TRANSVERSITY 2017

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INFN - FRASCATI NATIONAL LABORATORIES

Matching the TMD and collinear factorization framework

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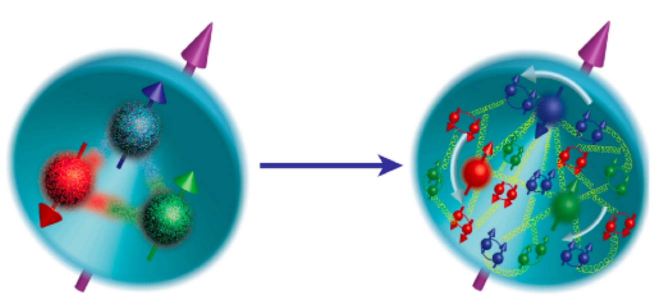
U.S. DEPARTMENT OF ENERGY | Office of Science

TMD Collaboration

PennState Berks

Overview comments

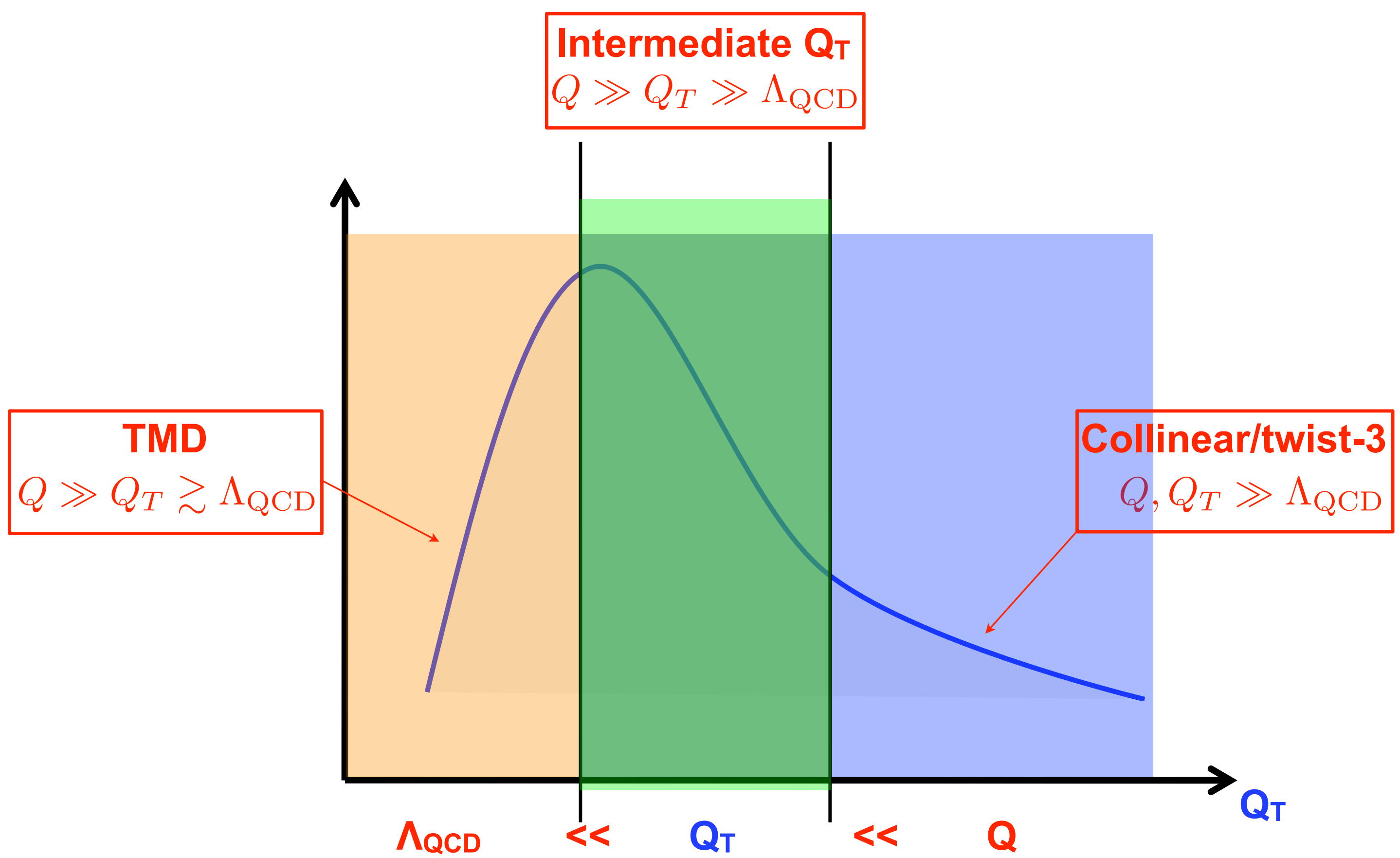
- Report implementation for combining TMD factorization and collinear factorization in studying nucleon structure in SIDIS
- Using an enhanced version of the CSS framework, we are able to re-derive at leading order the well-known relation between the (TMD) Sivers function and the (collinear twist-3) Qiu-Sterman function
- This relies on a modification of the so called "W+Y" construction of the q_T dependent SIDIS cross section (CSS based)
  - **Phys.Rev. D 94 (2016) Collins, Gamberg, Prokudin, Sato, Rogers, Wang**
- Extend treatment transversely polarized case, the Sivers Effect
  - **Gamberg , Metz, Pitonyak, Prokudin … 2017**



Overview comments

- This analysis comes from the modification of "*W+Y*" construction of SIDIS cross section used to match the TMD to collinear q_T dependent cross section as well as relating the TMD to collinear factorization within CSS

- By addressing the "*standard matching prescription*" traditionally used in CSS formalism relating low & high q_T behavior cross section @ moderate Q



Start w/ review of CSS $W + Y$ definition   Birds eye view

- Collins Soper Sterman NPB 1985
- Collins 2011 Cambridge Press

$$d\sigma(m \lesssim q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

- ***W*** describes the small transverse momentum behavior $q_T \ll Q$ and an additive correction term Y accounts for behavior at $q_T \sim Q$
- W is written in terms of TMD pdfs and/or TMD ffs and is constructed to be an accurate description in the limit of $q_T/Q \ll 1$. It includes all non-perturbative transverse momentum dependence
- The " ***Y*** -term " is described in terms of "collinear approximation" to the cross section: it is the correction term for large $q_T \sim Q$

Matching $W + Y$-schematic

✦Collins Soper Sterman NPB 1985

✦Collins 2011 Cambridge Press

- **This was *designed* with the aim to have a formalism that is valid to leading power in *m/Q* uniformly in q_T, where *m* is a typical hadronic mass scale**
- **and where there is a broad intermediate range of transverse momentum characterized by** $m \ll q_T \ll Q$

Implementations/studies

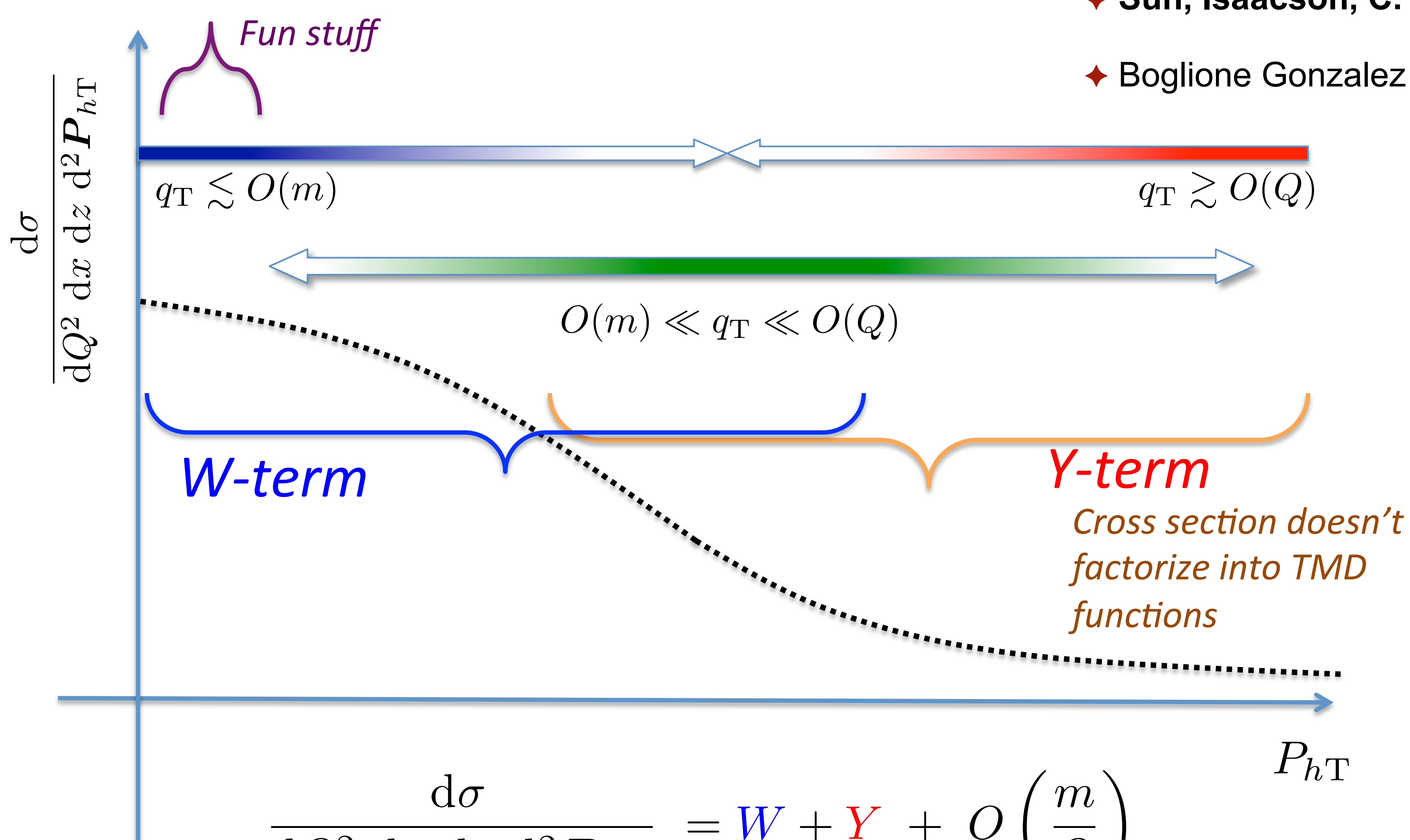
✦ Nadolsky Stump C.P. Yuan PRD 1999 **HERA data**

✦ Y. Koike, J. Nagashima, W. Vogelsang NPB (2006) **eRHIC**

✦ **Sun, Isaacson, C. -P. Yuan , F Yuan arXiv 2014**

✦ Boglione Gonzalez Melis Prokudin JHEP 2015 ….

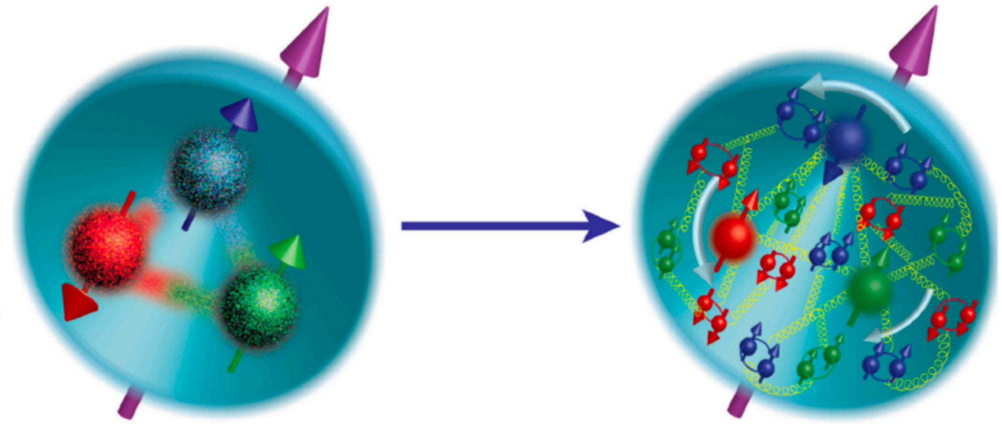
From Ted Rogers **W + Y**



note $P_{hT} = zq_T$

$$\frac{d\sigma}{dQ^2\, dx\, dz\, d^2\boldsymbol{P}_{hT}} = W + Y + O\left(\frac{m}{Q}\right)$$

TMD to collinear



EIC White Paper

Ted's Talk

$W(x, b_T, k_T)$

Wigner distributions

$\int d^2 b_T$ $\int d^2 k_T$

$f(x, k_T)$

transverse momentum distributions (TMDs)

semi–inclusive processes

$f(x, b_T)$

impact parameter distributions

$\int d^2 k_T$ $\int d^2 b_T$

$f(x)$

parton densities

inclusive and semi–inclusive processes

TMD to collinear

nb CSS TMD factorisation carried out in coordinate space: then FT back to momentum space

Must consider UV and IR Divergences and TMD evolution Studied in CSS formalism see Collins 2011 Cambridge Press

Parton model Semi-inclusive to Collinear integrate over q_T

$$W_{PM}(q_T, Q) = H_{LO,j',i'}(Q_0) \int d^2k_T f_{j'/A}(x, k_T) d_{B/i'}(z, q_T + k_T)$$

$$\int d^2q_T \, W_{PM}(q_T, Q) = H_{LO,j',i'}(Q_0) f_{j'/A}(x) d_{B/i'}(z)$$

Underlies Model building w/ and w/o evolution using TMD and collinear evolution approach
Anselmino Boglione D'Alesio Murgia Prokudin ...2005-2017

* Parton Model (expectation) from TMD *W*-term

Can such an interpretation be valid in an approximate manner from the QCD Standard CSS W-term ?

Can we preserve generalised parton model as an approximation to TMD evolution?

Analysis Relies heavily on **Phys.Rev. D 94 (2016) Collins,Gamberg,Prokudin, Sato, Rogers, Wang**

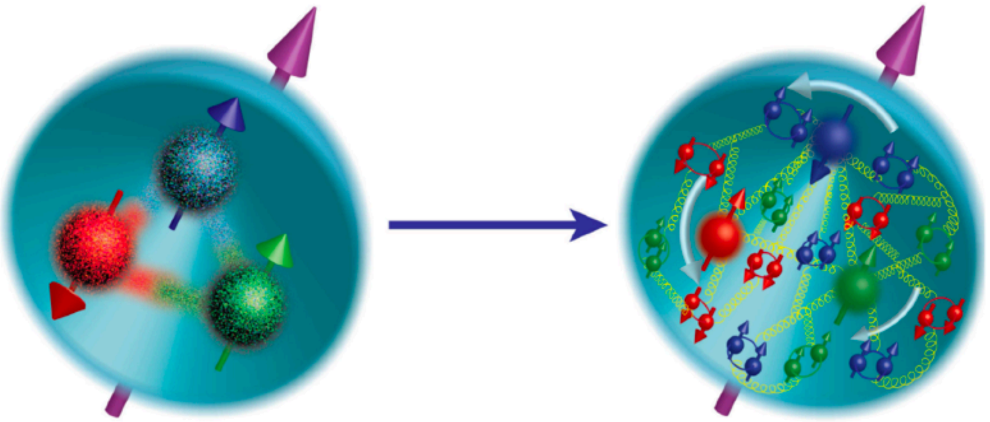
Reminder

- Parton Model Correlator Boer Mulders 1998 PRD, Bacchetta et al 2007 JHEP

Sivers 1989 PRD

$$\Phi^{[\gamma^+]}(x, p_T) = f(x, p_T) - \frac{\epsilon^{ij} k_T^i S^j}{M} f_{1T}^{\perp}(x, k_T)$$

- In CSS TMD Evolution/Factorization carried out in b-space



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- In CSS TMD Evolution/Factorization carried out in b-space

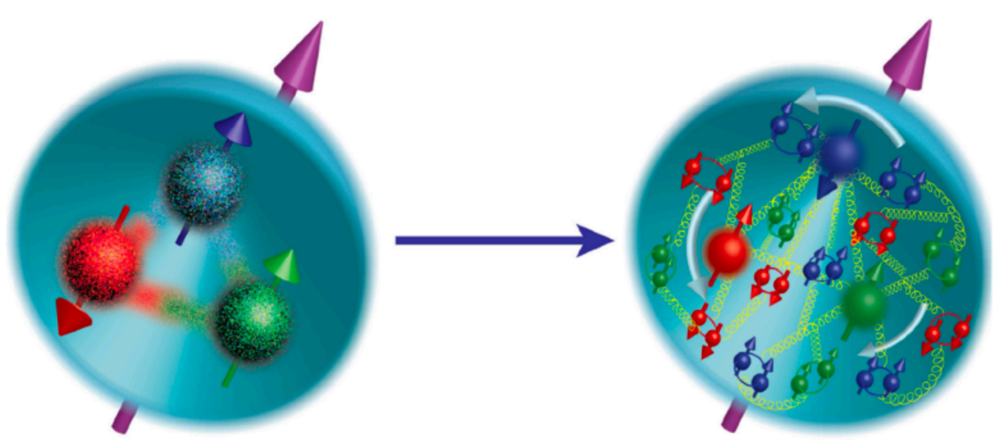
"b-space" correlator

$$\tilde{\Phi}^{[\gamma^+]}(x, \vec{b}_T; Q^2, \mu_Q) = \tilde{f}_1(x, b_T; Q^2, \mu_Q) - iM\epsilon^{ij} b_T^i S_T^j \left[-\frac{1}{M^2} \frac{1}{b_T} \frac{\partial}{\partial b_T} \tilde{f}_{1T}^{\perp}(x, b_T; Q^2, \mu_Q) \right]$$

Boer, Gamberg, Musch, Prokudin (2011) JHEP

Collins Aybat Rogers Qiu 2011, 2012 PRD

$$\equiv \tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q)$$



Unpolarized and Sivers evolve in same way

- Parton Model Correlator Mulders, Kotzinian, Bacchetta et al

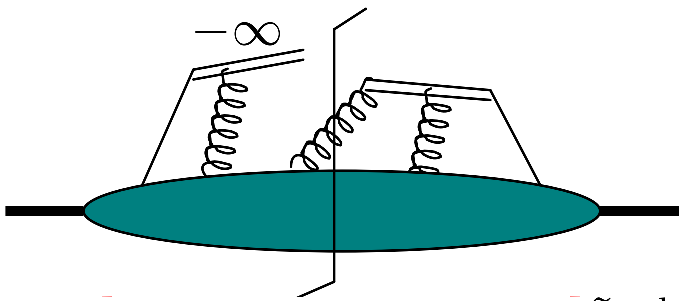
Recall the correlator in *b*-space Bessel Transform

$$\tilde{\Phi}^{[\gamma^+]}(x, \boldsymbol{b}_T) = \tilde{f}_1(x, \boldsymbol{b}_T^2) - i\,\epsilon_T^{\rho\sigma} b_{T\rho} S_{T\sigma}\, M \tilde{f}_{1T}^{\perp(1)}(x, \boldsymbol{b}_T^2)$$

The correlator Collins Soper Equation, thus unpolarised and Sivers evolve in similar manner

$$\frac{\partial \tilde{\phi}^i_{f/P}(x, \mathbf{b}_\mathrm{T}; \mu, \zeta_F)\epsilon_{ij} S_T^j}{\partial \ln\sqrt{\zeta_F}} = \tilde{K}(b_T; \mu)\tilde{\phi}^i_{f/P}(x, \mathbf{b}_\mathrm{T}; \mu, \zeta_F)\epsilon_{ij} S_T^j.$$

Evolution follows from their independence of rapidity scale



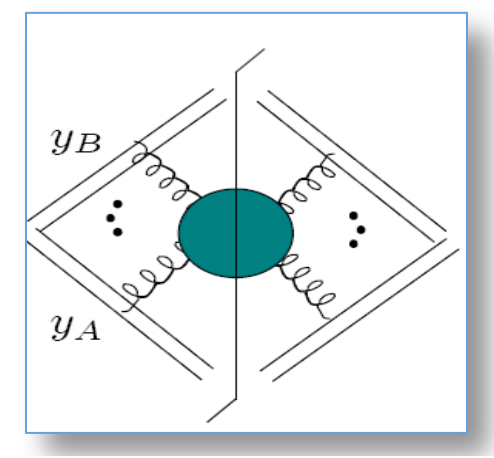
Collins Cambridge press 2011, Aybat & Rogers 2011 PRD

$$\tilde{F}_H^{\rm sub}(x, b_T; \mu, y_n) = \lim_{\substack{y_A \to \infty \\ y_B \to -\infty}} \tilde{F}_H^{\rm unsub}(x, b_T; \mu, y_P - y_B) \sqrt{\frac{\tilde{S}(b_T; y_A, y_n)}{\tilde{S}(b_T; y_A, y_B)\tilde{S}(b_T; y_n, y_B)}}$$

From operator definition get

Collins-Soper Equation:

$$-\quad \frac{\partial \ln \tilde{F}(x, b_T, \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$



$$\tilde{K}(b_T; \mu) = \frac{1}{2} \frac{\partial}{\partial y_n} \ln \frac{\tilde{S}(b_T; y_n, -\infty)}{\tilde{S}(b_T; +\infty, y_n)}$$

Along with .... Renormalization group Equations

$$\frac{d\tilde{K}}{d\ln\mu} = -\gamma_K(g(\mu))$$

$$\frac{d\ln\tilde{F}(x, b_T; \mu, \zeta)}{d\ln\mu} = -\gamma_F(g(\mu); \zeta/\mu^2)$$

RGE:
get anomalous
for *F* & *K*

Solve Collins Soper & RGE eqs. to obtain "evolved TMDs"

TMD Evolution-Solution for unpolarised & Sivers

$$\tilde{f}_1(x, b_T; Q^2, \mu_Q) \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes f_1(\hat{x}; \mu_{b_*})\right)$$

Collins (2011); ...

$$\times \ \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_1}(b_T, Q)\right]$$

Qiu & Sterman PRL 1991

$$\tilde{f}_{1T}^{\perp(1)}(x, b_T; Q^2, \mu_Q) \sim \left(\tilde{C}^{f_{1T}^{\perp}}(\hat{x}_1, \hat{x}_2, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes T_F(\hat{x}_1, \hat{x}_2; \mu_{b_*})\right)$$

$$\times \ \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_{1T}^{\perp}}(b_T, Q)\right]$$

Aybat, Collins, Qiu, Rogers (2012); Echevarria, Idilbi, Kang, Vitev (2014); ...

For Unpolarized FT-TMD

$$\tilde{f}_1(x, b_T; Q^2, \mu_Q) \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes f_1(\hat{x}; \mu_{b_*})\right)$$
$$\times \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_1}(b_T, Q)\right]$$

perturbative Sudakov factor

$$\underbrace{-\ln(Q/\mu_{b_*})\tilde{K}(b_*, \mu_{b_*}) - \int_{\mu_{b_*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu'); 1) - \gamma_K(\alpha_s(\mu'))\ln(Q/\mu')\right]}_{\text{same for unpol. and pol.}}$$

non-perturbative Sudakov factor

$$g_{f_1}(x, b_T) + g_K(b_T)\ln(Q/Q_0)$$

different for each TMD

universal

$$b_*(b_T) \equiv \sqrt{\frac{b_T^2}{1 + b_T^2/b_{\max}^2}} \qquad \mu_{b_*} = C_1/b_*(b_T)$$

For Unpolarized FT-TMD

$$\tilde{f}_1(x, b_T; Q^2, \mu_Q) \quad \sim \quad \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_T); \mu_{b_*}^2, \mu_{b_*}, \alpha_s(\mu_{b_*})) \otimes f_1(\hat{x}; \mu_{b_*})\right)$$
$$\times \ \exp\left[-S_{pert}(b_*(b_T); \mu_{b_*}, Q, \mu_Q) - S_{NP}^{f_1}(b_T, Q)\right]$$

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$$b_*(b_T) \equiv \sqrt{\frac{b_T^2}{1 + b_T^2/b_{\max}^2}} \qquad \mu_{b_*} = C_1/b_*(b_T)$$

Note: $b_*(0) = 0$ and $(\mu_{b_*})_{b_*\to 0} = \infty$ ➡ problematic large logarithms in S_{pert}

(Bozzi, Catani, de Florian, Grazzini (2006); Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

Consequence

When $b_T \to 0$, the b_T-space integrand goes zero. Thus, the integral over all transverse momentum of corresponding momentum-space contribution $f\,(x, k_T, Q)$ is zero.

- To understand this lets unpack perturbative part of CSS TMD evolution Kernel

Dependence driven by perturbative part of ev. Kernel

$$\exp\left[\int_{\mu_{b*}}^{\mu_Q} \frac{d\mu'}{\mu'}\left[\gamma(\alpha_s(\mu');1) - \ln\left(\frac{Q}{\mu'}\right)\gamma_K(\alpha_s(\mu'))\right]\right]$$

$$\begin{aligned}\tilde{f}(x, b_T \to 0, Q) \sim \exp\left[\frac{C_F}{\pi\beta_0}\int_{\ln\mu_b^2}^{\ln\mu_Q^2} \ln\mu'^2\right] &= \exp\left[-\frac{C_F}{\pi\beta_0}\ln\left(\frac{\mu_b^2}{\mu_Q^2}\right)\right] \\ &= \exp\left[-\frac{C_F}{\pi\beta_0}\ln\left(\frac{C_1^2}{b_T^2\mu_Q^2}\right)\right] \\ &= b_T^a \quad \text{where, } a = 2C_F/(\pi\beta_0) > 0 \\ &\to 0\end{aligned}$$

A little detail: dependence driven by perturbative part of ev. Kernel

$$f_{1CSS}(x, k_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{i k_T \cdot b_T} \tilde{f}_{1CSS}(x, b_T, Q)$$

$$\int d^2 k_T f_{1CSS}(x, k_T, Q) = \int d^2 b_T \delta^2(b_T) \, \tilde{f}_{1CSS}(x, b_T; Q)$$

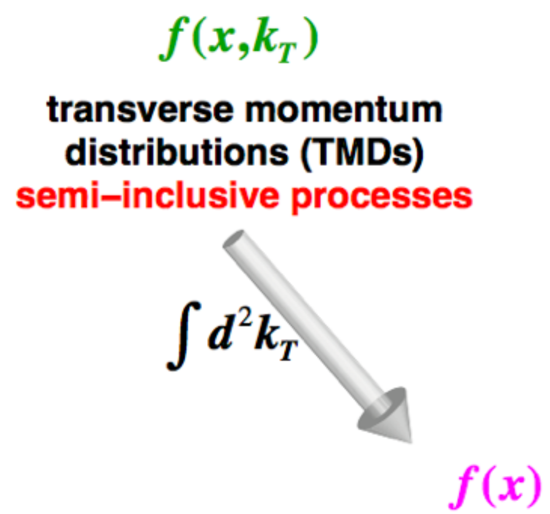
$$\int d^2 q_T f_{1CSS}(x, k_T, Q) = 0 \quad !$$

Phys. Rev. D 94 (2016) for details Collins,Gamberg, Prokudin, Sato, Rogers,Wang

Gamberg , Metz, Pitonyak, Prokudin … 2017

Collinear limit Original CSS

- ✦ *Collins, Soper, Sterman NPB 1985*
- ✦ *Ji Ma Yuan, PRD 2005*
- ✦ *Collins 2011*



Consequence is that physical interpretation of integrated TMDs as collinear pdfs is at odds with parton model intuition in original version of CSS

$$\int d^2k_T\, f_1(x,k_T;Q^2,\mu_Q) = \tilde{f}_1(x, b_T \to 0; Q^2, \mu_Q) = 0\,!$$

(Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

$$\int d^2k_T\, \frac{k_T^2}{2M^2}\, f_{1T}^{\perp}(x,k_T;Q^2,\mu_Q) \equiv f_{1T}^{\perp(1)}(x;Q^2,\mu_Q) = \tilde{f}_{1T}^{\perp(1)}(x, b_T \to 0; Q^2, \mu_Q) = 0\,!$$

(Gamberg, Metz, DP, Prokudin, to appear soon)

TMDs lose their physical interpretation in the "Original CSS" formalism!

Collinear limit Original CSS

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Consequence is that physical interpretation of integrated TMDs as collinear pdfs is at odds with parton model intuition in original version of CSS

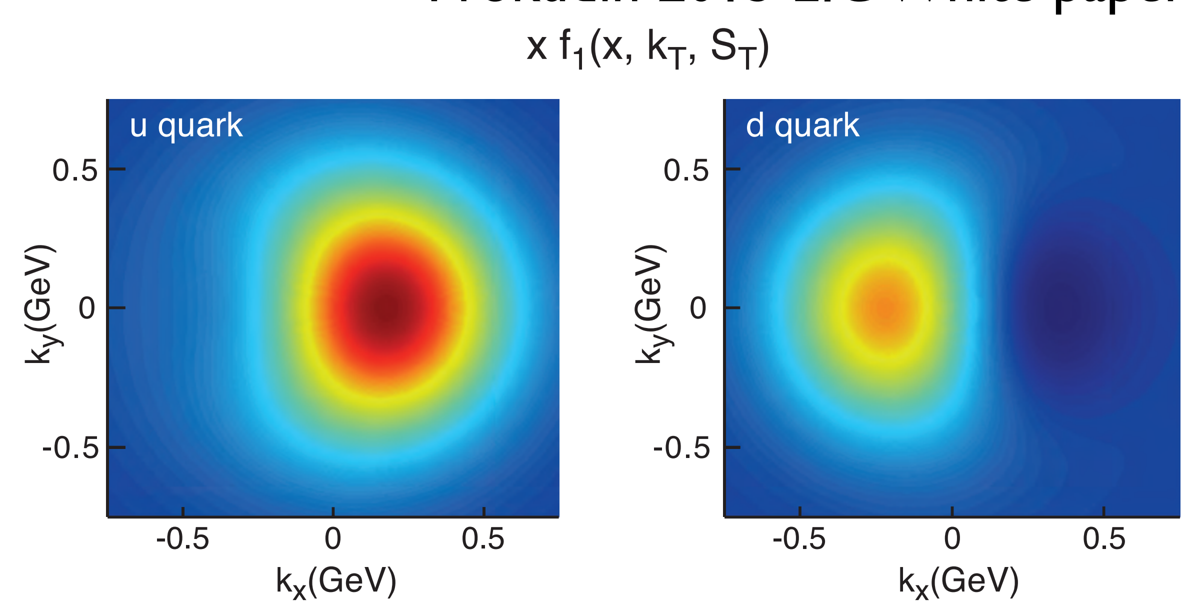
TMDs lose their physical interpretation in the "Original CSS" formalism!

$$\langle k_T^i(x)\rangle_{UT} = \int d^2k_T\, k_T^i \left(-\frac{\vec{k}_T \times \vec{S}_T}{M} f_{1T}^{\perp}(x, k_T)\right)$$

avg. TM of unpolarized quarks in a transversely polarized spin-1/2 target

Boer Mulders Teryaev PRD 1998
Burkhardt 2004,2013 PRD
Metz et al. 2013 PRD
And others …

Prokudin 2015 EIC White paper



$bQ >> 1$ contributions to the W *term*

- Issue has been addressed "q_T resummation" by Bozzi, Catani, de Florian, Grazzini, (2006) NPB, & "TMD CSS analysis" Collins, Gamberg, Prokudin, Rogers, Sato, Wang PRD 2016 studying the Fourier transform of the W term in the $W+Y$ matching in q_T of the SIDIS cross section from coordinate b-space to q_T momentum space

- In order to regulate the large logs(Q^2b^2) at small b in the FT they Bozzi et al. , replace logs(Q^2b^2) with logs(Q^2b^2+1) cutting off the b<<1/Q contribution

- Also Kulesza,Sterman,Vogelsang PRD 2002 in threshold resummation studies

We address these large logs by placing another boundary condition on now small b_T

"Improved CSS" (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))*

Place a lower cut-off on b_T: $b_T \to b_c(b_T)$ where $b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2}$

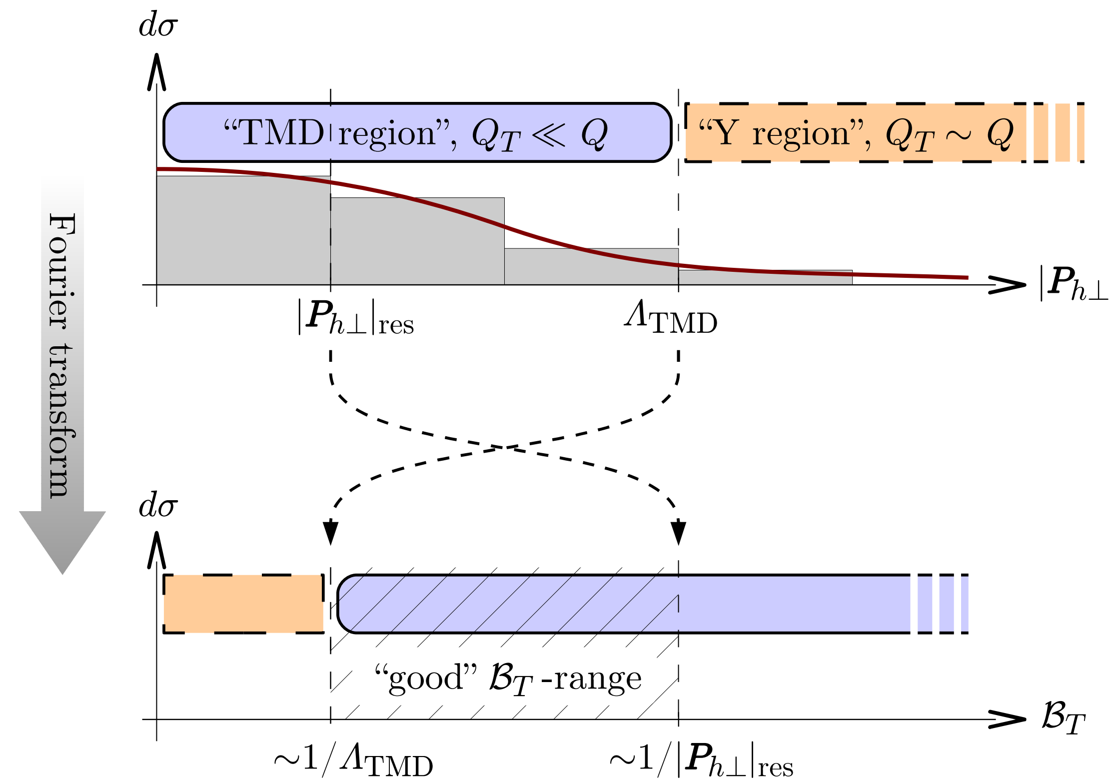
$$\Rightarrow \quad \mu_{b_*} \to \bar{\mu} \equiv \frac{C_1}{b_*(b_c(b_T))} \quad \text{so } \mu_{b_*} \text{ is cut off at } \mu_c \approx \frac{C_1 C_5 Q}{b_0}$$

B.C. Introduce small b-cuttoff

Similar to Catani et al. NPB 2006,
Bessel Weighting-Boer LG Musch Prokudin JHEP 2011

$$b_c(b_T) = \sqrt{b_T^2 + b_0^2/\left(C_5 Q\right)} \implies b_c(0) \sim 1/Q$$

Regulate unphysical divergences from in W term



Generalized B.C. when peforming Fourier transform

$$b_*(b_c(b_{\rm T})) \longrightarrow \begin{cases} b_{\rm min} & b_{\rm T} \ll b_{\rm min} \\ b_{\rm T} & b_{\rm min} \ll b_{\rm T} \ll b_{\rm max} \\ b_{\rm max} & b_{\rm T} \gg b_{\rm max}\,. \end{cases}$$

$$\tilde{W}_{New}(q_T, Q; \eta, C_5) = \Xi\left(\frac{q_T}{Q}, \eta\right) \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \tilde{W}^{OPE}\left(b_*(b_c(b_T)), Q\right) \tilde{W}_{NP}(b_c(b_T)), Q; b_{max})$$

Enhanced expression for $\tilde{W}(b_c, Q)$

$$\tilde{W}(b_c(b_{\mathrm{T}}), Q) = H(\mu_Q, Q) \sum_{j'i'} \int_{x_A}^{1} \frac{d\hat{x}}{\hat{x}} \tilde{C}^{\mathrm{pdf}}_{j/j'}(x_A/\hat{x}, b_*(b_c(b_{\mathrm{T}})); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) f_{j'/A}(\hat{x}; \bar{\mu}) \times$$
$$\times \int_{z_B}^{1} \frac{d\hat{z}}{\hat{z}^3} \tilde{C}^{\mathrm{ff}}_{i'/j}(z_B/\hat{z}, b_*(b_c(b_{\mathrm{T}})); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) d_{B/i'}(\hat{z}; \bar{\mu}) \times$$
$$\times \exp\left\{ \ln \frac{Q^2}{\bar{\mu}^2} \tilde{K}(b_*(b_c(b_{\mathrm{T}})); \bar{\mu}) + \int_{\bar{\mu}}^{\mu_Q} \frac{d\mu'}{\mu'} \left[2\gamma(\alpha_s(\mu'); 1) - \ln \frac{Q^2}{\mu'^2} \gamma_K(\alpha_s(\mu')) \right] \right\}$$
$$\times \exp\left\{ -g_A(x_A, b_c(b_{\mathrm{T}}); b_{\max}) - g_B(z_B, b_c(b_{\mathrm{T}}); b_{\max}) - 2 g_K(b_c(b_{\mathrm{T}}); b_{\max}) \ln\left(\frac{Q}{Q_0}\right) \right\}$$

Boundary conditions

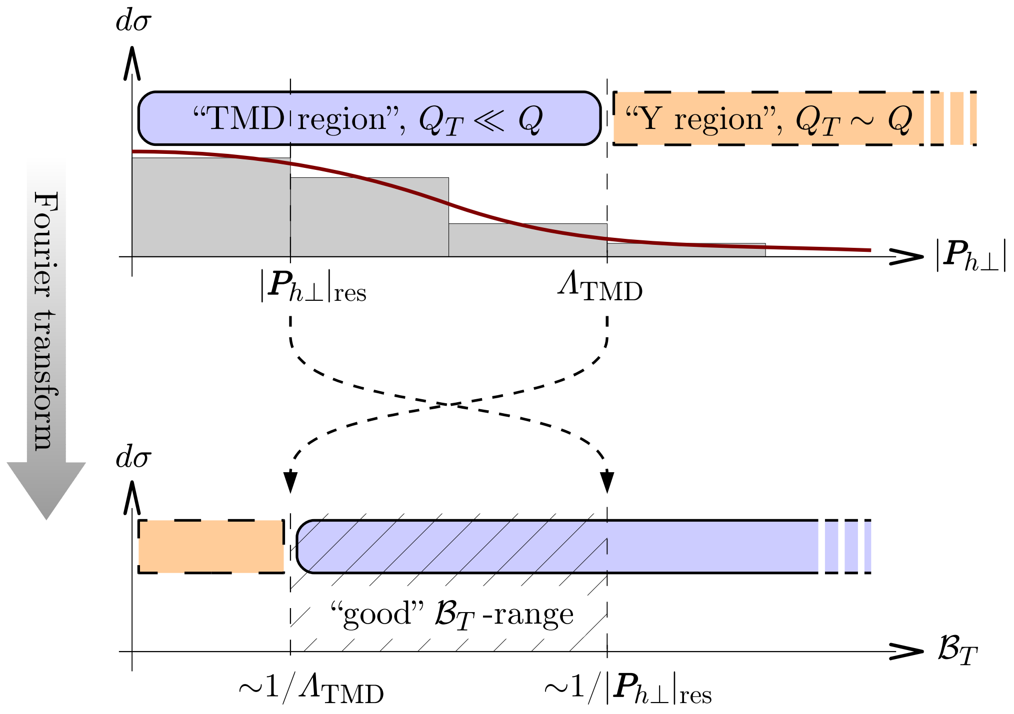
$$b_*(b_c(b_{\mathrm{T}})) \longrightarrow \begin{cases} b_{\min} & b_{\mathrm{T}} \ll b_{\min} \\ b_{\mathrm{T}} & b_{\min} \ll b_{\mathrm{T}} \ll b_{\max} \\ b_{\max} & b_{\mathrm{T}} \gg b_{\max} \,. \end{cases}$$

See Phys. Rev. D 94 (2016) for details J. Collins, L.Gamberg, A. Prokudin, N. Sato, T. Rogers, B. Wang

Impact on TMD definition in CSS

$$b_c(b_T) = \sqrt{b_T^2 + b_0^2/\left(C_5 Q\right)} \implies b_c(0) \sim 1/Q$$

$$b_*(b_c(b_\mathrm{T})) \longrightarrow \begin{cases} b_\mathrm{min} & b_\mathrm{T} \ll b_\mathrm{min} \\ b_\mathrm{T} & b_\mathrm{min} \ll b_\mathrm{T} \ll b_\mathrm{max} \\ b_\mathrm{max} & b_\mathrm{T} \gg b_\mathrm{max}\,. \end{cases}$$



Modified FT-TMD from enhanced CSS

"Improved CSS" (Unpolarized) (Collins, Gamberg, Prokudin, Rogers, Sato, Wang (2016))

Place a lower cut-off on b_T: $b_T \to b_c(b_T)$ where $b_c(b_T) = \sqrt{b_T^2 + b_0^2/(C_5 Q)^2}$

$$\Rightarrow \mu_{b_*} \to \bar{\mu} \equiv \frac{C_1}{b_*(b_c(b_T))} \text{ so } \mu_{b_*} \text{ is cut off at } \mu_c \approx \frac{C_1 C_5 Q}{b_0}$$

$$\tilde{\boldsymbol{f}}_1(\boldsymbol{x}, \boldsymbol{b_c(b_T)}; \boldsymbol{Q^2}, \boldsymbol{\mu_Q}) \sim \left(\tilde{C}^{f_1}(x/\hat{x}, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{f_1(\hat{x}; \bar{\mu})}\right) \times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_1}(b_c(b_T), Q)\right]$$

"Improved CSS" (Polarized) (Gamberg, Metz, DP, Prokudin, to appear soon)

$$\tilde{\Phi}^{[\gamma^+]}(x, \vec{b}_T; Q^2, \mu_Q) = \tilde{\boldsymbol{f}}_1(\boldsymbol{x}, \boldsymbol{b_T}; \boldsymbol{Q^2}, \boldsymbol{\mu_Q}) - iM\epsilon^{ij} b_T^i S_T^j \tilde{\boldsymbol{f}}_{1T}^{\perp(1)}(\boldsymbol{x}, \boldsymbol{b_T}; \boldsymbol{Q^2}, \boldsymbol{\mu_Q})$$

b_T -> $b_c(b_T)$

NO b_T -> $b_c(b_T)$ replacement – kinematic factor NOT associated with the scale evolution

b_T -> $b_c(b_T)$

and for Sivers first moment …

"Improved CSS" (Polarized) (Gamberg, Metz, DP, Prokudin, to appear soon)

$$\tilde{\Phi}^{[\gamma^+]}(x, \vec{b}_T, b_c(b_T); Q^2, \mu_Q) = \boldsymbol{\tilde{f}_1(x, b_c(b_T); Q^2, \mu_Q)} - iM\epsilon^{ij} b_T^i S_T^j \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_c(b_T); Q^2, \mu_Q)}$$

$$\boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x, b_c(b_T); Q^2, \mu_Q)} \sim \left(\tilde{C}^{f_{1T}^{\perp}}(\hat{x}_1, \hat{x}_2, b_*(b_c(b_T)); \bar{\mu}^2, \bar{\mu}, \alpha_s(\bar{\mu})) \otimes \boldsymbol{T_F(\hat{x}_1, \hat{x}_2; \mu_{b_*})}\right)$$

$$\times \exp\left[-S_{pert}(b_*(b_c(b_T)); \bar{\mu}, Q, \mu_Q) - S_{NP}^{f_{1T}^{\perp}}(b_c(b_T), Q)\right]$$

Enhanced CSS definitions of TMDs

$$f_1^j(x, k_T; Q^2, \mu_Q; C_5) \equiv \int \frac{db_T}{2\pi}\, b_T J_0(k_T b_T) \tilde{f}_1^j(x, b_c(b_T); Q^2, \mu_Q)\,,$$

$$D_1^j(z, p_T; Q^2, \mu_Q; C_5) \equiv \int \frac{db_T}{2\pi}\, b_T J_0(p_T b_T)\, \tilde{D}_1^{h/j}(z, b_c(b_T); Q^2, \mu_Q)\,,$$

$$\frac{k_T^2}{2M_P^2}\, f_{1T}^{\perp j}(x, k_T; Q^2, \mu_Q; C_5) \equiv k_T \int \frac{db_T}{4\pi} b_T^2 J_1(k_T b_T)\, \tilde{f}_{1T}^{\perp(1)j}(x, b_c(b_T); Q^2, \mu_Q)\,.$$

Which leads to

$$\int d^2\vec{k}_T\, \boldsymbol{f_1(x,k_T;Q^2,\mu_Q;C_5)} = \boldsymbol{\tilde{f}_1(x,b_c(0);Q^2,\mu_Q)} = \boldsymbol{f_1(x;\mu_c)} + O(\alpha_s(Q)) + O((m/Q)^p)$$

$$\int d^2\vec{p}_T\, \boldsymbol{D_1(z,p_T;Q^2,\mu_Q;C_5)} = \boldsymbol{\tilde{D}_1(z,b_c(0);Q^2,\mu_Q)} = \boldsymbol{D_1(z;\mu_c)} + O(\alpha_s(Q)) + O((m/Q)^p)$$

$$\int d^2\vec{k}_T\, \frac{\vec{k}_T^2}{2M^2}\, \boldsymbol{f_{1T}^{\perp}(x,k_T;Q^2,\mu_Q;C_5)} = \boldsymbol{\tilde{f}_{1T}^{\perp(1)}(x,b_c(0);Q^2,\mu_Q)} = -\frac{\boldsymbol{T_F(\hat{x}_1,\hat{x}_2;\mu_{b_*})}}{\boldsymbol{2M_P}} + O(\alpha_s(Q)) + O((m/Q)$$

$$\int d^2\vec{p}_T\, \frac{\vec{p}_T^2}{2z^2M_h^2}\, \boldsymbol{H_1^{\perp}(z,p_T;Q^2,\mu_Q;C_5)} = \boldsymbol{\tilde{H}_1^{\perp(1)}(z,b_c(0);Q^2,\mu_Q)} = \boldsymbol{H_1^{\perp(1)}(z;\mu_c)} + O(\alpha_s(Q)) + O((m/Q)^{p''})$$

At LO in the “Improved CSS” formalism we recover the relations one expects from the “naïve” operator definitions of the functions

The “Improved CSS” formalism (approximately) restores the physical interpretation of TMDs!

Agreement between TMD and Collinear results

- Relies on further modifications of W+Y construction see
- Collins, Gamberg, Prokudin, Sato, Rogers, Wang PRD 2016

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,dz} \equiv 2z^2 \int d^2\boldsymbol{q}_{\mathrm{T}}\,\Gamma(\boldsymbol{q}_{\mathrm{T}}, Q, S) = 2z^2\,\tilde{W}^{\mathrm{OPE}}_{\mathrm{UU}}(b'_{min}, Q)_{\mathrm{LO}} + O(\alpha_s(Q)) + O((m/Q)^p)$$

$$= \frac{2\alpha_{em}^2}{yQ^2}(1 - y + y^2/2) \sum_j e_j^2\, f_1^j(x;\mu_c)\, D_1^{h/j}(z;\mu_c) + O(\alpha_s(Q)) + O((m/Q)^p)$$

- Gamberg , Metz, Pitonyak, Prokudin … 2017

$$\frac{d\langle P_{h\perp}\,\Delta\sigma(S_T)\rangle}{dx\,dy\,dz} = -4\pi z^3 M_P\,\tilde{W}^{\mathrm{siv,OPE}}_{\mathrm{UT}}(b'_{min}, Q)_{\mathrm{LO}} + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

$$= \frac{2\pi\, z\,\alpha_{em}^2}{yQ^2}(1 - y + y^2/2)\sum_j e_j^2\, T_F^j(x, x;\mu_c)\, D_1^{h/j}(z;\mu_c) + O(\alpha_s(Q)) + O((m/Q)^{p'})$$

Agrees with collinear twist-3 result at leading ord

Z.-B.Kang,Vitev, Xing,PRD(2013)

Comments

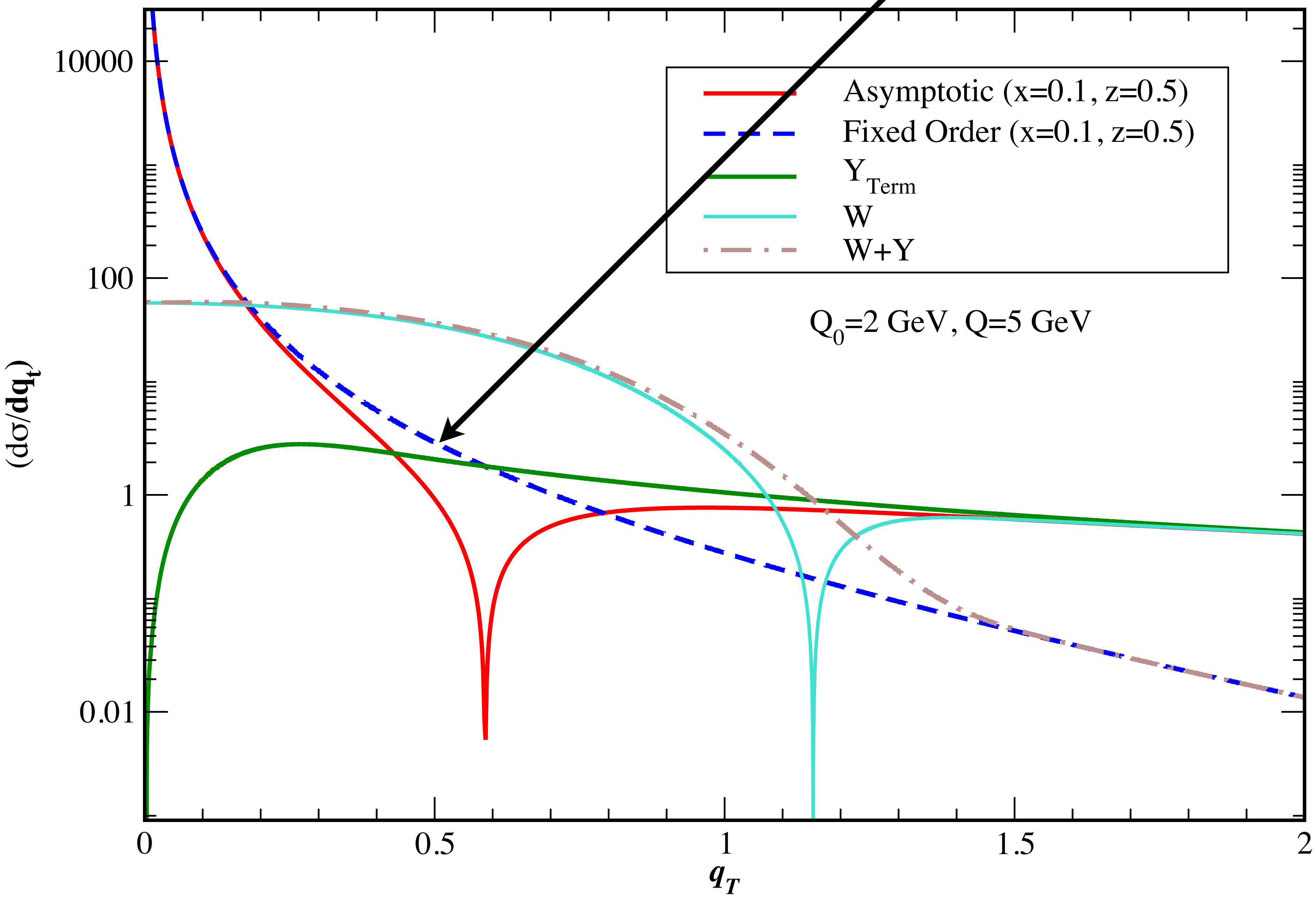
- With our method, the redefined W term allowed us to construct a relationship between integrated-TMD-factorization formulas and standard collinear factorization formulas, with errors relating the two being suppressed by powers of 1/Q
- Importantly, the exact definitions of the TMD pdfs and ffs are unmodified from the usual ones of factorization derivations. We preserve transverse-coordinate space version of the W term, but only modify the way in which it is used
- **We have a new now applied to transverse polarized phenomena**
- We are able to recover the well-known relations between TMD and collinear quantities one expects from the leading order parton model picture operator definition
- We recover the LO collinear twist 3 result from a weighted q_T integral of the differential cross section and derive the well known relation between the TMD Sivers function and the collinear twist 3 Qiu Sterman function from iCSS approach

Extras

Implementation of Collins, Gamberg, Prokudin, Sato, Rogers, Wang

- Now we can extend the power suppression error estimate down to $q_T = 0$ to get

$$d\sigma(q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$



Use analytic expressions for the collinear correlation functions, from GRV ZPC 1992 for up-quark pdf and from KKP NPB 2001 for the up-quark-to-pion ffs.

Phys. Rev. D 94 (2016) for details
Collins, Gamberg, Prokudin, Sato, Rogers, Wang

B.C. Introduce small b-cuttoff

$$b_c(b_T) = \sqrt{b_T^2 + b_0^2/\left(C_5 Q\right)} \implies b_c(0) \sim 1/Q$$

$$W_{New}(q_T, Q) = \int \frac{d^2 b_T}{(2\pi)^2} e^{i q_T \cdot b_T} \tilde{W}_{New}(b_T, Q)\,, \qquad b_{min} = b_0/(C_5 Q)$$

$$\int d^2 q_T W_{New}(q_T, Q) = \tilde{W}(b_{min}, Q) \neq 0$$

$$\int d^2 q_T\, W_{New}(q_T, Q) = H_{LO,j',i'} f_{j'/A}(x, \mu_c) d_{B/i'}(z, \mu_c) \; + O(\alpha_s(Q))$$

$$\mu_c \approx C_1 C_5 Q/b_0$$

Has a normal collinear factorization in terms of collinear pdfs w/ hard scale

$$\int d^2 q_T\, W_{New}(q_T, Q) + Y(q_T, Q) = H_{LO,j',i'} f_{j'/A}(x, \mu_c) d_{B/i'}(z, \mu_c) \; + O(\alpha_s(Q))$$

$$+ \text{ terms dominated by large } q_T \text{ contribution to } Y \text{ term}$$

With modified W+Y we can match to the collinear formalism

Has implications for modelling TMD and fitting

Review of Resummation

The "$W + Y$" prescription to describing the q_T dependent cross section now being intensely studied using the language of TMD factorization to SIDIS has its origin in the study of generic high mass systems (vector bosons, Higgs particles, . . .) produced in Drell Yan collisions (e.g. at the Tevatron and now at the LHC)

- *Collins, Soper, Sterman NPB 1985,*
- *Altarelli et al, NPB 1984*
- *Davies Webber, Stirling, NPB 1985,*
- *Arnold and Kauffman NPB 1991*
- *Nadolsky, Stump, Yuan zPRD 2000*
- *J.-W. Qiu, Zhang, PRL 2001,*
- *Berger, J.-W. Qiu, PRD 2003*
- *A. Bacchetta, D. Boer, M. Diehl, and P. J. Mulders, JHEP (2008)*
- *Sun, Isaacson, C.-P. Yuan, F. Yuan, arXiv:1406.3073*
- *Boglione, Gonzales, Melis, Prokudin JHEP 2014*
- *Bozzi, Catani et al. NPB 2006, JHEP 2015, …*
- *Collins, Gamberg, Prokudin, Sato, Rogers, Wang, PRD (2016)*

Review of Resummation

In the large-q_T region ($q_T \sim m_V$), where the transverse momentum is of the order of the vector boson mass m_V, one applies conventional perturbation theory to get at the q_T dependent cross section QCD corrections are known up to $O(\alpha_S^2)$ and in some case beyond…

However, the bulk of the vector boson cross section is produced in small-q_T region ($q_T \ll m_V$), where convergence of the fixed-order expansion is spoiled by the presence of large logarithmic corrections, $\alpha_S^n \ln^m(m^2_V / q_T^2)$ of soft & collinear origin

Review of Resummation

To obtain reliable predictions, these logarithmically-enhanced terms have to be evaluated and systematically "resummed" to all orders in perturbation theory

For large energy and Q^2 the "resummed" and fixed-order calculations, valid at small and large q_T, respectively, can be consistently matched at intermediate values of q_T to achieve a uniform theoretical accuracy for the entire range of transverse momenta

However at lower phenomenologically interesting values of Q, neither of the ratios q_T/Q or m/q_T are necessarily very small and matching can be problematic

It is this matching that I will focus on in the context of TMD factorization physics and its connection to collinear limit.

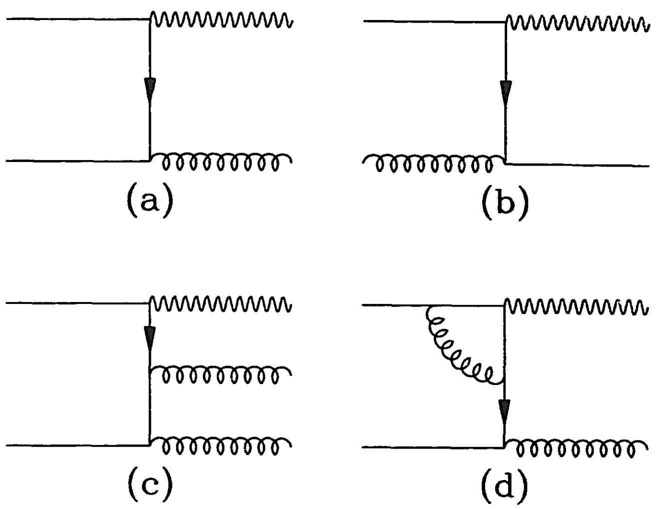
In recent years, the treatment ("resummation") of small-q_T logarithms has been reformulated by using SCET & and TMD factorization

Review of Resummation

At large transverse momentum q_T one calculates the cross section for W & Z production by factorized conventional pert. theory

$$\frac{d\sigma_F}{dq_T^2}(q_T, M, s) = \sum_{a,b} \int_0^1 dx_1 \int_0^1 dx_2\, f_{a/h_1}(x_1, \mu_F^2)\, f_{b/h_2}(x_2, \mu_F^2)\, \frac{d\hat{\sigma}_{Fab}}{dq_T^2}(q_T, M, \hat{s}; \alpha_S(\mu_R^2), \mu_R^2, \mu_F^2)$$

$$\frac{d\hat{\sigma}}{dq_T^2} = \alpha_W \alpha_s \left(u_1 + \alpha_s u_2 + \alpha_s^2 u_3 + \ldots\right)$$



Some examples of Feynman diagrams contributing to W or Z production at non-zero q_T: (a, d) $q\bar{q} \to Wg$, (b) $qg \to Wq$, (c) $q\bar{q} \to Wgg$.

Review of Resummation

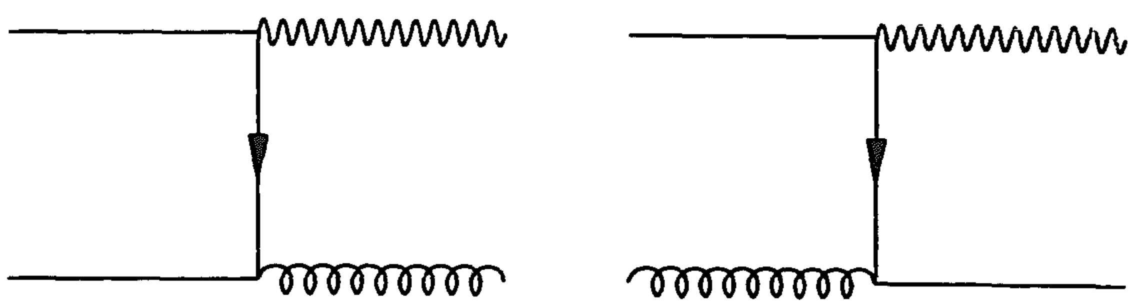
At low q_T , however, the convergence of the perturbation series deteriorates as dominant contributions have the form $\alpha_s \ln^2\left(\frac{Q^2}{q_T^2}\right)$

The convergence of the series is governed by $\alpha_s \ln^2\left(\frac{Q^2}{q_T^2}\right)$ rather than simply α_s

$$\frac{d\sigma}{dq_T^2} \sim \frac{\alpha_w \alpha_s}{q_T^2} \ln\left(\frac{Q^2}{q_T^2}\right)\left[v_1 + v_2 \alpha_s \ln^2\left(\frac{Q^2}{q_T^2}\right) + v_3 \alpha_s^2 \ln^4\left(\frac{Q^2}{q_T^2}\right) + \ldots\right]$$

The coefficients v_i of the "leading-logarithm" approximation are not independent and it is possible to sum the series exactly so that it may be applied even when $\alpha_s \ln^2\left(\frac{Q^2}{q_T^2}\right)$ is large

Fixed order theory calculation "asymptotically" diverges at low q_T *cannot by itself describe data*



$$\frac{1}{\sigma_0}\frac{d\sigma}{dq_T^2} \rightarrow \frac{\alpha_s}{q_T^2}\ln\left(\frac{M^2}{q_T^2} - \frac{3}{2}\right)$$

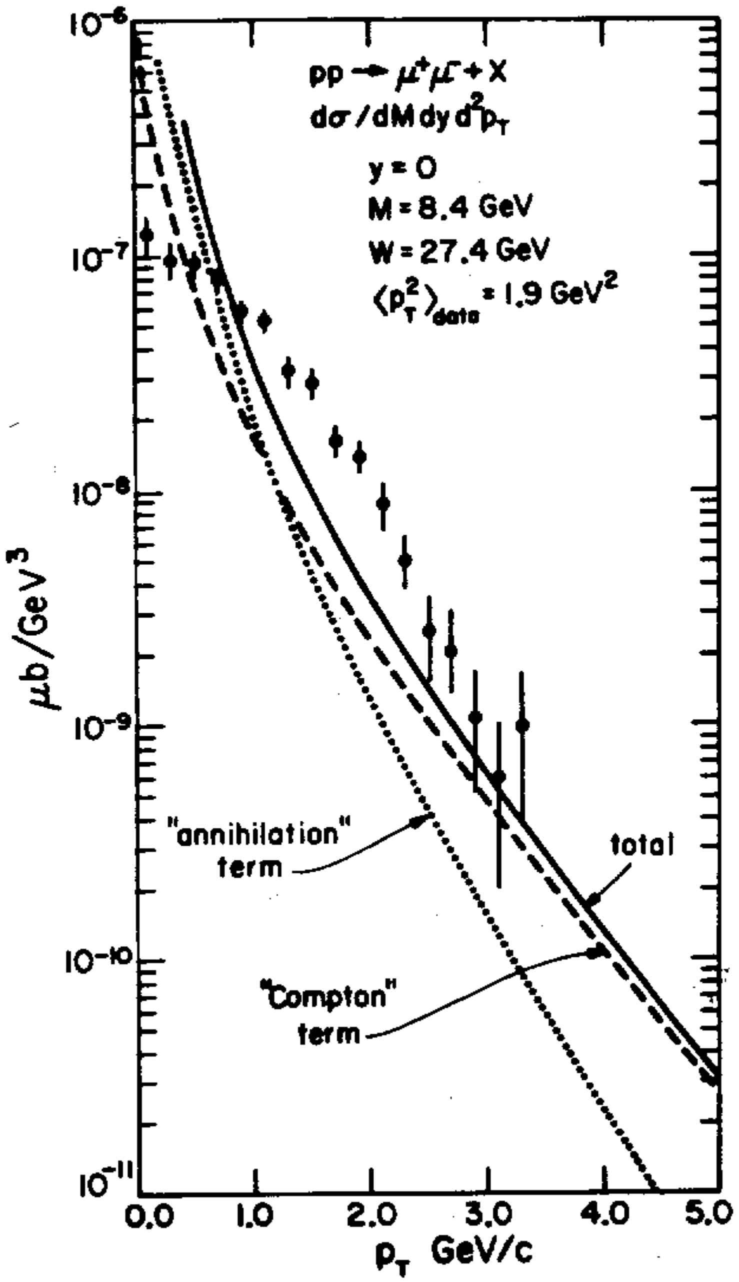


Figure 5.8 The distribution in transverse momentum, p_T, of muon pairs, $\mu^+\mu^-$ produced in pp collisions at $W = \sqrt{s} = 27.4$ GeV compared with the leading order perturbative QCD result. The "Compton" and "annihilation" contributions are given by the dashed and dotted curves, respectively (*taken from Ref. 9*).

From Resummation to CSS

This reorganization and "resummation" was carried out by Collins and Soper in b space; the result is

- *Collins Soper, NPB 1982*
- *Collins Soper Sterman NPB 1985*

$$\frac{d\sigma}{dq_T^2 dQ^2}(resum) \approx \frac{4\pi^3\alpha_w}{3s} e^2 \int \frac{d^2 b_T}{(2\pi)^2} e^{iq_T\cdot b_T} \sum_i \tilde{W}_i(b_T, Q)$$

$$\tilde{W}_i(b_T, Q) = H_i(Q) \left(\tilde{C}_i^{pdf}(x_A/\hat{x}, b_T) \otimes \tilde{f}_{i/A}(\hat{x}, \mu_b)\right) \left(\tilde{C}_j^{pdf}(x_B/\hat{z}, b_T) \otimes \tilde{\bar{f}}_{j/B}(\hat{x}, \mu_b)\right) e^{-S(b_T, Q)}$$

… TMD factorization

This expression contains the OPE of the Fourier transforms of the TMDs with soft gluon resummation in exponent. See Ted's talk …

II. GUIDING PRINCIPLES to enhanced CSS factorisation

The standard W + Y construction relies on the fact that, at very large Q, there is a broad range where m/qT and qT/Q are both good small expansion parameters. We suggest the following principles to guide the choice of an improved formalism:

1. When the W term is integrated over all q_T, it should obey an ordinary collinear factorization property. This implies that when the scales in the pdfs and ffs are set to $\mu = Q$, the result should agree with the ordinary factorization calculation for the integrated cross section to zeroth order in $\alpha_S(Q)$, thereby matching the parton-model result appropriately.

2. For $q_T \sim O(Q)$, the cross section given by W + Y should appropriately match fixed order collinear perturbation theory calculations for large trans- verse momentum.

3. For very large Q, the normal W + Y construction should automatically be recovered for the $m \ll q_T \ll Q$ region, to leading power in Q.

4. The modified W term should be expressed in terms of the same coordinate space quantity W(b) as before, in order that operator definitions of the pdfs and ffs can be used, together with their evolution equations.

5. W + Y should give a leading power approximation to the cross section over the whole range of q_T. Fixed order expansions of Y in collinear perturbation theory are suitable for calculating Y , while the usual solution of evolution equations is used for W .

Start w/ review of CSS $W + Y$ definitions

- ✦ Collins Soper Sterman NPB 1985
- ✦ Collins 2011 Cambridge Press

$$d\sigma(m \lesssim q_T \lesssim Q, Q) = W(q_T, Q) + Y(q_T, Q) + O\left(\frac{m}{Q}\right)^c d\sigma(q_T, Q)$$

- The CSS construction of $W + Y$ and the specific approximations are applied thru the **operation-approximators** ***T_{TMD}*** and ***T_{coll}*** that apply in their ***"design" regions*** $m \sim q_T \ll Q$ and $m \ll q_T \sim Q$ respectively which we emphasize by the ***range*** of the argument above

$$m \lesssim q_T \lesssim Q$$