On the extraction of Boer-Mulders functions

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The process

We consider SIDIS:

$$l + N \to l' + h^{\pm} + X$$

in the non-collinear picture of the parton model:

we consider **Boer-Mulders & Sivers functions:**

- The Boer-Mulders (BM) functions = $q^{\uparrow}$ in unpol. proton p:

$$p = (q + \underbrace{q^{\uparrow}}_{BM}) \quad \Rightarrow \quad \underbrace{\Delta f^{BM}_{q^{\uparrow}/p}(x_{_B}, k_{\perp})}_{BM} (\vec{s}_q \cdot \vec{P} \times \vec{k}_{\perp})$$

- The Sivers function = unpolarized q in transv. pol. proton $p^{\uparrow}$:

$$p^{\uparrow} = (q^{\uparrow} + \underbrace{q}_{Sivers}) \quad \Rightarrow \quad \underbrace{\Delta f^{Siv}_{q/p^{\uparrow}}(x_{_B}, k_{\perp})}_{Sivers} (\vec{S}_T \cdot \vec{P} \times \vec{k}_{\perp})$$

- BM and Siv. $\simeq (\vec{s} \cdot \vec{P} \times \vec{k}_{\perp})$ = T-odd:

At present:

- Δf^{BM} & Δf^{Siv} = both measured and $\neq 0$:

Δf^{BM} & Δf^{Siv} measured via the ϕ-dependence in SIDIS:

$$d\sigma^h(x,z,Q^2,P_T,\phi) = d\sigma_0^h \left\{1 + A^h_{\cos\phi}\cos\phi + A^h_{\cos 2\phi}\cos 2\phi + \ldots \right.$$
$$\left. + S_T\left[A^h_{Siv}\sin(\phi_s-\phi) + ..\right]\right\}$$

- $d\sigma_0^h \propto f_q(x) \otimes D_q^h(z)$

- $A^h_{\cos\phi}$ & $A^h_{\cos 2\phi} \simeq$ (Δf_{BM} + Cahn) contributions:

$$1)\quad A^h_{\cos\phi} \simeq \frac{1}{Q}\left[\underbrace{\Delta f^{BM}_{q^\uparrow/p}\otimes \Delta D^h_{q\uparrow}}_{BM} + \underbrace{f_q \otimes D^h_q}_{Cahn}\right]$$

$$2)\quad A^h_{\cos 2\phi} \simeq \underbrace{\Delta f^{BM}_{q^\uparrow/p}\otimes \Delta D^h_{q\uparrow}}_{BM} + \frac{1}{Q^2}\underbrace{f_q \otimes D^h_q}_{Cahn}$$

- $A^h_{Siv} \simeq \Delta f_q^{Siv} \otimes D_q^h$

Why do we discuss BM function?

a simplifying model assumption for $\Delta f^{BM}_{q^\uparrow/p}(x, k_\perp)$ is used:

BM is proportional to Siv. function: *(V. Barone, A. Prokudin & Bo Ma, 2008)*

$$\Delta f^{BM}_{q\Uparrow/p}(x_{_B}, k_\perp) = \lambda_q \, \Delta f^{Siv}_{q/p^\uparrow}(x_{_B}, k_\perp), \qquad \lambda_q = const$$

We ask:

1) Can we test this assumption using only measurable quantities - no TMD's!

Note: different assums. lead to diffn. TMD's!

2) Can we use available data to the suggested tests?

What is peculiar for Δf^{BM}?

$\Rightarrow$ Δf^{BM} **always enters the asymmetries together with Cahn effect!**

The Cahn effect

kinematic effect, calculated in pQCD $\propto f_q \otimes D_q^h$

depends on $\langle k_\perp^2 \rangle$ and $\langle p_\perp^2 \rangle$ = the "averg. transv. moment" of q and h

but controversial values for $\langle k_\perp^2 \rangle$ & $\langle p_\perp^2 \rangle$ at present:

1. old results:

- EMC (1987), Jlab (2009): $\langle k_\perp^2 \rangle \simeq 0.25\, GeV^2$, $\langle p_\perp^2 \rangle \simeq 0.20\, GeV^2$
- HERMES (2008): $\langle k_\perp^2 \rangle = 0.18\, GeV^2$, $\langle p_\perp^2 \rangle = 0.20\, GeV^2$

2. new results:

- COMPASS (2013): $\langle k_\perp^2 \rangle = 0.57 \pm 0.08\, GeV^2$, $\langle p_\perp^2 \rangle = 0.12 \pm 0.01\, GeV^2$
- HERMES (2013): $\langle k_\perp^2 \rangle = 0.61 \pm 0.20\, GeV^2$, $\langle p_\perp^2 \rangle = 0.19 \pm 0.02\, GeV^2$

– clear discrepancy of old and new values of $\langle k_\perp^2 \rangle, \langle p_\perp^2 \rangle$!

– Can we measure Cahn contribution independently?

We consider

$$l + N \to l + h^{\pm} + X, \qquad h = \pi^{\pm}, K^{\pm}, h^{\pm}$$

We use difference asymmetries:

$$A^{h^+-h^-} = \frac{\Delta\sigma^{h^+} - \Delta\sigma^{h^-}}{\sigma^{h^+} - \sigma^{h^-}}$$

$$\Delta\sigma \equiv \sigma^{\uparrow} - \sigma^{\downarrow}, \qquad \sigma \equiv \sigma^{\uparrow} + \sigma^{\downarrow}$$

• $A^{h^+-h^-}$ is not a new measurement:

$$A^{h^+-h^-} = \frac{1}{1-r}\left(A^{h^+} - r\, A^{h^-}\right),$$

$$A^{h^+} = \frac{\Delta\sigma^{h^+}}{\sigma^{h^+}}, \quad A^{h^-} = \frac{\Delta\sigma^{h^-}}{\sigma^{h^-}}, \; r = \frac{\sigma^{h^-}}{\sigma^{h^+}}$$

• advantage: $A^{h^+-h^-}$ determine only valence quark densities:

$$\Delta u_V = \Delta u - \Delta\bar{u}, \qquad \Delta d_V = \Delta d - \Delta\bar{d}$$

but with no dependence on $\bar{q}$

based on C-inv & SU(2)-inv. of strong ints:

$$C - inv: \qquad D^h_{\bar{q}} = D^{\bar{h}}_q, \qquad D^h_G = D^{\bar{h}}_G$$

• on $d = (p+n)$: $A^{h^+-h^-}_d$ determine only $Q_V = (u_V + d_V)$

We show

- if $\Delta f^{BM}_{Q_V}(x,k_\perp) = \lambda_{Q_V}\,\Delta f^{Siv}_{Q_V}(x,k_\perp), \qquad Q_V = u_V + d_V,$
- then

1. $$A^{h-\bar{h}}_{\cos\phi,d}(x) - C^h_{BM}\,\Phi(x)\,A^{h-\bar{h}}_{Siv,d}(x) = C^h_{Cahn}\,\Phi(x)$$

2. $$A^{h-\bar{h}}_{\cos 2\phi,d}(x) - \hat{C}^h_{BM}\,\hat{\Phi}(x)\,A^{h-\bar{h}}_{Siv,d}(x) = \frac{M^2}{\langle Q\rangle^2}\,\hat{C}^h_{Cahn}\,\hat{\Phi}(x)$$

- Q^2-dep. in PDFs and FFs is neglected

Only measurable quantities enter:

- $A^{h-\bar{h}}_{\cos\phi,\cos 2\phi,Siv,d}(x)$ = the difference asymms. on $d = p + n$, $h = \pi^\pm, K^\pm, h^\pm$:
- Φ & $\hat{\Phi}$ = known functions of kinem. varbls:

$$\Phi(x_B) = \frac{\sqrt{\pi}\,(2-\bar{y})\sqrt{1-\bar{y}}}{\langle Q\rangle\,[1+(1-\bar{y})^2]}, \qquad \hat{\Phi}(x_B) = \frac{2\,(1-\bar{y})}{[1+(1-\bar{y})^2]}, \qquad \bar{y} = \frac{\langle Q\rangle^2}{2M_d E\,x_B}$$

$\langle Q\rangle^2$ is some mean value of Q^2 for each x_B-bin

- C_i and $\hat{C}_i$ = consts, expressed in terms of $\Delta^N D^h_{q\uparrow}$ & $D^h_q(z,p^2_\perp)$

The expressions for C and $\hat{C}$

The expressions for C^h_{Cahn} and $C^h_{_{BM}}$:

$$C^h_{BM} \propto \lambda_{Q_V} \frac{\langle p_\perp^2\rangle^2_C}{M_C\langle p_\perp^2\rangle} \frac{\int_{0.2}^1 dz_h\,[z_h^2\langle k_\perp^2\rangle_S + 2\langle p_\perp^2\rangle_C]\,[\Delta D^h_{q_V\uparrow}(z_h)]\,/\langle P_T^2\rangle^{3/2}_{\widetilde{BM}}}{\int_{0.2}^1 dz_h\; z_h\;[D^h_{q_V}(z_h)]/\sqrt{\langle P_T^2\rangle_S}}$$

$$C^h_{Cahn} = -\langle k_\perp^2\rangle \frac{\int dz_h\, z_h [D^h_{q_V}(z_h)]/\sqrt{\langle P_T^2\rangle}}{\int dz_h\,[D^h_{q_V}(z_h)]}$$

$$\langle P_T^2\rangle = \langle p_\perp^2\rangle + z_h^2\langle k_\perp^2\rangle, \quad \langle P_T^2\rangle_{_{BM}} = \langle p_\perp^2\rangle_{_C} + z_h^2\langle k_\perp^2\rangle_{_{BM}}.$$

$$[\Delta D^h_{q_V\uparrow}(z_h)] \equiv e_u^2\,\Delta^N D^{h^+}_{u_V\uparrow} + e_d^2\,\Delta^N D^{h^+}_{d_V\uparrow}.$$

$$[D^h_{q_V}(z_h)] \equiv e_u^2\,D^{h^+}_{u_V} + e_d^2\,D^{h^+}_{d_V},$$

The expressions for $\hat{C}^h_{Cahn}$ and $\hat{C}^h_{_{BM}}$ are analogous.

but for us these are fitted consts!

Our tests on $\Delta f_{_{BM}}$:

$$\textbf{1.}\quad A^{h^+-h^-}_{\cos\phi,d}(x) - C_{_{BM}}\,\Phi(x)\,A^{h^+-h^-}_{Siv,d}(x) \;=\; C_{Cahn}\,\Phi(x)$$

$$\textbf{2.}\quad A^{h^+-h^-}_{\cos 2\phi,d}(x) - \hat{C}_{_{BM}}\,\hat{\Phi}(x)\,A^{h^+-h^-}_{Siv,d}(x) \;=\; \frac{M^2}{\langle Q\rangle^2}\,\hat{C}_{Cahn}\,\hat{\Phi}(x)$$

– we follow 2 approaches:

A) C_{BM}, C_{Cahn} / $\hat{C}_{BM}$, $\hat{C}_{Cahn}$ = fitted

B) C_{Cahn} / $\hat{C}_{Cahn}$ = calculated; C_{BM} / $\hat{C}_{BM}$ = fitted

- compare: C_{Cahn} (calc) $\Leftrightarrow$ C_{Cahn} (fitted $\equiv$ exp.)

  C_{Cahn} & $\hat{C}_{Cahn}$ (calc.) strongly depend on $\langle k_\perp^2\rangle$ and $\langle p_\perp^2\rangle$ $\Leftarrow$ indep. info

We shall use COMPASS data to both tests

Test with COMPASS data

COMPASS data, 2007, 2014, $E_\mu = 160$ GeV:

$$\mu + d \to \mu + h^\pm + X, \qquad \mu + d^\uparrow \to \mu + h^\pm + X$$

$$A^{h^\pm}_{\cos\phi}(x) = \frac{\int d\phi_h dz\, dP_T\; \cos(\phi_h)\, d\sigma^h}{\int d\phi_h\, dz\, dP_T\, d\sigma^h}, \qquad A^{h^\pm}_{\cos 2\phi}(x) = \frac{\int d\phi_h dz\, dP_T\; \cos(2\phi_h)\, d\sigma^h}{\int d\phi_h\, dz\, dP_T\, d\sigma^h}, \qquad 2014$$

$$A^{h^\pm}_{Siv}(x) = \frac{\int d\phi_h\, d\phi_s\, dz\, dP_T\; \sin(\phi_h - \phi_S)(d\sigma^h_\uparrow - \sigma^h_\downarrow)}{\int d\phi_h\, d\phi_s\, dz\, dP_T(d\sigma^h_\uparrow + d\sigma^h_\downarrow)}, \qquad 2002 - 2004\,(2007)$$

$$0,003 < x < 0,13, \qquad 1 < Q^2 < 16.7\, GeV^2$$

In 3 steps:

1. form the diff. asymmetries:

$$A^{h^+-h^-} = \frac{1}{1-r}\left(A^{h^+} - r A^{h^-}\right), \qquad r = \frac{\sigma^{h^+}}{\sigma^{h^-}}$$

2. choose the interval in Q^2: $Q_V(x,Q^2) \simeq Q_V(x)$, $\quad D_{u_V}(z,Q^2) \simeq D_{u_V}(z)$

$Q^2 \geq 1.8\,GeV^2$ is OK: $Q^2 = [1.77 - 16.27]GeV^2 \rightarrow x_{_B} = [0.014 - 0.13]$

consider 3 intervals: $x_{_B} = [0.006 - 0.13] \rightarrow Q^2 = [1.15 - 16.27]GeV^2$

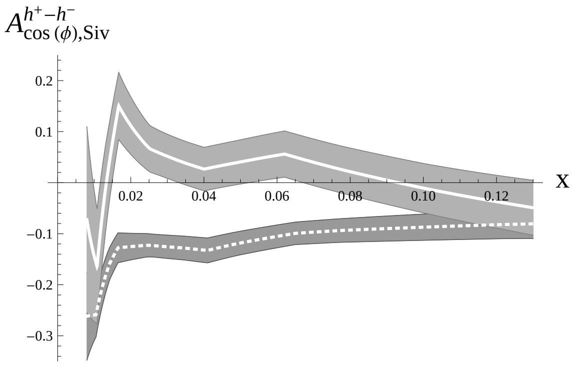
$x_{_B} = [0.014 - 0.13] \rightarrow Q^2 = [1.77 - 16.27]GeV^2$

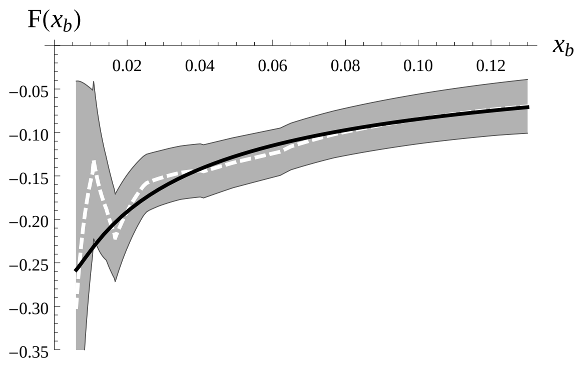
$x_{_B} = [0.022 - 0.13] \rightarrow Q^2 = [2.43 - 16.27]GeV^2$

3. χ^2-analysis $-$ C_i=?

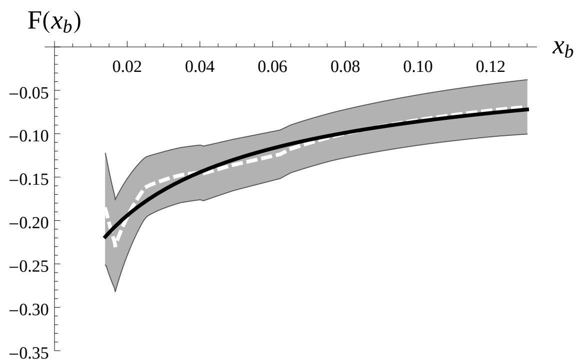
1. $A^{h^+-h^-}_{\cos\phi}(x) - C_{BM}\,\Phi(x)\,A^{h^+-h^-}_{Siv}(x) = C_{Cahn}\,\Phi(x)$

A) C_{BM}, C_{Cahn} = fitted:

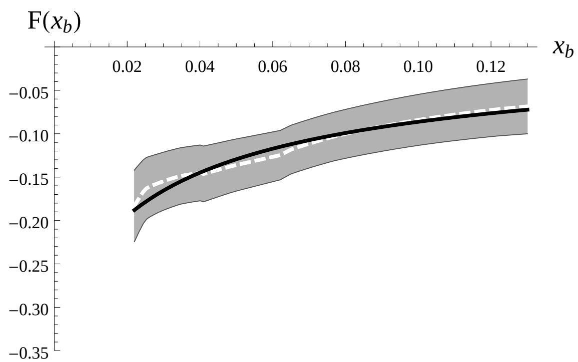




$x > 0.006$



$x > 0.014$



$x > 0.022$

- the relation b/n BM & Siv. is OK for $x > 0.014$!

B) C_{Cahn} = calculated; $C_{_{BM}}$ = fitted: a) $x > 0.014$; b) $x > 0.022$

$$C_{Cahn} = -\langle k_\perp^2 \rangle \frac{\int dz_h\, z_h [D_{q_V}^h(z_h)]/\sqrt{\langle P_T^2 \rangle}}{\int dz_h\, [D_{q_V}^h(z_h)]}, \qquad \langle P_T^2 \rangle = \langle p_\perp^2 \rangle + z_h^2 \langle k_\perp^2 \rangle$$

C_{Cahn} depends on the FF and $\langle k_\perp^2 \rangle$:

We tried 2 params. for FFs: LSS and AKK – no sensitivity, but strong dependence on $\langle k_\perp^2 \rangle$, $\langle p_\perp^2 \rangle$:

	$\mathcal{A}$	$\mathcal{B}$			
$\langle k_\perp^2 \rangle$ [GeV^2] $\langle p_\perp^2 \rangle$ [GeV^2]		0.25 0.20	0.18 0.20	0.57 ± 0.08 0.12 ± 0.01	0.61 ± 0.20 0.19 ± 0.02
C^h_{Cahn}	-0.167 ± 0.043	-0.21	-0.16	-0.49 ± 0.05	-0.4 ± 0.1
$C^h_{\widetilde{BM}}$	0.55 ± 0.80	1.43	0.44	13 ± 2	11 ± 4

A)

x_i	C^h_{Cahn} [GeV]	C_{BM} [GeV]
.014	-0.167 ± 0.043	0.55 ± 0.80
.022	-0.168 ± 0.050	0.57 ± 1.01

C^h_{Cahn} & C_{BM} =fitted

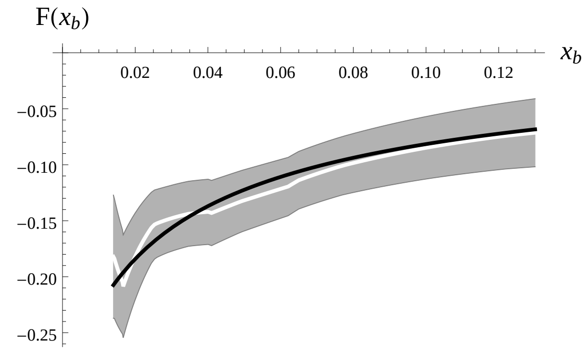
C_{BM} is badly determined – $A_{Siv,d} \simeq 0$!

C_{Cahn} (calc.) agrees with C_{Cahn} (fitted) for the old values:

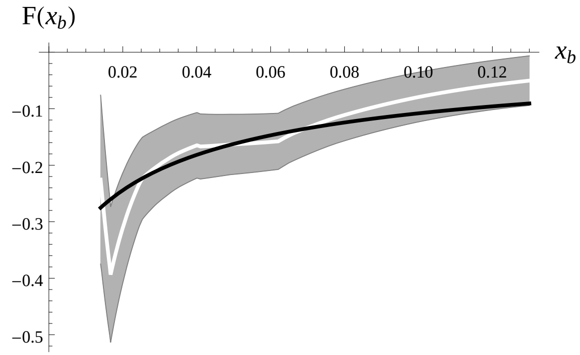
$\langle k_\perp^2 \rangle = 0.18 GeV^2$ and $\langle k_\perp^2 \rangle = 0.25 GeV^2$

$$A_{\cos\phi}^{h^+-h^-}(x) - C_{BM}\,\Phi(x)\,A_{Siv}^{h^+-h^-}(x) = C_{Cahn}\,\Phi(x)$$

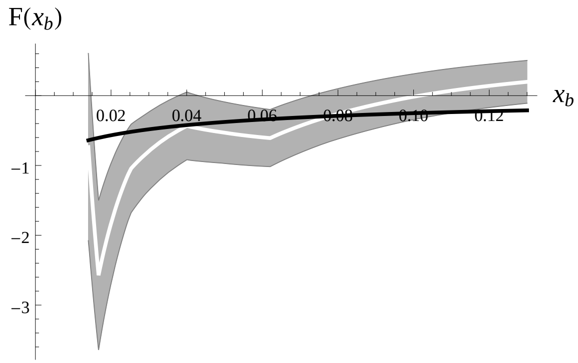
B) C_{Cahn} **= calculated;** C_{BM} **= fitted:** $x > 0.014;$



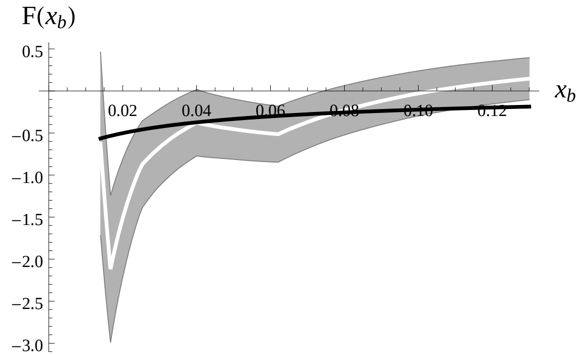
$\langle k_\perp^2 \rangle = 0.18\, GeV^2$



$\langle k_\perp^2 \rangle = 0.25\, GeV^2$



$\langle k_\perp^2 \rangle = 0.57\, GeV^2$



$\langle k_\perp^2 \rangle = 0.61\, GeV^2$

best fit for $\langle k_\perp^2 \rangle = 0.18\ GeV^2$

2. $A_{\cos 2\phi}^{h^+-h^-}(x) - \hat{C}_{BM}\,\hat{\Phi}(x) A_{Siv}^{h^+-h^-}(x) = \frac{M^2}{\langle Q\rangle^2}\,\hat{C}_{Cahn}\,\hat{\Phi}(x)$

The results are analogous but with bigger errors, due to the larger errors of $A^h_{\cos 2\phi}(x)$:

- **the relation b/n BM & Siv. is OK for $x > 0.014$!**

- **$\hat{C}_{Cahn}$ (calc.) agrees only with $\hat{C}_{Cahn}$ (fitted) for the old values:**

$\langle k_\perp^2\rangle = 0.18\,GeV^2$ and $\langle k_\perp^2\rangle = 0.25\,GeV^2$

Conclusions

We **performed 2 independent tests** for the relation b/n BM & Siv TMDs:

$$\Delta f_{Q_V}^{BM} = \lambda_{Q_V}\, \Delta f_{Q_V}^{Siv}, \qquad Q_V = u_V + d_V$$

using COMPASS data on $A_{\cos\phi}$, $A_{\cos 2\phi}$ and A_{Siv} on d in 2 different ways.

all tests agree with the assumption for the same inv: $x = [0.014, 0.13]$!

However:

1. the used assumption in Δf^{BM} is: [*V.Barone et al, 2008, 2010*]

$$\Delta f_q^{BM}(x, k_\perp) = \lambda_q\, \Delta f_q^{Siv}(x, k_\perp), \qquad \lambda_u \neq \lambda_d \neq \lambda_{\bar q}$$

Our result agrees with this assumption only if

$$\lambda_u = \lambda_d = \lambda_{\bar u} = \lambda_{\bar d}$$

- **But,** the obtained values are:

$$\lambda_u = 2.01 \pm 0.001, \quad \lambda_d = -1.111 \pm 0.001, \quad |\lambda_{\bar q}| = 1$$

$\Rightarrow$ these 2 sets of results are not consistent with each other.

Recall:

These contradictory results are obtained in 2 completely different analysis:

[*V.Barone et al, 2008, 2010*]: $A_{\cos 2\phi}$, using definite params. of Sivers and Collins TMDs, and definite Cahn effect

[*We*] use only measurable $A_{\cos\phi,\cos 2\phi,Siv}$, no TMDs, and we test the relation for $Q_V = u_V + d_V$, and we measure Cahn effect

What we can do in the future: on p-target we can test separately:

$$\Delta f_{u_V}^{BM}(x, k_\perp) = \lambda_{u_V}\, \Delta f_{u_V}^{Siv}(x, k_\perp),$$
$$\Delta f_{d_V}^{BM}(x, k_\perp) = \lambda_{d_V}\, \Delta f_{d_V}^{Siv}(x, k_\perp),$$

2. We determine Cahn contb. from fit to data & from calculations −

C_{Cahn} & $\hat{C}_{Cahn}$ = calculated depends strongly on $\langle k_\perp^2 \rangle$;

unexpectedly: Cahn contb. in both tests favours the old values of $\langle k_\perp^2 \rangle$:

$$\langle k_\perp^2 \rangle = 0.18\, GeV^2 \quad \& \quad \langle k_\perp^2 \rangle = 0.25\, GeV^2$$

THANK YOU!