Phenomenology of TMDs

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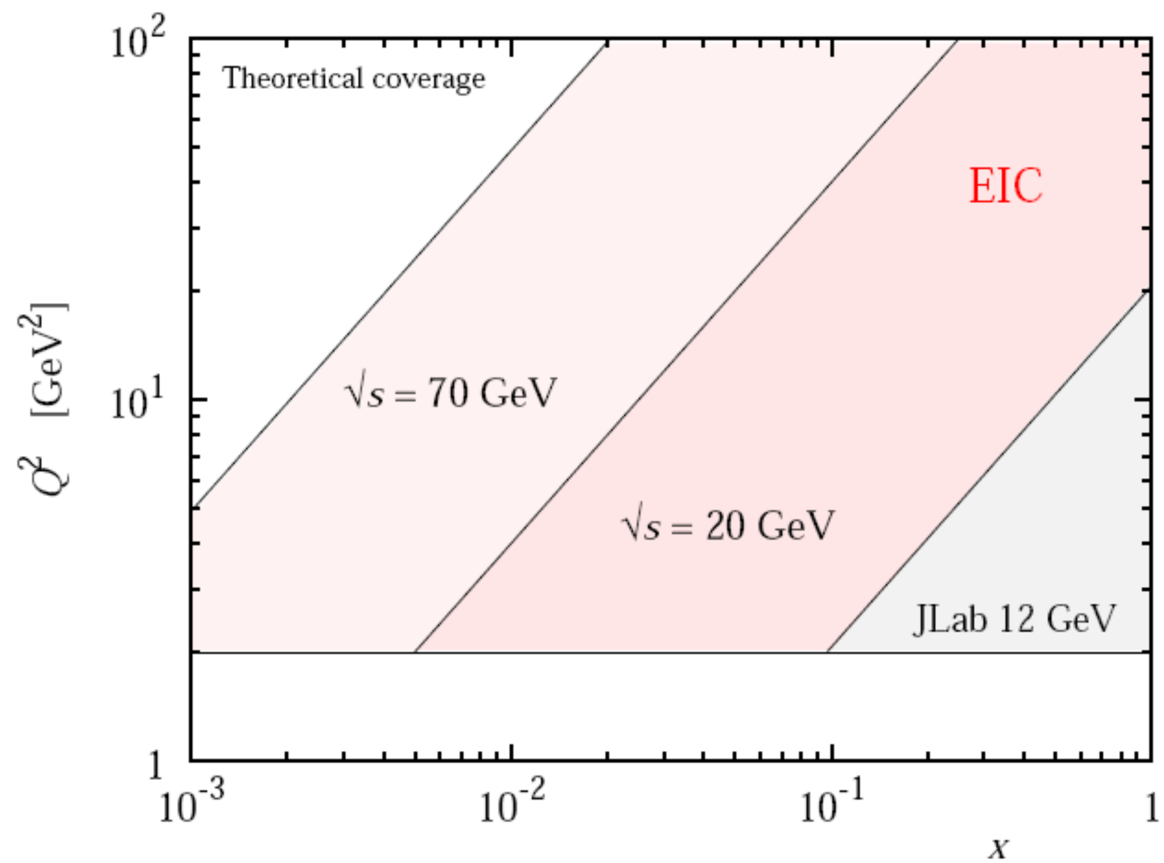
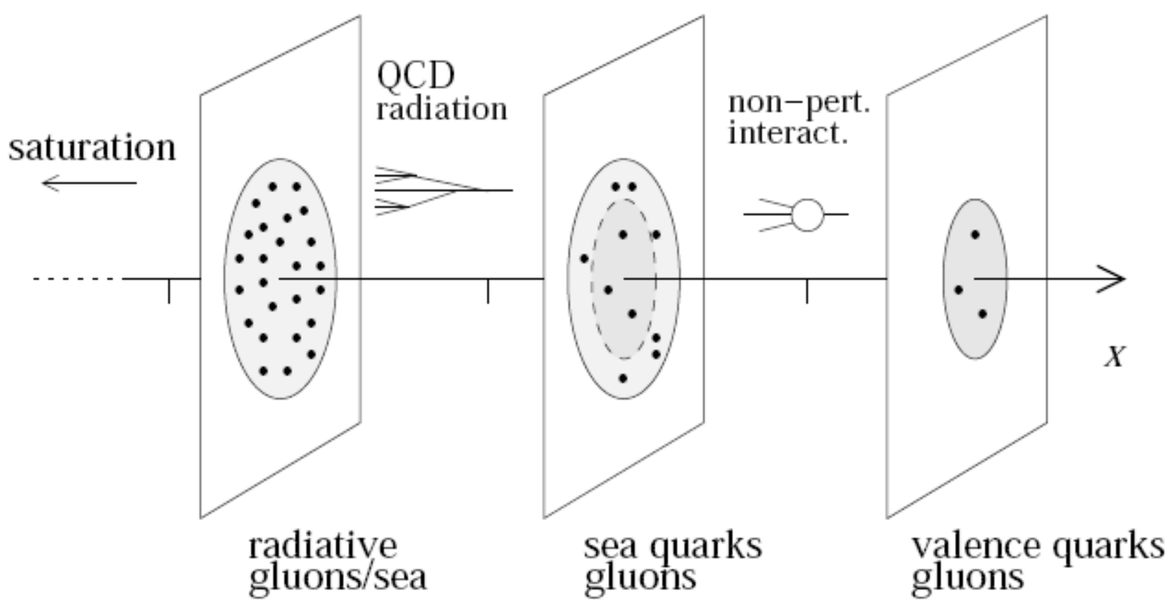




Nucleon is a many body dynamical system of quarks and gluons

Changing x we probe different aspects of nucleon wave function

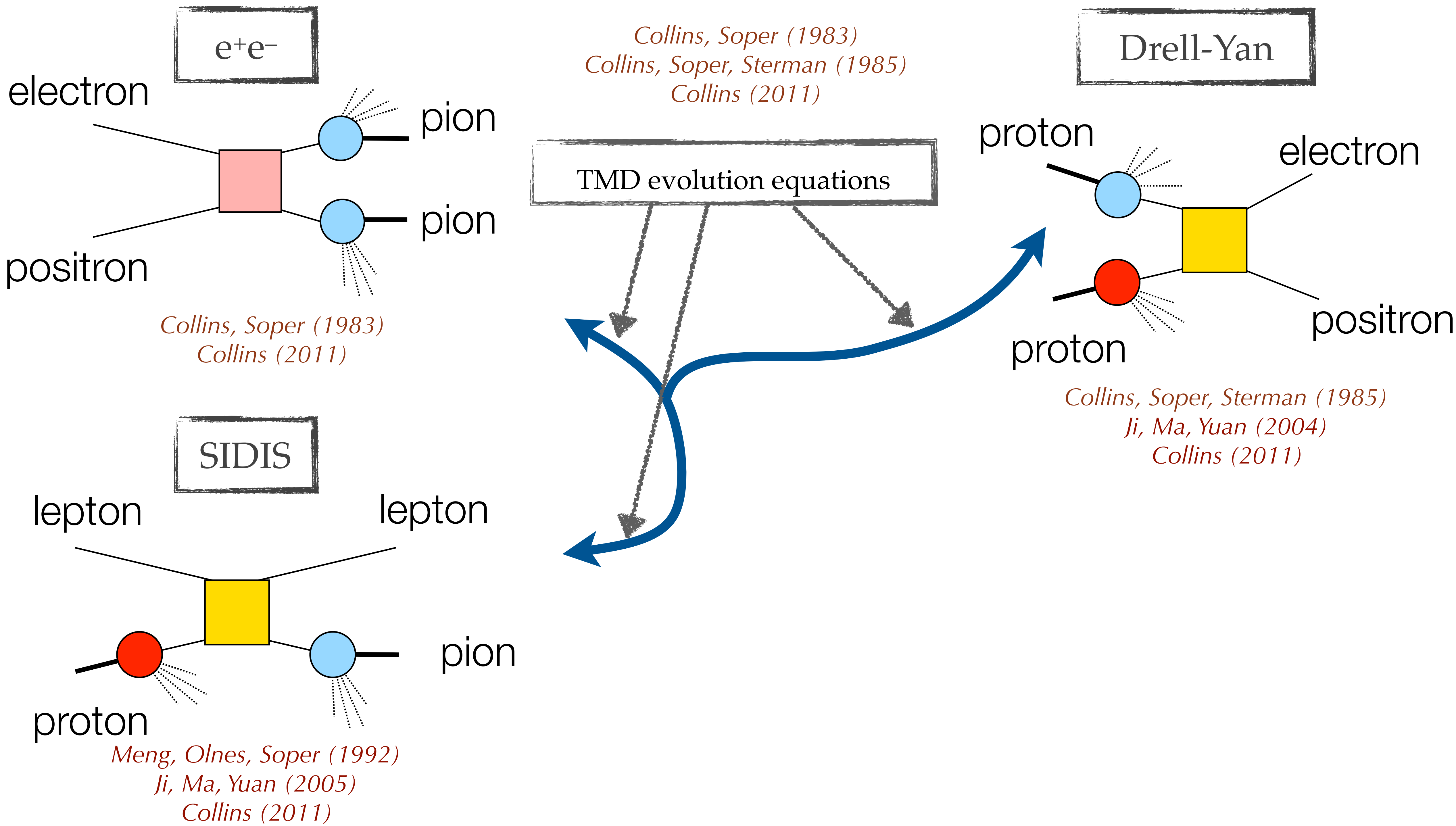
How partons move and how they are distributed in space is one of the directions of development of nuclear physics

Technically such information is encoded into Generalised Parton Distributions (GPDs) and Transverse Momentum Dependent distributions (TMDs)

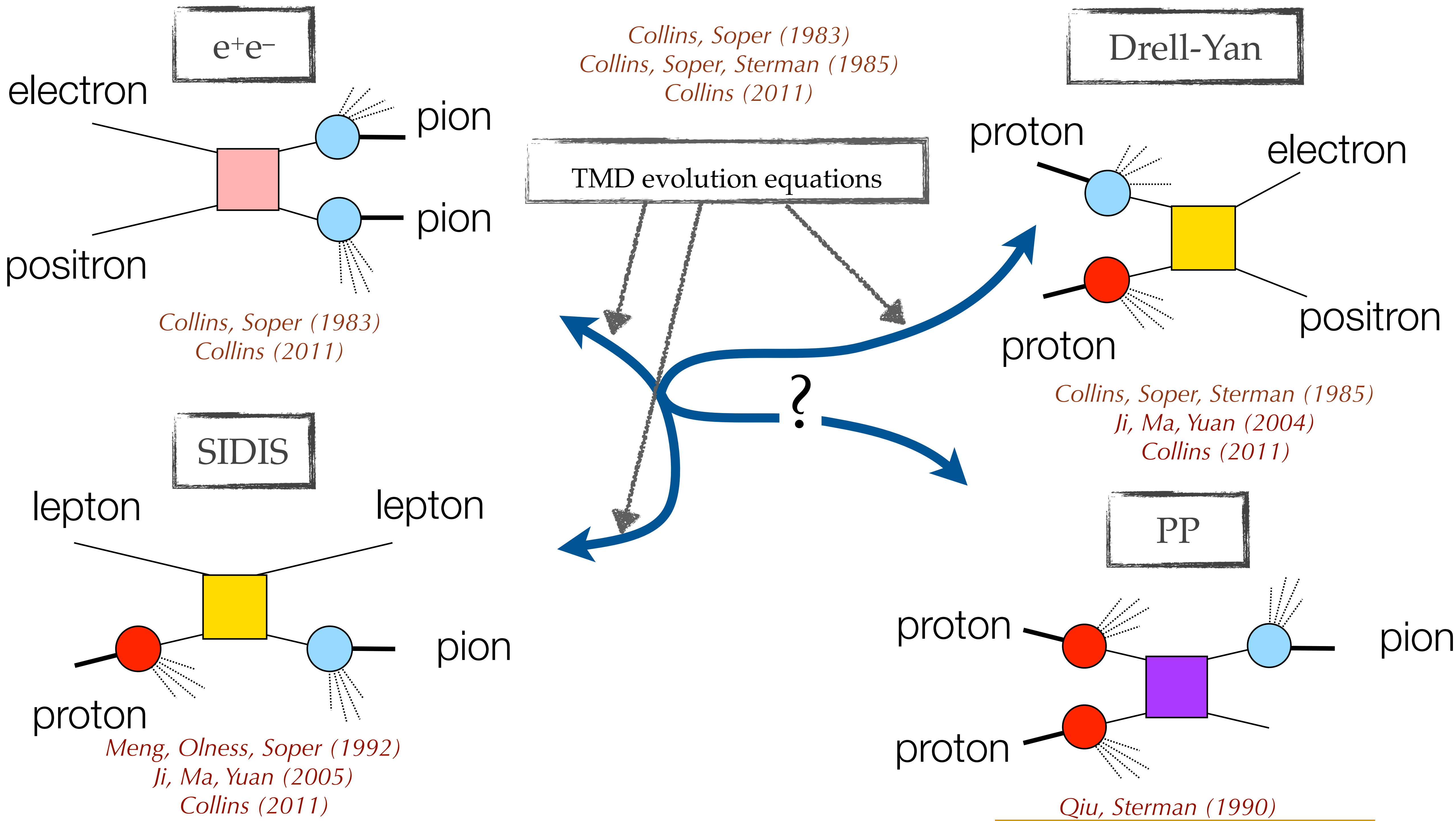
These distributions are also referred to as 3D (three-dimensional) distributions

Why TMDs, factorization, and evolution

TMD factorization

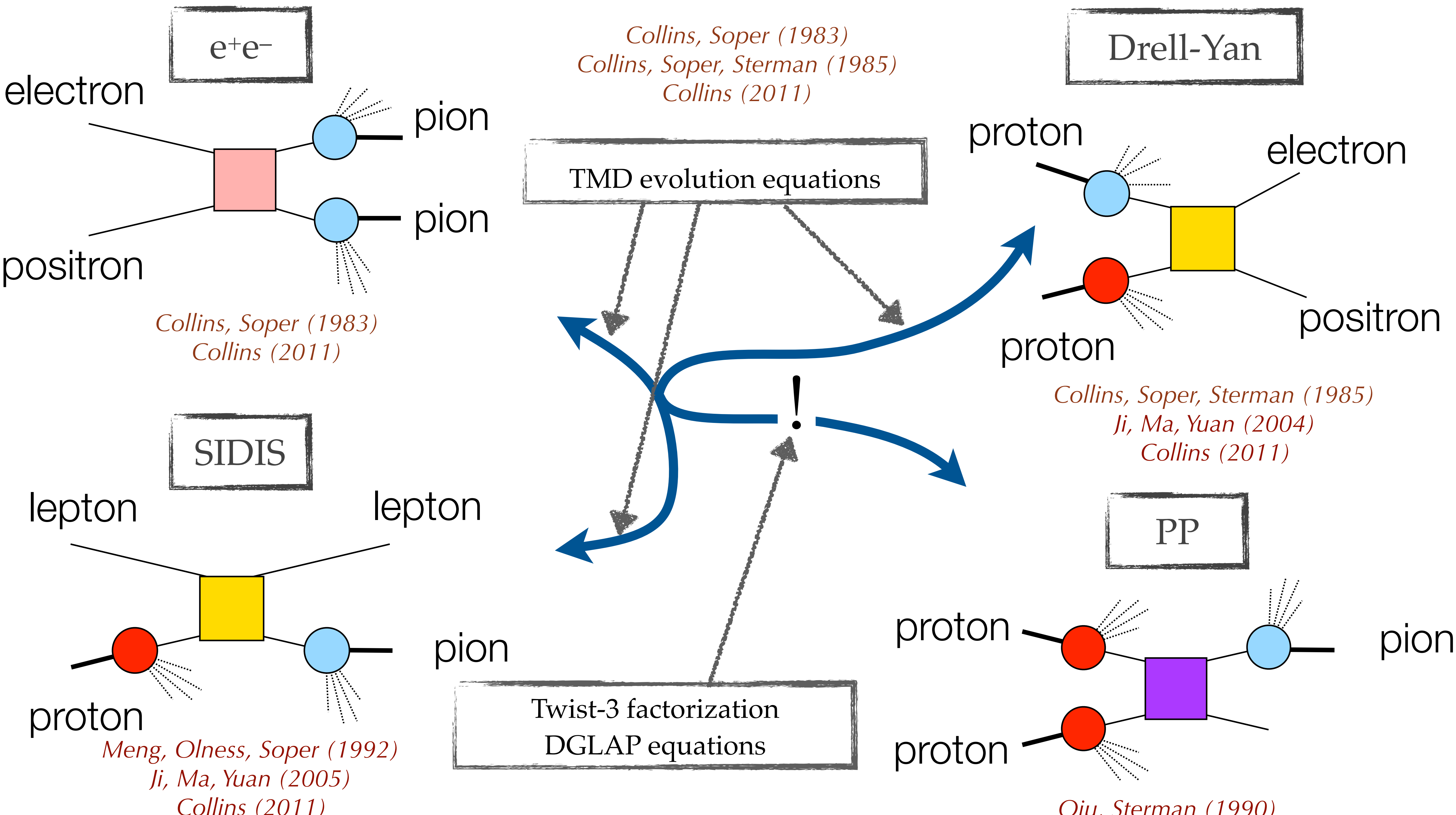


TMD factorization



Only one scale is measured in PP
TMD factorization is not applicable?

TMD factorization



- Twist-3 functions are related to TMD via OPE
- TMD and twist-3 factorizations are related in high QT region
- Global analysis of TMDs and twist-3 is possible:

All four processes can be used.

- Data are from HERMES, COMPASS, JLab,

BaBar, Belle, RHIC, LHC, Fermilab

Global fit is needed.
Work in progress

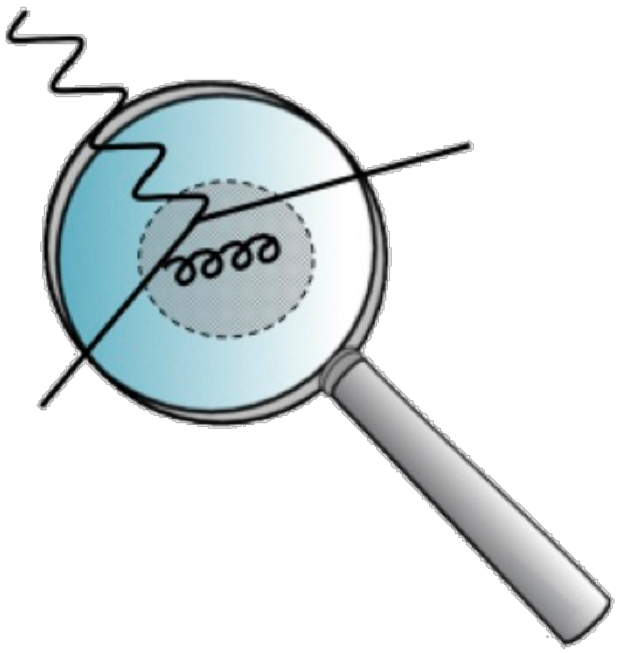
TMDs evolve

Just like collinear PDFs, TMDs also depend on the scale of the probe = evolution

Collinear PDFs

$F(x, Q)$

- ✓ DGLAP evolution
- ✓ Resum $\left[\alpha_s \ln(Q^2/\mu^2)\right]^n$
- ✓ Kernel: purely perturbative



TMDs

$F(x, k_\perp; Q)$

- ✓ Collins-Soper/rapidity evolution equation
- ✓ Resum $\left[\alpha_s \ln^2(Q^2/k_\perp^2)\right]^n$
- ✓ Kernel: can be non-perturbative when $k_\perp \sim \Lambda_{\rm QCD}$

$$F(x, Q_i) \;\downarrow\; R^{\rm coll}(x, Q_i, Q_f) \;\downarrow\; F(x, Q_f)$$

$$F(x, k_\perp, Q_i) \;\downarrow\; R^{\rm TMD}(x, k_\perp, Q_i, Q_f) \;\downarrow\; F(x, k_\perp, Q_f)$$

TMD evolution and non-perturbative component

- Fourier transform back to the momentum space, one needs the whole b region (large b): need some non-perturbative extrapolation
  - Many different methods/proposals to model this non-perturbative part

$$F(x, k_\perp; Q) = \frac{1}{(2\pi)^2} \int d^2 b e^{i k_\perp \cdot b} F(x, b; Q) = \frac{1}{2\pi} \int_0^\infty db\, b J_0(k_\perp b) F(x, b; Q)$$

Collins, Soper, Sterman 85, ResBos, Qiu, Zhang 99, Echevarria, Idilbi, Kang, Vitev, 14, Aidala, Field, Gamberg, Rogers, 14, Sun, Yuan 14, D'Alesio, Echevarria, Melis, Scimemi, 14, Rogers, Collins, 15, Vladimirov, Scimemi 17…

- Eventually evolved TMDs in b-space

$$F(x, b; Q) \approx C \otimes F(x, c/b^*) \times \exp\left\{ -\int_{c/b^*}^{Q} \frac{d\mu}{\mu} \left(A \ln \frac{Q^2}{\mu^2} + B \right) \right\} \times \exp\left(-S_{\text{non-pert}}(b, Q) \right)$$

longitudinal/collinear part

transverse part

✓ Non-perturbative: fitted from data
✓ The key ingredient – ln(Q) piece is spin-independent

Since the polarized scattering data is still limited kinematics, we can use unpolarized data to constrain/ extract key ingredients for the non-perturbative part

Quark TMDs

N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

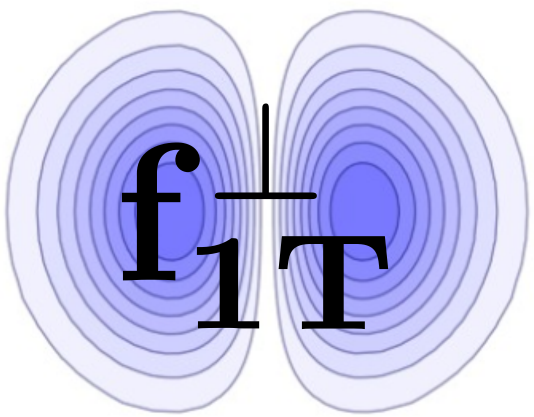
8 functions in total (at leading twist)

Each represents different aspects of partonic structure

Each depends on Bjorken-x, transverse momentum, the scale

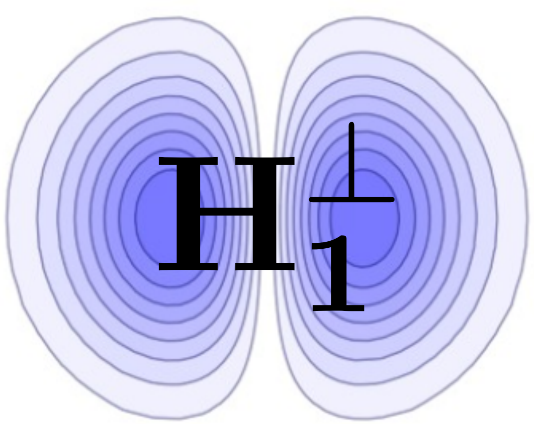
Each function is to be studied

Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)



Sivers function

Non universal



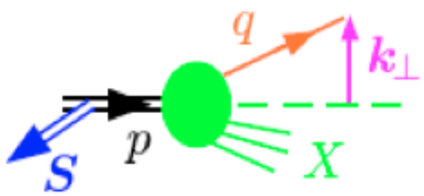
Collins function

Universal

Definitions

Sivers function: unpolarized quark distribution inside a transversely polarized nucleon

Sivers 1989



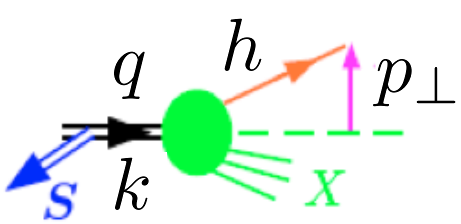
$$f_{q/h^\uparrow}(x, \vec{k}_\perp, \vec{S}) = f_{q/h}(x, k_\perp^2) - \frac{1}{M} f_{1T}^{\perp q}(x, k_\perp^2) \vec{S} \cdot (\hat{P} \times \vec{k}_\perp)$$

Spin independent

Spin dependent

Collins function: unpolarized hadron from a transversely polarized quark

Collins 1992



$$D_{q/h}(z, \vec{p}_\perp, \vec{s}_q) = D_{q/h}(z, p_\perp^2) + \frac{1}{zM_h} H_1^{\perp q}(z, p_\perp^2) \vec{s}_q \cdot (\hat{k} \times \vec{p}_\perp)$$

Sivers function: $f_{1T}^{\perp q}$ describes strength of correlation $\vec{S} \cdot (\hat{P} \times \vec{k}_\perp)$

Sivers 1989

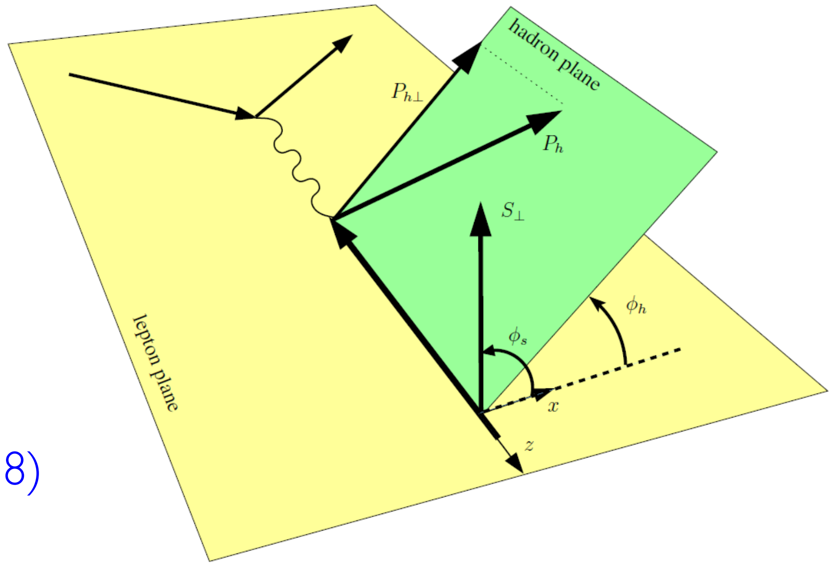
Collins function: $H_1^{\perp q}$ describes strength of correlation $\vec{s}_q \cdot (\hat{k} \times \vec{p}_\perp)$

Collins 1992

Both functions extensively studied experimentally, phenomenologically, theoretically

$$\ell P \to \ell' \pi X$$

Sivers function and Collins function can give rise to Single Spin Asymmetries in scattering processes. For instance in Semi Inclusive Deep Inelastic process



Kotzinian (1995), Mulders, Tangerman (1995), Boer, Mulders (1998)

$$d\sigma(S) \sim \sin(\phi_h + \phi_S) h_1 \otimes H_1^\perp + \sin(\phi_h - \phi_S) f_{1T}^\perp \otimes D_1 + \ldots$$

Sivers function

Large – N_c result $f_{1T}^{\perp u} = -f_{1T}^{\perp d}$ Pobylitsa 2003

→ Confirmed by phenomenological extractions

→ Confirmed by experimental measurements

Relation to GPDs (E) and anomalous magnetic moment Burkardt 2002

$$f_{1T}^{\perp q} \sim \kappa^q$$

→ Predicted correct sign of Sivers asymmetry in SIDIS

→ Shown to be model-dependent

Meissner, Metz, Goeke 2007

→ Used in phenomenological extractions

Bacchetta, Radici 2011

Sivers function

Sum rule

Burkardt 2004

→ Conservation of transverse momentum

→ Average transverse momentum shift of a quark inside a transversely polarized nucleon

$$\langle k_T^{i,q} \rangle = \varepsilon_T^{ij} S_T^j f_{1T}^{\perp(1)q}(x)$$

$$f_{1T}^{\perp(1)q}(x) = \int d^2 k_\perp \frac{k_\perp^2}{2M^2} f_{1T}^{\perp q}(x, k_\perp^2)$$

→ Sum rule

$$\sum_{a=q,g} \int_0^1 dx \langle k_T^{i,a} \rangle = 0 \qquad \sum_{a=q,g} \int_0^1 dx\, f_{1T}^{\perp(1)a}(x) = 0$$

Sivers function

Extractions

→ Many extractions without taking into account TMD evolution

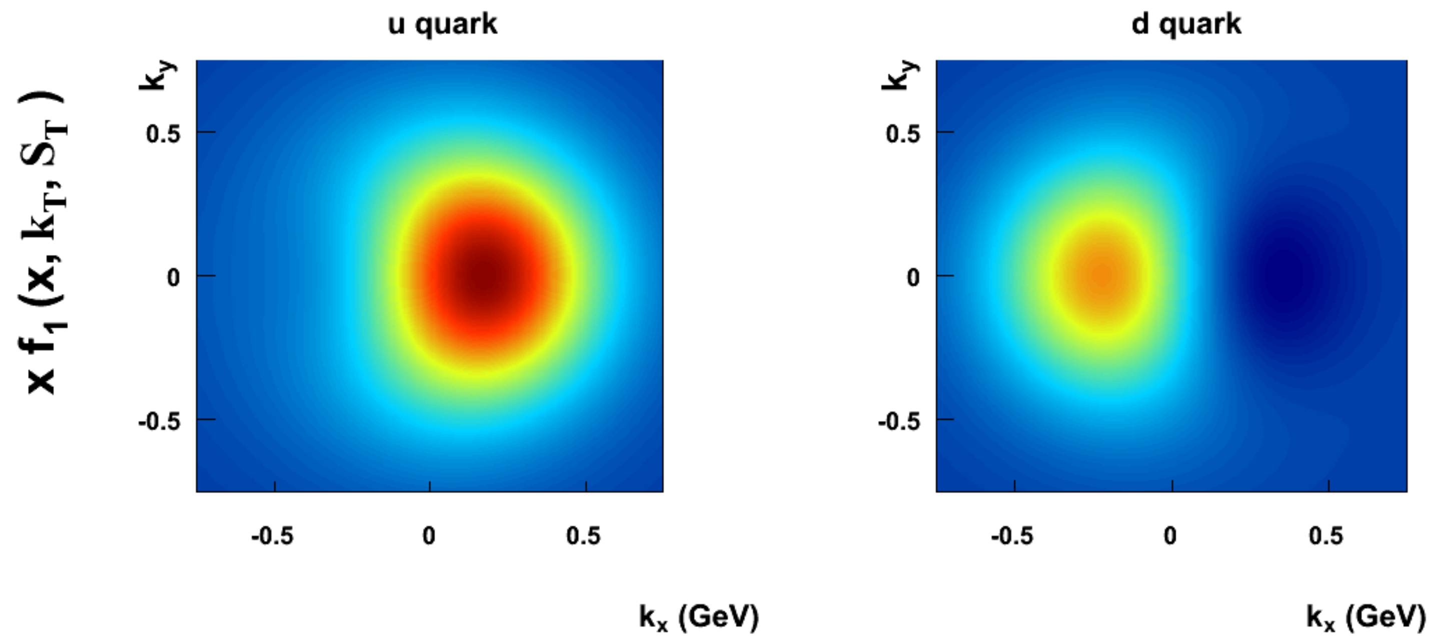
Efremov et al 2005, Vogelsang, Yuan 2005, Anselmino et al 2005, Collins et al 2006, Anselmino et al 2009, 2011, 2016, Bacchetta Radici 2011

→ Extractions with TMD evolution

Echevarria et al 2014, Sun Yuan 2013

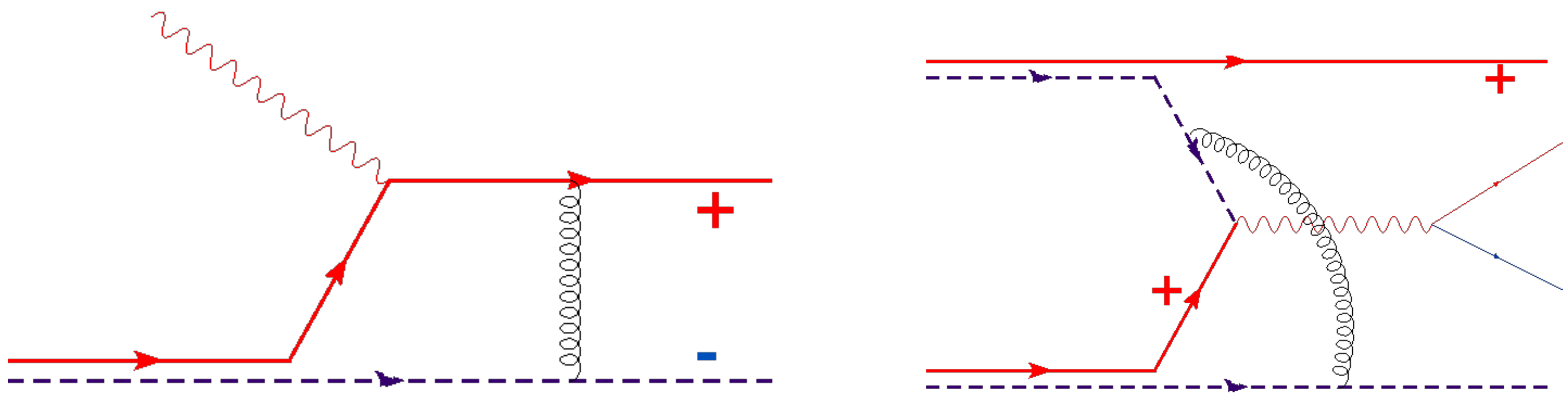
→ Relation to the tomography of the nucleon

Anselmino et al 2011



→ Agreement with the sum rule and large N_c prediction

Colored objects are surrounded by gluons, profound consequence of gauge invariance:
Sivers function has opposite sign when gluon couple after quark scatters (SIDIS) or before quark annihilates (Drell-Yan)



Brodsky,Hwang,Schmidt;
Belitsky,Ji,Yuan;
Collins;
Boer,Mulders,Pijlman;
Kang, Qiu;
Kovchegov, Sievert;
etc

$$f_{1T}^{\perp SIDIS} = -f_{1T}^{\perp DY}$$

Crucial test of TMD factorization and collinear twist-3 factorization
Several labs worldwide aim at measurement of Sivers effect in Drell-Yan
BNL, CERN, GSI, IHEP, JINR, FERMILAB etc
Barone et al., Anselmino et al., Yuan, Vogelsang, Schlegel et al., Kang, Qiu, Metz, Zhou etc
The verification of the sign change is an NSAC (DOE and NSF) milestone

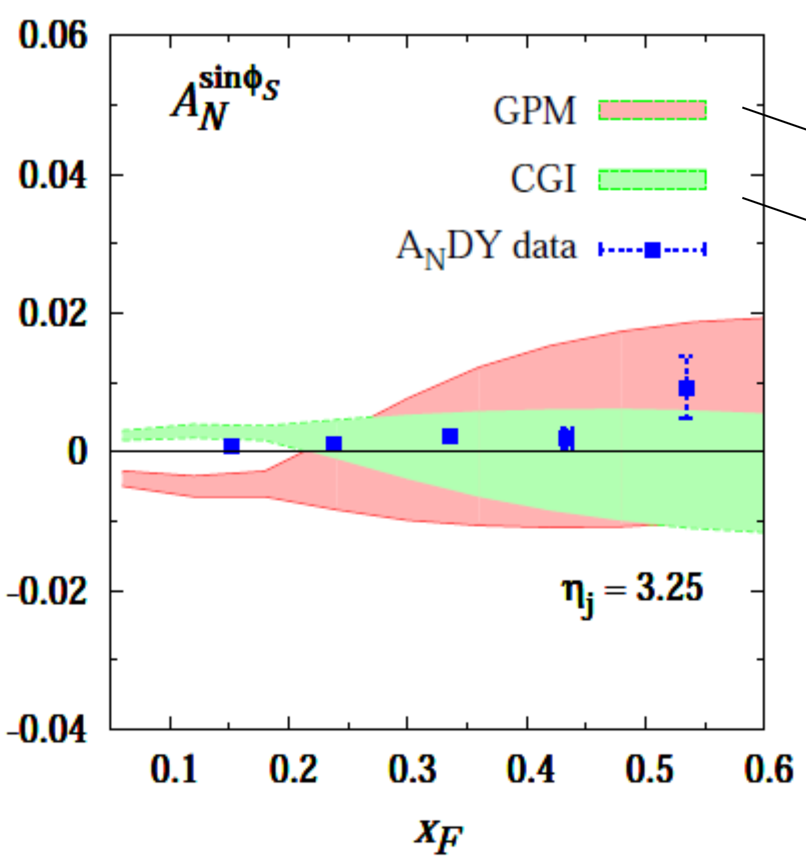
Process dependence of Sivers function

- Indication on process dependence of Sivers functions from analysis of A_N in $\ell N^{\uparrow} \to \ell X$

Metz et al 2012

- Indication on process dependence from AnDY data on A_N in $p^{\uparrow} p \to jet X$

Gamberg, Kang, AP 2013
D'Alesio et al 2013



No sign change

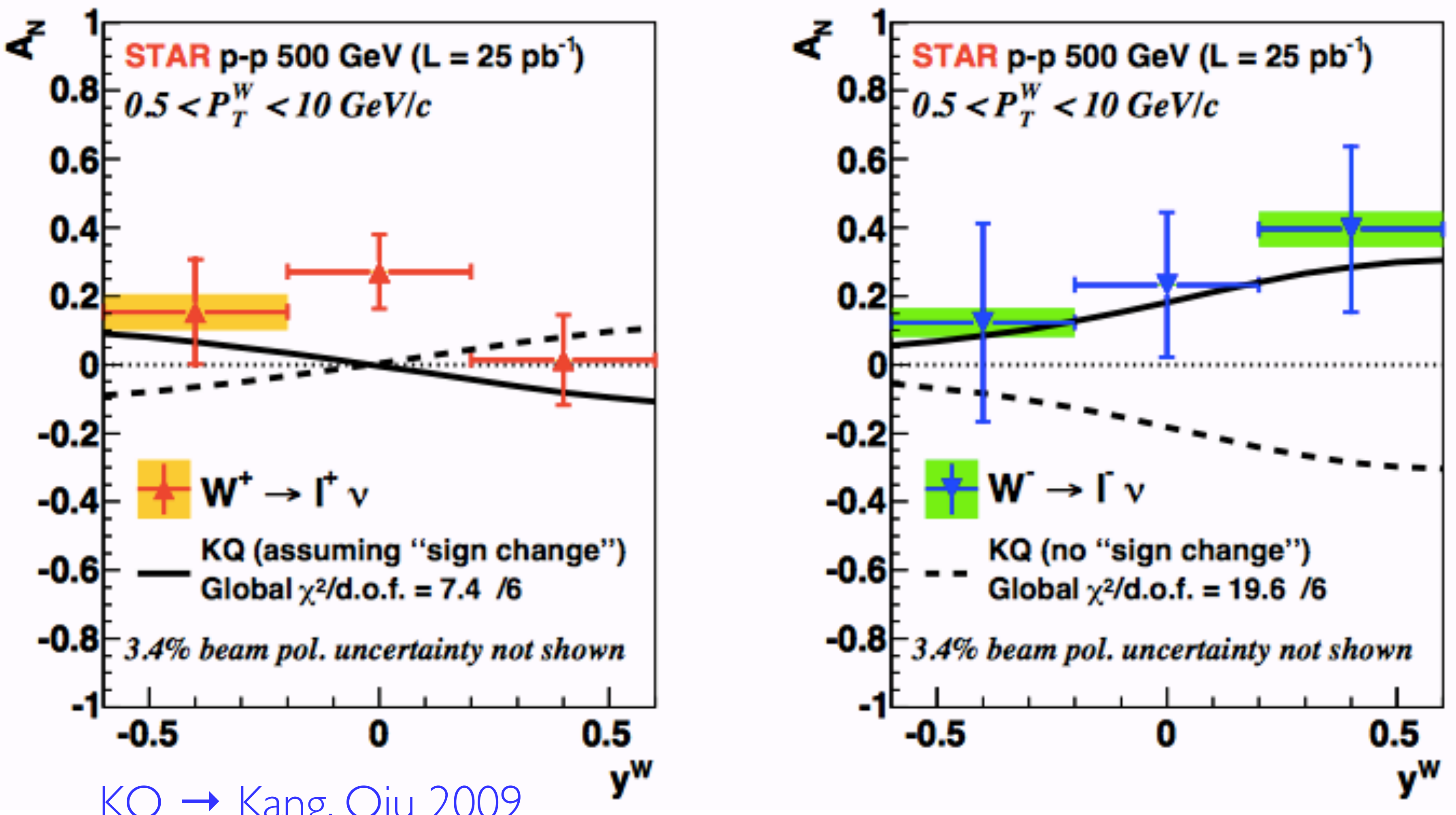
Sign change

Process dependence of Sivers function

STAR 2016

→ First experimental hint on the sign change: A_N in W and Z production

STAR Collab. Phys. Rev. Lett. 116, 132301 (2016)



KQ → Kang, Qiu 2009

$$p^\uparrow p \to W^\pm X$$
$$p^\uparrow p \to Z^0 X$$

→ No sign change

$$\chi^2/d.o.f \sim 1.2$$

$$\chi^2/d.o.f \sim 3.2$$

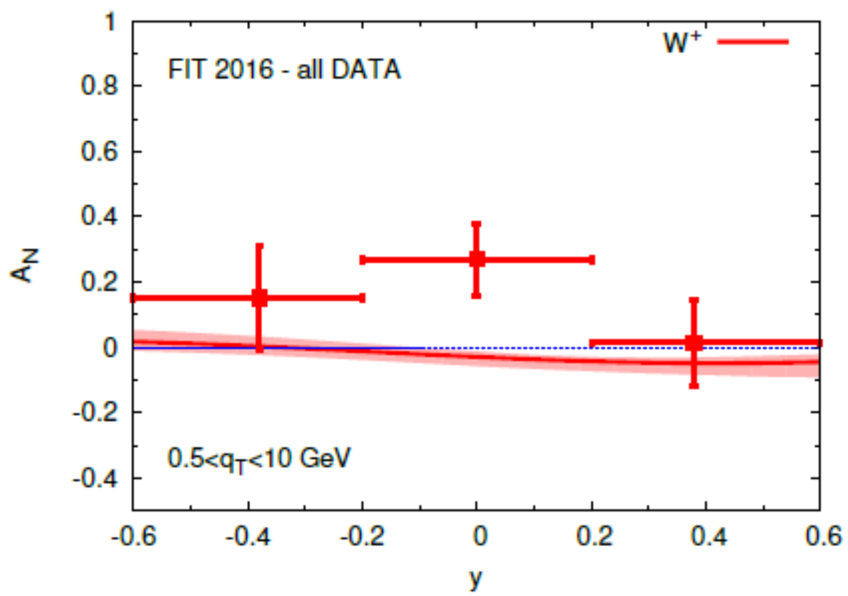
→ Large uncertainties of predictions

→ No antiquark Sivers functions

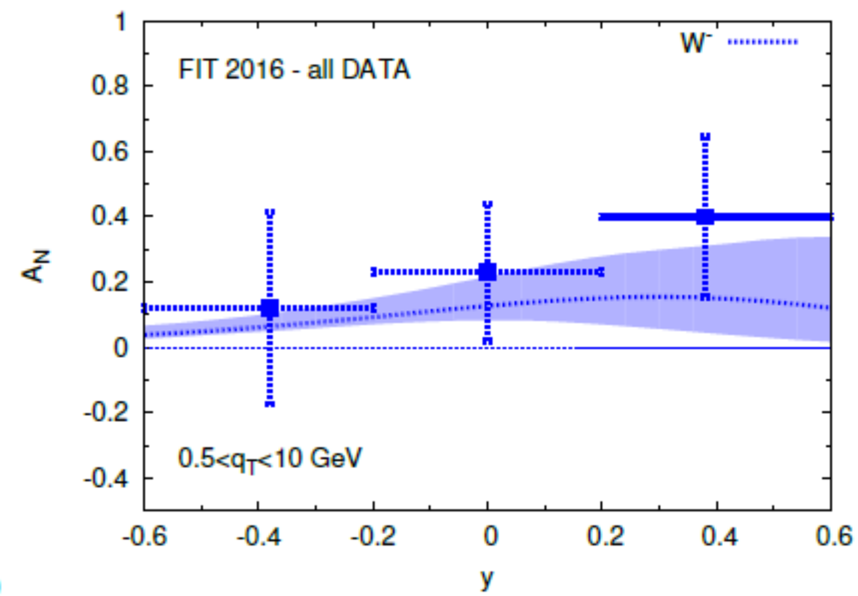
Process dependence of Sivers function

Anselmino et al 2016

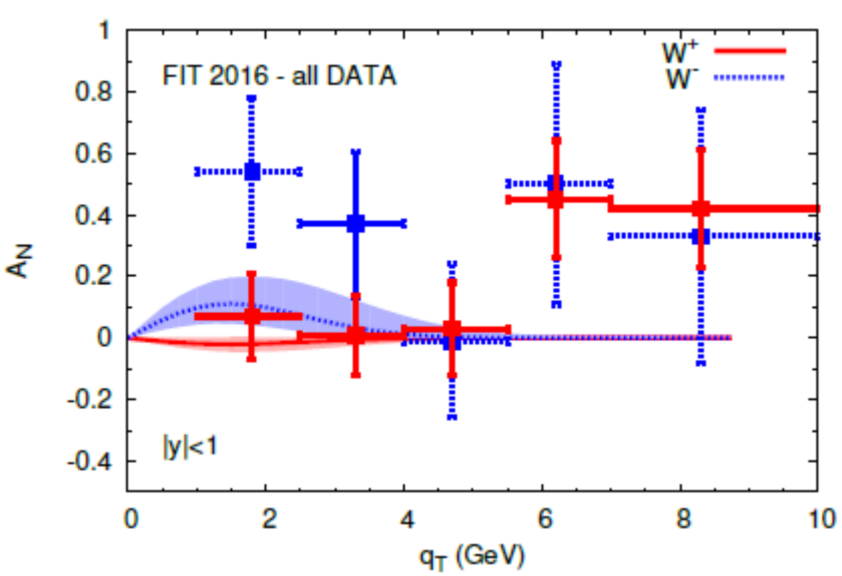
→ First experimental hint on the sign change: A_N in W and Z production



(a)



$$p^{\uparrow}p \rightarrow W^{\pm}X$$



→ Results with sign change

→ No TMD evolution

→ Antiquark Sivers functions included

→ STAR results hint on sign change

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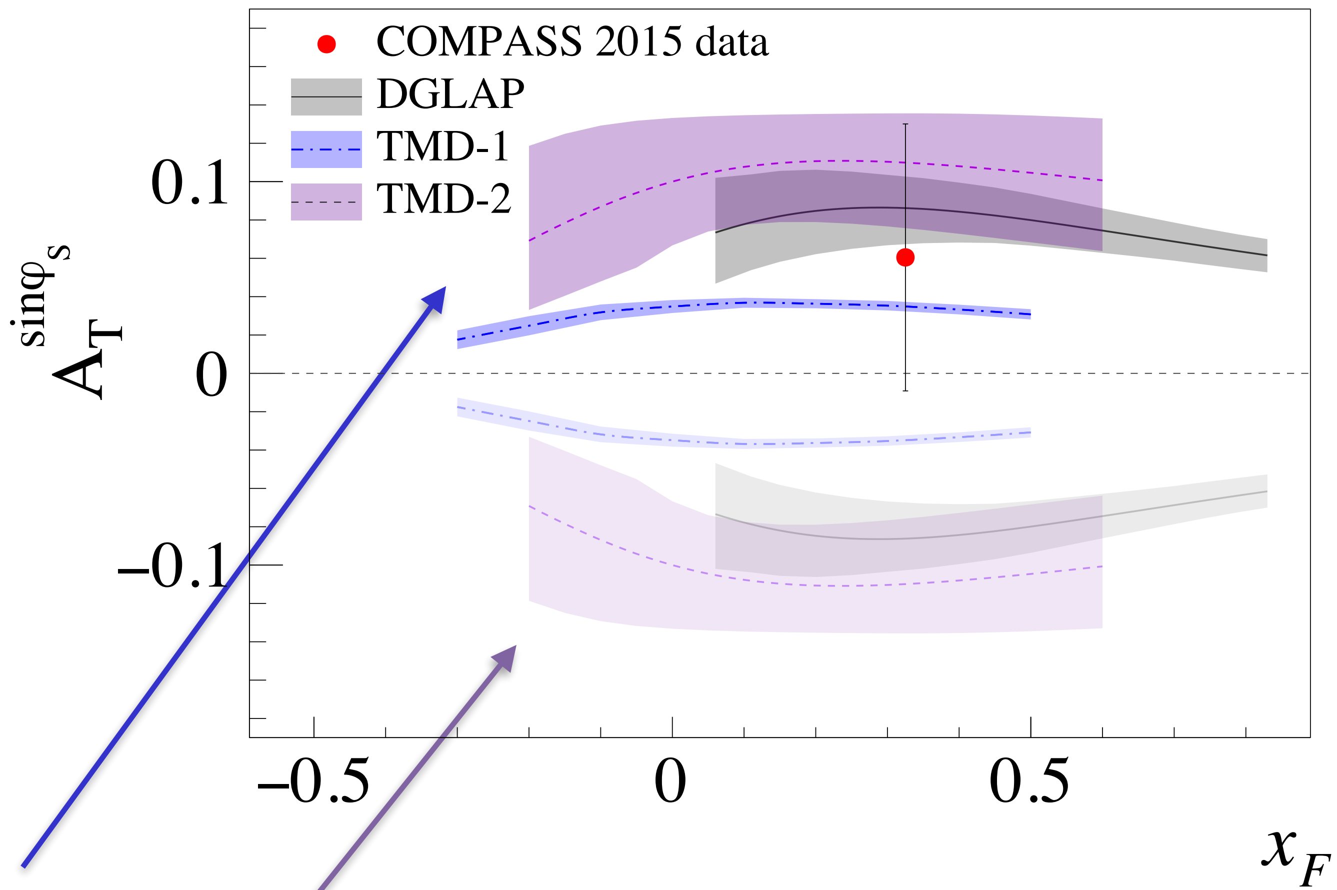
See talk by M. Boglione

Process dependence of Sivers function

COMPASS 2017

- First experimental hint on the sign change in Drell-Yan



- Sign change
- No sign change

- COMPASS results hint on sign change

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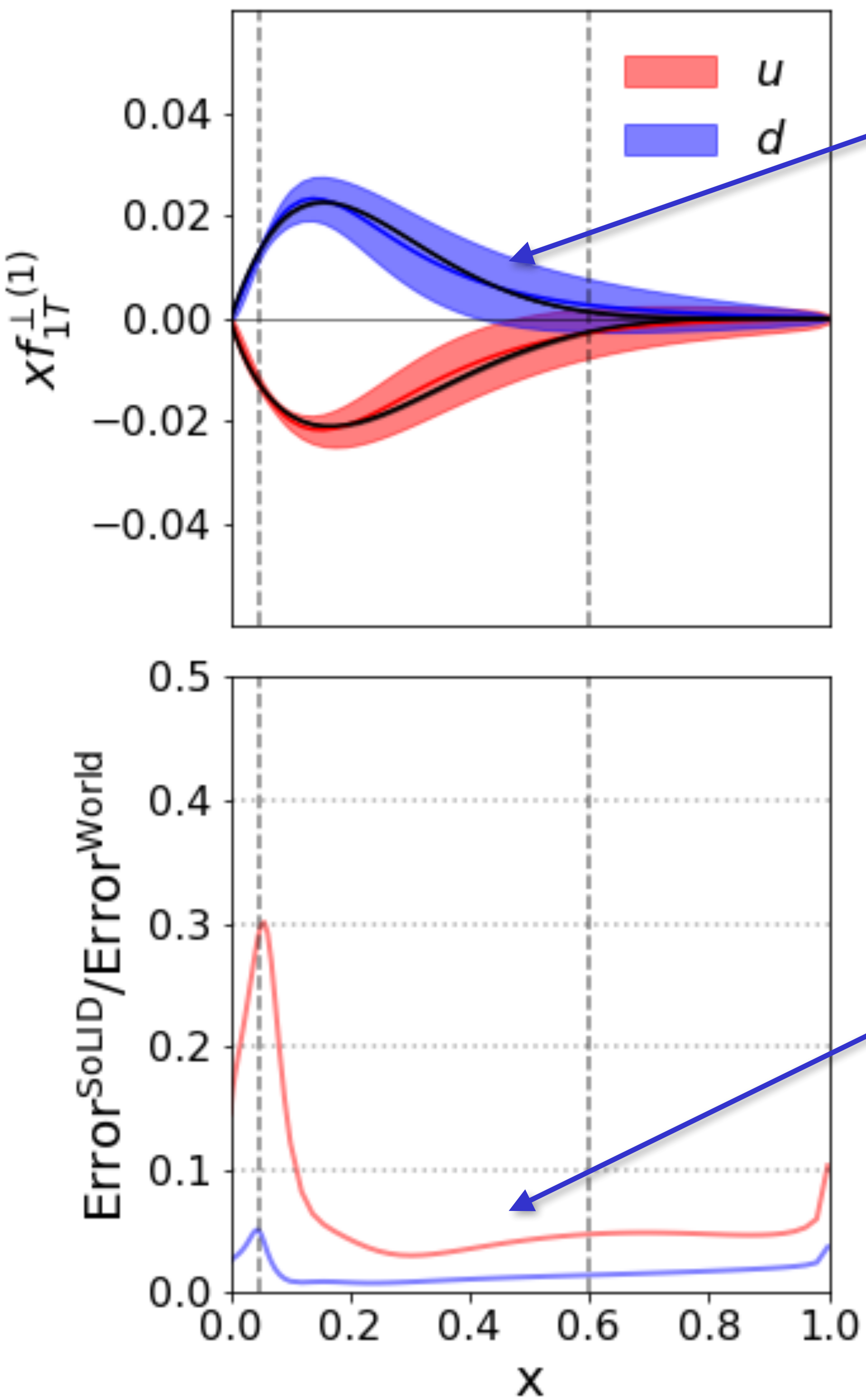
Sivers function

Expectation of JLab 12

Sato et al 2018



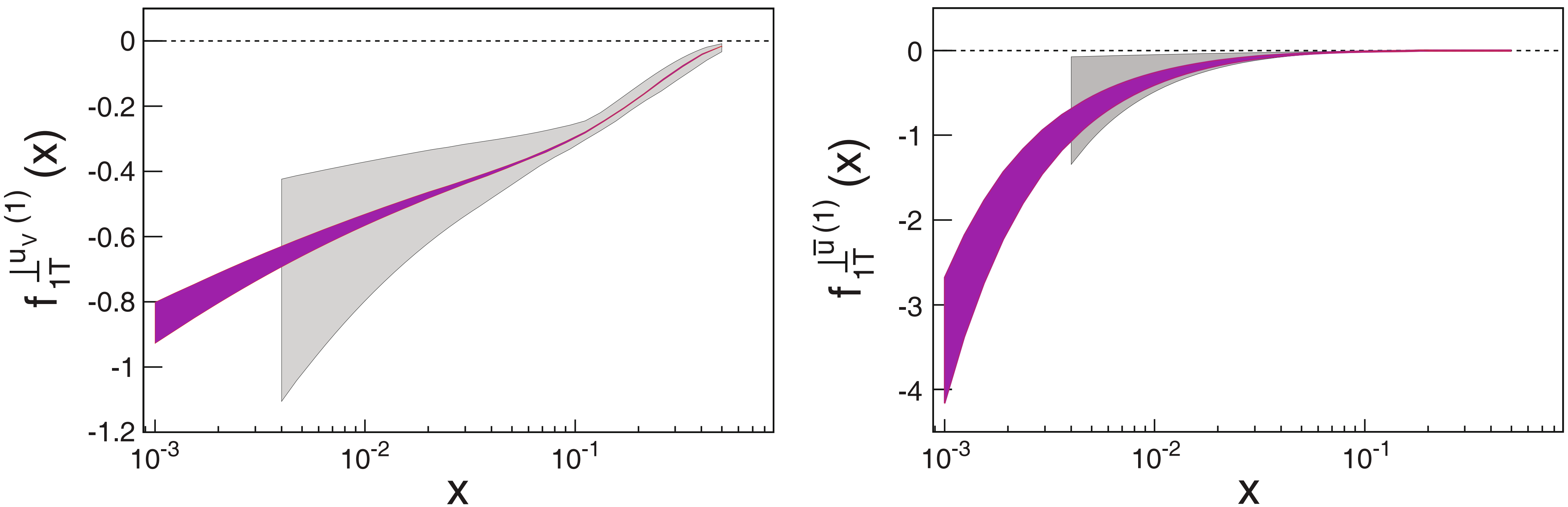
Current knowledge

Improvement from SoLID at JLab 12

Sivers function

Expectation of EIC

AP 2010



Update of this estimate is needed and work is in progress

Collins function

Schafer-Teryaev sum rule

Schafer Teryaev 1999
Meissner, Metz, Pitonyak 2010

→ Conservation of transverse momentum

$$\langle P_T^i(z)\rangle \sim H_1^{\perp(1)}(z) \qquad H_1^{\perp(1)}(z) = \int d^2p_\perp \frac{p_\perp^2}{2z^2M_h^2} H_1^\perp(z,p_\perp^2)$$

→ Sum rule

$$\sum_h \int_0^1 dz \langle P_T^i(z)\rangle = 0$$

→ If only pions are considered $H_1^{\perp fav}(z) \sim -H_1^{\perp unf}(z)$

Universality of TMD fragmentation functions

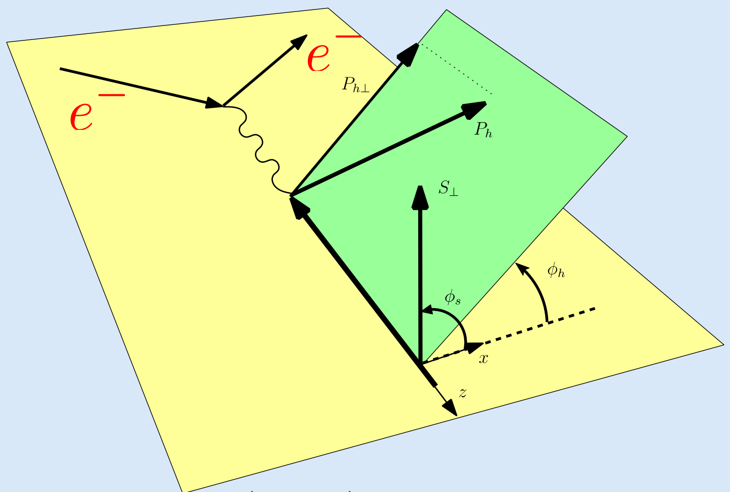
Metz 2002, Metz, Collins 2004, Yuan 2008
Gamberg, Mukherjee, Mulders 2011
Boer, Kang, Vogelsang, Yuan 2010

$$H_1^\perp(z)|_{SIDIS} = H_1^\perp(z)|_{e^+e^-} = H_1^\perp(z)|_{pp}$$

→ Very non trivial results

→ Agrees with phenomenology, allows global fits

- SIDIS and e+e-: combined global analysis

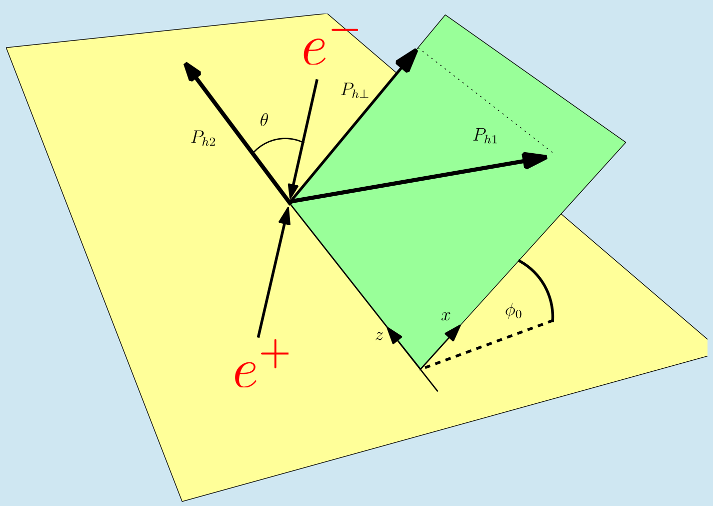


$$F_{UT}^{\sin(\phi_h+\phi_s)} \sim h_1(x_B, k_\perp) H_1^\perp(z_h, p_\perp)$$

transversity

Collins function

$$\frac{d\sigma(S_\perp)}{dx_B\, dy\, dz_h\, d^2P_{h\perp}} = \sigma_0(x_B, y, Q^2)\left[F_{UU} + \sin(\phi_h+\phi_s)\,\frac{2(1-y)}{1+(1-y)^2}\,F_{UT}^{\sin(\phi_h+\phi_s)} + \ldots\right]$$



$$Z_{\mathrm{collins}}^{h_1h_2} \sim H_1^\perp(z_1, p_{1\perp}) H_1^\perp(z_2, p_{2\perp})$$

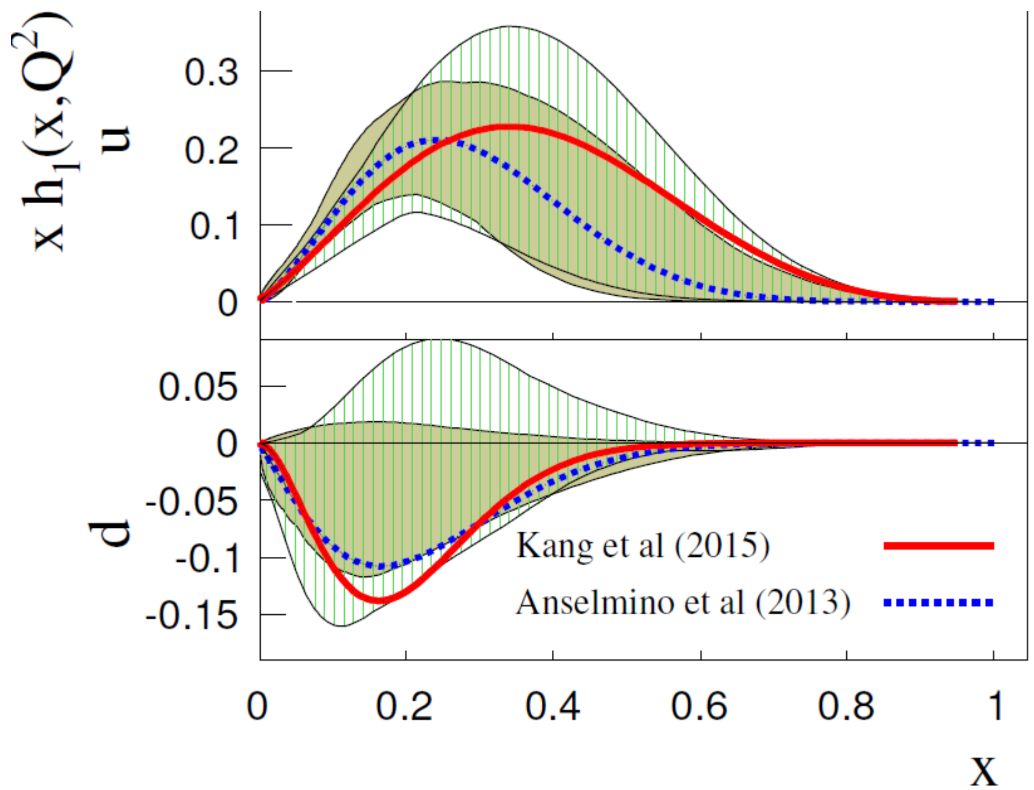
Collins function

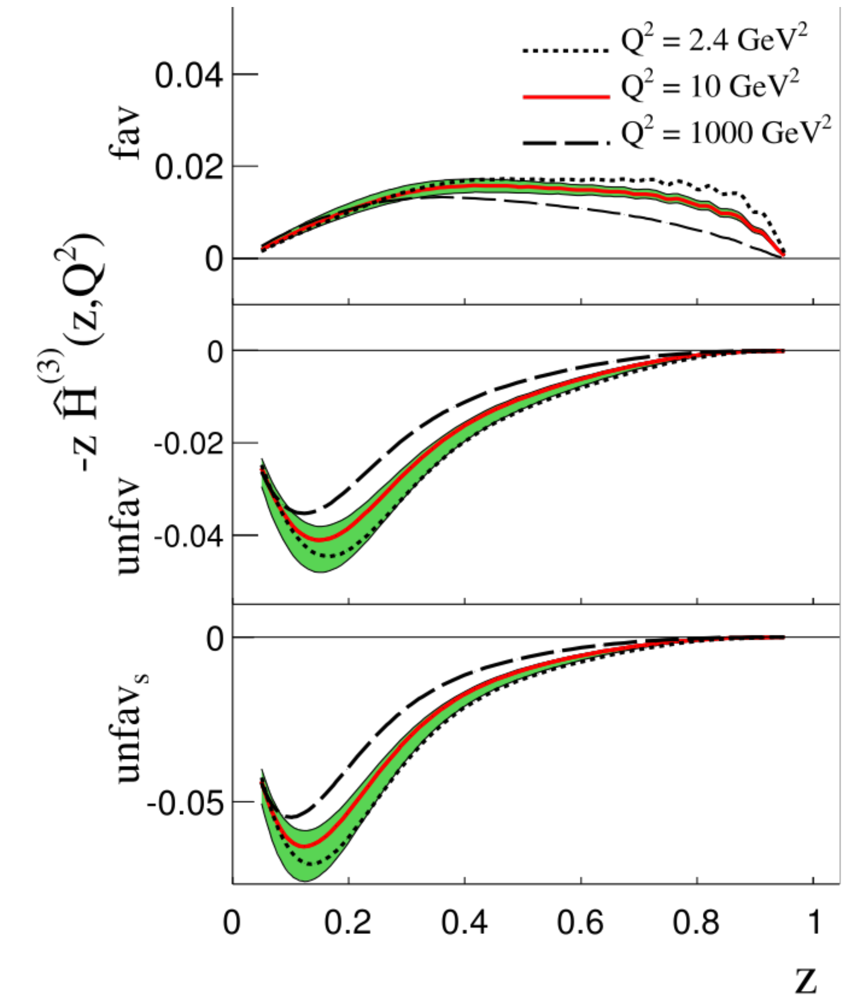
Collins function

$$\frac{d\sigma^{e^+e^-\to h_1h_2+X}}{dz_{h1}\, dz_{h2}\, d^2P_{h\perp}\, d\cos\theta} = \frac{N_c\pi\alpha_{\mathrm{em}}^2}{2Q^2}\left[\left(1+\cos^2\theta\right) Z_{uu}^{h_1h_2} + \sin^2\theta\cos(2\phi_0) Z_{\mathrm{collins}}^{h_1h_2}\right]$$

Kang-Prokudin-Sun-Yuan 2015
Anselmino et al 2015

- Fitted quark transversity and Collins function: x (z) -dependence



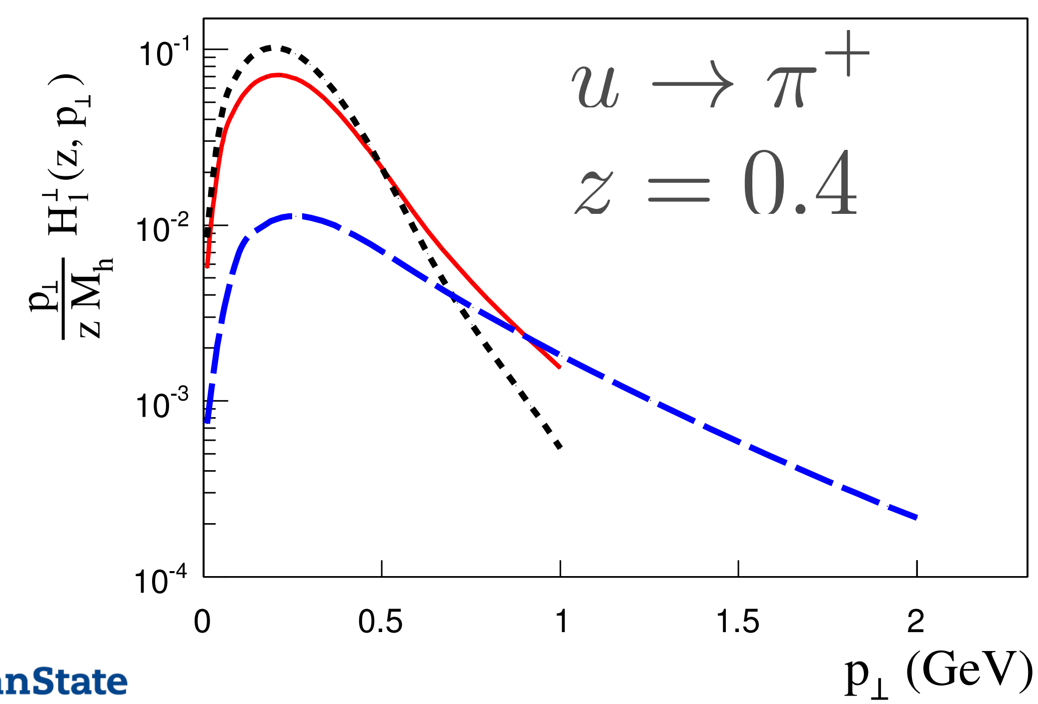


$\mathrm{fav}: \quad u \to \pi^+$

$\mathrm{unfav}: \quad d \to \pi^+$

$\mathrm{unfav_s}: \quad s \to \pi$

- Collins function: pt-dependence



Compatible with LO extraction
Anselmino et al 2009, 2013, 2015

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Precision of extraction depends on precision of calculations

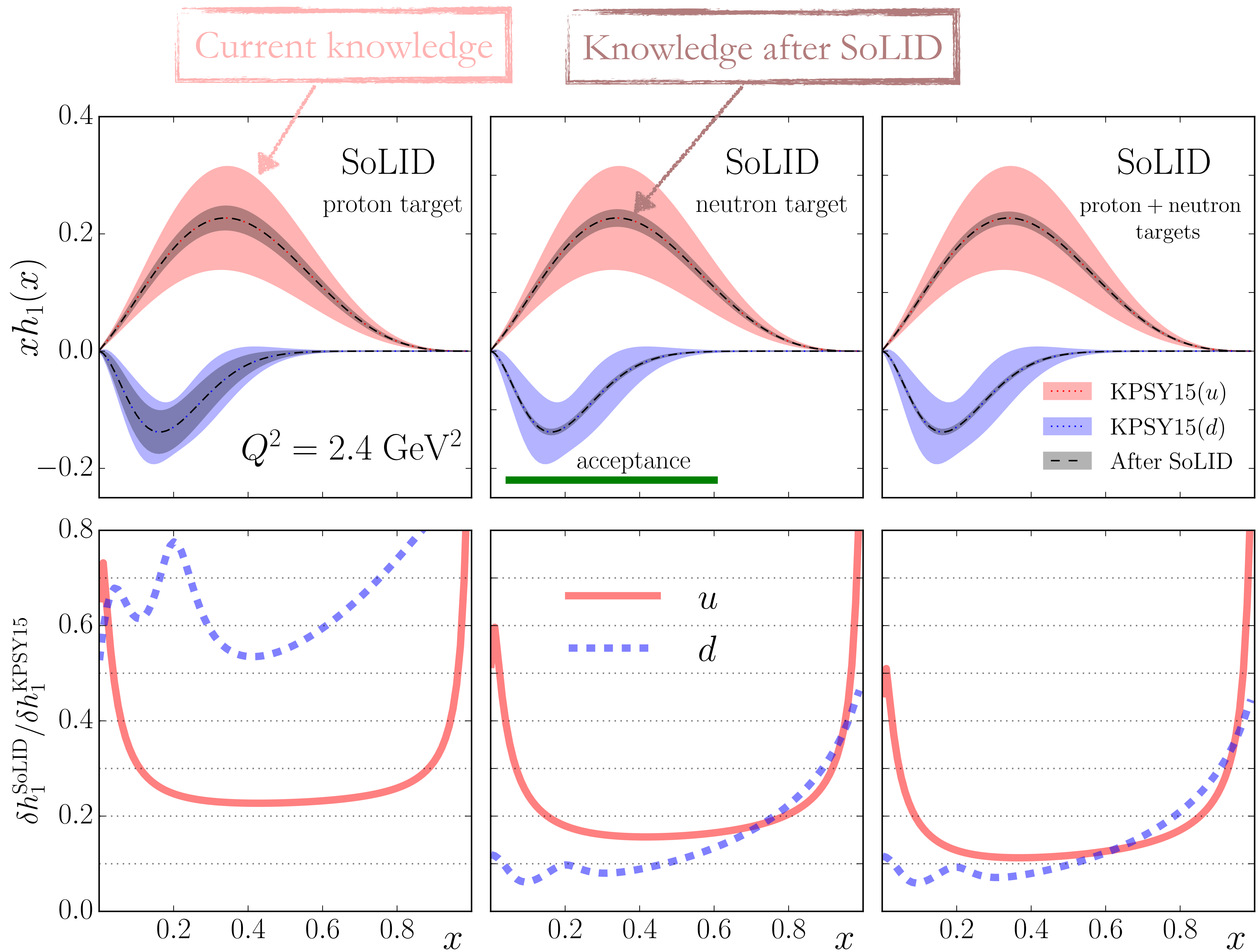
Leading Log (LL):	$A^{(1)}$		
Next-to Leading Log (NLL):	$A^{(1,2)}$	$B^{(1)}$	$C^{(1)}$
Next-to-Next-to Leading Log (NNLL):	$A^{(1,2,3)}$	$B^{(1,2)}$	$C^{(1)}$

Kang, AP, Sun, Yuan 2015
Echevarria, Scimemi, Vladimirov 2016

Precision is important!

$C^{(1)}$ means that one should use NLO collinear distributions

What do we expect from JLab 12?

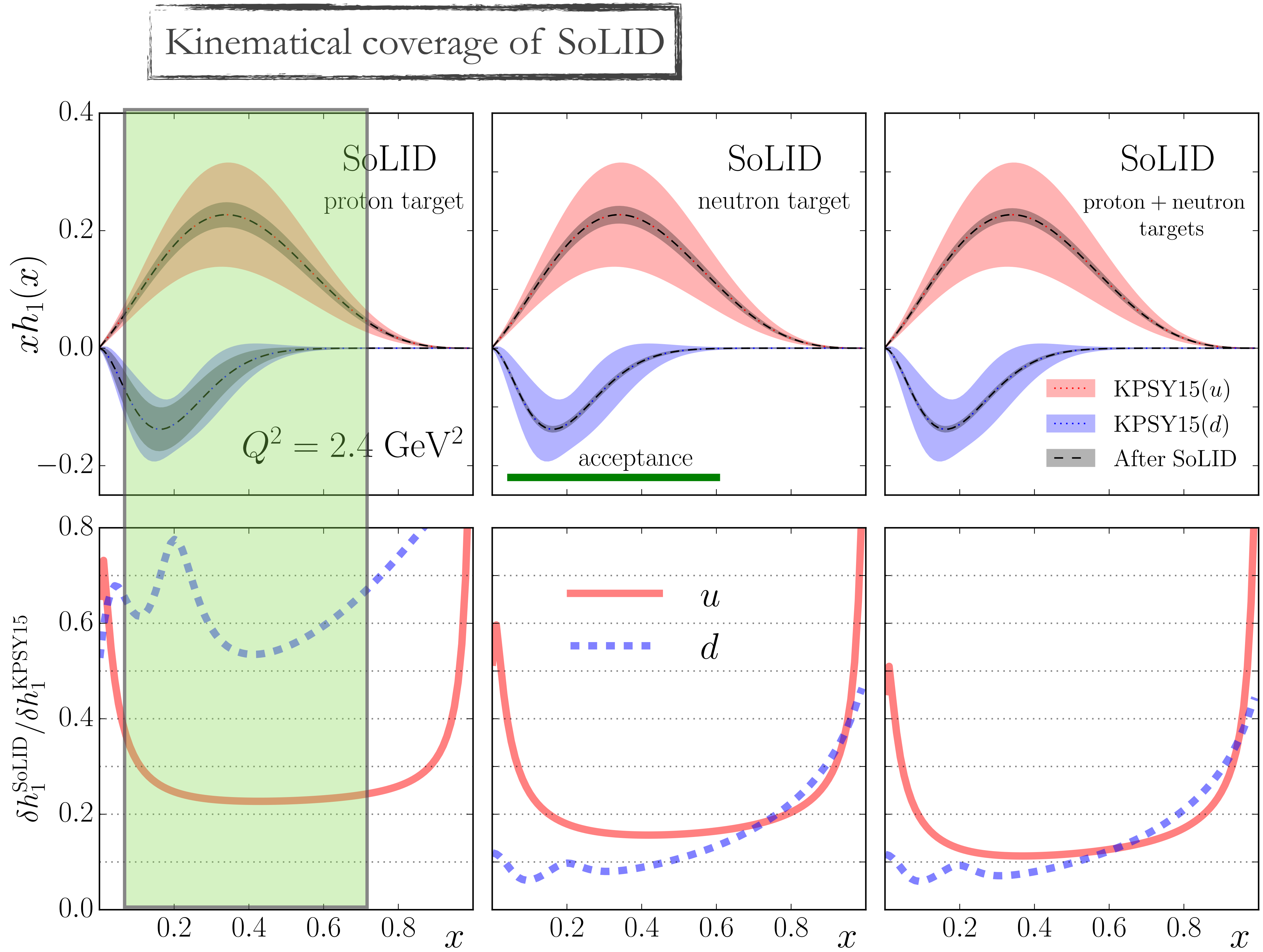


Bayesian statistics is used to estimate the improvement from new data

Current knowledge corresponds to a fit with TMD evolution *Kang et al., P.R. D**93** (16) 014009*

Ye et al Phys.Lett. B767 (2017) 91-98

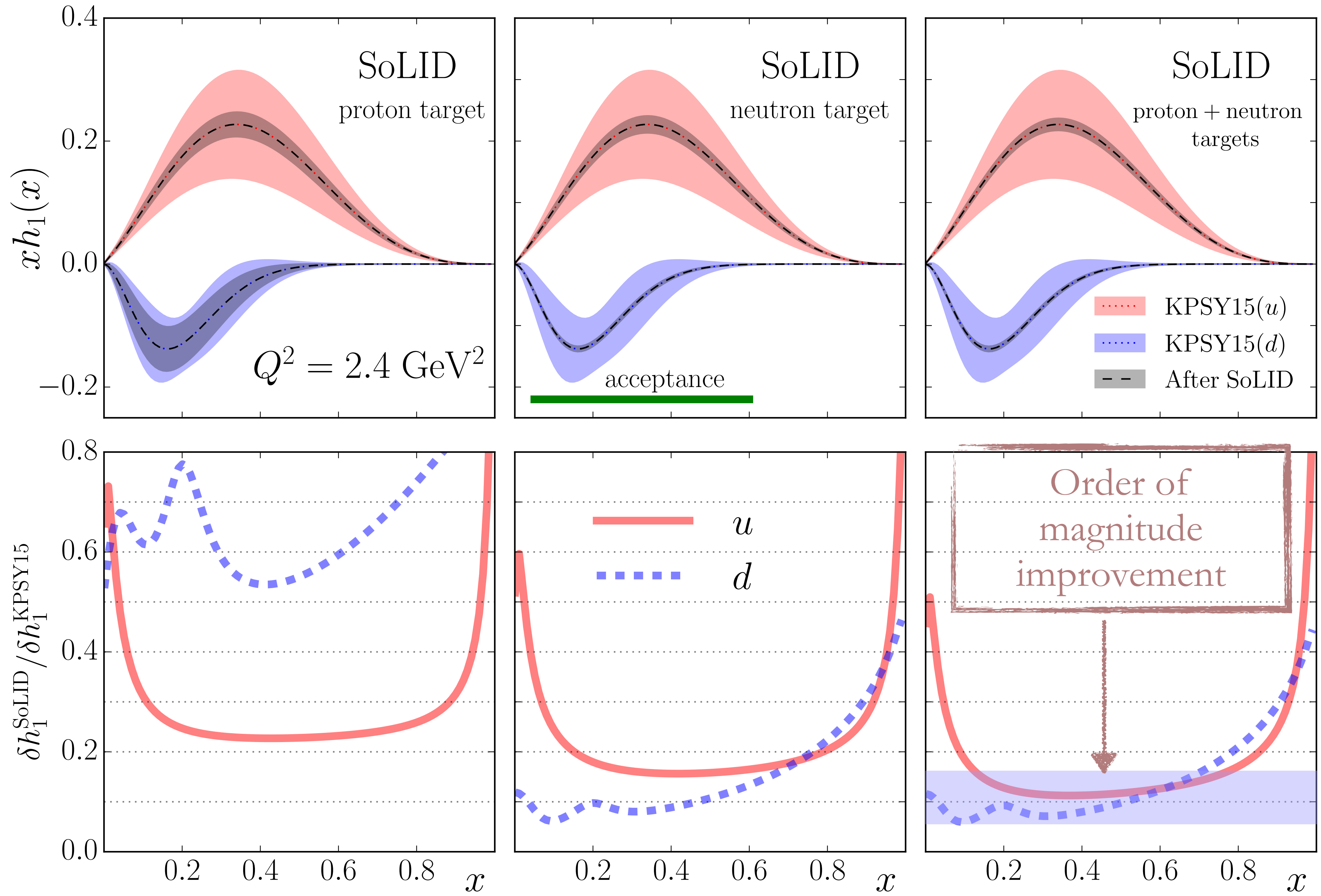
What do we expect from JLab 12?



The errors grow outside of the future data region as expected

Ye et al Phys.Lett. B767 (2017) 91-98

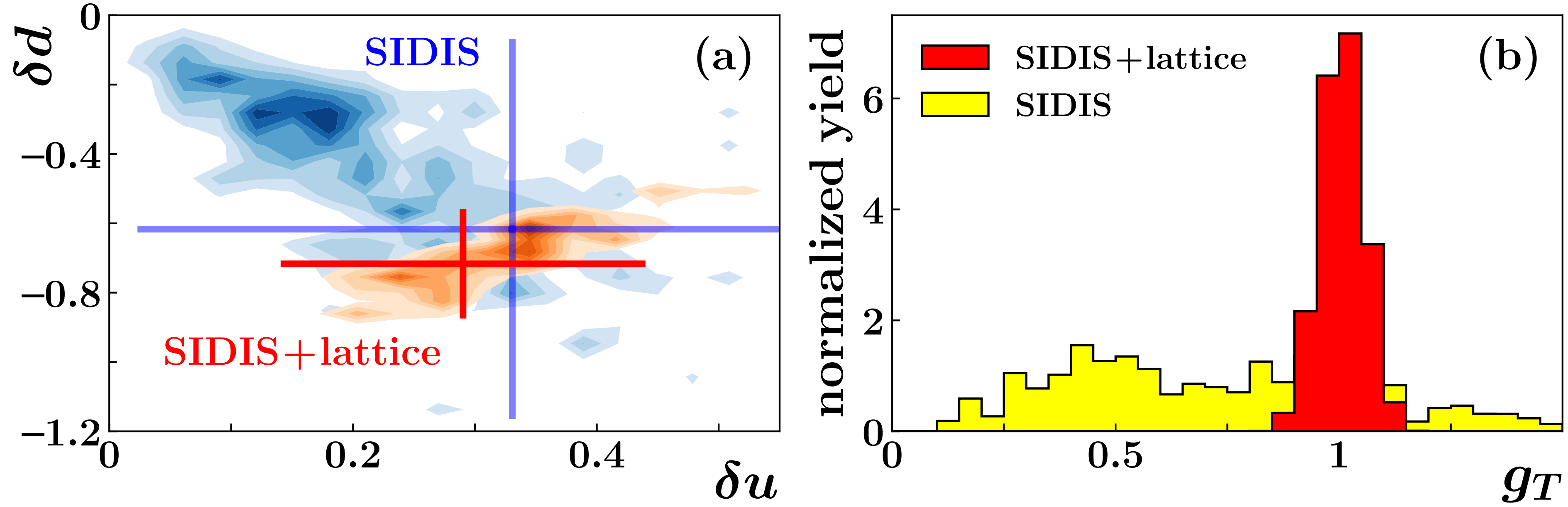
What do we expect from JLab 12?



Only combination of proton and neutron target measurements
will ensure similar improvement for both u and d quark transversity

Ye et al Phys.Lett. B767 (2017) 91-98

$$g_T = \delta u - \delta d$$ isovector tensor charge



First combined new methodology fir of SIDIS data using lattice constraints:
Lattice and SIDIS data are compatible and including lattice data improves extraction of g_T

Lin, Melnitchouk, AP, Sato 2017

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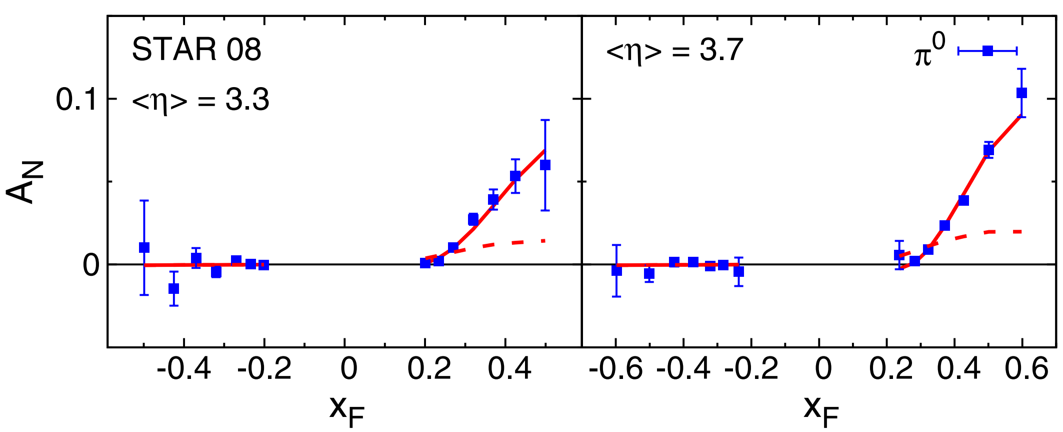
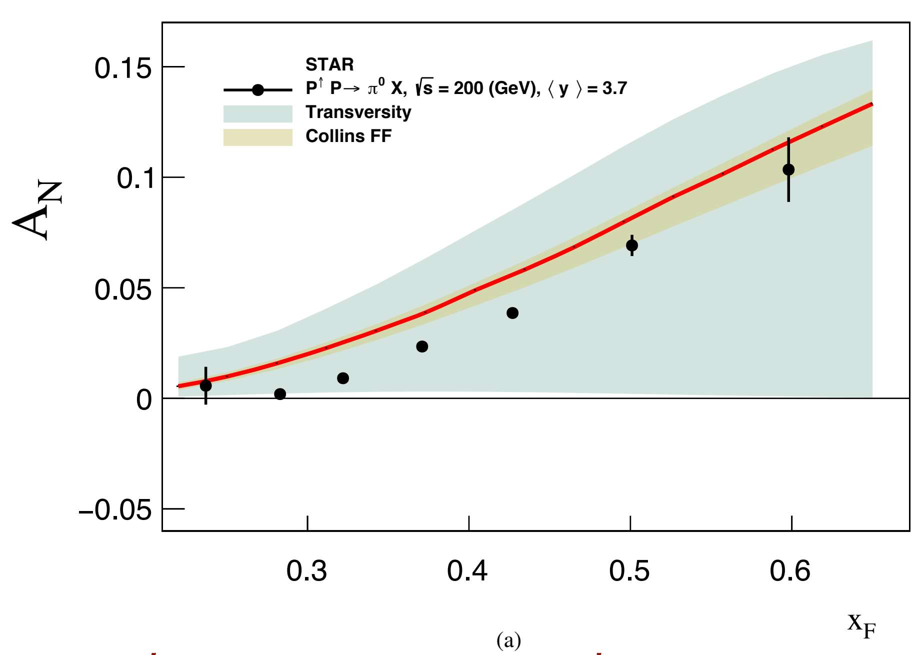
See talk by Jake Ethier

RHIC: the process $p + p^\uparrow \longrightarrow \pi + X$

Twist-3 factorization, fragmentation contributions



Kanazawa, Koike, Metz, Pitonyak (2014)



Gamberg, Kang, Pitonyak, AP PLB 770 (2017)

$$\frac{E_h d\sigma^{Frag}(S_P)}{d^3\vec{P}_h} = -\frac{4\alpha_s^2 M_h}{S}\epsilon^{P'PP_hS_P}\sum_i\sum_{a,b,c}\int_0^1\frac{dz}{z^3}\int_0^1 dx'\int_0^1 dx\,\delta(\hat s+\hat t+\hat u)$$
$$\times\frac{1}{\hat s(-x'\hat t - x\hat u)}\,h_1^a(x)\,f_1^b(x')\left\{\left[H_1^{\perp(1),\pi/c}(z) - z\frac{dH_1^{\perp(1),\pi/c}(z)}{dz}\right]S^i_{H_1^\perp} + \frac{1}{z}H^{\pi/c}(z)\,S^i_H\right.$$
$$\left.+\frac{2}{z}\int_z^\infty\frac{dz_1}{z_1^2}\frac{1}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2}\hat H_{FU}^{\pi/c,\Im}(z,z_1)\,S^i_{\hat H_{FU}}\right\},$$

Integration over **x** for transversity, conservation of momenta in *ab⟶cd:* $x_{min} = -(U/z)/(T/z + S)$

$$\int_{x_{min}}^1\frac{dx}{x}$$

RHIC data is sensitive to high-x behavior of transversity quark-gluon channel is dominant contribution for large x_F

More complicated structure of cross-section, additional functions to study

Improving errors in large-x region?
Analysis in progress.

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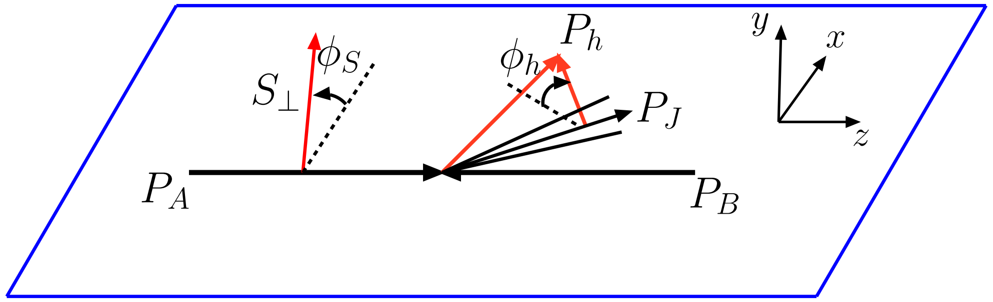
Hadron within a jet

- Consider a process P P scattering where jet is produced and a hadron is measured inside the jet

$$p^{\uparrow}\left[\vec{S}_{\perp}(\phi_S)\right] + p \to [\text{jet}\, h(\phi_H)] + X$$

Feng Yuan (2008), D'Alesio, Murgia, Pisano (2014)

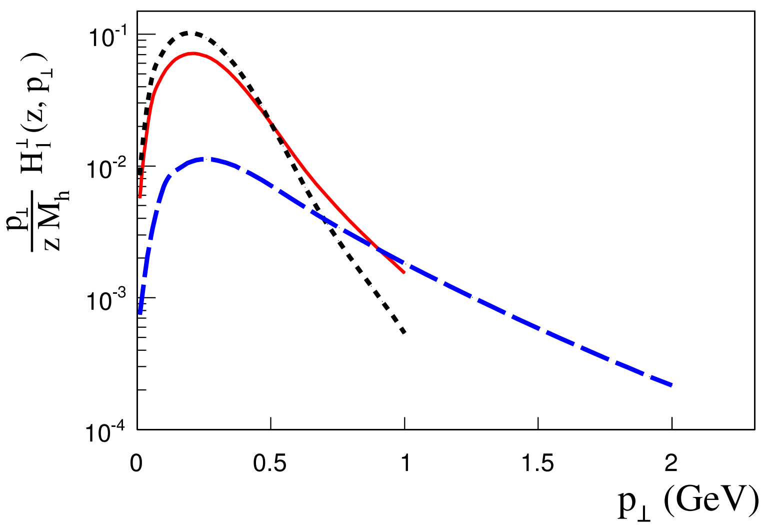


- Azimuthal modulation is related to convolution of Collins FF and transversity

Kang, Prokudin, Sun, Yuan (2015)

$$\frac{d\sigma}{dy\, d^2p_\perp^{\text{jet}}\, dz\, d^2 j_T} = F_{UU} + \sin(\phi_S - \phi_H) F_{UT}^{\sin(\phi_S - \phi_H)}$$

$$F_{UT}^{\sin(\phi_S-\phi_H)} \propto h_1^a(x_1) \otimes f_{b/B}(x_2) \otimes \frac{j_T}{zM_h} H_1^{\perp c}(z, j_T^2) \otimes H_{ab\to c}^{\text{Collins}}(\hat{s}, \hat{t}, \hat{u})$$



j_T : hadron transverse momentum with respect to the jet direction

Kang, Prokudin, Ringer, Yuan PLB 774 (2017)

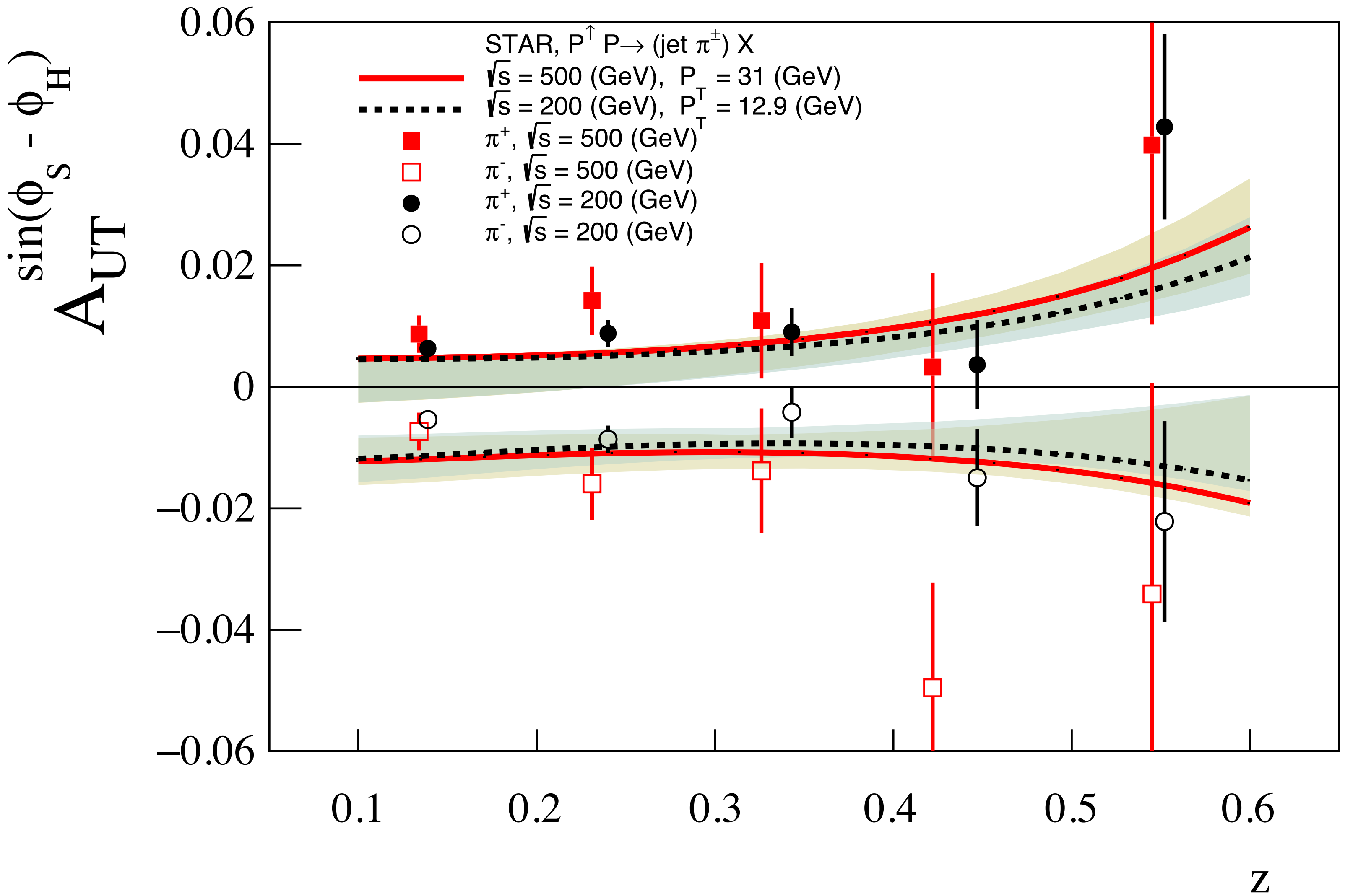
Asymmetry should depend on Q^2

PennState Berks

Kang, Prokudin, Ringer, Yuan (2017)

- Compute the asymmetry without TMD evolution

functions: Anselmino et al (2015)



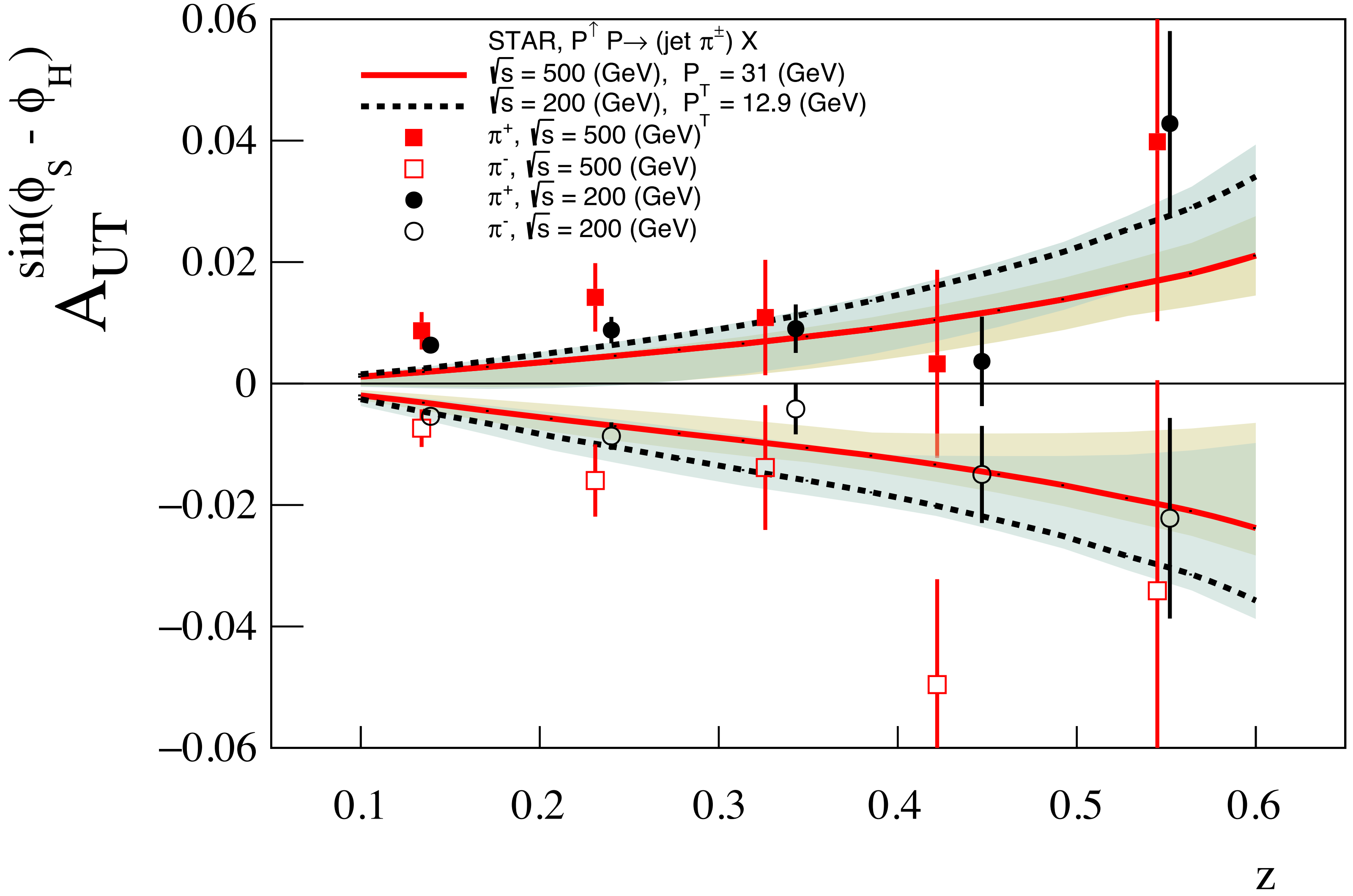
- Similar results for 200 GeV and 500 GeV
- Results are compatible with data within uncertainties

Test of QCD evolution

Kang, Prokudin, Ringer, Yuan (2017)

- Compute the asymmetry with TMD evolution

functions: Kang, Prokudin, Sun, Yuan (2015)



- Slight reduction for 500 compared to 200 GeV due to TMD evolution
- Results are compatible with data within uncertainties

Complementarity of SIDIS, e+e- and Drell-Yan, and hadron-hadron

Various processes allow study and test of evolution, universality and extractions of distribution and fragmentation functions. We need information from all of them

$$f(x) \otimes D(z)$$

Semi Inclusive DIS – convolution of distribution functions and fragmentation functions

$$\ell + P \to \ell' + h + X$$

$$f(x_1) \otimes f(x_2)$$

Drell-Yan – convolution of distribution functions

$$P_1 + P_2 \to \bar{\ell}\ell + X$$

$$D(z_1) \otimes D(z_2)$$

e+ e- annihilation – convolution of fragmentation functions

$$\bar{\ell} + \ell \to h_1 + h_2 + X$$

$$f(x_1) \otimes f(x_2) \otimes D(z)$$

Hadron-hadron – convolutions of PDF and fragmentation functions

$$h_1 + h_2 \to h_3(\gamma, jet, W, ...) + X$$

Combining measurements from all above is important

- TMD related studies have been extremely active in the past few years, lots of progress have been made
- We look forward to the future experimental results from COMPASS, RHIC, Jefferson Lab, LHC, Fermilab, future Electron Ion Collider
- Many TMD related groups are created throughout the world:

Italy, Netherlands, Belgium, Germany, Japan, China, Russia, and the USA

DOE funded Topical Collaboration for theory

TMD Collaboration

5 years of funding of **$2,160,000**
18 institutions
Theory, phenomenology, lattice QCD
Several postdoc positions.
2 tenure track positions: Temple, NMSU
Support of undergraduates.

The TMD Collaboration
Spokespersons: William Detmold (MIT) and Jianwei Qiu (BNL)

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- 5 years of funding
- 18 institutions
- Theory, phenomenology, lattice QCD
- Several postdoc and tenure track positions are created
- “To address the challenges of extracting novel quantitative information about the nucleon’s internal landscape”
- “To provide compelling research, training, and career opportunities for young nuclear theorists”