_\mu \equiv \nabla_\mu\phi$$

$$\phi_{\mu\nu} \equiv \nabla_\mu\nabla_\nu\phi$$

$$G_i(\phi, X) = \sum_{nm} c_{nm}\,\phi^n X^m$$

$$L_{c_T=1} = G_2(\phi, X) + G_3(\phi, X)\Box\phi + B_4(\phi, X)\,{}^{(4)}R$$

$$- \frac{4}{X}B_{4,X}(\phi, X)(\phi^\mu\phi^\nu\phi_{\mu\nu}\Box\phi - \phi^\mu\phi_{\mu\nu}\phi_\lambda\phi^{\lambda\nu})\,,$$

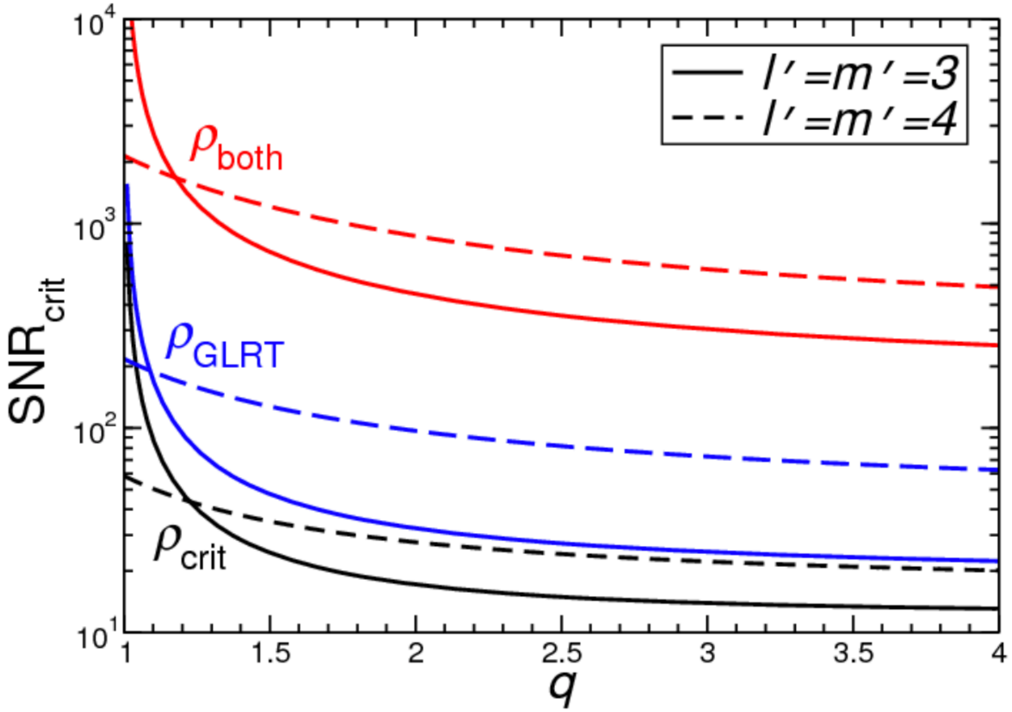
$$B_4 \equiv G_4 + XG_{5,\phi}/2.$$

Higher number of events + higher sensitivity

↓

Possibility to measure multiple frequencies

Signal to noise ratio (SNR) to detect multiple modes



*Berti E, Cardoso J, Cardoso V and Cavaglia M
Matched-filtering and parameter estimation of ringdown waveforms
2007 Phys. Rev. D76 104044

*Non spinning binary BH merger with masses ratio q (fundamental mode $l=m=2$)

$$\left((SNR)^2 = 4\int_0^\infty \frac{\tilde{h}^*(f)\tilde{h}(f)}{S_h(f)}df \right)$$