

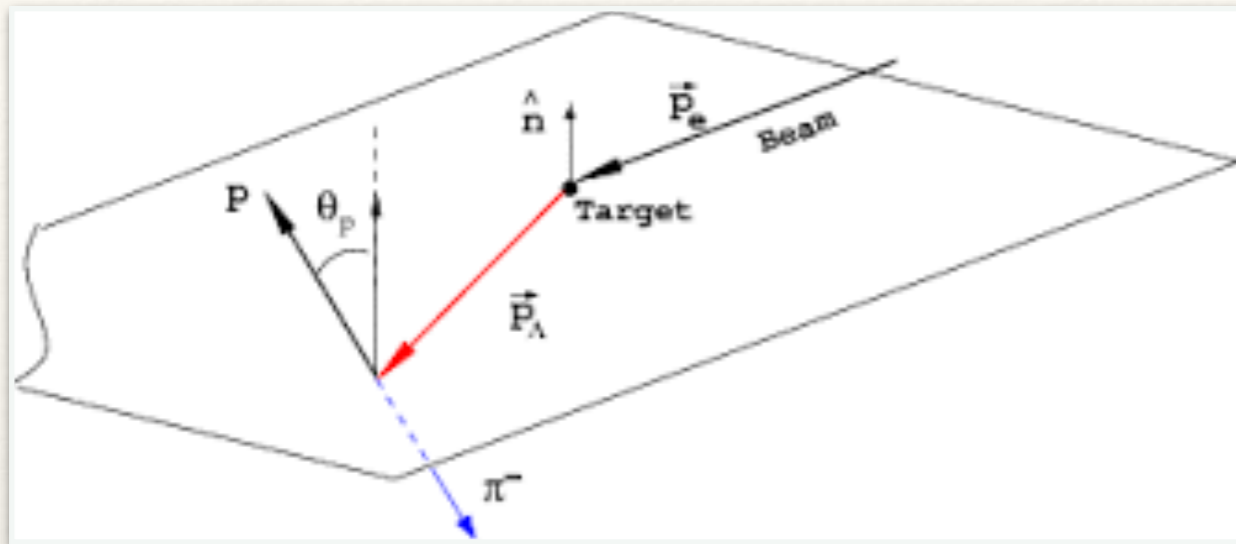
Workshop “Transversity 2017”, Dec. 14, 2017, Frascati, Italy

Higher Twists & Λ Polarization

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Institute for Theoretical Physics
University of Tübingen

in collaboration with
L. Gamberg, Z. Kang, D. Pionyak, S. Yoshida

Measurement of Λ -spin through decay $\Lambda \rightarrow p\pi^-$

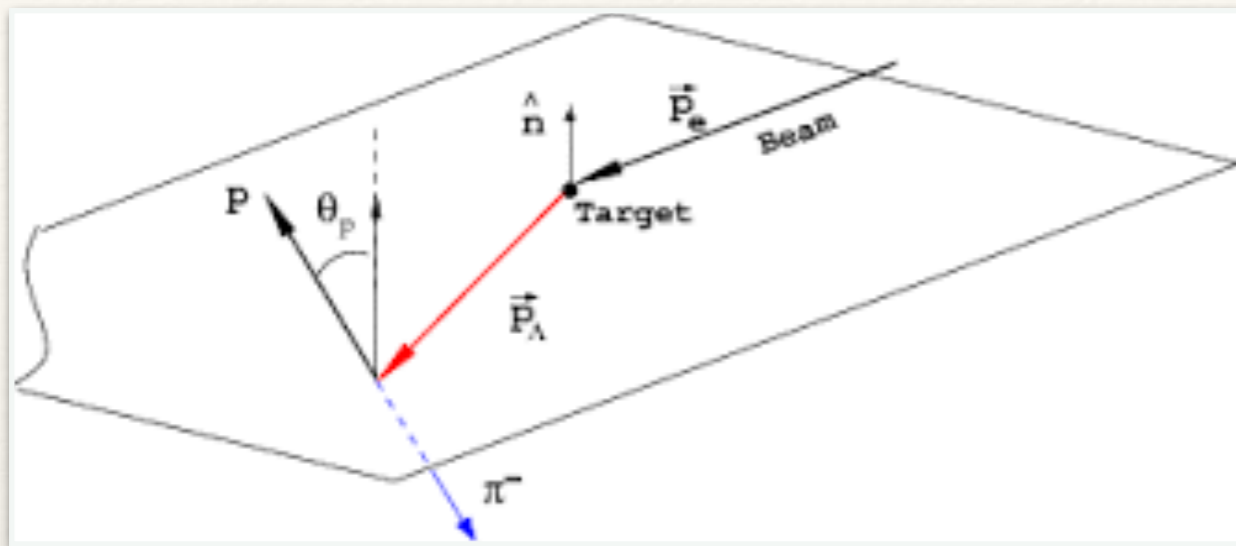


- Proton preferentially emitted along Λ -spin
- In Λ rest frame: pol. decay distribution

$$\left(\frac{dN}{d\Omega_p}\right)_{\text{pol}} = \left(\frac{dN}{d\Omega_p}\right)_{\text{unpol}} (1 + \alpha P_n^\Lambda \cos(\theta_p))$$

P^Λ : Transverse Lambda Polarization

Measurement of Λ -spin through decay $\Lambda \rightarrow p\pi^-$



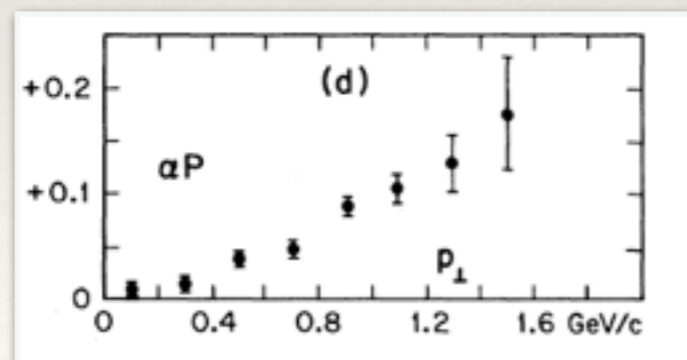
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P^Λ : Transverse Lambda Polarization

Transverse Λ polarization in pA: long history...

One of the first transverse spin effects at Fermilab (1976): $p+\text{Be} \rightarrow \Lambda^0 + X$
and many more follow-up measurements, also at CERN SPS (NA48), HERA-B



Λ polarization was found to be sizeable!

What about LHC? Is it feasible at a high energy collider?

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Recent ATLAS measurement at $\sqrt{S} = 7$ TeV

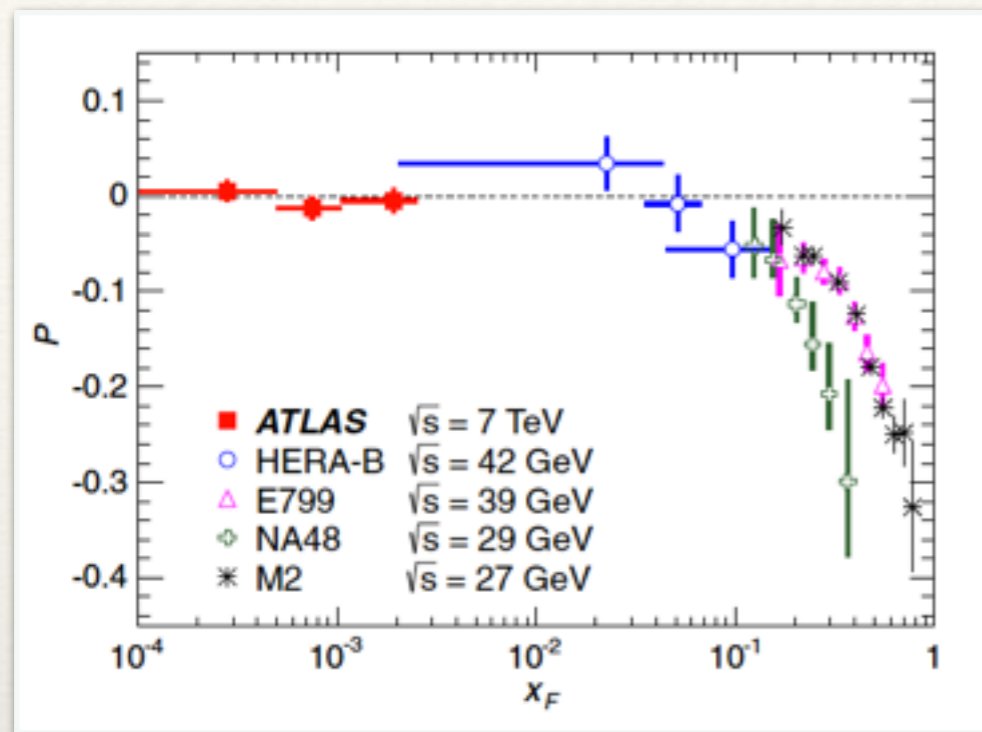
[ATLAS, PRD 91, 032004 (2015)]

Polarization small at mid-rapidity

Λ polarization at LHC possible

Can Λ polarization be useful for LHC physics?

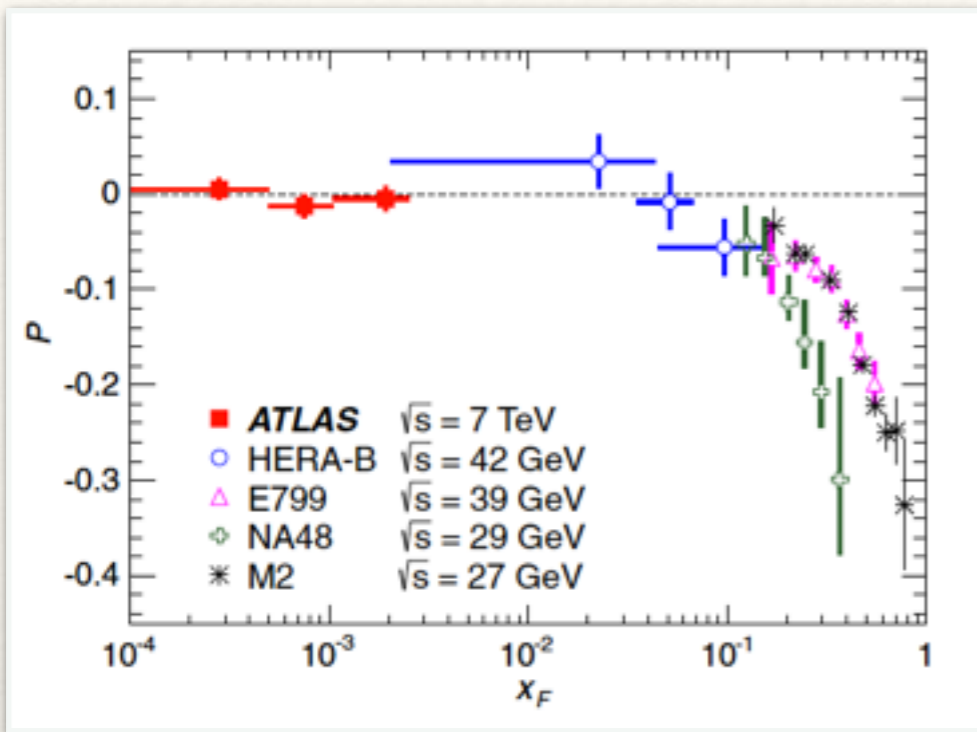
Tool in particle physics?



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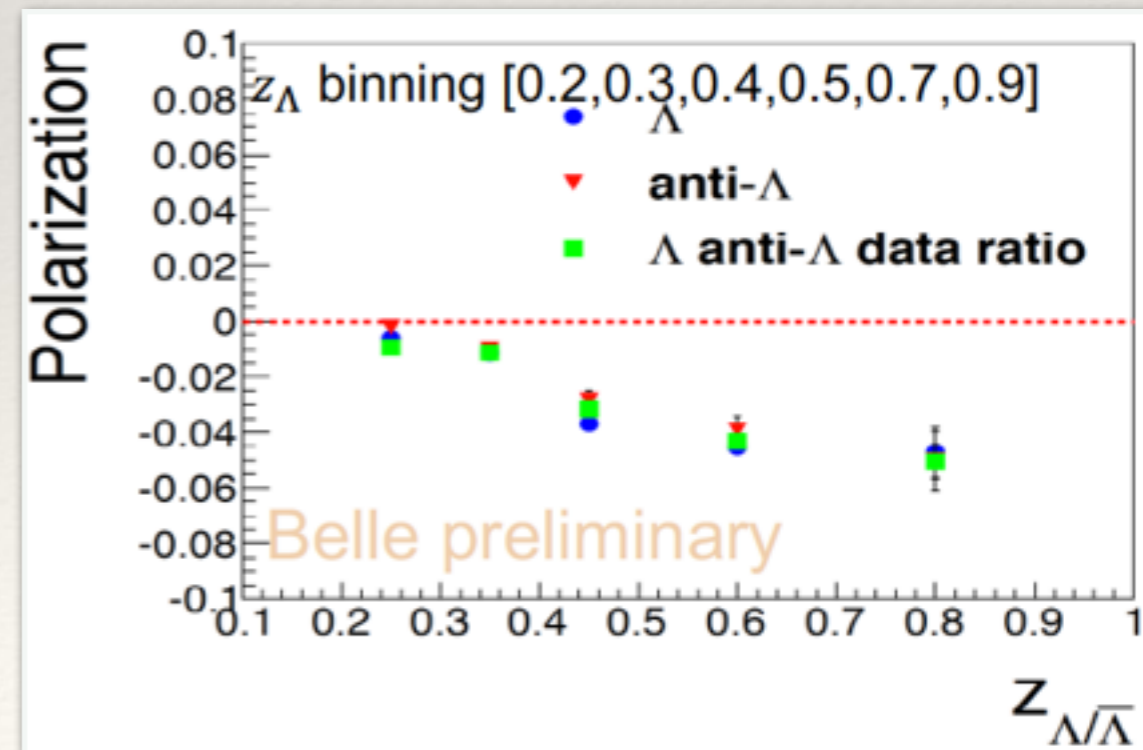


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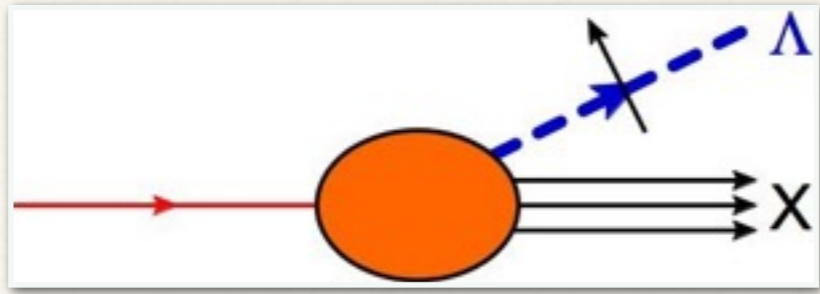
Can Λ polarization be useful for LHC physics?
 Tool in particle physics?

Simplest and cleanest process (like DIS): $e^+ e^- \longrightarrow \Lambda^\uparrow X$

- ❖ [OPAL at LEP on Z-pole \[Eur.Phys.J C2, 49 \(1998\)\]](#):
 Longitudinal Polarization,
 no significant Transverse Polarization
- ❖ [Preliminary Belle data: Transverse Polarization \[Yinghui Guan, SPIN 2016\]](#)
 \Rightarrow significant transverse polarization



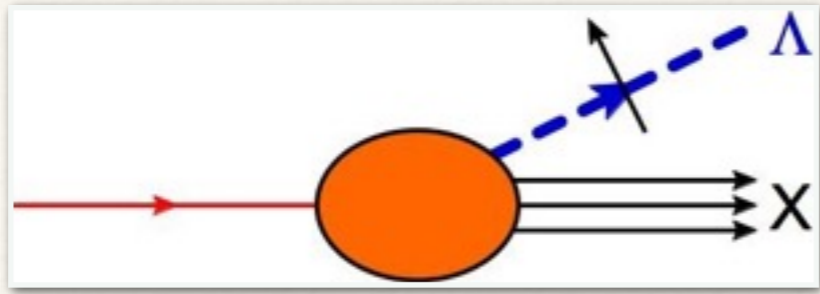
Perturbative QCD at leading twist: Λ fragmentation



parton $\longrightarrow \Lambda + X$ transition:

$$\langle P_\Lambda, S_\Lambda; X | \bar{q}(0) | 0 \rangle$$

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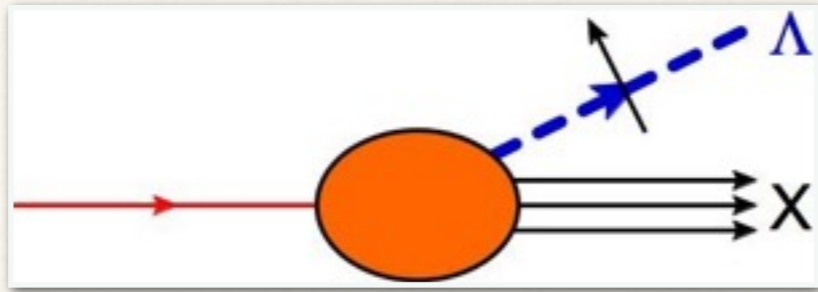
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‘square of the amplitude’

$$\Delta_{ij}(z) = \frac{1}{N_c} \sum_X \int \frac{d\lambda}{2\pi} e^{-i\frac{\lambda}{z}} \langle 0 | [\infty m, 0] q_i(0) | P_\Lambda, S_\Lambda; X \rangle \langle P_\Lambda, S_\Lambda; X | \bar{q}_j(\lambda m) [\lambda m, \infty m] | 0 \rangle$$

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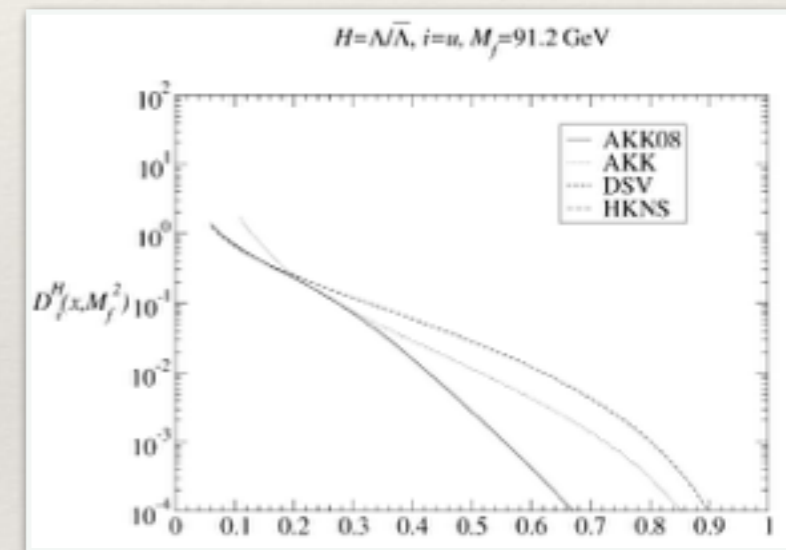
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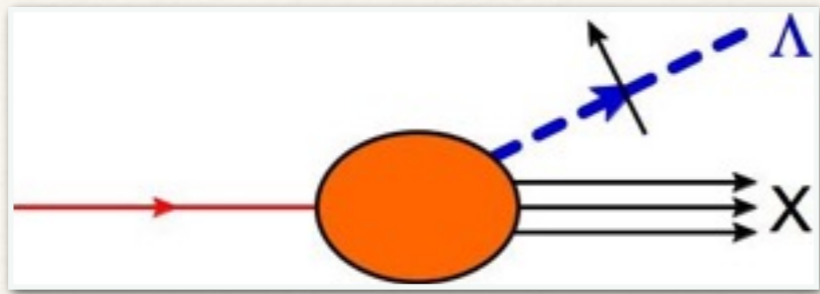
Λ fragmentation functions
at leading twist

$$D_1^{\Lambda/q}(z)$$

FF of unpolarized $q \rightarrow \Lambda$:
fairly known [fits by AKK08, DSV, ...]



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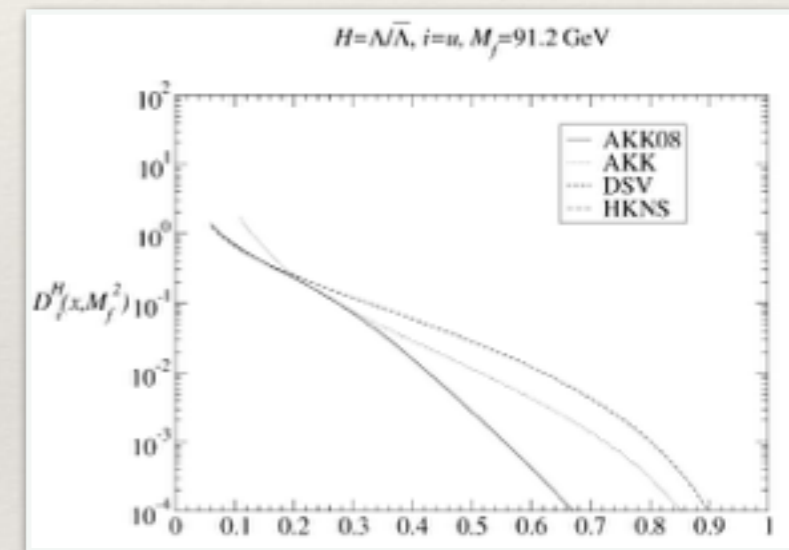
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Λ fragmentation functions at leading twist

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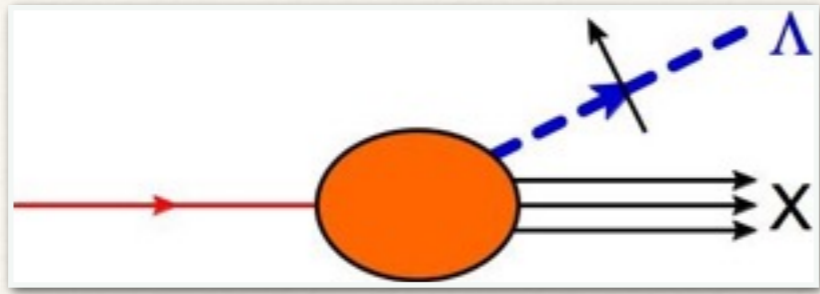
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$$G_1^{\Lambda/q}(z)$$

FF of longitudinally pol. $q \rightarrow \Lambda$:
poorly known [attempts by DSV to fit LEP data]

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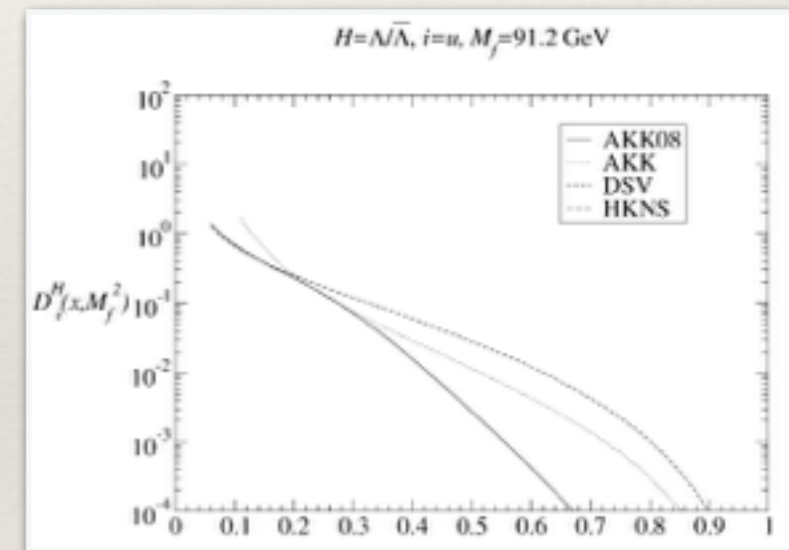
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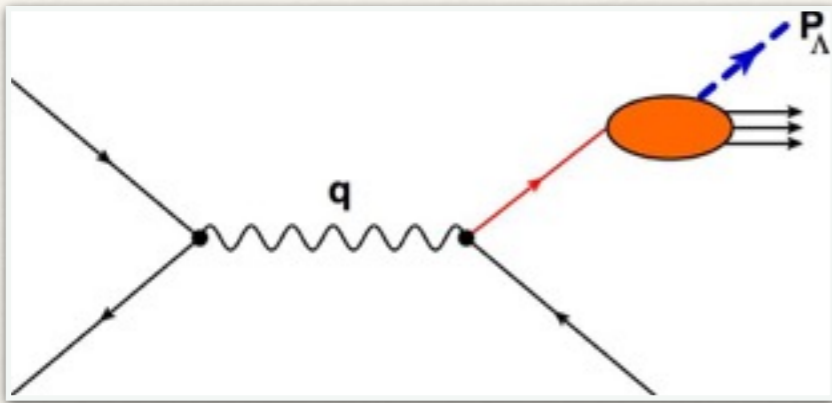
FF of longitudinally pol. $q \rightarrow \Lambda$:
poorly known [attempts by DSV to fit LEP data]

$$H_1^{\Lambda/q}(z)$$

FF of transversely pol. $q \rightarrow \Lambda$:
unknown, chiral-odd, hard to extract from single-inclusive processes
Candidate to explain large transverse Λ polarization?

Unpolarized $e^+ e^- \rightarrow \Lambda X$ cross section (leading twist) in pQCD

“Parton Model like” at LO

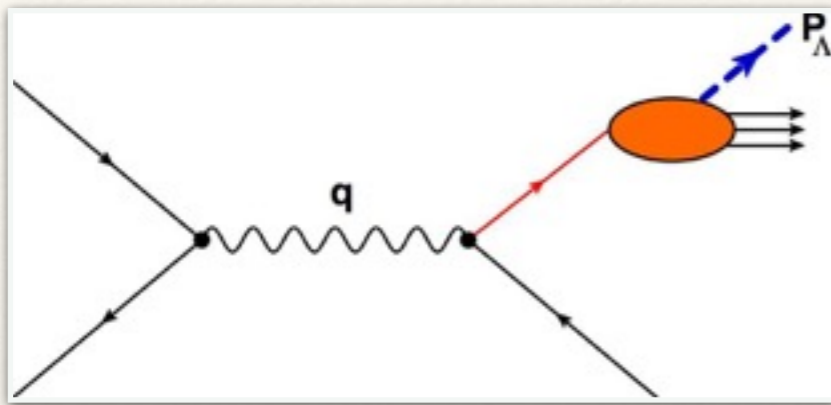


$$E_\Lambda \frac{d\sigma}{d^3 \vec{P}_\Lambda} \propto \sum_q e_q^2 D_1^{\Lambda/q}(z_h)$$

$$z_h = \frac{2P_\Lambda \cdot q}{q^2}$$

Unpolarized $e^+ e^- \rightarrow \Lambda X$ cross section (leading twist) in pQCD

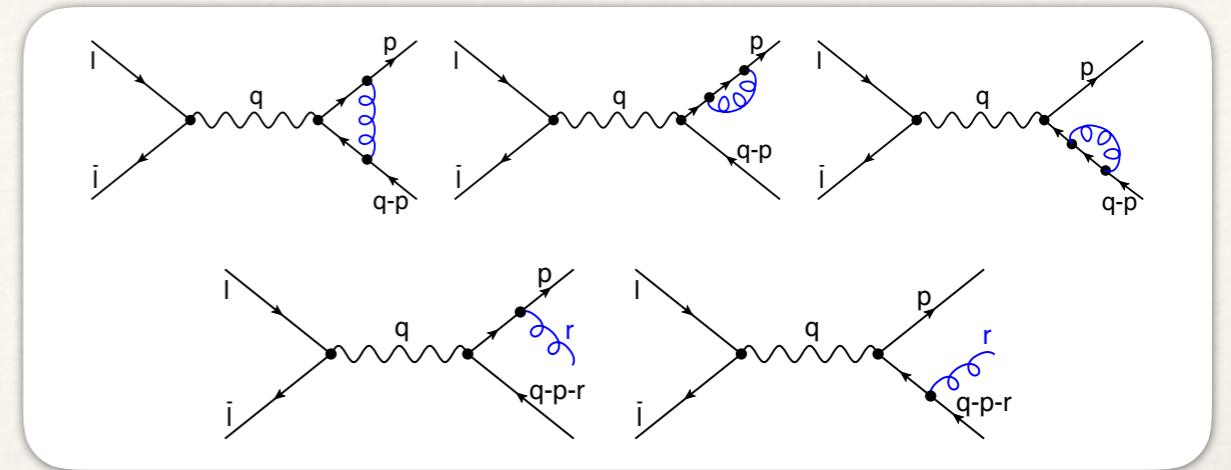
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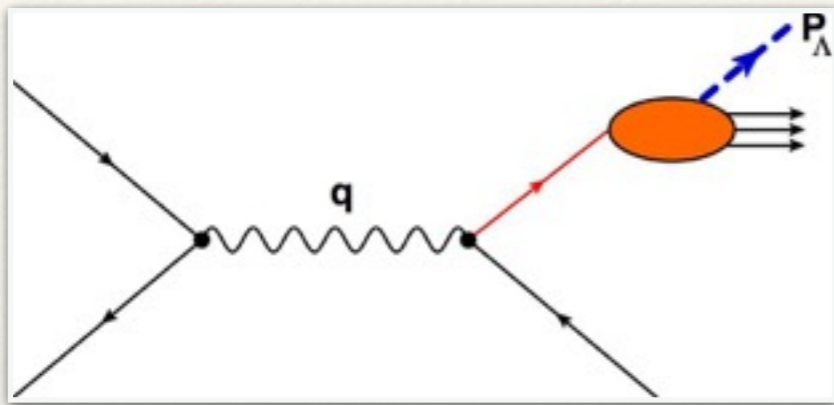
NLO



$$\left(E_\Lambda \frac{d\sigma}{d^3 \vec{P}_\Lambda} \right)_{\text{NLO}} \propto \sum_q e_q^2 \int_{z_h}^1 \frac{dw}{w} \left[\hat{\sigma}^{\text{MS},q}(w, s/\mu^2) D_1^{\Lambda/q}(z_h/w, \mu) + \hat{\sigma}^{\text{MS},g}(w, s/\mu^2) D_1^{\Lambda/g}(z_h/w, \mu) \right]$$

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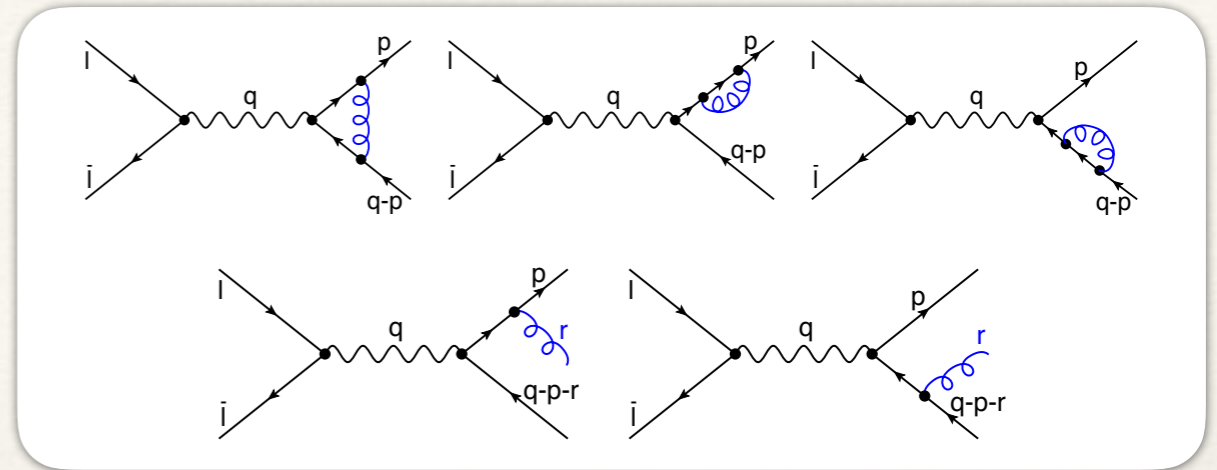
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Typical NLO features:

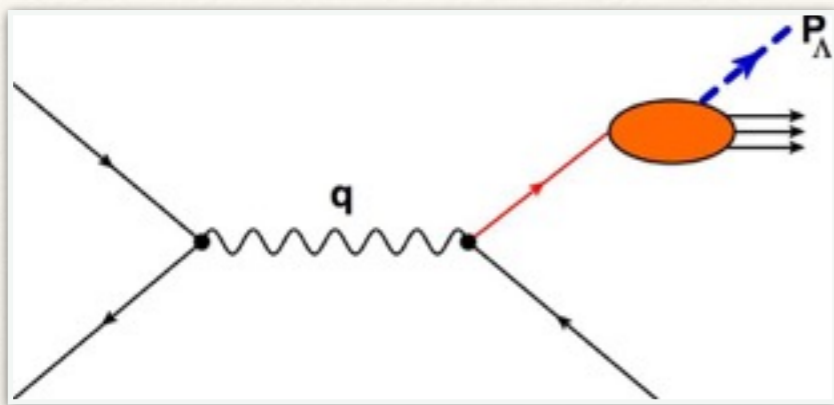
- ❖ infrared safe (cancellation of $1/\epsilon^2$ - poles in dim. reg.)

$$\hat{\sigma}_{\text{virt}} + \hat{\sigma}_{\text{real}} = \mathcal{O}(1/\epsilon)$$

$$\hat{\sigma}^{q/g} \propto -\frac{1}{\epsilon} P_{q/g} q(w) + \mathcal{O}(\epsilon^0)$$

Unpolarized $e^+ e^- \rightarrow \Lambda X$ cross section (leading twist) in pQCD

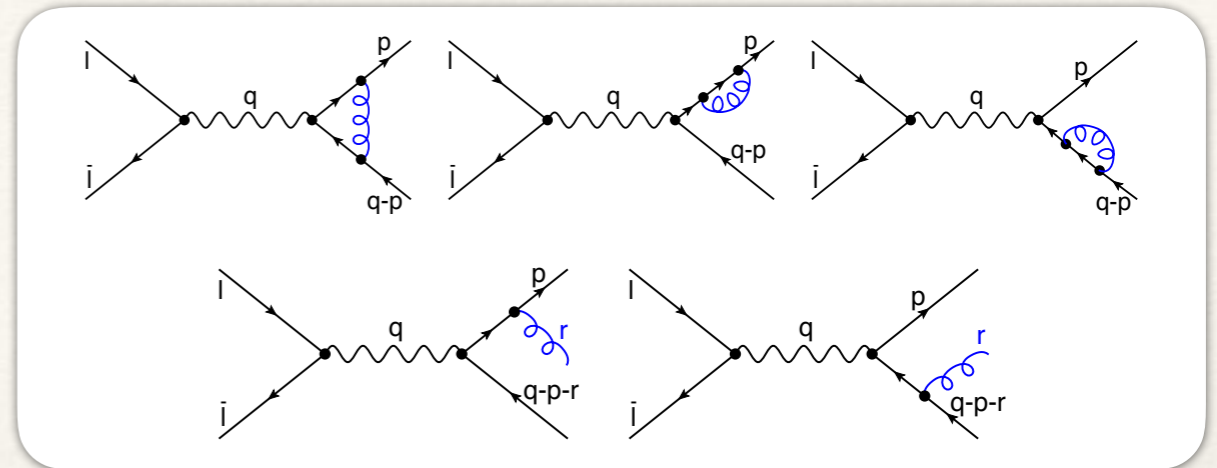
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NLO



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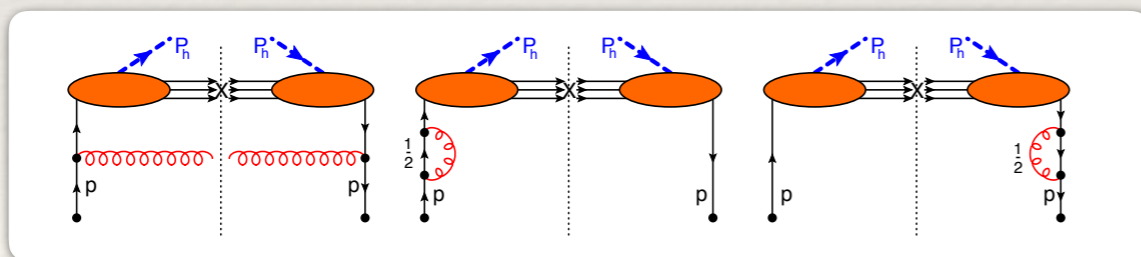
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- MSbar renormalization of fragmentation functions \rightarrow DGLAP evolution



$$D_{1,\text{bare}}^{\Lambda/q}(z) = D_{1,\text{ren}}^{\Lambda/q}(z) + \frac{\alpha_s}{2\pi} \frac{S_\epsilon^{\text{MS}}}{\epsilon} \sum_{i=q,g} \int_z^1 \frac{dw}{w} P_{iq}(w) D_{1,\text{ren}}^{\Lambda/i}\left(\frac{z}{w}\right) + \mathcal{O}(\alpha_s^2)$$

$\mathcal{O}(1/\epsilon)$ cancels,
necessary condition for
one-loop factorization!

Collinear Twist-3 formalism

'intrinsic' twist-3 FF with transverse spin:

$$G_T^{\Lambda/q}(z)$$

$$D_T^{\Lambda/q}(z)$$

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$$D_T^{\Lambda/q}(z)$$

'kinematic' twist-3 FF with transverse spin:

$$\Delta_{\partial}^{\alpha}(z) = \int d^2 p_T p_T^{\alpha} \Delta(z, z p_T)$$

→

$$G_{1T}^{\perp(1),\Lambda/q}(z)$$

$$D_{1T}^{\perp(1),\Lambda/q}(z)$$

Collinear Twist-3 formalism

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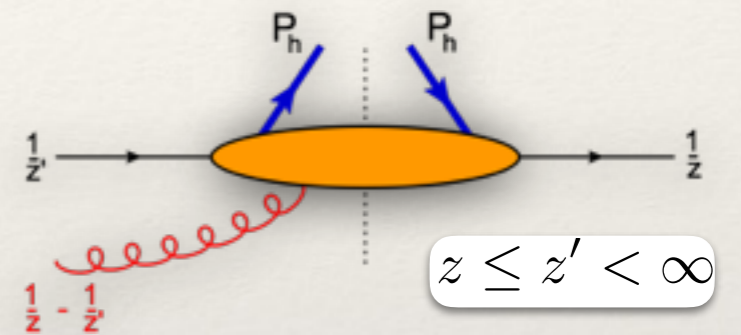
$$G_{1T}^{\perp(1),\Lambda/q}(z)$$

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'dynamical' twist-3 FF with transverse spin:

$$\Delta_F^{\alpha}(z, z') \sim \langle 0 | q(\lambda m) g F^{m\alpha}(\mu m) | P_{\Lambda}, S_{\Lambda}; X \rangle \langle P_{\Lambda}, S_{\Lambda}; X | \bar{q}(0) | 0 \rangle$$

$$\implies \hat{D}_{FT}^{\Lambda/q}(z, z'), \hat{G}_{FT}^{\Lambda/q}(z, z')$$



complex functions:

$$FF(z, z) = 0$$

$$FF(z, 0) = 0$$

$$\frac{\partial}{\partial z'} FF(z, z') \Big|_{z'=z} = 0$$

Collinear Twist-3 formalism

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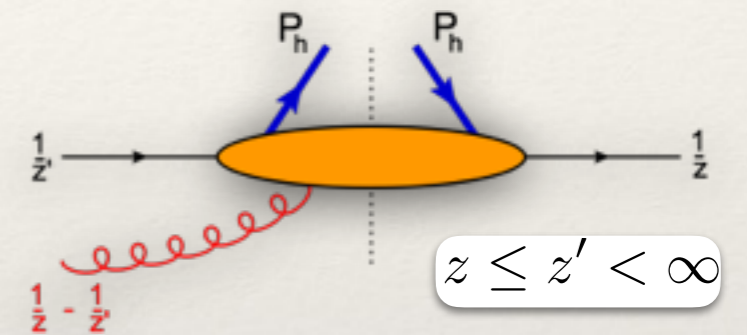
$$G_T^{\Lambda/q}(z) \quad D_T^{\Lambda/q}(z)$$

'kinematic' twist-3 FF with transverse spin:

$$\Delta_{\partial}^{\alpha}(z) = \int d^2 p_T p_T^{\alpha} \Delta(z, z p_T) \quad \longrightarrow \quad G_{1T}^{\perp(1), \Lambda/q}(z) \quad D_{1T}^{\perp(1), \Lambda/q}(z)$$

'dynamical' twist-3 FF with transverse spin:

$$\Delta_F^{\alpha}(z, z') \sim \langle 0 | q(\lambda m) g F^{m\alpha}(\mu m) | P_{\Lambda}, S_{\Lambda}; X \rangle \langle P_{\Lambda}, S_{\Lambda}; X | \bar{q}(0) | 0 \rangle \\ \implies \hat{D}_{FT}^{\Lambda/q}(z, z'), \hat{G}_{FT}^{\Lambda/q}(z, z')$$



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Relations: Equation of Motion & Lorentz-Invariance

[Kanazawa, Koike, Metz, Pitonyak, MS, PRD 93, 054024 (2016)]

$$D_{1T}^{\perp(1)}(z) + \frac{D_T(z)}{z} = \int_0^1 d\beta \frac{\Im[\hat{D}_{FT}(z, z/\beta)] - \Im[\hat{G}_{FT}(z, z/\beta)]}{1 - \beta}$$

$$G_{1T}^{\perp(1)}(z) - \frac{G_T(z)}{z} = \int_0^1 d\beta \frac{\Re[\hat{D}_{FT}(z, z/\beta)] - \Re[\hat{G}_{FT}(z, z/\beta)]}{1 - \beta}$$

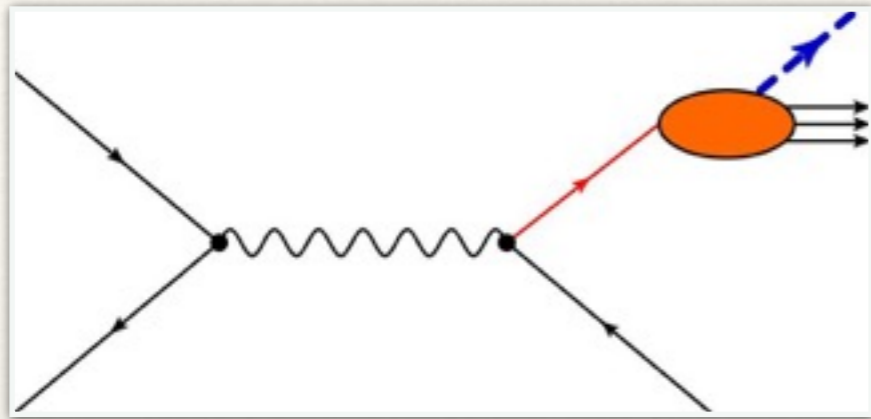
$$\frac{D_T(z)}{z} = - \left(1 - z \frac{d}{dz} \right) D_{1T}^{\perp(1)}(z) - 2 \int_0^1 d\beta \frac{\Im[\hat{D}_{FT}(z, z/\beta)]}{(1 - \beta)^2}$$

$$\frac{G_T(z)}{z} = \frac{G_1(z)}{z} + \left(1 - z \frac{d}{dz} \right) G_{1T}^{\perp(1)}(z) - 2 \int_0^1 d\beta \frac{\Re[\hat{G}_{FT}(z, z/\beta)]}{(1 - \beta)^2}$$

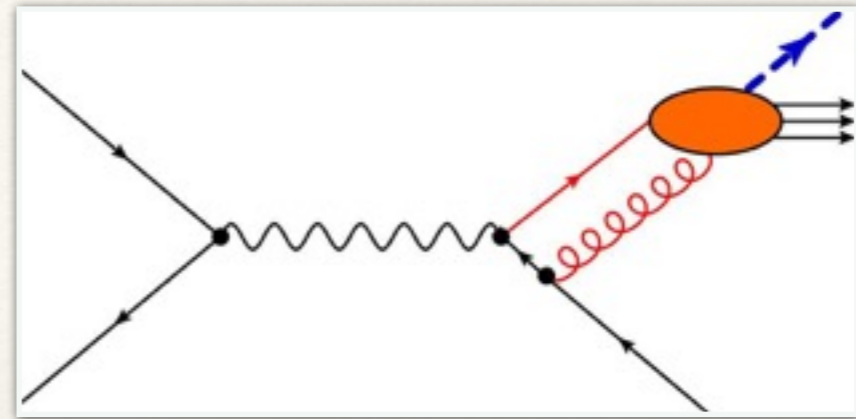
Two equations, three functions \rightarrow eliminate 'intrinsic & kinematical twist-3'

Transverse Λ polarization at LO

'intrinsic' & 'kinematical' twist-3 FF:



'dynamical' twist-3 FF:



$$\frac{d\sigma(S_{\Lambda T})}{dz_h d\phi} = C |S_{\Lambda T}| \sin(\phi_S) \sum_q e_q^2 \left[\frac{D_T^{\Lambda/q}(z_h)}{z_h} - D_{1T}^{\perp(1)\Lambda/q}(z_h) + \int_0^1 d\beta \frac{\Im[\hat{D}_{FT} - \hat{G}_{FT}]^{\Lambda/q}(z_h, z_h/\beta)}{1-\beta} \right]$$



Equation of Motion:

$$\frac{d\sigma(S_{\Lambda T})}{dz_h d\phi} = C |S_{\Lambda T}| \sin(\phi_S) \sum_q e_q^2 \left[2 \frac{D_T^{\Lambda/q}(z_h)}{z_h} \right]$$

or:

$$\frac{d\sigma(S_{\Lambda T})}{dz_h d\phi} = C |S_{\Lambda T}| \sin(\phi_S) \sum_q e_q^2 \left[-2 D_{1T}^{\perp(1)\Lambda/q}(z_h) + 2 \int_0^1 d\beta \frac{\Im[\hat{D}_{FT} - \hat{G}_{FT}]^{\Lambda/q}(z_h, z_h/\beta)}{1-\beta} \right]$$

Single-Transverse Λ spin asymmetry

- ❖ Unique effect driven by a single fragmentation function $D_T \rightarrow$ absent in DIS (1γ)
- ❖ Belle data \rightarrow first information on D_T

Transverse Λ polarization at NLO

[Gamberg, Kang, Pitonyak, M.S., Yoshida, work in progress]

- ❖ Study the NLO dynamics for twist-3 fragmentation in the simplest process
- ❖ Different compared to twist-3 distributions (no pole contributions)

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Virtual & Real diagrams (qg/q - channel only)

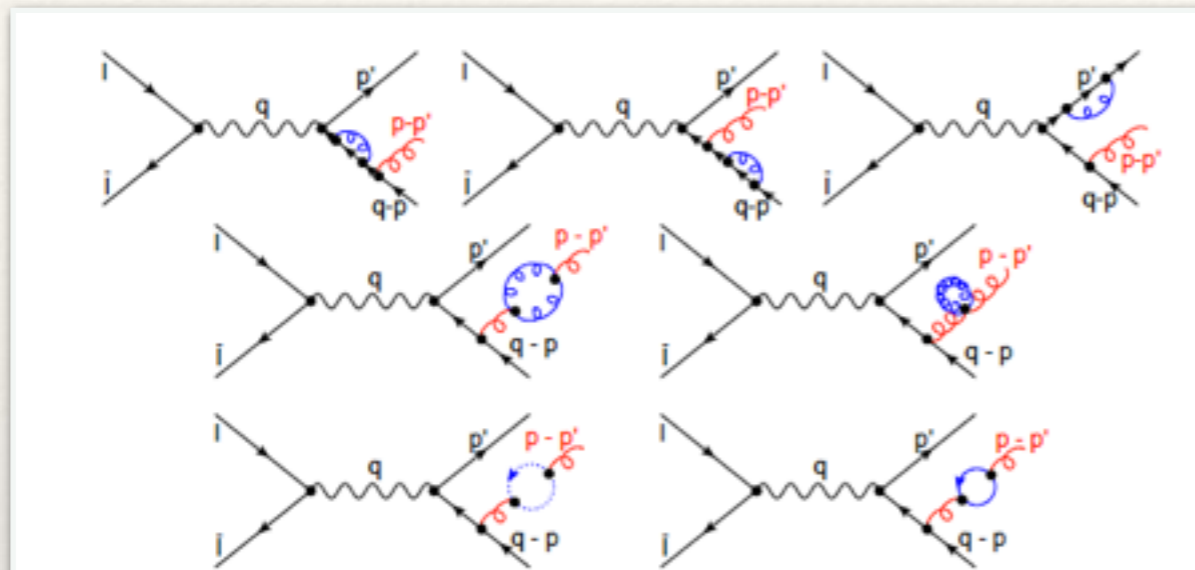


Figure 6: Self-energy corrections

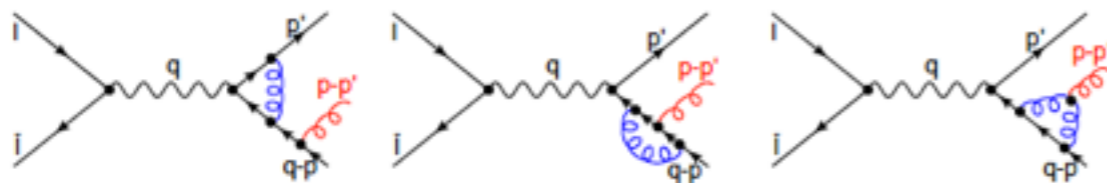


Figure 7: Vertex corrections

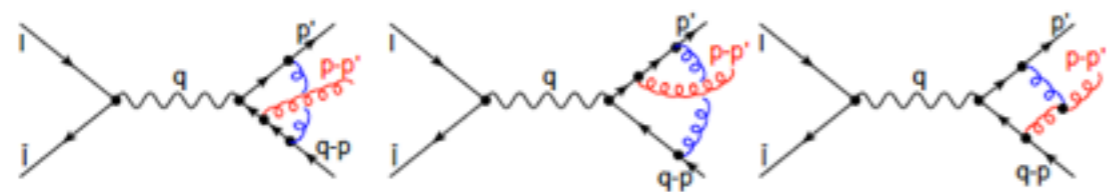


Figure 8: Box corrections

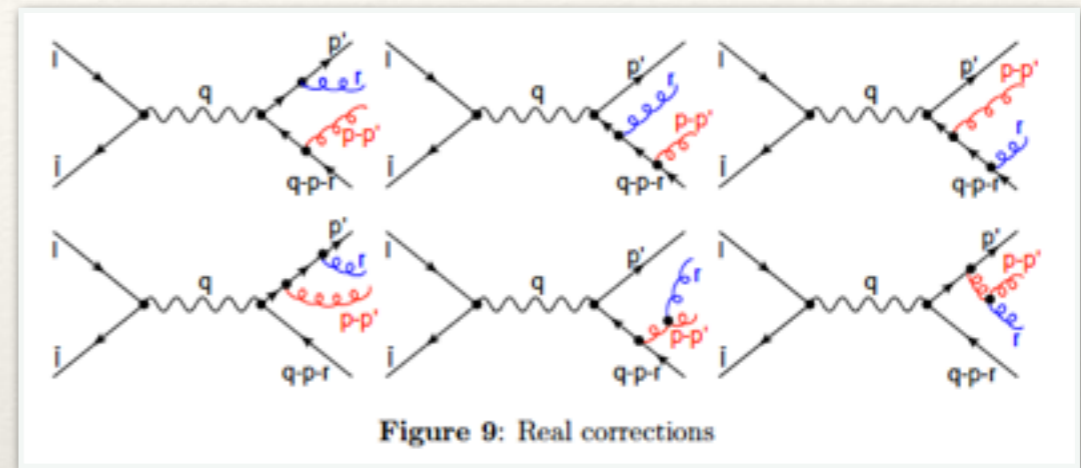


Figure 9: Real corrections

- ❖ E.o.M. - relations are crucial:
- ❖ Eliminate 'intrinsic' twist-3 contributions
- ❖ Imaginary parts: In the dynamical fragmentation process & loop diagrams
- ❖ Infrared $1/\epsilon^2$ - poles cancel ✓
- ❖ $1/\epsilon$ - poles of imaginary parts of loops cancel through E.o.M. ✓
- ❖ $1/\epsilon$ - collinear poles of real parts of loops through MSbar - renormalization (?)

Complete structure of the NLO result w/o intrinsic twist-3

$$\begin{aligned}
 E_h \frac{d\sigma_U^{\text{EoM},2}}{d^{d-1}\vec{P}_h}(S_h) &= (4\pi^2 z_h^2)^\epsilon \frac{2\alpha_{\text{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l} S_h}{s} (2v-1) \times \\
 &\sum_{q=u,\bar{u},\dots} e_q^2 \left[-2 D_{1T}^{\perp(1),q}(z_h) + 2 \int_0^1 d\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta} \right. \\
 &+ \frac{\alpha_s}{2\pi} S_\epsilon \int_{z_h}^1 \frac{dw}{w^2} \int_0^1 d\beta \left\{ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{q;\text{EoM}}(w) D_{1T}^{\perp(1),q}\left(\frac{z_h}{w}\right) \right. \\
 &+ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{g;\text{EoM}}(w) D_{1T}^{\perp(1),g}\left(\frac{z_h}{w}\right) + \hat{\sigma}_{H_1^{(1)}}^{g;\text{EoM}}(w) H_1^{(1)g}\left(\frac{z_h}{w}\right) \\
 &+ \hat{\sigma}_{\Im D_{FT}}^{qg;\text{EoM}}(w, \beta) \frac{\Im[\hat{D}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} + \hat{\sigma}_{\Im G_{FT}}^{qg;\text{EoM}}(w, \beta) \frac{\Im[\hat{G}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} \\
 &+ \hat{\sigma}_1^{gg;\text{EoM}}(w, \beta) \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\epsilon) \hat{H}_{FT}^{gg}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \\
 &+ \hat{\sigma}_3^{gg;\text{EoM}}(w, \beta) \Im[(1-\epsilon) \hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\epsilon}{2} \hat{H}_{FT}^{gg}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \\
 &+ \hat{\sigma}_{D_{FT}}^{q\bar{q};\text{EoM}}(w) \left(\sum_{q=u,d,\dots} \Im[\hat{D}_{FT}^{q\bar{q}}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \right) \\
 &\left. + \hat{\sigma}_{\Re}^q(w, \beta) \frac{\Re[\hat{D}_{FT}^q - \hat{G}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} \right\} + \mathcal{O}(\Lambda^2/s),
 \end{aligned}$$

Complete structure of the NLO result w/o intrinsic twist-3

LO

$$\begin{aligned}
 E_h \frac{d\sigma_U^{\text{EoM},2}}{d^{d-1}\vec{P}_h}(S_h) &= (4\pi^2 z_h^2)^\epsilon \frac{2\alpha_{\text{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l} S_h}{s} (2v-1) \times \\
 &\sum_{q=u,\bar{u},\dots} e_q^2 \left[-2 D_{1T}^{\perp(1),q}(z_h) + 2 \int_0^1 d\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta} \right. \\
 &+ \frac{\alpha_s}{2\pi} S_\epsilon \int_{z_h}^1 \frac{dw}{w^2} \int_0^1 d\beta \left\{ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{q;\text{EoM}}(w) D_{1T}^{\perp(1),q}\left(\frac{z_h}{w}\right) \right. \\
 &+ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{g;\text{EoM}}(w) D_{1T}^{\perp(1),g}\left(\frac{z_h}{w}\right) + \hat{\sigma}_{H_1^{(1)}}^{g;\text{EoM}}(w) H_1^{(1)g}\left(\frac{z_h}{w}\right) \\
 &+ \hat{\sigma}_{\Im D_{FT}}^{qg;\text{EoM}}(w, \beta) \frac{\Im[\hat{D}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} + \hat{\sigma}_{\Im G_{FT}}^{qg;\text{EoM}}(w, \beta) \frac{\Im[\hat{G}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} \\
 &+ \hat{\sigma}_1^{gg;\text{EoM}}(w, \beta) \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\epsilon) \hat{H}_{FT}^{gg}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \\
 &+ \hat{\sigma}_3^{gg;\text{EoM}}(w, \beta) \Im[(1-\epsilon) \hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\epsilon}{2} \hat{H}_{FT}^{gg}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \\
 &+ \hat{\sigma}_{D_{FT}}^{q\bar{q};\text{EoM}}(w) \left(\sum_{q=u,d,\dots} \Im[\hat{D}_{FT}^{q\bar{q}}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \right) \\
 &\left. + \hat{\sigma}_{\Re}^q(w, \beta) \frac{\Re[\hat{D}_{FT}^q - \hat{G}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} \right\} + \mathcal{O}(\Lambda^2/s),
 \end{aligned}$$

Complete structure of the NLO result w/o intrinsic twist-3

$$\begin{aligned}
 E_h \frac{d\sigma_U^{\text{EoM},2}}{d^{d-1}\vec{P}_h}(S_h) &= (4\pi^2 z_h^2)^\epsilon \frac{2\alpha_{\text{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l} S_h}{s} (2v-1) \times \\
 &\sum_{q=u,\bar{u},\dots} e_q^2 \left[-2 D_{1T}^{\perp(1),q}(z_h) + 2 \int_0^1 d\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta} \right. \\
 &+ \frac{\alpha_s}{2\pi} S_\epsilon \int_{z_h}^1 \frac{dw}{w^2} \int_0^1 d\beta \left\{ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{q;\text{EoM}}(w) D_{1T}^{\perp(1),q}\left(\frac{z_h}{w}\right) \right. \\
 &+ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{g;\text{EoM}}(w) D_{1T}^{\perp(1),g}\left(\frac{z_h}{w}\right) + \hat{\sigma}_{H_1^{(1)}}^{g;\text{EoM}}(w) H_1^{(1)g}\left(\frac{z_h}{w}\right) \\
 &+ \hat{\sigma}_{\Im D_{FT}}^{qg;\text{EoM}}(w, \beta) \frac{\Im[\hat{D}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} + \hat{\sigma}_{\Im G_{FT}}^{qg;\text{EoM}}(w, \beta) \frac{\Im[\hat{G}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} \\
 &+ \hat{\sigma}_1^{gg;\text{EoM}}(w, \beta) \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\epsilon)\hat{H}_{FT}^{gg}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \\
 &+ \hat{\sigma}_3^{gg;\text{EoM}}(w, \beta) \Im[(1-\epsilon)\hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\epsilon}{2}\hat{H}_{FT}^{gg}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \\
 &+ \hat{\sigma}_{D_{FT}}^{q\bar{q};\text{EoM}}(w) \left(\sum_{q=u,d,\dots} \Im[\hat{D}_{FT}^{q\bar{q}}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \right) \\
 &\left. + \hat{\sigma}_{\Re}^q(w, \beta) \frac{\Re[\hat{D}_{FT}^q - \hat{G}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} \right\} + \mathcal{O}(\Lambda^2/s),
 \end{aligned}$$

LO

NLO

2-quark correlation w/ EoM

Complete structure of the NLO result w/o intrinsic twist-3

$$\begin{aligned}
 E_h \frac{d\sigma_U^{\text{EoM},2}}{d^{d-1}\vec{P}_h}(S_h) &= (4\pi^2 z_h^2)^\epsilon \frac{2\alpha_{\text{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l} S_h}{s} (2v-1) \times \\
 &\sum_{q=u,\bar{u},\dots} e_q^2 \left[-2 D_{1T}^{\perp(1),q}(z_h) + 2 \int_0^1 d\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta} \right. \\
 &+ \frac{\alpha_s}{2\pi} S_\epsilon \int_{z_h}^1 \frac{dw}{w^2} \int_0^1 d\beta \left\{ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{q;\text{EoM}}(w) D_{1T}^{\perp(1),q}\left(\frac{z_h}{w}\right) \right. \\
 &+ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{g;\text{EoM}}(w) D_{1T}^{\perp(1),g}\left(\frac{z_h}{w}\right) + \hat{\sigma}_{H_1^{(1)g}}^{g;\text{EoM}}(w) H_1^{(1)g}\left(\frac{z_h}{w}\right) \\
 &+ \hat{\sigma}_{\Im D_{FT}}^{qg;\text{EoM}}(w, \beta) \frac{\Im[\hat{D}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} + \hat{\sigma}_{\Im G_{FT}}^{qg;\text{EoM}}(w, \beta) \frac{\Im[\hat{G}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} \\
 &+ \hat{\sigma}_1^{gg;\text{EoM}}(w, \beta) \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\epsilon) \hat{H}_{FT}^{gg}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \\
 &+ \hat{\sigma}_3^{gg;\text{EoM}}(w, \beta) \Im[(1-\epsilon) \hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\epsilon}{2} \hat{H}_{FT}^{gg}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \\
 &+ \hat{\sigma}_{D_{FT}}^{q\bar{q};\text{EoM}}(w) \left(\sum_{q=u,d,\dots} \Im[\hat{D}_{FT}^{q\bar{q}}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \right) \\
 &\left. + \hat{\sigma}_{\Re}^q(w, \beta) \frac{\Re[\hat{D}_{FT}^q - \hat{G}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} \right\} + \mathcal{O}(\Lambda^2/s),
 \end{aligned}$$

LO

NLO

2-quark correlation w/ EoM

NLO

2-gluon correlation w/ EoM

Complete structure of the NLO result w/o intrinsic twist-3

$$\begin{aligned}
 E_h \frac{d\sigma_U^{\text{EoM},2}}{d^{d-1}\vec{P}_h}(S_h) &= (4\pi^2 z_h^2)^\epsilon \frac{2\alpha_{\text{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l} S_h}{s} (2v-1) \times \\
 &\sum_{q=u,\bar{u},\dots} e_q^2 \left[-2 D_{1T}^{\perp(1),q}(z_h) + 2 \int_0^1 d\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta} \right. \\
 &+ \frac{\alpha_s}{2\pi} S_\epsilon \int_{z_h}^1 \frac{dw}{w^2} \int_0^1 d\beta \left\{ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{q;\text{EoM}}(w) D_{1T}^{\perp(1),q}\left(\frac{z_h}{w}\right) \right. \\
 &+ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{g;\text{EoM}}(w) D_{1T}^{\perp(1),g}\left(\frac{z_h}{w}\right) + \hat{\sigma}_{H_1^{(1)g}}^{g;\text{EoM}}(w) H_1^{(1)g}\left(\frac{z_h}{w}\right) \\
 &+ \hat{\sigma}_{\Im D_{FT}}^{qg;\text{EoM}}(w, \beta) \frac{\Im[\hat{D}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} + \hat{\sigma}_{\Im G_{FT}}^{qg;\text{EoM}}(w, \beta) \frac{\Im[\hat{G}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} \\
 &+ \hat{\sigma}_1^{gg;\text{EoM}}(w, \beta) \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\epsilon)\hat{H}_{FT}^{gg}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \\
 &+ \hat{\sigma}_3^{gg;\text{EoM}}(w, \beta) \Im[(1-\epsilon)\hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\epsilon}{2}\hat{H}_{FT}^{gg}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \\
 &+ \hat{\sigma}_{D_{FT}}^{q\bar{q};\text{EoM}}(w) \left(\sum_{q=u,d,\dots} \Im[\hat{D}_{FT}^{q\bar{q}}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \right) \\
 &\left. + \hat{\sigma}_{\Re}^q(w, \beta) \frac{\Re[\hat{D}_{FT}^q - \hat{G}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} \right\} + \mathcal{O}(\Lambda^2/s),
 \end{aligned}$$

LO

NLO

2-quark correlation w/ EoM

NLO

2-gluon correlation w/ EoM

NLO

q-gluon-q correlation w/ EoM

Complete structure of the NLO result w/o intrinsic twist-3

$$\begin{aligned}
 E_h \frac{d\sigma_U^{\text{EoM},2}}{d^{d-1}\vec{P}_h}(S_h) &= (4\pi^2 z_h^2)^\epsilon \frac{2\alpha_{\text{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l} S_h}{s} (2v-1) \times \\
 &\sum_{q=u,\bar{u},\dots} e_q^2 \left[-2 D_{1T}^{\perp(1),q}(z_h) + 2 \int_0^1 d\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta} \right. \\
 &+ \frac{\alpha_s}{2\pi} S_\epsilon \int_{z_h}^1 \frac{dw}{w^2} \int_0^1 d\beta \left\{ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{q;\text{EoM}}(w) D_{1T}^{\perp(1),q}\left(\frac{z_h}{w}\right) \right. \\
 &+ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{g;\text{EoM}}(w) D_{1T}^{\perp(1),g}\left(\frac{z_h}{w}\right) + \hat{\sigma}_{H_1^{(1)g}}^{g;\text{EoM}}(w) H_1^{(1)g}\left(\frac{z_h}{w}\right) \\
 &+ \hat{\sigma}_{\Im D_{FT}}^{qg;\text{EoM}}(w, \beta) \frac{\Im[\hat{D}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} + \hat{\sigma}_{\Im G_{FT}}^{qg;\text{EoM}}(w, \beta) \frac{\Im[\hat{G}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} \\
 &+ \hat{\sigma}_1^{gg;\text{EoM}}(w, \beta) \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\epsilon)\hat{H}_{FT}^{gg}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \\
 &+ \hat{\sigma}_3^{gg;\text{EoM}}(w, \beta) \Im[(1-\epsilon)\hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\epsilon}{2}\hat{H}_{FT}^{gg}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \\
 &+ \hat{\sigma}_{D_{FT}}^{q\bar{q};\text{EoM}}(w) \left(\sum_{q=u,d,\dots} \Im[\hat{D}_{FT}^{q\bar{q}}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \right) \\
 &\left. + \hat{\sigma}_{\Re}^q(w, \beta) \frac{\Re[\hat{D}_{FT}^q - \hat{G}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} \right\} + \mathcal{O}(\Lambda^2/s),
 \end{aligned}$$

LO

NLO

2-quark correlation w/ EoM

NLO

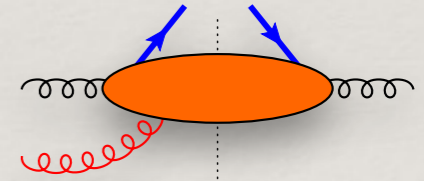
2-gluon correlation w/ EoM

NLO

q-gluon-q correlation w/ EoM

NLO

triple-gluon correlation w/ EoM



Complete structure of the NLO result w/o intrinsic twist-3

$$\begin{aligned}
 E_h \frac{d\sigma_U^{\text{EoM},2}}{d^{d-1}\vec{P}_h}(S_h) = & (4\pi^2 z_h^2)^\epsilon \frac{2\alpha_{\text{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l} S_h}{s} (2v-1) \times \\
 & \sum_{q=u,\bar{u},\dots} e_q^2 \left[-2 D_{1T}^{\perp(1),q}(z_h) + 2 \int_0^1 d\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta} \right. \\
 & + \frac{\alpha_s}{2\pi} S_\epsilon \int_{z_h}^1 \frac{dw}{w^2} \int_0^1 d\beta \left\{ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{q;\text{EoM}}(w) D_{1T}^{\perp(1),q}\left(\frac{z_h}{w}\right) \right. \\
 & + \hat{\sigma}_{D_{1T}^{\perp(1)}}^{g;\text{EoM}}(w) D_{1T}^{\perp(1),g}\left(\frac{z_h}{w}\right) + \hat{\sigma}_{H_1^{(1)g}}^{g;\text{EoM}}(w) H_1^{(1)g}\left(\frac{z_h}{w}\right) \\
 & + \hat{\sigma}_{\Im D_{FT}}^{qg;\text{EoM}}(w, \beta) \frac{\Im[\hat{D}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} + \hat{\sigma}_{\Im G_{FT}}^{qg;\text{EoM}}(w, \beta) \frac{\Im[\hat{G}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} \\
 & + \hat{\sigma}_1^{gg;\text{EoM}}(w, \beta) \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\epsilon)\hat{H}_{FT}^{gg}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \\
 & + \hat{\sigma}_3^{gg;\text{EoM}}(w, \beta) \Im[(1-\epsilon)\hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\epsilon}{2}\hat{H}_{FT}^{gg}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \\
 & + \hat{\sigma}_{D_{FT}}^{q\bar{q};\text{EoM}}(w) \left(\sum_{q=u,d,\dots} \Im[\hat{D}_{FT}^{q\bar{q}}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \right) \\
 & \left. + \hat{\sigma}_{\Re}^q(w, \beta) \frac{\Re[\hat{D}_{FT}^q - \hat{G}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} \right\} + \mathcal{O}(\Lambda^2/s),
 \end{aligned}$$

LO

NLO

2-quark correlation w/ EoM

NLO

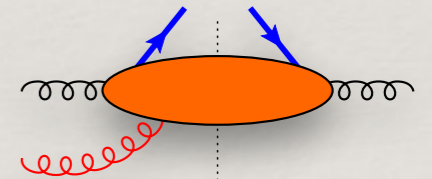
2-gluon correlation w/ EoM

NLO

q-gluon-q correlation w/ EoM

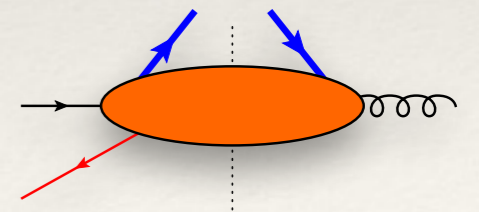
NLO

triple-gluon correlation w/ EoM



NLO

qq-gluon correlation w/ EoM



Complete structure of the NLO result w/o intrinsic twist-3

$$\begin{aligned}
 E_h \frac{d\sigma_U^{\text{EoM},2}}{d^{d-1}\vec{P}_h}(S_h) = & (4\pi^2 z_h^2)^\epsilon \frac{2\alpha_{\text{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l} S_h}{s} (2v-1) \times \\
 & \sum_{q=u,\bar{u},\dots} e_q^2 \left[-2 D_{1T}^{\perp(1),q}(z_h) + 2 \int_0^1 d\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta} \right. \\
 & + \frac{\alpha_s}{2\pi} S_\epsilon \int_{z_h}^1 \frac{dw}{w^2} \int_0^1 d\beta \left\{ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{q;\text{EoM}}(w) D_{1T}^{\perp(1),q}\left(\frac{z_h}{w}\right) \right. \\
 & + \hat{\sigma}_{D_{1T}^{\perp(1)}}^{g;\text{EoM}}(w) D_{1T}^{\perp(1),g}\left(\frac{z_h}{w}\right) + \hat{\sigma}_{H_1^{(1)g}}^{g;\text{EoM}}(w) H_1^{(1)g}\left(\frac{z_h}{w}\right) \\
 & + \hat{\sigma}_{\Im D_{FT}}^{qg;\text{EoM}}(w, \beta) \frac{\Im[\hat{D}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} + \hat{\sigma}_{\Im G_{FT}}^{qg;\text{EoM}}(w, \beta) \frac{\Im[\hat{G}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} \\
 & + \hat{\sigma}_1^{gg;\text{EoM}}(w, \beta) \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\epsilon)\hat{H}_{FT}^{gg}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \\
 & + \hat{\sigma}_3^{gg;\text{EoM}}(w, \beta) \Im[(1-\epsilon)\hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\epsilon}{2}\hat{H}_{FT}^{gg}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \\
 & + \hat{\sigma}_{D_{FT}}^{q\bar{q};\text{EoM}}(w) \left(\sum_{q=u,d,\dots} \Im[\hat{D}_{FT}^{q\bar{q}}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \right) \\
 & \left. + \hat{\sigma}_{\Re}^q(w, \beta) \frac{\Re[\hat{D}_{FT}^q - \hat{G}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} \right\} + \mathcal{O}(\Lambda^2/s),
 \end{aligned}$$

LO

NLO

2-quark correlation w/ EoM

NLO

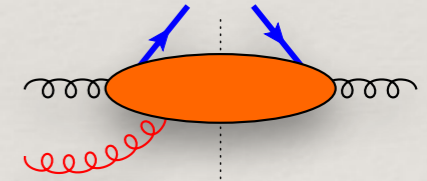
2-gluon correlation w/ EoM

NLO

q-gluon-q correlation w/ EoM

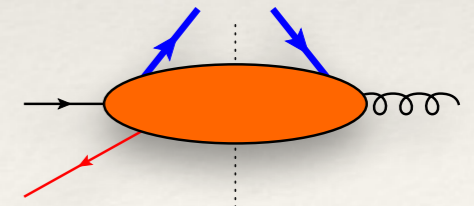
NLO

triple-gluon correlation w/ EoM



NLO

qq-gluon correlation w/ EoM



NLO

imaginary parts of loops

Complete structure of the NLO result w/o intrinsic twist-3

$$\begin{aligned}
 E_h \frac{d\sigma_U^{\text{EoM},2}}{d^{d-1}\vec{P}_h}(S_h) = & (4\pi^2 z_h^2)^\epsilon \frac{2\alpha_{\text{em}}^2 N_c}{z_h s^2} \frac{2M_h \epsilon^{P_h m l} S_h}{s} (2v-1) \times \\
 & \sum_{q=u,\bar{u},\dots} e_q^2 \left[-2 D_{1T}^{\perp(1),q}(z_h) + 2 \int_0^1 d\beta \frac{\Im[\hat{D}_{FT}^q - \hat{G}_{FT}^q](z_h, \frac{z_h}{\beta})}{1-\beta} \right. \\
 & + \frac{\alpha_s}{2\pi} S_\epsilon \int_{z_h}^1 \frac{dw}{w^2} \int_0^1 d\beta \left\{ \hat{\sigma}_{D_{1T}^{\perp(1)}}^{q;\text{EoM}}(w) D_{1T}^{\perp(1),q}\left(\frac{z_h}{w}\right) \right. \\
 & + \hat{\sigma}_{D_{1T}^{\perp(1)}}^{g;\text{EoM}}(w) D_{1T}^{\perp(1),g}\left(\frac{z_h}{w}\right) + \hat{\sigma}_{H_1^{(1)g}}^{g;\text{EoM}}(w) H_1^{(1)g}\left(\frac{z_h}{w}\right) \\
 & + \hat{\sigma}_{\Im D_{FT}}^{qg;\text{EoM}}(w, \beta) \frac{\Im[\hat{D}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} + \hat{\sigma}_{\Im G_{FT}}^{qg;\text{EoM}}(w, \beta) \frac{\Im[\hat{G}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} \\
 & + \hat{\sigma}_1^{gg;\text{EoM}}(w, \beta) \Im[\hat{D}_{FT}^{gg} - \hat{G}_{FT}^{gg} + (1-\epsilon)\hat{H}_{FT}^{gg}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \\
 & + \hat{\sigma}_3^{gg;\text{EoM}}(w, \beta) \Im[(1-\epsilon)\hat{D}_{FT}^{gg} + \hat{G}_{FT}^{gg} + \frac{\epsilon}{2}\hat{H}_{FT}^{gg}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \\
 & + \hat{\sigma}_{D_{FT}}^{q\bar{q};\text{EoM}}(w) \left(\sum_{q=u,d,\dots} \Im[\hat{D}_{FT}^{q\bar{q}}]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right) \right) \\
 & \left. + \hat{\sigma}_{\Re}^q(w, \beta) \frac{\Re[\hat{D}_{FT}^q - \hat{G}_{FT}^q]\left(\frac{z_h}{w}, \frac{z_h}{w\beta}\right)}{1-\beta} \right\} + \mathcal{O}(\Lambda^2/s),
 \end{aligned}$$

LO

NLO

2-quark correlation w/ EoM

NLO

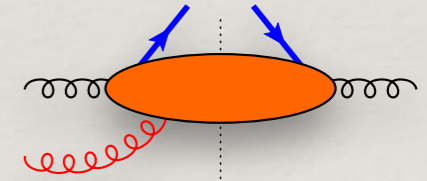
2-gluon correlation w/ EoM

NLO

q-gluon-q correlation w/ EoM

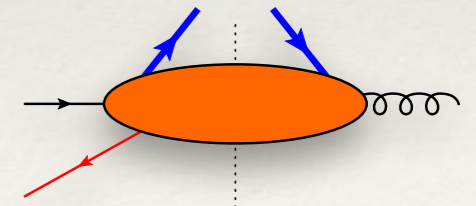
NLO

triple-gluon correlation w/ EoM



NLO

qq-gluon correlation w/ EoM



NLO

imaginary parts of loops

partonic factors in Feynman gauge

⇒ repeat calculation in light cone gauge as a check

Summary & Outlook

- ❖ Λ Polarization: Long history, measured in pp-collisions, recently at ATLAS \rightarrow feasible at a high-energy collider
- ❖ Recent measurement at Belle in e^+e^- : clean processes to determine polarized Λ fragmentation functions
- ❖ Theory for e^+e^- : Transverse Λ single-spin asymmetry through (LO) D_T , consequence of missing T-reversal \rightarrow unique feature
- ❖ Outlook/Implication: NLO underway (but complicated...),
 \rightarrow calculate 'splitting functions' for polarized Λ fragmentation function

Then: more processes in e^+e^- to be studied ($\Lambda+\pi$ - final state)