Polarized quark TMD fragmentation functions: formalism and MC results

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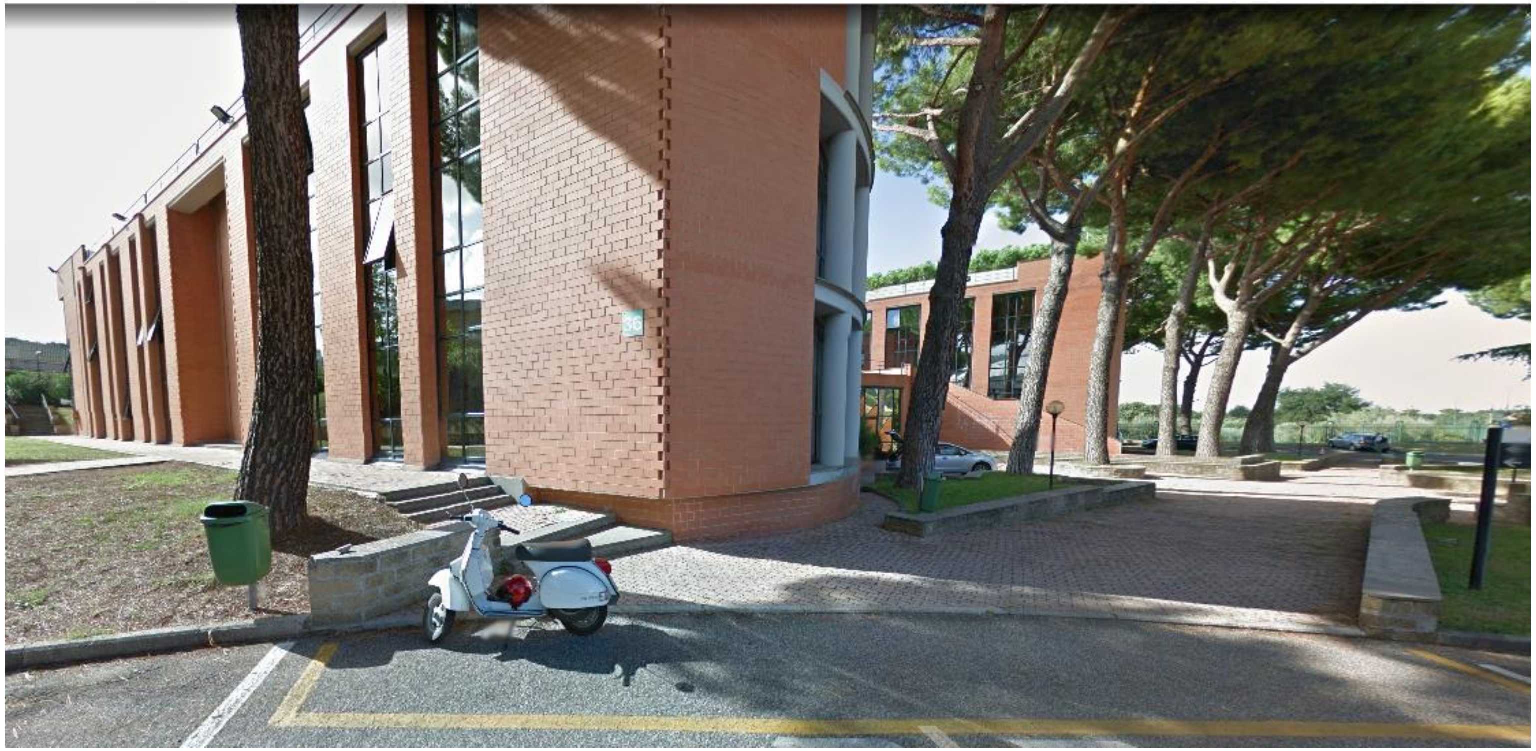
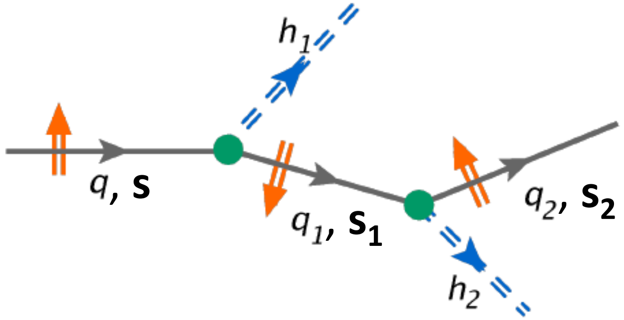


Outlook

- Fragmentation functions discussed in talks by Aschenauer, Avagyan, Ethier, Bressan (Artru *et al* results), Nocera, Radici, Schnell, Vossen… => I'll skip general introduction
- Recursive model for quark fragmentation
  - Collinear Field Feynman model
  - Generalization to spin and transverse momentum (STMD) case
    - Pions production, only quarks are polarized
- Monte Carlo implementation
  - Validation
  - Results for one and two hadron FFs
- Conclusions

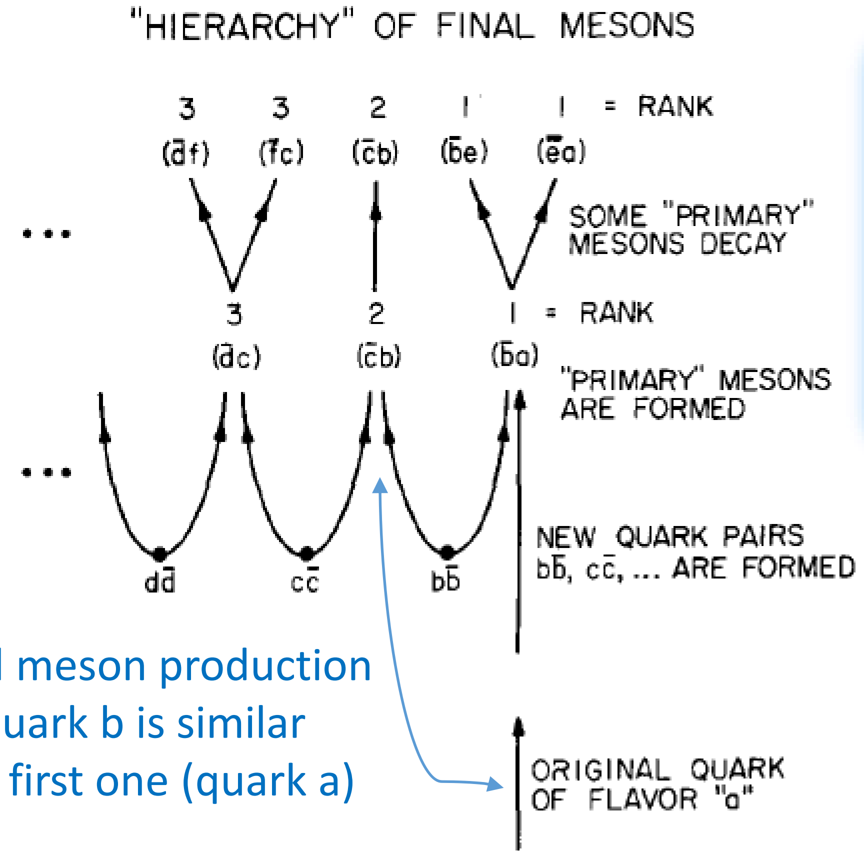
Generalization of recursive mechanism to STMD FFs



- H. Matevosyan, AK, A.W. Thomas & collaborators
  - First MC study with constant spin transfer: Matevosyan, AK, Thomas: **PLB 731(2014)208**
  - Theory framework: include polarization and transverse momentum flow in recursive FF approach: Bentz, AK, Matevosyan, Ninomiya, Thomas, Yazaki: **PR D94, 034004 (2016)**
  - MC implementation and results: validation and examples
    - One hadron production , Matevosyan, AK, Thomas: **PRD 95, 014021 (2017),**
    - Two hadron production, Matevosyan, AK, Thomas:
      - Longitudinally polarized quark, **arXiv:1707.04999, PRD 96, 074010 (2017),**
      - Transversely polarized quark , **arXiv:1709.08643**, submitted to **PRD**

Modeling FFs: Recursive FF model

Field, Feynman PRD 15(1977)2590, NPB 136(1078)1 (**A PARAMETRIZATION OF THE PROPERTIES OF QUARK JETS**)



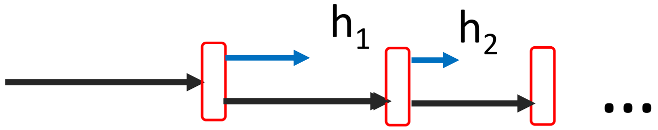
Second meson production by quark b is similar to the first one (quark a)

Fig. 1. Illustration of the "hierarchy" structure of the final mesons produced when a quark of type "a" fragments into hadrons. New quark pairs $b\bar{b}$, $c\bar{c}$, etc., are produced and "primary" mesons are formed. The "primary" meson $\bar{b}a$ that contains the original quark is said to have "rank" one and primary meson $\bar{c}b$ rank two, etc. Finally, some of the primary mesons decay and we assign all the decay products to have the rank of the parent. The order in "hierarchy" is *not* the same as order in momentum or rapidity.

assumed that for very high momenta, all distributions scale so that they depend only on ratios of the hadron momenta to the quark momenta. Given these assumptions, complete knowledge of the structure of a quark jet is determined by one unknown function $f(\eta)$ and three parameters describing flavor, primary meson spin, and transverse momentum to be discussed later. The function $f(\eta)$ is defined by

$$f(\eta)\,d\eta = \text{the probability that the first hierarchy (rank-1) primary meson } \textit{leaves} \text{ the fraction of momentum } \eta \text{ to the remaining cascade,} \quad (2.1)$$

f(η) – elementary q→q' fragmentation or splitting function



Recursive FF: integral equation

Geometric series: $S = 1 + x + x^2 + x^3 + \cdots = 1 + x\left(1 + x + x^2 + \cdots\right) = 1 + xS \;\Rightarrow\; S = 1/(1-x)$

2.2. Single-particle decay distribution F(z)

The above ansatz leads to an obvious and simple Monte Carlo calculation of a jet as well as to a straightforward recursive integral equation. For example, if we define a single-particle distribution in the quark jet as

$F(z)\,\mathrm{d}z$ = the probability of finding any primary meson (independent of hierarchy) with fractional momentum z within $\mathrm{d}z$ in a quark jet, (2.4)

then $F(z)$ must satisfy the following integral equation (take $W_0 = 1$)

$$F(z) = f(1-z) + \int_z^1 f(\eta)\, F(z/\eta)\, \mathrm{d}\eta/\eta\,, \tag{2.5}$$

where the limits are automatic since we define $f(1-z) = 0$ and $F(z) = 0$ for $z > 1$ or $z < 0$. Eq. (2.5) arises because the primary meson might be the first in rank (with probability $f(1-z)\,\mathrm{d}z$) or if not, then the first-rank primary meson has left a momentum fraction η with probability $f(\eta)\,\mathrm{d}\eta$, and in this remaining cascade the probability to find z in $\mathrm{d}z$ is $F(z/\eta)\,\mathrm{d}z/\eta$ by the scaling principle. Dividing out the $\mathrm{d}z$ leaves eq. (2.5).

Only longitudinal scaled momentum flow is taken into account

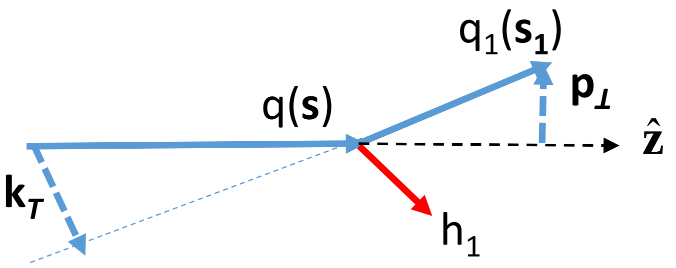
Nucleon 3D partonic structure: Twist-2 STMD qDFs

Nucleon Polarization \ Quark polarization	U	L	T
U	$f_1^q(x, k_T^2)$		$\frac{\epsilon_T^{ij} k_T^{\ j}}{M} h_1^{\perp q}(x, k_T^2)$
L		$S_L g_{1L}^q(x, k_T^2)$	$S_L \frac{\mathbf{k}_T}{M} h_{1L}^{\perp q}(x, k_T^2)$
T	$\frac{\mathbf{k}_T \times \mathbf{S}_T}{M} f_{1T}^{\perp q}(x, k^2)$	$\frac{\mathbf{k}_T \cdot \mathbf{S}_T}{M} g_{1T}^{\perp q}(x, k_T^2)$	$\mathbf{S}_T h_{1T}^q(x, k_T^2) + \frac{\mathbf{k}_T}{M} \frac{(\mathbf{k}_T \cdot \mathbf{S}_T)}{M} h_{1T}^{\perp q}(x, k_T^2)$

All azimuthal dependences are in prefactors. TMDs do not depend on them

Twist-2 quark to quark and pion STMD FFs

Initial quark polarization	Final quark polarization: U	L	T
U	$D(z, p_\perp^2)$		$-\frac{\mathbf{k}_T \times \hat{\mathbf{z}}}{\mathcal{M}} D_T^\perp(z, p_\perp^2)$
L		$s_L G_L(z, p_\perp^2)$	$s_L \frac{\mathbf{k}_T}{\mathcal{M}} G_T(z, p_\perp^2)$
T	$-\frac{(\mathbf{k}_T \times \mathbf{s}_T) \cdot \hat{\mathbf{z}}}{\mathcal{M}} H^\perp(z, p_\perp^2)$	$\frac{\mathbf{k}_T \cdot \mathbf{s}_T}{\mathcal{M}} H_L^\perp(z, p_\perp^2)$	$\mathbf{s}_T H_T(z, p_\perp^2) + \frac{\mathbf{k}_T}{\mathcal{M}} \frac{(\mathbf{k}_T \cdot \mathbf{s}_T)}{\mathcal{M}} H_T^\perp(z, p_\perp^2)$



$$\mathbf{k}_T = -\mathbf{p}_T / z$$

$$z = \frac{p_{q_1}^0 + p_{q_1}^3}{p_q^0 + p_q^3}$$

STMD splitting function (SF) probability distribution

Polarized quark to unpolarized hadron SF distribution

$$F^{q\to h_1}\left(z,\mathbf{p}_\perp;\mathbf{s}\right)=F^{q\to q_1}\left(1-z,-\mathbf{p}_\perp;\mathbf{s}_1=0,\mathbf{s}\right)=D(1-z,\mathbf{p}_\perp^2)+\frac{1}{M}\left(\mathbf{k}_T\times\mathbf{s}_T\right)\cdot\hat{\mathbf{z}}H^\perp(1-z,\mathbf{p}_\perp^2)$$

Polarized quark to polarized quark SF distribution

$$\begin{aligned}F^{q\to q_1}\left(z,\mathbf{p}_\perp;\mathbf{s}_1,\mathbf{s}\right)=&\ D(z,\mathbf{p}_\perp^2)-\frac{1}{M}\left(\mathbf{k}_T\times\mathbf{s}_{1T}\right)\cdot\hat{\mathbf{z}}D_T^\perp(z,\mathbf{p}_\perp^2)\\&+\left(\mathbf{s}_T\cdot\mathbf{s}_{1T}\right)H_T(z,\mathbf{p}_\perp^2)+\frac{1}{M}s_{1L}\left(\mathbf{k}_T\cdot\mathbf{s}_T\right)H_L^\perp(z,\mathbf{p}_\perp^2)\\&+\frac{1}{M^2}\left(\mathbf{s}_{1T}\cdot\mathbf{k}_T\right)\left(\mathbf{s}_T\cdot\mathbf{k}_T\right)H_T^\perp(z,\mathbf{p}_\perp^2)-\frac{1}{M}\left(\mathbf{k}_T\times\mathbf{s}_T\right)\cdot\hat{\mathbf{z}}H^\perp(z,\mathbf{p}_\perp^2)\\&+\left(s_{1L}s_L\right)G_L(z,\mathbf{p}_\perp^2)+\frac{1}{M}s_L\left(\mathbf{s}_{1T}\cdot\mathbf{k}_T\right)G_T(z,\mathbf{p}_\perp^2)\end{aligned}$$

All additional terms related to final quark polarization $\mathbf{s}_1$

Quark polarization after hadron emission

$$F^{q\to q_1}\left(z,\mathbf{p}_\perp;\mathbf{s}_1,\mathbf{s}\right)=\alpha\left(z,\mathbf{p}_\perp;\mathbf{s}\right)+\boldsymbol{\beta}\left(z,\mathbf{p}_\perp;\mathbf{s}\right)\cdot\mathbf{s}_1,\quad \mathbf{s}=\left(\mathbf{s}_T,s_L\right)$$

α and $\boldsymbol{\beta}$ are linear functions of $\mathbf{s}$

$$\mathbf{k}_T^{'}=(-k_y,k_x)$$

$$\alpha\left(z,\mathbf{p}_\perp;\mathbf{s}\right)=D(z,\mathbf{p}_\perp^2)-\frac{1}{M}\left(\mathbf{k}_T\times\mathbf{s}_T\right)\cdot\hat{\mathbf{z}}H^\perp(z,\mathbf{p}_\perp^2)$$

$$\beta_L\left(z,\mathbf{p}_\perp;\mathbf{s}\right)=s_L G_L(z,\mathbf{p}_\perp^2)-\frac{1}{M}\left(\mathbf{k}_T\cdot\mathbf{s}_T\right)H_L^\perp(z,\mathbf{p}_\perp^2)$$

$$\boldsymbol{\beta}_\perp\left(z,\mathbf{p}_\perp;\mathbf{s}\right)=-\frac{\mathbf{k}_T^{'}}{M}D_T^\perp(z,\mathbf{p}_\perp^2)+s_L\frac{\mathbf{k}_T}{M}G_T(z,\mathbf{p}_\perp^2)$$

$$+\mathbf{s}_T H_T(z,\mathbf{p}_\perp^2)+\frac{\mathbf{k}_T}{M^2}\left(\mathbf{s}_T\cdot\mathbf{k}_T\right)H_T^\perp(z,\mathbf{p}_\perp^2)$$

Unpol. SF

Collins effect

Long. pol. transfer

Tran. pol. rotation

Polarizing SF

Long. pol. rotation

Tran. pol. transfer

Pretzelosity like

The final quark spin is completely determined by elementary splitting functions and depends on z, $\mathbf{p}_\perp$ and initial quark polarization **s**

$$\langle\mathbf{s}_1\rangle=\frac{\boldsymbol{\beta}\left(z,\mathbf{p}_\perp;\mathbf{s}\right)}{\alpha\left(z,\mathbf{p}_\perp;\mathbf{s}\right)}$$

Coupled integral equations for single hadron production in STMD quark jet model

Bentz, AK, Matevosyan, Ninomiya, Thomas, Yazaki: **PR D94, 034004 (2016)**

$$D^{(q\to\pi)}(z,p_T^2) = \hat{D}^{(q\to q_1)}(z,p_T^2) + \hat{D}^{(q\to q_1)} \otimes D^{(q_1\to\pi)} + \hat{D}_T^{\perp(q\to q_1)} \otimes H^{\perp(q_1\to\pi)}$$

$$H^{\perp(q\to\pi)}(z,p_T^2) = \hat{H}^{\perp(q\to\pi)}(z,p_T^2) + \hat{H}^{\perp(q\to q_1)} \otimes D^{(q_1\to\pi)}$$

$$+\hat{H}_T^{(q\to q_1)} \otimes H^{\perp(q_1\to\pi)} + \hat{H}_T^{\perp(q\to q_1)} \otimes H^{\perp(q_1\to\pi)}$$

hatted $\hat{D}^{q\to\pi}$ *etc* - elementary SF

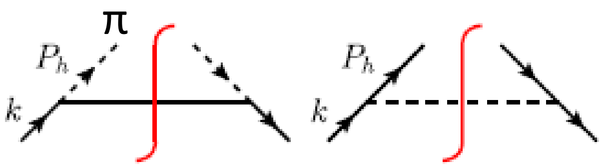
Sum rules for longitudinal and transverse momentum (Schäfer-Teryaev) conservation

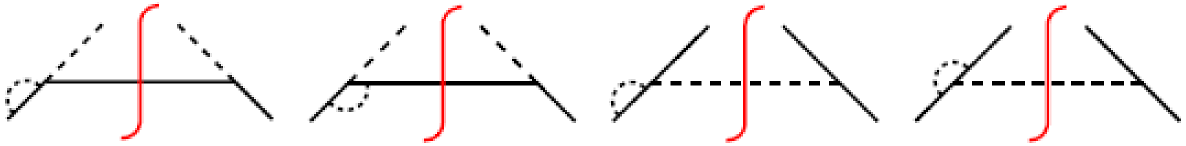
$$\sum_h \gamma_h \int_0^1 \mathrm{d}z\, z \int \mathrm{d}^2 p_\perp\, D^{(q\to h)}(z, \mathrm{p}_\perp^2) = 1\,, \qquad \text{(II.22)}$$

$$\sum_h \gamma_h \int_0^1 \frac{\mathrm{d}z}{2z\,M_h} \int \mathrm{d}^2 p_\perp \cdot \mathrm{p}_\perp^2\, H^{\perp(q\to h)}(z, \mathrm{p}_\perp^2) = 0\,, \qquad \text{(II.23)}$$

where γ_h is the spin degeneracy factor of the hadron and M_h its mass. A similar derivation can be given for the

Spectator model for elementary SFs





T-even

$$\hat{D}(z,p_\perp^2) = C[p_\perp^2 + (1-z)^2M^2],$$

$$\hat{G}_L(z,p_\perp^2) = C[-p_\perp^2 + (1-z)^2M^2],$$

$$\hat{G}_T(z,p_\perp^2) = C[2z(1-z)M^2],$$

$$\hat{H}_T(z,p_\perp^2) = -\hat{D}(z,p_\perp^2),$$

$$\hat{H}_L^\perp(z,p_\perp^2) = \hat{G}_T(z,p_\perp^2),$$

$$\hat{H}_T^\perp(z,p_\perp^2) = C[2z^2M^2],$$

$$C(z,p_\perp^2) = \frac{1-z}{12}\frac{g_\pi^2}{(2\pi)^3}\frac{1}{(p_\perp^2 + M^2(1-z)^2 + zm_\pi^2)^2}.$$

T-even SFs saturate positivity bounds

T-odd: $\hat{\mathrm{H}}^\perp$ and $\hat{\mathrm{D}}_T^\perp$

Positivity bound violation

For MC implementation we use :

$$\hat{D}(z) = 1.1\hat{D}_{\mathrm{tree}}(z)$$

$$\frac{p_\perp}{zM}\frac{\hat{H}^{\perp(q\to h)}(z,p_\perp^2)}{\hat{D}^{(q\to h)}(z,p_\perp^2)} = 0.4\frac{2p_\perp M_Q}{p_\perp^2 + M_Q^2},$$

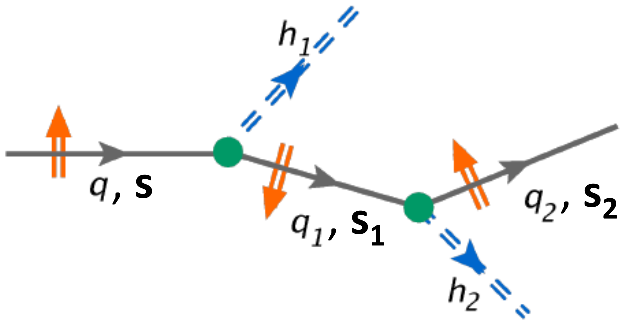
$$\hat{D}^{(q\to h)}(z,p_\perp^2) = \hat{D}^{(q\to q_1)}(1-z,p_\perp^2),$$

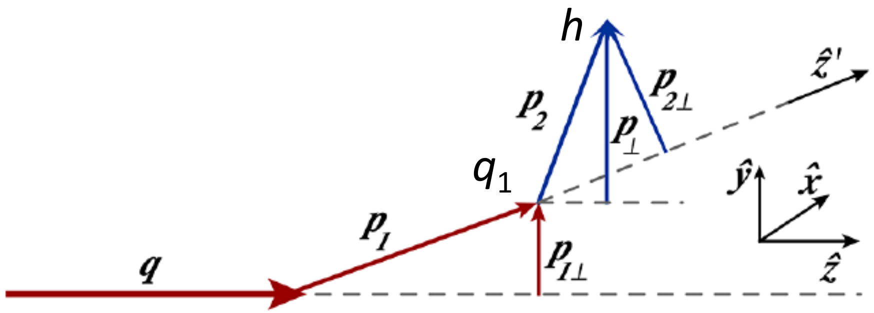
$$\hat{H}^{\perp(q\to h)}(z,p_\perp^2) = -\hat{H}^{\perp(q\to q_1)}(1-z,p_\perp^2),$$

$$\hat{H}^{\perp(q\to q_1)}(z,p_\perp^2) = -\hat{D}_T^{\perp(q\to q_1)}(z,p_\perp^2),$$

Monte Carlo implementation of STMD quark jet model : Validation, two-step process, 1

Matevosyan, AK, Thomas: **PRD 95, 014021 (2017)**





$$z = z_1 z_2,$$

$$\boldsymbol{p}_\perp = z_2 \boldsymbol{p}_{1\perp} + \boldsymbol{p}_{2\perp}$$

$$D^{(2)}_{q\to h}(z, p_\perp^2) = 2\sum_{q_1}\int_0^1 dz_1 \int_0^1 dz_2 \int d^2\boldsymbol{p}_{1\perp}\int d^2\boldsymbol{p}_{2\perp} \times \delta(z - z_1 z_2)\delta^2(\boldsymbol{p}_\perp - z_2\boldsymbol{p}_{1\perp} - \boldsymbol{p}_{2\perp})$$
$$\times\left[\hat{D}^{q\to q_1}(z, p_{1\perp}^2)\hat{D}^{q_1\to h}(z_2, p_{2\perp}^2) + \frac{1}{z\mathcal{M}m_h}(\boldsymbol{p}_{1\perp}\cdot\boldsymbol{p}_{2\perp})\hat{D}_T^{\perp(q\to q_1)}(z_1, p_{1\perp}^2)\hat{H}^{\perp(q_1\to h)}(z_2, p_{2\perp}^2)\right],$$

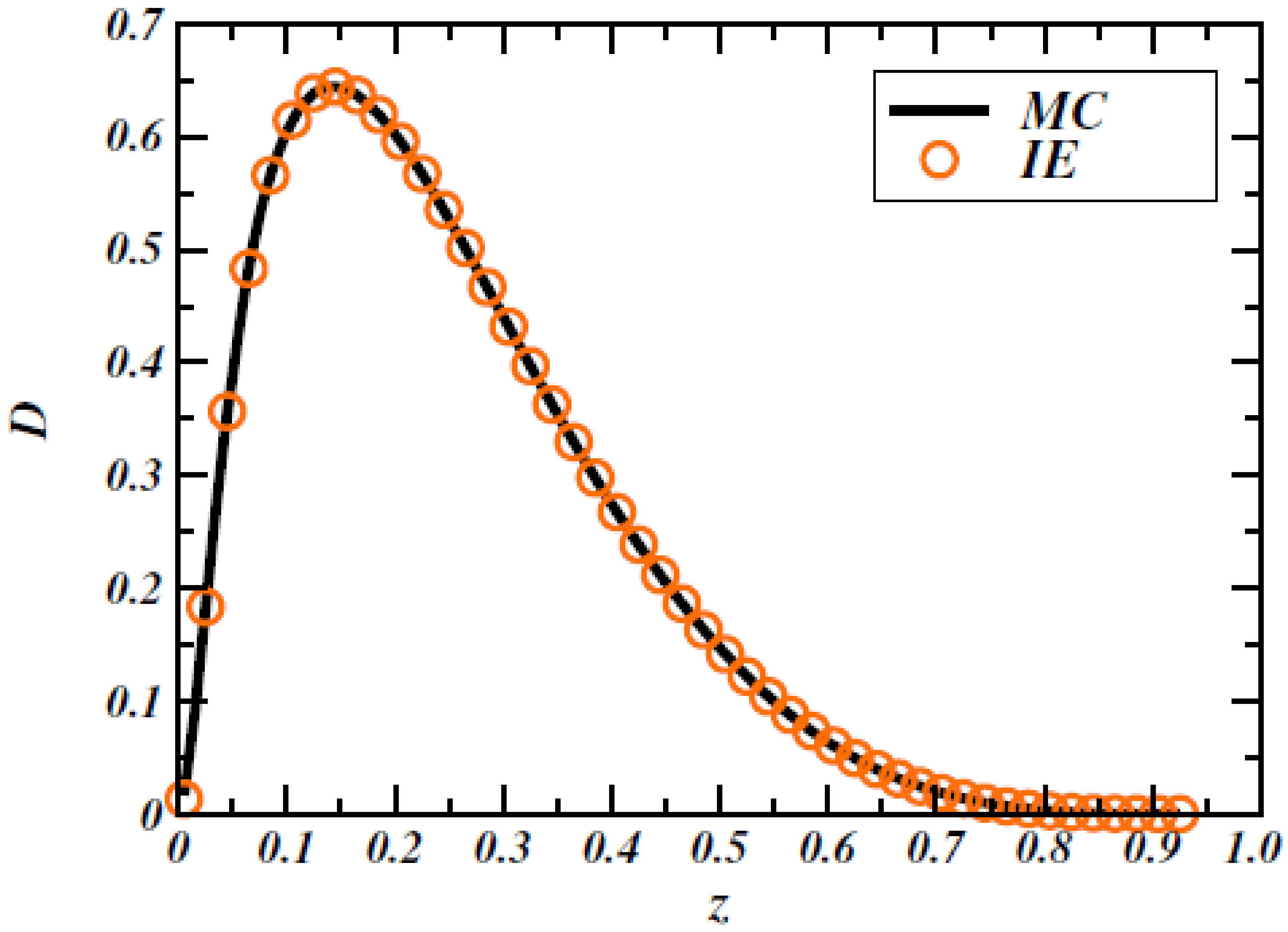
$$H^{\perp(2)}_{q\to h}(z, p_\perp^2) = 2\frac{z m_h}{(\boldsymbol{p}_\perp\times\boldsymbol{s}_T)\cdot\hat{z}}\sum_{q_1}\int_0^1 dz_1\int_0^1 dz_2\int d^2\boldsymbol{p}_{1\perp}\int d^2\boldsymbol{p}_{2\perp}\times\delta(z - z_1 z_2)\delta^2(\boldsymbol{p}_\perp - z_2\boldsymbol{p}_{1\perp} - \boldsymbol{p}_{2\perp})$$
$$\times\left[\frac{1}{z_1\mathcal{M}}(\boldsymbol{p}_{1\perp}\times\boldsymbol{s}_T)\cdot\hat{\boldsymbol{z}}\hat{H}^{\perp(q\to q_1)}(z_1, p_{1\perp}^2)\hat{D}^{(q_1\to h)}(z_2, p_{2\perp}^2)\right.$$
$$\left.+\frac{1}{z_2 m_h}\left(\boldsymbol{p}_{2\perp}\times\left\{\boldsymbol{s}_T\hat{H}_T^{(q\to q_1)}(z_1, p_{1\perp}^2) + \boldsymbol{p}_{1\perp}(\boldsymbol{p}_{1\perp}\cdot\boldsymbol{s}_T)\frac{1}{z_1^2\mathcal{M}^2}\hat{H}_T^{\perp(q\to q_1)}(z_1, p_{1\perp}^2)\right\}\right)\cdot\hat{\boldsymbol{z}}\hat{H}^{\perp(q_1\to h)}(z_2, p_{2\perp}^2)\right]$$

Recoil quark

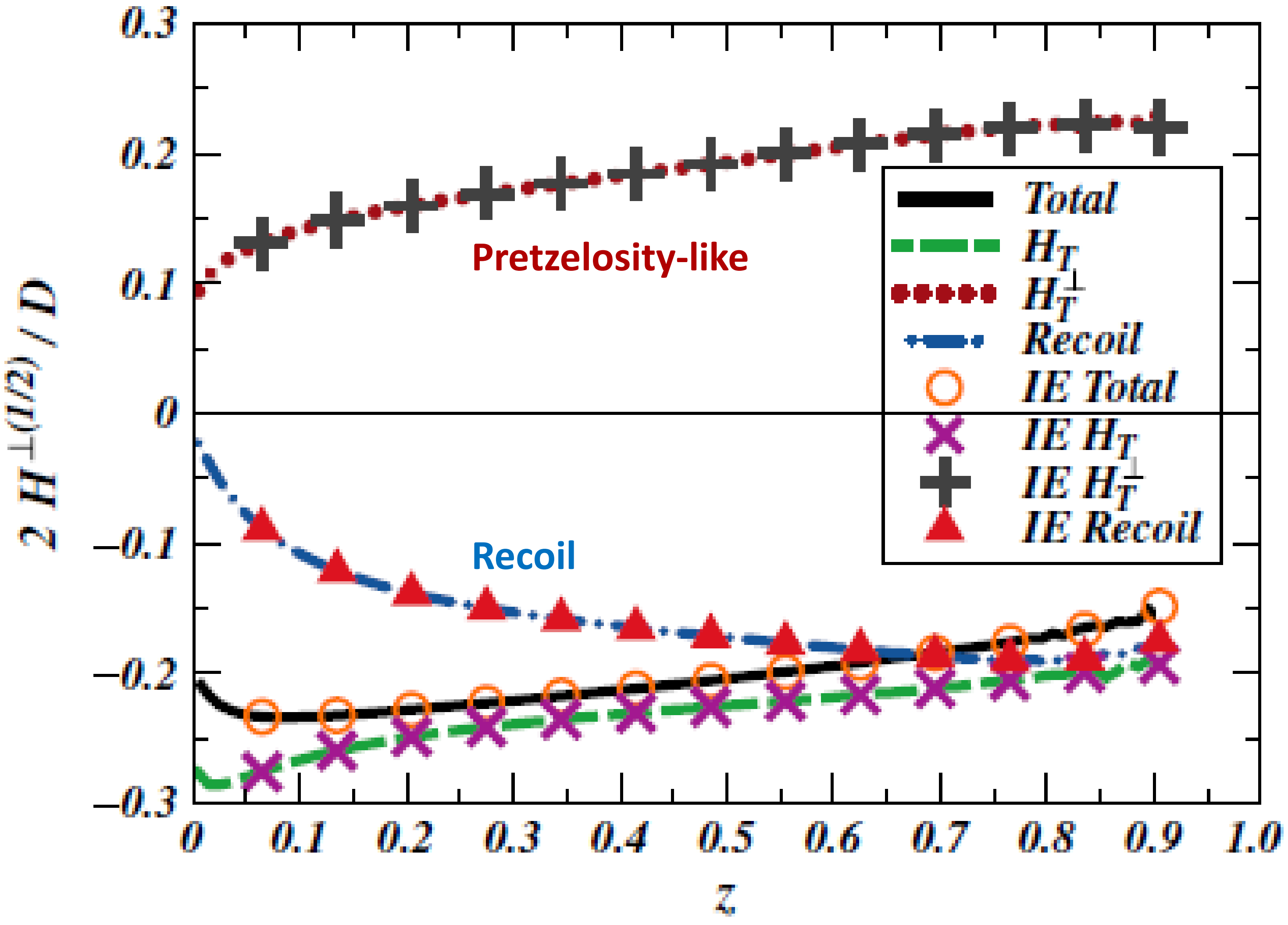
Collins effect

Second hadron

Monte Carlo implementation: Validation, two-step u → π^+, 2.1



Monte Carlo implementation: Validation, two-step $u \to \pi^+$, 2.2



Recoil: $\hat{H}^{\perp(q\to q_1)} \otimes \hat{D}^{(q_1\to h)}$

$\hat{H}_T : \hat{H}_T^{(q\to q_1)} \otimes \hat{H}^{\perp(q\to h)}$

$\hat{H}_T^\perp : \hat{H}_T^{\perp(q\to q_1)} \otimes \hat{H}^{\perp(q\to h)}$

Monte Carlo implementation: Validation, two-step process, 2.3

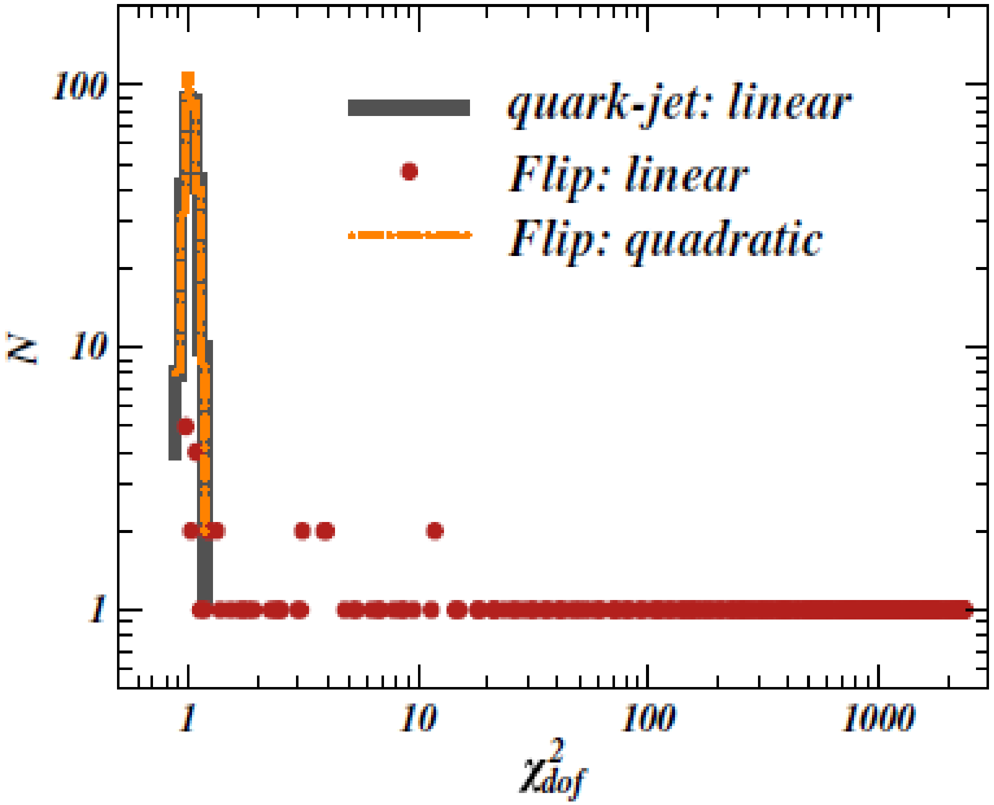


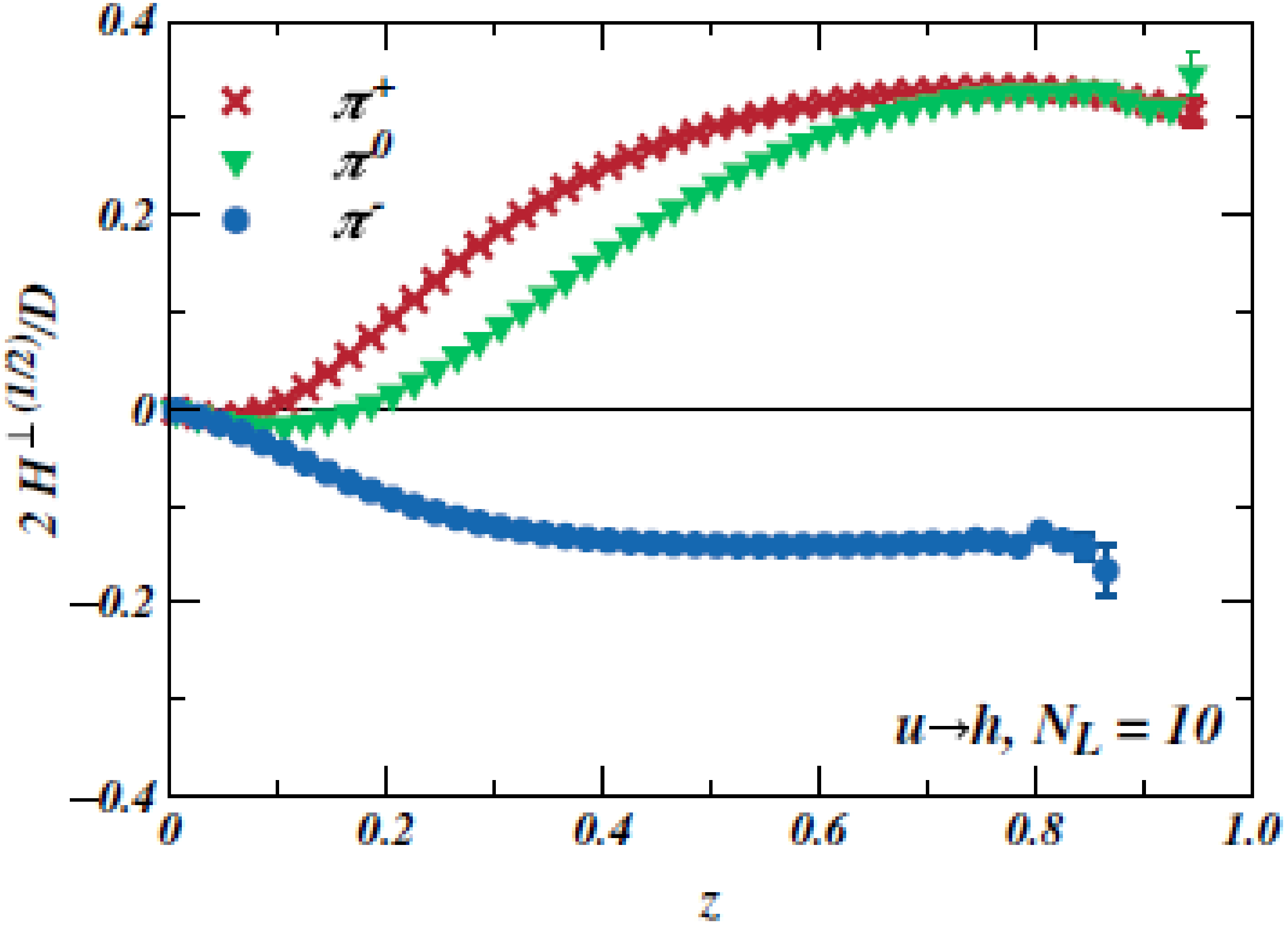
FIG. 4. Histogram of the values of $\chi^2_{\rm dof}$ for fits of all polarized fragmentation functions of the u quark to rank-2 pions, fitted with linear and quadratic polynomials in $\sin(\varphi_C)$ of Eqs. (42) and (43) for MC simulations of rank-2 hadrons. The label "Flip" denotes the simulations where the transverse polarization of the quark is simply flipped after each hadron emission step.

Fit functions

$$P_1(\phi_C) = c_0 + c_1 \sin(\phi_C)$$

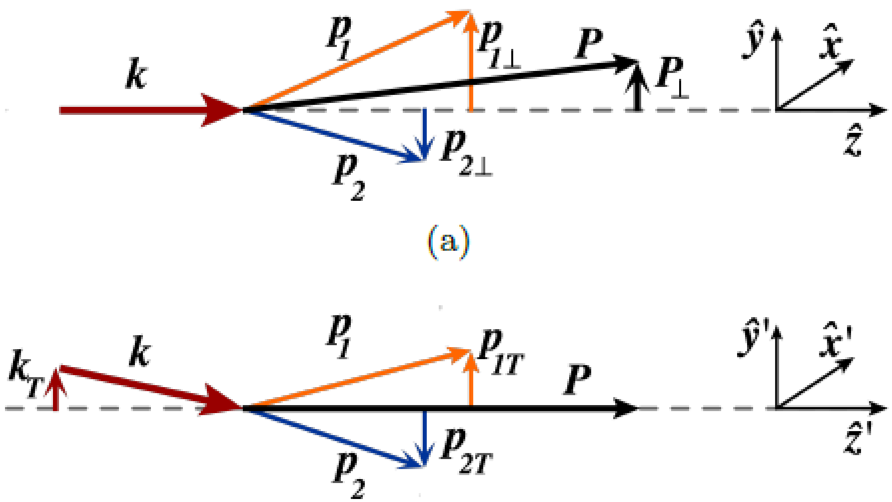
$$P_2(\phi_C) = c_0 + c_1 \sin(\phi_C) + c_2 \sin^2(\phi_C)$$

Results for Collins effect



Favored FF has larger magnitude and opposite sign with respect to unfavored one

Dihadron FFs: definition



$$P \equiv P_h = P_1 + P_2,$$

$$R = \frac{1}{2}(P_1 - P_2),$$

$$z = z_1 + z_2,$$

$$\xi = \frac{z_1}{z} = 1 - \frac{z_2}{z}$$

$$z_i = P_i^- / k^-$$

$$\boldsymbol{P}_{1T} = \boldsymbol{P}_{1\perp} + z_1 \boldsymbol{k}_T,$$

$$\boldsymbol{P}_{2T} = \boldsymbol{P}_{2\perp} + z_2 \boldsymbol{k}_T .$$

$$\boldsymbol{k}_T = -\frac{\boldsymbol{P}_\perp}{z},$$

$$\boldsymbol{R}_T = \frac{z_2 \boldsymbol{P}_{1\perp} - z_1 \boldsymbol{P}_{2\perp}}{z} = (1-\xi)\boldsymbol{P}_{1\perp} - \xi \boldsymbol{P}_{2\perp}.$$

$$\Delta_{ij}(k; P_1, P_2) = \sum_X \int d^4\zeta e^{ik\cdot\zeta} \langle 0|\psi_i(\zeta)|P_1 P_2, X\rangle \langle P_1 P_2, X|\bar{\psi}_j(0)|0\rangle.$$

$$\Delta^\Gamma(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T) = \frac{1}{4z} \int dk^+ \mathrm{Tr}[\Gamma \Delta(k, P_1, P_2)]|_{k^- = P_h^-/z}.$$

$$\Delta^{[\gamma^-]} = D_1(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T),$$

$$\Delta^{[\gamma^-\gamma_5]} = \frac{\epsilon_T^{ij} R_{Ti} k_{Tj}}{M_1 M_2} G_1^\perp(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T),$$

$$\Delta^{[i\sigma^{i-}\gamma_5]} = \frac{\epsilon_T^{ij} R_{Tj}}{M_1 + M_2} H_1^\sphericalangle(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T) + \frac{\epsilon_T^{ij} k_{Tj}}{M_1 + M_2} H_1^\perp(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

Number density distribution

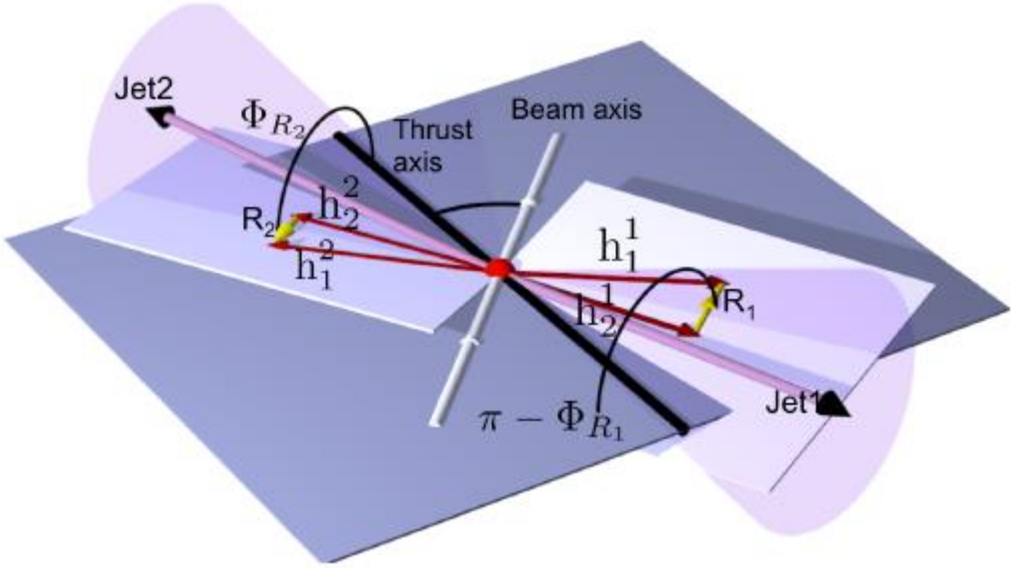
q pol.	U	L	T
DiFF	D_1	$G_1^{\perp}$	$H_1^{\sphericalangle}, H_1^{\perp}$

Longitudinal handedness

$$
\begin{aligned}
F\left(z,\xi,\mathbf{k}_T,\mathbf{R}_T;s\right) &= D_1\left(z,\xi,k_T^2,R_T^2,\cos\left(\varphi_{RK}\right)\right) \\
&-s_L \frac{R_T k_T \sin\left(\varphi_{RK}\right)}{M_1 M_2} G_1^{\perp}\left(z,\xi,k_T^2,R_T^2,\cos\left(\varphi_{RK}\right)\right) \\
&+s_T \frac{R_T \sin\left(\varphi_R-\varphi_S\right)}{M_1+M_2} H_1^{\sphericalangle}\left(z,\xi,k_T^2,R_T^2,\cos\left(\varphi_{RK}\right)\right) \\
&+s_T \frac{k_T \sin\left(\varphi_k-\varphi_S\right)}{M_1+M_2} H_1^{\perp}\left(z,\xi,k_T^2,R_T^2,\cos\left(\varphi_{RK}\right)\right)
\end{aligned}
$$

$$\cos\left(\varphi_{RK}\right) \doteq \cos\left(\varphi_R-\varphi_k\right)$$

Dihadron production from longitudinally polarized quark



Theory: Boer, Jakob, Radici, PR D **67**, 094003 (2003)

Exp: BELLE: arXiv:1505.08020v1 [hep-ex] 29 May 2015

$$\langle \cos(2(\Phi_{R_1'} - \Phi_{R_2'})) \rangle \propto \sum_{q,\bar{q}} e_q^2 G_1^{\perp,q}(z, M^2) G_1^{\perp,\bar{q}}(\bar{z}, \bar{M}^2)$$

$$D_1(z, M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T \, D_1(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T),$$

$$G_1^{\perp}(z, M_h^2) = \int d\xi \int d\varphi_R \int d^2\boldsymbol{k}_T (\boldsymbol{k}_T \cdot \boldsymbol{R}_T) \, G_1^{\perp}(z, \xi, \boldsymbol{k}_T^2, \boldsymbol{R}_T^2, \boldsymbol{k}_T \cdot \boldsymbol{R}_T)$$

Fourier series (FS) c_1

$$G_1^{\perp}\left(z, \xi, k_T^2, R_T^2 \cos\left(\phi_R - \phi_k\right)\right) =$$
$$= c_0 + c_1 \cos\left(\phi_R - \phi_k\right) + c_2 \cos 2\left(\phi_R - \phi_k\right) + \cdots$$

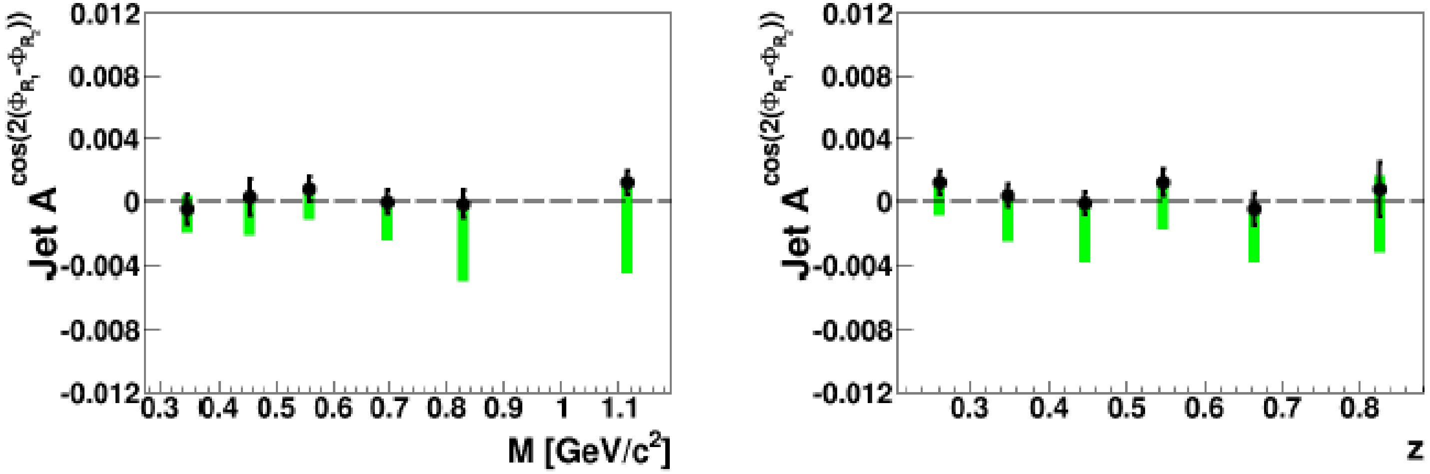
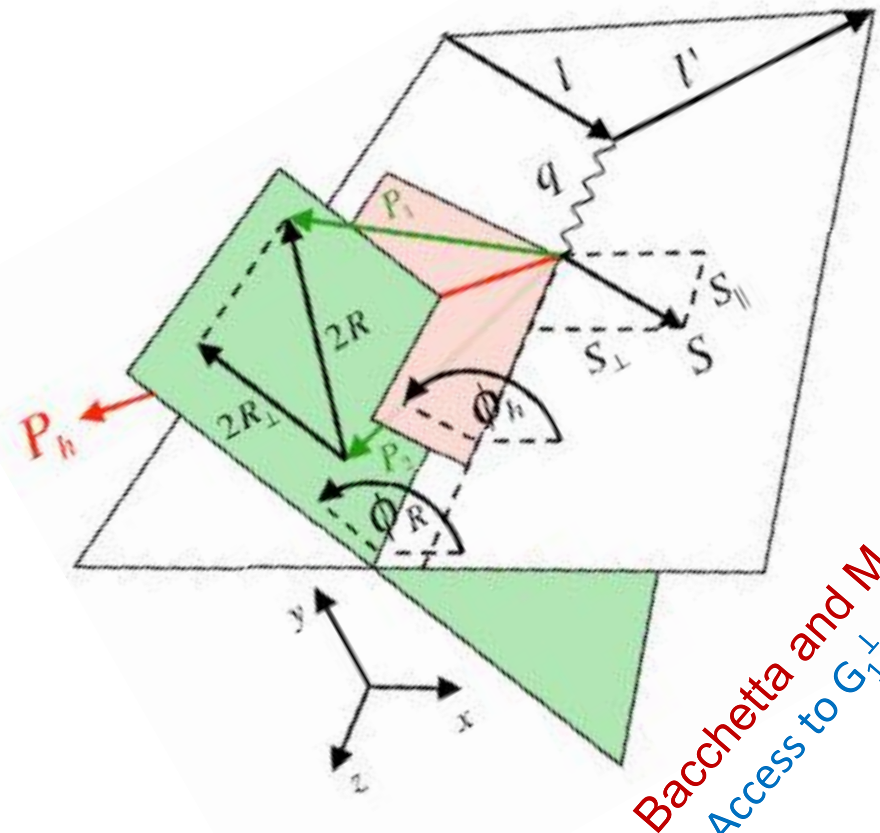


FIG. 2: Results for $A^{\cos(2(\Phi_{R_1}-\Phi_{R_2}))}$ binned in M and z. The black error bars are statistical and the green bands show the systematic uncertainty.

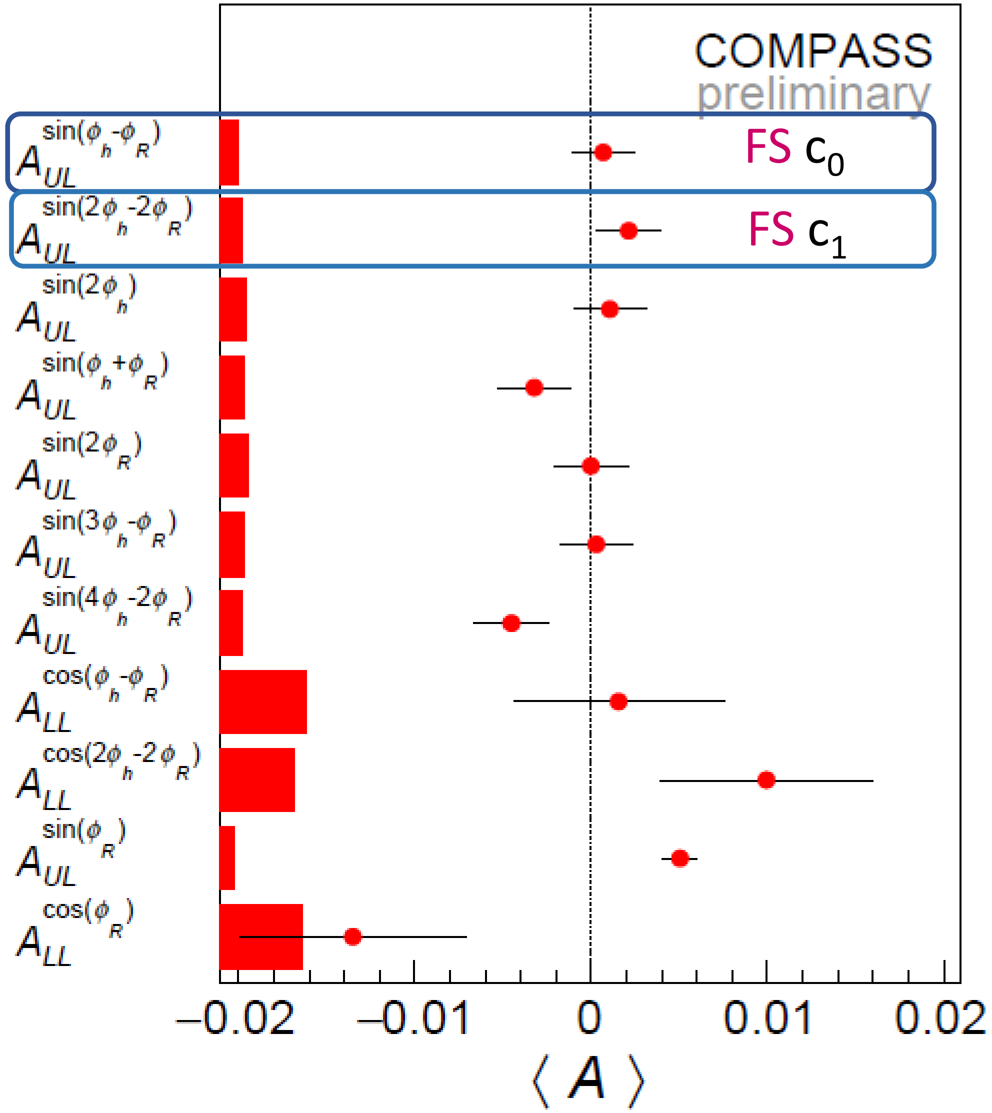
COMPASS preliminary results: longitudinal polarized target

COMPASS (preliminary): arXiv:1702.07317v1 [hep-ex] 23 Feb 2017



S. Gliske, A. Bacchetta and M. Radici: PR D 90, 114027 (2014)

Access to $G_1^\perp$ spherical harmonics



Longitudinally polarized quark fragmentation to two hadrons
Extracting D_1 and $G_1^{\perp}$ from number densities

Matevosyan, AK, Thomas: arXiv:1707.04999, **PRD 96, 074010 (2017)**

Number density: $F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;s_L) = D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\cos(\varphi_{RK})) - s_L \frac{R_T k_T \sin(\varphi_{RK})}{M_1 M_2} G_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\cos(\varphi_{RK}))$

As an example we consider M^2 integrated Dihadron FFs

$$D_1(z) = \int d\xi \int d^2\boldsymbol{R}_T \int d^2\boldsymbol{k}_T \times D_1(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T),$$

$$G_1^{\perp}(z) = \int d\xi \int d^2\boldsymbol{R}_T \int d^2\boldsymbol{k}_T \times (\boldsymbol{k}_T\cdot\boldsymbol{R}_T)\ G_1^{\perp}(z,\xi,\boldsymbol{k}_T^2,\boldsymbol{R}_T^2,\boldsymbol{k}_T\cdot\boldsymbol{R}_T)$$

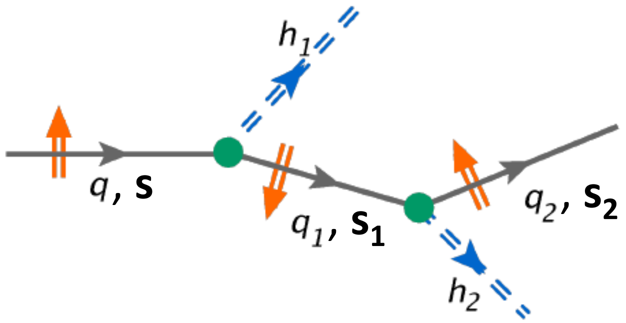
In terms of number density

$$D_1(z) = \int d\xi \int d^2\boldsymbol{R}_T \int d^2\boldsymbol{k}_T \times F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;s_L),$$

$$G^{\perp}(z) = -\frac{M_1 M_2}{s_L} \int d\xi \int d^2\boldsymbol{R}_T \int d^2\boldsymbol{k}_T \times \boxed{\cot(\varphi_{RK})} F(z,\xi,\boldsymbol{k}_T,\boldsymbol{R}_T;s_L)$$

Calculate number density F from 10^{12} events generated using extended quark jet model

Validation: two step (N_L=2) u → $\pi^+ \pi^-$ case



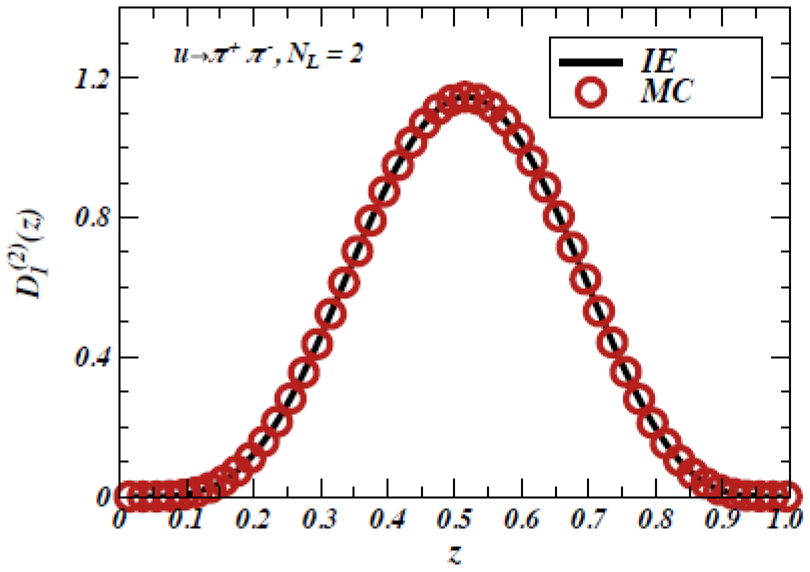
$$D_1^{(2)}(z) \propto \hat{D}^{q\to q_1} \otimes \hat{D}^{q_1 \to h_2}$$

$$G_1^{\perp(2)}(z) \propto \hat{G}_T^{q\to q_1} \otimes \hat{H}^{\perp q_1 \to qh}$$

quark long. pol. rotation

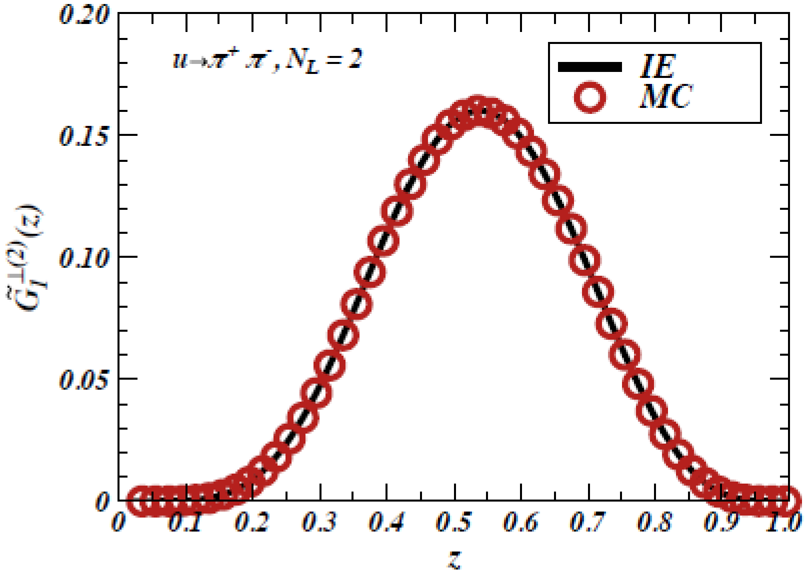
Collins effect

Twist-2 STMD one-hadron splitting functions generates nonzero quark longitudinal polarization dependent FF without interference effects among produced hadrons

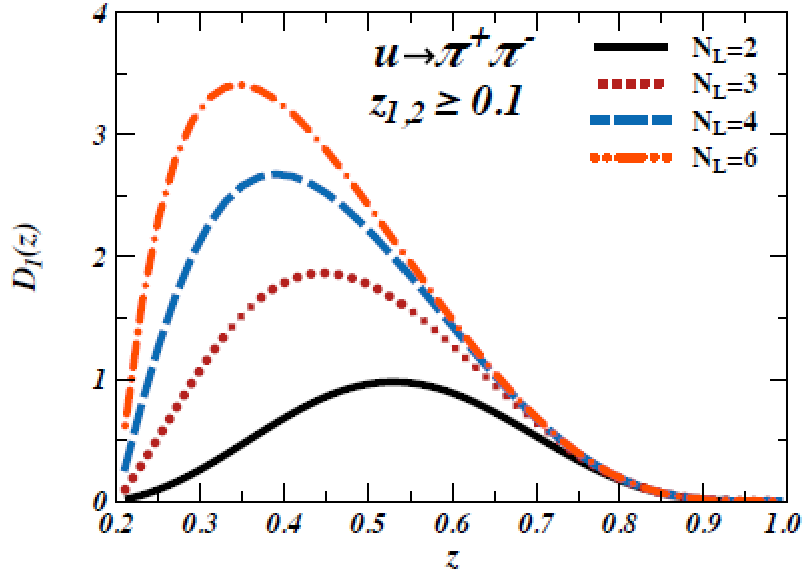


(a)

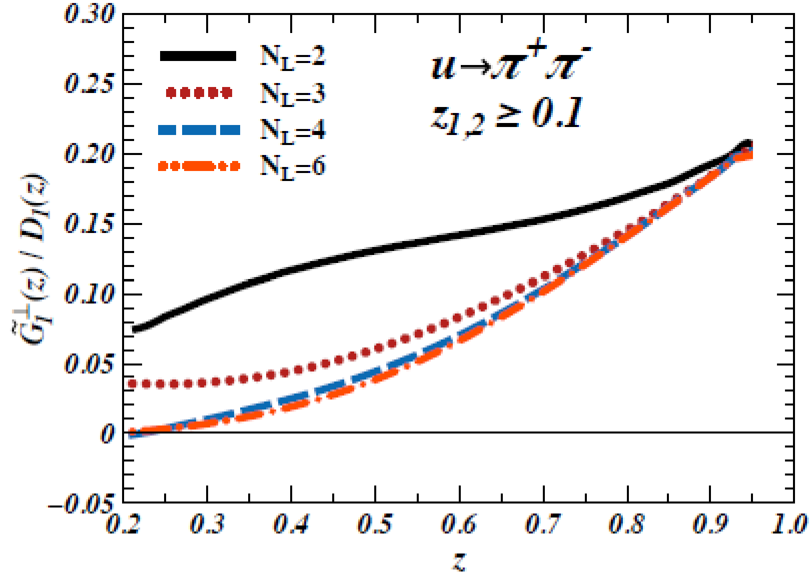
$$\tilde{G}_1^{\perp}(z) \equiv \frac{1}{M_1 M_2} G_1^{\perp}(z)$$



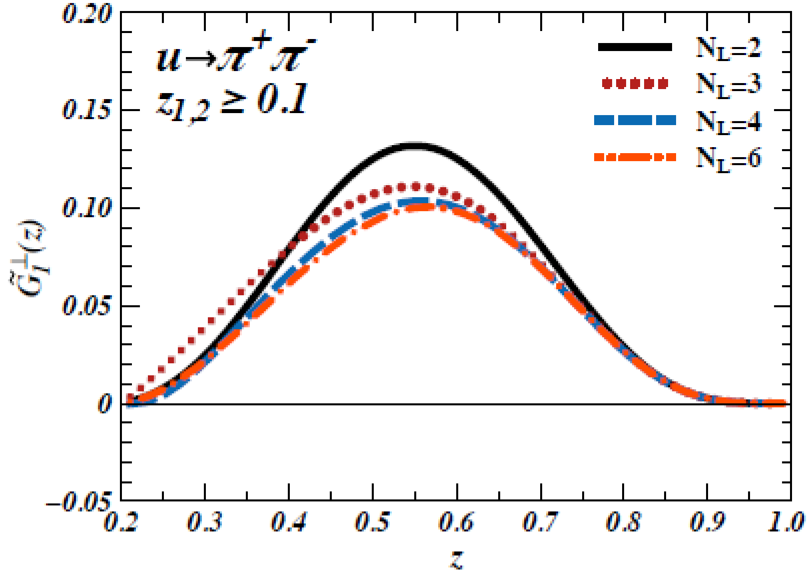
Results for $G_1^{\perp}$ FFs

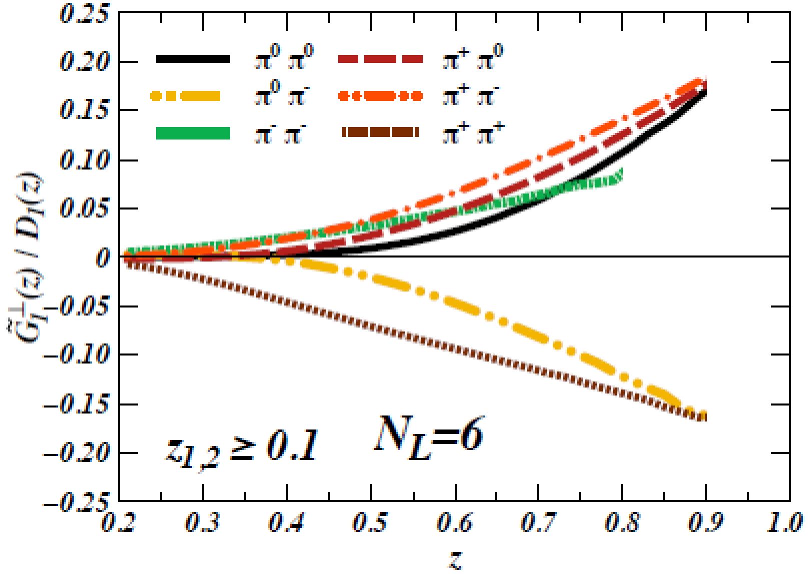


(a)



Fast saturation when increasing number of produced pions





Small (~2–3%) analyzing power for z about 0.4–0.5.

Analyzing power of Collins effect is about 20% using the same model for SFs

Dihadron production from transversely polarized quark

$$F\left(z,\xi,\mathbf{k}_T,\mathbf{R}_T;s\right)=D_1\left(z,\xi,\mathrm{k}_T^2,R_T^2,\cos\left(\varphi_{RK}\right)\right)$$

$$-s_L\frac{R_T k_T\sin\left(\varphi_{RK}\right)}{M_1M_2}G_1^{\perp}\left(z,\xi,\mathrm{k}_T^2,R_T^2,\cos\left(\varphi_{RK}\right)\right)$$

$$+s_T\frac{R_T\sin\left(\varphi_R-\varphi_S\right)}{M_1+M_2}H_1^{\sphericalangle}\left(z,\xi,\mathrm{k}_T^2,R_T^2,\cos\left(\varphi_{RK}\right)\right)$$

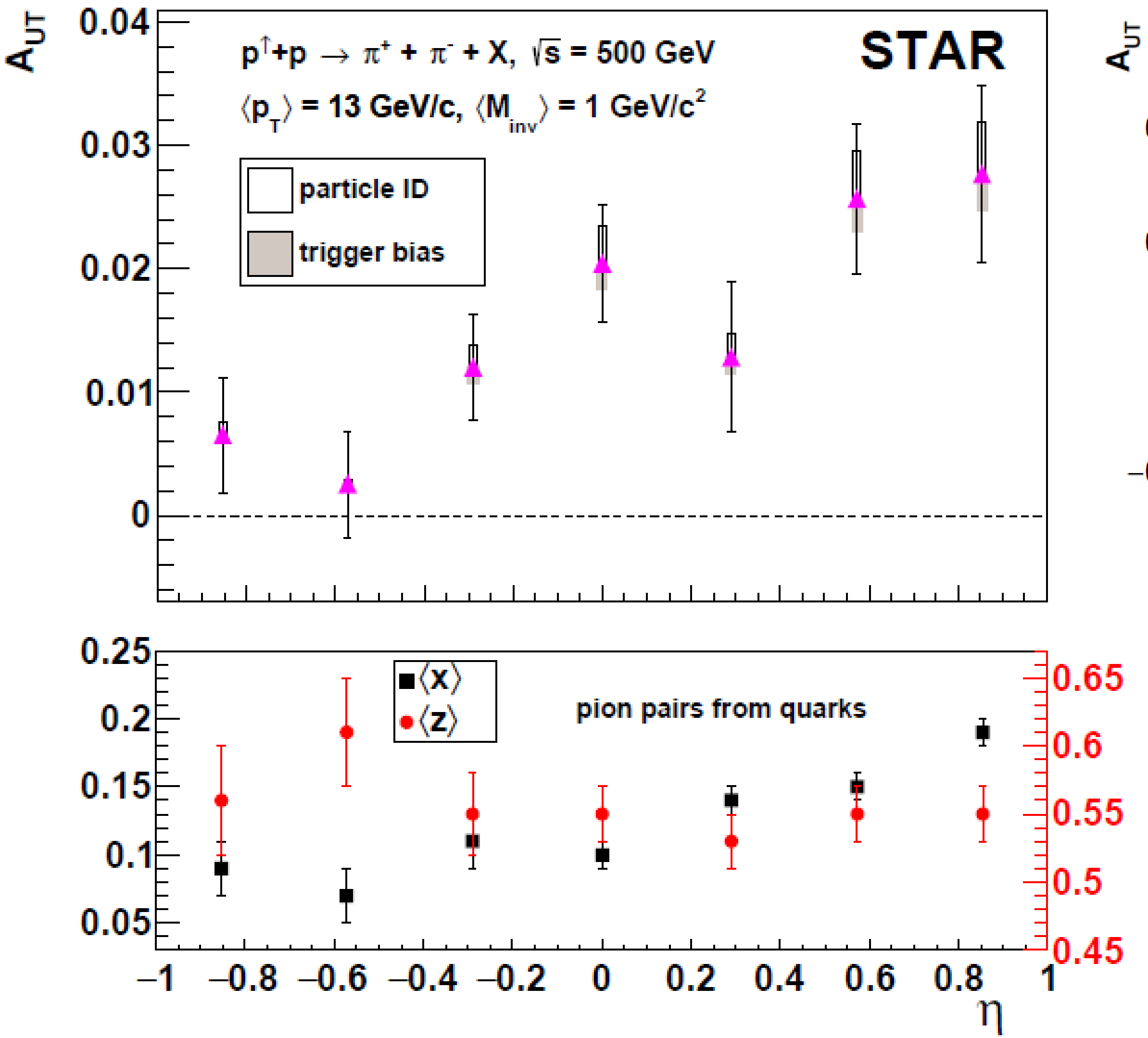
$$+s_T\frac{k_T\sin\left(\varphi_k-\varphi_S\right)}{M_1+M_2}H_1^{\perp}\left(z,\xi,\mathrm{k}_T^2,R_T^2,\cos\left(\varphi_{RK}\right)\right)$$

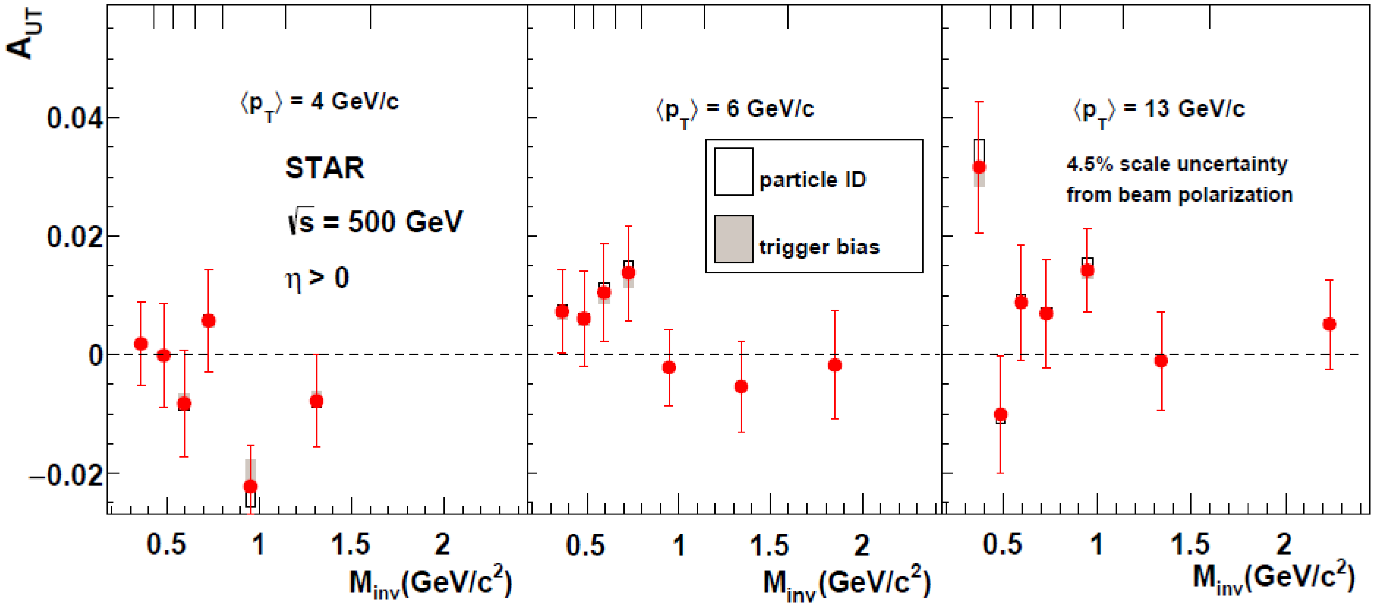
HERMES
COMPASS
BELLE
} **Well known, used for nucleon transversity extraction**

STAR (new)

STAR, arXiv:1710.10215v1 [hep-ex] 27 Oct 2017

Theory: Radici, Ricci, Bacchetta, Mukherjee, **Phys. Rev. D94 (3) (2016) 034012.**





The same-charge, momentum-ordered $(|\vec{p}_{h,1}| > |\vec{p}_{h,2}|)$

Same charge pairs

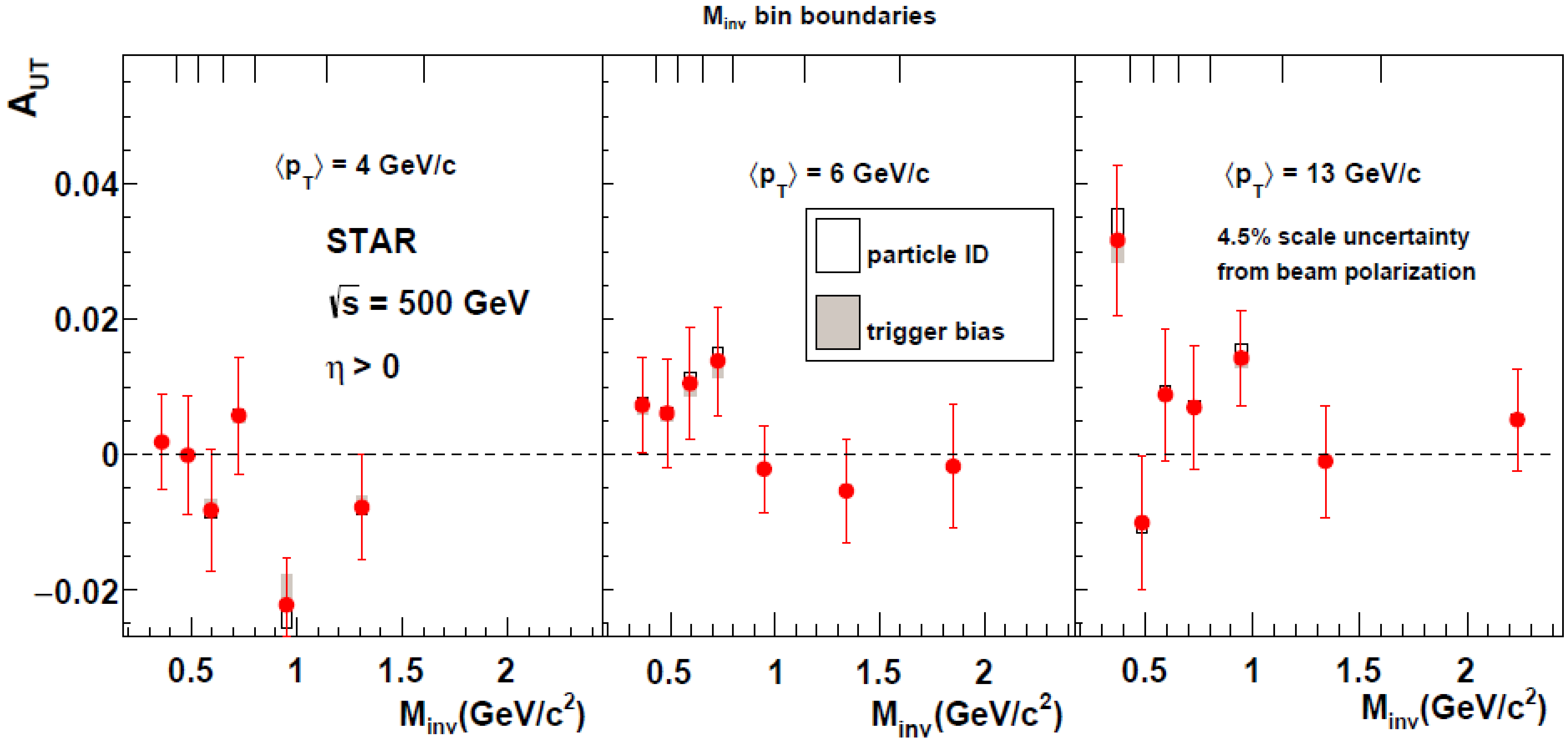


Figure 5: The same-charge, momentum-ordered ($|\vec{p}_{h,1}| > |\vec{p}_{h,2}|$) asymmetry A_{UT} as a function of M_{inv} for the lowest p_T bin, mid-p_T bin, and the highest p_T bin used in Fig. 4. Statistical uncertainties are represented by the error bars, the open rectangles are the systematic uncertainties originating from the particle identification, and the solid one represent the trigger bias systematic uncertainties. The M_{inv} bin boundaries are shown at the top of the figure.

Combinations entering in the cross sections

$$H_1^{\sphericalangle,SIDIS}\left(z,M_h^2\right) \doteq H_1^{\sphericalangle,[0]}\left(z,M_h^2\right) + H_1^{\perp,[1]}\left(z,M_h^2\right)$$

$$H_1^{\perp,SIDIS}\left(z,M_h^2\right) \doteq H_1^{\perp,[0]}\left(z,M_h^2\right) + H_1^{\sphericalangle,[1]}\left(z,M_h^2\right)$$

Fourier n-th cosine moment of integrated FFs are defined as:

$$F^{[n]}\left(z,M_h^2\right) \doteq \int d\xi d\varphi_R d^2k_T \cos(n\varphi_{RK})F\left(z,M_h^2,k_T^2,R_T^2,\mathbf{k}_T\cdot\mathbf{R}_T\right)$$

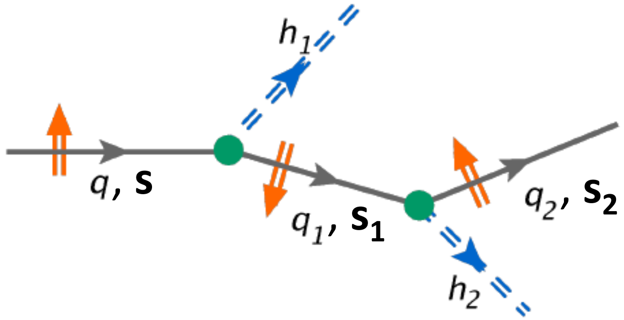
$$D_1(z) \equiv \int dM_h^2\; D_1(z,M_h^2),$$

$$\tilde{G}_1^{\perp}(z) \equiv \frac{1}{M_1M_2}\int dM_h^2\; G_1^{\perp}(z,M_h^2)),$$

$$\tilde{H}_1^{\sphericalangle}(z) \equiv \frac{1}{M_1+M_2}\int dM_h^2\; H_1^{\sphericalangle}(z,M_h^2),$$

$$\tilde{H}_1^{\perp}(z) \equiv \frac{1}{M_1+M_2}\int dM_h^2\; H_1^{\perp}(z,M_h^2).$$

Validation for $q \rightarrow h_1 + q_1 \rightarrow h_1 + h_2 + q_2$

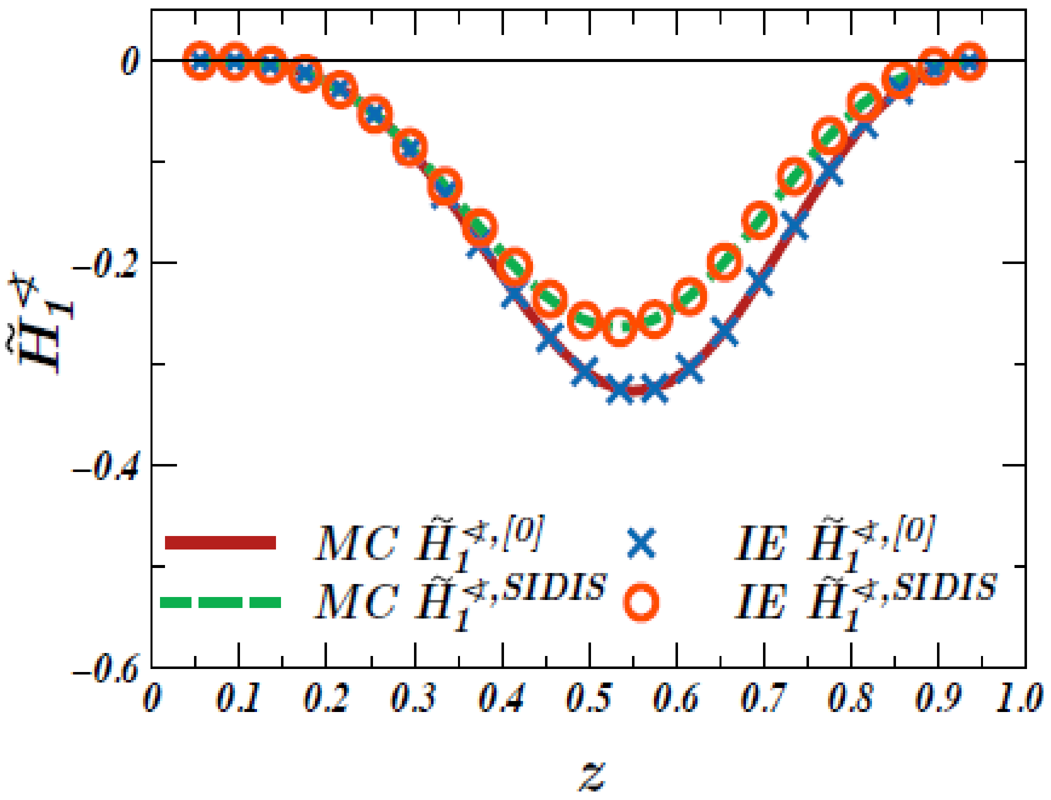


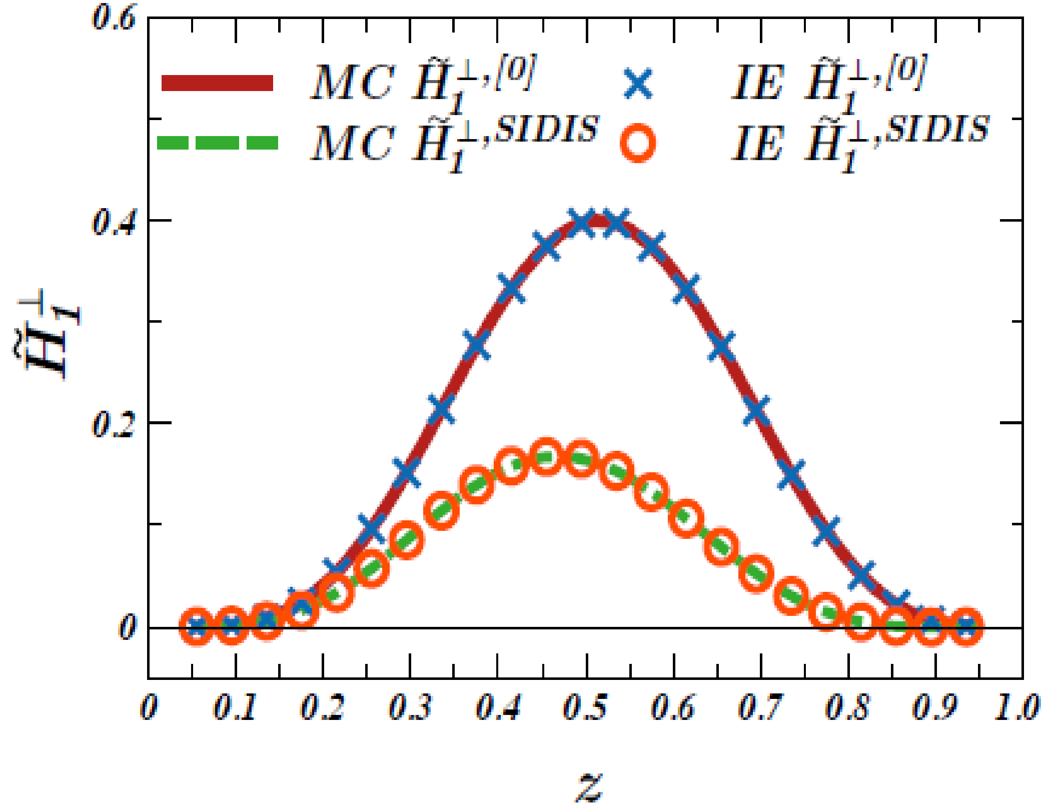
$$H_1^{\sphericalangle(2)} \propto \hat{H}^{\perp,q\to q_1} \otimes \hat{D}^{q_1\to h_2} + \hat{H}_T^{q\to q_1} \otimes \hat{H}^{\perp,q_1\to h_2} + \hat{H}_T^{\perp,q\to q_1} \otimes \hat{H}^{\perp,q_1\to h_2}$$

Collins effect for first splitting

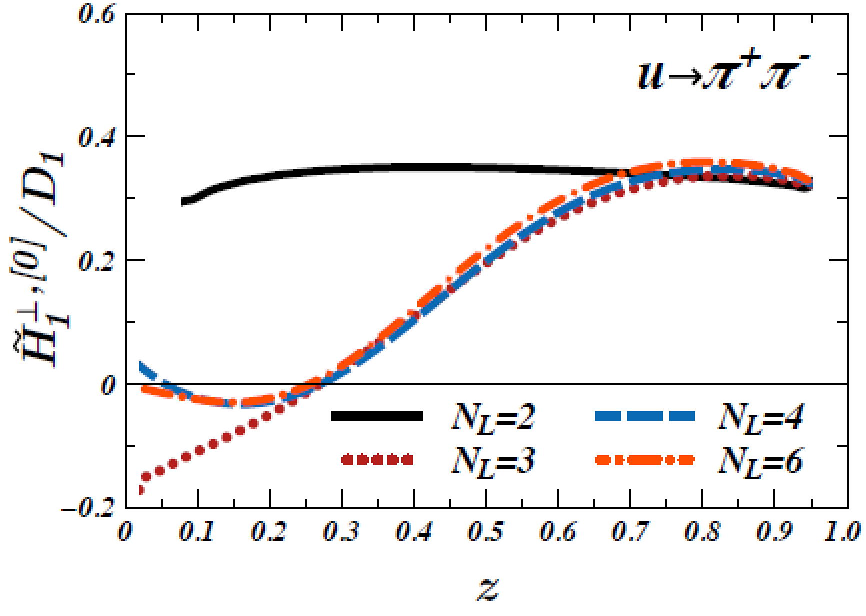
tran. spin transfer

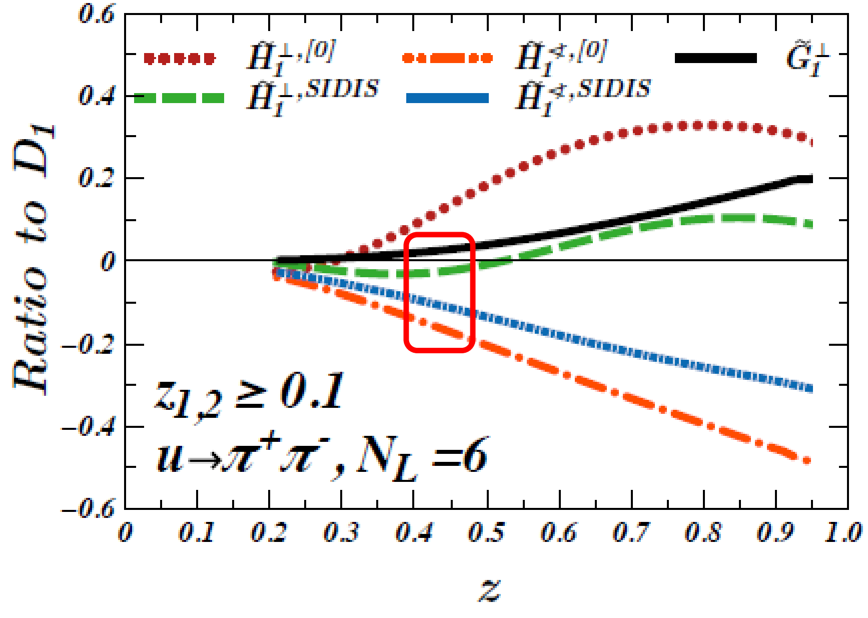
Collins effect for second splitting

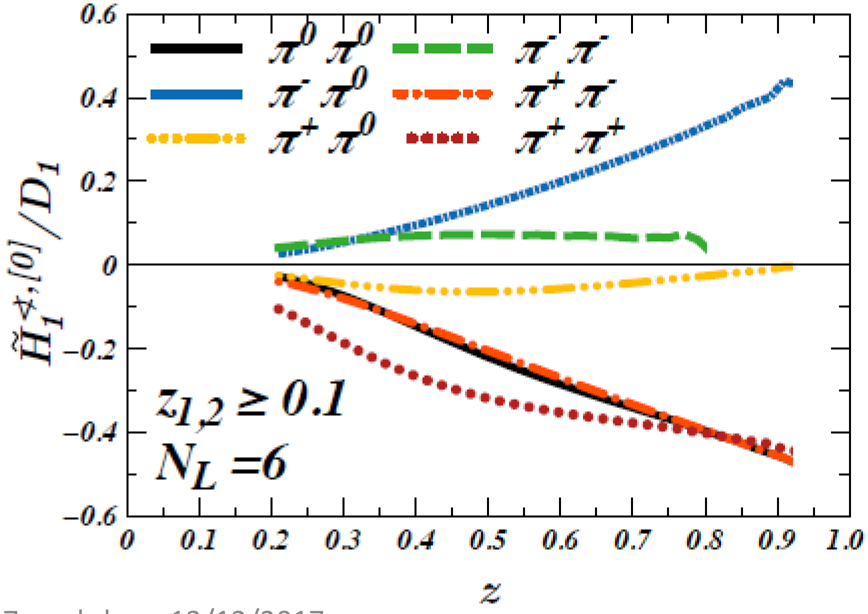


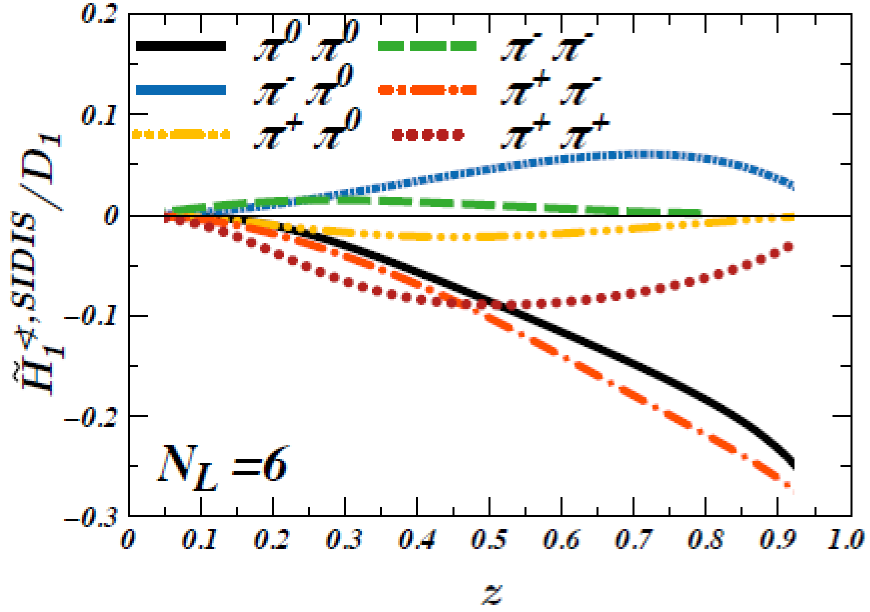


Results for zero and first Fourier moments









Comments on the abstract of my talk

We also note that in general, different combinations of the fully unintegrated transverse polarization-dependent DiFFs enter into the azimuthal asymmetries for the dihadron production in semi-inclusive electron-positron annihilation and lepton nucleon scattering. This might potentially have significant implications for the phenomenological extractions of the transversity via interference fragmentation functions. Our model predicts a factor of two difference between the two definitions. (Nov. 13 2017)

The situation has since changed. We have found sign mistakes in the original derivation of back-to-back dihadron production cross section in e^+e^- annihilation.

The corrected expression contains the same DiFF (IFF) as in SIDIS (and pp):

$$H_1^{\sphericalangle,SIDIS}\left(z,M_h^2\right) \doteq H_1^{\sphericalangle,[0]}\left(z,M_h^2\right) + H_1^{\perp,[1]}\left(z,M_h^2\right)$$

This means that there is no problem in transversity extraction using IFF.

Second observation concerns the "longitudinal handedness asymmetry" (access to $G_1^{\perp}$):

$$\left\langle \cos\left(2\left(\Phi_{R_1'} - \Phi_{R_2'}\right)\right)\right\rangle \equiv 0 \text{ at twist two}$$

Confirmed by zero results of BELLE measurement !!!

Matevosyan, Bacchetta, Courtoy, AK, Radici, Thomas, *article in preparation*

Conclusions

- Complete framework for polarized quark fragmentation has been developed
- MC implementation and extraction of STMD FFs:
  - Validation
  - Results for one and two hadron production
    - Collins effect in one-hadron production
    - Longitudinally polarized quark fragmentation for two-hadron production
    - Transversely polarized quark fragmentation for two-hadron production
      - Nonzero results without interference between two produced hadrons !
    - Nonzero Collins splitting function plays a key role!
  - For the moment only light quarks and pions are included in MC
  - Generalization for s quark fragmentation and kaon production is straightforward
- To do
  - Include vector meson production and strong decay for polarized case
    - Already done for unpolarized TMD FFs also taking into account vector meson production, Matevosyan *et al*
  - Develop MC framework for STMD case for full hadronization of quark-antiquark and quark-diquark system to describe back-to-back production asymmetries in e^+e^- annihilation and target fragmentation in SIDIS