JAM extractions of quark helicity and transversity distributions [PRL 119 132001 arXiv:1710.09858]

Jacob Ethier

on behalf of Jefferson Lab Angular Momentum (JAM) collaboration

Transversity Workshop

December 12th, 2017

WILLIAM & MARY

CHARTERED 1693

Jefferson Lab

Thomas Jefferson National Accelerator Facility

JAM

Proton spin structure from DIS

- Measured via longitudinal and transverse double spin asymmetries

$$A_{\parallel} = \frac{\sigma^{\uparrow\Downarrow} - \sigma^{\uparrow\Uparrow}}{\sigma^{\uparrow\Downarrow} + \sigma^{\uparrow\Uparrow}} = D\left(A_1 + \eta A_2\right) \qquad A_{\perp} = \frac{\sigma^{\uparrow\Rightarrow} - \sigma^{\uparrow\Leftarrow}}{\sigma^{\uparrow\Rightarrow} + \sigma^{\uparrow\Leftarrow}} = d\left(A_2 + \zeta A_1\right)$$

→ Virtual photoproduction asymmetries: $A_1 = \frac{(g_1 - \gamma^2 g_2)}{F_1}$ $A_2 = \gamma \frac{(g_1 + g_2)}{F_1}$ $\gamma^2 = \frac{4M^2x^2}{Q^2}$

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- First moment of polarized structure function g_1:

$$\int_0^1 dx g_1^p(x,Q^2) = \frac{1}{36}\left[8\underline{\Delta\Sigma} + 3g_A + a_8\right]\left(1 - \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2)\right) + \mathcal{O}(\frac{1}{Q^2})$$

Quark contribution: $\Delta\Sigma(Q^2) = \int_0^1 dx\left(\Delta u^+(x,Q^2) + \Delta d^+(x,Q^2) + \Delta s^+(x,Q^2)\right)$

"Plus" helicity distributions: $\Delta q^+ = \Delta q + \Delta\bar{q}$

→ DIS requires assumptions about triplet and octet axial charges

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"Plus" helicity distributions: $\Delta q^+ = \Delta q + \Delta\bar{q}$

→ DIS requires assumptions about triplet and octet axial charges

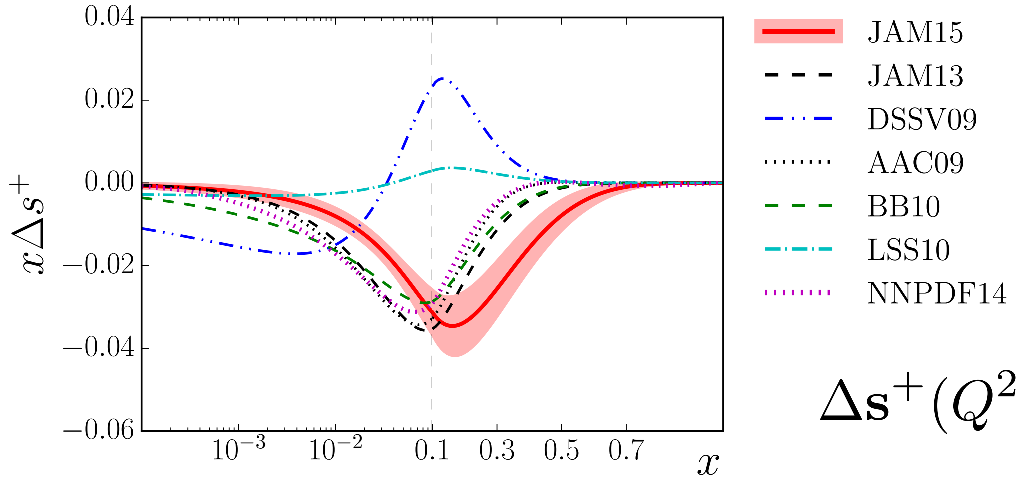
- Assuming exact $SU(3)_f$ values from weak baryon decays

$$\int dx\left(\Delta u^+ - \Delta d^+\right) = g_A \sim 1.269 \qquad \int dx\left(\Delta u^+ + \Delta d^+ - 2\Delta s^+\right) = a_8 \sim 0.586$$

$$\Delta\Sigma_{[10^{-3},0.8]} \sim 0.3$$

Proton spin structure from DIS

- Still much we don't know about collinear helicity distributions!
  - → Minimal information about sea and glue helicity from DIS



N. Sato et. al. Phys. Rev. D94 114004 (2016)

$$\Delta \mathbf{s}^+(Q^2) = \int_0^1 dx \Delta s^+(x, Q^2)$$

JAM15: $\Delta \mathbf{s}^+ = -0.1 \pm 0.01$ DSSV09: $\Delta \mathbf{s}^+ = -0.11$ $Q^2 = 1\,\mathrm{GeV}^2$

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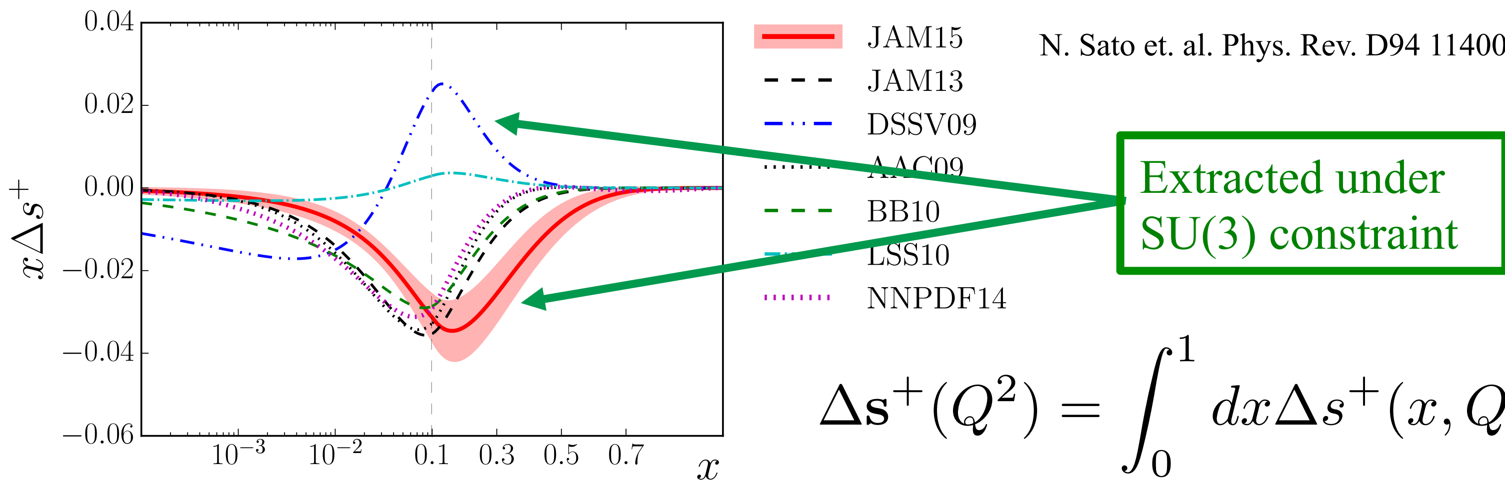

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- Assuming ~20% SU(3)$_f$ symmetry breaking in value of a_8

$$\Delta \mathbf{s}^+ \sim -0.03 \pm 0.03$$

C. Aidala et. al. Rev. Mod. Phys. 85 655 (2013)

- How does semi-inclusive DIS affect the shape of Δs+?

  → More general: what can SIDIS tell us about sea quark contributions?

Proton spin structure from SIDIS

- Measured via longitudinal double spin asymmetries

$$A_1^h(x,z,Q^2) = \frac{g_1^h(x,z,Q^2)}{F_1^h(x,z,Q^2)}$$

- Polarized structure function at NLO defined in terms of 2-D convolution

$$g_1^h(x,z,Q^2) = \frac{1}{2}\sum_q e_q^2 \Big\{ \Delta q(x,\mu_F) D_q^h(z,\mu_{FF}) + \frac{\alpha_s(\mu_R)}{2\pi} \times \Big(\Delta q \otimes \Delta C_{qq} \otimes D_q^h + \Delta q \otimes \Delta C_{gq} \otimes D_g^h + \Delta g \otimes \Delta C_{qg} \otimes D_q^h \Big) \Big\}$$

- SIDIS allows separation of quark and anti-quark helicity distributions – however, valence is still the dominant contribution in most asymmetries

$$g_{1,p}^{K^+} \sim 4\Delta u D_u^{K^+} + \Delta\bar{s} D_{\bar{s}}^{K^+}$$
$$g_{1,p}^{K^-} \sim 4\Delta\bar{u} D_{\bar{u}}^{K^-} + \Delta s D_s^{K^-} + 4\Delta u D_u^{K^-}$$
$$g_{1,d}^{K^+} \sim 4(\Delta u + \Delta d) D_u^{K^+} + 2\Delta\bar{s} D_{\bar{s}}^{K^+}$$
$$g_{1,d}^{K^-} \sim 4(\Delta\bar{u} + \Delta\bar{d}) D_{\bar{u}}^{K^-} + 2\Delta s D_s^{K^-} + 4(\Delta u + \Delta d) D_u^{K^-}$$

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Dominate terms in intermediate to large-*x* region

Low-x sensitivity

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Transverse spin structure from SIDIS

- Measured via Collins single spin asymmetries

$$A_{UT}^{\sin(\phi_h+\phi_s)} = \frac{2(1-y)}{1+(1-y)^2} \frac{F_{UT}^{\sin(\phi_h+\phi_s)}}{F_{UU}}$$

- Structure functions defined in terms of TMD convolution operator

$$F_{UU} = \mathcal{C}\left[f_1 \otimes D_1\right]$$

$$F_{UT}^{\sin(\phi_h+\phi_s)} = \mathcal{C}\left[\frac{\hat{\boldsymbol{h}} \cdot \boldsymbol{p}_\perp}{z m_h} \otimes h_1 \otimes H_1^\perp\right]$$

Unpolarized TMD PDF	Unpolarized TMD FF
$f_1(x, k_\perp)$	$D_1(z, p_\perp)$
TMD transversity PDF	**Collins FF**
$h_1(x, k_\perp)$	$H_1^\perp(z, p_\perp)$

} Q^2 evolution governed by Collins-Soper equations

Recent JAM Analyses

First simultaneous extraction of spin-dependent parton distributions and fragmentation functions from a global QCD analysis

J. J. Ethier,[1,2] Nobuo Sato,[3] and W. Melnitchouk[2]

[1] *College of William and Mary, Williamsburg, Virginia 23187, USA*
[2] *Jefferson Lab, Newport News, Virginia 23606, USA*
[3] *University of Connecticut, Storrs, Connecticut 06269, USA*

Jefferson Lab Angular Momentum (JAM) Collaboration

(Dated: October 4, 2017)

- Emphasis on SIDIS impact to sea quark helicity distributions
- SU(2) and SU(3) constraints used in DIS only analyses are released

$$\int_0^1 dx\,(\Delta u^+ - \Delta d^+) \stackrel{?}{=} g_A$$

→ Direct test of QCD

$$\int_0^1 dx\,(\Delta u^+ + \Delta d^+ - 2\Delta s^+) \stackrel{?}{=} a_8$$

→ Combined DIS+SIDIS can determine values for g_A and a_8

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Jefferson Lab Angular Momentum (JAM) Collaboration

(Dated: October 4, 2017)

First Monte Carlo global analysis of nucleon transversity with lattice QCD constraints

H.-W. Lin,[1] W. Melnitchouk,[2] A. Prokudin,[2,3] N. Sato,[4] and H. Shows III[5]

[1] *Michigan State University, East Lansing, Michigan 48824, USA*
[2] *Jefferson Lab, Newport News, Virginia 23606, USA*
[3] *Penn State Berks, Reading, Pennsylvania 19610, USA*
[4] *University of Connecticut, Storrs, Connecticut 06269, USA*
[5] *Louisiana State University, Baton Rouge, Louisiana 70803, USA*

Jefferson Lab Angular Momentum (JAM) Collaboration

(Dated: November 26, 2017)

JAM Fitting Methodology

- Based on Bayesian statistical methods – robust determination of "observables" O (PDFs,FFs,etc.) and their uncertainties

$$E[\mathcal{O}] = \int d^n a \mathcal{P}(\vec{a}|data)\mathcal{O}(\vec{a})$$

$$V[\mathcal{O}] = \int d^n a \mathcal{P}(\vec{a}|data)\left[\mathcal{O}(\vec{a}) - E[\mathcal{O}]\right]^2$$

- Bayes' theorem defines probability $\mathcal{P}$ as

$$\mathcal{P}(\vec{a}|data) = \frac{1}{Z}\mathcal{L}(data|\vec{a})\pi(\vec{a})$$

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↑

Likelihood function

$$\mathcal{L} = \exp\left(-\frac{1}{2}\chi^2(\vec{a})\right)$$ → Gaussian form in data with $\chi^2 = \sum_e^{N_{exp}} \sum_i^{N_{data}} \frac{(D_i^e - T_i)^2}{(\sigma_i^e)^2}$

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Priors

"Evidence" $Z = \int d^n a \mathcal{L}(data|\vec{a})\pi(\vec{a})$

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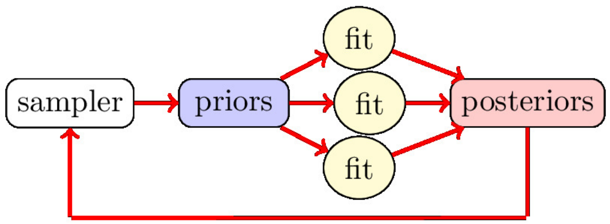
- JAM uses Monte Carlo techniques to evaluate expectation value and variance integrals

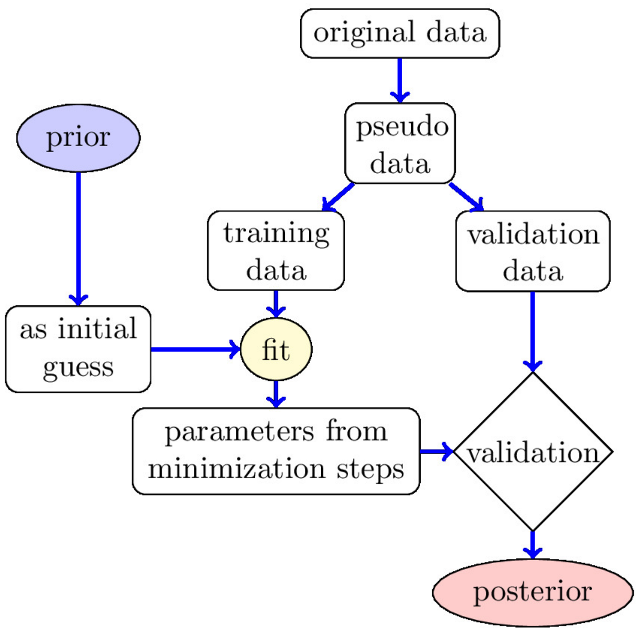
→ samples parameter space and assigns weights w_k to each parameter a_k such that

$$E[\mathcal{O}(\vec{a})] = \sum_k w_k \mathcal{O}(\vec{a}_k) \qquad V[\mathcal{O}(\vec{a})] = \sum_k w_k (\mathcal{O}\left(\vec{a}_k\right) - E[\mathcal{O}])^2$$

Iterative Monte Carlo (IMC)

(Used in JAM17 combined analysis)





→ Samples wide region of parameter space

→ Data is partitioned for cross-validation – training set is fitted via chi-square minimization

→ Posteriors sent through sampler – Kernel density estimation (KDE): estimates the multi-dimensional probability density function of the parameters

→ Procedure iterated until converged

$$\mathrm{E}[\mathcal{O}] = \frac{1}{n}\sum_{k=1}^{n} \mathcal{O}(\boldsymbol{a}_k)$$

$$\mathrm{V}[\mathcal{O}] = \frac{1}{n}\sum_{k=1}^{n} (\mathcal{O}(\boldsymbol{a}_k) - \mathrm{E}[\mathcal{O}])^2$$

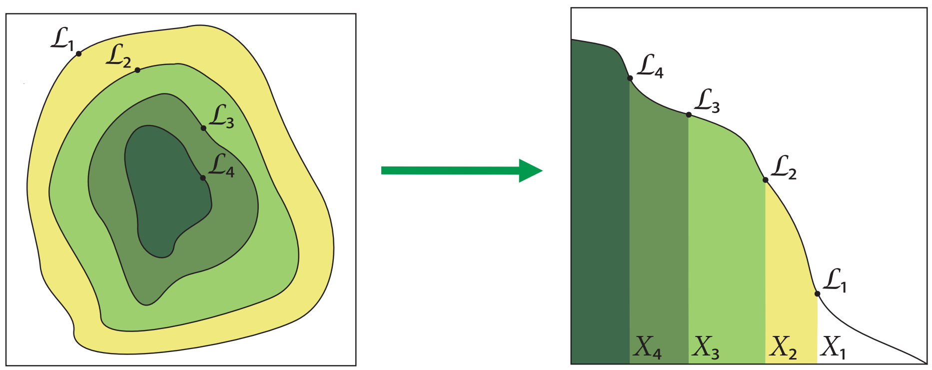
Nested Sampling

(Used in JAM transversity analysis)

- Statistical mapping of multidimensional integral to 1-D

$$Z = \int d^n a \mathcal{L}(data|\vec{a})\pi(\vec{a}) = \int_0^1 dX \mathcal{L}(X)$$

where the *prior volume* $dX = \pi(\vec{a})d^n a$



$$Z_i \sim \sum_i \mathcal{L}_i w_i$$

where $w_i = \frac{1}{2}(X_{i-1} - X_{i+1})$

Feroz et al. arXiv:1306.2144 [astro-ph]

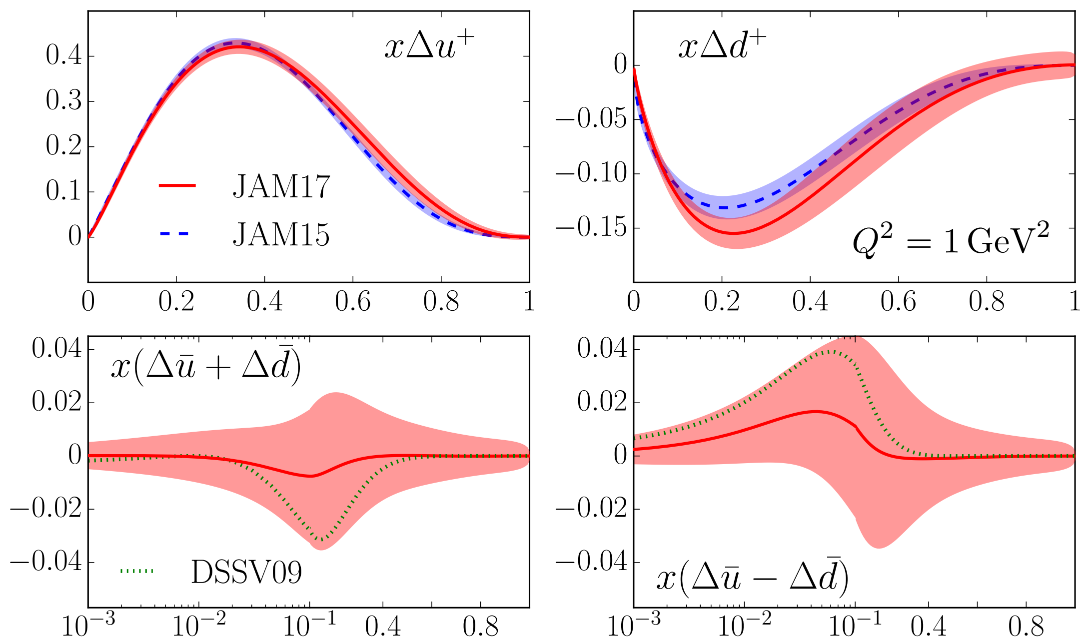
- Algorithm:

→ Initialize $X_0 = 1$, $L = 0$ and choose N active points $X_1, X_2, \ldots, X_N$ from prior

→ For each iteration, sample new point and remove lowest L_i, replacing with point such that L is monotonically increasing

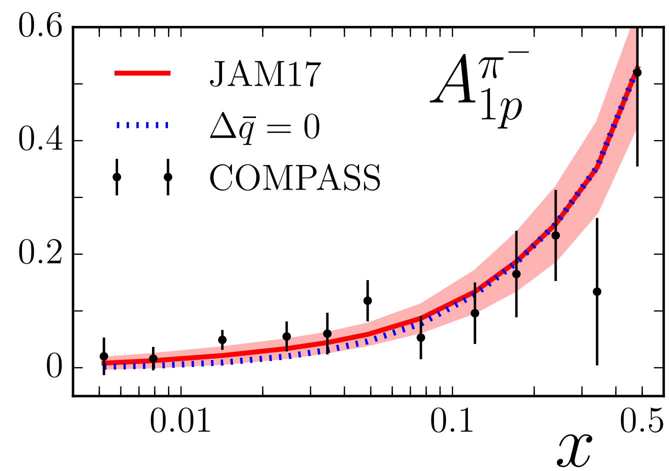
→ Repeat until entire parameter space has been explored

Polarized PDF Distributions



- Δu+ consistent with previous analysis
- Δd+ slightly larger in magnitude

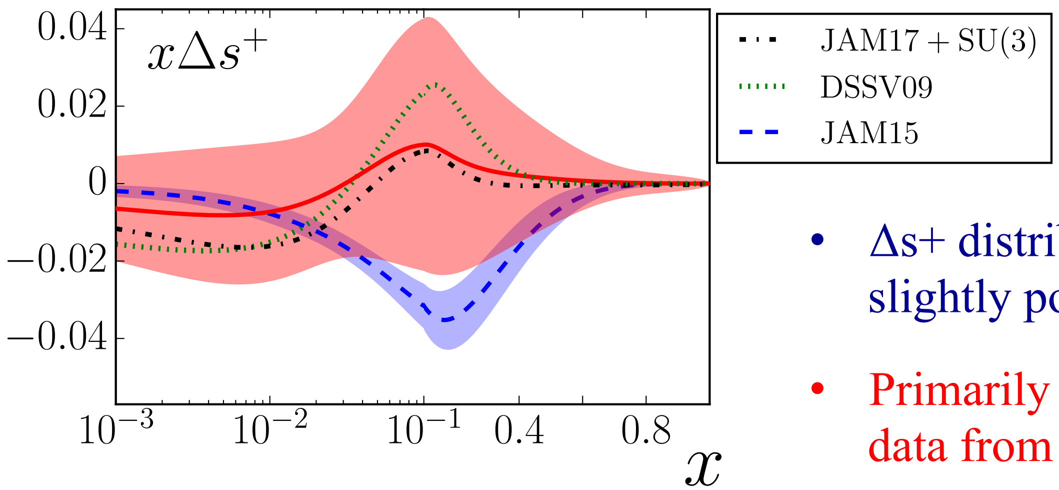
→ Anti-correlation with Δs+, which is less negative than JAM15 at $x \sim 0.2$



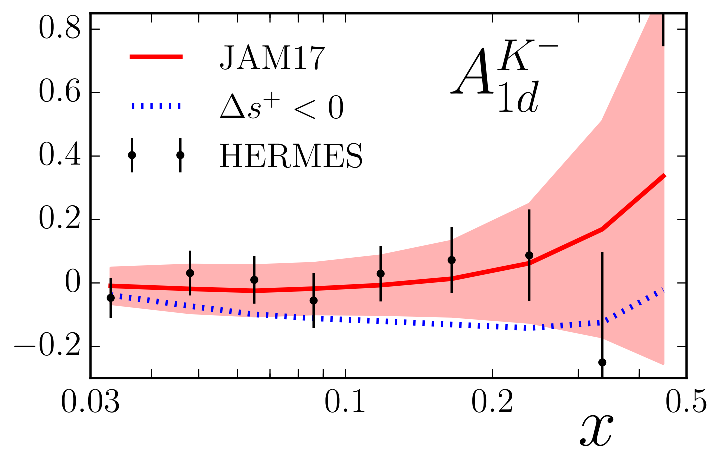
- Isoscalar sea distribution consistent with zero
- Isovector sea slightly prefers positive shape at low *x*

→ Non-zero asymmetry given by small contributions from SIDIS asymmetries

Strange polarization



- Δs+ distribution consistent with zero, slightly positive in intermediate *x* range
- Primarily influenced by HERMES K- data from deuterium target



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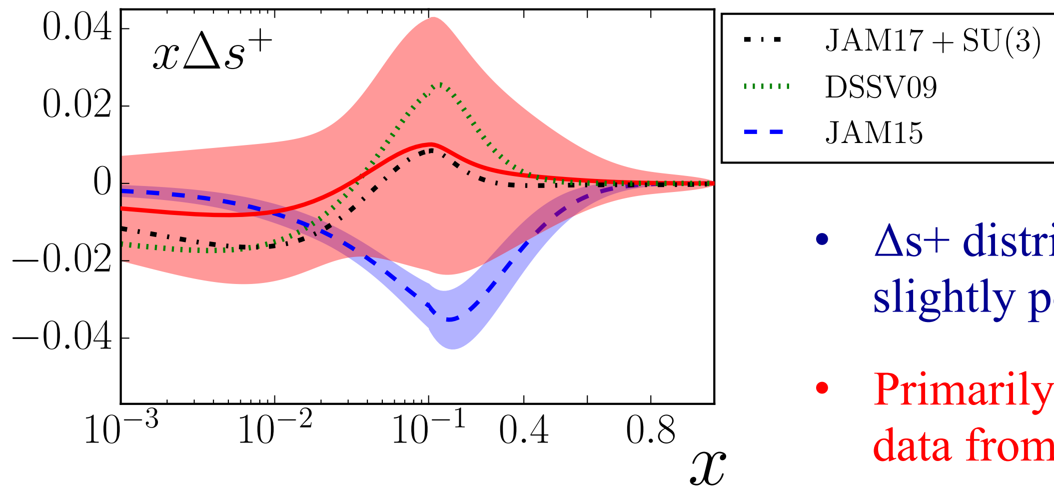

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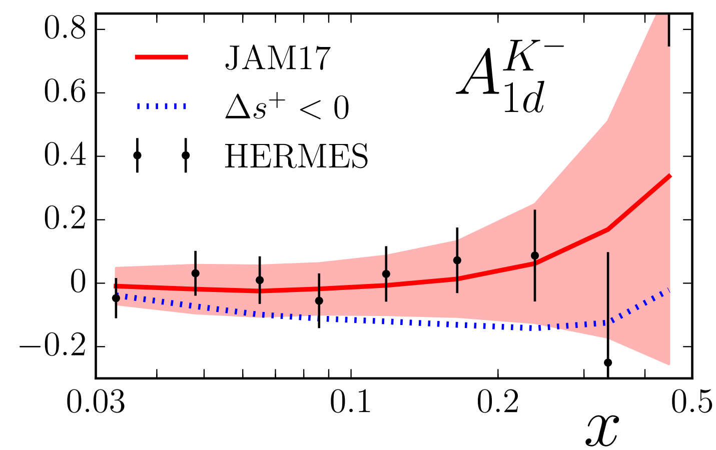
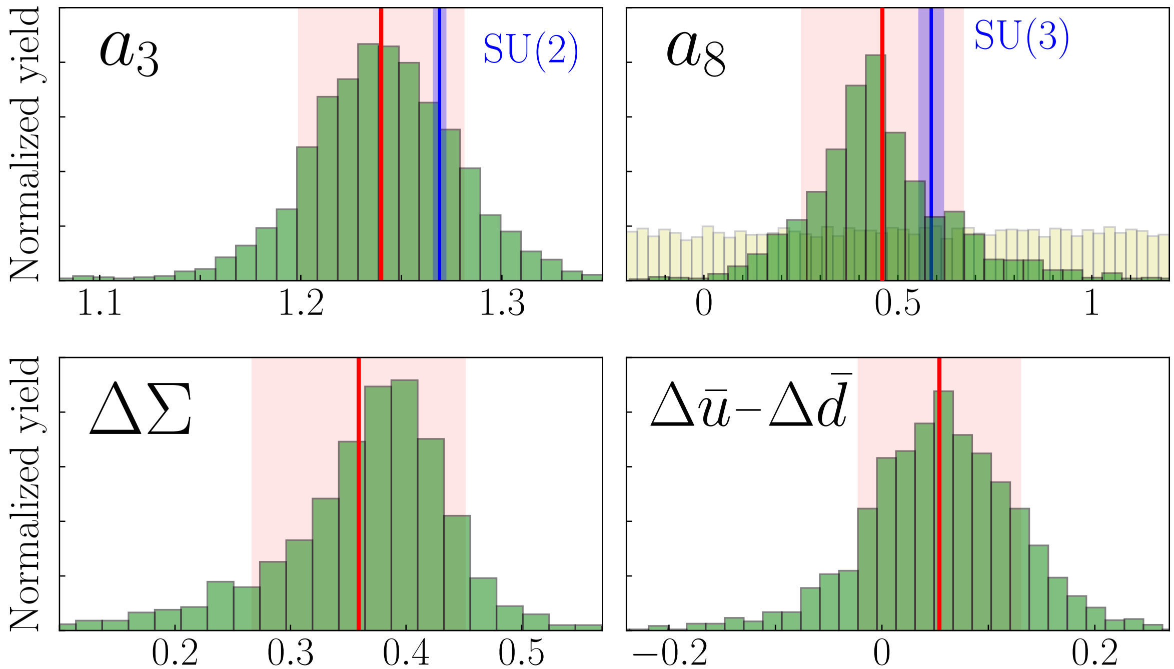
Why does DIS+SU(3) give large negative Δs+?

- Low *x* DIS deuterium data from COMPASS prefers small negative Δs^+
- Needs to be more negative in intermediate region to satisfy SU(3) constraint
- Large-*x* shape parameter for Δs+ typically fixed, producing a peak at x~0.1



Helicity Analysis - Moments



$$g_A = 1.24 \pm 0.04$$ Confirmation of SU(2) symmetry to ~2%

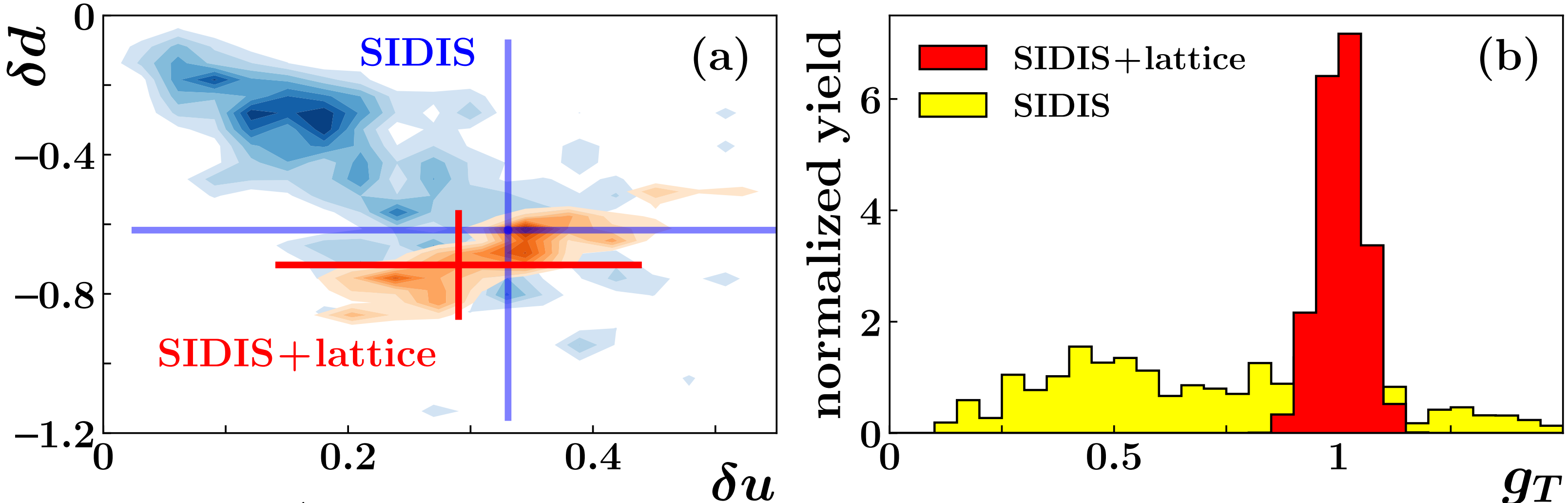
$$a_8 = 0.46 \pm 0.21$$ ~20% SU(3) breaking ± ~20%; large uncertainty

- Need better determination of Δs^+ moment to reduce a_8 uncertainty!

$$\Delta \mathbf{s}^+ = -0.03 \pm 0.09$$

Transversity Analysis - Tensor charge

H.-W. Lin *et al*. arXiv:1710.09858



$$\delta q = \int_0^1 dx \left(h_1^q - h_1^{\bar{q}}\right)$$

Isovector moment: $g_T \equiv \delta u - \delta d$

- Significant reduction of peak widths with lattice input $g_T^{\rm latt} = 1.01(6)$

Lin *et al* analysis:

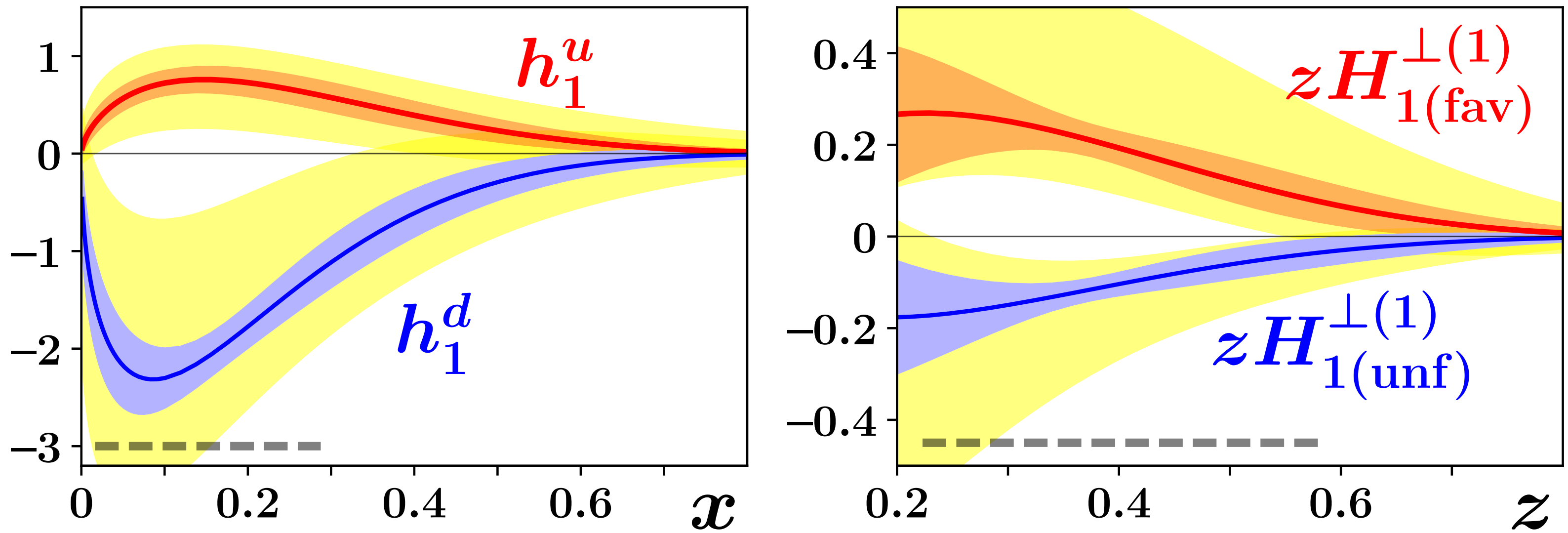
2 GeV²
$$\delta u = 0.3(3) \rightarrow 0.3(2)$$
$$\delta d = -0.6(5) \rightarrow -0.7(2)$$
$$g_T = 0.9(8) \rightarrow 1.0(1)$$

Kang *et al*:

10 GeV²
$$\delta u = 0.39(11)$$
$$\delta d = -0.22(14)$$
$$g_T = 0.61(25)$$

Transversity distributions

H.-W. Lin *et al*. arXiv:1710.09858



- Distributions computed at 2 GeV^2 – yellow bands indicate SIDIS only fit, colored are SIDIS + Lattice fit
- Significant reduction of uncertainties with Lattice data
- Larger $|h_1|$ for down flavor w.r.t up comes from larger π^- asymmetry
- Fitted anti-quark distributions consistent with zero

Summary and Outlook

- Monte Carlo statistical methods important for rigorous determination of non-perturbative functions and their uncertainties

  → Will be needed in future global analyses that contain large data sets and require many fit parameters (TMDs, GPDs)

- JAM extraction of helicity distributions from DIS+SIDIS resolves strange polarization puzzle

  → Large uncertainties on sea distributions – need to include other observables sensitive to sea (W production)

  → Difficult to determine a_8 with DIS+SIDIS, but results confirm SU(2) symmetry to ~2%

- JAM extraction of transversity distributions first to use Monte Carlo fitting methodology – shows compatibility between SIDIS data and lattice results

  → Significant reduction of uncertainties with lattice input

Backup Slides

Parameterizations and Chi-square

Template function: $$\mathrm{T}(x;\boldsymbol{a}) = \frac{M\, x^{a}(1-x)^{b}(1+c\sqrt{x})}{B(n+a,1+b)+cB(n+\frac{1}{2}+a,1+b)}$$

- PDFs: n = 1 $\Delta q^{+}, \Delta\bar{q}, \Delta g = \mathrm{T}(x;\boldsymbol{a})$
- FFs: n = 2, c = 0

  Favored: $D^{h}_{q^{+}} = \mathrm{T}(z;\boldsymbol{a}) + \mathrm{T}(z;\boldsymbol{a}')$

  Unfavored: $D^{h}_{q^{+},g} = \mathrm{T}(z;\boldsymbol{a})$

Pions:

$$D_{\bar{u}}^{\pi^{+}} = D_{d}^{\pi^{+}} = \mathrm{T}(z;\boldsymbol{a})$$

$$D_{s}^{\pi^{+}} = D_{\bar{s}}^{\pi^{+}} = \frac{1}{2}D_{s^{+}}^{\pi^{+}}$$

Kaons:

$$D_{\bar{u}}^{K^{+}} = D_{d}^{K^{+}} = \frac{1}{2}D_{d^{+}}^{K^{+}}$$

$$D_{s}^{K^{+}} = \mathrm{T}(z;\boldsymbol{a})$$

- Chi-squared definition:

$$\chi^{2}(\boldsymbol{a}) = \sum_{e}\left[\sum_{i}\left(\frac{\mathcal{D}_{i}^{(e)}N_{i}^{(e)} - T_{i}^{(e)}(\boldsymbol{a})}{\alpha_{i}^{(e)}N_{i}^{(e)}}\right)^{2} + \sum_{k}\left(r_{k}^{(e)}\right)^{2}\right] + \sum_{\ell}\left(\frac{a^{(\ell)}-\mu^{(\ell)}}{\sigma^{(\ell)}}\right)^{2}$$

Parameterizations and Chi-square

Template function: $$\mathrm{T}(x;\boldsymbol{a}) = \frac{M\,x^{a}(1-x)^{b}(1+c\sqrt{x})}{B(n+a,1+b)+cB(n+\frac{1}{2}+a,1+b)}$$

- PDFs: n = 1 $\Delta q^{+}, \Delta\bar{q}, \Delta g = \mathrm{T}(x;\boldsymbol{a})$
- FFs: n = 2, c = 0

$$\text{Favored: } D^{h}_{q^{+}} = \mathrm{T}(z;\boldsymbol{a}) + \mathrm{T}(z;\boldsymbol{a}')$$

$$\text{Unfavored: } D^{h}_{q^{+},g} = \mathrm{T}(z;\boldsymbol{a})$$

Pions:

$$D^{\pi^+}_{\bar{u}} = D^{\pi^+}_{d} = \mathrm{T}(z;\boldsymbol{a})$$

$$D^{\pi^+}_{s} = D^{\pi^+}_{\bar{s}} = \frac{1}{2}D^{\pi^+}_{s^+}$$

Kaons:

$$D^{K^+}_{\bar{u}} = D^{K^+}_{d} = \frac{1}{2}D^{K^+}_{d^+}$$

$$D^{K^+}_{s} = \mathrm{T}(z;\boldsymbol{a})$$

- Chi-squared definition:

$$\chi^2(\boldsymbol{a}) = \sum_{e}\left[\sum_{i}\left(\frac{\mathcal{D}^{(e)}_{i}N^{(e)}_{i} - T^{(e)}_{i}(\boldsymbol{a})}{\alpha^{(e)}_{i}N^{(e)}_{i}}\right)^2 + \sum_{k}\left(r^{(e)}_{k}\right)^2\right] + \sum_{\ell}\left(\frac{a^{(\ell)}-\mu^{(\ell)}}{\sigma^{(\ell)}}\right)^2$$

Penalty for fitting normalizations

Parameterizations and Chi-square

Template function: $$\mathrm{T}(x;\boldsymbol{a}) = \frac{M\, x^{a}(1-x)^{b}(1+c\sqrt{x})}{B(n+a,1+b)+cB(n+\frac{1}{2}+a,1+b)}$$

- PDFs: n = 1 $\Delta q^{+}, \Delta\bar{q}, \Delta g = \mathrm{T}(x;\boldsymbol{a})$
- FFs: n = 2, c = 0

Favored: $D^{h}_{q^{+}} = \mathrm{T}(z;\boldsymbol{a}) + \mathrm{T}(z;\boldsymbol{a}')$

Unfavored: $D^{h}_{q^{+},g} = \mathrm{T}(z;\boldsymbol{a})$

Pions:

$$D^{\pi^{+}}_{\bar{u}} = D^{\pi+}_{d} = \mathrm{T}(z;\boldsymbol{a})$$

$$D^{\pi^{+}}_{s} = D^{\pi^{+}}_{\bar{s}} = \frac{1}{2}D^{\pi^{+}}_{s+}$$

Kaons:

$$D^{K^{+}}_{\bar{u}} = D^{K^{+}}_{d} = \frac{1}{2}D^{K^{+}}_{d+}$$

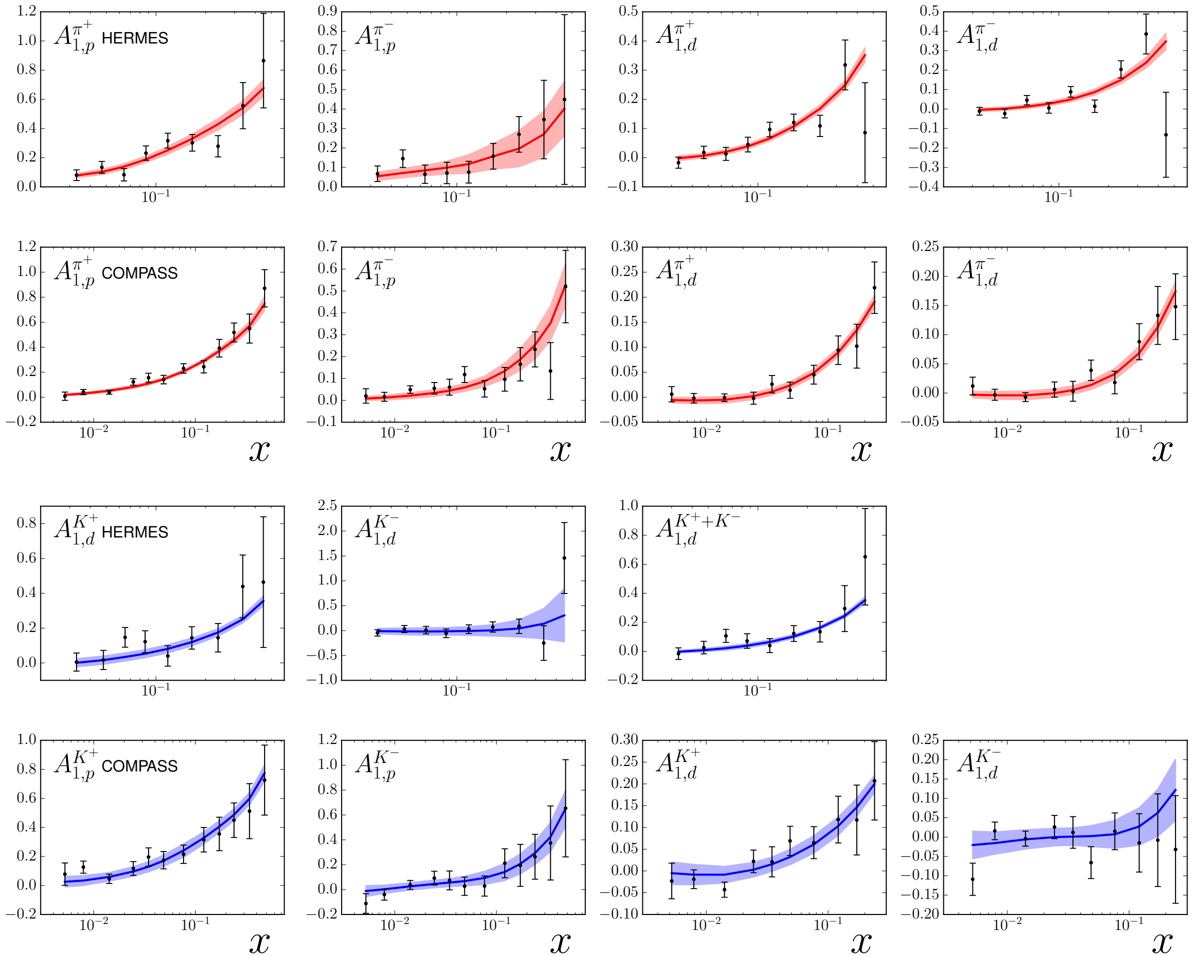
$$D^{K^{+}}_{s} = \mathrm{T}(z;\boldsymbol{a})$$

- Chi-squared definition:

$$\chi^{2}(\boldsymbol{a}) = \sum_{e}\left[\sum_{i}\left(\frac{\mathcal{D}^{(e)}_{i}N^{(e)}_{i} - T^{(e)}_{i}(\boldsymbol{a})}{\alpha^{(e)}_{i}N^{(e)}_{i}}\right)^{2} + \sum_{k}\left(r^{(e)}_{k}\right)^{2}\right] + \sum_{\ell}\left(\frac{a^{(\ell)}-\mu^{(\ell)}}{\sigma^{(\ell)}}\right)^{2}$$

Modified likelihood to include prior information

Data vs Theory – SIDIS

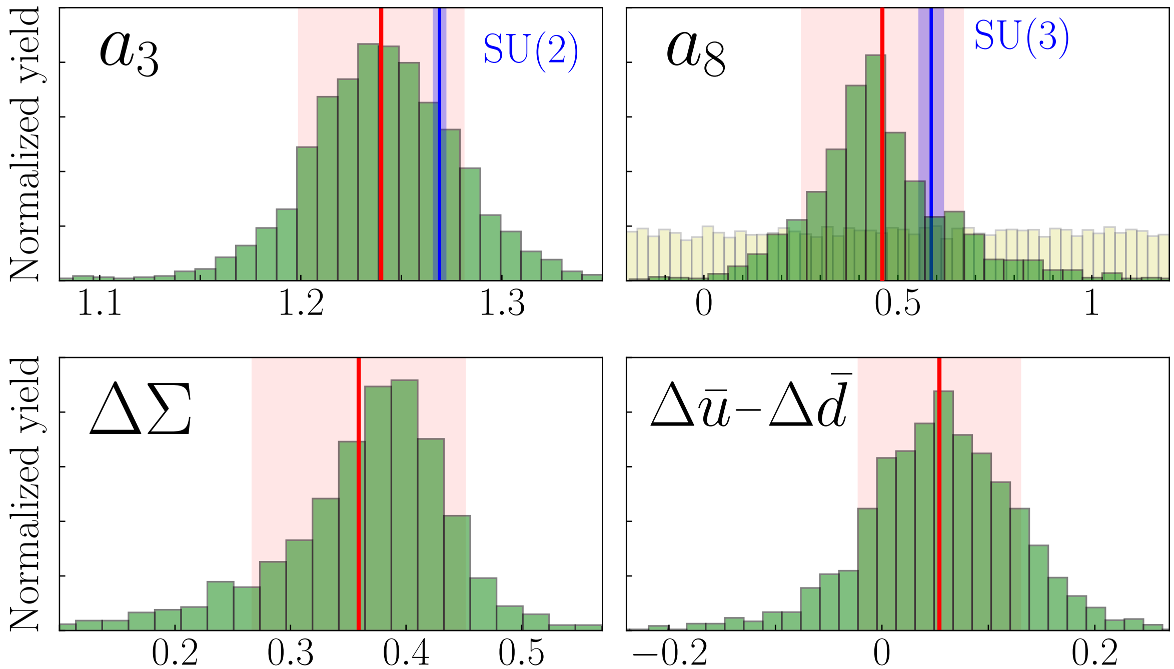


$$A_1^h = \frac{g_1^h}{F_1^h}$$

process	target	$N_{\rm dat}$	χ^2
DIS	$p, d, {}^3$He	854	854.8
SIA $(\pi^\pm, K^\pm)$		850	997.1
SIDIS $(\pi^\pm)$			
HERMES	d	18	28.1
HERMES	p	18	14.2
COMPASS	d	20	8.0
COMPASS	p	24	18.2
SIDIS $(K^\pm)$			
HERMES	d	27	18.3
COMPASS	d	20	18.7
COMPASS	p	24	12.3
Total:		**1855**	**1969.7**

Good agreement with all SIDIS data!

Moments



$$\Delta\Sigma = 0.36 \pm 0.09$$

Slightly larger central value than previous analyses, but consistent within uncertainty

Preference for slightly positive sea asymmetry; not very well constrained by SIDIS

$$\Delta\bar{u} - \Delta\bar{d} = 0.05 \pm 0.08$$

Transversity Parameterizations

Factorized form:

$$f^q(x, k_\perp^2) = f^q(x)\mathcal{G}_f^q(k_\perp^2) \qquad\qquad f^q = f_1^q \text{ or } h_1^q$$

where

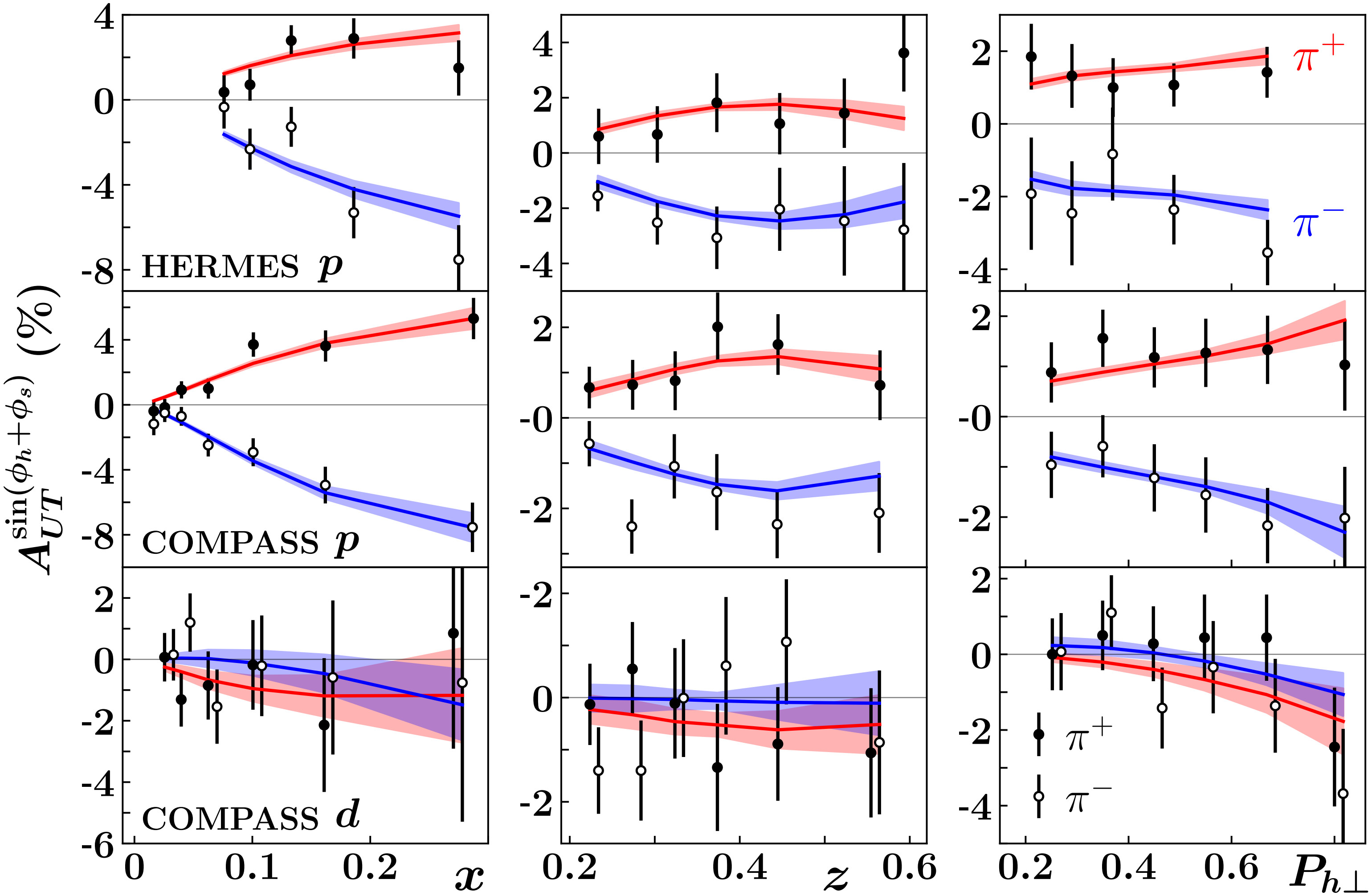
$$\mathcal{G}_f^q(k_\perp^2) = \frac{1}{\pi \langle k_\perp^2 \rangle_f^q} \exp\left[-\frac{k_\perp^2}{\langle k_\perp^2 \rangle_f^q} \right]$$

Similarly for the TMD FFs:

$$D_1^{h/q}(z, p_\perp^2) = D_1^{h/q}(z)\ \mathcal{G}_{D_1}^{h/q}(p_\perp^2)$$

$$H_1^{\perp h/q}(z, p_\perp) = \frac{2z^2 m_h^2}{\langle p_\perp^2 \rangle_{H_1^\perp}^{h/q}}\ H_{1\,h/q}^{\perp(1)}(z)\ \mathcal{G}_{H_1^\perp}^{h/q}(p_\perp^2)$$

Data vs Theory – Single Spin Asymmetries



H.-W. Lin *et al*. arXiv:1710.09858