Weighted Sivers asymmetry in SIDIS at COMPASS

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On behalf of the COMPASS Collaboration

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Outline

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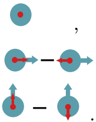
Sivers asymmetry in SIDIS: Sivers effect

COMPASS

- Parton intrinsic momentum $\boldsymbol{k}_{\mathrm{T}}$ integrated over:
  - Hadron structure described by 3 PDFs:
  - Number density $f_1(x)$, or $q(x)$,
  - helicity $g_1(x)$, or $\Delta q(x)$,
  - transversity $h_1(x)$, or $\Delta_{\mathrm{T}} q(x)$.



- Often parton intrinsic $\boldsymbol{k}_{\mathrm{T}}$ cannot be neglected, e.g. if hadron produced by the struck quark is observed.
- Transverse Momentum Dependent (TMD) PDFs:
  - Two-dimensional objects $f(x, k_{\mathrm{T}}^2)$.
  - Integration over $\boldsymbol{k}_{\mathrm{T}} \rightarrow$ 'collinear' PDFs or zero.

		Parent hadron polarization		
		Unpolarised	Longitudinal	Transverse
Parton p.	U	$f_1(x, k_{\mathrm{T}}^2)$ (number density)		$f_{1T}^{\perp}(x, k_{\mathrm{T}}^2)$ (Sivers)
	L		$g_1(x, k_{\mathrm{T}}^2)$ (helicity)	$g_{1T}(x, k_{\mathrm{T}}^2)$
	T	$h_1^{\perp}(x, k_{\mathrm{T}}^2)$ (Boer–Mulders)	$h_{1L}^{\perp}(x, k_{\mathrm{T}}^2)$	$h_1(x, k_{\mathrm{T}}^2)$ (transversity) $h_{1T}^{\perp}(x, k_{\mathrm{T}}^2)$

Sivers PDF

Left-right asymmetry in distribution of quarks. Result: asymmetry in production of hadrons.

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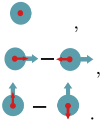

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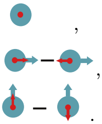

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Sivers PDF

It correlates unpolarised parton $\boldsymbol{k}_{\mathbf{T}}$ with transverse polarisation of the nucleon.

Left-right asymmetry in distribution of quarks.
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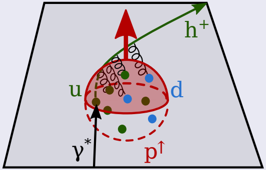


Sivers effect in SIDIS as interpreted by M. Burkardt [M. Burkardt, Nucl.Phys.A735 (2004) 185.].

Sivers asymmetry in SIDIS: Cross-section

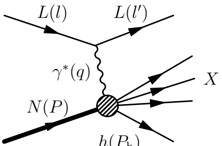
- Semi-inclusive DIS (SIDIS) of μ^+ on transversely polarised H in an NH_3 target,

$$\mu(l) + \mathrm{p}(P, \boldsymbol{S}_\mathbf{T}) \to \mu(l') + h(P_\mathrm{h}) + X.$$

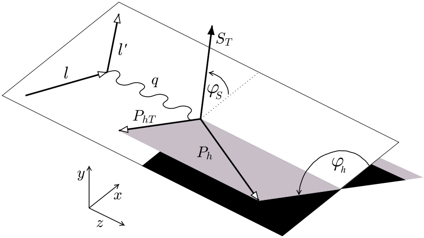
- Cross-section (LO in $1/Q$, $P_{\mathrm{hT}} \ll Q$) [A. Bacchetta *et al.*, JHEP 0702 (2007) 093]:

$$\frac{\mathrm{d}\sigma_{\mathrm{SIDIS}}}{\mathrm{d}x\mathrm{d}y\mathrm{d}z\mathrm{d}\phi_\mathrm{S}\mathrm{d}\phi_\mathrm{h}\mathrm{d}P_{\mathrm{hT}}^2} = \frac{\alpha^2}{xyQ^2}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{\frac{2-2y+y^2}{2}F_{\mathrm{UU,T}}$$
$$+ (2-y)\sqrt{1-y}\cos\phi_\mathrm{h}F_{\mathrm{UU}}^{\cos\phi_\mathrm{h}} + (1-y)\cos(2\phi_\mathrm{h})F_{\mathrm{UU}}^{\cos 2\phi_\mathrm{h}}$$
$$+ |\boldsymbol{S}_\mathbf{T}|\Bigg[\frac{2-2y+y^2}{2}\sin(\phi_\mathrm{h}-\phi_\mathrm{S})F_{\mathrm{UT,T}}^{\sin(\phi_\mathrm{h}-\phi_\mathrm{S})} + (1-y)\sin(\phi_\mathrm{h}+\phi_\mathrm{S})F_{\mathrm{UT}}^{\sin(\phi_\mathrm{h}+\phi_\mathrm{S})}$$
$$+ (1-y)\sin(3\phi_\mathrm{h}-\phi_\mathrm{S})F_{\mathrm{UT}}^{\sin(3\phi_\mathrm{h}-\phi_\mathrm{S})}\Bigg]\Bigg\}.$$

- Sivers asymmetry: $\sin(\phi_\mathrm{h}-\phi_\mathrm{S})$ modulation amplitude.



SIDIS process at tree level.



SIDIS process in the $\gamma^* N$ frame.

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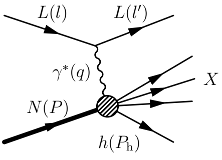

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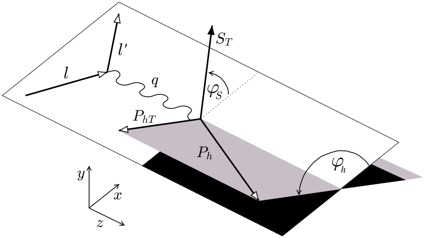

SIDIS process in the $\gamma^* N$ frame.

Sivers asymmetry in SIDIS: Convolution of TMDs

COMPASS

- Cross-section, substitution $\phi_S = \Phi_{Siv} + \phi_h$, integration over ϕ_h:

$$\frac{d\sigma_{SIDIS}}{dx\,dy\,dz\,d\Phi_{Siv}\,dP_{hT}^2} = C(x,y,Q^2)\left[F_{UU,T}(x,z,P_{hT}^2,Q^2) + |\boldsymbol{S}_{\mathbf{T}}|\sin(\Phi_{Siv})F_{UT,T}^{\sin(\phi_h-\phi_S)}(x,z,P_{hT}^2,Q^2)\right],$$

- Sivers asymmetry (we omit the dependency of F on the scale Q^2 from now on):

$$A_{UT,T}^{\sin(\phi_h-\phi_S)}(x,z,P_{hT}^2) = \frac{F_{UT,T}^{\sin(\phi_h-\phi_S)}(x,z,P_{hT}^2)}{F_{UU,T}(x,z,P_{hT}^2)}.$$

- TMD factorisation [A. Bacchetta *et al.*, JHEP 0702 (2007) 093]:

$$F_{UU,T}(x,z,P_{hT}^2) = \mathcal{C}\left[f_1(x,k_T^2)D_1(z,p_\perp^2)\right]$$

$$F_{UT,T}^{\sin(\phi_h-\phi_S)}(x,z,P_{hT}^2) = \mathcal{C}\left[\frac{\boldsymbol{P}_{hT}\cdot\boldsymbol{k}_T}{P_{hT}M}f_{1T}^{\perp}(x,k_T^2)D_1(z,p_\perp^2)\right],$$

- where $\mathcal{C}[wfD]$ denotes convolution over intrinsic transverse momenta

$$\mathcal{C}[wfD] = x\sum_q e_q^2\int d^2\boldsymbol{p}_\perp d^2\boldsymbol{k}_T\,\delta^{(2)}(\boldsymbol{P}_{hT} - \boldsymbol{p}_\perp - z\boldsymbol{k}_T) \times w(\boldsymbol{p}_\perp,\boldsymbol{k}_T,\boldsymbol{P}_{hT})\,f^q(x,k_T^2)\,D^q(z,p_\perp^2).$$

Clear signal for π^+, K^+ on protons (HERMES, COMPASS) $\Rightarrow f_{1T}^{\perp q} \neq 0$,
no signal on deuterons (COMPASS) $\Rightarrow f_{1T}^{\perp u} \approx -f_{1T}^{\perp d}$,
(see the talks of H. Avagyan, A. Bressan, C. Van Hulse, Z. Meziani...).

Sivers asymmetry in SIDIS: Convolution of TMDs

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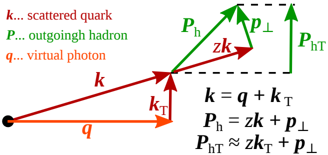

Addition of momenta in the $\gamma^* N$ frame.

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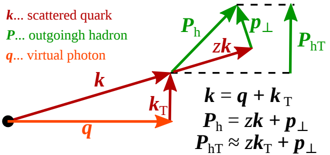

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Sivers asymmetry in SIDIS: Convolution of TMDs

- The convolution over intrinsic transverse momenta

$$\mathcal{C}[wfD] = x\sum_q e_q^2 \int \mathrm{d}^2\boldsymbol{p}_\perp \mathrm{d}^2\boldsymbol{k}_\mathbf{T}\,\delta^{(2)}(\boldsymbol{P}_\mathbf{hT} - \boldsymbol{p}_\perp - z\boldsymbol{k}_\mathbf{T}) w(\boldsymbol{p}_\perp, \boldsymbol{k}_\mathbf{T}, \boldsymbol{P}_\mathbf{hT})\, f^q(x, k_\mathrm{T}^2)\, D^q(z, p_\perp^2).$$

- It can be easily shown that the integration of $F_{\mathrm{UU,T}}(x, z, P_{\mathrm{hT}}^2)$ over $\mathrm{d}^2\boldsymbol{P}_\mathbf{hT}$ gives

$$F_{\mathrm{UU,T}}(x,z) = \int \mathrm{d}^2\boldsymbol{P}_\mathbf{hT}\,\mathcal{C}\left[f_1(x, k_\mathrm{T}^2) D_1(z, p_\perp^2)\right] = x\sum_q e_q^2 f_1^q(x) D_1^q(z).$$

- On the contrary, the integration of $F_{\mathrm{UT,T}}^{\sin(\phi_\mathrm{h}-\phi_\mathrm{S})(x,z,P_{\mathrm{hT}}^2)}$ over $\mathrm{d}^2\boldsymbol{P}_\mathbf{hT}$

$$F_{\mathrm{UT,T}}^{\sin(\phi_\mathrm{h}-\phi_\mathrm{S})}(x,z) = \int \mathrm{d}^2\boldsymbol{P}_\mathbf{hT}\,\mathcal{C}\left[\frac{\boldsymbol{P}_\mathbf{hT}\cdot\boldsymbol{k}_\mathbf{T}}{P_\mathrm{hT}M} f_{1\mathrm{T}}^\perp(x, k_\mathrm{T}^2) D_1(z, p_\perp^2)\right] = ?$$

  can be solved only if assumptions are made on k_T^2 and $p_\perp^2$ dependence of $f_{1\mathrm{T}}^\perp$ and D_1.
- Popular solution: Gaussian model

$$f_{1\mathrm{T}}^\perp(x, k_\mathrm{T}^2) = f_{1\mathrm{T}}^\perp(x)\frac{\mathrm{e}^{-k_\mathrm{T}^2/\langle k_\mathrm{T}^2\rangle_\mathrm{Siv}}}{\pi\langle k_\mathrm{T}^2\rangle_\mathrm{Siv}} \qquad D_1(x, p_\perp^2) = D_1(z)\frac{\mathrm{e}^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle},$$

  yielding (assuming flavour independent Gaussian widths)

$$F_{\mathrm{UT,T}}^{\sin(\phi_\mathrm{h}-\phi_\mathrm{S})}(x,z) = a_\mathrm{G} x\sum_q e_q^2 f_{1\mathrm{T}}^{\perp q(1)}(x) D_1^q(z).$$

  where the Gaussian factor a_G and the first k_T^2-moment of the Sivers function are

$$a_\mathrm{G} = \frac{\sqrt{\pi}M}{\sqrt{\langle k_\mathrm{T}^2\rangle_\mathrm{Siv} + \langle p_\perp^2\rangle/z^2}} \qquad f_{1\mathrm{T}}^{\perp q(1)}(x) = \int \mathrm{d}^2\boldsymbol{k}_\mathbf{T}\frac{k_\mathrm{T}^2}{2M^2} f_{1\mathrm{T}}^{\perp q}(x, k_\mathrm{T}^2).$$

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$$a_\mathrm{G} = \frac{\sqrt{\pi}M}{\sqrt{\langle k_\mathrm{T}^2\rangle_\mathrm{Siv} + \langle p_\perp^2\rangle/z^2}} \qquad f_{1\mathrm{T}}^{\perp q(1)}(x) = \int \mathrm{d}^2\boldsymbol{k}_\mathbf{T}\frac{k_\mathrm{T}^2}{2M^2} f_{1\mathrm{T}}^{\perp q}(x, k_\mathrm{T}^2).$$

Outline

Transverse momentum weighting:

- However, the Gaussian assumption is strong...
- Possible alternative: weighting with powers of the transverse momentum
  - [A. Kotzinian and P. Mulders, Phys.Lett. B406 (1997) 373]
  - [D. Boer and P. Mulders, Phys.Rev. D57 (1998) 5780]
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$$\int \mathrm{d}^2\boldsymbol{P}_{\mathbf{hT}} \frac{P_{\mathrm{hT}}}{zM} F_{\mathrm{UT,T}}^{\sin(\phi_\mathrm{h}-\phi_\mathrm{S})}(x,z) = \int \mathrm{d}^2\boldsymbol{P}_{\mathbf{hT}} \frac{P_{\mathrm{hT}}}{zM} \mathcal{C}\left[\frac{\boldsymbol{P}_{\mathbf{hT}}\cdot\boldsymbol{k}_{\mathbf{T}}}{P_{\mathrm{hT}}M} f_{1\mathrm{T}}^{\perp}(x,k_{\mathrm{T}}^2) D_1(z,p_\perp^2)\right]$$

$$= x\sum_q e_q^2 \int \mathrm{d}^2\boldsymbol{k}_{\mathbf{T}}\mathrm{d}^2\boldsymbol{p}_\perp \frac{\boldsymbol{p}_\perp\cdot\boldsymbol{k}_{\mathbf{T}} + zk_{\mathrm{T}}^2}{zM^2} f_{1\mathrm{T}}^{\perp q}(x,k_{\mathrm{T}}^2) D_1^q(z,p_\perp^2)$$

$$= 2x\sum_q e_q^2 f_{1\mathrm{T}}^{\perp q(1)}(x) D_1^q(z)$$

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Measurements: (preliminary) HERMES [I. Gregor (HERMES), Acta Phys.Polon. B36 (2005) 209], COMPASS (this talk).

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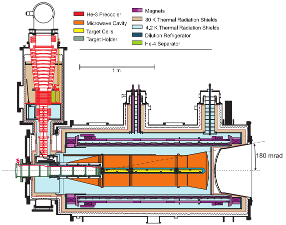
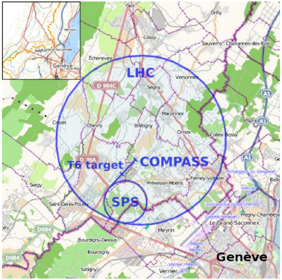
Outline

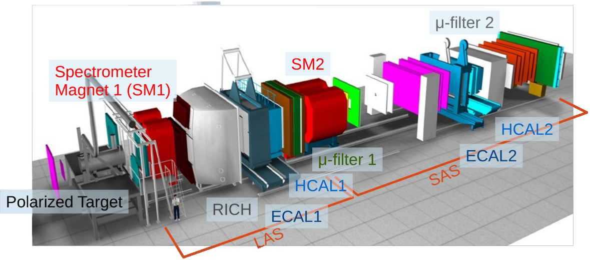
- COMPASS Collaboration.
- Multi-purpose apparatus. SIDIS 2010 setup:
  - Transversely polarised p (NH_3) target
    polarisation ≈ 85%, 3 oppositely-pol. cells.
  - 160 GeV/c μ^+ beam, about 10^9 μ^+/spill of 10 s
  - Two-stage spectrometer, about 350 detector planes.
  - Particle identification RICH for hadrons, μ filters.



Polarised target cryostat.



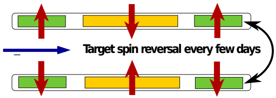
Location of the site at CERN's SPS



COMPASS SIDIS setup.

Measurement: Asymmetry calculation

- Data and event selection the same as for published Sivers asymmetry [C. Adolph *et al.*(COMPASS), Phys.Lett. B717 (2012) 383].
- Polarised target with 2 cells $c = \mathrm{O}$ ('outer'), I ('inner'),
- and periods $p = 1, 2$ with opposite polarisation $\uparrow\downarrow\uparrow, \downarrow\uparrow\downarrow$.
- Weighted Sivers asymmetry:



$$A_{\mathrm{UT,T}}^{\sin(\phi_{\mathrm{h}}-\phi_{\mathrm{S}})W}(x,z) = \frac{\int \mathrm{d}^2 \boldsymbol{P}_{\mathbf{hT}} W F_{\mathrm{UT,T}}^{\sin(\phi_{\mathrm{h}}-\phi_{\mathrm{S}})}(x,z)}{\int \mathrm{d}^2 \boldsymbol{P}_{\mathbf{hT}} F_{\mathrm{UU,T}}(x,z)}.$$

- Only the spin-dependent part of the cross-section is weighted!
  $\rightarrow$ different methods from the standard asymmetries.
- $N_{cp}(\phi_{\mathrm{h}} - \phi_{\mathrm{S}})$ – number of events
- $N_{cp}^{W}(\phi_{\mathrm{h}} - \phi_{\mathrm{S}})$ – sum of weights of events
- We calculate the ratio:

$$\frac{N_{\mathrm{O1}}^{W} N_{\mathrm{I2}}^{W} - N_{\mathrm{O2}}^{W} N_{\mathrm{I1}}^{W}}{\sqrt{(N_{\mathrm{O1}}^{W} N_{\mathrm{I2}}^{W} + N_{\mathrm{O2}}^{W} N_{\mathrm{I1}}^{W})(N_{\mathrm{O1}} N_{\mathrm{I2}} + N_{\mathrm{O2}} N_{\mathrm{I1}})}} \approx 2\, \overline{S_{\mathrm{T}}}\, A_{\mathrm{T}}^{\sin(\phi_{\mathrm{h}}-\phi_{\mathrm{S}})W} \sin(\phi_{\mathrm{h}} - \phi_{\mathrm{S}}),$$

- Acceptance $a(\phi_{\mathrm{h}} - \phi_{\mathrm{S}})$ is cancelled.

$$\overline{S_{\mathrm{T}}} = \langle f_{\mathrm{dil.}} \rangle \langle P_{\mathrm{targ.}} \rangle.$$

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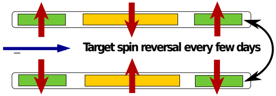

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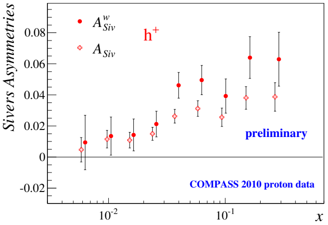
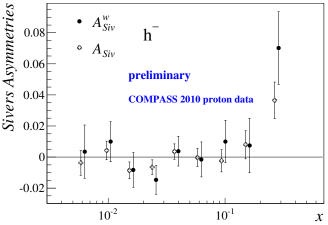
Outline

Results: $P_{\mathrm{hT}}/(zM)$-weighted Sivers asymmetry





The Sivers asymmetries in production of h^+, h^- with $z > 0.2$ weighted with $P_{\mathrm{hT}}/(zM)$[1], compared with the 'standard', published Sivers asymmetry[2]. And their ratio for h^+.

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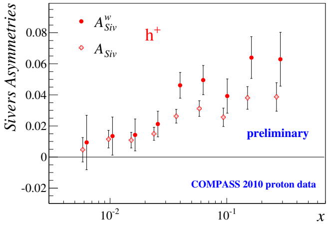

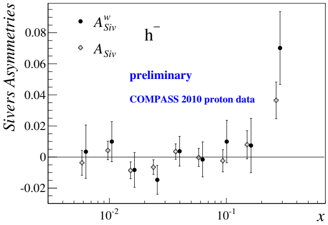

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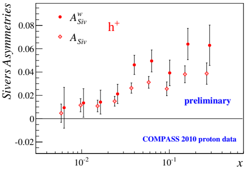

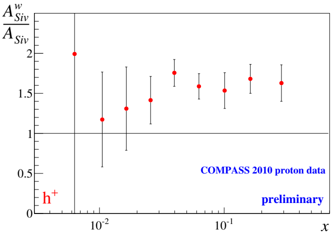

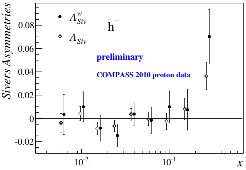

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- Dependence of the weighted asymmetry on z:

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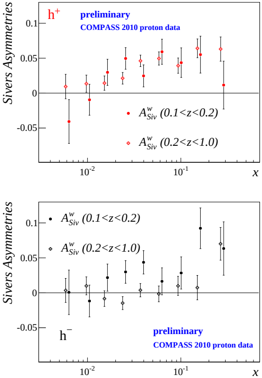

$P_{\mathrm{hT}}/(zM)$-weighted Sivers asymmetries, $h^\pm$ with $0.1 < z < 0.2$ and $z > 0.2$ in bins of x.

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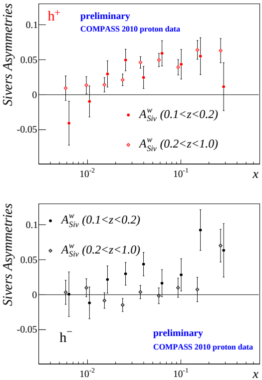

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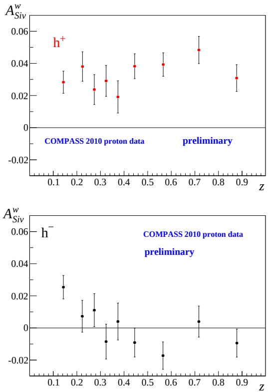

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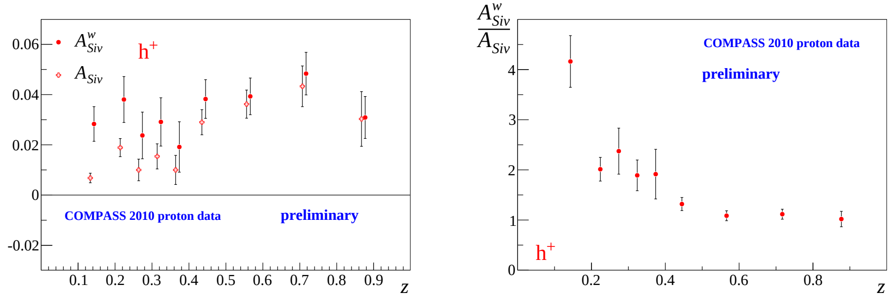

The comparison and the ratio of the $P_{hT}/(zM)$-weighted Sivers asymmetries and the 'standard', published ones[3] for h^+ in bins of z.

- Note: the weighted asymmetry for h^+:

$$A^w_{\rm Siv}(z) \approx 2\frac{\int {\rm d}x C(x) f_{1T}^{\perp u(1)}(x)}{\int {\rm d}x C(x) f_1^u(x)} \qquad \text{(note : independent of } z\text{)},$$

- the 'standard' asymmetry + the Gaussian model for k_T^2 + assuming $\langle k_T^2\rangle_{\rm Siv} = \langle k_T^2\rangle$:

$$A_{\rm Siv}(x) \approx a_G(z)\frac{\int {\rm d}x C(x) f_{1T}^{\perp u(1)}(x)}{\int C(x) {\rm d}x f_1^u(x)}, \qquad a_G(z) = \frac{\sqrt{\pi}M}{\sqrt{\langle k_T^2\rangle_{\rm Siv} + \langle p_\perp^2\rangle / z^2}} \approx \frac{\pi M z}{2\langle P_{hT}\rangle}.$$

[3] [C. Adolph *et al.*(COMPASS), Phys.Lett. B717 (2012) 383]

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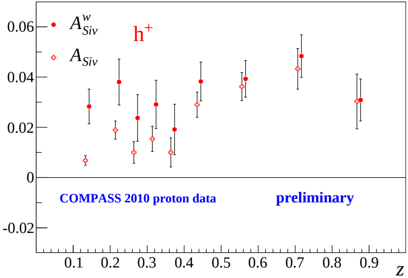

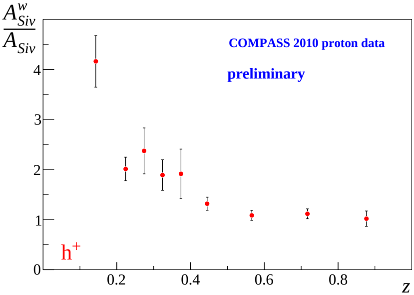

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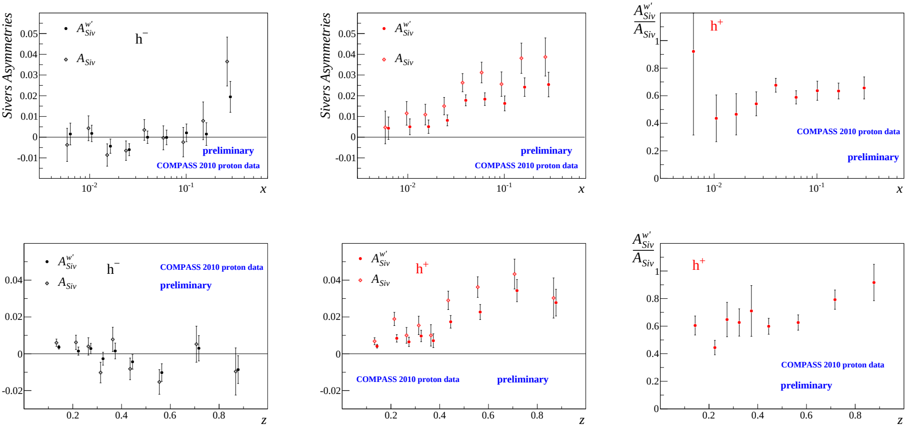
Results: P_{hT}/M-weighted Sivers asymmetry

- We remove z from the weight to better compare with the Gaussian assumption:

$$A_{\mathrm{Siv}}^{w'} = A_{\mathrm{UT,T}}^{\sin(\phi_h-\phi_S)\frac{P_{hT}}{M}} = \frac{\int d^2 \boldsymbol{P}_{\mathbf{hT}} \frac{P_{hT}}{M} F_{\mathrm{UT,T}}^{\sin(\phi_h-\phi_S)}(x,z)}{\int d^2 \boldsymbol{P}_{\mathbf{hT}} F_{\mathrm{UU,T}}(x,z)} = 2\frac{\sum_q e_q^2 x f_{1T}^{\perp q(1)}(x) z D_1^q(z)}{\sum_q e_q^2 x f_1^q(x) D_1^q(z)}.$$



P_{hT}/M-weighted Sivers asymmetries compared with the 'standard', published ones for $h^{\pm}$ and in bins of x and z, and the ratio $A_{\mathrm{Siv}}^{w'}/A_{\mathrm{Siv}}$ for h^+.

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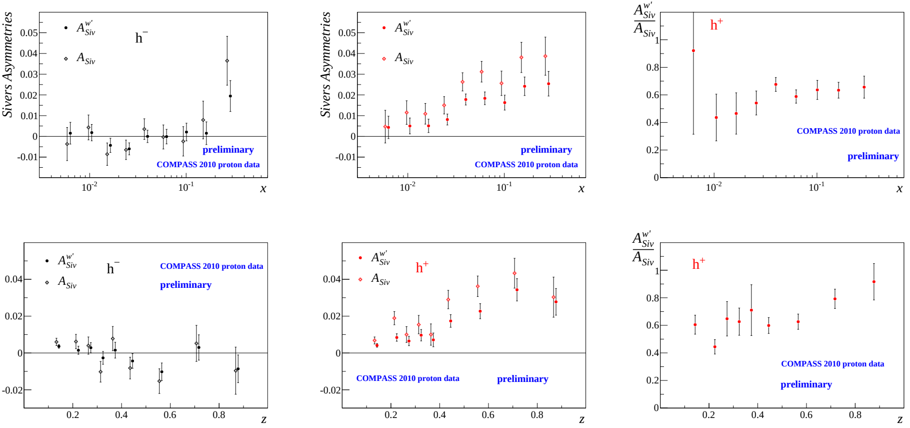

P_{hT}/M-weighted Sivers asymmetries compared with the 'standard', published ones for $h^{\pm}$ and in bins of x and z, and the ratio $A_{\mathrm{Siv}}^{w'}/A_{\mathrm{Siv}}$ for h^+.

Outline

- The expression for the $P_{\mathrm{hT}}/(zM)$-weighted Sivers asymmetry is very straightforward – it is natural to try the extraction!
- Having measured the $P_{\mathrm{hT}}/(zM)$-weighted Sivers asymmetry in SIDIS (> 0) and the q_{T}-weighted asymmetry in Drell–Yan (≈ 0) (see the talk of B. Parsamyan, [J. Matoušek (COMPASS), arXiv:1710.06497 [hep-ex], to appear in proc. of DSPIN-17]).
- Obvious question: How large asymmetry can we expect in Drell–Yan?
- Inspiration:
  - Extraction of Sivers function from SIDIS [A. Martin *et al.*, Phys.Rev. D95 (2017) 094024],
  - projection of the weighted asymmetry for DY [A. Efremov *et al.*, Phys.Lett. B612 (2005) 233].

Extraction of Sivers 1st moment: Motivation

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Extraction of Sivers 1st moment: PDFs and FFs

- SIDIS events, for $h^{\pm}$ with $z > 0.2$ in bins of x.
- u, d, s, ū, d̄, s̄ quarks for the unpolarised PDFs,
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$$A_{\mathrm{UT,T},h^{\pm}}^{\sin(\phi_{\mathrm{h}}-\phi_{\mathrm{S}})\frac{P_{\mathrm{hT}}}{zM}}(x,Q^2) = 2\,\frac{\frac{4}{9}\,f_{1\mathrm{T}}^{\perp(1)\mathrm{u}}(x,Q^2)\,\tilde{D}_{1,\mathrm{u}}^{h^{\pm}}(Q^2) + \frac{1}{9}\,f_{1\mathrm{T}}^{\perp(1)\mathrm{d}}(x,Q^2)\,\tilde{D}_{1,\mathrm{d}}^{h^{\pm}}(Q^2)}{\sum_q e_q^2\, f_1^q(x,Q^2)\tilde{D}_{1,q}^{h^{\pm}}(Q^2)},$$

- PDFs – from CTEQ 5D global fit
  [H. Lai *et al.* (CTEQ), Eur.Phys.J. C12 (2000) 375]
- The FFs from DSS 07 LO global fit
  [D. de Florian *et al.*, Phys.Rev. D75 (2007) 114010]

$$\tilde{D}_{1,q}^{h^{\pm}}(Q^2) = \int_{0.2}^{1} \mathrm{d}z D_{1,q}^{h^{\pm}}(z,Q^2)$$

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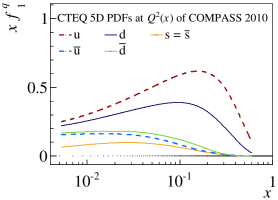

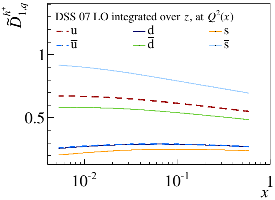

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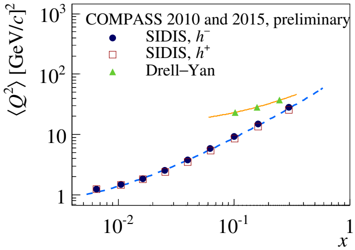
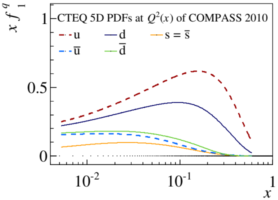
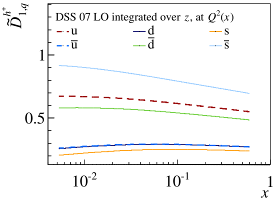






Extraction of Sivers 1st moment: Fit results

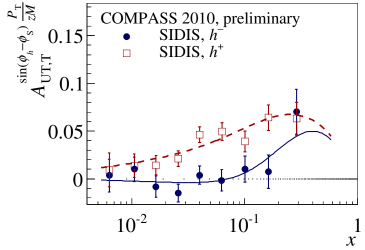
- The only 2 unknowns – Sivers 1st k_T^2-moment of u and d. We use parametrisation

$$x f_{1\mathrm{T}}^{\perp(1)q}(x) = a_q\, x^{b_q}\, (1-x)^{c_q}, \qquad q = \mathrm{u}, \mathrm{d}.$$

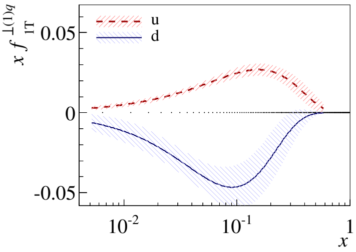
- The asymmetry for h^- and h^+ is simultaneously fitted.

$$A_{\mathrm{UT,T},h^\pm}^{\sin(\phi_\mathrm{h}-\phi_\mathrm{S})\frac{P_\mathrm{hT}}{zM}}(x, Q^2) = 2\,\frac{\frac{4}{9}\, f_{1\mathrm{T}}^{\perp(1)\mathrm{u}}(x, Q^2)\, \tilde{D}_{1,\mathrm{u}}^{h^\pm}(Q^2) + \frac{1}{9}\, f_{1\mathrm{T}}^{\perp(1)\mathrm{d}}(x, Q^2)\, \tilde{D}_{1,\mathrm{d}}^{h^\pm}(Q^2)}{\sum_q e_q^2\, f_1^q(x, Q^2) \tilde{D}_{1,q}^{h^\pm}(Q^2)},$$

- Error bands: 1σ, only stat. error of the data and fit.

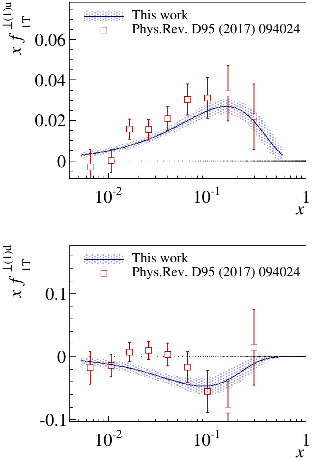


Fit of the $P_\mathrm{hT}/(zM)$-weighted Sivers asymmetry in SIDIS.



The 1st k_T^2-moment of the Sivers function at $Q^2 = Q^2_\mathrm{SIDIS}(x)$.

Extraction of Sivers 1st moment: Comparison with other works



The Sivers 1st moments obtained from the weighted asymmetries (curves) and the point-by-point extraction from the 'standard' asymmetry from the same data [A. Martin *et al.*, Phys.Rev. D95 (2017) 094024].

- The magnitude of our function for u is somewhat smaller.
- Note: both our function and the points are $f_{1T}^{\perp q(1)}(x, Q^2(x))$.
- Our uncertainty band is narrow because of the restrictive ansatz.

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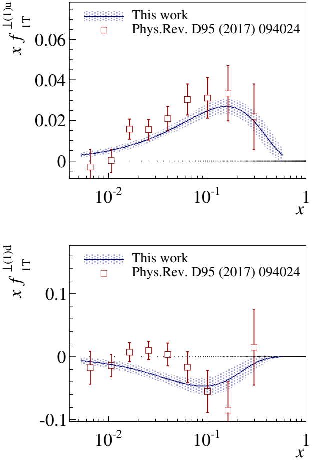

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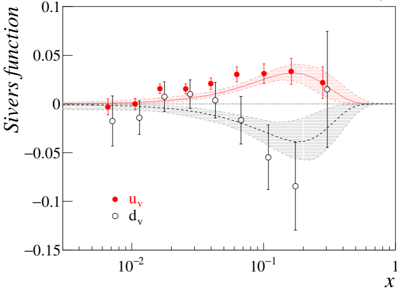

Comparison of the point-by-point extraction with a recent fit considering TMD evolution [M. Anselmino *et al.*,, Phys.Rev. D86 (2012) 014028.], taken at $Q^2 = 4\ (\mathrm{GeV}/c)^2$.
Fig. from A. Martin *et al.*

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Extraction of Sivers 1st moment: Projection for Drell–Yan

- $f_{1T}^{\perp q}\big|_{DY} = -f_{1T}^{\perp q}\big|_{SIDIS}$ [J. Collins, Phys.Lett. B536 (2002) 43]
- We assume valence quark dominance:

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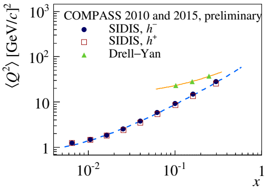
- Collinear evolution of f_1, $Q^2 = Q^2_{DY}(x_N)$ from fit.
- No evolution of the Sivers function first moment between $Q^2_{SIDIS}(x)$ and $Q^2_{DY}(x_N)$



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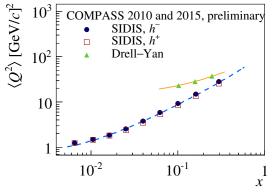
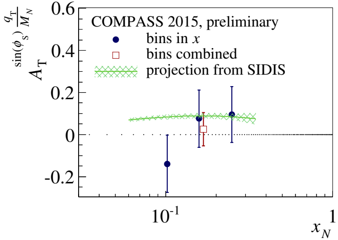
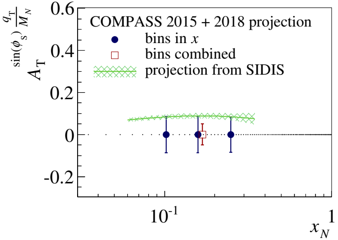




Weighted Sivers asymmetry in Drell–Yan measured in 2015 data and the projection from SIDIS. Statistical errors only.



Projection for combined 2015 and 2018 data (assuming 1.5 times larger statistics in 2018).

Outline

- The transverse momentum weighted asymmetries are interesting!
  - A model-independent way to overcome the convolution over intrinsic $\boldsymbol{k}_{\mathrm{T}}$,
  - i.e. provide direct access to the k_{T}^2-moments of TMD PDFs.
- COMPASS has measured in bins of x and z the Sivers asymmetries in SIDIS weighted with
  - $P_{\mathrm{hT}}/(zM)$ – easier interpretation in terms of TMDs,
  - P_{hT}/M – easier comparison with the results of the the Gaussian assumption.
- Interesting comparisons with the results of the the Gaussian assumption.
- A first straightforward attempt on Sivers 1st k_{T}^2-moment extraction gives quite reasonable result.

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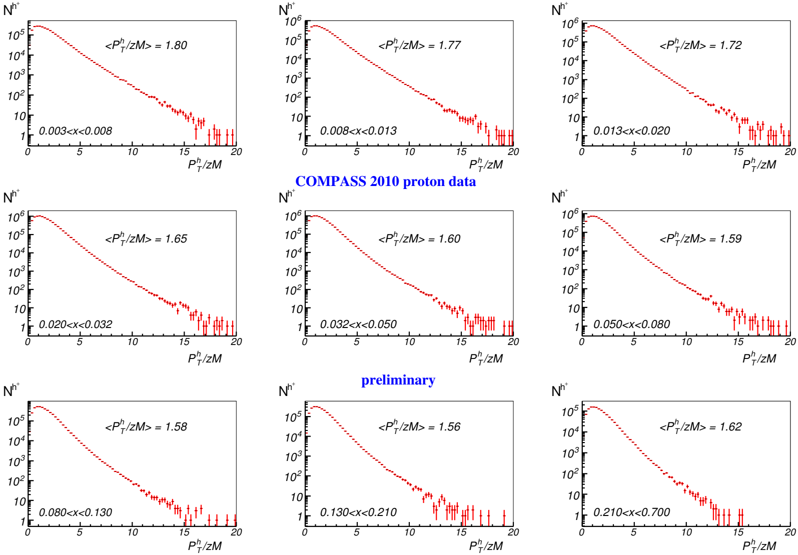
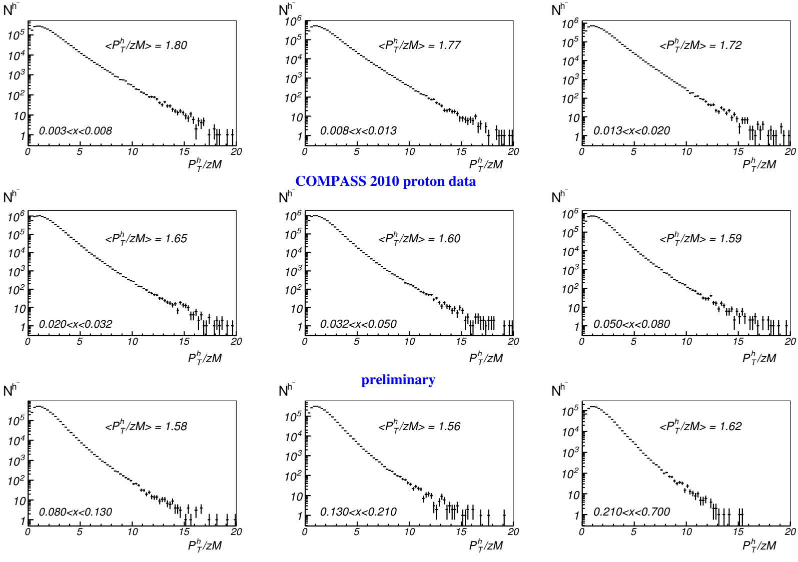
Thank you for your attention!

Backup: Distribution of $P_{hT}/(zM)$ in bins of x for h+



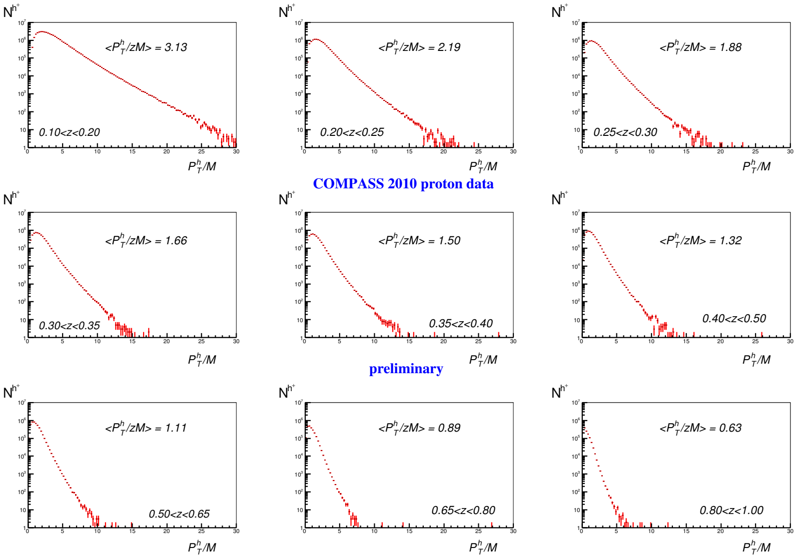
Positive hadrons, $z > 0.2$.

Backup: Distribution of $P_{hT}/(zM)$ in bins of x for h–



Negative hadrons, $z > 0.2$.

Backup: Distribution of $P_{hT}/(zM)$ in bins of z for h+



Positive hadrons.

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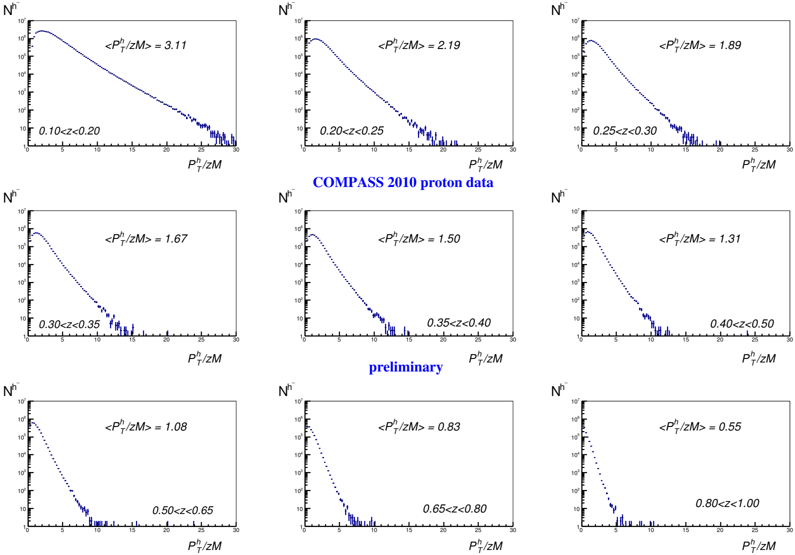
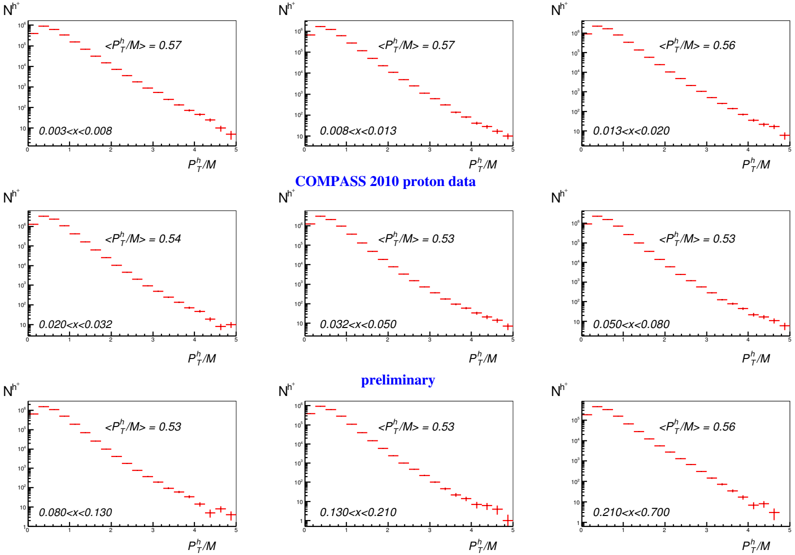


Negative hadrons.

Backup: Distribution of P_{hT}/M in bins of x for h+



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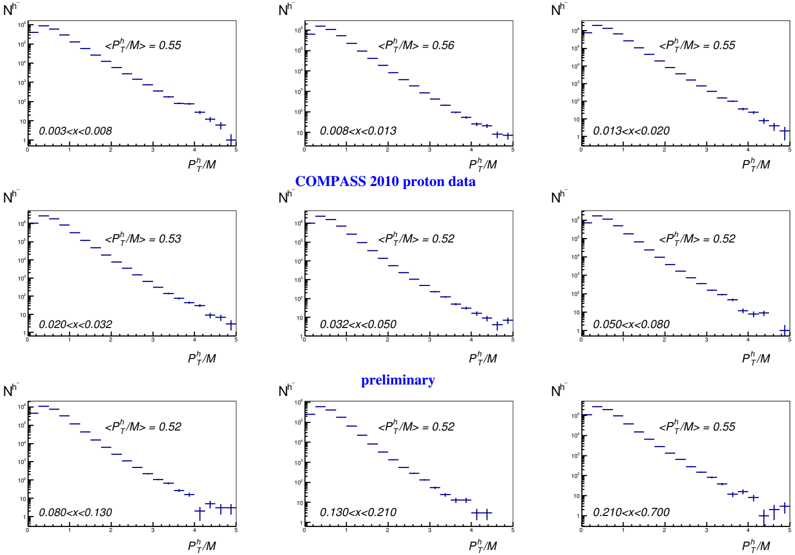

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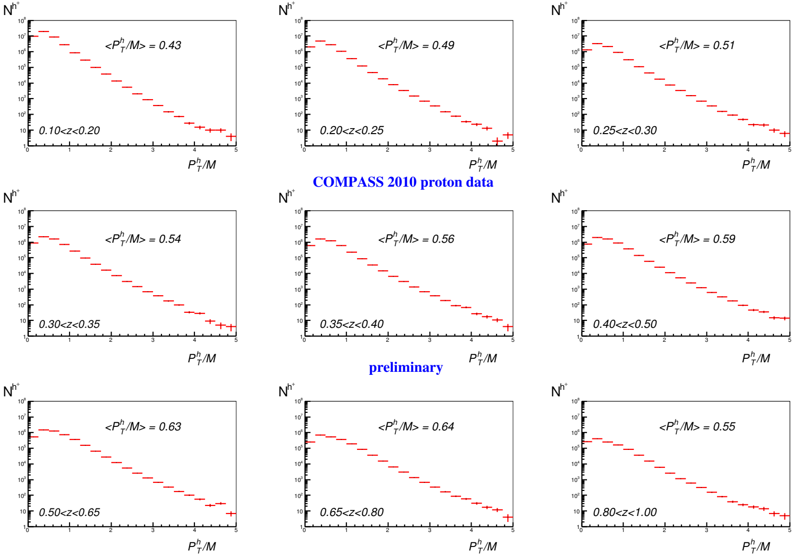

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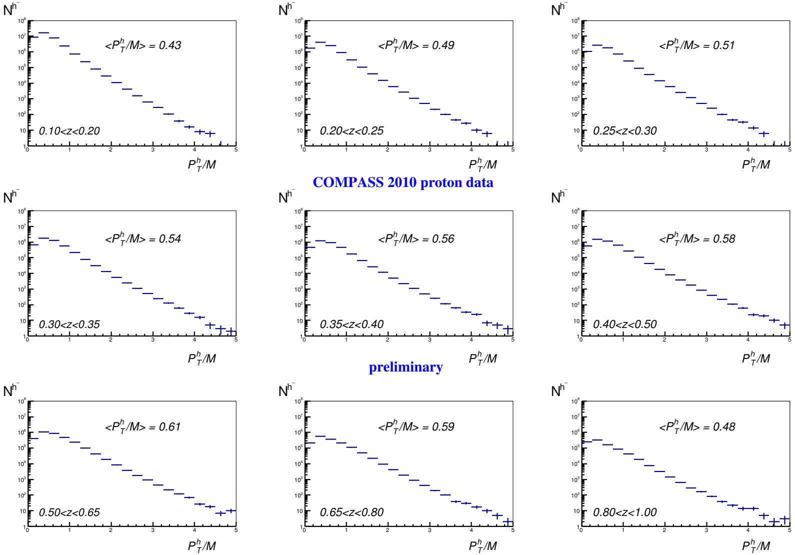

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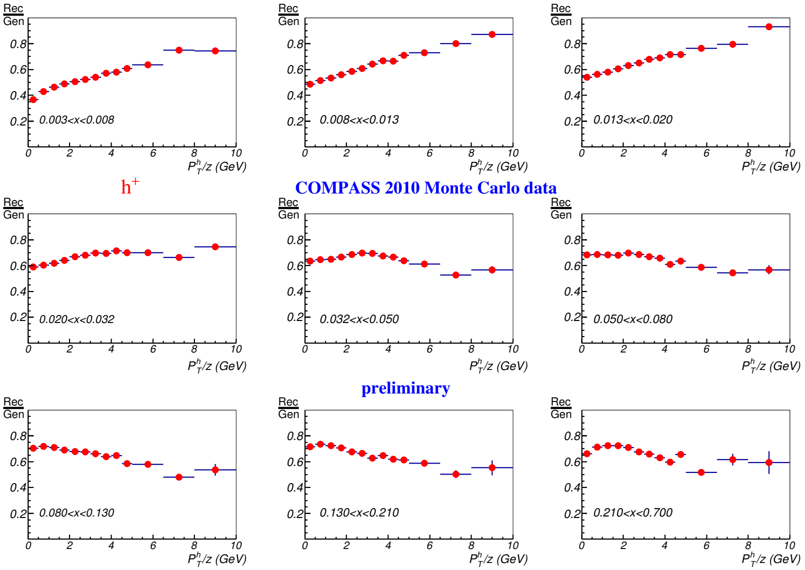
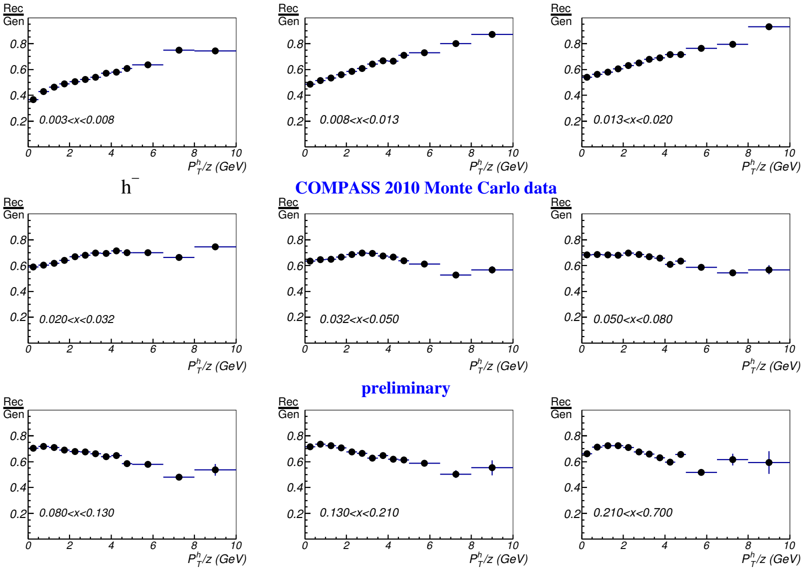
Backup: Acceptance in P_{hT}/z for h+



Negative hadrons.

Backup: Acceptance in P_{hT}/z for h–

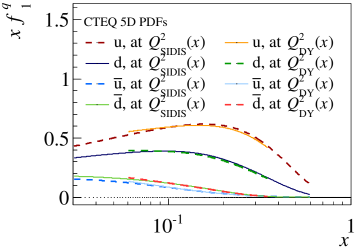


Negative hadrons.

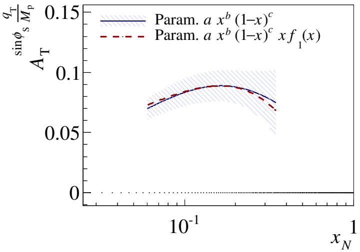
$$\text{(A)} \qquad x f_{1\mathrm{T}}^{\perp(1)q}(x) = a_q\, x^{b_q}\, (1-x)^{c_q},$$

$$\text{(B)} \qquad x f_{1\mathrm{T}}^{\perp(1)q}(x, Q^2) = a_q\, x^{b_q}\, (1-x)^{c_q}\, x f_1^q(x, Q^2).$$

- PDFs at $Q^2_{\mathrm{SIDIS}}(x)$ is almost the same as at $Q^2_{\mathrm{DY}}(x)$ in the valence region.

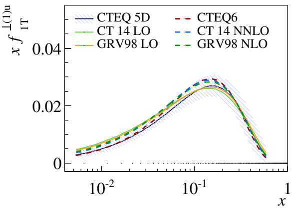


PDFs at $Q^2_{\mathrm{SIDIS}}(x)$ and $Q^2_{\mathrm{DY}}(x)$.

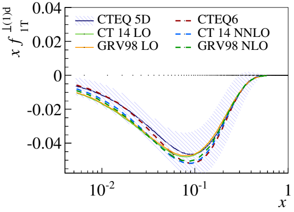


Projection for Drell–Yan.

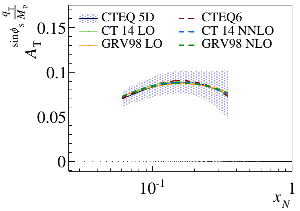
- Alternative PDF sets have been tested.
- The differences lie within 1σ, except at small x.
- The impact on the DY projection cancels in the ratio of $f_{1\mathrm{T}}^{\perp(1)}$ and f_1



Sivers, u

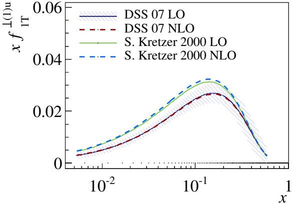


Sivers, d

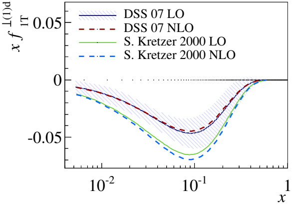


Projections for Drell–Yan.

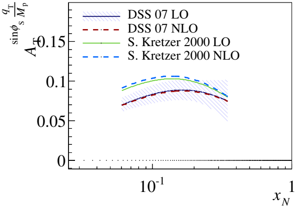
- Alternative FF set from S. Kretzer has been tested.



Sivers, u



Sivers, d



Projection for Drell–Yan.

Differences rather large in lower x-range.

- Test of the quality of the valence dominance approximation

$$A_{\mathrm{T}}^{\sin\phi_{\mathrm{S}}\frac{q_{\mathrm{T}}}{M_{\mathrm{p}}}}(x_N, Q^2) \approx 2\,\frac{f_{1\mathrm{T,p}}^{\perp(1)\mathrm{u}}(x_N)}{f_{1,\mathrm{p}}^{\mathrm{u}}(x_N, Q^2)}.$$

- More precise formula, pion PDFs from GRV-PI0 [M. Glück *et al.*,Z.Phys. C53 (1992) 651]

$$A_{\mathrm{T}}^{\sin\phi_{\mathrm{S}}\frac{q_{\mathrm{T}}}{M_{\mathrm{p}}}}(x_N) = 2\,\frac{\frac{4}{9} f_{1\mathrm{T,p}}^{\perp(1)\mathrm{u}}(x_N) f_{1,\pi^-}^{\bar{\mathrm{u}}}(x_\pi) + \frac{1}{9} f_{1\mathrm{T,p}}^{\perp(1)\mathrm{d}}(x_N) f_{1,\pi^-}^{\bar{\mathrm{d}}}(x_\pi)}{\sum_{q=\mathrm{u,d,\bar{u},\bar{d}}} e_q^2 f_{1,\mathrm{p}}^{q}(x_N) f_{1,\pi^-}^{\bar{q}}(x_\pi)}.$$