Analogue Hawking radiation in a "flow of light"

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EPL, **92**, 14002 (2010)
Phys.Rev.A, **86**, 063821 (2012)
Phys.Rev. A **85**, 045602 (2012)
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Analogue gravity by an optical vortex. Resonance enhancement of Hawking radiation

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PRA - submitted

GIF German-Israeli Foundation for Scientific Research and Development

- Black hole
- Analogue gravity-Transonic flow.
- Maxwell equations and hydrodynamics.
- Mimicking horizon and curved space, Hawking radiation
- Fluctuations in transonic regime – linear geometry
- Vortex as a background
- Superradiance and resonant enhancement

Black hole – naïve approach

$$G\frac{Mm}{R} = mv^2 < mc^2$$

Escape velocity cannot exceed speed of light

$$R > R_s = G\frac{M}{c^2}$$

Radius of event horizon

$$S = 4\pi M^2$$

Entropy = surface area

$$\frac{1}{T} = \frac{\partial S}{\partial E} = \frac{\partial S}{\partial M} = 8\pi M$$

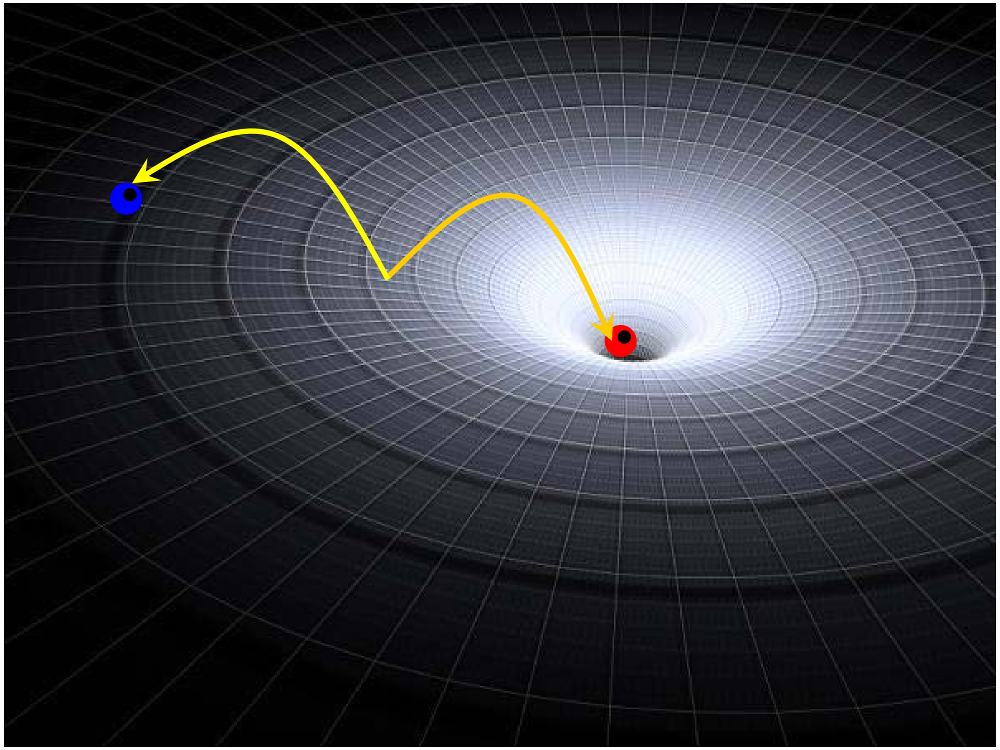
Temperature

Black hole Hawking radiation

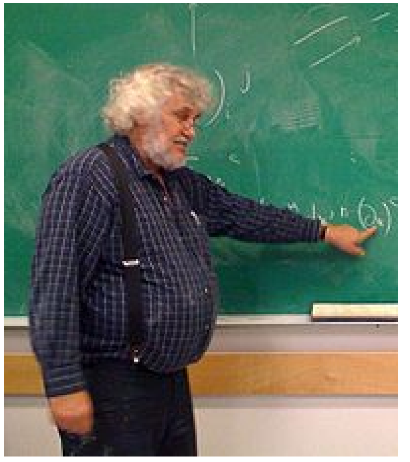
Black hole radiates according to
the Planck law for black body radiation

$$T_H = \frac{\hbar c^2}{8\pi G M k_B} = \frac{1.227 \cdot 10^{23} kg}{M} K$$

$$= \frac{6 \cdot 10^{-8} M_s}{M} K$$



Modeling black hole in a flow – Unruh, 1981



$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$ Continuity equation

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\frac{1}{\rho} \nabla P$$ Euler equation

Laminar solution for a transonic flow

plus fluctuations near this solution

$$\rho_0 = \bar{\rho} e^{\psi_0}$$

$$\mathbf{v}_0 = \nabla \varphi_0$$

$$\psi_0 \to \psi_0 + \psi$$

$$\varphi_0 \to \varphi_0 + \varphi$$

W. G. Unruh, Phys. Rev. Lett. **46**, 1351 (1981)

Equation for fluctuations

$$\rho_0 \frac{\partial \psi}{\partial t} + \nabla \cdot (\rho_0 \nabla \varphi) + \nabla \cdot (\rho_0 \mathbf{v}_0 \psi) = 0$$

$$\frac{\partial \varphi}{\partial t} + \mathbf{v}_0 \cdot \nabla \varphi + s^2 \psi = 0$$

Excluding ψ

$$\frac{\partial^2 \varphi}{\partial \mathrm{t}^2} + \frac{\partial}{\partial \mathrm{t}}(\mathbf{v}_0 \cdot \nabla \varphi) + \nabla \cdot \left(\mathbf{v}_0 \frac{\partial \varphi}{\partial \mathrm{t}} \right) + \nabla \cdot [(\mathbf{v}_0 \cdot \nabla \varphi)\mathbf{v}_0] - \nabla \cdot [s^2 \nabla \varphi] = 0$$

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^i}\left(\sqrt{-g}\,g^{ij}\frac{\partial}{\partial x^j}\right)\varphi = 0 \qquad g = 1/\det\left(g^{ij}\right) = -s^{-2}$$

Contravariant metric
in a “curvilinear space”

$$g^{ij} = \frac{1}{s}\begin{pmatrix} 1 & v_{0x} & v_{0y} & v_{0z} \\ v_{0x} & v_{ox}^2 - s^2 & v_{0x}v_{0y} & v_{0x}v_{0z} \\ v_{0y} & v_{0x}v_{0y} & v_{ox}^2 - s^2 & v_{0y}v_{0z} \\ v_{0x} & v_{0x}v_{0z} & v_{0y}v_{0z} & v_{ox}^2 - s^2 \end{pmatrix}$$

Connection between contravariant
and covariant metrics

$$g_{il}g^{lj} = \delta_i^j$$

$$d\sigma^2 = g_{ij}dx^i dx^j = \frac{1}{s}\left(\left[s^2 - \mathbf{v}_0 \cdot \mathbf{v}_0\right]dt^2 + 2dt\mathbf{v}_0 \cdot d\mathbf{r} - d\mathbf{r} \cdot d\mathbf{r}\right)$$

$$\tau = t + \int \frac{v_0^{\perp}(r)dr}{s^2 - {v_0^{\perp}}^2} \approx t + \frac{1}{2a_s}\ln(r - r_s)$$

1 + 1 Klein-Gordon equation

$$\frac{\mathbf{1}}{\sqrt{-g}}\frac{\partial}{\partial^i x}\sqrt{-g}\,g^{ij}\frac{\partial}{\partial^j x}\psi = 0$$

$$g^{ij} = \frac{1}{s}\begin{pmatrix} 1 & v_x \\ v_x & v_x^2 - s^2 \end{pmatrix}$$

$$d\sigma^2 = \frac{1}{s}\left(-(s^2 - v_x^2)dt^2 + 2v_x dt dx + dx^2\right)$$

$$dt = d\tau - \frac{v_x}{v_x^2 - s^2}dx$$

$$g_{ij} = \frac{1}{s}\begin{pmatrix} v_x^2 - s^2 & 0 \\ 0 & \frac{1}{v_x^2 - s^2} \end{pmatrix}$$

$$d\sigma^2 = -\frac{s^2 - v_x^2}{s}d\tau^2 + \frac{s}{s^2 - v_x^2}dx^2$$

$$s^2 - v_x^2 = a_s x$$

Schwarzschild metric

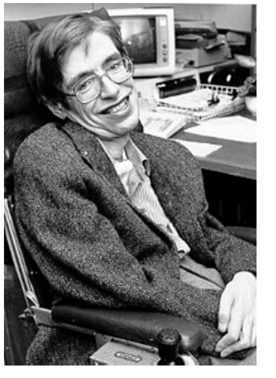
Hawking radiation

$$d\sigma^2 = g_{ij}dx^i dx^j = \frac{1}{s}\left(\left[s^2 - \mathbf{v}_0\cdot\mathbf{v}_0\right]d\tau^2 - \frac{sdr^2}{s^2 - \mathbf{v}_0\cdot\mathbf{v}_0}\right)$$

"Schwarzschild metric"

$$\tau = t + \int \frac{v_0^{\perp}(r)dr}{s^2 - {v_0^{\perp}}^2} \approx t + \frac{1}{2a_s}\ln r$$

The corresponding equation for fluctuation is equivalent to that considered by Hawking.



Hawking radiation in a laboratory

$$t=\int_{x_0}^{x_1}\frac{dx}{s+v(x)}+\int_{x_1}^{x_0}\frac{dx}{s-v(x)}\approx\int_{x_0}^{x_1}\frac{dx}{2s+ax}+\int_{x_1}^{x_0}\frac{dx}{-ax}\approx$$

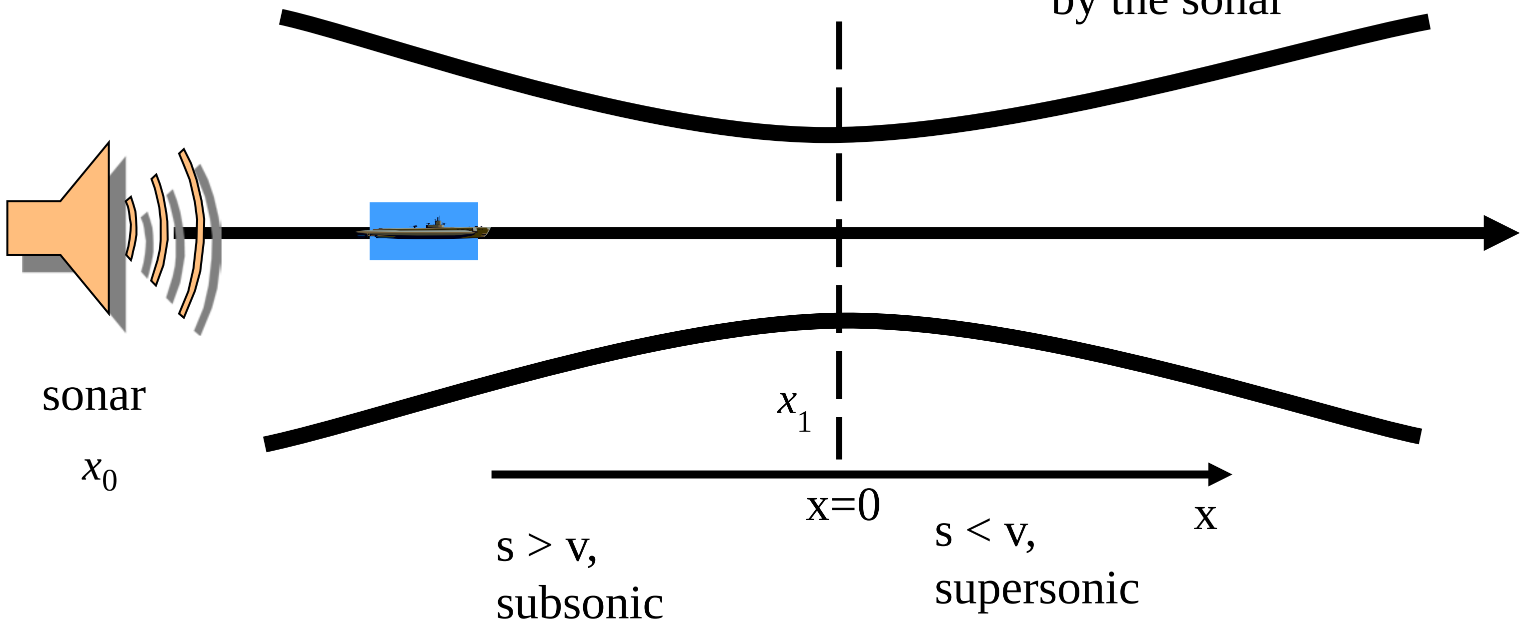
$$\frac{1}{a}\ln\frac{2s+ax_1}{2s+ax_0}+\frac{1}{a}\ln\frac{x_0}{x_1};\qquad v(x)=s+ax$$

Submarine can be located by the sonar

Submarine can still be located by the sonar but with a very long waiting time. Time slowing.

“Mach horizon”

Submarine cannot be located by the sonar



Bose-Einstein Condensate

Gross-Pitaevskii Eq.:

$$i\partial_t \psi = -\frac{1}{2m}\partial_x^2 \psi + g\,|\psi|^2\,\psi$$

Madelung transformation: $\psi = \sqrt{\rho}\exp(-i\varphi)$

$$\Rightarrow \quad \partial_t \rho + \nabla(\rho v) = 0$$

$$\partial_t v + \frac{1}{2}\nabla v^2 = -\frac{1}{m}\nabla\left(-\frac{1}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} + g\rho\right)$$

Euler flow equations for light

E. Madelung, *Quantum theory in hydrodynamic form*, Z. Phys. **40**, 322 (1927).

Nonlinear Schrödinger equation. I

$$\nabla \times \mathbf{H} = \frac{\partial}{\partial t}\mathbf{D}, \qquad \nabla \bullet \mathbf{D} = 0,$$

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}\mathbf{B}, \qquad \nabla \bullet \mathbf{B} = 0.$$

Maxwell equations

$$D(r,t) = \hat{\varepsilon}E(r,t) = \int_0^\infty \varepsilon(\tau)E(r,t-\tau)d\tau,$$

$$B = \mu H$$

$$-\nabla^2 \boldsymbol{E}(\boldsymbol{r},\boldsymbol{t}) + \frac{\partial^2}{\partial \boldsymbol{t}^2}(\boldsymbol{\mu}\hat{\boldsymbol{\varepsilon}}\boldsymbol{E}) = \mathbf{0}$$

Helmholtz equation

Non-Linear Schrödinger (NLS) equation

$$\frac{\partial^2 E}{\partial z^2} + \frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \alpha |E|^2 E - \hat{\varepsilon} \frac{\partial^2 E}{\partial t^2} = 0$$

Helmholtz equation deduced from the Maxwell equations

$$E(x, y, z; t) = \int \frac{d\omega}{2\pi} A(x, y, z, \omega) \exp(i\beta_0 z - i\omega t)$$

paraxial approximation

Wave packet around the frequency ω_0 and wave vector β_0

I.Carusotto, C. Ciuti
RMP, 2013

Non-Linear Schrödinger (NLS) equation

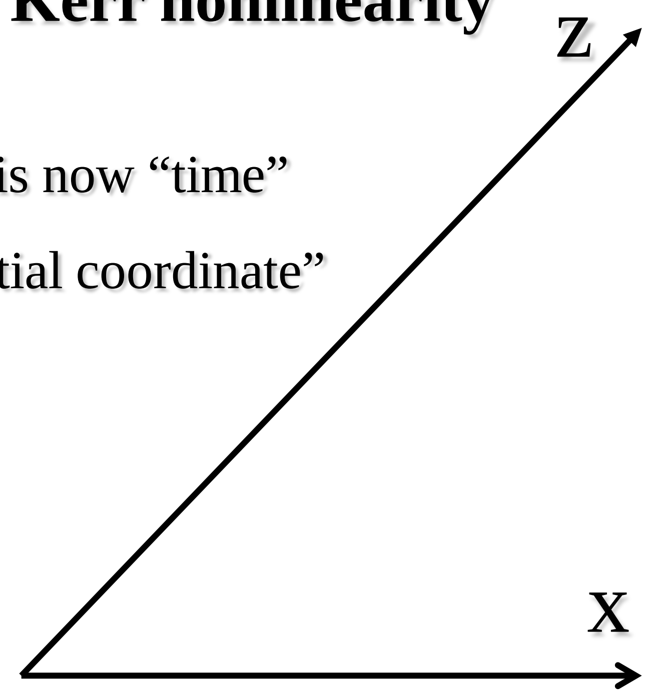
$$i\frac{\partial}{\partial z}A = -\frac{1}{2\beta_0}\Delta A + [\beta_0^2 - n^2(x,y)]A + g|A|^2 A$$

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + 2\beta_0 \pi D \frac{\partial^2}{\partial \tilde{t}^2}$$

Kerr nonlinearity

Spatial coordinate ‘z’ is now “time”

Time ‘t’ is now a “spatial coordinate”



Refraction coefficient plays the role of potential

Hydrodynamic representation

$$A = fe^{-i\varphi}$$

Substituted into

The density field → $\rho = |A|^2 = f^2$

The velocity field → $\beta_0 v = -\nabla\varphi$

$$\left(-\frac{1}{2\beta_0}\nabla^2 + U_{ext}\right)A + g|A|^2 A = i\frac{\partial A}{\partial z}$$

Continuity equation

$$\frac{\partial\rho}{\partial z} + \nabla(\rho\mathbf{v}) = 0$$

Euler equation

$$\frac{\partial\mathbf{v}}{\partial z} + \frac{1}{2}\nabla\mathbf{v}^2 = -\frac{1}{\hbar\beta_0}\nabla\left(-\frac{\hbar^2}{2\hbar\beta_0}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} + U_{ext} + g\rho\right)$$

$g\rho = s^2$ sound velocity

Quantum potential

Behavior of fluctuation modes near the "horizon". QP neglected

$$\xi_1 = e^{i\nu\tau} e^{i\frac{2\nu}{3s\alpha}\ln(-x)}$$

"Left" mover

$$\xi_2 = e^{i\nu\tau - ik(\nu)\xi}; \quad k(\nu) = \frac{\nu}{2s}\frac{1 - i\frac{\alpha}{\nu}}{1 - i\frac{3\alpha}{2\nu}}$$

Right mover

The first mode oscillates like crazy near the horizon (x=0) and acquires the factor $e^{\frac{2\pi\nu}{3\alpha s}}$ when crossing it.

This factor defines the "Hawking temperature".

$$e^{\frac{2\pi\nu}{3\alpha s}} \Rightarrow e^{\frac{\hbar\nu}{T_H}}$$

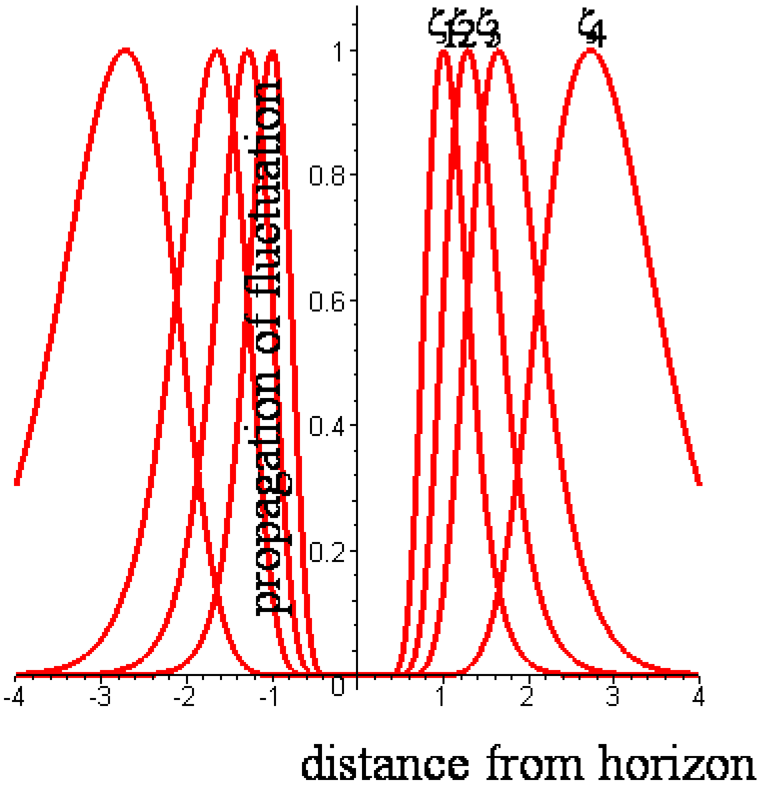
$$T_H = \frac{3\alpha s\hbar}{2\pi}$$

Straddled fluctuations

$$f(x;z) = \int d\omega g(\omega) e^{-i\omega z} \xi_\omega(x)$$

$$g(\omega) = \frac{1}{\Gamma\sqrt{2\pi}} e^{-\frac{(\omega-\omega_g)^2}{2\Gamma^2}}$$

Zero at the horizon



Derivation of Hawking temperature

The fluctuation is cut in two parts $\varphi_l + \varphi_r e^{\frac{2\pi\nu}{3\alpha s}}$

KG normalization requires that $\left\| \varphi_l + \varphi_r e^{\frac{2\pi\nu}{3\alpha s}} \right\| = \|\varphi_r\| e^{\frac{4\pi\nu}{3\alpha s}} - \|\varphi_l\| = -1$

Assuming a symmetry $\|\varphi_r\| = \|\varphi_l\|$

The relative weight of the left part is $\dfrac{1}{e^{\frac{4\pi\nu}{3\alpha s}} - 1}$

It is identified as Planck distribution $\dfrac{1}{e^{\frac{\hbar\nu}{T_H}} - 1}; \quad T_H = \dfrac{3\hbar s\alpha}{4\pi}; \quad \lambda_{\mathrm{H}} = \dfrac{8\pi^2}{3s\alpha}$

Hawking “temperature” corresponds to the wave length

Close to the horizon. Role of QP

$$\left(-i\nu + i\bar{s}k\right)\chi_k - \alpha\bar{s}\partial_k(k\partial_k)\chi_k + i\alpha\frac{k}{\beta}\xi_k + \frac{k^2}{\beta}\xi_k = 0$$

$$-\frac{1}{4\beta}\left(i\alpha k + ik^2\right)\chi_k - \beta\bar{s}^2(1 - i\alpha\partial_k)\chi_k + [i(-\nu + \bar{s}k)\xi_k - \alpha\bar{s}\partial_k(k\xi_k)] = 0$$

$$\chi(x,z) = e^{-i\nu z}\int_C dk k^{\gamma_1}(k - \frac{2}{3}\nu - \frac{i}{3}\alpha)^{\gamma_2}\exp\{\Lambda(k,\nu) + ikx\}$$

$$\Lambda(k,\nu) = \frac{l_n^2}{\alpha}\left\{-\frac{i}{18}k^3 + ...\right\}; \quad l_n^2 = \frac{1}{2\beta^2\bar{s}^2} = \frac{1}{2\beta^2 g|A|^2}$$

$$\gamma_1 = \frac{1}{4} - \frac{i\nu}{2\alpha}; \; \gamma_2 = -\frac{1}{4} - \frac{i\nu}{6\alpha} + \frac{14}{81}l_n^2\nu^2 - \frac{4i\nu}{81\alpha}l_n^2\nu^3$$

Close to the horizon

$$\chi_s \approx x^{\gamma-1}, \quad \gamma = -\gamma_1 - \gamma_2 = i\frac{2\nu}{3\bar{s}\alpha} - \frac{2}{27}\frac{\nu^2 l_n^2}{\bar{s}^2} + \frac{4i}{81}\frac{\nu^3 l_n^2}{\alpha\bar{s}^3}$$

Corrections due to QP

This solution holds in the window

$$\min\left\{\frac{\bar{s}}{\nu}, \frac{1}{\alpha}\right\} > |x| > l_r = \frac{l_n}{(l_n\alpha)^{1/3}}$$

For smaller x fluctuations are regular and do not zero on the horizon

$$N(\omega) = \left(e^{\frac{\hbar\nu}{T_H}} - 1\right)^{-1};$$

$$\frac{1}{T_H(\nu)} = \frac{4\pi}{3\bar{s}\alpha\hbar} + \frac{8\pi}{81}\frac{\nu^2 l_n^2}{\alpha\bar{s}^3\hbar}$$

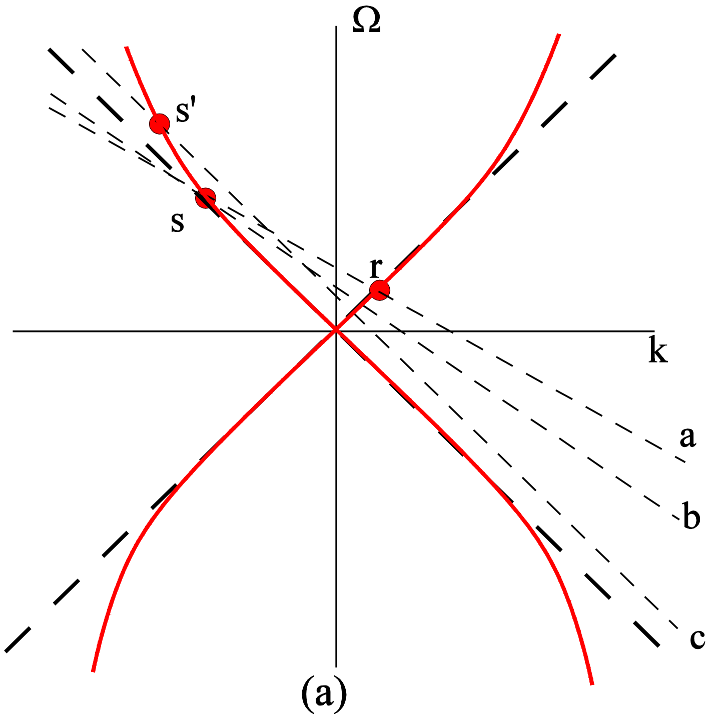
Frequency distribution of the HR deviates from the Planck function at high frequencies.

Parentani etal, 2009,2011 (numerically)

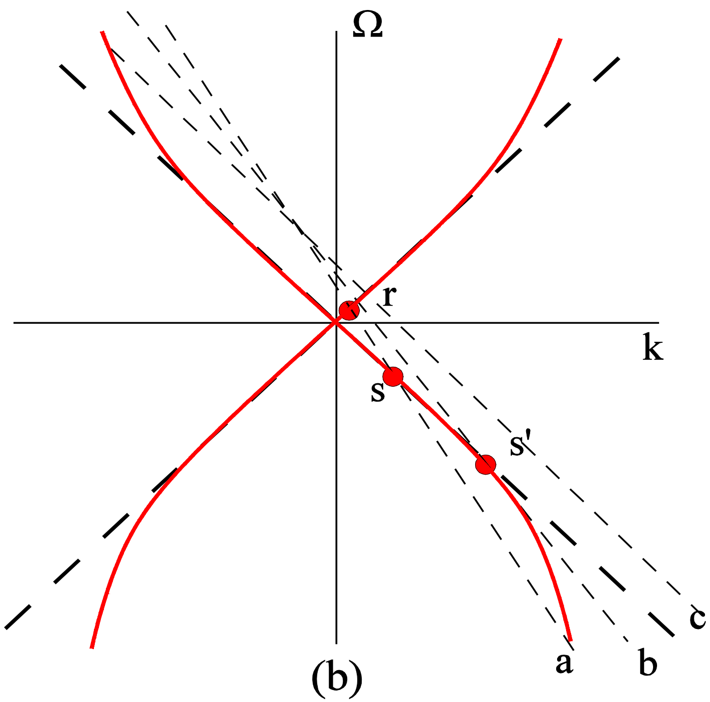
VF, R. Schilling, 2012 (analytically)

Saddle point equation

$$\partial_k \ln \chi_k = -ix \;\; \Rightarrow \;\; \Omega^2(k,x) = \frac{l_n^2}{2} k^4 \bar{s}^2 + k^2 s^2(x) = [\nu - kv(x)]^2$$



subsonic



supersonic

J.Macher, R.Parentani PRD79,124008(2009), PRA80, 043601(2009)

Quantization of fluctuations

$$L = \frac{i}{4} f_0^2 [\vartheta^+ \sigma_y (\overline{D}\vartheta) - (\overline{D}\vartheta^+) \sigma_y \vartheta]$$

$$- \frac{1}{2} g f_0^4 \vartheta^+ (1 + \sigma_z) \vartheta - \frac{1}{4\beta_0} f_0^2 (\nabla \vartheta^+) \nabla \vartheta$$

$$\vartheta = \begin{pmatrix} \frac{1}{\sqrt{2}} \chi \\ \sqrt{2} \xi \end{pmatrix}$$

$$p_x(z) = -f_0^2 \xi$$ Canonical momentum

$$\chi(x, z)$$ Canonical coordinate

$$f_0^2 [\xi(x', z), \chi(x, z)] = i N \delta(x' - x)$$

Y.Vinish, VF, IJMP, 2016

P. E. Larre and I. Carusotto, PRA 2015

Experimental realizations in progress

1. U. Leonhardt etal – light reflected by a soliton in a waveguide
2. D. Fazzio etal – light reflected by a soliton in a waveguide
3. J. Steinhauer etal – BEC observation of a horizon and radiation
4. S. Bar-Ad etal – “luminous fluid” in a nonlinear medium
5. S. Weinfurthner – “bathtub” experiments

Optical vortex – Laguerre-Gauss beam

$$P(r,z) = \frac{2}{\pi\,|n|!\,w^2(z)}\left(\frac{2r^2}{w^2(z)}\right)^{|n|} e^{-i\frac{r^2}{R(z)}} e^{-2r^2/w^2(z)} e^{in\phi}$$

$$\mathbf{v} = -\frac{1}{\beta}\nabla_{\perp}\varphi(r,z,f) = \frac{r}{f}\hat{r} - \frac{n}{\beta r}\hat{\phi}$$

$$e^{-i\frac{\beta r^2}{R(z)}} e^{in\phi}$$

Optical vortex – Laguerre-Gauss beam

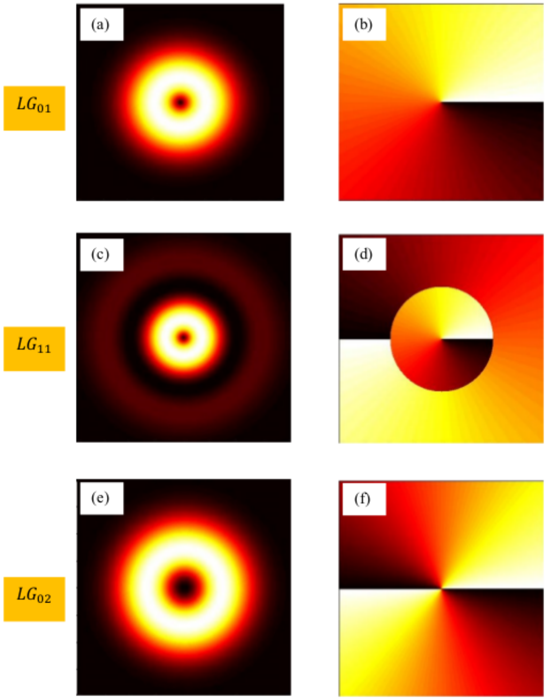


Figure 2.1 Intensity distributions (a,c,e) and corresponding helical phase pattern (b,d,f) at focal plane for $\boldsymbol{LG_{01}}$, $\boldsymbol{LG_{11}}$ and $\boldsymbol{LG_{02}}$ mode of vortex beams.

6

“Kerr” metric

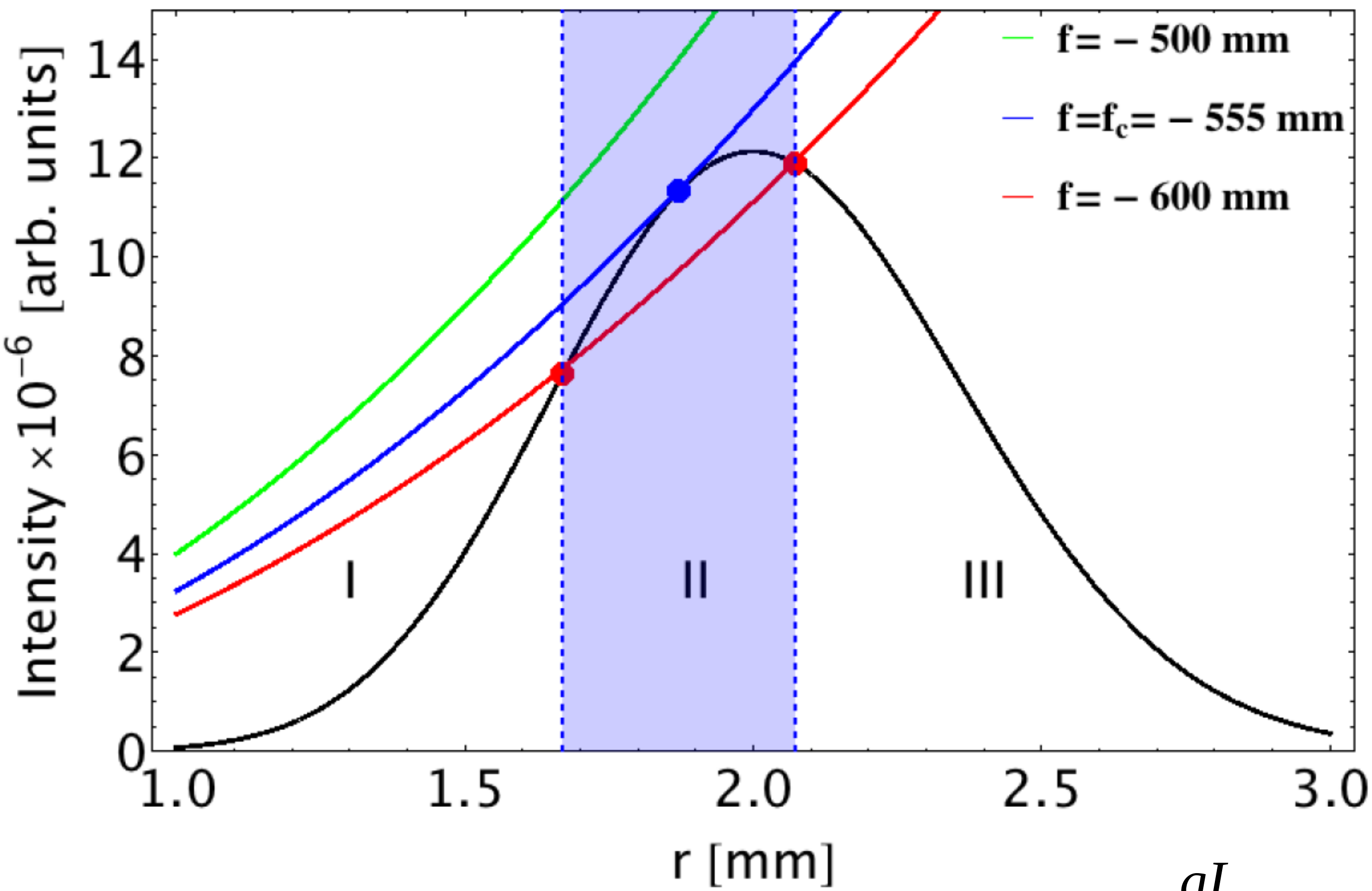
$$\frac{1}{\sqrt{\det(-g_{\mu\nu})}}\partial_\mu\left(g^{\mu\nu}\sqrt{\det(-g_{\mu\nu})}\partial_\nu\xi\right)=0$$

$$g^{\mu\nu}=\begin{pmatrix} s^2-v_0^2 & v_r & rv_\phi & 0 \\ v_r & -1 & 0 & 0 \\ rv_\phi & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$dl^2=\frac{1}{s}\left[\left(s^2-v_0^2\right)dz^2-\frac{s^2}{s^2-v_r^2}dr^2-r^2d\phi^2+2rv_\phi dzd\phi\right]$$

$$s^2-v_0^2=0,\qquad s^2-v_r^2=0$$

Position of event horizon

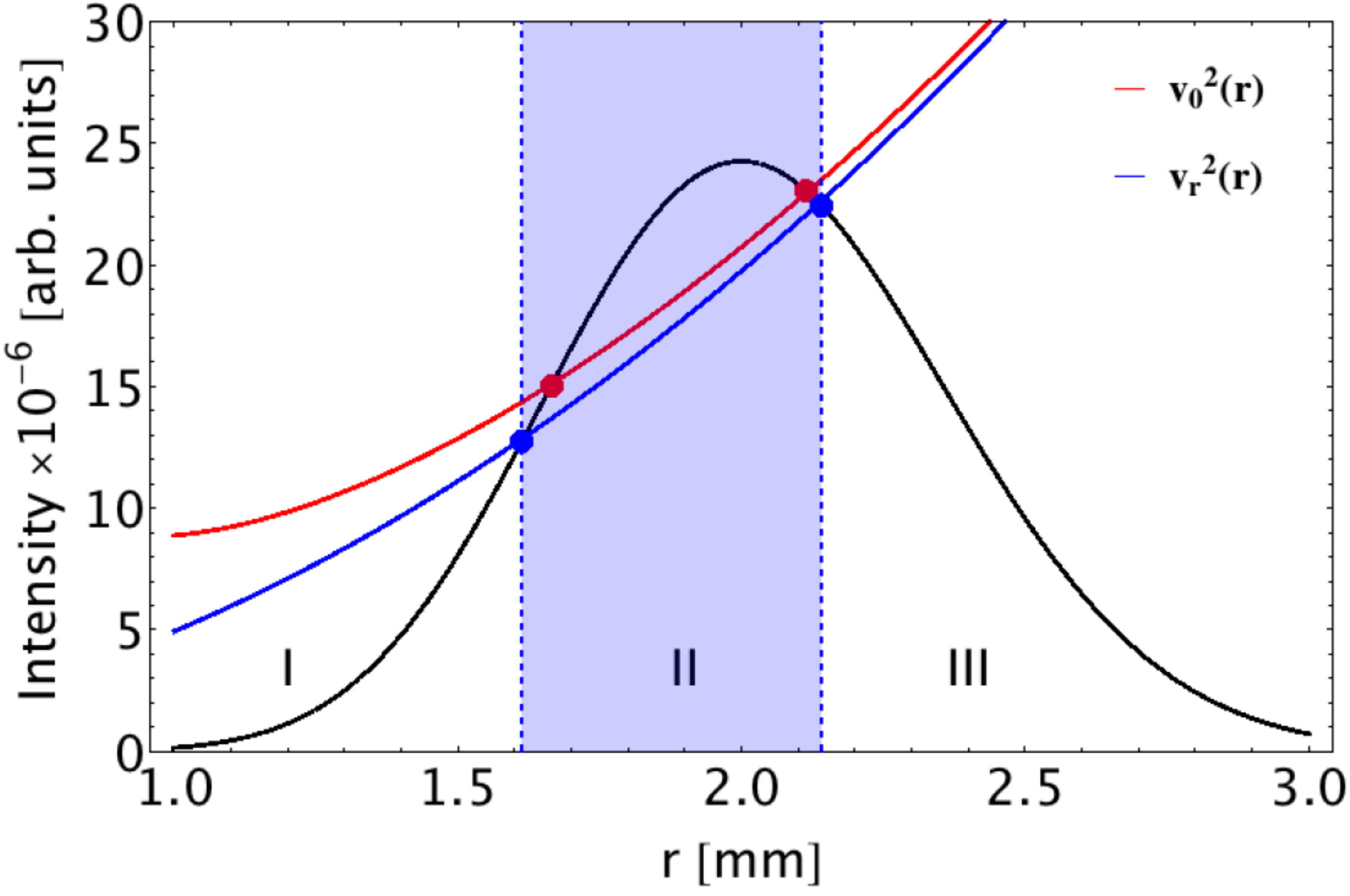


II– subsonic

I, III - supersonic

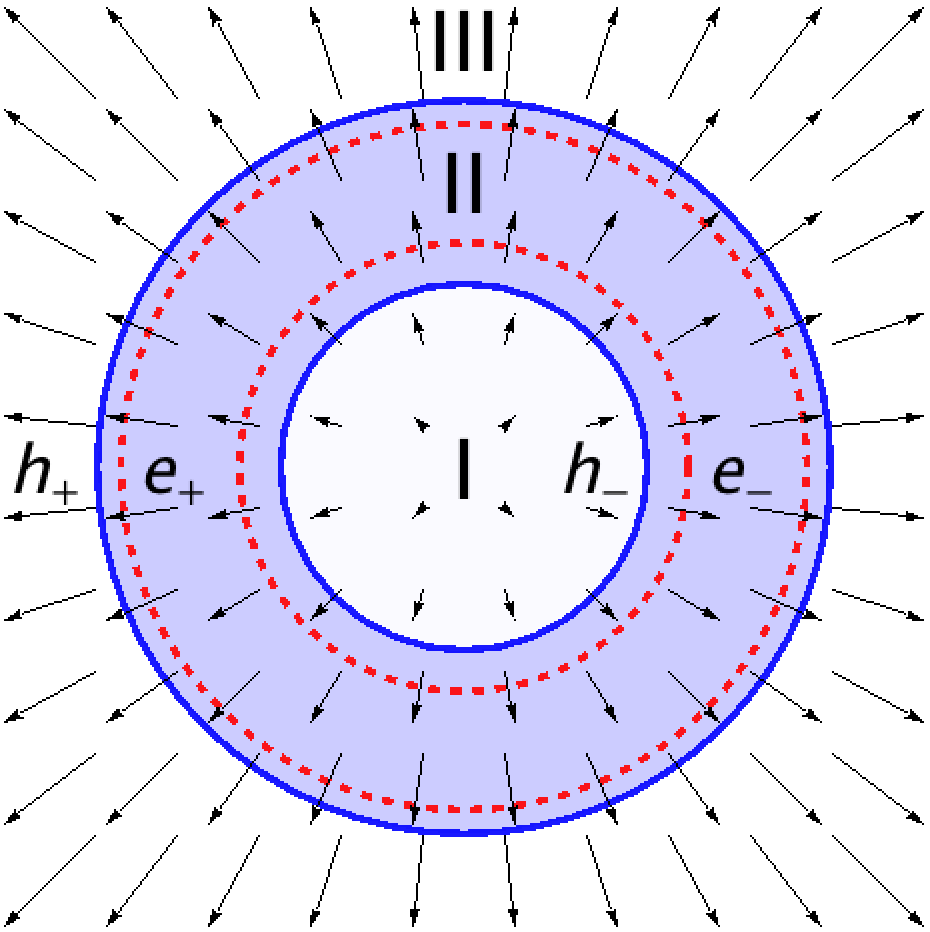
$$\frac{gI}{\beta_0}P(r)=\frac{r^2}{f^2}$$

Ergoregion



$$\frac{gI}{\beta_0}P(r)=\frac{r^2}{f^2}+\frac{n^2}{\beta_0^2 r}$$

Geometry of the flow



F. Marino PPA2008.2009

Superradiance

$$\psi = e^{-i\nu r} + R\, e^{i\nu r}$$

$$\psi = T e^{-i\left(\nu - \frac{mn}{\beta r_n^2}\right)}$$

Scattering of a cylindrical wave

$$1 - |R|^2 = \frac{1}{|\nu|} |T|^2 \left(\nu - \frac{mn}{\beta r_h^2}\right)$$

$$\frac{1}{\nu} |T|^2 \left(\nu - \frac{mn}{\beta r_h^2}\right) < 0, \quad |R|^2 > 1$$

The scattered wave is stronger than the incident

Hawking radiation-resonance

$$\bar{\nu} = \nu - \frac{nm}{\beta r_h^2}$$

$$N(\nu) = \left(e^{\frac{2\pi}{3s\alpha}\bar{\nu}} - 1 \right)^{-1} \frac{1}{\nu} |T|^2 \left(\nu - \frac{mn}{\beta r_h^2} \right)$$

Spectrum of Hawking radiation

$$N(\nu) = \frac{3s\alpha}{2\pi} \frac{1}{|\nu|} |T|^2$$

$$\nu - \frac{nm}{\beta r_h^2} = 0 \Rightarrow m\lambda_\nu = \tau_\phi$$

$$\lambda_\nu = 2\pi / \nu, \qquad \tau_\phi = 2\pi r_h / v_\phi$$

Resonance condition:
An integer number of wavelengths coincide with the time (propagation distance) necessary for one full rotation of the vortex

Does white horizon play a role?

$$\tilde{\nu}_{h_+} = \left(\nu - \frac{mn}{\beta r_{h_+}^2} \right)$$ - black horizon $\quad r_{h_-} < r_{h_+}$

$$\tilde{\nu}_{h_-} = \left(\nu - \frac{mn}{\beta r_{h_-}^2} \right)$$ - white horizon $\quad \nu_{r_{h_-}} > \nu_{r_{h_+}}$

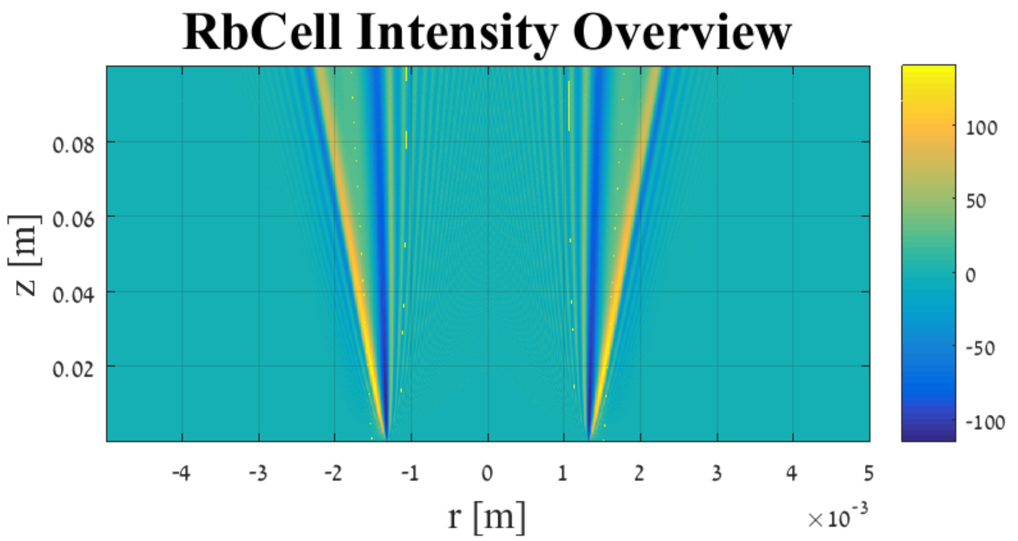
$$\partial_r^2 \psi = \frac{1}{s^2}\left(\nu - \frac{mn}{\beta r^2}\right)^2 - \frac{s^2 - v^2}{sr^2}$$

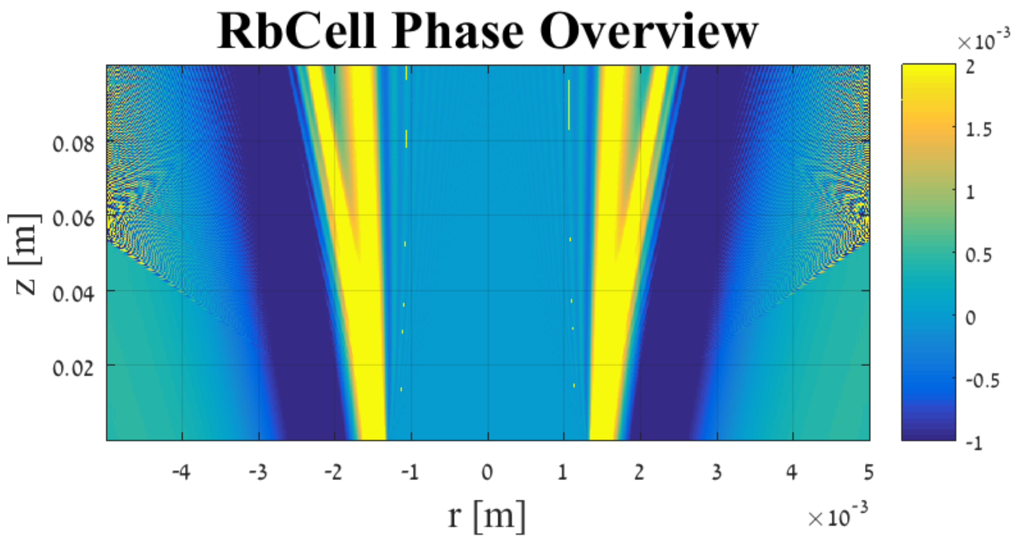
$\nu_{r_{h_-}} < \nu$ regular regime

$\nu_{r_{h_+}} < \nu < \nu_{r_{h-}}$

$\nu < \nu_{r_{h_-}}$

There is a barrier between the horizons reflecting the radiation. Can it be a BH bomb?
Press, Teukolsky, Nature, 1972


RbCell Intensity Overview
z [m]
0.08
0.06
0.04
0.02
-4
-3
-2
-1
0
1
2
3
4
5
r [m]
×10-3
100
50
0
-50
-100


RbCell Phase Overview
z [m]
0.08
0.06
0.04
0.02
-4
-3
-2
-1
0
1
2
3
4
5
r [m]
×10-3
×10-3
2
1.5
1
0.5
0
-0.5
-1

Concluding remarks

- Hawking radiation can be studied in laboratory.
- A possible way is to create an optical transonic flow. Currently in progress.
- Studying classical fluctuations may be feasible and presents a special interest, especially superradiance.
- Resonance with a vortex can make observation of Hawking radiation by the vortex feasible.