JAM extractions of quark helicity and transversity distributions

Jacob Ethier

on behalf of Jefferson Lab Angular Momentum (JAM) collaboration
Transversity Workshop
December 12th, 2017
Proton spin structure from DIS

- Measured via longitudinal and transverse double spin asymmetries

\[ A_{||} = \frac{\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow}} = D (A_1 + \eta A_2) \quad A_{\perp} = \frac{\sigma^{\uparrow\Rightarrow} - \sigma^{\uparrow\Leftarrow}}{\sigma^{\uparrow\Rightarrow} + \sigma^{\uparrow\Leftarrow}} = d (A_2 + \zeta A_1) \]

\[ \Gamma \] Virtual photoproduction asymmetries: 
\[ A_1 = \frac{(g_1 - \gamma^2 g_2)}{F_1} \quad A_2 = \gamma \frac{(g_1 + g_2)}{F_1} \quad \gamma^2 = \frac{4M^2x^2}{Q^2} \]
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A_{\perp} = \frac{\sigma^{\uparrow\Rightarrow} - \sigma^{\uparrow\leftarrow}}{\sigma^{\uparrow\Rightarrow} + \sigma^{\uparrow\leftarrow}} = d \left( A_2 + \zeta A_1 \right)
\]

- Virtual photoproduction asymmetries: 
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  \( \gamma^2 = \frac{4M^2 x^2}{Q^2} \)

- First moment of polarized structure function \( g_1 \):

\[
\int_0^1 dx g_1^p(x, Q^2) = \frac{1}{36} \left[ 8\Delta \Sigma + 3g_A + a_8 \right] \left( 1 - \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \right) + \mathcal{O}\left( \frac{1}{Q^2} \right)
\]

**Quark contribution:** \( \Delta \Sigma(Q^2) = \int_0^1 dx \left( \Delta u^+(x, Q^2) + \Delta d^+(x, Q^2) + \Delta s^+(x, Q^2) \right) \)

“Plus” helicity distributions: \( \Delta q^+ = \Delta q + \Delta \bar{q} \)

- DIS requires assumptions about triplet and octet axial charges
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- First moment of polarized structure function \(g_1\):

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→ DIS requires assumptions about triplet and octet axial charges

- Assuming exact \(SU(3)_f\) values from weak baryon decays

\[
\int dx \left( \Delta u^+ - \Delta d^+ \right) = g_A \sim 1.269 \quad \int dx \left( \Delta u^+ + \Delta d^+ - 2\Delta s^+ \right) = a_8 \sim 0.586
\]

\[
\Delta \Sigma_{[10^{-3}, 0.8]} \sim 0.3
\]
Proton spin structure from DIS

• Still much we don’t know about collinear helicity distributions!
  → Minimal information about sea and glue helicity from DIS

\[ \Delta s^+(Q^2) = \int_0^1 dx \Delta s^+(x, Q^2) \]

JAM15: \( \Delta s^+ = -0.1 \pm 0.01 \)  
DSSV09: \( \Delta s^+ = -0.11 \)  
\( Q^2 = 1 \text{ GeV}^2 \)
Proton spin structure from DIS

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JAM15: \( \Delta s^+ = -0.1 \pm 0.01 \)

DSSV09: \( \Delta s^+ = -0.11 \quad Q^2 = 1 \text{GeV}^2 \)

- Assuming \( \sim 20\% \) SU(3)\(_f\) symmetry breaking in value of \( a_8 \)
  \[ \Delta s^+ \sim -0.03 \pm 0.03 \]

- How does semi-inclusive DIS affect the shape of \( \Delta s^+ \)?
  → More general: what can SIDIS tell us about sea quark contributions?
Proton spin structure from SIDIS

- Measured via longitudinal double spin asymmetries

\[
A_1^h(x, z, Q^2) = \frac{g_1^h(x, z, Q^2)}{F_1^h(x, z, Q^2)}
\]

- Polarized structure function at NLO defined in terms of 2-D convolution

\[
g_1^h(x, z, Q^2) = \frac{1}{2} \sum_q e_q^2 \left\{ \Delta q(x, \mu_F) D_q^h(z, \mu_{FF}) + \frac{\alpha_s(\mu_R)}{2\pi} \right. \\
\left. \times \left( \Delta q \otimes \Delta C_{qq} \otimes D_q^h + \Delta q \otimes \Delta C_{qg} \otimes D_g^h + \Delta g \otimes \Delta C_{qg} \otimes D_q^q \right) \right\}
\]

- SIDIS allows separation of quark and anti-quark helicity distributions – however, valence is still the dominant contribution in most asymmetries

\[
\begin{align*}
g_{1,p}^{K^+} &\sim 4\Delta u D_{u}^{K^+} + \Delta \bar{s} D_{\bar{s}}^{K^+} \\
g_{1,p}^{K^-} &\sim 4\Delta \bar{u} D_{u}^{K^-} + \Delta s D_{s}^{K^-} + 4\Delta u D_{u}^{K^-} \\
g_{1,d}^{K^+} &\sim 4(\Delta u + \Delta d) D_{u}^{K^+} + 2\Delta \bar{s} D_{\bar{s}}^{K^+} \\
g_{1,d}^{K^-} &\sim 4(\Delta \bar{u} + \Delta \bar{d}) D_{u}^{K^-} + 2\Delta s D_{s}^{K^-} + 4(\Delta u + \Delta d) D_{u}^{K^-}
\end{align*}
\]
Proton spin structure from SIDIS

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\[ \times \left( \Delta q \otimes \Delta C_{qq} \otimes D_q^h + \Delta q \otimes \Delta C_{gq} \otimes D_g^h + \Delta g \otimes \Delta C_{qg} \otimes D_q^h \right) \]

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| \( g_{1,p}^{K^+} \) | \( \sim 4\Delta u D_u^{K^+} \) | \( \Delta \bar{s}D_{\bar{s}}^{K^+} \) | \( \Delta s D_s^{K^+} \) | \( 4\Delta u D_u^{K^-} \) |
| \( g_{1,p}^{K^-} \) | \( \sim 4\Delta \bar{u} D_{\bar{u}}^{K^-} \) | \( \Delta s D_s^{K^-} \) | \( \Delta \bar{s}D_{\bar{s}}^{K^-} \) | \( 4\Delta u D_u^{K^-} \) |
| \( g_{1,d}^{K^+} \) | \( \sim 4(\Delta u + \Delta d) D_u^{K^+} \) | \( 2\Delta \bar{s}D_{\bar{s}}^{K^+} \) | \( 2\Delta s D_s^{K^+} \) | \( 4(\Delta u + \Delta d) D_u^{K^-} \) |
| \( g_{1,d}^{K^-} \) | \( \sim 4(\Delta \bar{u} + \Delta \bar{d}) D_{\bar{u}}^{K^-} \) | \( 2\Delta s D_s^{K^-} \) | \( 2\Delta \bar{s}D_{\bar{s}}^{K^-} \) | \( 4(\Delta u + \Delta d) D_u^{K^-} \) |

Dominate terms in intermediate to large-\( x \) region

Low-\( x \) sensitivity
Proton spin structure from SIDIS

- Measured via longitudinal double spin asymmetries

\[ A^h_1(x, z, Q^2) = \frac{g^h_1(x, z, Q^2)}{F^h_1(x, z, Q^2)} \]

- Polarized structure function at NLO defined in terms of 2-D convolution

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g^h_1(x, z, Q^2) = \frac{1}{2} \sum_q e_q^2 \left\{ \Delta q(x, \mu_F) D^h_q(z, \mu_{FF}) + \frac{\alpha_s(\mu_R)}{2\pi} \right. \\
\times \left. \left( \Delta q \otimes \Delta C_{qq} \otimes D^h_q + \Delta q \otimes \Delta C_{gq} \otimes D^h_g + \Delta g \otimes \Delta C_{qg} \otimes D^h_q \right) \right\} \]

- SIDIS allows separation of quark and anti-quark helicity distributions – however, valence is still the dominant contribution in most asymmetries

\[
\begin{align*}
g^{K^+}_{1,p} & \sim 4\Delta u D^{K^+}_u + \Delta \bar{s} D^{K^+}_{\bar{s}} \\
g^{K^-}_{1,p} & \sim 4\Delta \bar{u} D^{K^-}_{\bar{u}} + \Delta s D^{K^-}_s + 4\Delta u D^{K^-}_u \\
g^{K^+}_{1,d} & \sim 4(\Delta u + \Delta d) D^{K^+}_u + 2\Delta \bar{s} D^{K^+}_{\bar{s}} \\
g^{K^-}_{1,d} & \sim 4(\Delta \bar{u} + \Delta \bar{d}) D^{K^-}_{\bar{u}} + 2\Delta s D^{K^-}_s + 4(\Delta u + \Delta d) D^{K^-}_u
\end{align*}
\]
Transverse spin structure from SIDIS

- Measured via Collins single spin asymmetries:

\[ A_{UT}^{\sin(\phi_h + \phi_s)} = \frac{2(1 - y)}{1 + (1 - y)^2} \frac{F_{UT}^{\sin(\phi_h + \phi_s)}}{F_{UU}} \]

- Structure functions defined in terms of TMD convolution operator:

\[ F_{UU} = C \left[ f_1 \otimes D_1 \right] \]
\[ F_{UT}^{\sin(\phi_h + \phi_s)} = C \left[ \frac{\hat{h} \cdot p_{\perp}}{zm_h} \otimes h_1 \otimes H_1^{\perp} \right] \]

Unpolarized TMD PDF

\[ f_1(x, k_{\perp}) \]

Unpolarized TMD FF

\[ D_1(z, p_{\perp}) \]

TMD transversity PDF

\[ h_1(x, k_{\perp}) \]

Collins FF

\[ H_1^{\perp}(z, p_{\perp}) \]

\( Q^2 \) evolution governed by Collins-Soper equations
Recent JAM Analyses

First simultaneous extraction of spin-dependent parton distributions and fragmentation functions from a global QCD analysis

J. J. Ethier, Nobuo Sato, and W. Melnitchouk

1 College of William and Mary, Williamsburg, Virginia 23187, USA
2 Jefferson Lab, Newport News, Virginia 23606, USA
3 University of Connecticut, Storrs, Connecticut 06269, USA

Jefferson Lab Angular Momentum (JAM) Collaboration

(Dated: October 4, 2017)

• Emphasis on SIDIS impact to sea quark helicity distributions
• SU(2) and SU(3) constraints used in DIS only analyses are released

\[
\int_0^1 dx \left( \Delta u^+ - \Delta d^+ \right) \cong g_A \\
\int_0^1 dx \left( \Delta u^+ + \Delta d^+ - 2\Delta s^+ \right) \cong a_8
\]

\(\rightarrow\) Direct test of QCD

\(\rightarrow\) Combined DIS+SIDIS can determine values for \(g_A\) and \(a_8\)
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Jefferson Lab Angular Momentum (JAM) Collaboration

(Dated: October 4, 2017)

First Monte Carlo global analysis of nucleon transversity with lattice QCD constraints


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5 Louisiana State University, Baton Rouge, Louisiana 70803, USA

Jefferson Lab Angular Momentum (JAM) Collaboration

(Dated: November 26, 2017)
JAM Fitting Methodology

- Based on Bayesian statistical methods – robust determination of “observables” $O$ (PDFs, FFs, etc.) and their uncertainties

\[
E[O] = \int d^n a \mathcal{P}(\vec{a} | data) O(\vec{a})
\]

\[
V[O] = \int d^n a \mathcal{P}(\vec{a} | data) [O(\vec{a}) - E[O]]^2
\]

- Bayes’ theorem defines probability $\mathcal{P}$ as

\[
\mathcal{P}(\vec{a} | data) = \frac{1}{Z} \mathcal{L}(data | \vec{a}) \pi(\vec{a})
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**JAM Fitting Methodology**

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\]

**Likelihood function**

\[
\mathcal{L} = \exp \left( -\frac{1}{2} \chi^2(\vec{a}) \right) \rightarrow \text{Gaussian form in data with} \quad \chi^2 = \sum_{e}^{N_{\text{exp}}} \sum_{i}^{N_{\text{data}}} \frac{(D_i^e - T_i)^2}{(\sigma_i^e)^2}
\]
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$$\mathcal{P}(\bar{a}|data) = \frac{1}{Z} \mathcal{L}(data|\bar{a}) \pi(\bar{a})$$

“Evidence” $Z = \int d^n a \mathcal{L}(data|\bar{a}) \pi(\bar{a})$
JAM Fitting Methodology

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$$E[O] = \int d^n a \mathcal{P}(\bar{a}|data) O(\bar{a})$$

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- Bayes’ theorem defines probability $\mathcal{P}$ as

$$\mathcal{P}(\bar{a}|data) = \frac{1}{Z} \mathcal{L}(data|\bar{a}) \pi(\bar{a})$$

- JAM uses Monte Carlo techniques to evaluate expectation value and variance integrals

  $\rightarrow$ samples parameter space and assigns weights $w_k$ to each parameter $a_k$ such that

$$E[O(\bar{a})] = \sum_k w_k O(\bar{a}_k) \quad V[O(\bar{a})] = \sum_k w_k (O(\bar{a}_k) - E[O])^2$$
Iterative Monte Carlo (IMC) (Used in JAM17 combined analysis)

- Samples wide region of parameter space
- Data is partitioned for cross-validation – training set is fitted via chi-square minimization
- Posteriors sent through sampler – **Kernel density estimation (KDE):** estimates the multi-dimensional probability density function of the parameters
- Procedure iterated until converged

\[
E[\mathcal{O}] = \frac{1}{n} \sum_{k=1}^{n} \mathcal{O}(a_k)
\]

\[
V[\mathcal{O}] = \frac{1}{n} \sum_{k=1}^{n} (\mathcal{O}(a_k) - E[\mathcal{O}])^2
\]
**Nested Sampling**

(Used in JAM transversity analysis)

- Statistical mapping of multidimensional integral to 1-D

\[ Z = \int d^n a \mathcal{L}(\text{data}|\bar{a}) \pi(\bar{a}) = \int_0^1 dX \mathcal{L}(X) \]

where the prior volume \( dX = \pi(\bar{a}) d^n a \)

- Algorithm:
  - Initialize \( X_0 = 1, L = 0 \) and choose \( N \) active points \( X_1, X_2, \ldots, X_N \) from prior
  - For each iteration, sample new point and remove lowest \( L_i \), replacing with point such that \( L \) is monotonically increasing
  - Repeat until entire parameter space has been explored

\[ Z_i \sim \sum_i \mathcal{L}_i w_i \]

where \( w_i = \frac{1}{2} (X_{i-1} - X_{i+1}) \)

**Polarized PDF Distributions**

- $\Delta u^+$ consistent with previous analysis
- $\Delta d^+$ slightly larger in magnitude

$\Rightarrow$ Anti-correlation with $\Delta s^+$, which is less negative than JAM15 at $x \sim 0.2$

- Isoscalar sea distribution consistent with zero
- Isovector sea slightly prefers positive shape at low $x$

$\Rightarrow$ Non-zero asymmetry given by small contributions from SIDIS asymmetries
Strange polarization

- Δs+ distribution consistent with zero, slightly positive in intermediate x range
- Primarily influenced by HERMES K-data from deuterium target
Strange polarization

Why does DIS+SU(3) give large negative $\Delta s^+$?

- Low $x$ DIS deuterium data from COMPASS prefers small negative $\Delta s^+$
- Needs to be more negative in intermediate region to satisfy SU(3) constraint
- Large-$x$ shape parameter for $\Delta s^+$ typically fixed, producing a peak at $x \sim 0.1$

- $\Delta s^+$ distribution consistent with zero, slightly positive in intermediate $x$ range
- Primarily influenced by HERMES $K^-$ data from deuterium target
Helicity Analysis - Moments

\[ a_3 \]
\[ a_8 \]
\[ \Delta \Sigma \]
\[ \Delta \bar{u} - \Delta \bar{d} \]

\[ g_A = 1.24 \pm 0.04 \] Confirmation of SU(2) symmetry to \( \sim 2\% \)

\[ a_8 = 0.46 \pm 0.21 \] \( \sim 20\% \) SU(3) breaking \( \pm \sim 20\% \); large uncertainty

- Need better determination of \( \Delta s^+ \) moment to reduce \( a_8 \) uncertainty!

\[ \Delta s^+ = -0.03 \pm 0.09 \]
Transversity Analysis - Tensor charge

\[ \delta q = \int_0^1 dx \left( h_1^q - \bar{h}_1^q \right) \]

Isovector moment: \( g_T \equiv \delta u - \delta d \)

- Significant reduction of peak widths with lattice input \( g_T^{\text{latt}} = 1.01(6) \)

**Lin et al analysis:**
\[
\begin{align*}
\delta u &= 0.3(3) \rightarrow 0.3(2) \\
\delta d &= -0.6(5) \rightarrow -0.7(2) \\
g_T &= 0.9(8) \rightarrow 1.0(1)
\end{align*}
\]

**Kang et al:**
\[
\begin{align*}
\delta u &= 0.39(11) \\
\delta d &= -0.22(14) \\
g_T &= 0.61(25)
\end{align*}
\]

Transversity distributions

- Distributions computed at 2 GeV$^2$ – yellow bands indicate SIDIS only fit, colored are SIDIS + Lattice fit
- Significant reduction of uncertainties with Lattice data
- Larger $|h_1|$ for down flavor w.r.t up comes from larger $\pi^-$ asymmetry
- Fitted anti-quark distributions consistent with zero

Summary and Outlook

• Monte Carlo statistical methods important for rigorous determination of non-perturbative functions and their uncertainties
  → Will be needed in future global analyses that contain large data sets and require many fit parameters (TMDs, GPDs)

• JAM extraction of helicity distributions from DIS+SIDIS resolves strange polarization puzzle
  → Large uncertainties on sea distributions – need to include other observables sensitive to sea (W production)
  → Difficult to determine $a_8$ with DIS+SIDIS, but results confirm SU(2) symmetry to ~2%

• JAM extraction of transversity distributions first to use Monte Carlo fitting methodology – shows compatibility between SIDIS data and lattice results
  → Significant reduction of uncertainties with lattice input
Backup Slides
Parameterizations and Chi-square

Template function: 
\[
T(x; \alpha) = \frac{M x^a (1 - x)^b (1 + c \sqrt{x})}{B(n + a, 1 + b) + c B(n + \frac{1}{2} + a, 1 + b)}
\]

- PDFs: \( n = 1 \) \( \Delta q^+, \Delta \bar{q}, \Delta g = T(x; \alpha) \)
- FFs: \( n = 2, c = 0 \)
  - Favored: \( D^{h}_{q^+} = T(z; \alpha) + T(z; \alpha') \)
  - Unfavored: \( D^{h}_{q^+,g} = T(z; \alpha) \)

\begin{align*}
\text{Pions:} & \\
D^{\pi^+}_{u} &= D^{\pi^+}_{d} = T(z; \alpha) \\
D^{\pi^+}_{s} &= D^{\pi^+}_{\bar{s}} = \frac{1}{2} D^{\pi^+}_{s+} \\
\text{Kaons:} & \\
D^{K^+}_{u} &= D^{K^+}_{d} = \frac{1}{2} D^{K^+}_{d+} \\
D^{K^+}_{s} &= T(z; \alpha) \\
\end{align*}

- Chi-squared definition:
\[
\chi^2(\alpha) = \sum_e \left[ \sum_i \left( \frac{D^{(e)}_i N^{(e)}_i - T^{(e)}_i(\alpha)}{\alpha_i^{(e)} N_i^{(e)}} \right)^2 + \sum_k \left( r^{(e)}_k \right)^2 + \sum_{\ell} \left( \frac{a^{(\ell)} - \mu^{(\ell)}}{\sigma^{(\ell)}} \right)^2 \right]
\]
Parameterizations and Chi-square

Template function: \[ T(x; \alpha) = \frac{M x^a (1 - x)^b (1 + c \sqrt{x})}{B(n + a, 1 + b) + c B(n + \frac{1}{2} + a, 1 + b)} \]

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  - Favored: \( D_{q^+}^h = T(z; \alpha) + T(z; \alpha') \)
  - Unfavored: \( D_{q^+,g}^h = T(z; \alpha) \)

**Pions:**
\[
\begin{align*}
D_{u^+}^{\pi^+} &= D_{d^+}^{\pi^+} = T(z; \alpha) \\
D_{s^+}^{\pi^+} &= D_{s^+}^{\pi^+} = \frac{1}{2} D_{s^+}^{\pi^+}
\end{align*}
\]

**Kaons:**
\[
\begin{align*}
D_{u^+}^{K^+} &= D_{d^+}^{K^+} = \frac{1}{2} D_{d^+}^{K^+} \\
D_{s^+}^{K^+} &= T(z; \alpha)
\end{align*}
\]

- Chi-squared definition:
\[
\chi^2(\alpha) = \sum_e \left[ \sum_i \left( \frac{D_i^{(e)} N_i^{(e)} - T_i^{(e)}(\alpha)}{\alpha_i^{(e)} N_i^{(e)}} \right)^2 + \sum_k \left( r_k^{(e)} \right)^2 \right] + \sum_\ell \left( \frac{a^{(\ell)} - \mu^{(\ell)}}{\sigma^{(\ell)}} \right)^2
\]

Penalty for fitting normalizations
Parameterizations and Chi-square

Template function: \[ T(x; \alpha) = \frac{M x^a (1 - x)^b (1 + c \sqrt{x})}{B(n + a, 1 + b) + c B(n + \frac{1}{2} + a, 1 + b)} \]

- PDFs: \( n = 1 \)  \( \Delta q^+, \Delta \bar{q}, \Delta g = T(x; \alpha) \)
- FFs: \( n = 2, c = 0 \)  
  Favored: \( D_{q^+}^h = T(z; \alpha) + T(z; \alpha') \)
  Unfavored: \( D_{q^+}^h, g = T(z; \alpha) \)

- Chi-squared definition:
  \[ \chi^2(\alpha) = \sum_e \left[ \sum_i \left( \frac{D_i^{(e)} N_i^{(e)} - T_i^{(e)}(\alpha)}{\alpha_i^{(e)} N_i^{(e)}} \right)^2 + \sum_k \left( \frac{r_k^{(e)}}{\sigma_k^{(e)}} \right)^2 \right] + \sum_\ell \left( \frac{a^{(\ell)} - \mu^{(\ell)}}{\sigma^{(\ell)}} \right)^2 \]

Modified likelihood to include prior information
Data vs Theory – SIDIS

\[ A^h_1 = \frac{g^h_1}{F^h_1} \]

<table>
<thead>
<tr>
<th>process</th>
<th>target</th>
<th>( N_{\text{dat}} )</th>
<th>( \chi^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIS ((\pi^\pm, K^\pm))</td>
<td>( p, d, ^3\text{He} )</td>
<td>854</td>
<td>854.8</td>
</tr>
<tr>
<td>SIA ((\pi^\pm, K^\pm))</td>
<td>( d )</td>
<td>18</td>
<td>28.1</td>
</tr>
<tr>
<td>HERMES (d)</td>
<td>( p )</td>
<td>18</td>
<td>14.2</td>
</tr>
<tr>
<td>HERMES (d)</td>
<td>( p )</td>
<td>24</td>
<td>8.0</td>
</tr>
<tr>
<td>COMPASS (d)</td>
<td>( d )</td>
<td>20</td>
<td>18.2</td>
</tr>
<tr>
<td>COMPASS (p)</td>
<td>( p )</td>
<td>24</td>
<td>12.3</td>
</tr>
<tr>
<td>Total:</td>
<td></td>
<td>1855</td>
<td>1969.7</td>
</tr>
</tbody>
</table>

Good agreement with all SIDIS data!
Moments

\[ \Delta \Sigma = 0.36 \pm 0.09 \]

Slightly larger central value than previous analyses, but consistent within uncertainty

Preference for slightly positive sea asymmetry; not very well constrained by SIDIS

\[ \Delta \bar{u} - \Delta \bar{d} = 0.05 \pm 0.08 \]
Transversity Parameterizations

Factorized form:

\[ f^q(x, k^2_\perp) = f^q(x) G^q_f(k^2_\perp) \]

where

\[ G^q_f(k^2_\perp) = \frac{1}{\pi \langle k^2_\perp \rangle_q f} \exp \left[ -\frac{k^2_\perp}{\langle k^2_\perp \rangle_q f} \right] \]

Similarly for the TMD FFs:

\[ D^{h/q}_1(z, p^2_\perp) = D^{h/q}_1(z) G^{h/q}_{D_1}(p^2_\perp) \]

\[ H^{h/q}_1(z, p_\perp) = \frac{2z^2 m_h^2}{\langle p^2_\perp \rangle_{H^{h/q}_1}} H^{(1)}_{1 h/q}(z) G^{h/q}_{H^{h/q}_1}(p^2_\perp) \]
Data vs Theory – Single Spin Asymmetries