QFC2017 - Quantum gases, fundamental interactions and cosmology - Pisa

Inflation & non-Gaussianity

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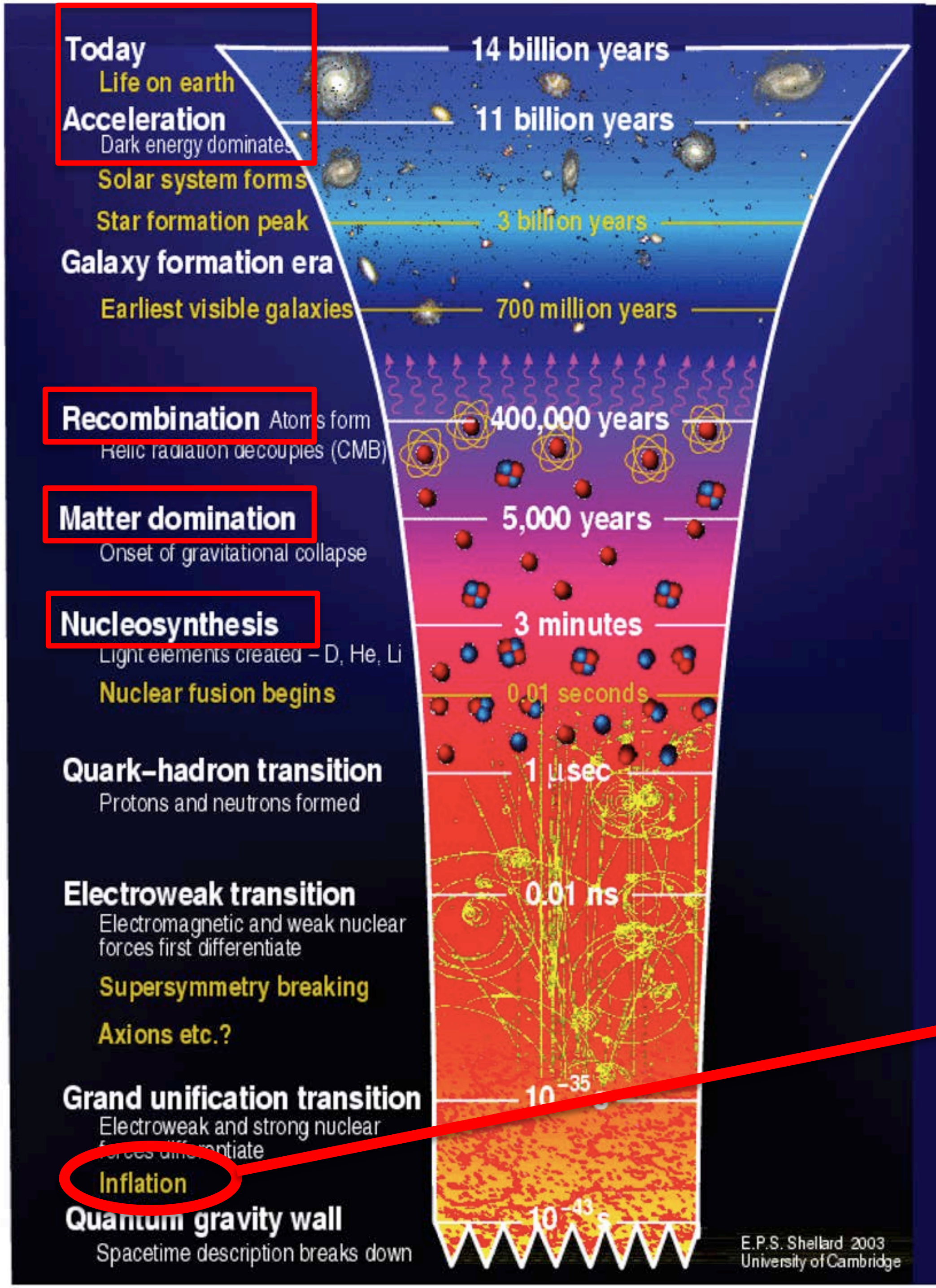


We are here

Z_{rec}~1100

Z_{eq}~3500

T~1 MeV

We seek information about very early times and very high energies E~10^{16} GeV

... did we get it?

Inflation is a self-consistent model of the very early universe

Inflation in the early Universe

- Inflation (Brout et al. 1978; Starobinski 1980; Kazanas 1980; Sato 1981; Guth 1981; Linde 1982, Albrecht & Steinhardt 1982; etc. ...) is an epoch of accelerated expansion in the early Universe (~ 10^{-34} s after the "Big Bang") which allows to solve two inconsistencies of the standard Big Bang model.
  - **horizon**: why is the Universe so homogeneous and isotropic on average?
  - **flatness**: why is the Universe spatial curvature so small even ~ 14 billion years after the Big Bang?)
- Inflation is based upon the idea that the **vacuum energy** of a scalar quantum field, dubbed the "inflaton", dominates over other forms of energy, hence giving rise to a quasi-exponential (de Sitter) expansion, with scale-factor

$$a(t) \approx \exp(Ht)$$

Inflation is the generator of cosmological perturbations that give rise to CMB anisotropies and LSS formation

Inflation predictions

- Quantum vacuum oscillations of the inflaton (or other scalar fields) give rise to classical fluctuations in the energy density, which provide the seeds for **Cosmic Microwave Background (CMB)** radiation temperature anisotropies and polarization, as well as for the formation of **Large Scale Structures (LSS)** in the present Universe. It also gives rise to a *yet-undetected* stochastic background of **gravitational waves**.

- All the matter and radiation which we see today must have been generated after inflation (during "reheating"), since all previous forms of matter and radiation have been tremendously diluted by the accelerated expansion ("Cosmic no-hair conjecture").

Inflation model predictions have already been confirmed by several observations!

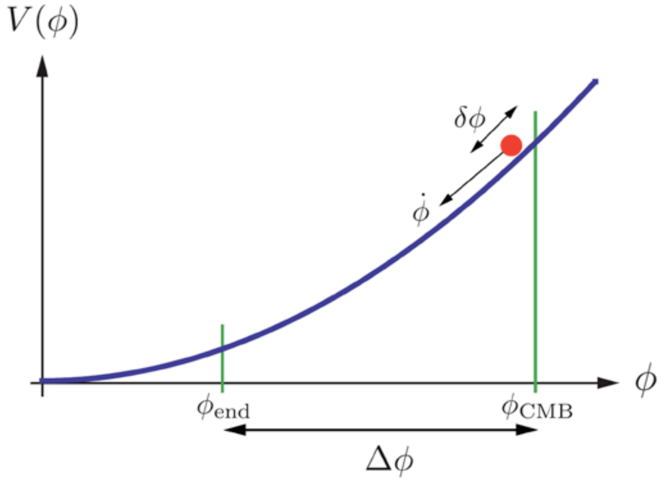
Inflation and the Inflaton

$$\mathcal{L}_\phi[\phi, g_{\mu\nu}] = \frac{1}{2} g^{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V(\phi)$$

Standard kinetic term

Inflaton potential: describes the self-interactions of the inflaton field and its interactions with the rest of the world

Think the inflaton mean field as a particle moving under a force induced by the potential V



Ex: $$V(\phi) = \frac{m^2}{2} \phi^2$$

Two simple but very important examples

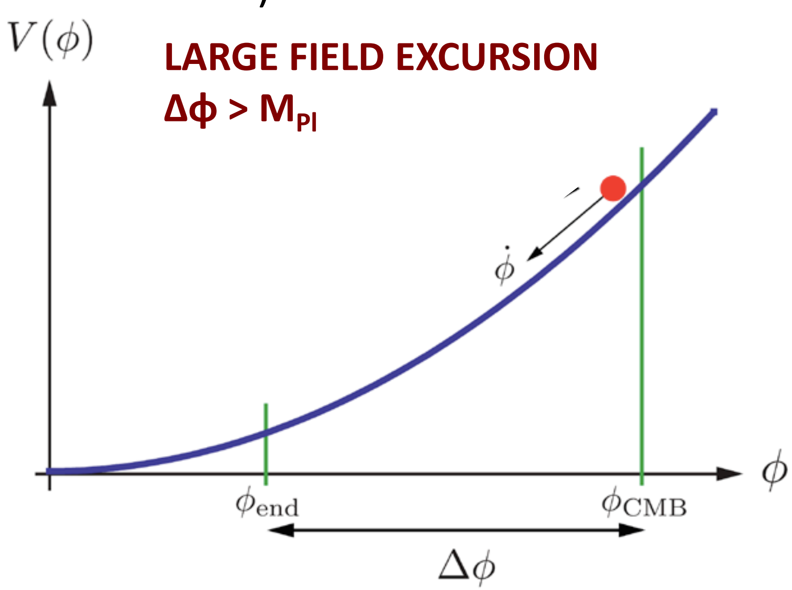
``Large field'' models

$$V(\phi) \propto \phi^{\alpha}$$

typical of ``caothic inflation scenario''
(Linde `83)

$$V(\phi) \propto \exp[\phi/\mu]$$

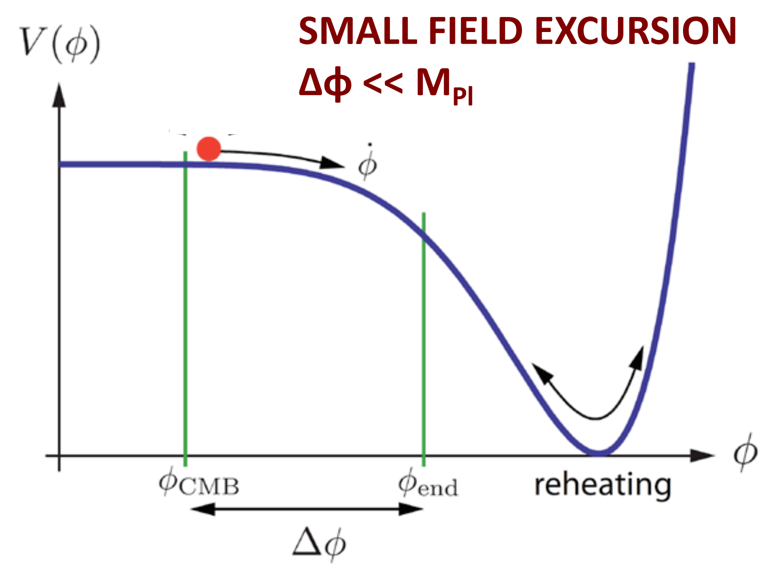
``power law inflation'' (Lucchin, Matarrese '85)



``Small field'' models

$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{\mu}\right)^p\right] \qquad \phi < \mu < M_{\rm Pl}$$

from spontaneous symmetry breaking or Goldstone, axion models (Linde; Albrecht, Steinhardt `82; Freese et al '90)



Observational predictions of inflation

- **Primordial density (scalar) perturbations**

$$\mathcal{P}_\zeta(k) = \frac{16}{9}\frac{V^2}{M_{\rm Pl}^4\dot{\phi}^2}\left(\frac{k}{k_0}\right)^{n-1}$$

amplitude

spectral index: $n - 1 = 2\eta - 6\epsilon$
(or ``tilt'')

$$\epsilon = \frac{M_{\rm Pl}^2}{16\pi}\left(\frac{V'}{V}\right)^2 \ll 1; \quad \eta = \frac{M_{\rm Pl}^2}{8\pi}\left(\frac{V''}{V}\right) \ll 1$$

- **Primordial (tensor) gravitational waves**

$$\mathcal{P}_{\rm T}(k) = \frac{128}{3}\frac{V}{M_{\rm Pl}^4}\left(\frac{k}{k_0}\right)^{n_{\rm T}}$$

Tensor spectral index: $n_{\rm T} = -2\epsilon$

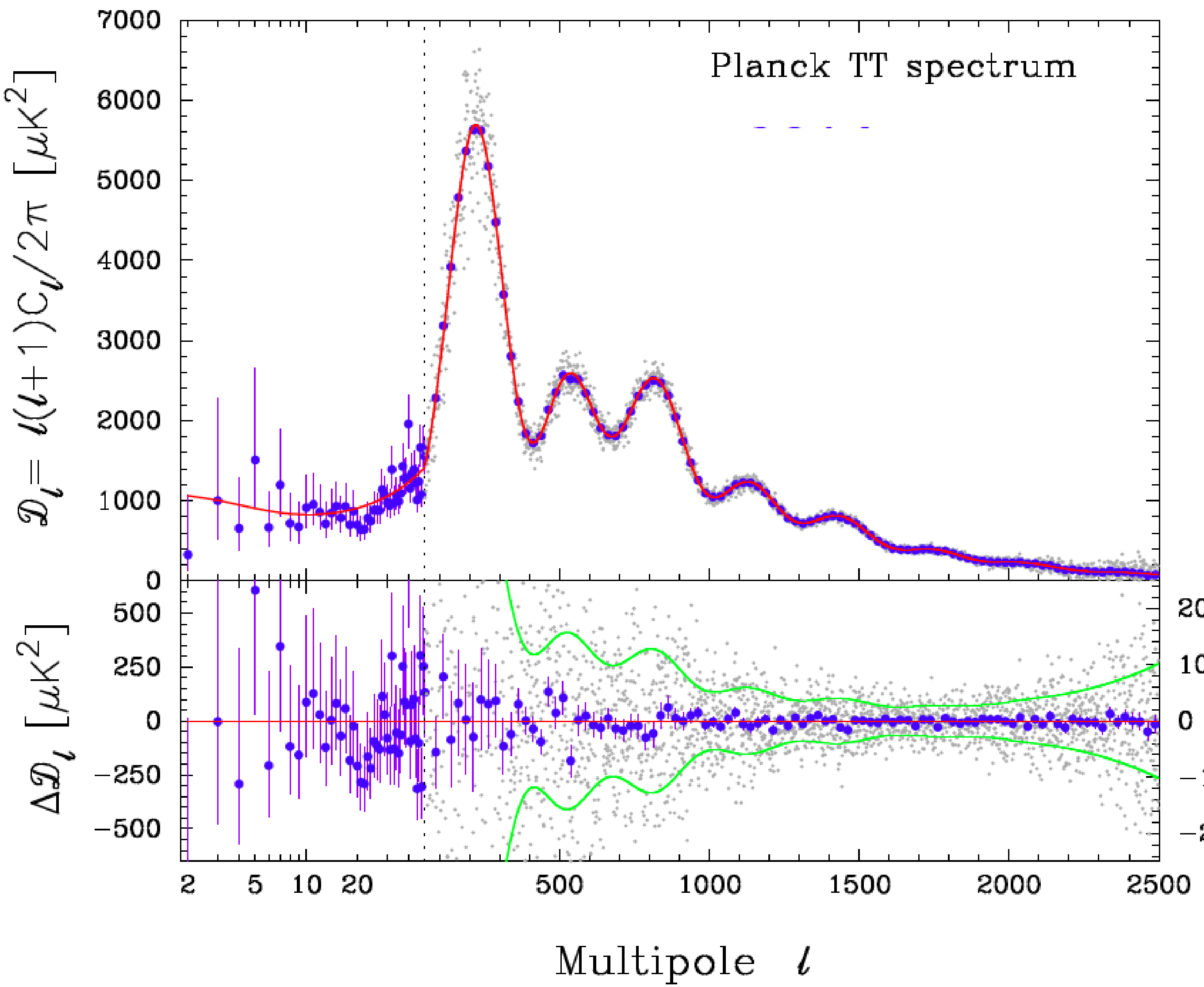
- **Tensor-to-scalar ratio**

$$r = \frac{\mathcal{P}_{\rm T}}{\mathcal{P}_\zeta} = 16\epsilon$$

- **Consistency relation** (valid for *all* single field slow-roll inflation, easily generalizable to non-canonical kinetic term)

$$r = -8n_T$$

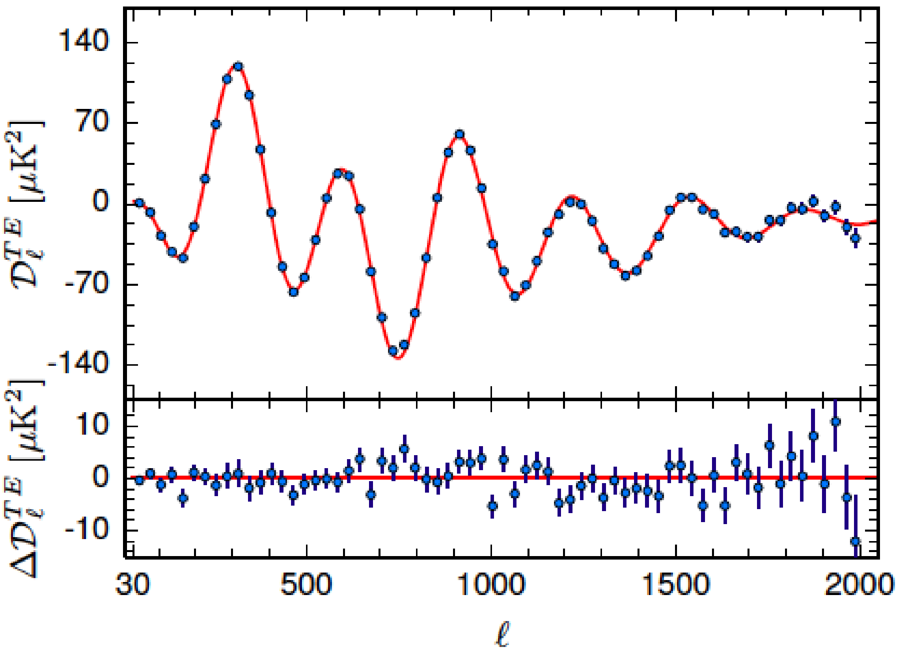
Planck 2015 TT-spectrum



and ... including polarization

Planck 2015

TE



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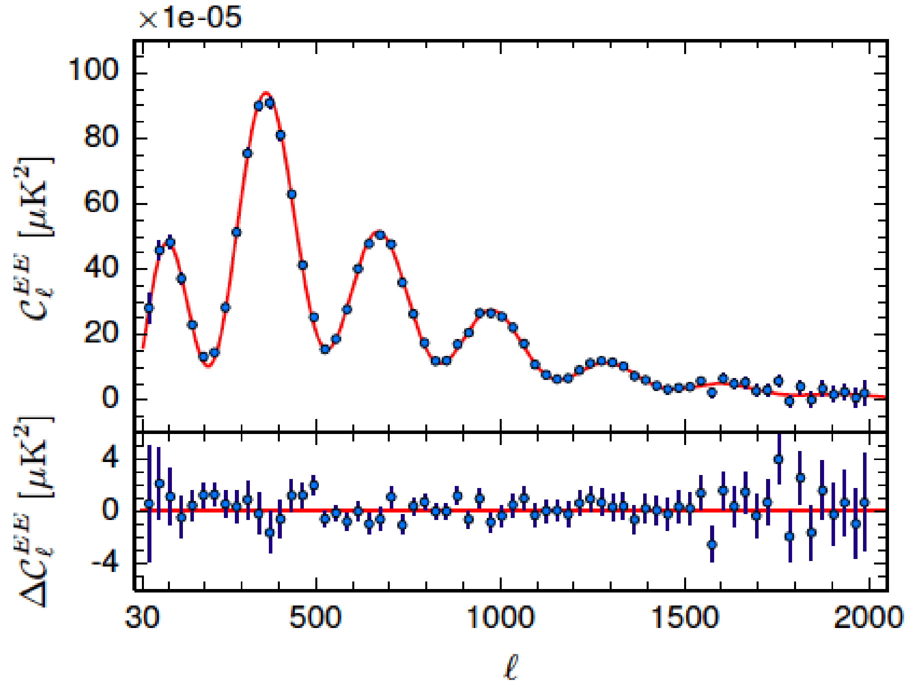
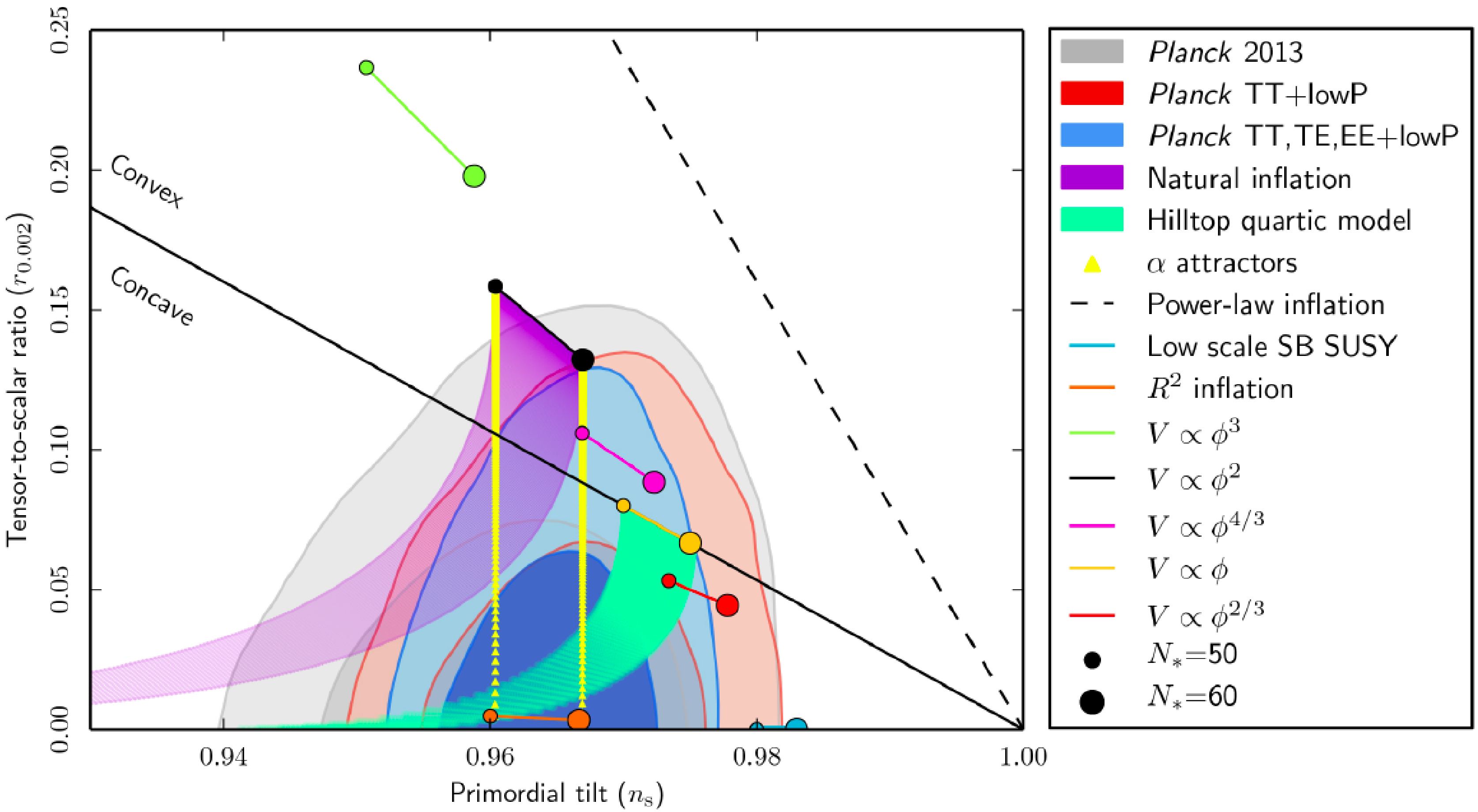


Planck 2015 constraints on inflation models



Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from Planck in combination with other datasets, vs. theoretical prediction of selected inflation models.

Inflation & cosmic observables

Parameter	Meaning	Physical Origin	Current Status
$A_{\rm s}$	Scalar amplitude	H, $\dot{H}$, $c_{\rm s}$	$(2.13 \pm 0.05) \times 10^{-9}$
$n_{\rm s}$	Scalar tilt	$\dot{H}$, $\ddot{H}$, $\dot{c}_{\rm s}$	0.965 ± 0.005
$\mathrm{d}n_{\rm s}/\mathrm{d}\ln k$	Scalar running	$\dddot{H}$, $\ddot{c}_{\rm s}$	only upper limits
$A_{\rm t}$	Tensor amplitude	H	only upper limits
$n_{\rm t}$	Tensor tilt	$\dot{H}$	only upper limits
r	Tensor-to-scalar ratio	$\dot{H}$, $c_{\rm s}$	only upper limits
$\Omega_{\rm k}$	Curvature	Initial conditions	only upper limits
$f_{\rm NL}$	Non-Gaussianity	Extra fields, sound speed, $\cdots$	only upper limits
S	Isocurvature	Extra fields	only upper limits
$G\mu$	Topological defects	End of inflation	only upper limits

Table 1: Summary of key parameters in inflationary cosmology, together with their likely physical origins and current observational constraints. At present, only upper limits exist for all parameters except $A_{\rm s}$ and $n_{\rm s}$ [5].

credits: Finelli et al. 2016

Primordial (i.e. inflationary) Non-Gaussianity

Why (non-) Gaussian?

Gaussian ⟺ free (i.e. non-interacting) field

large-scale phase coherence ⟺ non-linear gravitational dynamics

Testable predictions of inflation

- Cosmological aspects
  - Critical density Universe
  - Almost scale-invariant and **nearly Gaussian**, adiabatic density fluctuations
  - Almost scale-invariant stochastic background of relic gravitational waves
- Particle physics aspects
  - **Nature of the inflaton**
  - Inflation energy scale

PNG probes the physics of the Early Universe (a "cosmological collider")

- PNG amplitude and shape measures deviations from standard inflation, perturbation generating processes after inflation, initial state before inflation, ...
- Inflation models which would yield the same predictions for scalar spectral index and tensor-to-scalar ratio might be distinguishable in terms of PNG.
- Some specific features and shapes may also probe mass and spin of new particles.
- We should aim at "reconstructing" the inflationary action, starting from measurements of a few observables (like n_S, r, n_T, f_{NL}, g_{NL}, etc. ...), just like in the nineties we were aiming at a reconstruction of the inflationary potential (see e.g. revival of the latter industry after the Bicep2 claim of PGW detection, ...).

NG requires higher-order statistics (than the power-spectrum)

- The simplest statistics (but not fully general) measuring NG is the 3-point function or its Fourier transform, the "bispectrum":

$$<\phi(\mathbf{k}_1)\phi(\mathbf{k}_2)\phi(\mathbf{k}_3)> = (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\ B_\phi(k_1,k_2,k_3)$$

which carries shape information.

- In our simple linear + quadratic model above, the bispectrum of the gravitational potential reads:

$$B_\phi(k_1,k_2,k_3) = 2f_{NL}\ [P_\phi(k_1)P_\phi(k_2) + \text{cyclic terms}]$$

(by direct application of Wick's theorem), where

$$<\phi(\mathbf{k}_1)\phi(\mathbf{k}_2)> = (2\pi)^3\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2)\ P_\phi(k_1)$$

Bispectrum & Primordial non-Gaussianity (PNG)

- PNG probes fundamental physics during inflation, being sensitive to the ***interactions*** of fields present during inflation (different inflationary models predict different ***amplitudes and shapes*** of the bispectrum)

- Searching for deviations from this *standard paradigm* is interesting *per-se* for theoretically well-motivated models of inflation and, as shown in *Planck 2013 results, can* ***severely limit*** various classes of inflationary models beyond the simplest paradigm. PNG probes interactions among particles at inflation energy scales. See recent literature on probing string-theory via oscillatory PNG (Arkani-Hamed & Maldacena 2015 "Cosmological collider physics"; Silverstein 2017 "The dangerous irrelevance of string theory").

Where does NG come from (in standard inflation)?

- *Falk et al. (1993)* found $f_{NL} \sim \xi \sim \varepsilon^2$ (from non-linearity in the inflaton potential in a fixed de Sitter space) in the standard single-field slow-roll scenario

- *Gangui et al. (1994)*, using stochastic inflation found $f_{NL} \sim \varepsilon, \eta$ (from second-order gravitational corrections during inflation). *Acquaviva et al. (2003)* and *Maldacena (2003)* confirmed this estimate (up to numerical factors and momentum-dependent terms) with a full second-order approach. Weinberg extended the calculation of the bispectrum to 1-loop. One of these terms gives rise to the so-called "consistency relation", according to which found $f_{NL} = - 5/12(n_s-1)$ It has been shown that this term can be gauged away by a non-linear rescaling of coordinates, up to sub-leading terms. Hence the only residual term is proportional to ε i.e. to the amplitude of tensor modes.

Starting point: the curvature (or gravitational potential) bispectrum

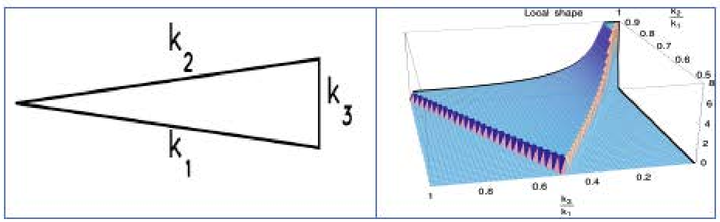
Bispectrum of primordial curvature perturbations

Amplitude

Shape

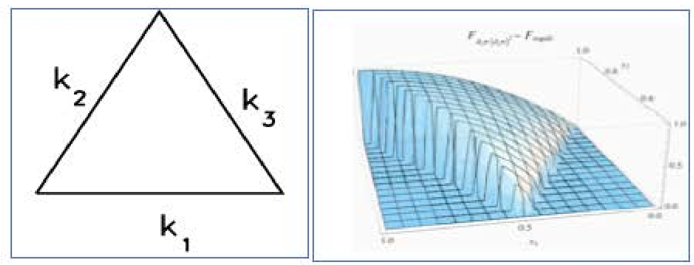
$$\langle \Phi(\vec{k}_1)\Phi(\vec{k}_2)\Phi(\vec{k}_3)\rangle = (2\pi)^3\delta^{(3)}(\vec{k}_1+\vec{k}_2+\vec{k}_3) f_{\rm NL} F(k_1,k_2,k_3)$$

Local NG



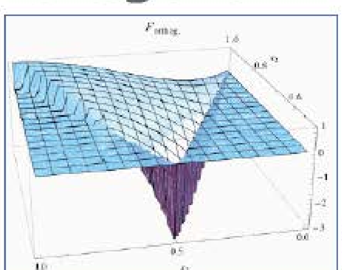
Multi-field models of inflation;
Cuvaton models;
Ekpyrotic/cyclic models

Equilateral NG



Single inflaton with non-standard kinetic term;
higher derivative interactions

Orthogonal NG



Single inflaton with non-standard kinetic term;
higher derivative interactions

Also: directionally dependent bispectra, tensor bispectra and many others.

Late nineties: simplest NG model

Many primordial (inflationary) models of non-Gaussianity can be represented in configuration space by the simple formula (Salopek & Bond 1990; Gangui et al. 1994; Verde et al. 1999; Komatsu & Spergel 2001)

$$\Phi = \phi_L + f_{NL} * (\phi_L^2 - \langle\phi_L^2\rangle) + g_{NL} * (\phi_L^3 - \langle\phi_L^2\rangle \phi_L) + \ldots$$

where Φ is the large-scale gravitational potential (more precisely $\Phi = 3/5\, \zeta$ on superhorizon scales, where ζ is the gauge-invariant comoving curvature perturbation), ϕ_L its linear Gaussian contribution and f_{NL} the dimensionless <u>*non-linearity parameter*</u> (or more generally *non-linearity function*). The percent of non-Gaussianity in CMB data implied by this model is

$$\text{NG \%} \sim 10^{-5} \;|f_{NL}|$$

< 10^{-5} from CMB & LSS

$$\sim 10^{-10} \;|g_{NL}|$$

< 10^{-5} from CMB & LSS

"non-Gaussian = non-dog"
(Ya.B. Zel'dovich)

Non-Gaussianity &
Cosmic Microwave Background (CMB)

Planck 2015 results XVII: *Planck* collaboration: A&A 594, A17 (2016)

PNG Planck project (Coordinators: S. Matarrese & B. Wandelt)

- Constrain (with high precision) and/or detect primordial non-Gaussianity (NG) as due to (non-standard) inflation (NG amplitude and shape measure deviations from standard inflation, perturbation generating processes after inflation, initial state before inflation, ...)

- We test: ***local, equilateral, orthogonal*** shapes (+ many more) for the bispectrum and constrain primordial trispectrum parameter g_{NL} (τ_{NL} constrained in previous release).

- Currently we are working at a final, *Planck* legacy release, which will improve the 2015 results in terms of more refined treatment of E-mode polarization (including lower and higher l.

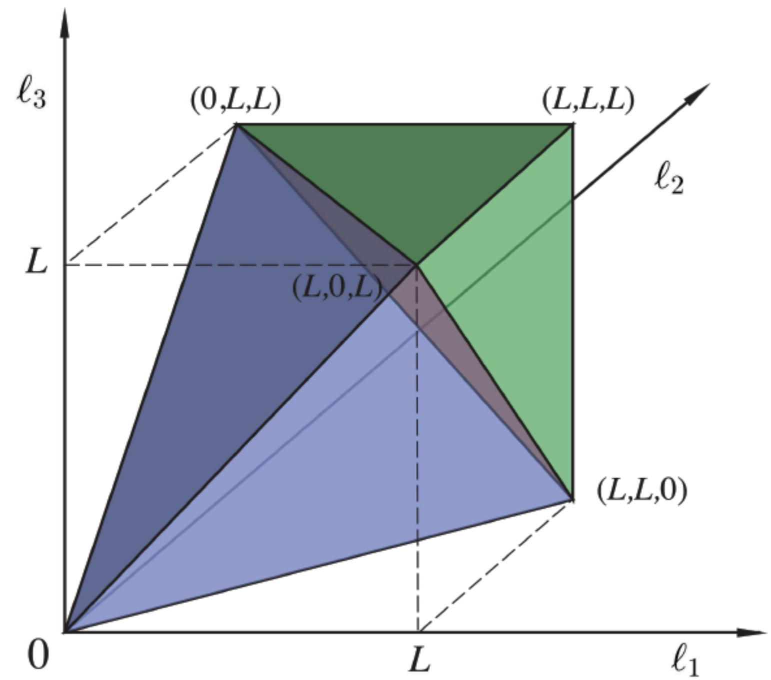
CMB bispectrum representation

$$B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} \equiv \langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle$$

$$= \mathcal{G}^{\ell_1 \ell_2 \ell_3}_{m_1 m_2 m_3} \, b_{\ell_1 \ell_2 \ell_3}$$

Gaunt integrals

$$\mathcal{G}^{\ell_1 \ell_2 \ell_3}_{m_1 m_2 m_3} \equiv \int Y_{\ell_1 m_1}(\hat{\boldsymbol{n}})\, Y_{\ell_2 m_2}(\hat{\boldsymbol{n}})\, Y_{\ell_3 m_3}(\hat{\boldsymbol{n}})\, d^2\hat{\boldsymbol{n}}$$
$$= h_{\ell_1 \ell_2 \ell_3} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix},$$

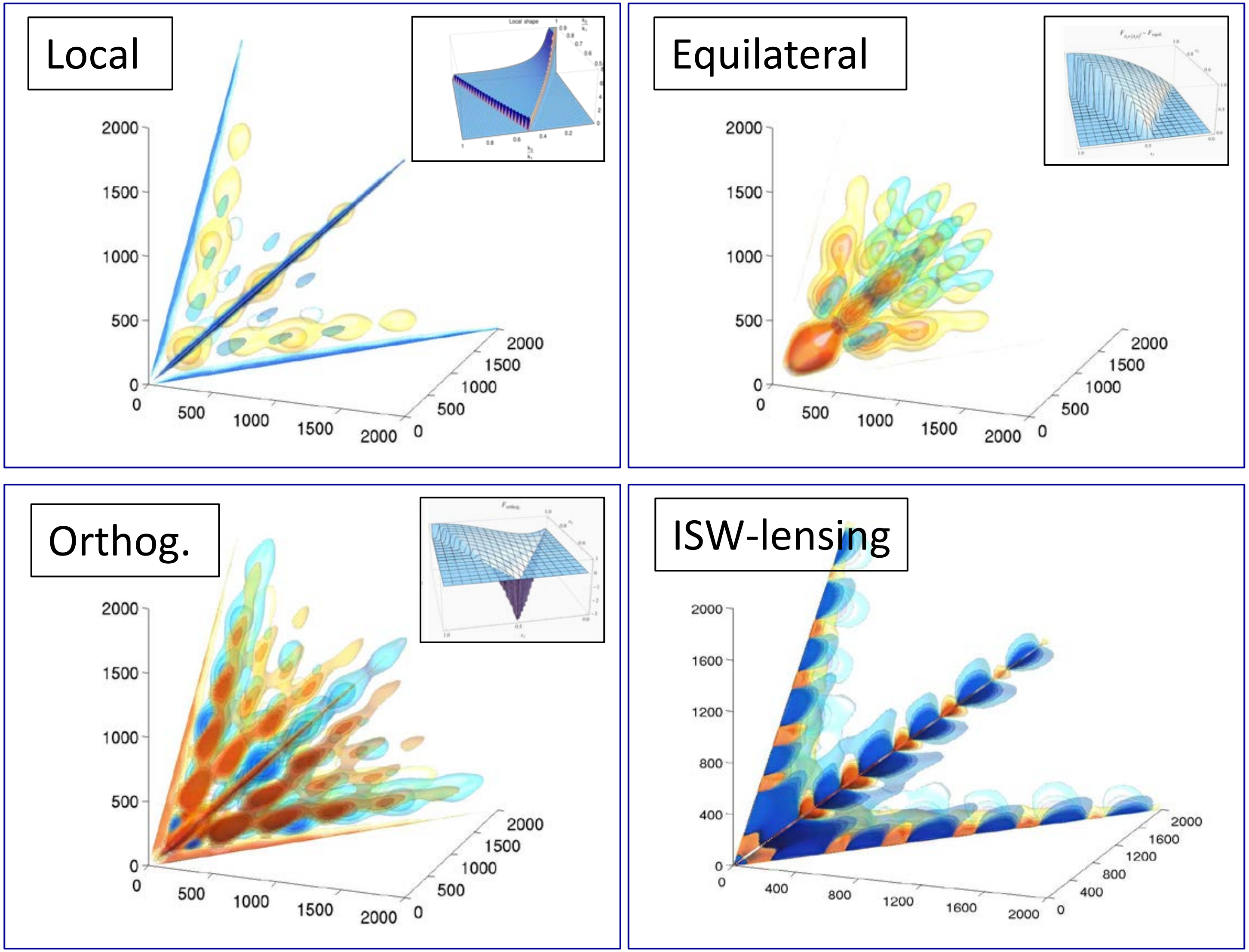


Triangle condition: $\ell_1 \leq \ell_2 + \ell_3$ for $\ell_1 \geq \ell_2, \ell_3$, +perms.

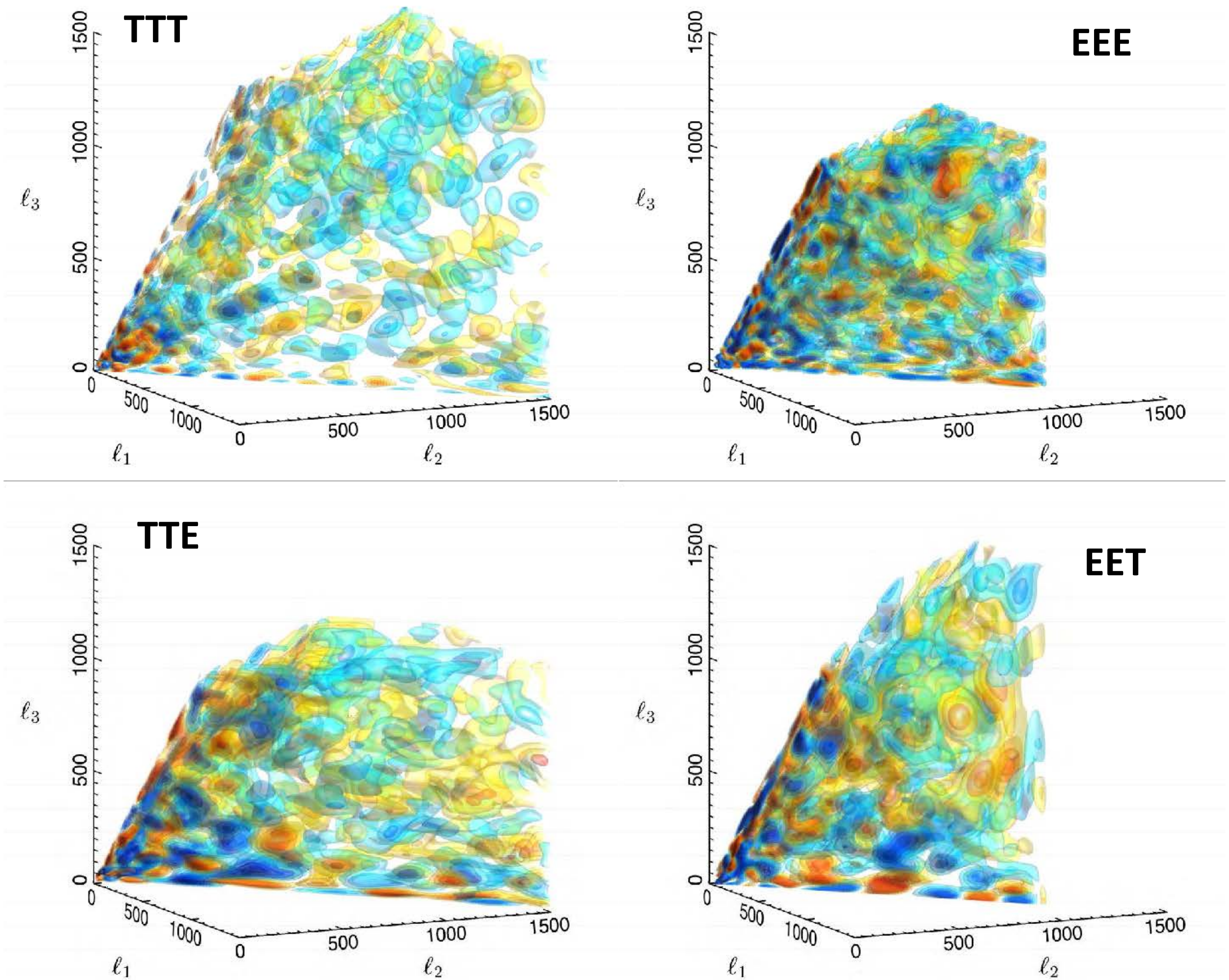
Parity condition: $\ell_1 + \ell_2 + \ell_3 = 2n\,, \quad n \in \mathbb{N}\,,$

Resolution: $\ell_1, \ell_2, \ell_3 \leq \ell_{\max}\,, \quad \ell_1, \ell_2, \ell_3 \in \mathbb{N}\,.$

Bispectrum shapes (modal representation)



The 2015 *Planck* bispectrum (modal)



(S/N weighted)

f_{NL} from *Planck* bispectrum (KSW)

Standard inflation still alive ... and in very good shape!

Standard inflation i.e.

- single scalar field (*single clock*)
- canonical kinetic term
- slow-roll dynamics
- Bunch-Davies initial vacuum state
- Einstein gravity

predicts tiny (up to $O(10^{-2})$) PNG signal

→ no (presently) detectable PNG

Beyond “standard” shapes

In 2015 we constrained f_{NL} for a large number of primordial models beyond the standard local, equilateral, orthogonal shapes, including

- ✓ Equilateral family (DBI, EFT, ghost)
- ✓ Flattened shapes (non-Bunch Davies)
- ✓ Feature models (oscillatory bispectra, scale-dependent)
- ✓ Direction dependence
- ✓ Quasi-single-field
- ✓ Parity-odd models

- No evidence for NG found, constraints on parameters from the models above

- Extended survey of feature models with respect to 2013, 600 -> 2000 modes, including polarization.

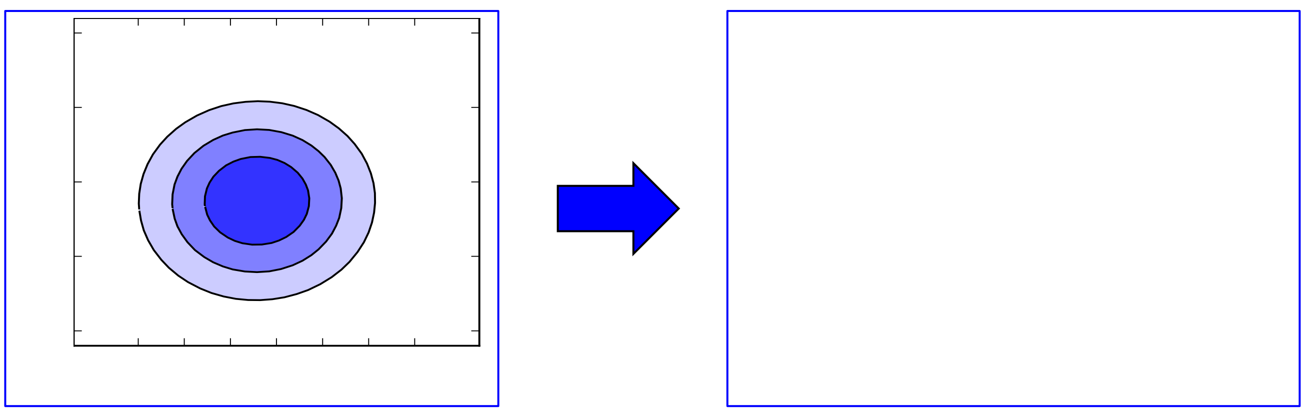
Implications for inflation

- No evidence for primordial NG of the local, equilateral, orthogonal type. consistent with the simplest scenario: standard single-field slow roll.

- Other possibilities are however not ruled out. Constraints on f_{NL} are converted into constraints on relevant model parameters, for example:

  - Curvaton decay fraction $r_D > 19\%$ (from local $f_{NL,}$ T+E)

  - Speed of sound in Effective Field Theory $c_S > 0.024$ (from equil. + ortho. f_{NL})



  - DBI inflation: $c_S > 0.087$ (T+E)

Non-Gaussianity &
Large-Scale Structure (LSS) of the Universe

(= primordial NG + NG from gravitational instability)

PNG and LSS

PNG in LSS (to make contact with the CMB definition) can be defined through a potential Φ defined starting from the DM density fluctuation δ through Poisson's equation (use comoving gauge for density fluctuation, Bardeen 1980)

$$\delta = -\left(\frac{3}{2}\Omega_m H^2\right)^{-1} \nabla^2 \Phi$$

Assuming the same model

$$\Phi = \phi_L + f_{NL}(\phi_L^2 - \langle \phi_L^2 \rangle) + g_{NL}(\phi_L^3 - \langle \phi_L^2 \rangle \phi_L) + \ldots$$

Φ on sub-horizon scales reduces to minus the large-scale gravitational potential, ϕ_L is the linear Gaussian contribution and f_{NL} and g_{NL} are dimensionless non-linearity parameters (or more generally non-linearity functions).

CMB and LSS conventions may differ by a factor 1.3 for f_{NL}, $(1.3)^2$ for g_{NL}

Searching for PNG with rare events

- Besides using standard statistical estimators, like (mass) bispectrum, trispectrum, three and four-point function, skewness, etc. …, one can look at the tails of the distribution, i.e. at rare events.

- Rare events have the advantage that they often maximize deviations from what predicted by a Gaussian distribution, but have the obvious disadvantage of being rare! But remember that, according to Press-Schechter-like schemes, all collapsed DM halos correspond to (rare) peaks of the underlying density field.

- Matarrese, Verde & Jimenez (2000) and Verde, Jimenez, Kamionkowski & Matarrese showed that clusters at high redshift ($z>1$) can probe NG down to $f_{NL} \sim 10^2$. Alternative approach by LoVerde et al. (2007). Determination of mass function using stochastic approach (first-crossing of a diffusive barrier) Maggiore & Riotto 2009. Ellispsoidal collapse used by Lam & Sheth 2009. Saddle-point + diffusive barrier (Paranjape et al. 2010). Log-Edgeworth expantion: LoVerde & Smith 2011. Excursion sets studied with correlated steps: Paranjape, Lam & Sheth 2011; Paranjape & Sheth 2011, ... and many, many more. Excellent agreement of analytical formulae with N-body simulations found by Grossi et al. 2009; Desjacques et al. 2009; Pillepich et al. 2010; … and many others afterwards.

- Halo (galaxy) clustering and halo (galaxy) higher-order correlation functions represent further and more powerful implementations of this general idea.

Bias: halos (→galaxies) do not trace the underlying (dark) matter distribution

- Following the original proposal by Kaiser (1984), introduced for galaxy clusters and later for galaxies, we are used to parametrize our ignorance about the way in which DM halos clusters in space w.r.t. the underlying DM, via some "bias" parameters, e.g. (Eulerian bias)

$$\delta_{halo}(x) = b_1\, \delta_{matter}(x) + b_2\, \delta^2_{matter}(x) + \ldots$$

- or via some non-linear and non-local expression (e.g. as a function of the Lagrangian position of the proto-halo center of mass.
- The resulting non-linear and non-local affects the statistical distribution of the halos introducing further NG effects.
- The various bias parameters can be generally dealt with either as purely phenomenological ones (i..e. to be fitted to observations) or predicted by a theory (e.g. Press-Schecter + Lagrangian PT).

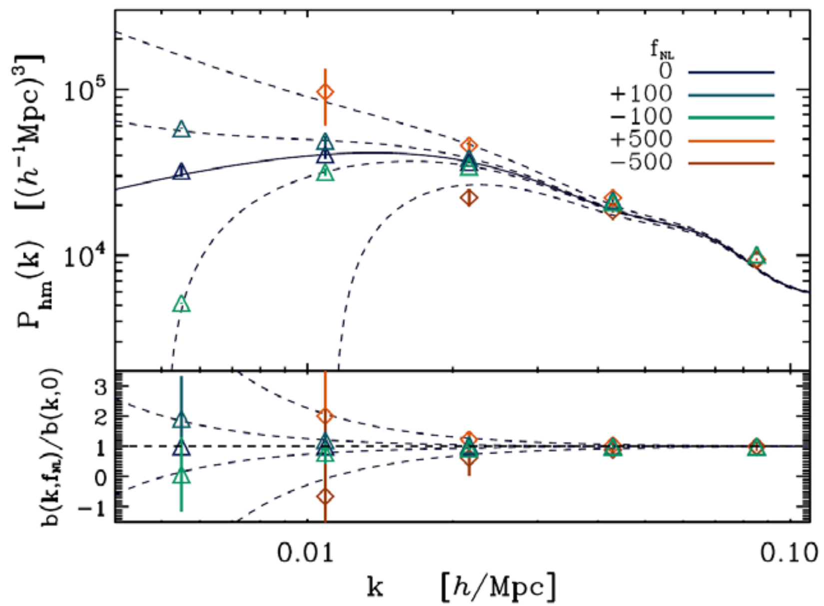
Dark matter halo clustering as a powerful constraint on PNG

$$\delta_{halo} = b\, \delta_{matter}$$

Dalal, Dore', Huterer & Shirokov 2007

Dalal et al. (2007) have shown that halo bias is sensitive to primordial non-Gaussianity through a scale-dependent correction term

$$\Delta b(k)/b \;\; \alpha \;\; 2\, f_{NL} \delta_c / k^2$$



This opens interesting prospects for constraining or measuring NG in LSS but demands for an accurate evaluation of the effects of (general) NG on halo biasing.

Clustering of peaks (DM halos) of NG density field

Start from results obtained in the 80's by Grinstein & Wise 1986, ApJ, 310, 19; Matarrese, Lucchin & Bonometto 1986, ApJ, 310, L21 giving the general expression for the peak 2-point function as a function of N-point connected correlation functions of the background linear (i.e. Lagrangian) mass-density field

$$\xi_{h,M}(|\mathbf{x}_1 - \mathbf{x}_2|) = -1 +$$

$$\exp\left\{ \sum_{N=2}^{\infty} \sum_{j=1}^{N-1} \frac{\nu^N \sigma_R^{-N}}{j!(N-1)!} \xi^{(N)} \left[\underbrace{\mathbf{x}_1,\ldots,\mathbf{x}_1}_{j\ times}, \underbrace{\mathbf{x}_2,\ldots\ldots,\mathbf{x}_2}_{(N-j)\ times} \right] \right\}$$

(requires use of path-integral, cluster expansion, multinomial theorem and asymptotic expansion). The analysis of NG models was motivated by a paper by Vittorio, Juszkiewicz and Davis (1986) on bulk flows.

THE ASTROPHYSICAL JOURNAL, 310:L21–L26, 1986 November 1

A PATH-INTEGRAL APPROACH TO LARGE-SCALE MATTER DISTRIBUTION ORIGINATED BY NON-GAUSSIAN FLUCTUATIONS

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ABSTRACT

The possibility that, in the framework of a biased theory of galaxy clustering, the underlying matter distribution be non-Gaussian itself, because of the very mechanisms generating its present status, is explored. We show that a number of contradictory results, seemingly present in large-scale data, in principle can recover full coherence, once the requirement that the underlying matter distribution be Gaussian is dropped. For example, in the present framework the requirement that the two-point correlation functions vanish at the same scale (for different kinds of objects) is overcome. A general formula, showing the effects of a non-Gaussian background on the expression of three-point correlations in terms of two-point correlations, is given.
Subject heading: galaxies: clustering

THE ASTROPHYSICAL JOURNAL, 310:19–22, 1986 November 1

NON-GAUSSIAN FLUCTUATIONS AND THE CORRELATIONS OF GALAXIES OR RICH CLUSTERS OF GALAXIES[1]

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ABSTRACT

Natural primordial mass density fluctuations are those for which the probability distribution, for mass density fluctuations averaged over the horizon volume, is independent of time. This criterion determines that the two-point correlation of mass density fluctuations has a Zeldovich power spectrum (i.e., a power spectrum proportional to k at small wavenumbers) but allows for many types of reduced (connected) higher correlations. Assuming galaxies or rich clusters of galaxies arise wherever suitably averaged natural mass density fluctuations are unusually large, we show that the two-point correlation of galaxies or rich clusters of galaxies can have significantly more power at small wavenumbers (e.g., a power spectrum proportional to $1/k$ at small wavenumbers) than the Zeldovich spectrum. This behavior is caused by the non-Gaussian part of the probability distribution for the primordial mass density fluctuations.
Subject headings: cosmology — galaxies: clustering

Halo bias in PNG models

- Matarrese & Verde 2008 applied this relation to the case of PNG of the gravitational potential, obtaining the power-spectrum of dark matter halos modeled as high "peaks" (up-crossing regions) of height $\nu=\delta_c/\sigma_R$ of the underlying mass density field (Kaiser's model). Here $\delta_c(z)$ is the critical overdensity for collapse (at redshift a) and σ_R is the *rms* mass fluctuation on scale R ($M \sim R^3$).

- Account for motion of peaks (going from Lagrangian to Eulerian space), which implies (Catelan et al. 1998)

$$1+\delta_h(\mathbf{x}_{Eulerian}) = (1+\delta_h(\mathbf{x}_{Lagrangian}))(1+\delta_R(\mathbf{x}_{Eulerian}))$$

and (to linear order) $b=1+b_L$ (Mo & White 1996) to get the scale-dependent halo bias in the presence of NG initial conditions. *Corrections may arise from second-order bias and GR terms.*

- Alternative approaches (e.g. based on 1-loop calculations) by Taruya et al. 2008; Matsubara 2009; Jeong & Komatsu 2009. Giannantonio & Porciani 2010 improve fit to N-body simulations by assuming dependence on gravitational potential) → extension to bispectrum by Baldauf et al. 2011. Leistedt et al. (2014) include g_{NL} and f_{NL} in analysis os QSO clustering.

Halo bias in PNG models

- Extension to general (scale and configuration dependent) NG is Straightforward (Matarrese & Verde 2008)

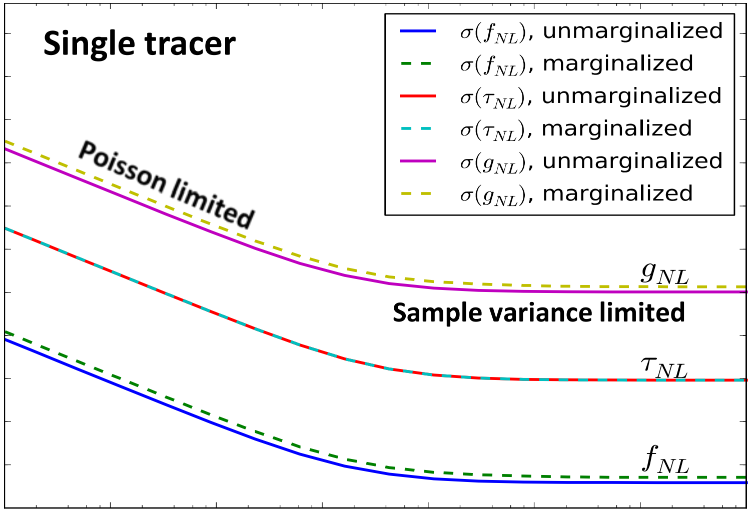
- In full generality write the f bispectrum as $B_f(k_1,k_2,k_3)$. The relative NG correction to the halo bias is

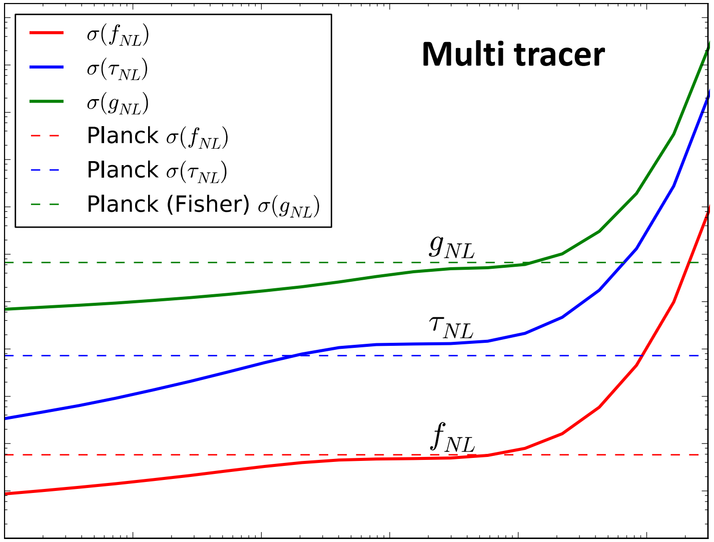
$$\frac{\Delta b_h}{b_h} = \frac{\Delta_c(z)}{D(z)} \frac{1}{8\pi^2\sigma_R^2} \int dk_1 k_1^2 \mathcal{M}_R(k_1) \times \int_{-1}^{1} d\mu \mathcal{M}_R\left(\sqrt{\alpha}\right) \frac{B_\phi(k_1,\sqrt{\alpha},k)}{P_\phi(k)} \times \frac{1}{M_R(k)}$$

$$\alpha = k_1^2 + k^2 + 2k_1 k\mu$$

- It also applies to non-local (e.g. “equilateral”) PNG (DBI, ghost inflation, etc..) and universal PNG term!! (→ see also Schmidt & Kamionkowski 2010).

PNG with LSS: 2-point function





Ferraro & Smith 2014

$$\Delta b(k) = 2(b-1) f_{NL} \delta_c \frac{3\Omega_m}{2a\, g(a) r_H^2 k^2}$$

- Single tracer, V = 25 Gpc^3 h^{-3}, statistical power ~ Planck
- Multi-tracer techniques have the power to reach σ_{fNL} ~ 1 (local)
- Significant degeneracies between f_{NL}, g_{NL}, τ_{NL}

Credits: M. Liguori

PNG with LSS: Bispectrum

Sample	Power Spectrum $\sigma_{f_{\rm NL}}$ bias float	Power Spectrum $\sigma_{f_{\rm NL}}$ bias fixed	Bispectrum $\sigma_{f_{\rm NL}}$ bias float	Bispectrum $\sigma_{f_{\rm NL}}$ bias fixed
BOSS	21.30	13.28	$1.04^{(0.65)}_{(2.47)}$	$0.57^{(0.35)}_{(1.48)}$
eBOSS	14.21	11.12	$1.18^{(0.82)}_{(2.02)}$	$0.70^{(0.48)}_{(1.29)}$
Euclid	6.00	4.71	$0.45^{(0.18)}_{(0.71)}$	$0.32^{(0.12)}_{(0.35)}$
DESI	5.43	4.37	$0.31^{(0.17)}_{(0.48)}$	$0.21^{(0.12)}_{(0.37)}$
BOSS + Euclid	5.64	4.44	$0.39^{(0.17)}_{(0.59)}$	$0.28^{(0.11)}_{(0.34)}$

Tellarini et al. 2016

- Fisher matrix forecast. Tree-level bispectrum. Local NG initial conditions. In redshift space. Covariance between different triangles neglected (optimistic!).
- Bispectrum could do better than power-spectrum.
- f_{NL} ~ 1 achievable with forthcoming surveys?
- Many issues, e.g. full covariance, accurate bias model, GR effects, survey geometry, estimator implementation … Still, great potential: 3D vs 2D (CMB).

GR effects on PS and bispectrum

- In full generality GR effects (including also redshift-space distortions, lensing, etc …) have to be taken into account both in the galaxy power-spectrum and bispectrum, as well as in the DM evolution.
- Recently, Bertacca, Raccanelli, Bartolo, Liguori, Matarrese & Verde (2017) obtained for the first time the complete expression for the galaxy bispectrum (which is obviously VERY complex) to be soon compared with observations.

Controversial issues on non-Gaussianity

Is the single-field consistency relation observable?

The cispectrum for single-field inflation (Gangui et al. 1995; Acquaviva et al. 2001; Maldacena 2001) can be represented as:

$$B_\zeta(k_1,k_2,k_3) \propto \frac{(\Delta_\zeta^2)^2}{(k_1k_2k_3)^2}\left[(1-n_s)\,S_{\rm loc.}(k_1,k_2,k_3) + \frac{5}{3}\varepsilon\,S_{\rm equil.}(k_1,k_2,k_3)\right]$$

$$n_s - 1 = -\eta - 2\varepsilon, \text{ with } \varepsilon \equiv -\frac{\dot{H}}{H^2},\ \eta \equiv \frac{\dot{\varepsilon}}{H\varepsilon}$$

The observability of the so-called "Maldacena consistency relation", related to the above bispectrum for single field inflation, in CMB and LSS data has led to a long-standing controversy. Recently, various groups have argued that the $(1\text{-}n_s)$ term is <u>totally unobservable </u>(for single-clock inflation), as, in the *<u>strictly squeezed limit </u>*(one of the wave-numbers going to 0), this term can be gauged away by a suitable coordinate tranformation. Cabass, Schmidt and Pajer (2017) argued that the term survives up to a "renormalization" which further reduces it by a factor of ~ 0.1 if one applies Conformal Fermi Coordinates to get rid of such a "gauge mode".

- Is this (CFC approach) the only way to deal with this term?
- Can we aim at an **<u>exact</u>** description, which is not affected by "spurious PNG"?

Observability of GR non-linearities

- In the halo bias case the effect is unobservable. Indeed, as pointed out by Dai, Pajer & Schmidt 2015 and de Putter, Doré & Green 2015, a local physical redefinition of the mass, gauges way such a NG effect (*in the pure squeezed limit*), similarly to Maldacena's $f_{NL} = - 5/12(n_s-1)$ single-field NG contribution ("consistency relation").

- This is true *provided the halo bias definition is strictly local*. Are there significant exceptions? Are all non-linear GR effects fully accounted for by "projection effects"?

- However, this dynamically generated GR non-linearity is physical and cannot be gauged away by any local mass-rescaling, provided it involves scales larger than the patch required to define halo bias, but smaller than the separation between halos (and the distance of the halo to the observer).

- Hence one would expect it to be in principle detectable in the matter bispectrum. Similarly, the observed galaxy bispectrum obtained via a full GR calculation must include all second-order GR non-linearities on such scales (*only as projection effects?*)

Concluding remarks

Short term goals

- Improve f_{NL} limits from CMB (*Planck*) with polarization & full data
- Look for more non-Gaussian shapes, scale-dependenf f_{NL}, etc. ...
- Make use of bispectrum in 3D data
- Improve constraints on g_{NL}

Long term goals

- reconstruct inflationary action
- if (quadratic) NG turns out to be small for all shapes go on and search for $f_{NL} \sim 1$ non-linear GR effects and second-order radiation transfer function contributions. For LSS resort to GR-based N-body simulations!

- ✓ Inflation provides a causal mechanism for the generation of cosmological perturbations

- ✓ CMB and LSS data fully support the detailed predictions of inflation

- ✓ The direct detection of:

  - ☞ primordial **gravitational waves**

  - ☞ primordial **non-Gaussianity**

  with the specific features predicted by inflation would provide strong independent support to the model