TRANSVERSITY 2017

Frascati - 11-15 December

A global fit of partonic Transverse Momentum Dependent distributions

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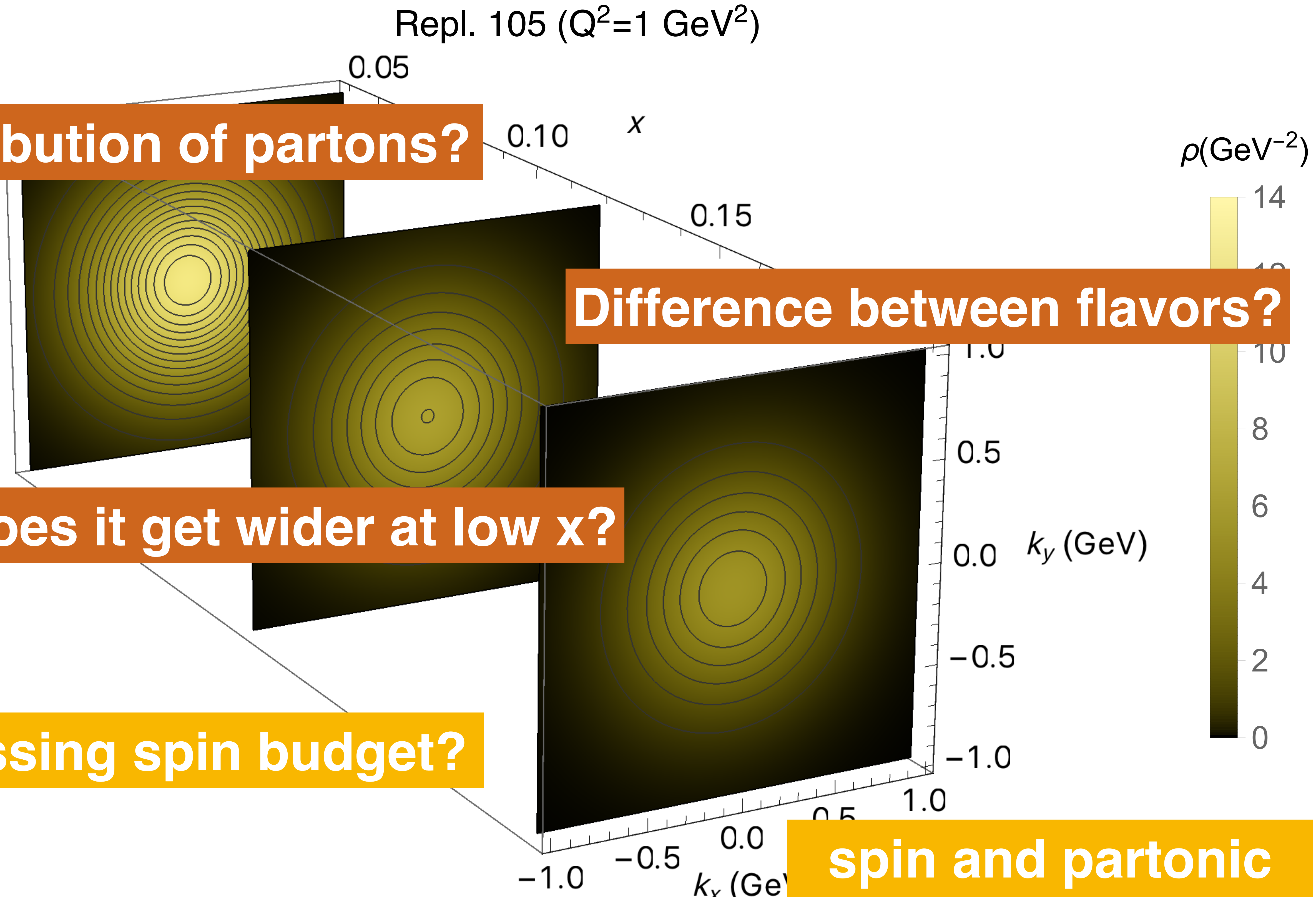
INFN

3DSPIN
MAPPING THE PROTON IN 3D

erc

3DSPIN: structure of the nucleon



distribution of partons?

Difference between flavors?

Does it get wider at low x?

missing spin budget?

spin and partonic motion correlation?

Transverse Momentum Distributions: PDF

quark pol.

nucleon pol.

	U	L	T
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1,\ h_{1T}^\perp$

Unpolarized

TMD Parton Distribution Functions
(TMD PDFs)

Transverse Momentum Distributions: PDF

quark pol. / nucleon pol.

Unpolarized

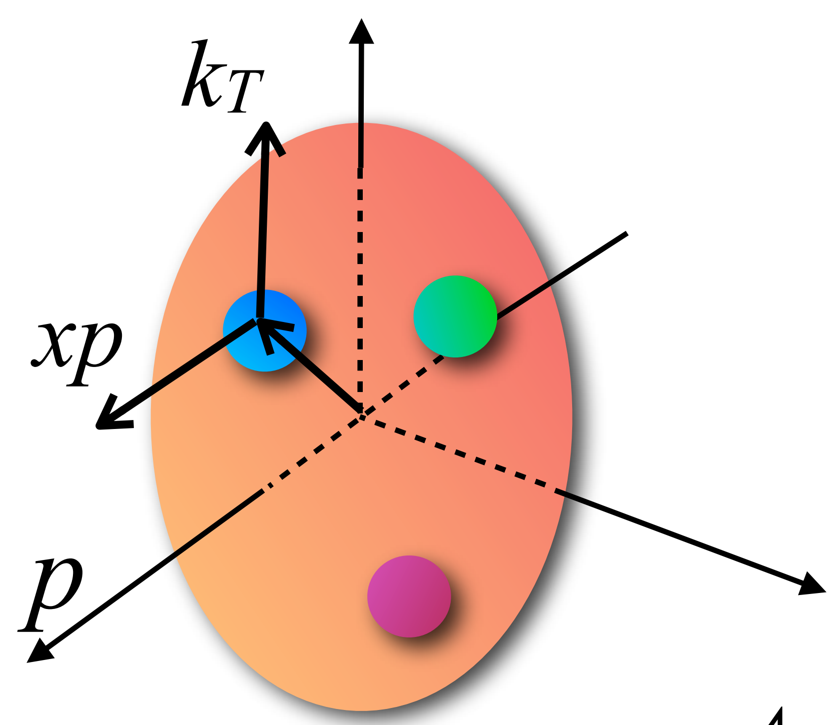
	U	L	T
U	f_1		$h_1^{\perp}$
L		g_{1L}	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	$h_1,\ h_{1T}^{\perp}$

dependence on:

longitudinal momentum fraction x

transverse momentum $\boldsymbol{k}_{\perp}$

energy scale



Why studying unpolarized TMDs?

nucleon tomography

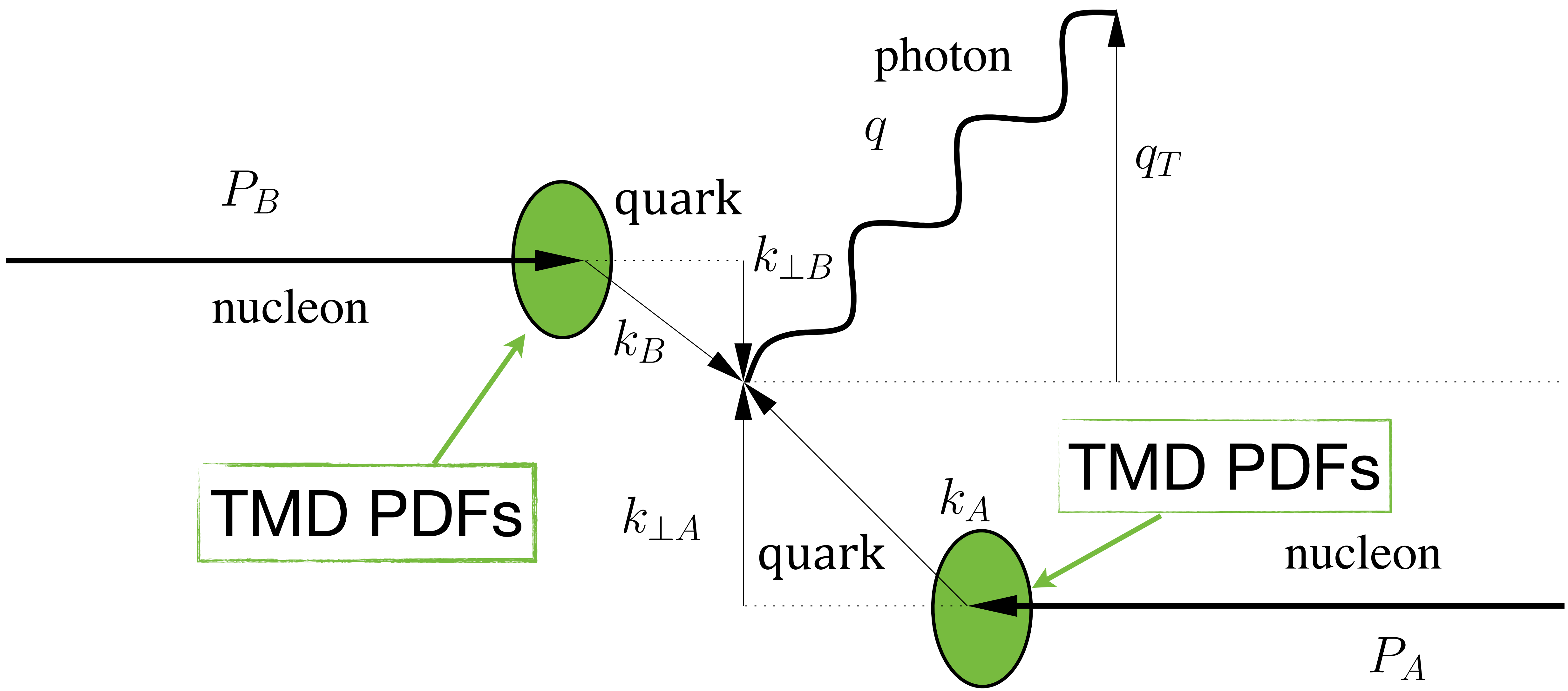
➢ improve our knowledge of 1D and 3D hadron structure
➢ have a reliable baseline to investigate polarized TMDs via spin asymmetries

High-energy phenomenology

➢ fundamental to predict qT spectra and to improve our investigations of BSM physics

Extraction from SIDIS & Drell-Yan

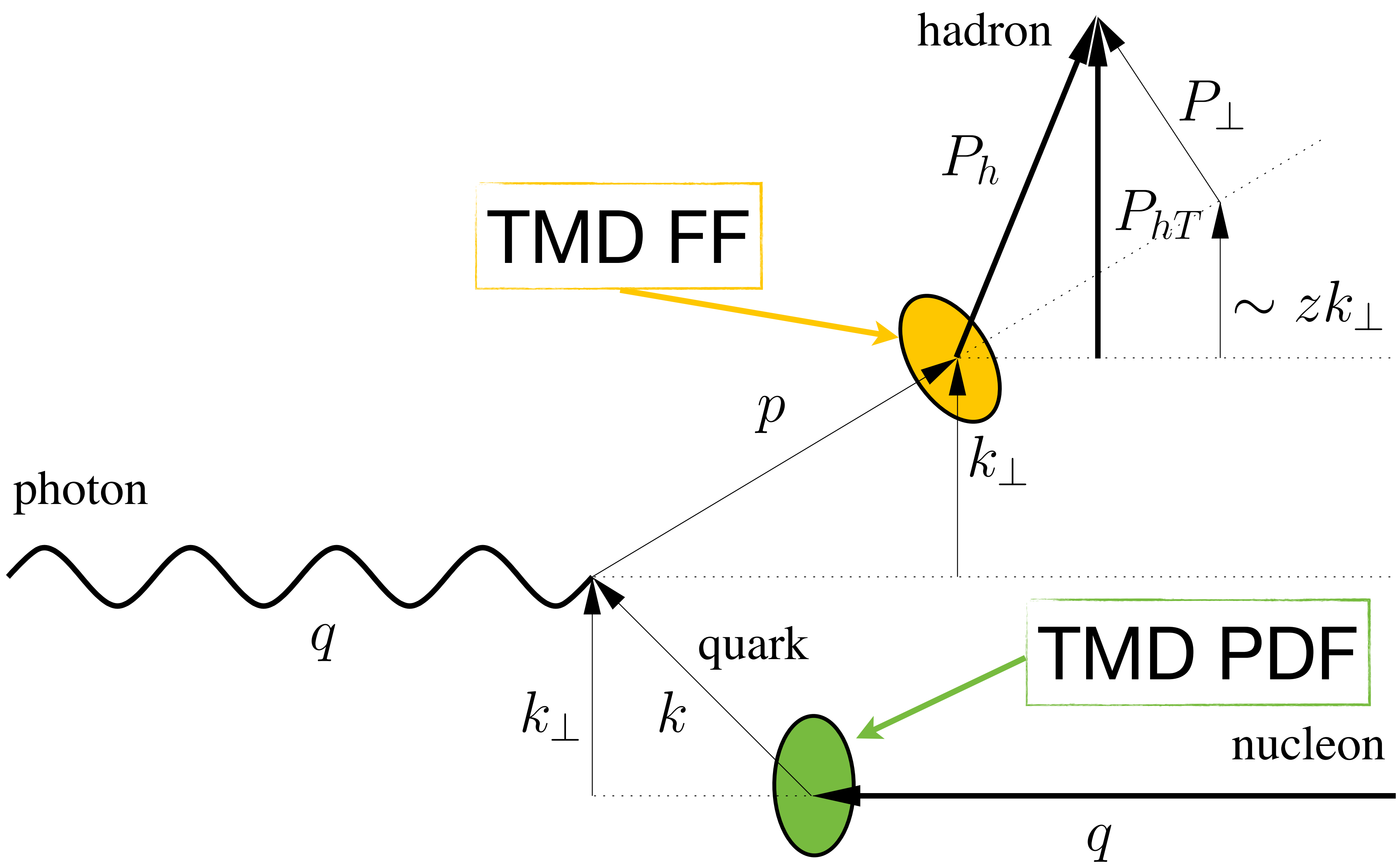
Drell-Yan \ Z production



$$A + B \to \gamma^* \to l^+ l^- \qquad A + B \to Z \to l^+ l^-$$

Extraction from SIDIS & Drell-Yan

Semi-inclusive Deep Inelastic Scattering



$$l(\ell) + N(\mathcal{P}) \to l(\ell') + h(\mathcal{P}_h) + X$$

TMDs: Fragmentation Function

quark pol.

U	L	T
D_1		$H_1^{\perp}$

Unpolarized

TMD Fragmentation Functions
(TMD FFs)

dependence on:

longitudinal momentum fraction z

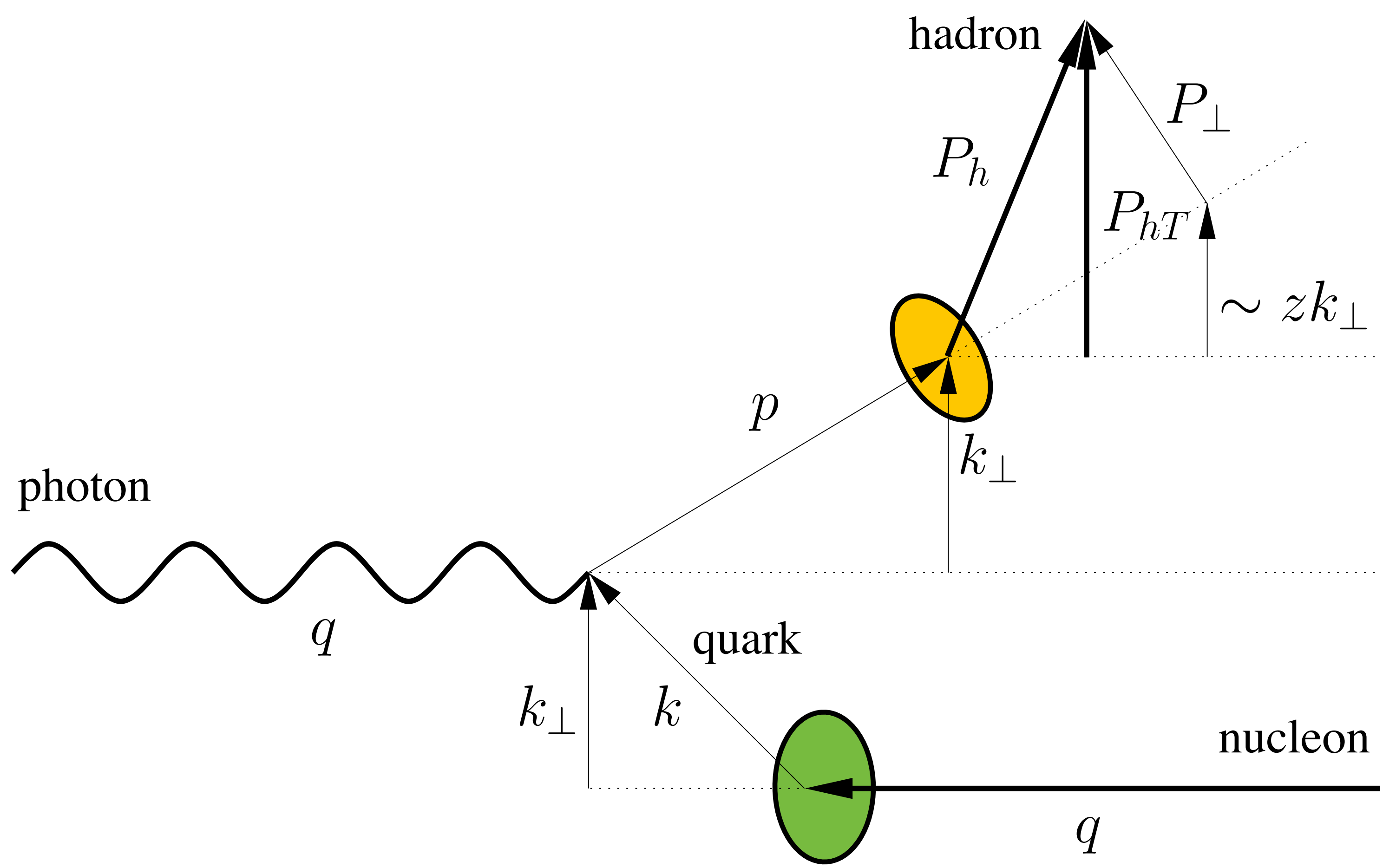
transverse momentum $\boldsymbol{P}_{\perp}$

energy scale

Structure functions and TMDs

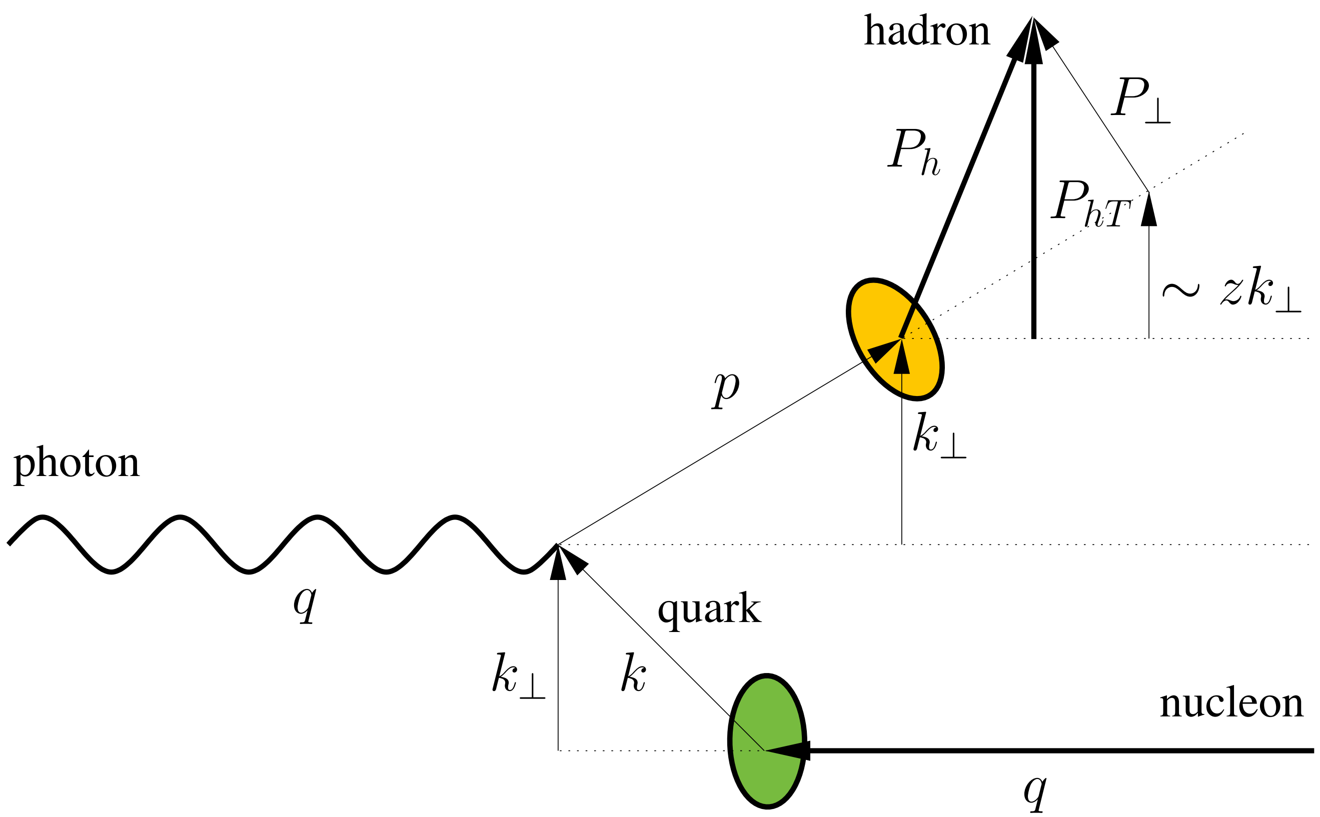
multiplicities

$$m_N^h\left(x,z,\boldsymbol{P}_{hT}^2,Q^2\right)=\frac{d\sigma_N^h/\left(dxdzd\boldsymbol{P}_{hT}^2dQ^2\right)}{d\sigma_{DIS}/\left(dxdQ^2\right)}\approx\frac{\pi F_{UU,T}\left(x,z,\boldsymbol{P}_{hT}^2,Q^2\right)}{F_T(x,Q^2)}$$



$$F_{UU,T}\left(x,z,P_{hT}^2,Q^2\right)=\sum_a\mathcal{H}_{UU,T}^a\left(Q^2;\mu^2\right)\int d^2k_T\,d^2P_T\,f_1^a\left(x,k_T^2;\mu^2\right)D_1^{h/a}\left(z,P_T^2;\mu^2\right)$$
$$\cdot\delta^2(zk_T-P_{hT}+P_T)+Y_{UU,T}(Q^2,P_{hT}^2)+\mathcal{O}\left(M^2/Q^2\right)$$

Structure functions and TMDs



$$F_{UU,T}\left(x,z,P_{hT}^2,Q^2\right)=\sum_a \mathcal{H}^a_{UU,T}\left(Q^2;\mu^2\right)\int d^2k_T\, d^2P_T\, f_1^a\left(x,k_T^2;\mu^2\right) D_1^{h/a}\left(z,P_T^2;\mu^2\right)$$

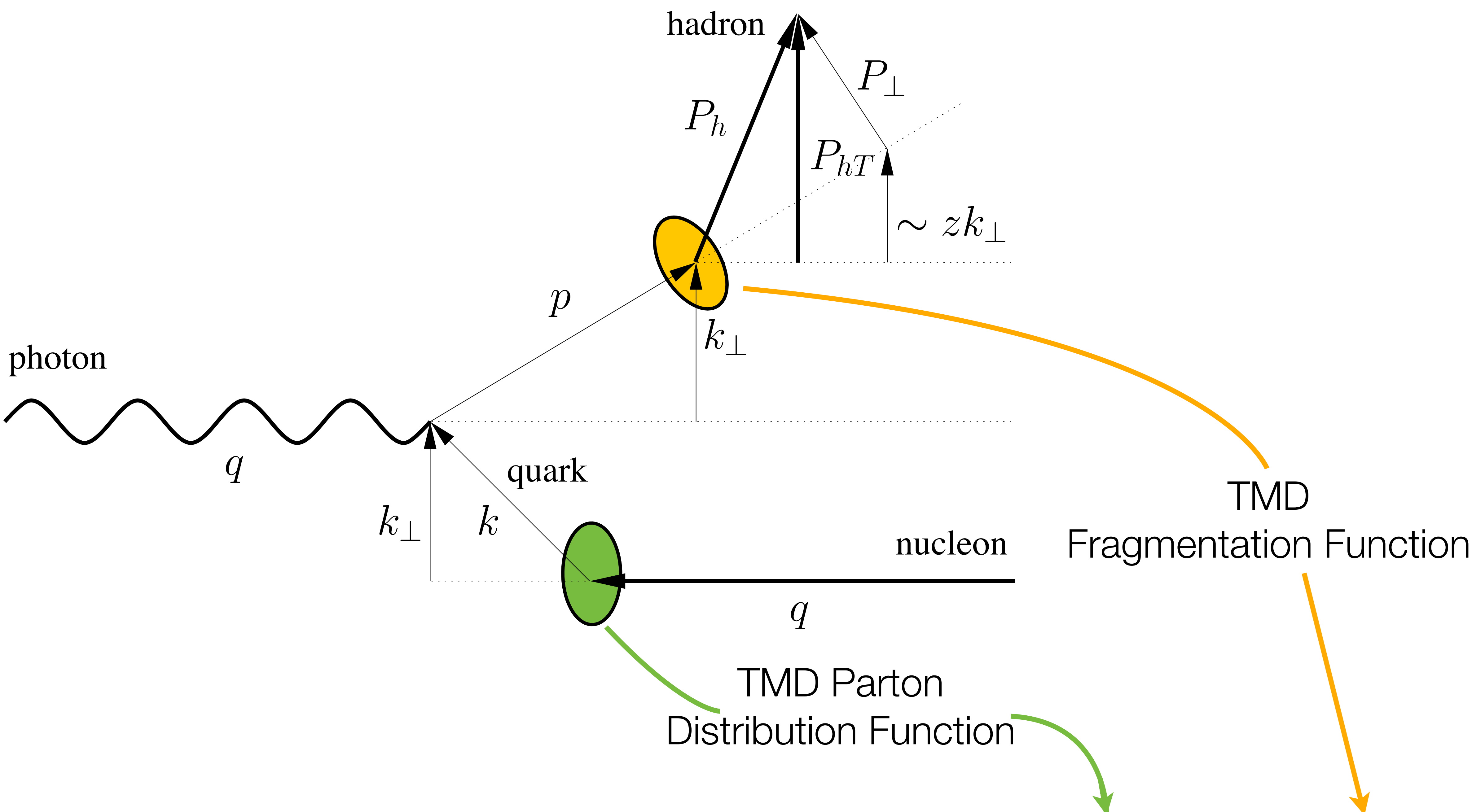
$$\cdot\delta^2(zk_T-P_{hT}+P_T)+Y_{UU,T}(Q^2,P_{hT}^2)+\mathcal{O}\left(M^2/Q^2\right)$$

At our accuracy level (LO-NLL):

$$\mathcal{H}_{UU,T}\simeq\mathcal{O}\left(\alpha_s^0\right)$$

$$Y_{UU,T}\left(Q^2,P_h^2T\right)\simeq 0$$

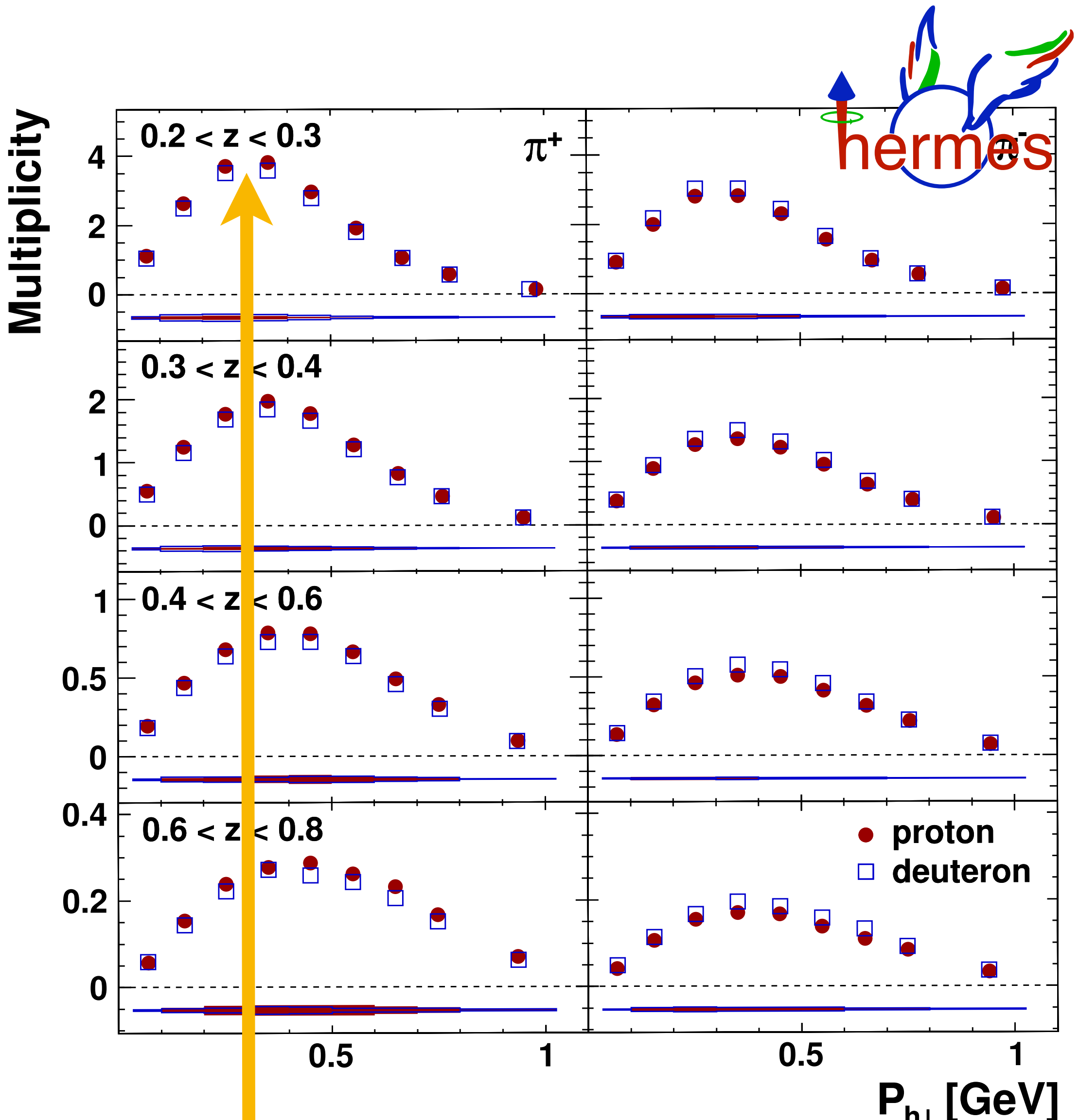
Structure functions and TMDs



$$F_{UU,T}\left(x,z,P_{hT}^2,Q^2\right) \simeq \sum_a \left(Q^2;\mu^2\right)\int d^2k_T\, d^2P_T\, f_1^a\left(x,k_T^2;\mu^2\right) D_1^{h/a}\left(z,P_T^2;\mu^2\right)$$
$$\cdot\delta^2\left(zk_T - P_{hT} + P_T\right)$$

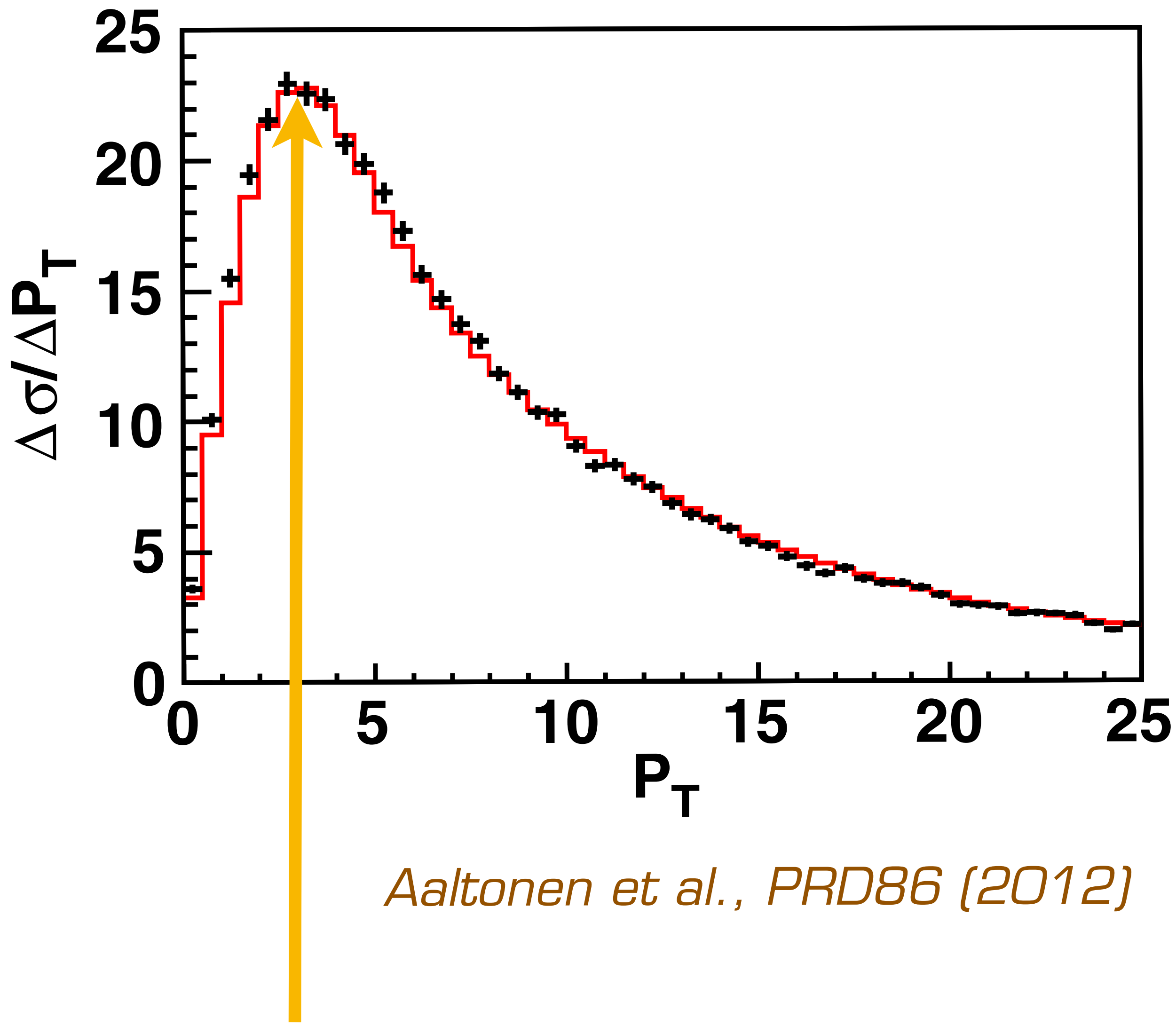
TMD Evolution

HERMES, Q ≈ 1.5 GeV



Airapetian et al., PRD87 (2013)

CDF, Q ≈ 91 GeV



Aaltonen et al., PRD86 (2012)

Width of TMDs changes of one order of magnitude → Evolution

Evolved TMDs

Fourier transform: ξ_T space

$$\tilde{f}_1^a\left(x,\xi_T;\mu^2\right) = \\ = \sum_i \left(\tilde{C}_{a/i}\otimes f_1^i\right)\left(x,\bar{\xi}_*;\mu_b\right)\, e^{\tilde{S}(\bar{\xi}_*;\mu_b,\mu)}\, e^{g_K(\xi_T)\ln(\mu/\mu_0)}\, \hat{f}^a_{NP}\left(x,\xi_T\right)$$

nonperturbative part of evolution

collinear PDF

pQCD

(Wilson Coefficient)

(Sudakov form factor)

non-perturbative part of TMD

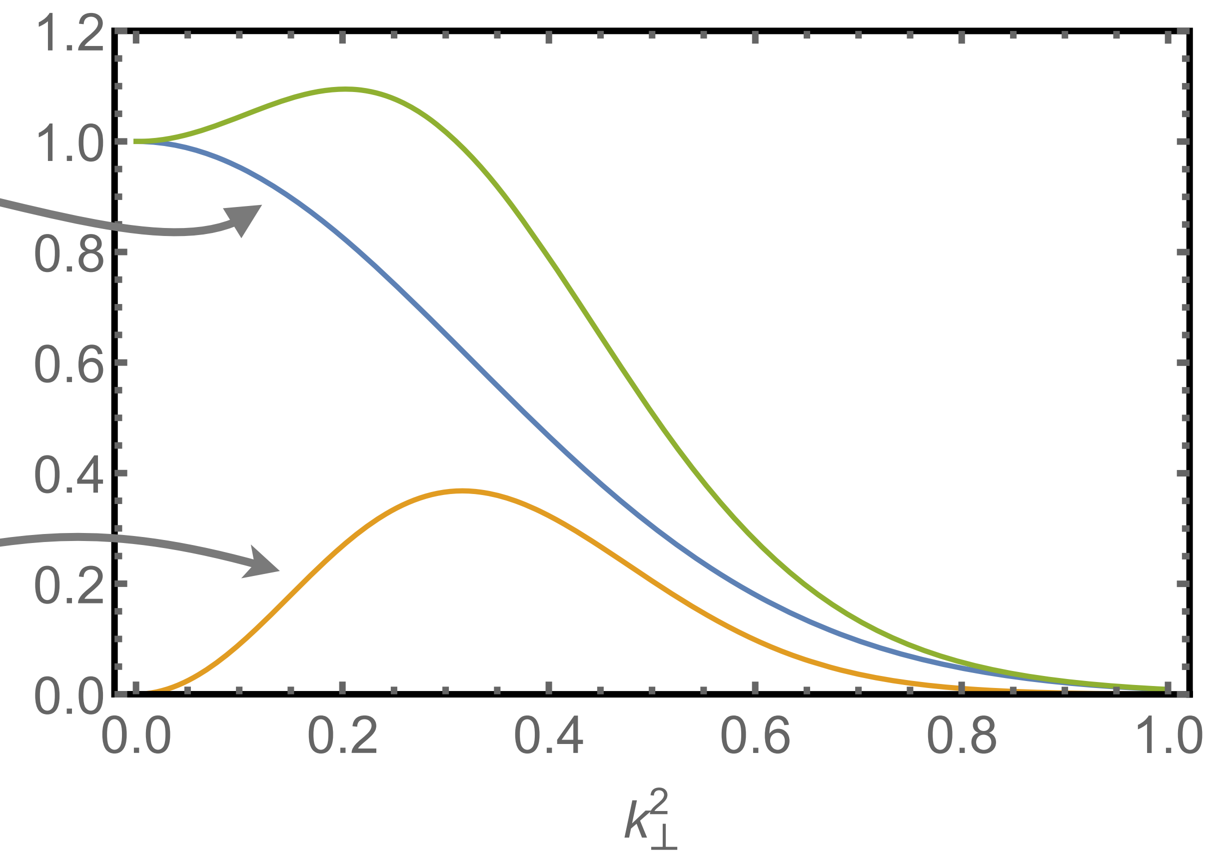
Non-perturbative contributions have to be extracted from experimental data, after parametrization

Model: non perturbative elements

input TMD PDF ($Q^2=1\,\mathrm{GeV}^2$)

$$\hat{f}^a_{NP} = \mathcal{F}.\mathcal{T}.\ \text{of}$$

$$\left(e^{-\frac{k_T^2}{g1a}} + \lambda k_T^2 e^{-\frac{k_T^2}{g1a}} \right)$$



sum of two different gaussians
with kinematic dependence on transverse momenta

width x-dependence

$$g_1(x) = N_1 \frac{(1-x)^\alpha\, x^\sigma}{(1-\hat{x})^\alpha\, \hat{x}^\sigma}$$

where

$$N_1 \equiv g_1(\hat{x})$$
$$\hat{x} = 0.1$$

Model: non perturbative elements

input TMD FF ($Q^2=1\,GeV^2$)

$$\hat{D}^{a\to h}_{1NP} = \text{ F.T. of } \quad \frac{1}{g_{3a\to h} + \left(\lambda_F/z^2\right) g^2_{4a\to h}} \left(e^{-\frac{\boldsymbol{P}^2_\perp}{g_{3a\to h}}} + \lambda_F \frac{\boldsymbol{P}^2_\perp}{z^2}\, e^{-\frac{\boldsymbol{P}^2_\perp}{g_{4a\to h}}} \right)$$

sum of two different gaussians
with different variance
with kinematic dependence on transverse momenta

width z-dependence

$$g_{3,4}(z) = N_{3,4}\, \frac{(z^\beta + \delta)\ (1-z)^\gamma}{(\hat{z}^\beta + \delta)\ (1-\hat{z})^\gamma}$$

where

$$N_{3,4} \equiv g_{3,4}(\hat{z})$$

$$\hat{z} = 0.5$$

Average transverse momenta

$$\langle \boldsymbol{k}^2_\perp \rangle(x) = \frac{g_1(x) + 2\lambda g_1^2(x)}{1 + \lambda g_1(x)}$$

$$\langle \boldsymbol{P}^2_\perp \rangle(z) = \frac{g_3^2(z) + 2\lambda_F g_4^3(z)}{g_3(z) + \lambda_F g_4^2(z)}$$

Model: non perturbative elements

Free parameters

$N_1, \alpha, \sigma, \lambda$ 4 for TMD PDF

$N_3, N_4, \beta, \delta, \gamma, \lambda_F$ 6 for TMD FF

$$g_K = -g_2 \frac{b_T^2}{2}$$

1 for NP contribution to TMD evolution

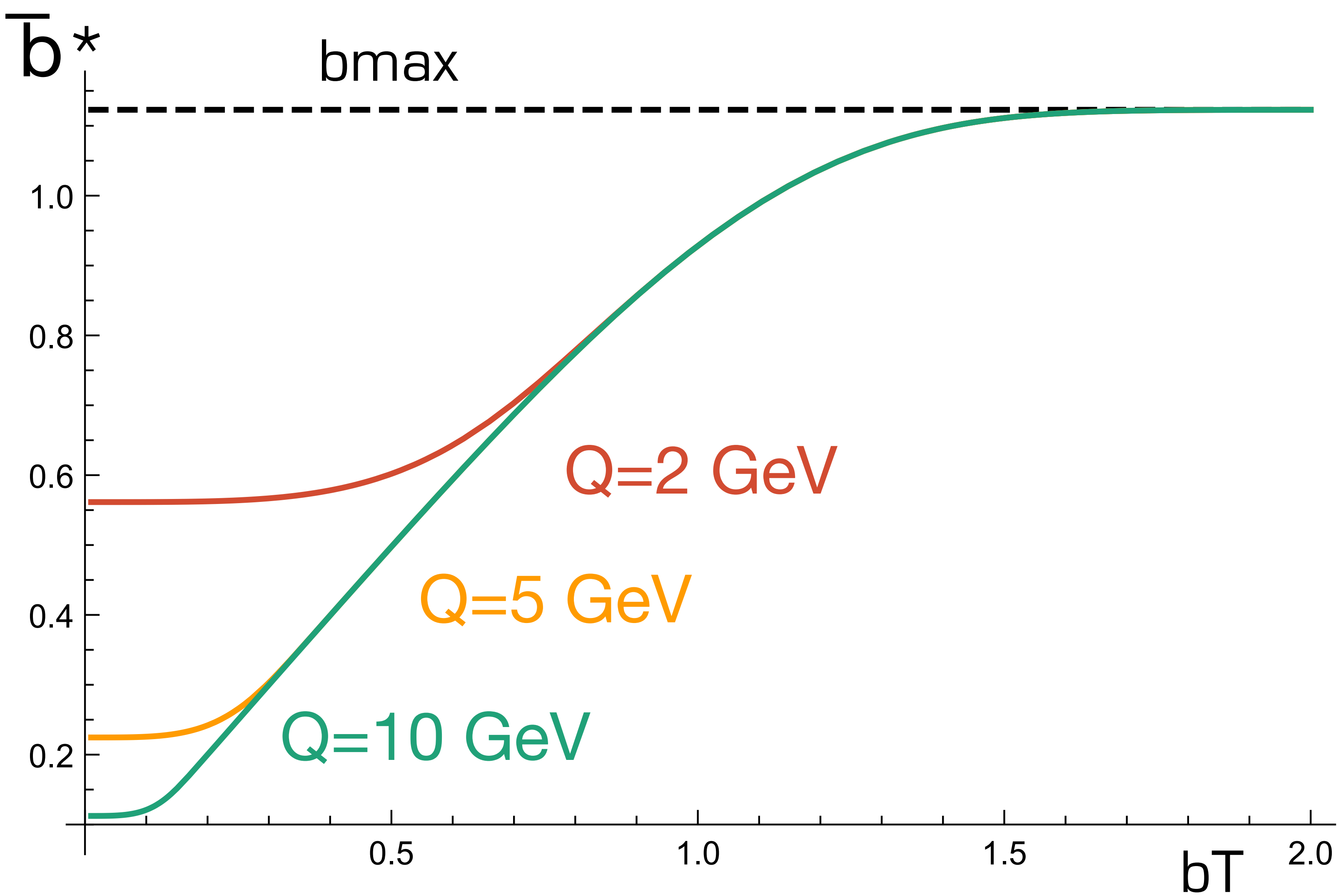
In total we have 11 parameters, for intrinsic transverse momentum (4 PDFs, 6 FFs) and evolution (g2)

Evolution and b_T regions

alternative notation: ξ_T

$$\mu_b = 2e^{-\gamma_E}/b_*$$

$$\bar{b}_\star(b_T; b_{\min}, b_{\max}) = b_{\max}\left(\frac{1-e^{-b_T^4/b_{\max}^4}}{1-e^{-b_T^4/b_{\min}^4}}\right)^{1/4}$$



$$b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = 2e^{-\gamma_E}/Q$$

The phenomenological importance of bmin is a signal that, especially in SIDIS data at **low Q,** we are exiting the proper TMD region and approaching the region of collinear factorization

Experimental data

COMPASS SIDIS μN

6252
data points

E288 E605 Drell-Yan

203
data points

hermes SIDIS eN

1514
data points

DØ CDF Z Production

90
data points

Data selection and analysis

$Q2 > 1.4$ GeV^2

$0.2 < z < 0.7$

P_{hT} , q_T < Min[0.2Q , 0.7Qz] + 0.5 GeV

Motivations behind kinematical cuts

TMD factorization ($Ph_T/z << Q^2$)

Avoid target fragmentation (low z)

and exclusive contributions (high z)

Experimental data

SIDIS μN

6252
data points

E288

E605

Drell-Yan

203
data points

Total: 8059 data

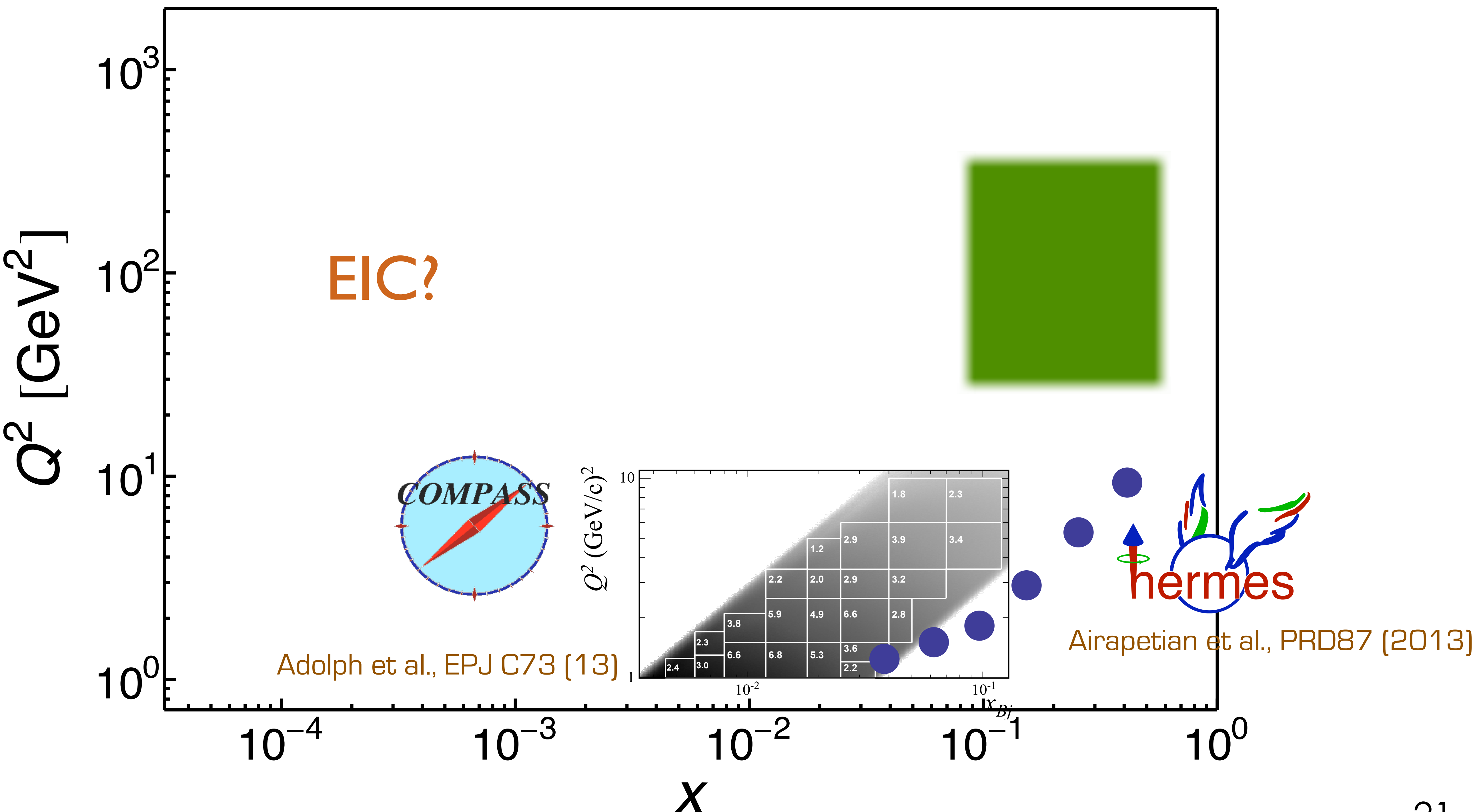
SIDIS eN

1514
data points

Z Production

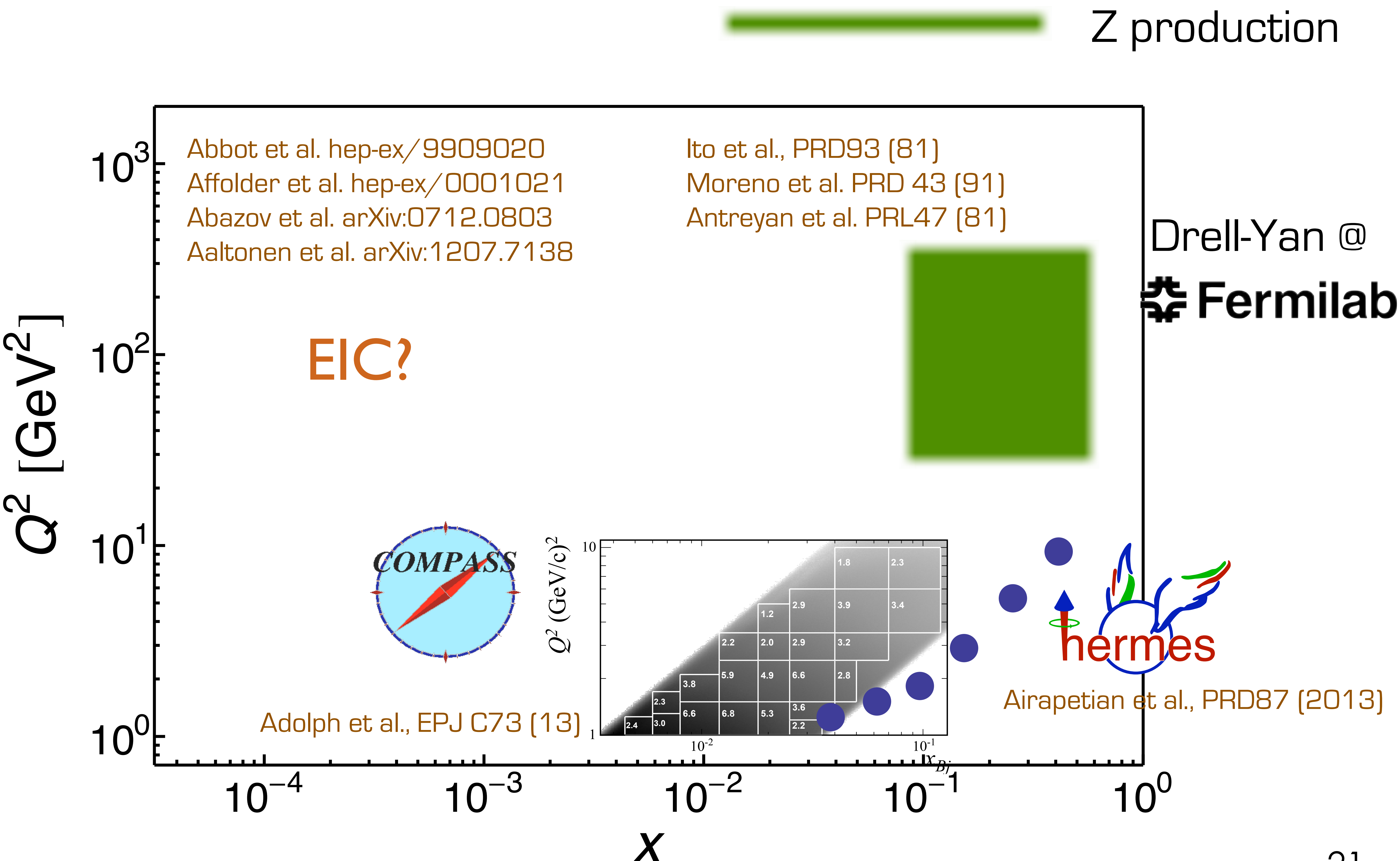
90
data points

Data region



Q^2 [GeV^2]
10^3
10^2
10^1
10^0
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0
x
EIC?
COMPASS
Adolph et al., EPJ C73 (13)
hermes
Airapetian et al., PRD87 (2013)

Data region



Z production
Drell-Yan @
Fermilab
Abbot et al. hep-ex/9909020
Affolder et al. hep-ex/0001021
Abazov et al. arXiv:0712.0803
Aaltonen et al. arXiv:1207.7138
Ito et al., PRD93 (81)
Moreno et al. PRD 43 (91)
Antreyan et al. PRL47 (81)
EIC?
COMPASS
Adolph et al., EPJ C73 (13)
hermes
Airapetian et al., PRD87 (2013)
Q^2 [GeV^2]
10^0
10^1
10^2
10^3
x
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^0

	Framework	**SIDIS** HERMES	**SIDIS** COMPASS	**DY**	**Z production**	**# points**
KN 2006	NLL/NLO	✗	✗	✓	✓	98
Pavia 2013	No Evo	✓	✗	✗	✗	1539
Torino 2014	No Evo	✓ (separately)	✓ (separately)	✗	✗	576 (H) 6284 (C)
DEMS 2014	NNLL/NLO	✗	✗	✓	✓	223
Pavia 2017	NLL/LO	✓	✓	✓	✓	8059
SV 2017	NNLL/NNLO	✗	✗	✓	✓	309

An almost global fit

	Framework	HERMES	COMPASS	DY	Z production	N of points
Pavia 2017 (+ JLab)	LO-NLL	✔	✔	✔	✔	8059

Features

heading towards a **global fit** of quark **unpolarized TMDs**

Flexible functional form (beyond gaussians)

includes TMD **evolution**

replica methodology

An almost global fit

	Framework	HERMES	COMPASS	DY	Z production	N of points
Pavia 2017 (+ JLab)	LO-NLL	✔	✔	✔	✔	8059

Cons

no “pure” info on TMD FFs

(would require e+e- annihilation)

TMD accuracy: not the state of the art (LO-NLL)

still undetermined flavor dependence

An almost global fit

	Framework	HERMES	COMPASS	DY	Z production	N of points
Pavia 2017 (+ JLab)	LO-NLL	✔	✔	✔	✔	8059

[JHEP06(2017)081]

Summary of results

Total number of data points: 8059

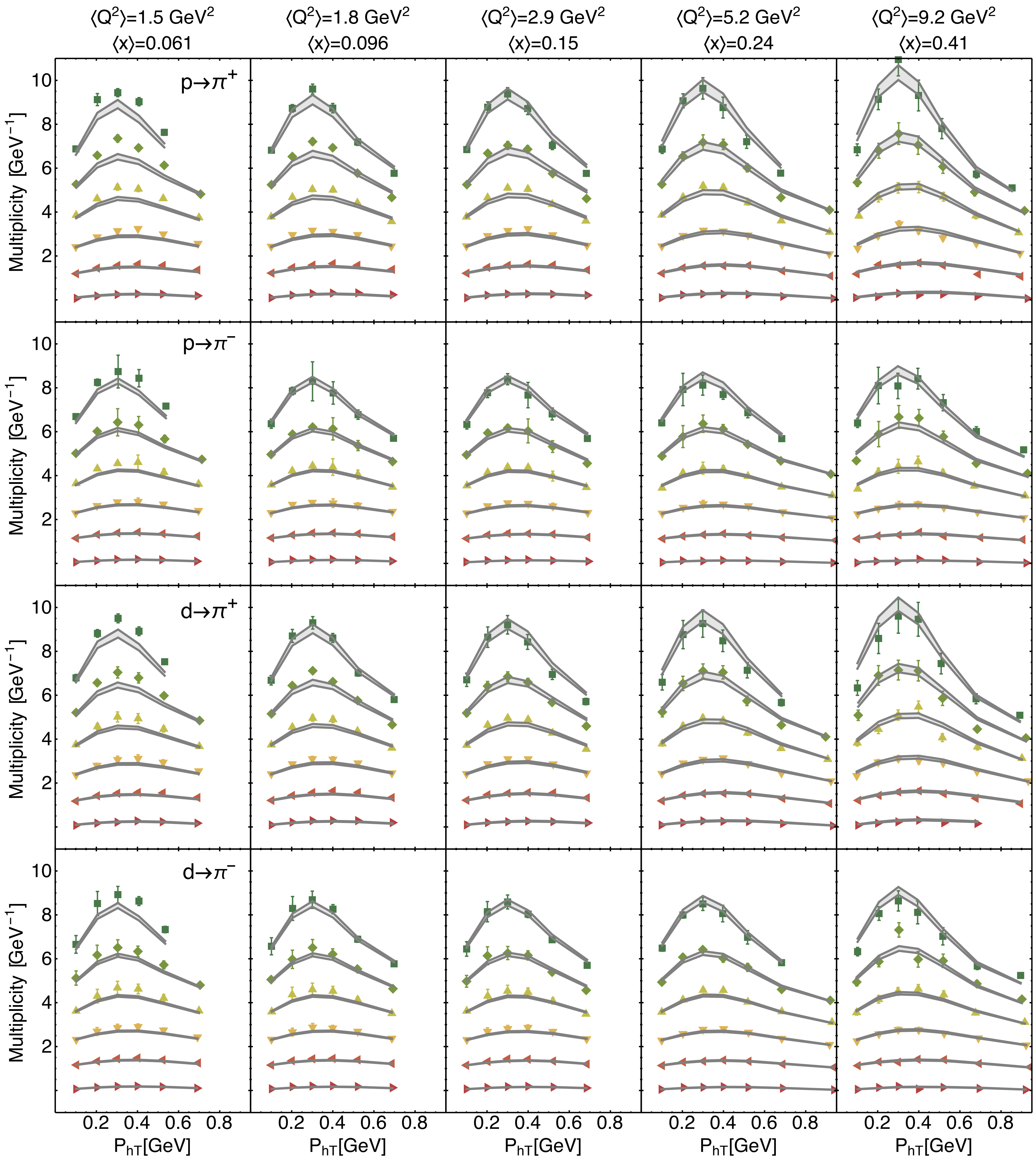
Total number of free parameters: 11

➛ 4 for TMD PDFs ➛ 6 for TMD FFs

➛ 1 for TMD evolution

$$\chi^2/d.o.f. = 1.55 \pm 0.05$$

Hermes data
pion production



π

χ^2/dof

4.83

2.47

3.46

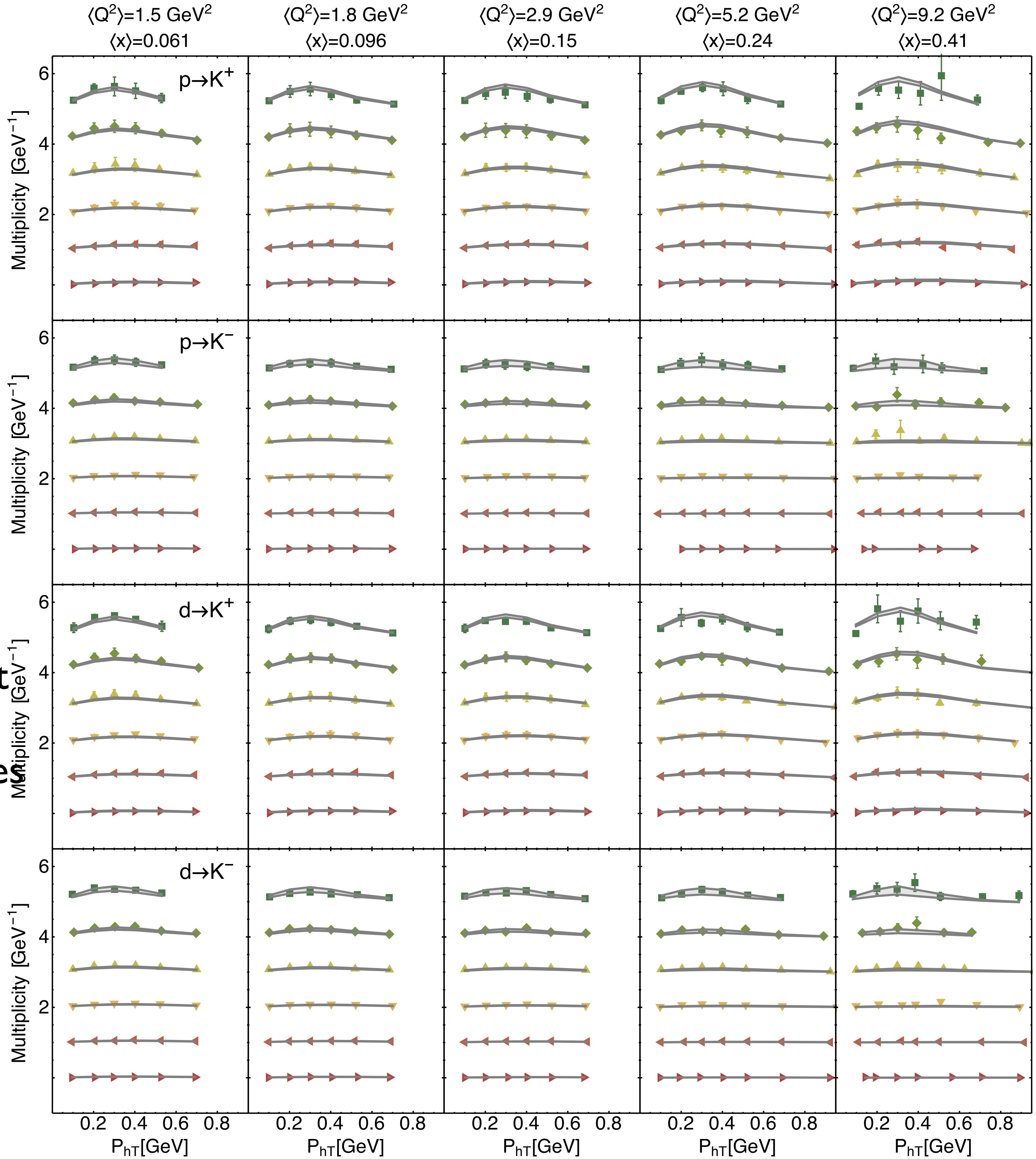
2.00

Hermes data

kaon production

hermes

K



better agreement than pions: larger uncertainties form FFs

- ⟨z⟩=0.24 (offset=5)
- ⟨z⟩=0.28 (offset=4)
- ⟨z⟩=0.34 (offset=3)
- ⟨z⟩=0.43 (offset=2)
- ⟨z⟩=0.54 (offset=1)
- ⟨z⟩=0.70 (offset=0)

χ^2/dof

0.91

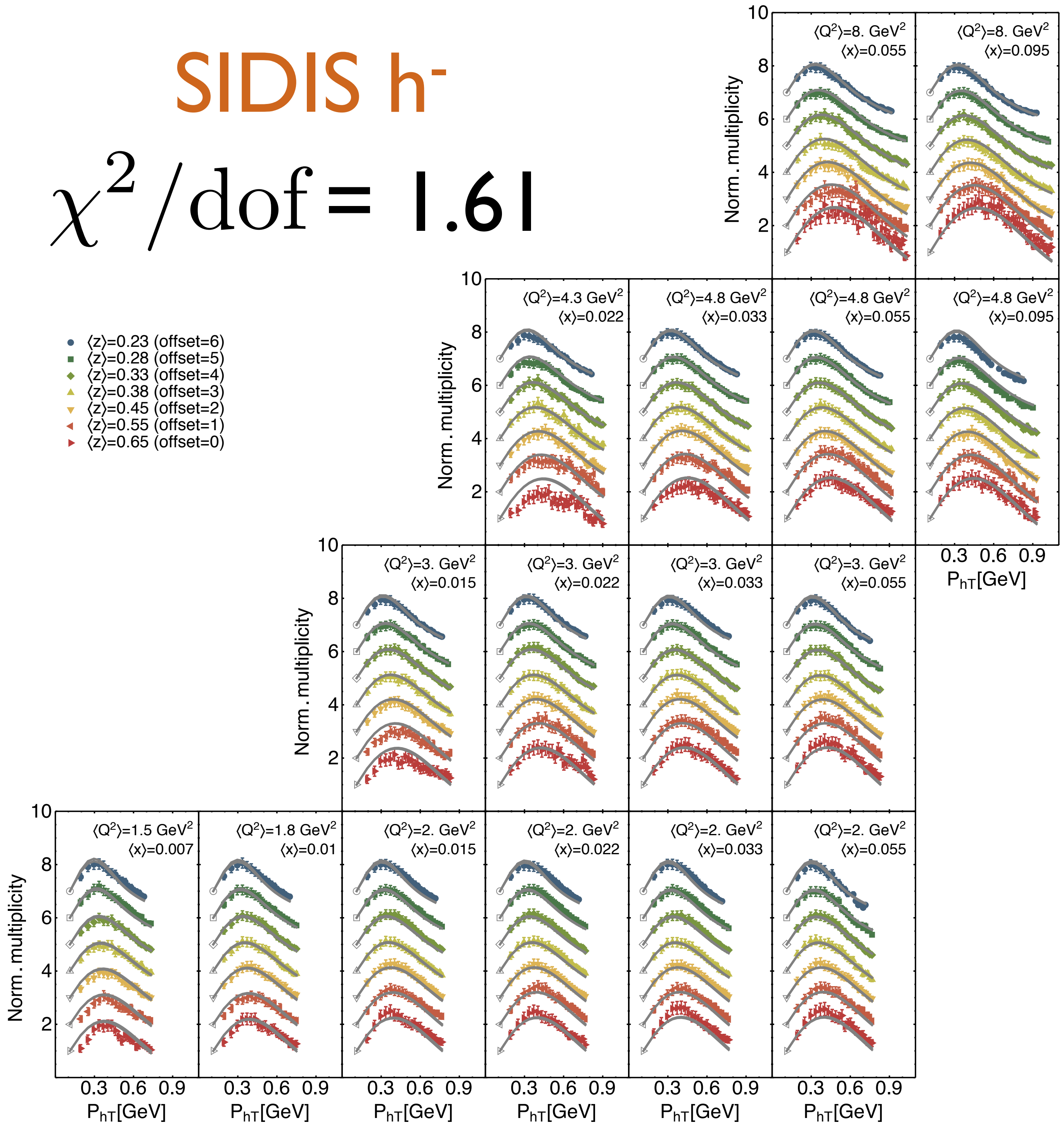
0.82

1.31

2.54

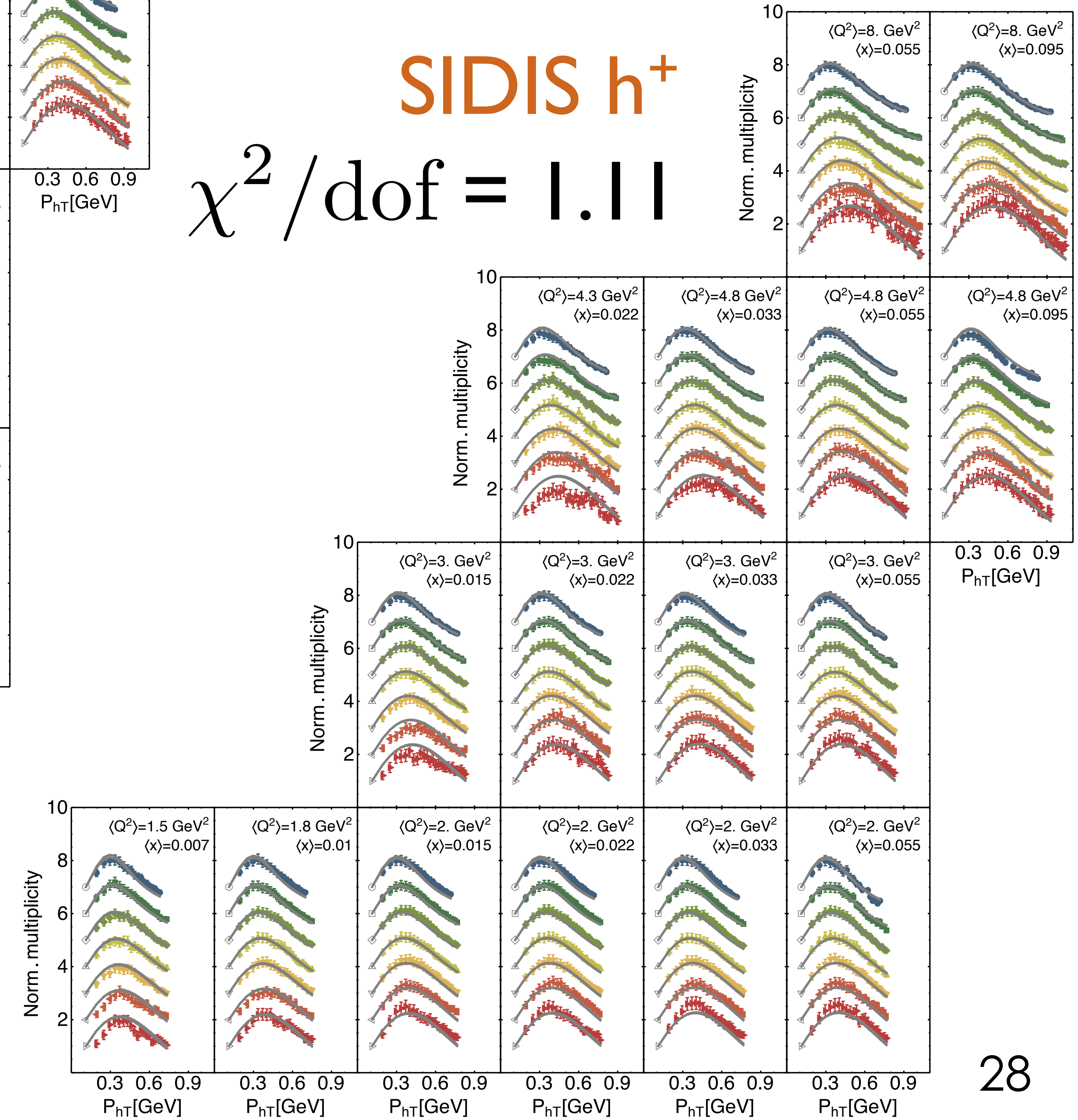
SIDIS h^-

$\chi^2/\mathrm{dof} = 1.61$

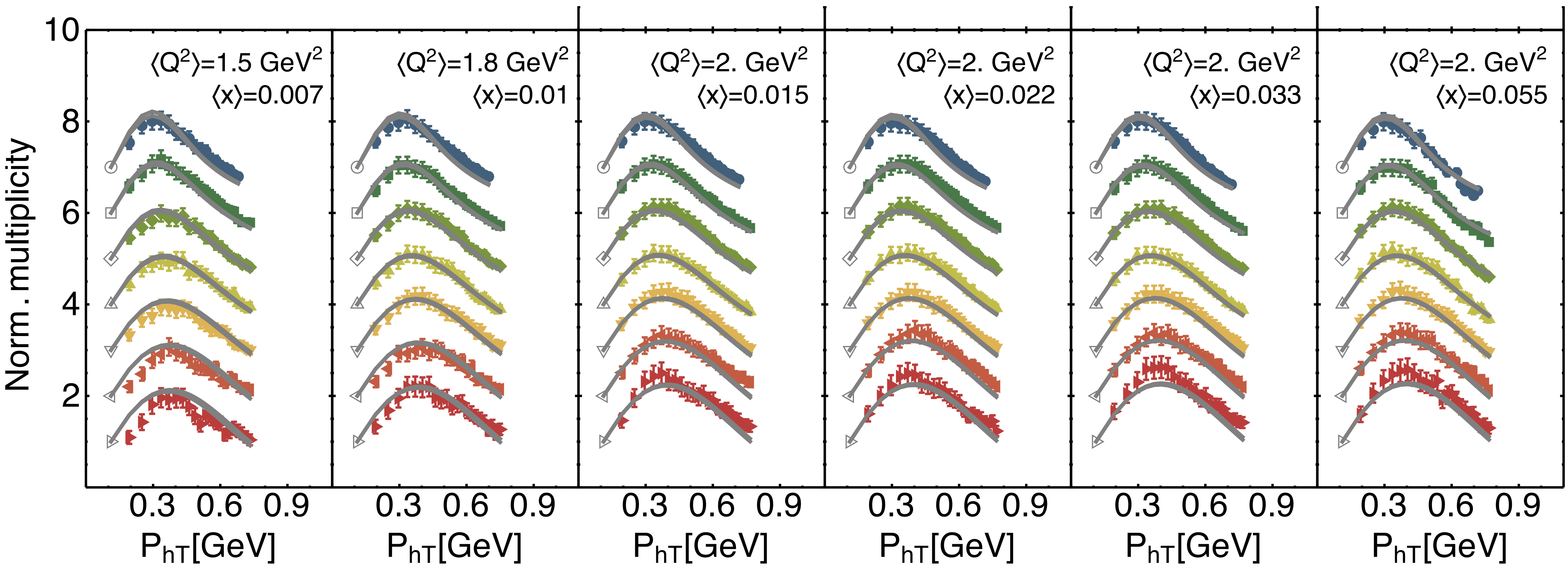


SIDIS h^+

$\chi^2/\mathrm{dof} = 1.11$



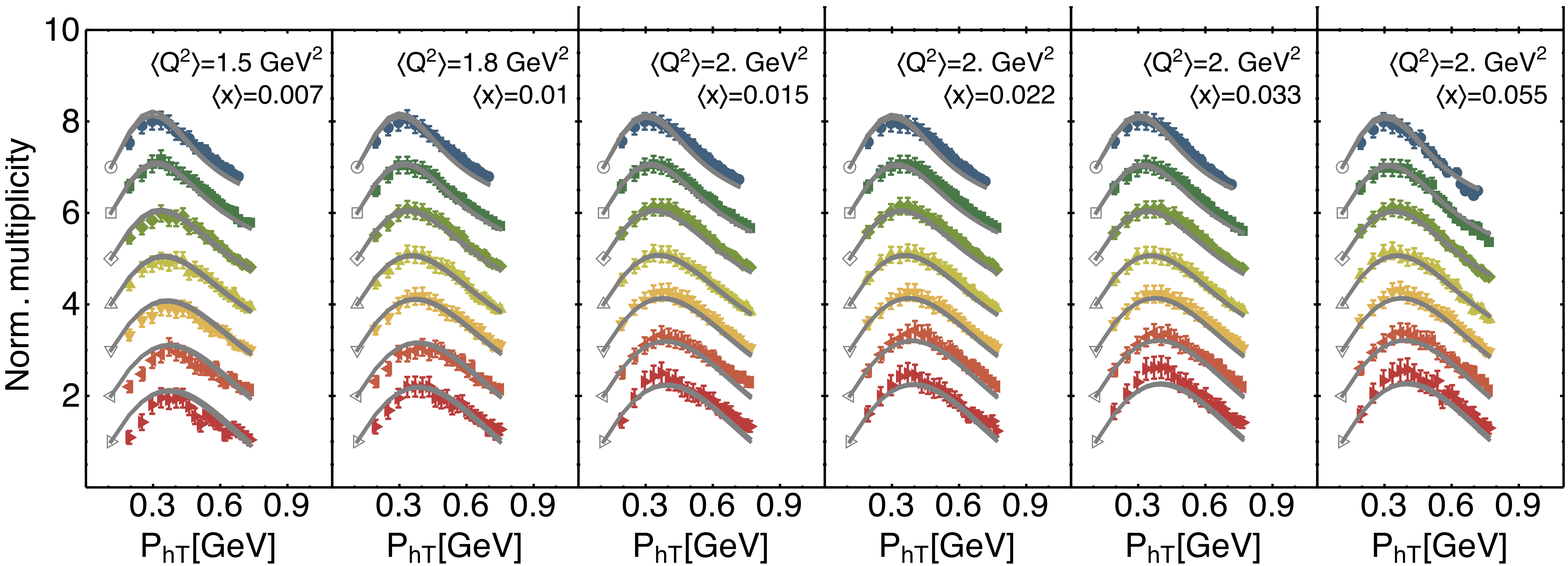
COMPASS data
SIDIS h^+



to avoid known problems
with Compass data normalization:

Observable $$\frac{m_N^h\left(x, z, \boldsymbol{P}_{hT}^2, Q^2\right)}{m_N^h\left(x, z, \min[\boldsymbol{P}_{hT}^2], Q^2\right)}$$

COMPASS data
SIDIS h^+

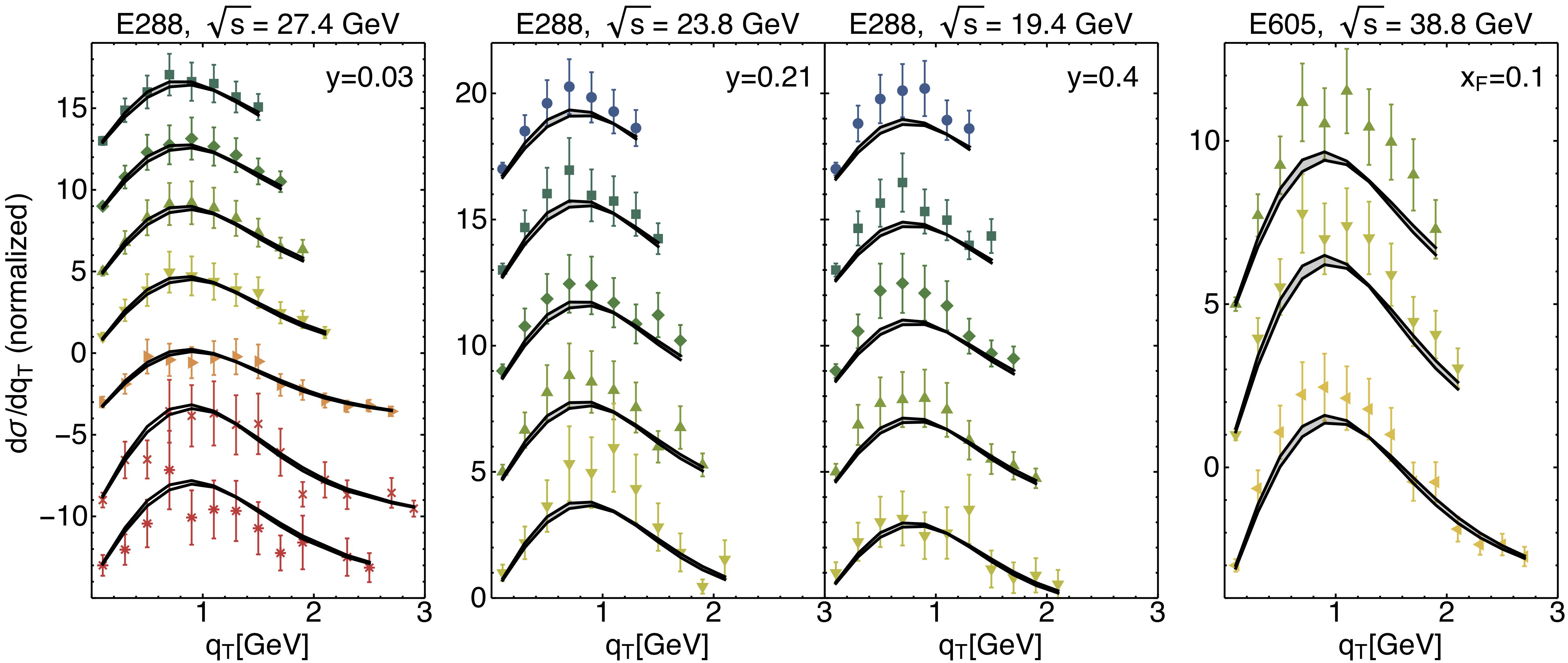


Revised Data:
arXiv:1709.07374

Observable: $$\frac{m_N^h\left(x,z,\boldsymbol{P}_{hT}^2,Q^2\right)}{m_N^h\left(x,z,\min[\boldsymbol{P}_{hT}^2],Q^2\right)}$$

Drell-Yan data

χ^2/dof **0.32** **0.84** **0.99** **1.12**



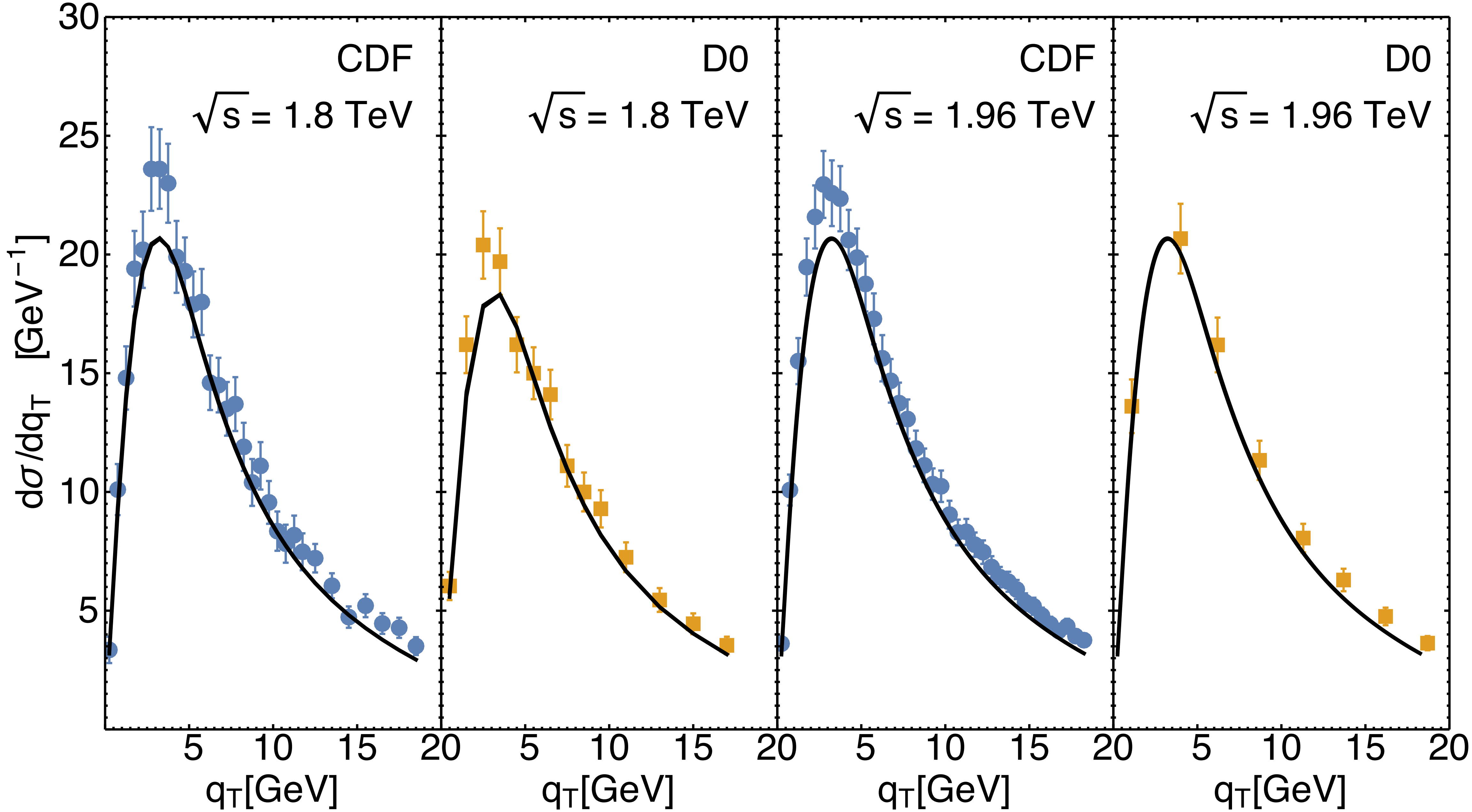
Fermilab

Q[2] Evolution: The peak is now at about 1 GeV, it was at 0.4 GeV for SIDIS

Z-boson production data

normalization : fixed from DEMS fit, different from exp.
(not really relevant for TMD parametrizations)

χ^2/dof 1.36 1.11 2.00 1.73



Q² Evolution: The peak is now at about 4 GeV

Best fit values

TMD PDFs	N_1 [GeV^2]	α	σ		λ [GeV^{-2}]	
All replicas	0.28 ± 0.06	2.95 ± 0.05	0.17 ± 0.02		0.86 ± 0.78	
Replica 105	0.285	2.98	0.173		0.39	
TMD FFs	N_3 [GeV^2]	β	δ	γ	λ_F [GeV^{-2}]	N_4 [GeV^2]
All replicas	0.21 ± 0.02	1.65 ± 0.49	2.28 ± 0.46	0.14 ± 0.07	5.50 ± 1.23	0.13 ± 0.01
Replica 105	0.212	2.10	2.52	0.094	5.29	0.135

TABLE XI: 68% confidence intervals of best-fit values for parametrizations of TMDs at $Q = 1$ GeV.

Flavor independent scenario:

$$N_1 = 0.28 \pm 0.06 \text{ GeV}^2$$

$$N_3 = 0.21 \pm 0.02 \text{ GeV}^2$$

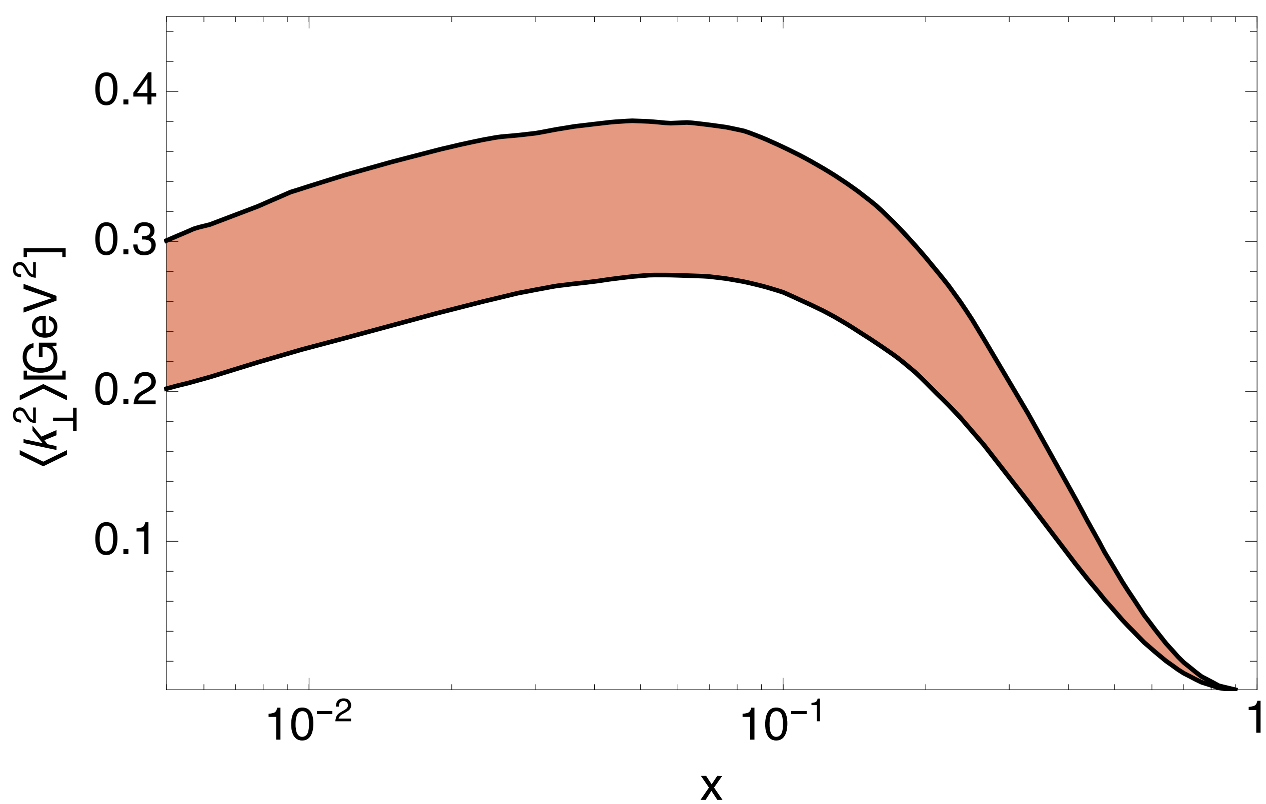
$$N_4 = 0.13 \pm 0.01 \text{ GeV}^2$$

$$g_2 = 0.13 \pm 0.01 \text{ GeV}^2$$

best value from 200 replicas

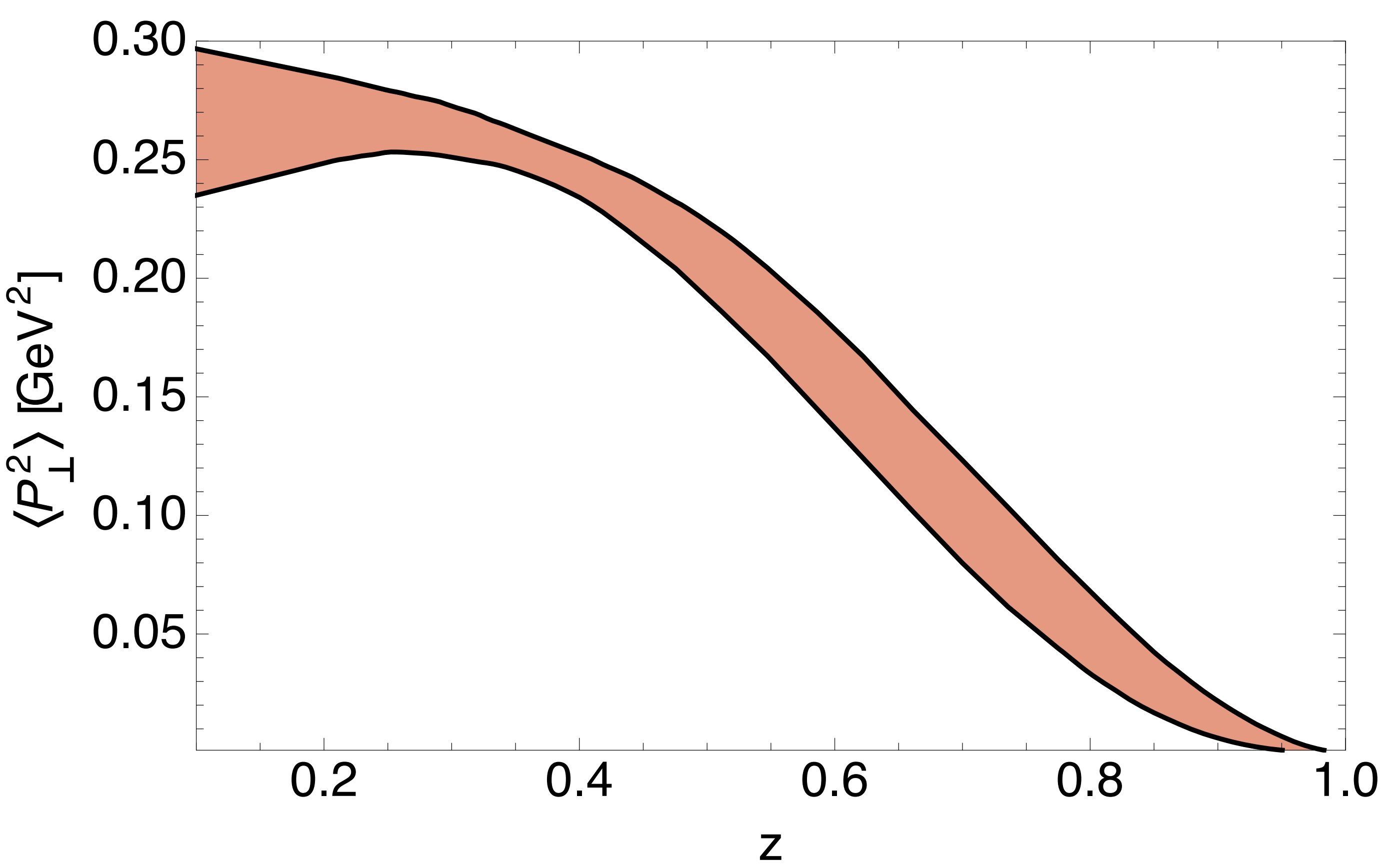
compatible with other extractions

Mean transverse momentum



Change in TMD width
x-dependence

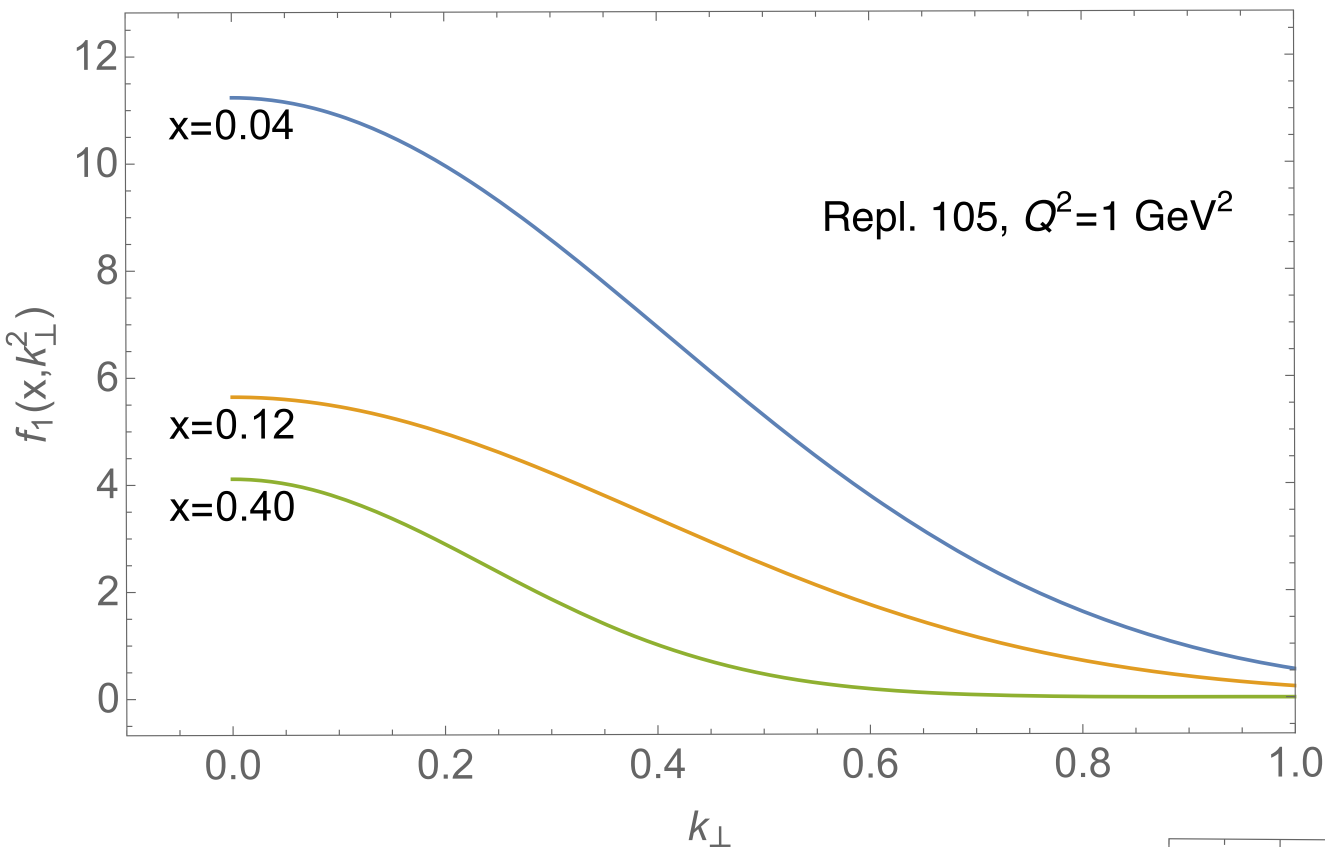
In TMD PDF



In TMD FF

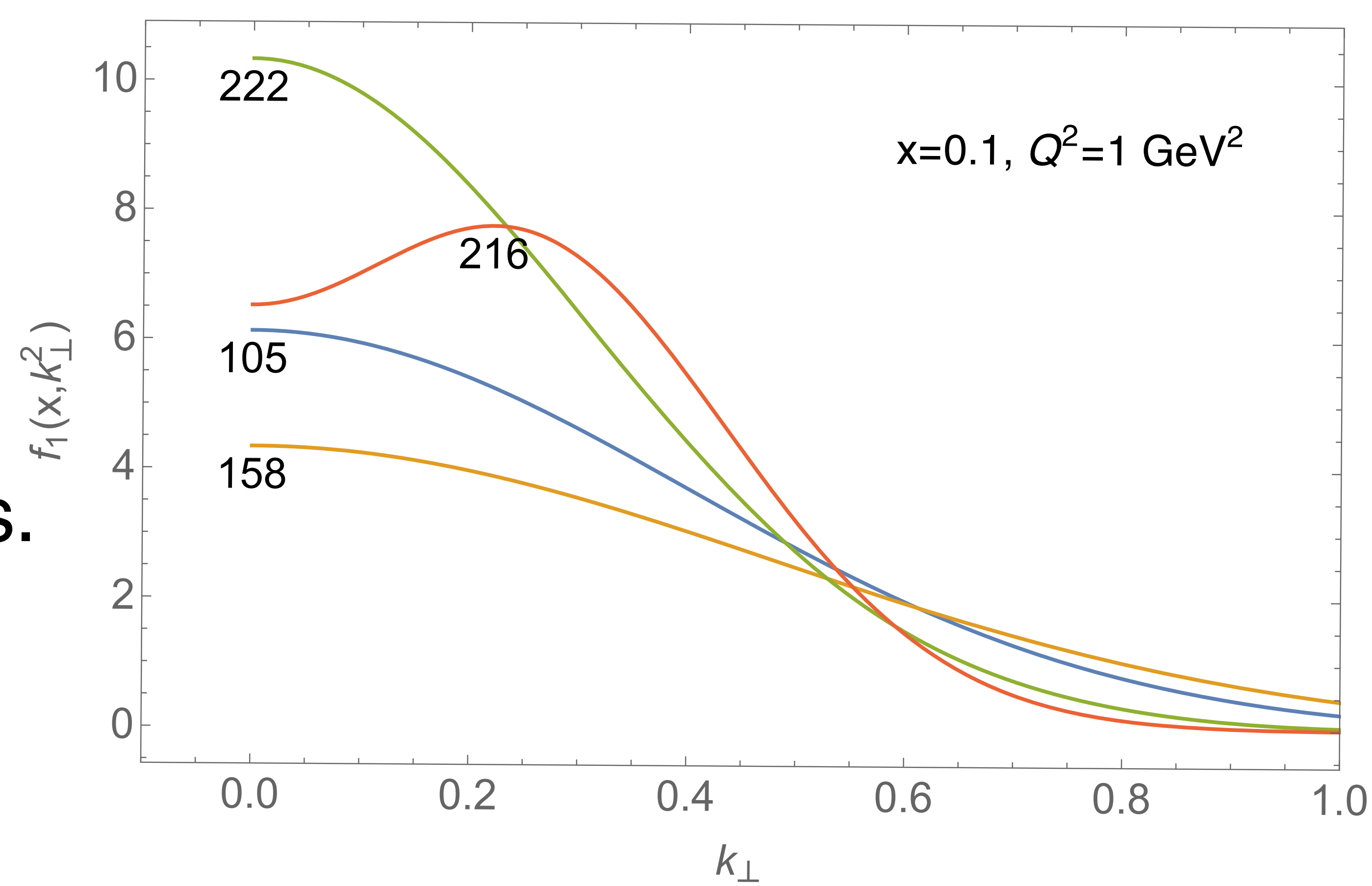
Q^2=1GeV^2

Shape uncertainties in replicas



x-dependence of a single replica.
Most of them are similar.

Shape of four selected replicas.
Still huge uncertainties.



Stability of our results

Test of our default choices

How does the χ^2 of a single replica change if we modify them?

Original χ^2/dof = 1.51

Normalization of HERMES data as done for COMPASS:
χ^2/dof = 1.27

Parametrizations for collinear PDFs (NLO GJR 2008 default choice):
NLO MSTW 2008 (1.84), NLO CJ12 (1.85)

More stringent cuts (TMD factorization better under control)
χ^2/dof $\rightarrow$ 1
Ex: $Q2 > 1.5$ GeV^2; $0.25 < z < 0.6$; $PhT < 0.2Qz \Rightarrow \chi^2$/dof = 1.02 (477 bins)

Conclusions

For the first time we demonstrated that it is possible to fit simultaneously SIDIS, DY and Z boson

We extracted a reasonable functional form for TMD from more than 8000 data points

We tested the universality and applicability of the TMD framework and it works quite well
(most of the discrepancies come from normalization)

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Conclusions and open issues

For the first time we demonstrated that it is possible to fit simultaneously SIDIS, DY and Z boson

We extracted TMDs from more than 8000 data points

We tested the universality and applicability of the TMD framework and it works quite well

TO DO:

- NLO+NLL calculation in progress
- problems with normalizations theory/experiment
- flavor dependence and more flexible forms
- new data sets

.....

BACKUP

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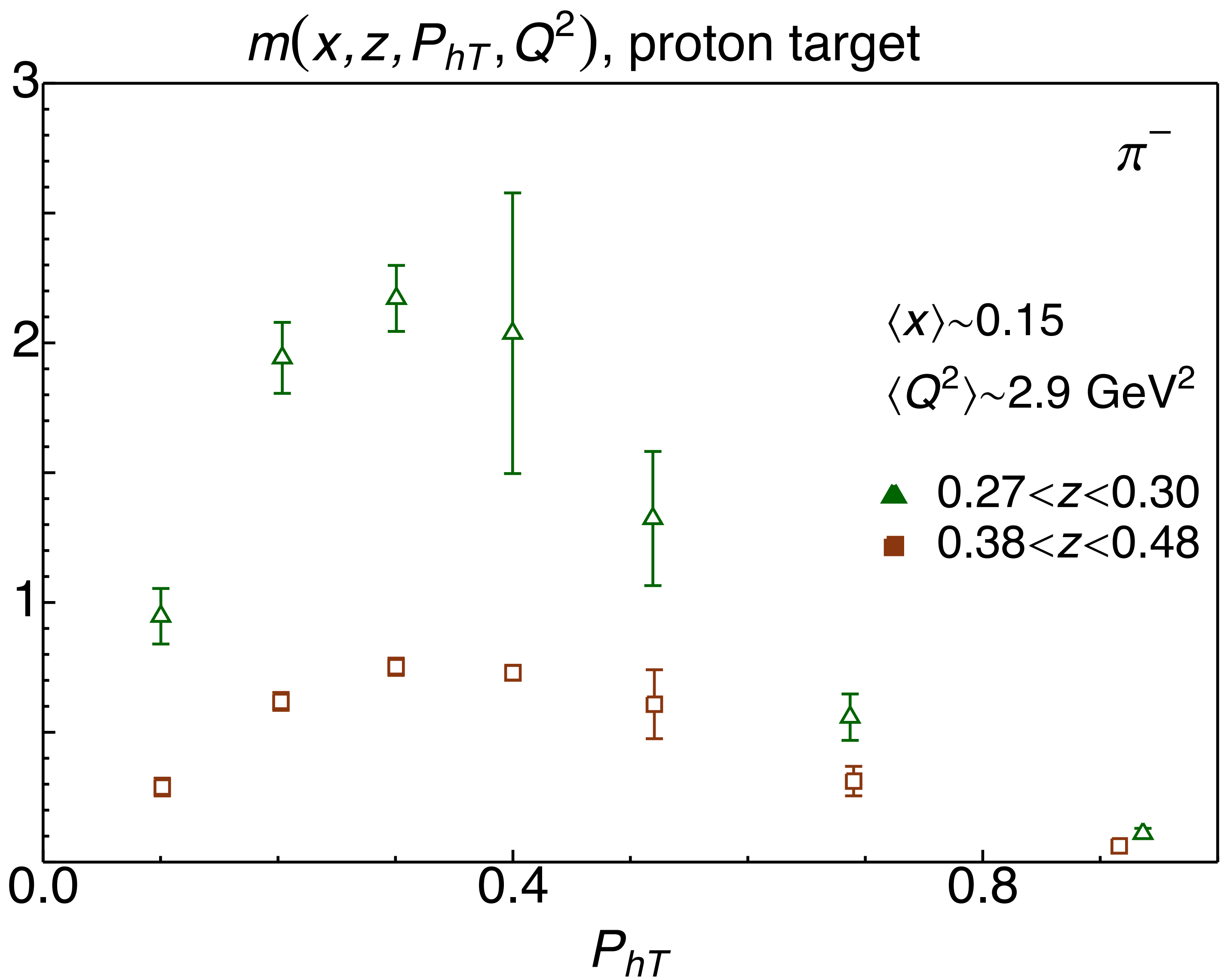
best value from 200 replicas

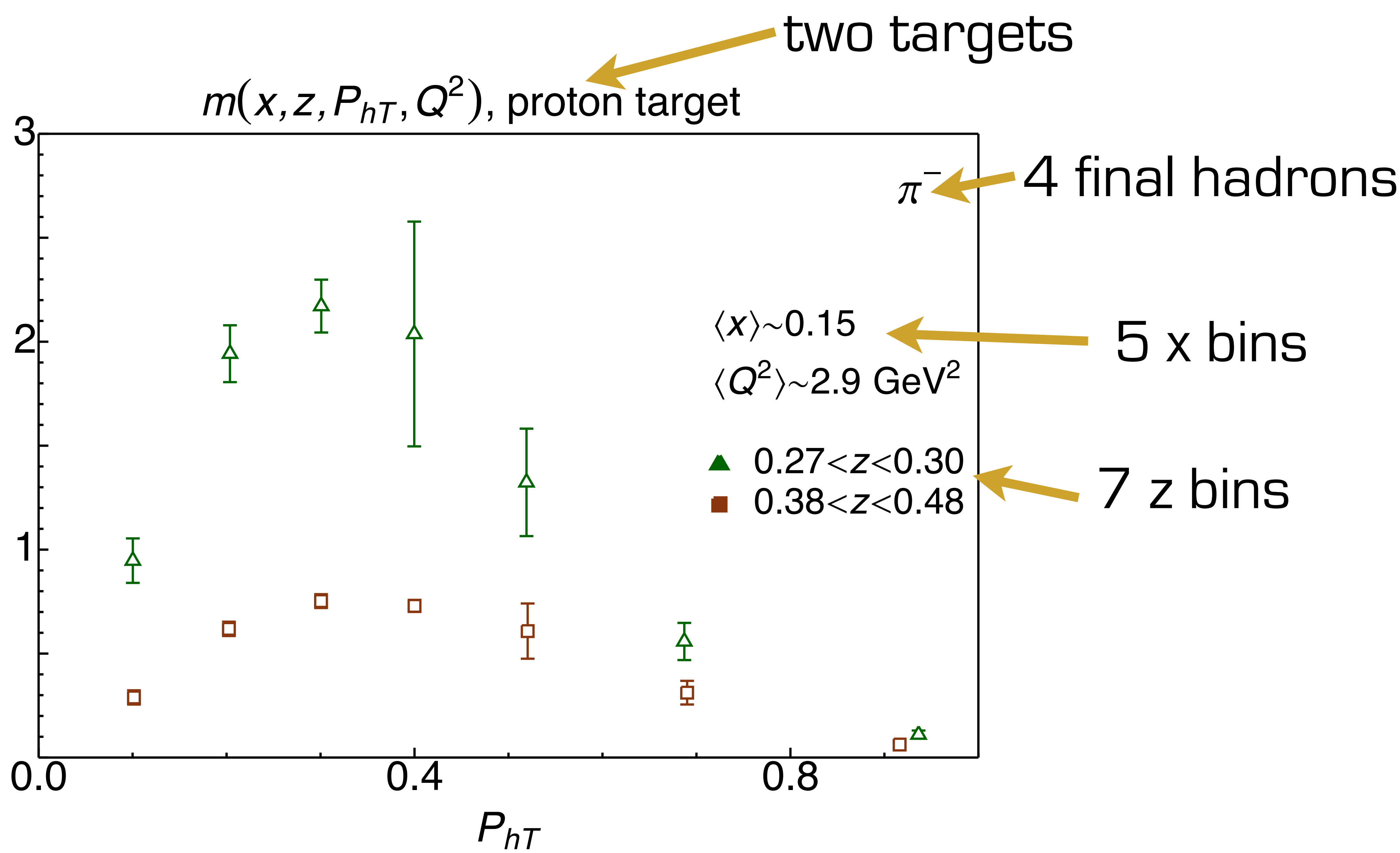
compatible with other extractions

The replica method



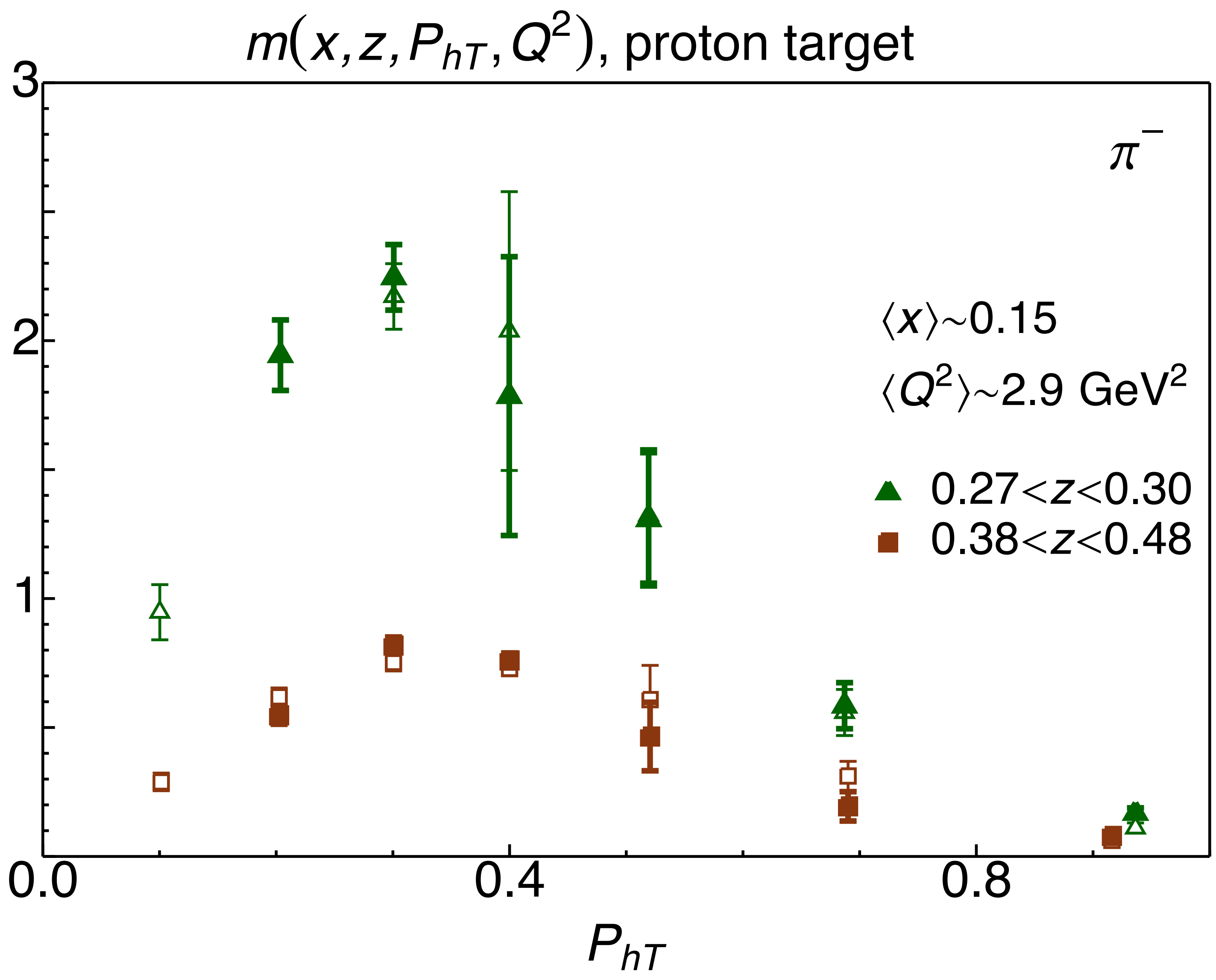
Example of original data

The replica method



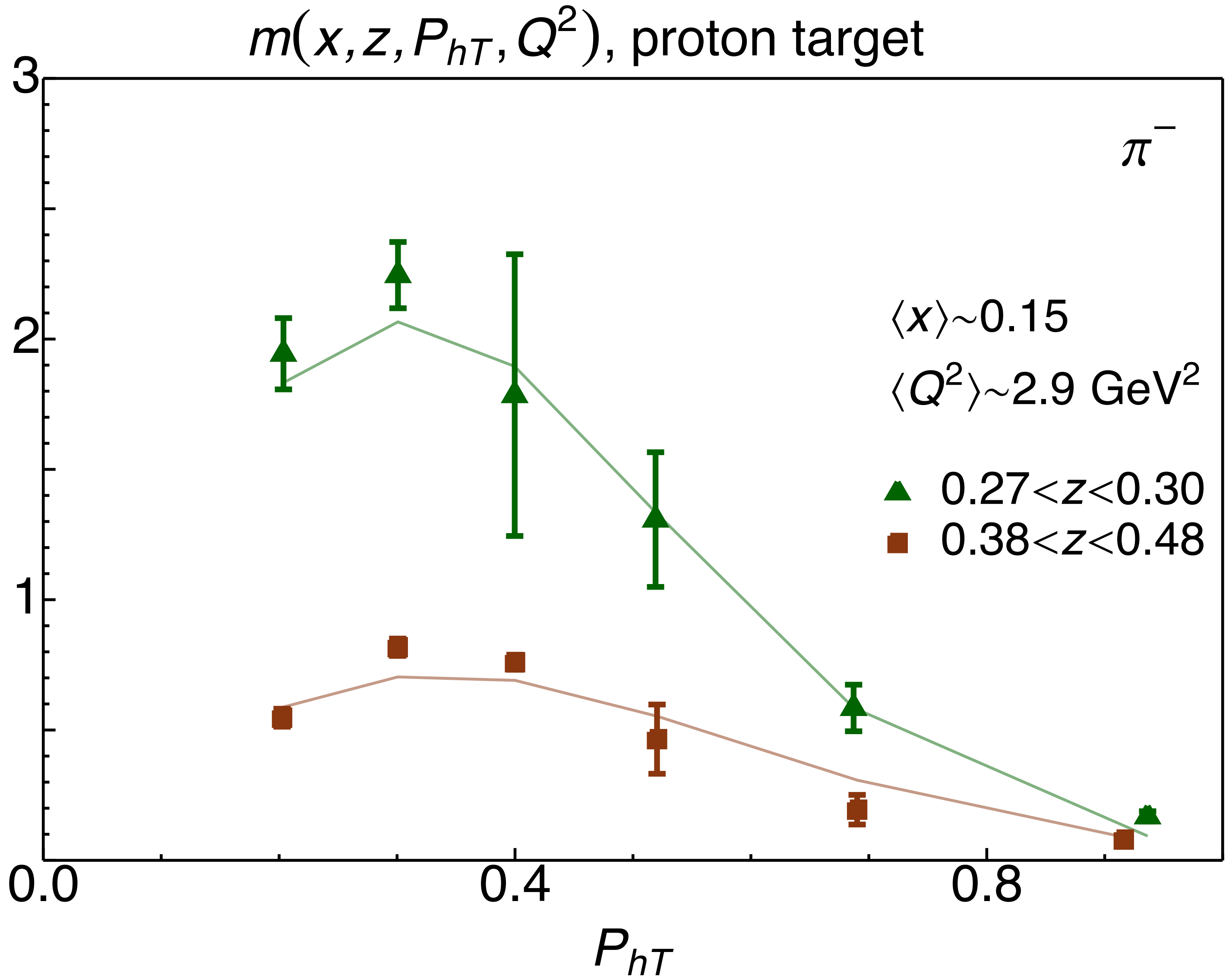
Example of original data

The replica method



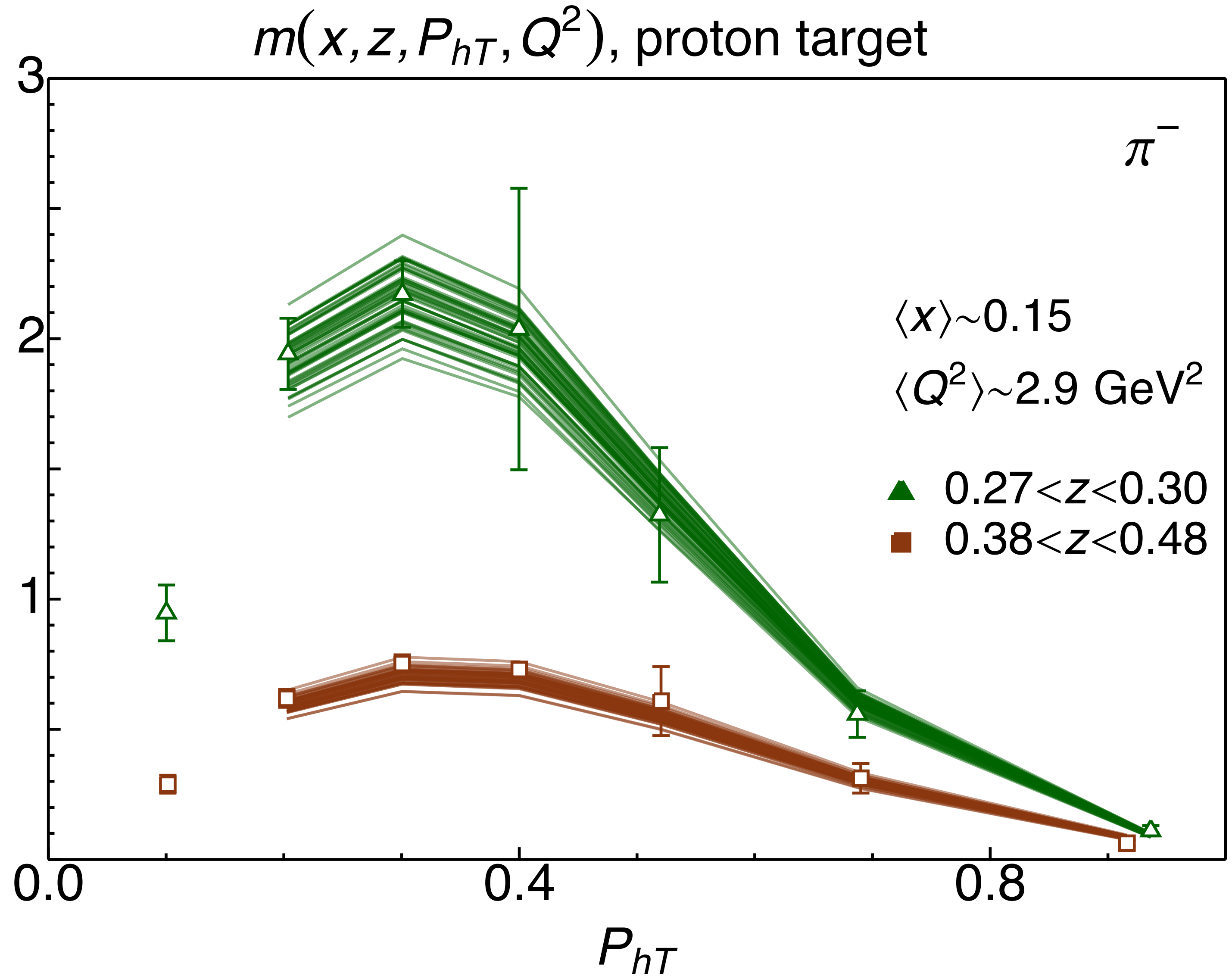
Data are replicated (with Gaussian distribution)

The replica method



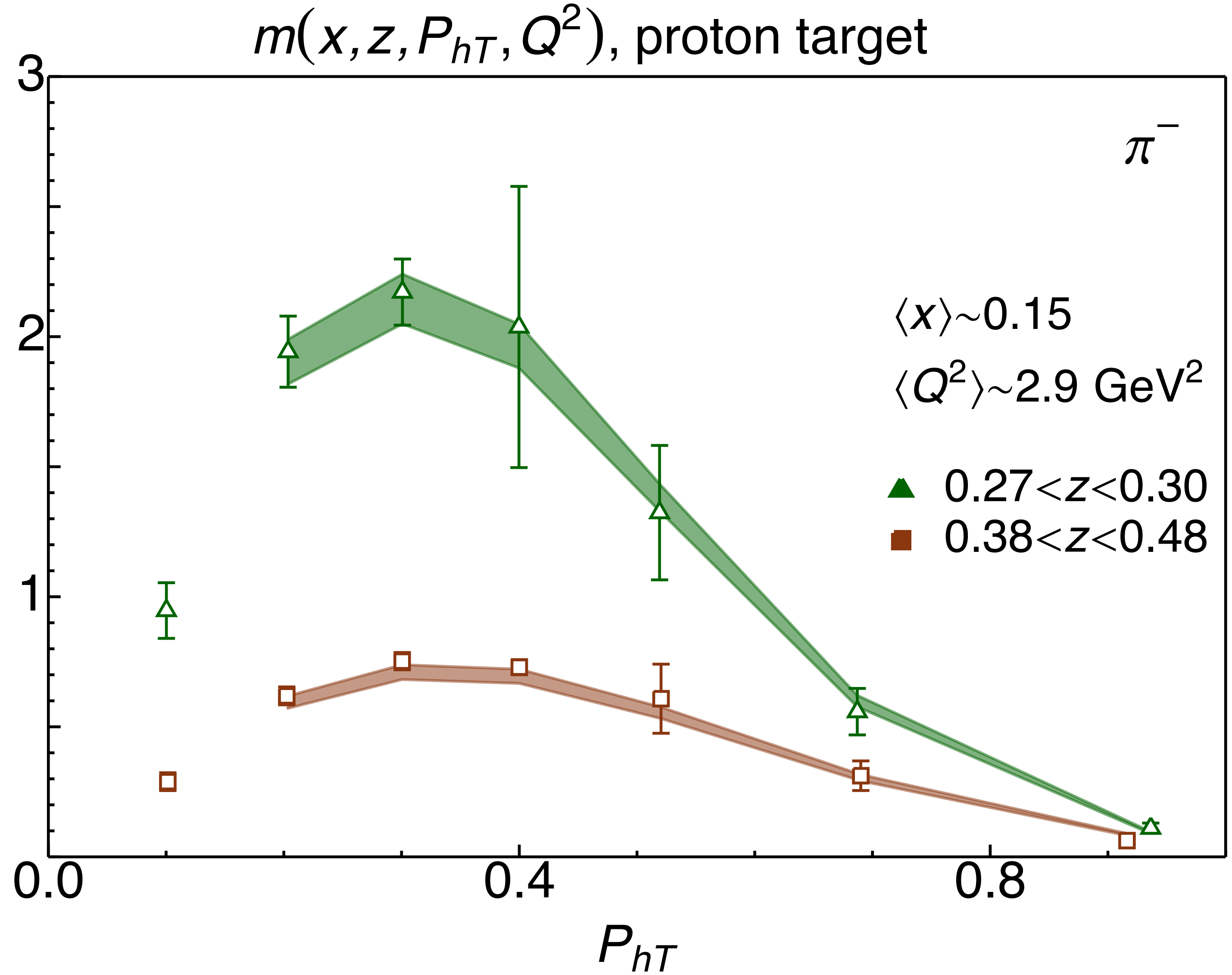
The fit is performed on the replicated data

The replica method



The procedure is repeated 200 times

The replica method



For each point, a central 68% confidence interval is identified

Previous fit studies

	Framework	HERMES	COMPASS	DY	Z production	N of points
KN 2006 hep-ph/0506225	LO-NLL	✘	✘	✔	✔	98
Pavia 2013 (+Amsterdam, Bilbao) arXiv:1309.3507	No evo (QPM)	✔	✘	✘	✘	1538
Torino 2014 (+JLab) arXiv:1312.6261	No evo (QPM)	✔ (separately)	✔ (separately)	✘	✘	576 (H) 6284 (C)
DEMS 2014 arXiv:1407.3311	NLO-NNLL	✘	✘	✔	✔	223
EIKV 2014 arXiv:1401.5078	LO-NLL	1 (x,Q^2) bin	1 (x,Q^2) bin	✔	✔	500 (?)
Pavia 2017 (+ JLab)	LO-NLL	✔	✔	✔	✔	8059

Data selection

SIDIS proton-target data

	HERMES $p \to \pi^+$	HERMES $p \to \pi^-$	HERMES $p \to K^+$	HERMES $p \to K^-$
Reference				
Cuts	$Q^2 > 1.4$ GeV^2 $0.2 < z < 0.7$ $P_{hT} < \text{Min}[0.2\ Q, 0.6\ Qz] + 0.5$ GeV			
Points	188	186	187	185
Max. Q^2	9.2 GeV^2			
x range	$0.06 < x < 0.4$			
Notes				

Motivations behind kinematical cuts

TMD factorization ($Ph_T/z << Q^2$)

Avoid target fragmentation (low z)

and exclusive contributions (high z)

Data selection

SIDIS
deuteron-target
data

	HERMES $D \to \pi^+$	HERMES $D \to \pi^-$	HERMES $D \to K^+$	HERMES $D \to K^-$	COMPASS $D \to h^+$	COMPASS $D \to h^-$
Cuts	$Q^2 > 1.4$ GeV^2 $0.2 < z < 0.7$ $P_{hT} < \text{Min}[0.2\ Q, 0.6\ Qz] + 0.5$ GeV					
Points	188	188	186	187	3024	3021
Max. Q^2	9.2 GeV^2				10 GeV^2	
x range	$0.06 < x < 0.4$				$0.006 < x < 0.12$	
Notes					Observable: $\dfrac{m_N^h(x, z, \boldsymbol{P}_{hT}^2, Q^2)}{m_N^h(x, z, \text{Min}[\boldsymbol{P}_{hT}^2], Q^2)}$	

to avoid problems
with Compass data normalization

Data selection

	E288 200	E288 300	E288 400	E605
Cuts	$q_T < 0.2\ Q + 0.5$ GeV			
Points	45	45	78	35
$\sqrt{s}$	19.4 GeV	23.8 GeV	27.4 GeV	38.8 GeV
Q range	4-9 GeV	4-9 GeV	5-9, 11-14 GeV	7-9, 10.5-18 GeV
Kin. var.	y=0.4	y=0.21	y=0.03	$-0.1 < x_F < 0.2$

Drell-Yan data

Z production data

	CDF Run I	D0 Run I	CDF Run II	D0 Run II
Cuts	$q_T < 0.2\ Q + 0.5$ GeV $= 18.7$ GeV			
Points	31	14	37	8
$\sqrt{s}$	1.8 TeV	1.8 TeV	1.96 TeV	1.96 TeV
Normalization	1.114	0.992	1.049	1.048

fixed from DEMS fit, different from exp. (not really relevant for TMD parametrizations)

μ and b_* prescriptions

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

μ and b* prescriptions

Choice **Choice**

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T)\ln\frac{\mu}{\mu_0}} \hat{f}^a_{\rm NP}(x, b_T)$$

μ and b_* prescriptions

Choice **Choice**

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$$\mu_b = 2e^{-\gamma_E}/b_* \qquad b_* \equiv \frac{b_T}{\sqrt{1 + b_T^2/b_{\rm max}^2}}$$

Collins, Soper, Sterman, NPB250 (85)

$$\mu_b = 2e^{-\gamma_E}/b_* \qquad b_* \equiv b_{\rm max}\left(1 - e^{-\frac{b_T^4}{b_{\rm max}^4}}\right)^{1/4}$$

Bacchetta, Echevarria, Mulders, Radici, Signori
arXiv:1508.00402

$$\mu_b = Q_0 + q_T \qquad b_* = b_T$$

DEMS 2014

μ and b_* prescriptions

Choice **Choice**

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i (\tilde{C}_{a/i} \otimes f_1^i)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}^a_{\rm NP}(x, b_T)$$

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Collins, Soper, Sterman, NPB250 (85)

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Bacchetta, Echevarria, Mulders, Radici, Signori arXiv:1508.00402

$$\mu_b = Q_0 + q_T \qquad b_* = b_T$$

DEMS 2014

Complex-b prescription

Laenen, Sterman, Vogelsang, PRL 84 (00)

Nonperturbative ingredients 1

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

Nonperturbative ingredients 1

Choice

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}^a_{\rm NP}(x, b_T)$$

Nonperturbative ingredients 1

Choice

$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

$$e^{-\frac{b_T^2}{\langle b_T^2 \rangle}}$$

almost everybody

$$e^{-\frac{b_T^2}{\left\langle b_T^2(x) \right\rangle_a}}$$

Pavia 2013, KN 2006

$$e^{-\lambda_1 b_T} \left(1 + \lambda_2 b_T^2\right)$$

DEMS 2014

Low-b_T modifications

$$\log\left(Q^2 b_T^2\right) \to \log\left(Q^2 b_T^2 + 1\right)$$

see, e.g., Bozzi, Catani, De Florian, Grazzini
hep-ph/0302104

see talks by Collins, Boglione, (Rogers?)

Low-b_T modifications

$$\log\left(Q^2 b_T^2\right) \to \log\left(Q^2 b_T^2 + 1\right)$$

see, e.g., Bozzi, Catani, De Florian, Grazzini
hep-ph/0302104

$$b_*(b_c(b_\mathrm{T})) = \sqrt{\frac{b_\mathrm{T}^2 + b_0^2/(C_5^2 Q^2)}{1 + b_\mathrm{T}^2/b_\mathrm{max}^2 + b_0^2/(C_5^2 Q^2 b_\mathrm{max}^2)}}$$

$$b_\mathrm{min} \equiv b_*(b_c(0)) = \frac{b_0}{C_5 Q}\sqrt{\frac{1}{1 + b_0^2/(C_5^2 Q^2 b_\mathrm{max}^2)}}$$

Collins et al.
arXiv:1605.00671

see talks by Collins, Boglione, (Rogers?)

Data selection

$Q^2 > 1.4 \text{ GeV}^2$

$0.2 < z < 0.7$

$P_{hT}, q_T < 0.2\, Q + 0.5 \text{ GeV}$ $\qquad$ $P_{hT} < 0.8 \text{ GeV (if } z < 0.3)$

Data selection

$Q^2 > 1.4\ \mathrm{GeV}^2$

$0.2 < z < 0.7$

$P_{hT}, q_T < 0.2\,Q + 0.5\ \mathrm{GeV}$ $\qquad$ $P_{hT} < 0.8\ \mathrm{GeV}\ (\mathrm{if}\ z < 0.3)$

Total number of data points: 8156

Total χ^2/dof = 1.45

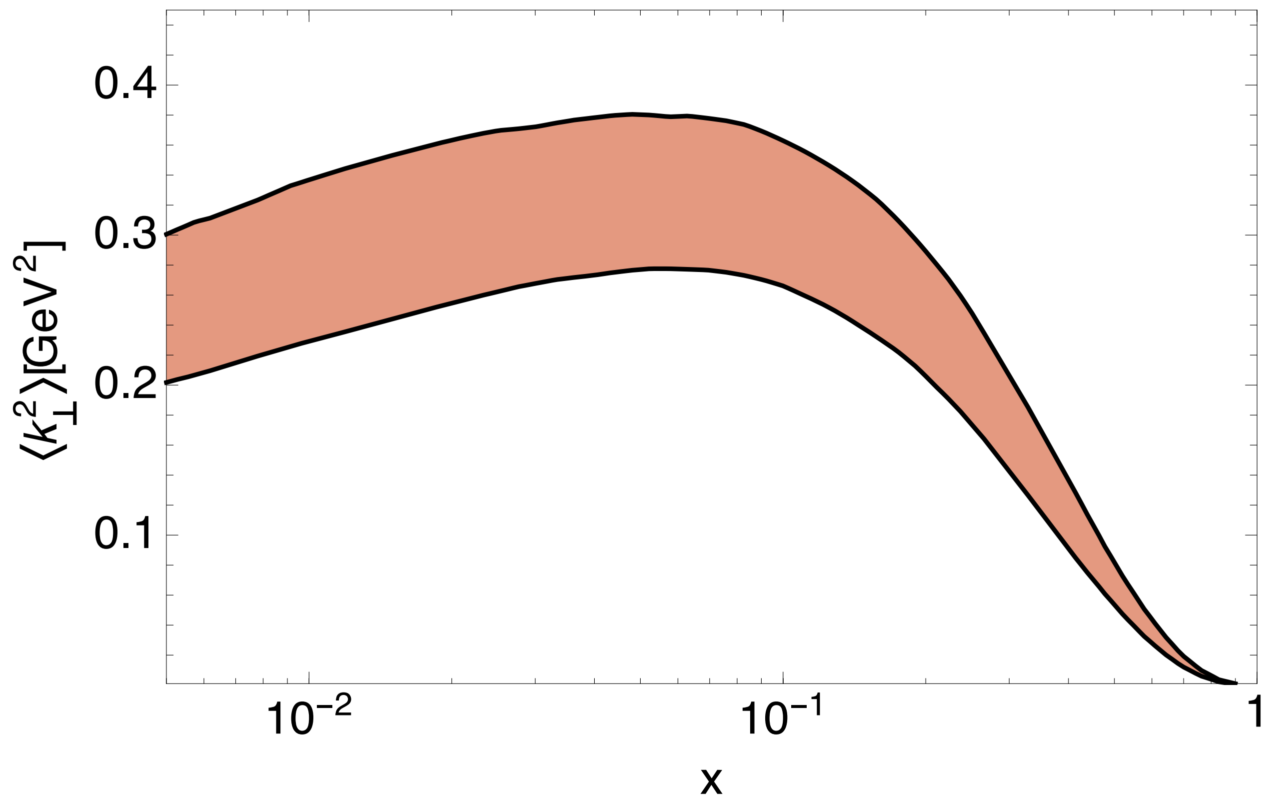
Preliminary

Pavia 2016 perturbative ingredients

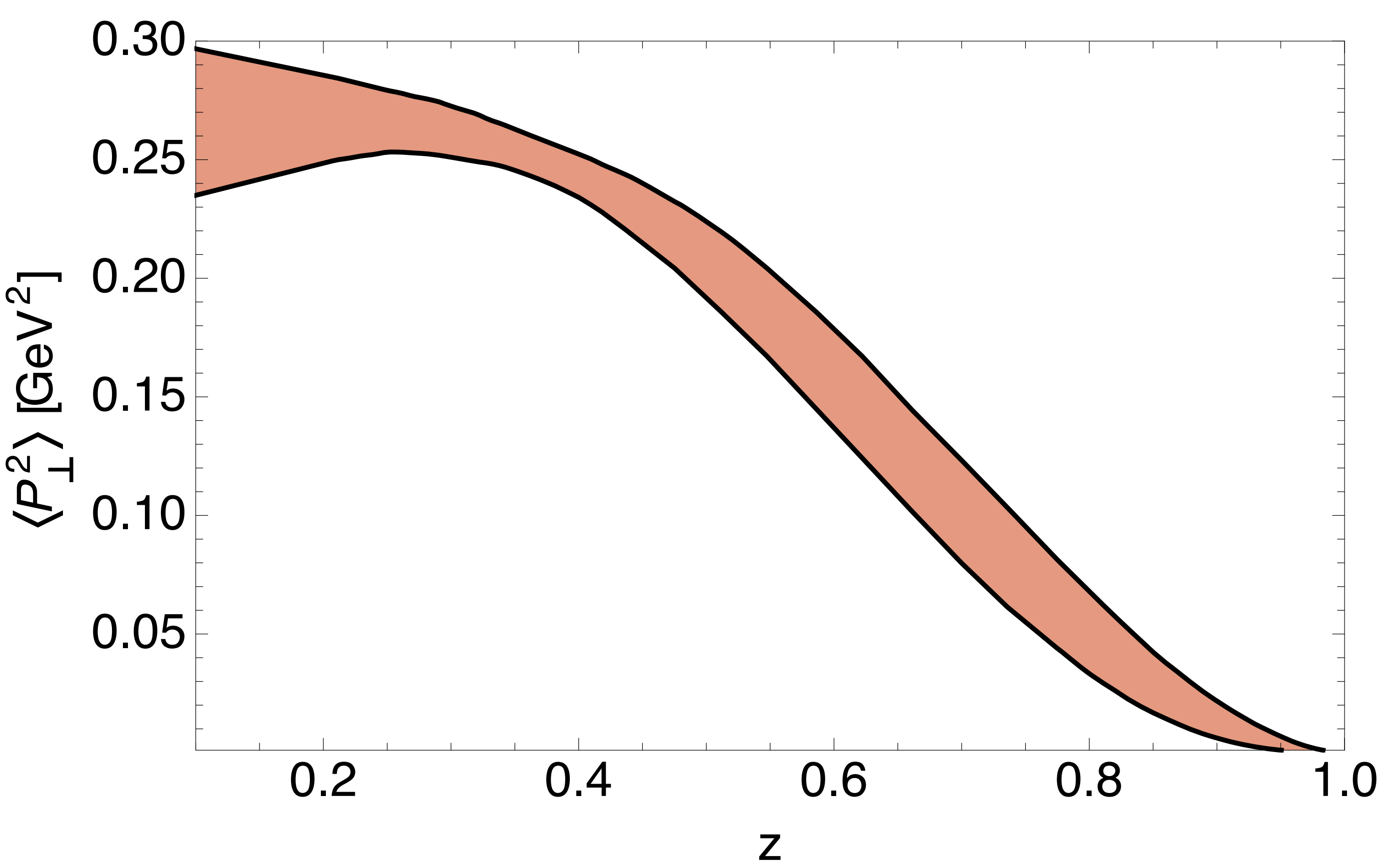
$$\widetilde{f}_1^a(x, b_T; \mu^2) = \sum_i \big(\tilde{C}_{a/i} \otimes f_1^i\big)(x, b_*; \mu_b) e^{\tilde{S}(b_*;\mu_b,\mu)} e^{g_K(b_T) \ln \frac{\mu}{\mu_0}} \hat{f}_{\rm NP}^a(x, b_T)$$

$A_1(\mathcal{O}(\alpha_S^1))$ ✔	$A_2(\mathcal{O}(\alpha_S^2))$ ✔	$A_3(\mathcal{O}(\alpha_S^3))$	$\dots$
	$B_1(\mathcal{O}(\alpha_S^1))$ ✔	$B_2(\mathcal{O}(\alpha_S^2))$	$\dots$
$C_0(\mathcal{O}(\alpha_S^0))$ ✔	$C_1(\mathcal{O}(\alpha_S^1))$	$C_2(\mathcal{O}(\alpha_S^2))$	$\dots$
$H_0(\mathcal{O}(\alpha_S^0))$ ✔	$H_1(\mathcal{O}(\alpha_S^1))$	$H_2(\mathcal{O}(\alpha_S^2))$	$\dots$
	$Y_1(\mathcal{O}(\alpha_S^1))$	$Y_2(\mathcal{O}(\alpha_S^2))$	$\dots$

Mean transverse momentum



In TMD PDF



In TMD FF

Q^2=1GeV^2