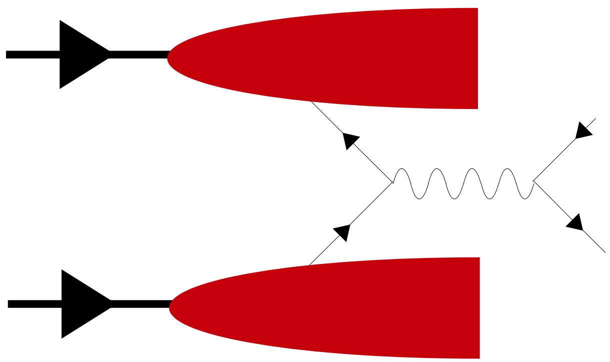
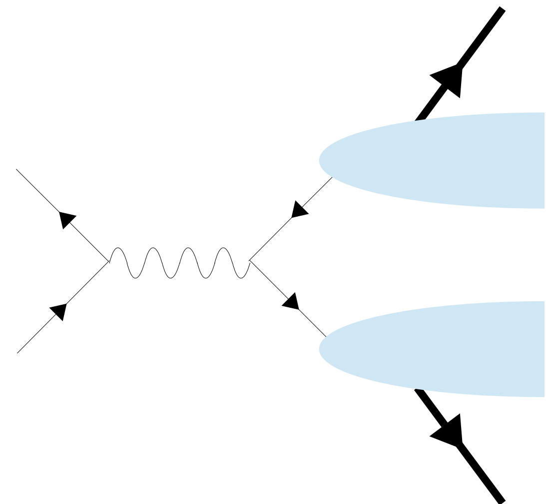
Transverse Momentum Effects in e+e− inclusive single-hadron production

J. Osvaldo Gonzalez-Hernandez
University of Turin

Phenomenology

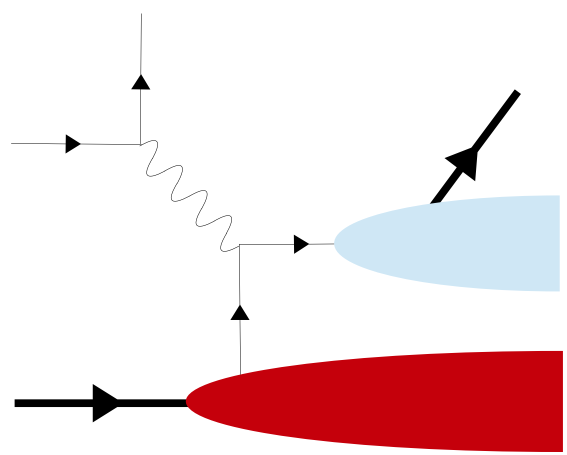
Drell Yan





e+e-

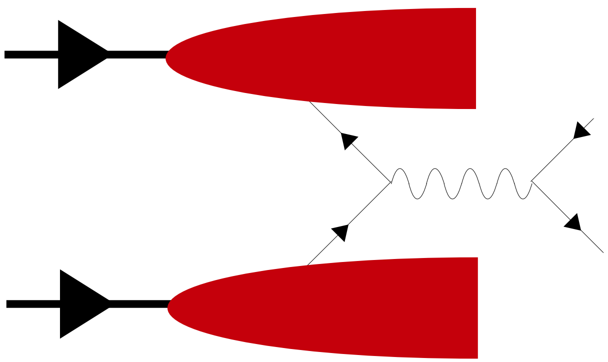
PDFs



Fragmentation Functions

SIDIS

Drell Yan



Under control, high precision phenomenology:

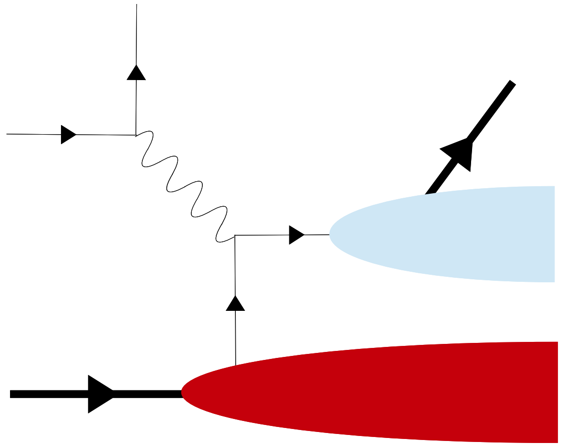
See for example:
arXiv:1706.01473
Ignazio Scimemi, Alexey Vladimirov

Must still address some issues.

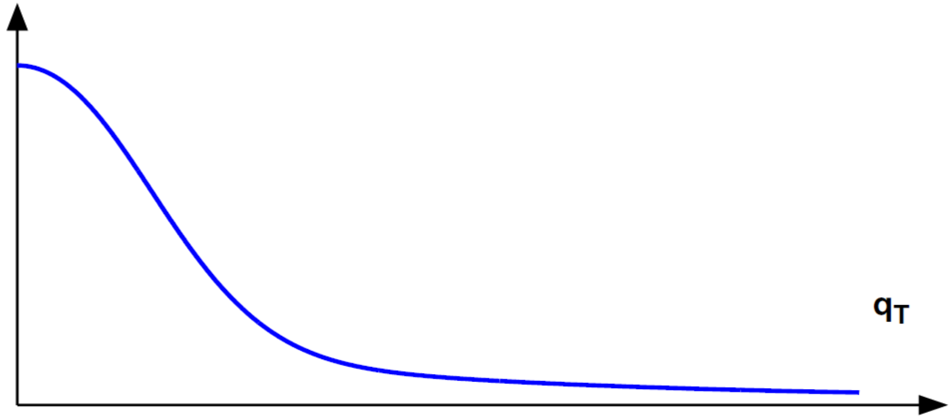
The matching between low and large transverse momentum regimes

See for example

- Boglione, JOGH, Melis, Prokudin JHEP 1502 (2015) 095
- M. Boglione, J. Collins, L. Gamberg, JOGH, T. C. Rogers, and N. Sato, Phys. Lett.B766, 245 (2017),1611.10329.



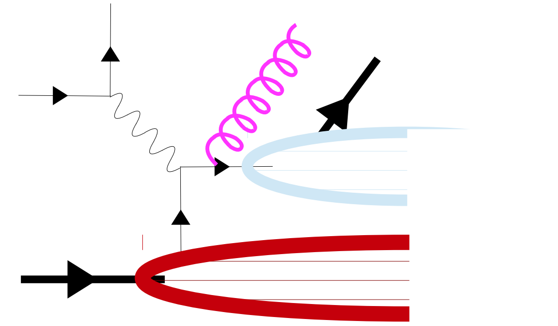
SIDIS



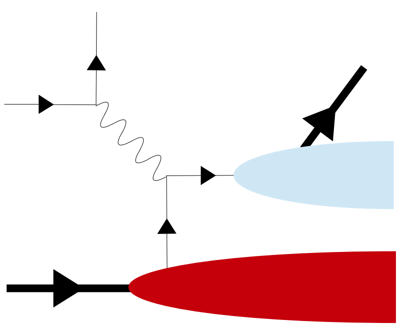
q_T<<Q
TMD region

q_T~Q
Matching region

q_T>>Q
pQCD



Hard gluon radiation



$$\frac{d\sigma^{\rm NLO}}{dx\,dy\,dz\,dq_T^2} = \frac{d\sigma^{\rm ASY}}{dx\,dy\,dz\,dq_T^2} + Y$$

$$\frac{d\sigma^{\rm total}}{dx\,dy\,dz\,dq_T^2} = \pi\sigma_0^{\rm DIS}\int\frac{d^2\boldsymbol{b}_T e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}}{(2\pi)^2} W^{\rm SIDIS}(x,z,b_T,Q) + Y^{\rm SIDIS}(x,z,q_T,Q)$$

Source of Errors?

Unpolarized SIDIS cross section (current region)

$$\frac{d\sigma^{\ell+p\to\ell' hX}}{dx_B\, dQ^2\, dz_h\, dP_T^2} = \frac{2\,\pi^2\alpha^2}{(x_B\, s)^2}\,\frac{\left[1+(1-y)^2\right]}{y^2}\,F_{UU}$$

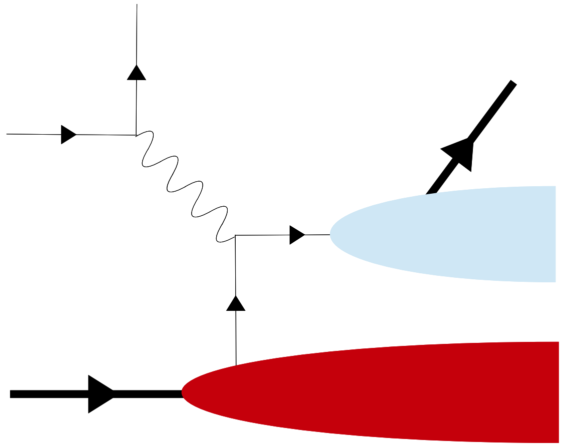
$$F_{UU} = \sum_q \mathcal{H}_q\ \text{F.T.}\left\{\tilde{D}_{h/q}(z, z\,\boldsymbol{b}_\perp; Q)\ \tilde{f}_{q/P}(x, \boldsymbol{b}_\perp; Q)\right\}$$

$$+\ \text{large } q_T \text{ corrections} + \text{power suppressed terms}$$

Perturbation Theory **Factorization**

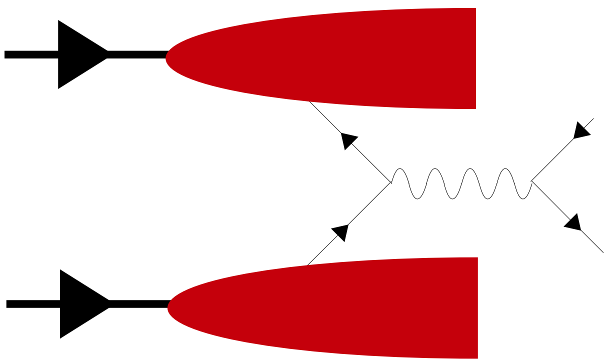
Must still address some issues.

A lot of work is being done
to improve Phenomenology
in SIDIS



SIDIS

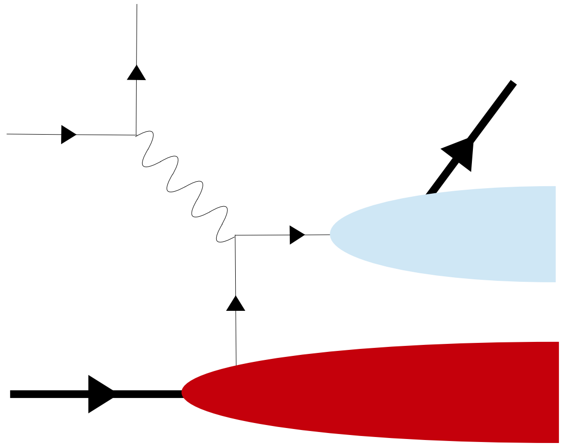
Drell Yan



Combined analysis at LO

A. Bacchetta, F. Delcarro,
C. Pisano, M. Radici , A.Signori
JHEP 1706 (2017) 081

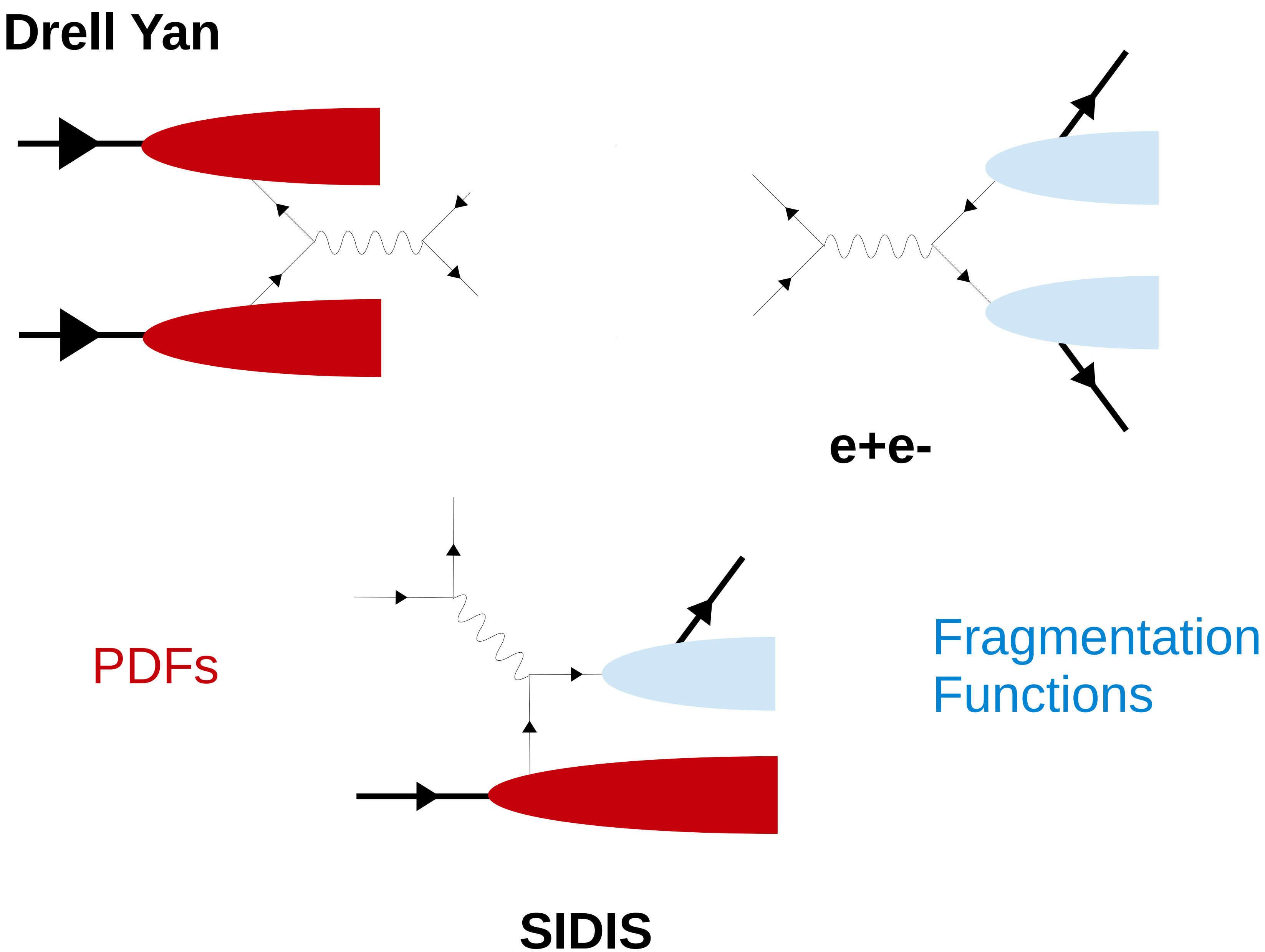
PDFs



Fragmentation
Functions

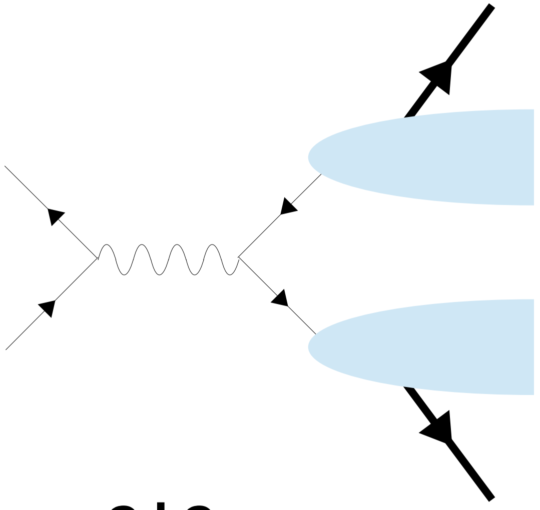
SIDIS

Phenomenology



Recently, BELLE, BaBar, BES III
Collins asymmetries.

No modern unpolarized
measurements are available.



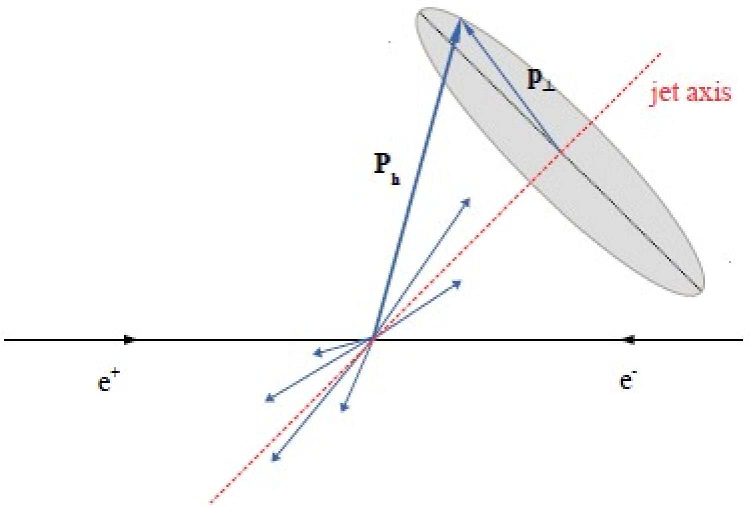
e+e-

Recently, BELLE, BaBar, BES III Collins asymmetries.

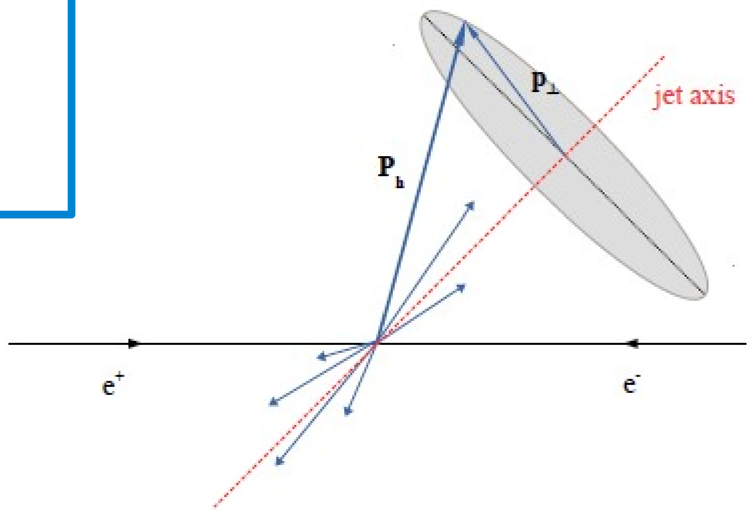
No modern unpolarized measurements are available.

TASSO, MARK II available for
e+e- → X h

- **pT** distributions
- different energies
- integrated over **z**



Boglione, JOGH, R. Taghavi
Phys.Lett. B772 (2017) 78
arXiv:1704.08882



TASSO, MARK II available for
e+e- → X h

- **pT** distributions
- different energies
- integrated over **z**

Big Limitation

New analysis:
TMD effects in one hadron production?

Use this...

Assuming factorization

$$D_{h/q}(z, p_\perp) = d_{h/q}(z)\, h_d(p_\perp)$$

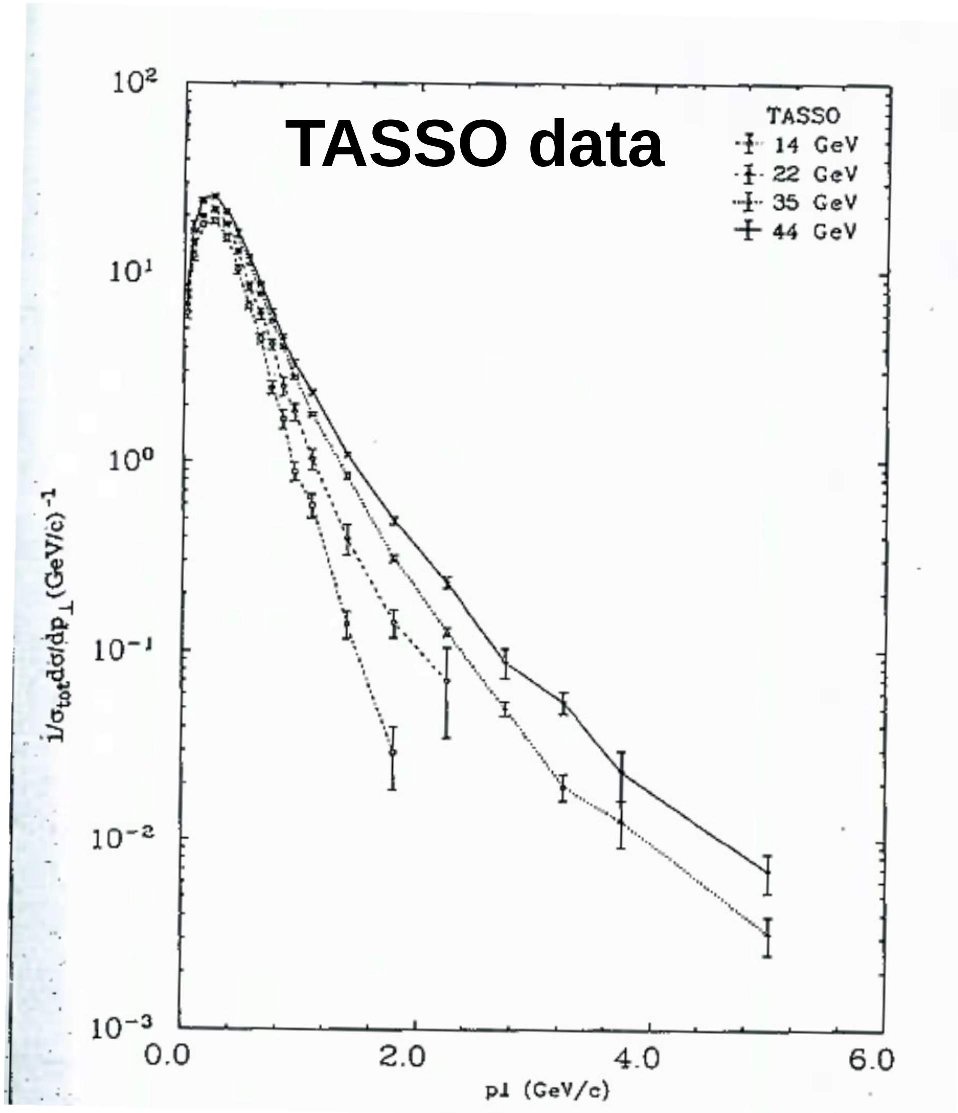
To get information about this

TMD picture

$$\tilde{D}_{h/q}(z, \boldsymbol{b}_\perp; Q) = \sum_j \left[\left(\tilde{C}_{j/q} \otimes \frac{d_{h/j}}{z^2} \right) e^{\Gamma_D(Q)} \right] \exp\left\{ g_{j/P}(x, b_\perp) + g_K(b_\perp) \log\left(\frac{Q}{Q_0} \right) \right\}$$

Things to investigate:

- appropriate functional form for $\mathbf{g}_{j/P}$
- scale evolution regulated by $\mathbf{g}_K$



$$\tilde{D}_{h/q}(z, \boldsymbol{b}_\perp; Q) = \sum_j \left[\left(\tilde{C}_{j/q} \otimes \frac{d_{h/j}}{z^2} \right) e^{\Gamma_D(Q)} \right] \exp\left\{ g_{j/P}(x, b_\perp) + g_K(b_\perp) \log\left(\frac{Q}{Q_0} \right) \right\}$$

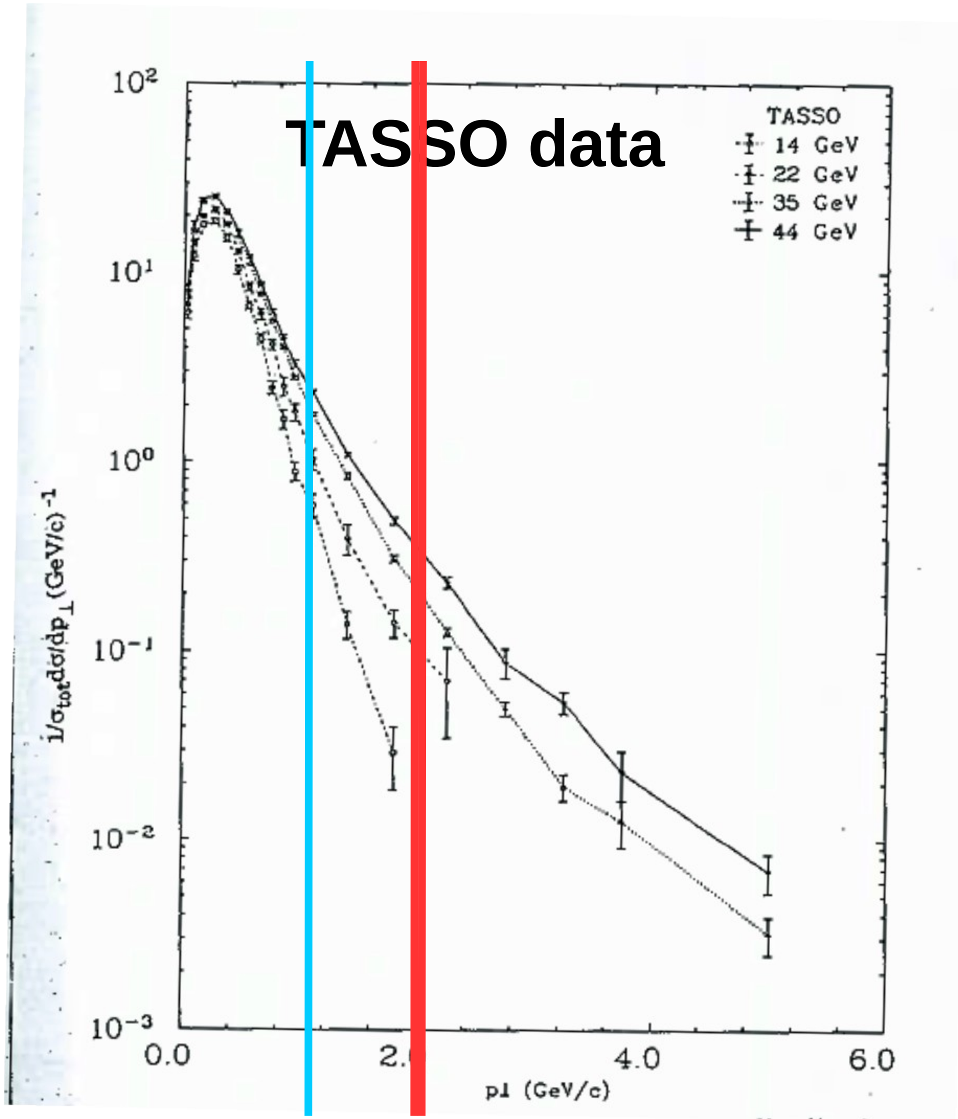
Identify region where TMD Effects would dominate:

For fully differential cross sections, matching region is Expected to be at

$$p_{\perp} \sim zQ$$

Use experimental **<z>** to make an estimate

$$p_{\perp} \sim 2\,\mathrm{GeV}$$



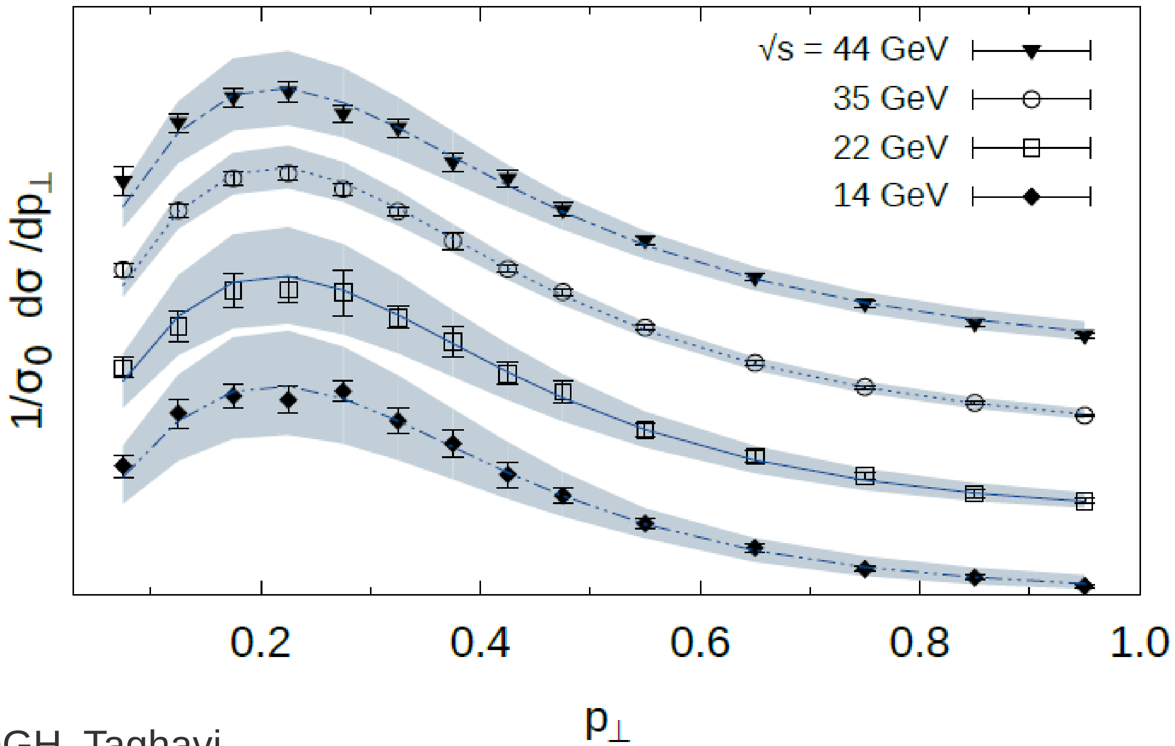
We looked at a restricted range: $p_{\perp} < 1\ \mathrm{GeV}$

Power law to model transverse momentum dependence

$$D_{h/q}(z, p_\perp) = d_{h/q}(z)\, h_d(p_\perp)$$

$$h(p_\perp) = 2(\alpha - 1)\mathrm{M}^{2\,(\alpha-1)} \frac{1}{\left(p_\perp^2 + \mathrm{M}^2\right)^\alpha}$$

FIT TO TASSO

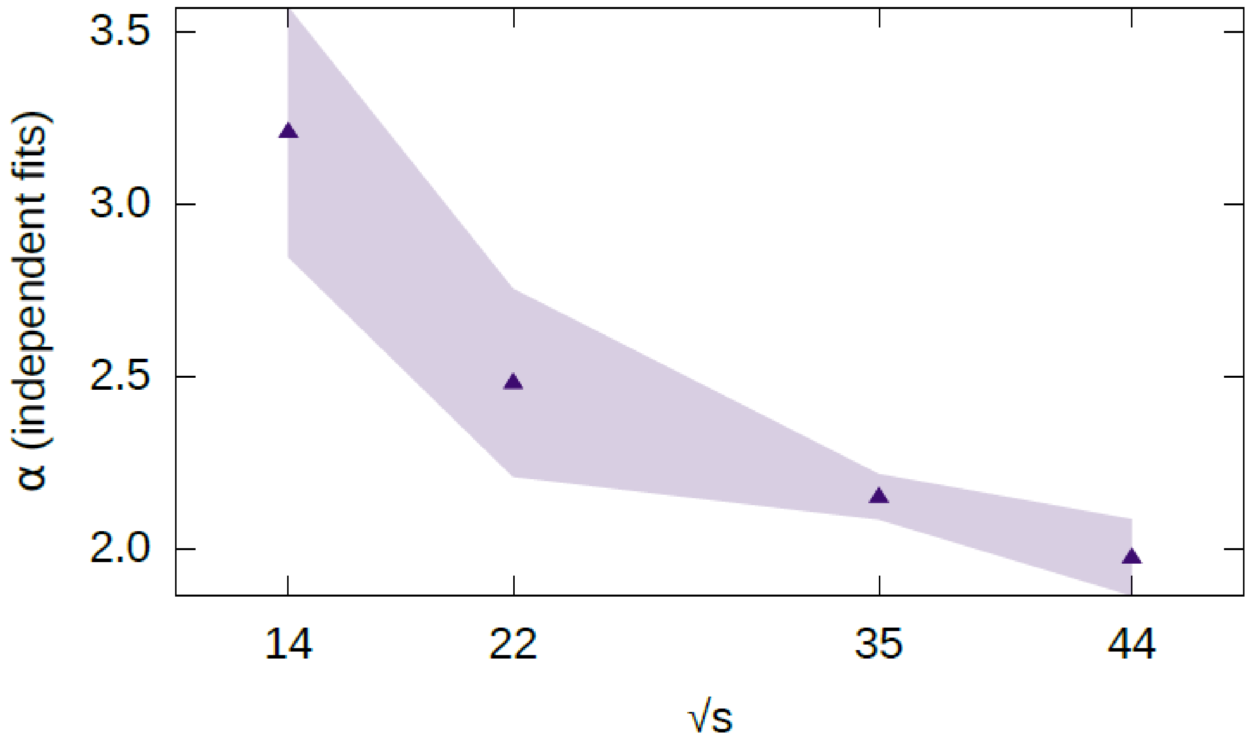


Boglione, JOGH, Taghavi

Phys.Lett. B772 (2017) 78-86

Power law parameters follow a logarithmic trend

$$h(p_\perp) = 2(\alpha - 1)M^{2\,(\alpha-1)} \frac{1}{\left(p_\perp^2 + M^2\right)^\alpha}$$



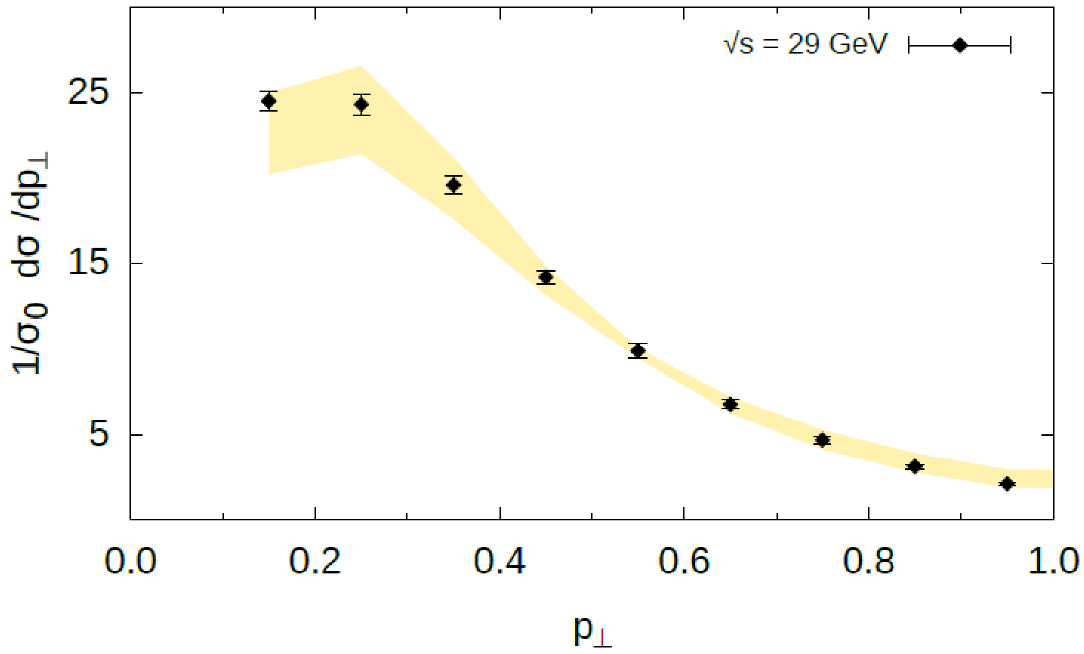
Boglione, JOGH, Taghavi

Phys.Lett. B772 (2017) 78-86

Power law parameters follow a logarithmic trend
Consistent with MARK II data.

$$h(p_\perp) = 2(\alpha - 1)M^{2(\alpha-1)} \frac{1}{\left(p_\perp^2 + M^2\right)^\alpha}$$

COMPARISON TO MARK II



Boglione, JOGH, Taghavi

Phys.Lett. B772 (2017) 78-86

TMD

$$\mathcal{F}^{-1}\left\{\frac{d\sigma^h}{dz\,d^2\boldsymbol{p}_\perp}\right\} \propto \exp\left\{\left(\lambda_\Gamma(b_*) + g_K(b_\perp)\right)\log\left(\frac{Q}{Q_0}\right)\right\}\Big|_{b_\perp \to z\, b_\perp}$$

$$\lambda_\Gamma(b_*) \equiv \frac{32}{27}\log\left(\log\frac{2e^{-\gamma_E}}{\Lambda_{QCD}\, b_*}\right)$$

MODEL

$$h(p_\perp) = 2(\alpha-1)\mathrm{M}^{2\,(\alpha-1)}\frac{1}{\left(p_\perp^2 + \mathrm{M}^2\right)^\alpha}$$

$$\mathcal{F}^{-1}\left\{\frac{1}{\left(p_\perp^2+\mathrm{M}^2\right)^\alpha}\right\} \xrightarrow{\text{large}\, b_\perp} \frac{1}{2^\alpha\,\pi\,\Gamma(\alpha)}\left(\frac{b_\perp}{\mathrm{M}}\right)^{\alpha-1}\sqrt{\frac{\pi}{2}}\,\frac{e^{-b_\perp \mathrm{M}}}{\sqrt{b_\perp \mathrm{M}}}\left[1+\mathcal{O}\left(\frac{1}{b_\perp \mathrm{M}}\right)\right]$$

gK: Slow growing at large bT

$$g_K(b_\perp) \xrightarrow{\text{large } b_\perp} \tilde{\alpha} \log(\nu\, b_\perp)$$

J. Collins, T. Rogers

Phys.Rev. D91 (2015) no.7, 074020

TMD

$$\mathcal{F}^{-1}\left\{\frac{d\sigma^h}{dz\, d^2\boldsymbol{p}_\perp}\right\} \propto \exp\left\{\Big(\lambda_\Gamma(b_*) + g_K(b_\perp)\Big)\log\left(\frac{Q}{Q_0}\right)\right\}\Bigg|_{b_\perp\to z\, b_\perp}$$

$$\lambda_\Gamma(b_*) \equiv \frac{32}{27}\log\left(\log\frac{2e^{-\gamma_E}}{\Lambda_{QCD}\, b_*}\right)$$

Logarithmic behavior of alpha may be interpreted as a consequence of the **Log** in the definition of the **TMD FF**.

$$\alpha = \alpha_0 + \tilde{\alpha}\log\left(\frac{Q}{Q_0}\right)$$

$$g_K(b_\perp) \xrightarrow{\text{large } b_\perp} \tilde{\alpha}\log(\nu\, b_\perp)$$

TMD

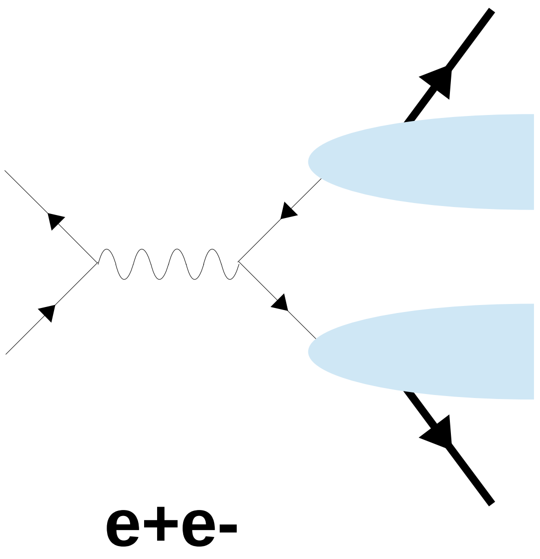
There are caveats on this interpretation, while consistent with theoretical expectations, it's not the only possibility.
(loss of information through z-integration)

Logarithmic behavior of alpha may be interpreted as a consequence of the **Log** in the definition of the **TMD FF**.

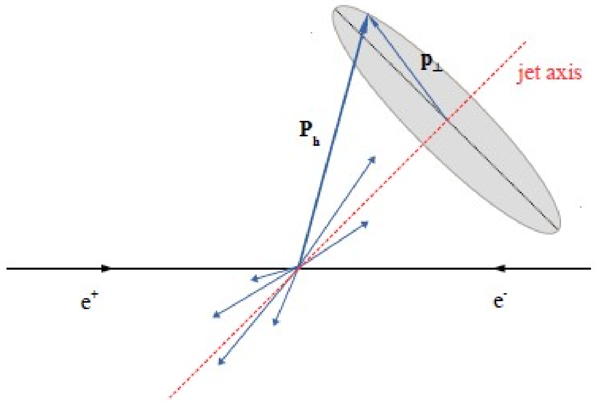
$$\alpha = \alpha_0 + \tilde{\alpha} \log\left(\frac{Q}{Q_0}\right)$$

$$g_K(b_\perp) \xrightarrow{\text{large } b_\perp} \tilde{\alpha} \log(\nu\, b_\perp)$$

The lack of information about **z** hinders a full TMD extraction of the FF.

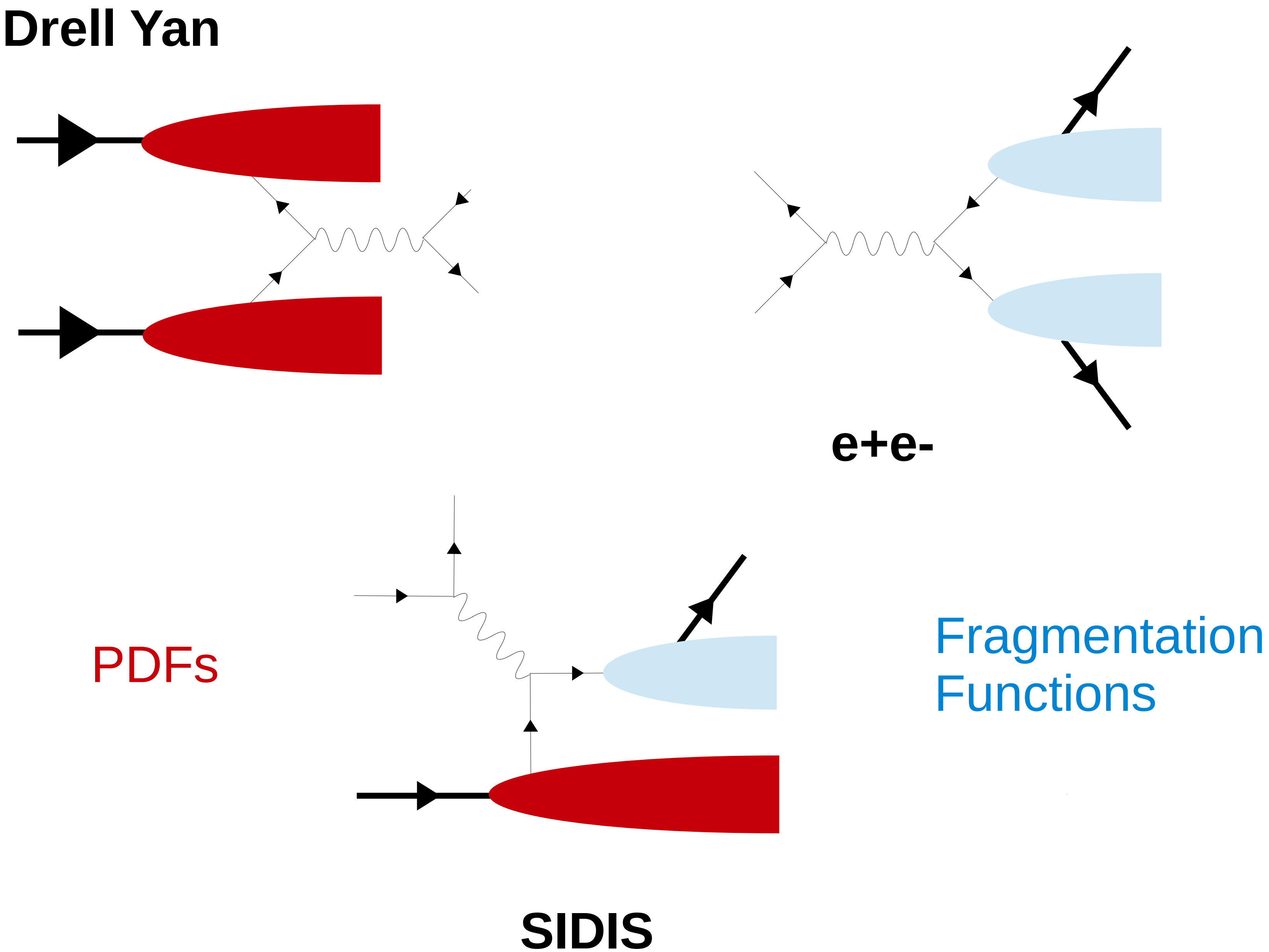


Future upcoming data by BELLE on unpolarized one-hadron production may allow for a combined analysis with TASSO and MARK II data.



Phenomenological Test (factorization?)

Final Remarks



Thank you.

Source of Errors?

Unpolarized SIDIS cross section (current region)

$$\frac{d\sigma^{\ell+p\to\ell' hX}}{dx_B\, dQ^2\, dz_h\, dP_T^2} \propto \sum_q \mathcal{H}_q \text{ F.T.} \left\{ \tilde{D}_{h/q}(z, z\, \boldsymbol{b}_\perp; Q)\ \tilde{f}_{q/P}(x, \boldsymbol{b}_\perp; Q) \right\}$$

$$+ \text{ large } q_T \text{ corrections} + \text{power suppressed terms}$$

Perturbation Theory **Factorization**

(Re)Calculation of large qT SIDIS cross section at O(α_s^2)

Work in progress:

J.O.G.H., T. Rogers, N. Sato, B. Wang

$$\frac{d\sigma^{\ell+p\to\ell' hX}}{dx_B\, dQ^2\, dz_h\, dP_T^2} \propto \sum_q \mathcal{H}_q \text{ F.T.} \left\{ \tilde{D}_{h/q}(z, z\,\boldsymbol{b}_\perp; Q)\ \tilde{f}_{q/P}(x, \boldsymbol{b}_\perp; Q) \right\}$$

$$+ \text{ large } q_T \text{ corrections} + \text{power suppressed terms}$$

Perturbation Theory

Kinematics of TMD regime

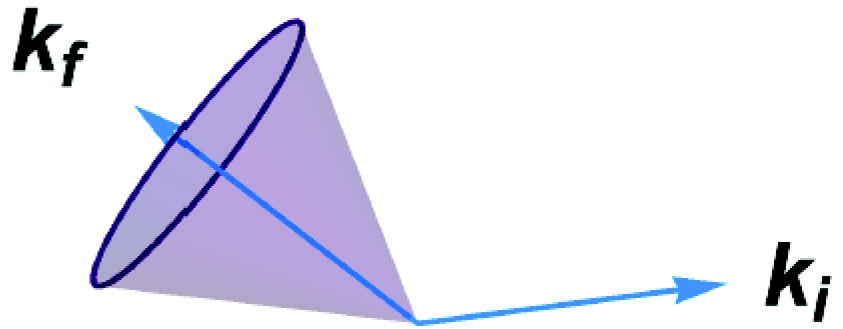
Mixing with other physics?

$$\frac{d\sigma^{\ell+p\to\ell' hX}}{dx_B\, dQ^2\, dz_h\, dP_T^2} \propto \sum_q \mathcal{H}_q \ \mathrm{F.T.}\left\{\tilde{D}_{h/q}(z, z\,\boldsymbol{b}_\perp; Q)\ \tilde{f}_{q/P}(x, \boldsymbol{b}_\perp; Q)\right\}$$

$$+ \text{ large } q_T \text{ corrections } + \boxed{\text{power suppressed terms}}$$

Factorization

Power counting and kinematics of the current region



small masses

$$P_h \cdot k_{\mathrm{f}} = O\left(m^2\right)$$

$$P_h \cdot k_{\mathrm{i}} = O\left(Q^2\right)$$

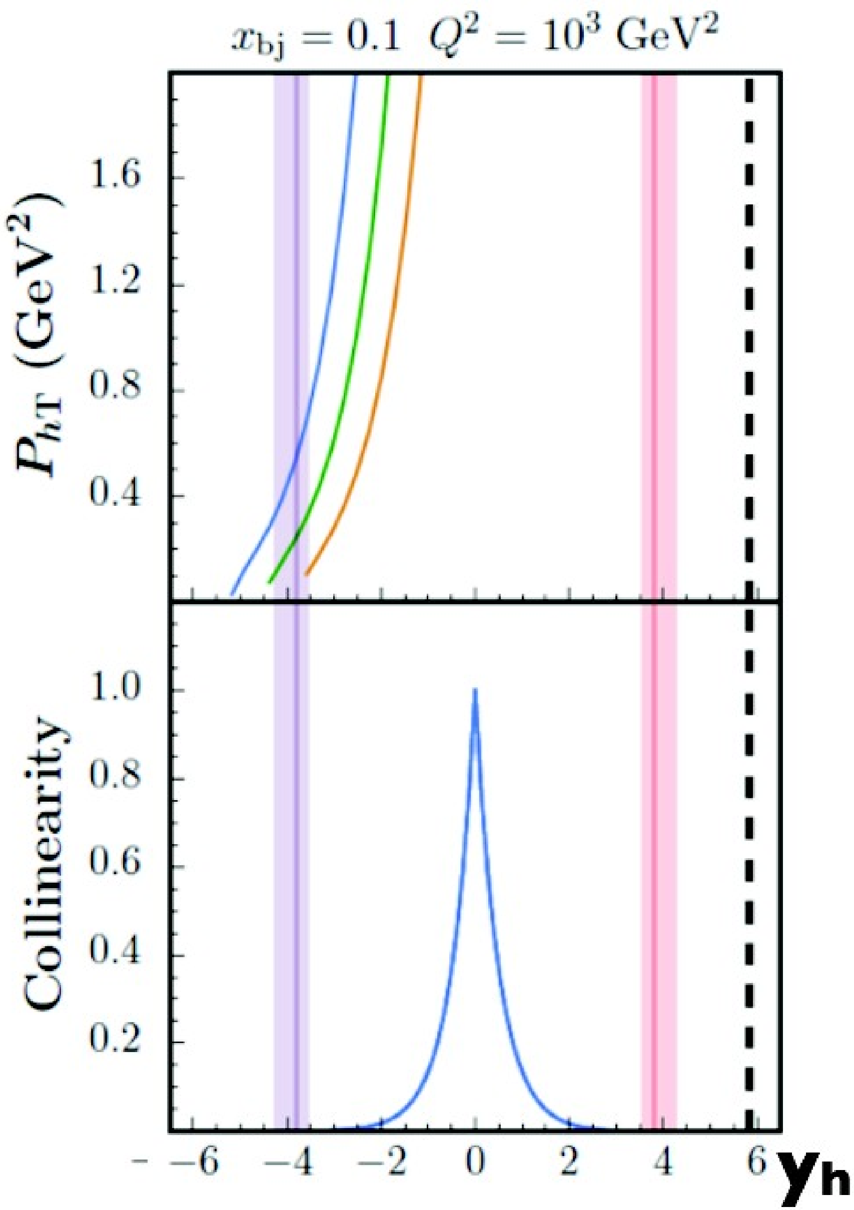
hard scale

require small values for

$$R \equiv \frac{P_h \cdot k_{\mathrm{f}}}{P_h \cdot k_{\mathrm{i}}}$$

notice quark momenta have to be estimated

M. Boglione, J. Collins, L. Gamberg, JOGH, T. C. Rogers, and N. Sato,Phys. Lett.B766, 245 (2017),1611.10329.

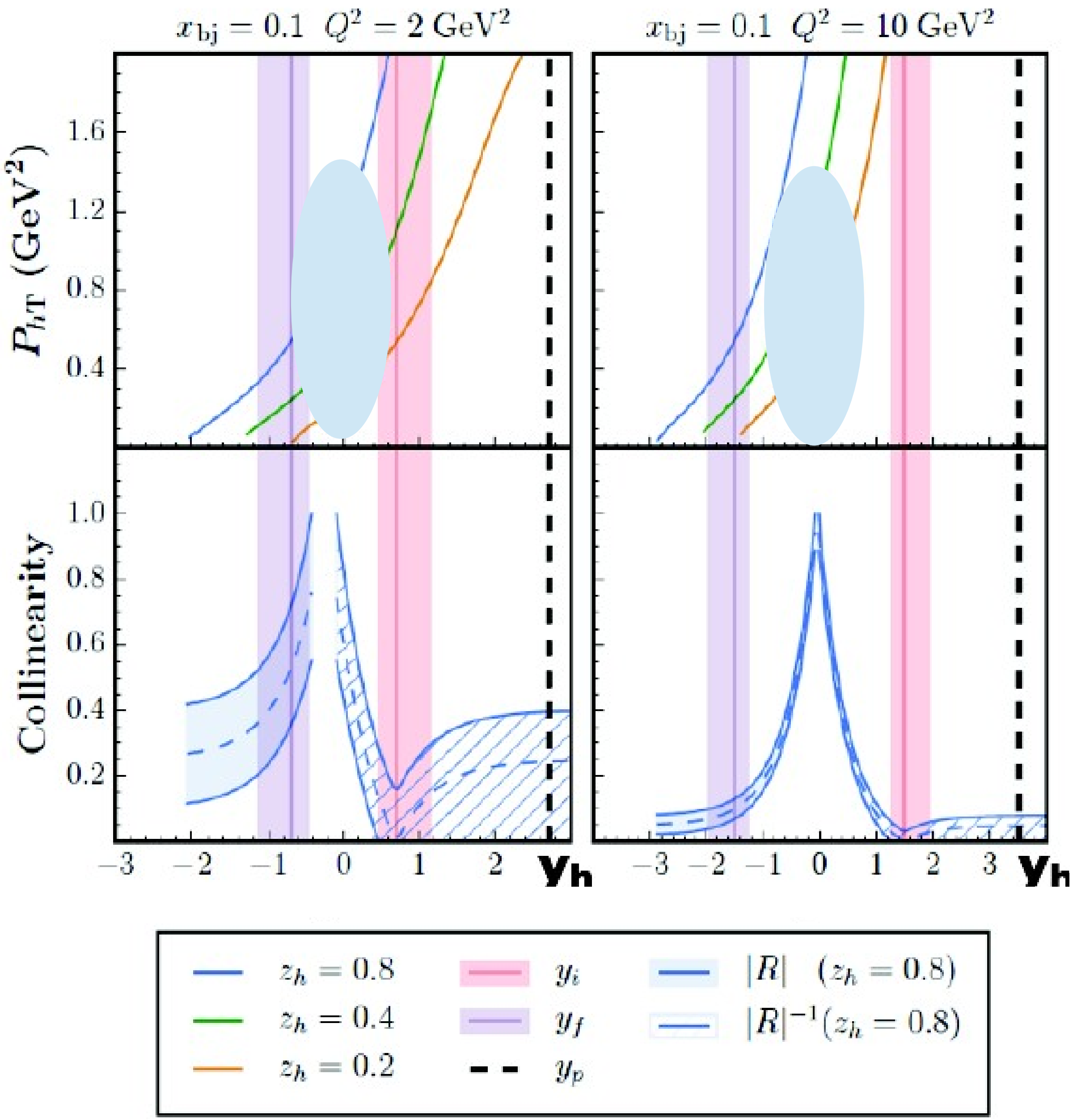


$$y_h \equiv \frac{1}{2} \log \frac{P_h^+}{P_h^-}$$

$$R \equiv \frac{P_h \cdot k_{\rm f}}{P_h \cdot k_{\rm i}}$$

TMD:Current Region **Target Fragmentation**

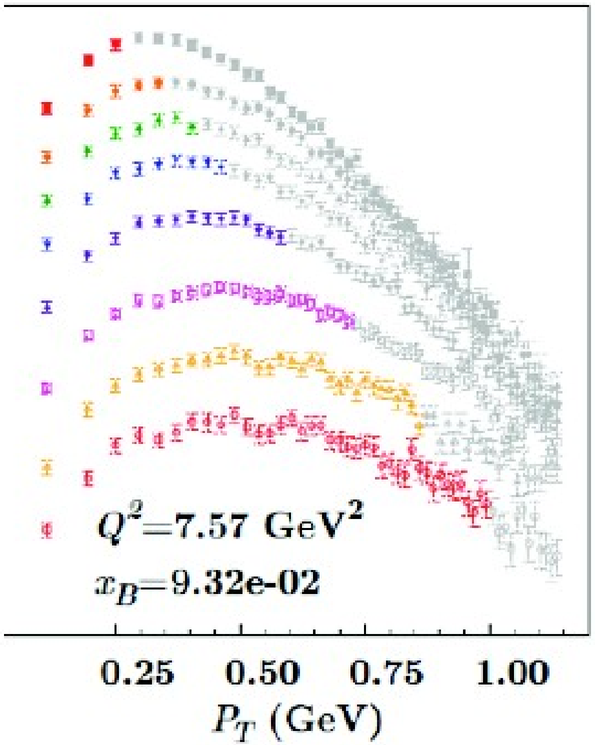
M. Boglione, J. Collins, L. Gamberg, JOGH, T. C. Rogers, and N. Sato,Phys. Lett.B766, 245 (2017),1611.10329.



Available data is likely to receive contributions from non-TMD physics.

$$R \equiv \frac{P_h \cdot k_{\rm f}}{P_h \cdot k_{\rm i}}$$

M. Boglione, J. Collins, L. Gamberg, JOGH, T. C. Rogers, and N. Sato,Phys. Lett.B766, 245 (2017),1611.10329.



precise implementation of the R criterion on data is work in progress

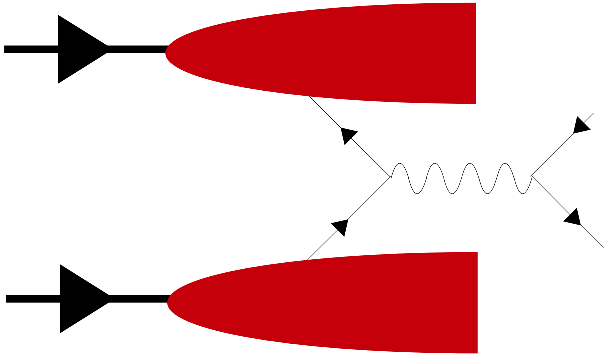
a better set of variables?

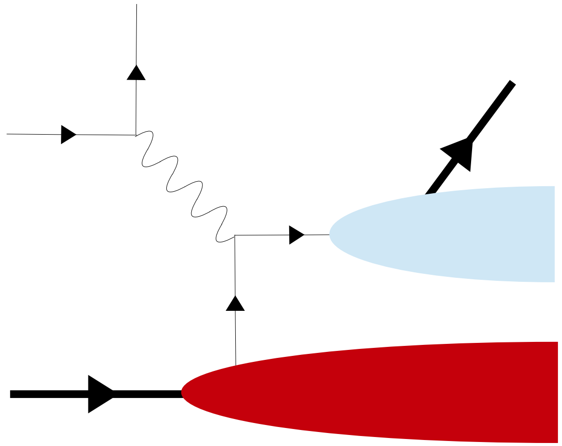
$\{Q^2, x_B, P_{hT}, z_h\}$

$q_T = P_{hT} / z_h$ y_h

***ONLY AN EXAMPLE**

Drell Yan



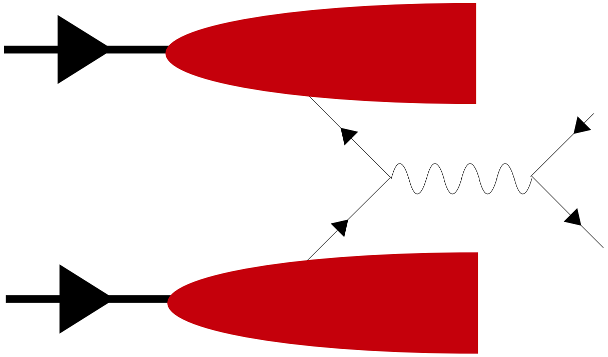


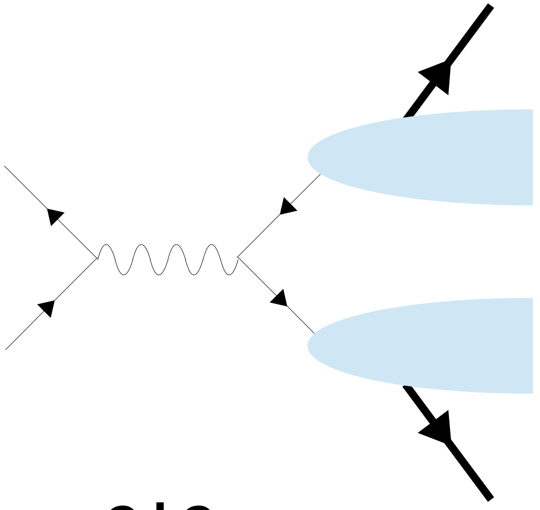
?

SIDIS

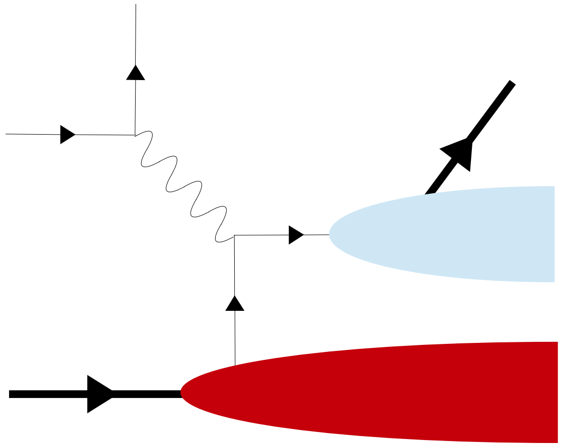
Final Remarks

Drell Yan





e+e-

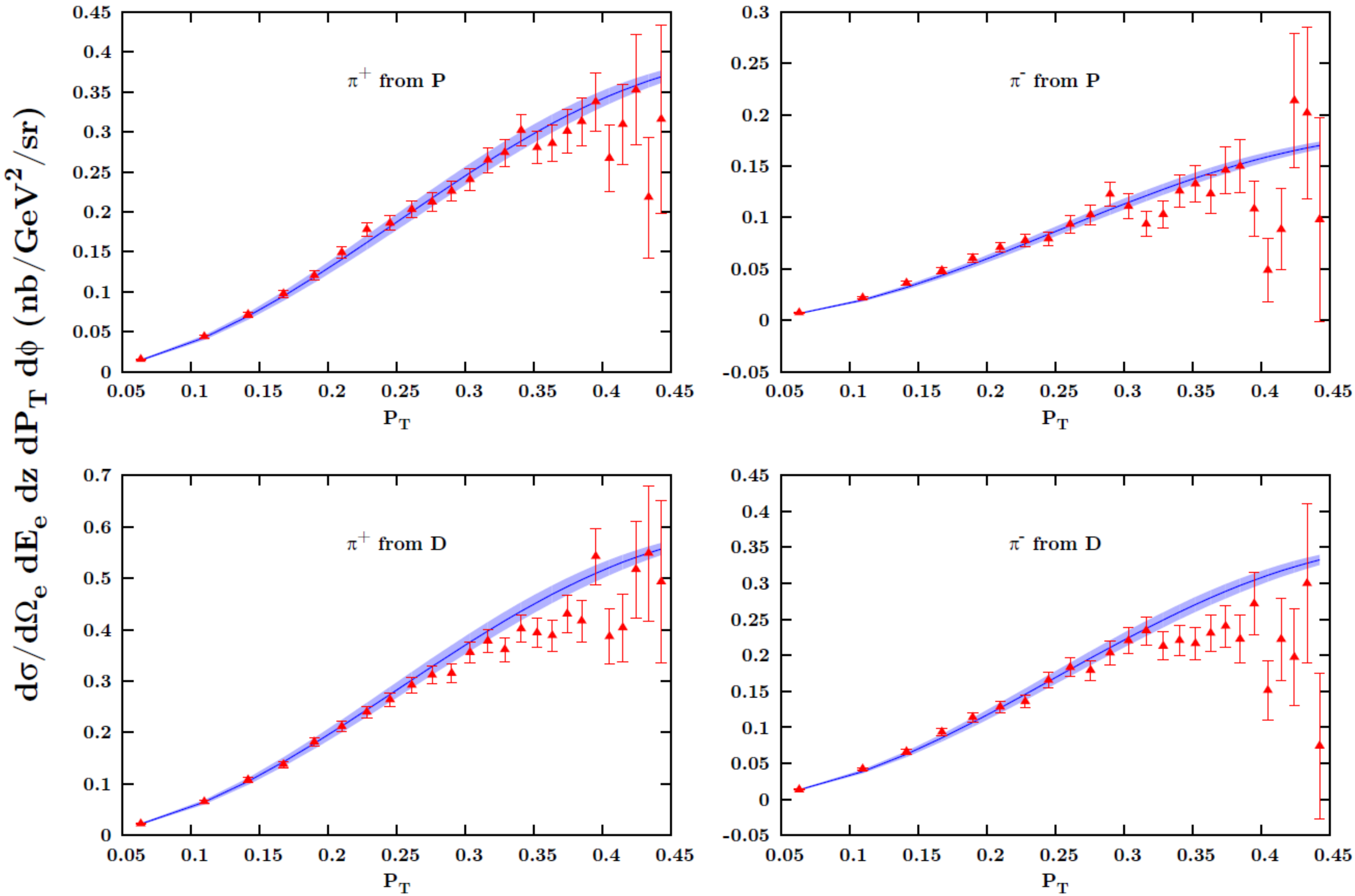


PDFs

Fragmentation Functions

SIDIS

Jlab SIDIS data (2012) (Parameters from HERMES extraction).



Ingredients for extraction of Collins function.

$e^+e^- \rightarrow \pi\,\pi\,X$

Unpolarized TMDFF → $D_{h_1 h_2}$; Collins TMDFF → $N_{h_1 h_2}$

$$\frac{d\sigma^{e^+e^-\to h_1 h_2 X}}{dz_1\, dz_2\, d^2\boldsymbol{P}_{1T}\, d\cos\theta_2} = \frac{3\pi\alpha^2}{2s}\left\{D_{h_1\,h_2} + N_{h_1\,h_2}\cos 2\phi_1\right\}$$

$$P_0^{U,L,C} = \frac{N^{U,L,C}}{D^{U,L,C}}$$

Ratio

$$D^U = D_{\pi^+\pi^-} + D_{\pi^-\pi^+} \qquad N^U = N_{\pi^+\pi^-} + N_{\pi^-\pi^+}$$

$$D^L = D_{\pi^+\pi^+} + D_{\pi^-\pi^-} \qquad N^L = N_{\pi^+\pi^+} + N_{\pi^-\pi^-}$$

$$D^C = D^U + D^L \qquad N^C = N^U + N^L\,,$$

$$\frac{A_0^U}{A_0^{L(C)}} \equiv 1 + \cos(2\phi_1) A_0^{UL(C)}$$

Double Ratio

SIDIS

$$\frac{d\sigma^{\ell(S_\ell)+p(S)\to\ell' hX}}{dx_B dQ^2 dz_h d^2\boldsymbol{P}_T d\phi_S} =$$

$$\frac{2\alpha^2}{Q^4}\left\{\frac{1+(1-y)^2}{2}F_{UU} + \quad \cdots\right.$$

$$\left. + S_T(1-y)(\sin(\phi_h+\phi_S)F_{UT}^{\sin(\phi_h+\phi_S)}\right\}.$$

$$A_{UT}^{\sin(\phi_h+\phi_S)} \sim \frac{F_{UT}^{\sin(\phi_h+\phi_S)}}{F_{UU}}$$

Ratio

Unpolarized TMDFF &TMDPDF → F_{UU}

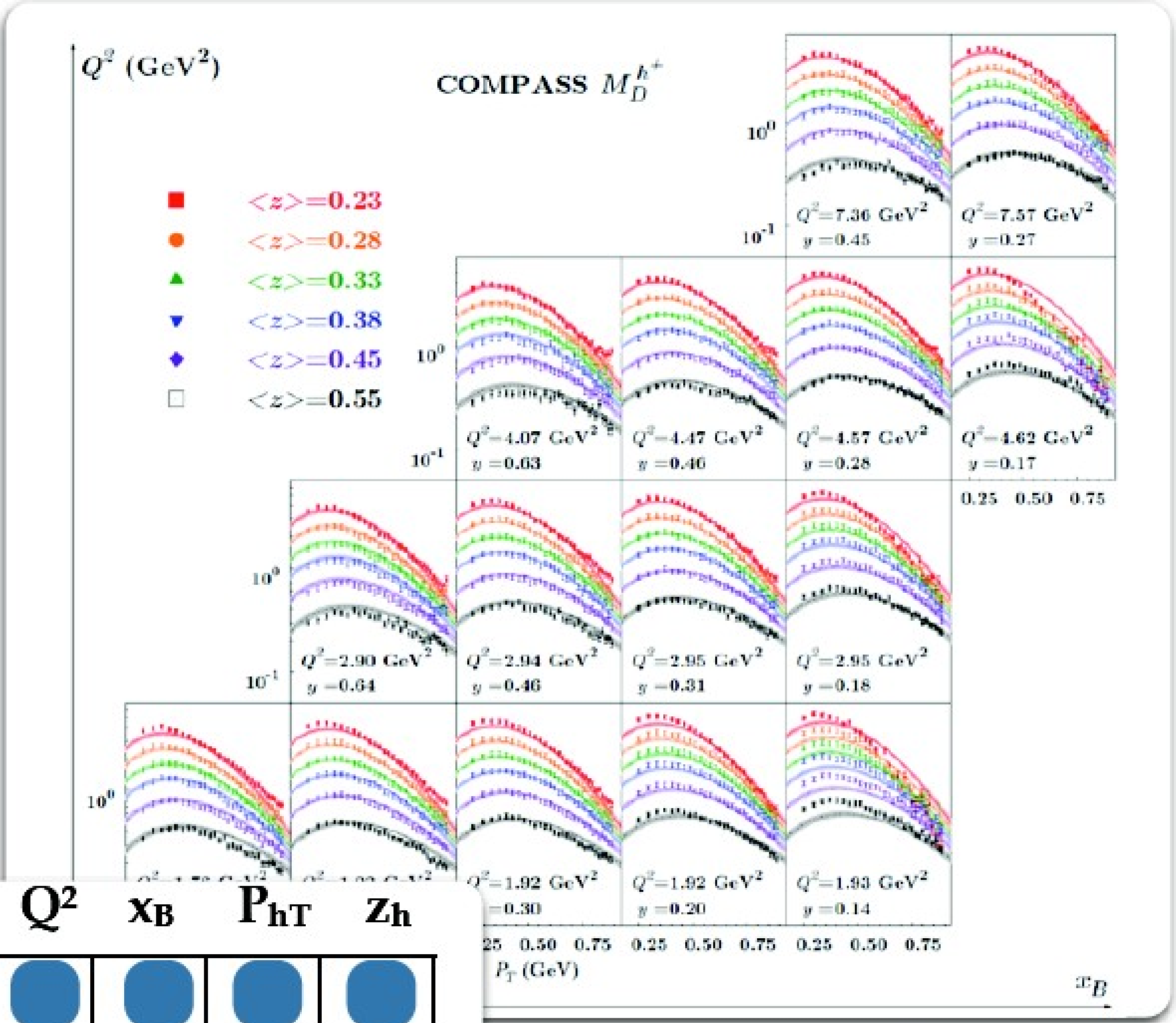
TMD Transversity & Collins function → $F_{UT}^{\sin(\phi_h+\phi_S)}$

Unpolarized SIDIS cross section (current region)

$$\frac{d\sigma^{\ell+p\to\ell' hX}}{dx_B\, dQ^2\, dz_h\, dP_T^2} = \frac{2\,\pi^2\alpha^2}{(x_B\, s)^2}\, \frac{\left[1+(1-y)^2\right]}{y^2}\, F_{UU}$$

$$F_{UU} = \sum_q \mathcal{H}_q \;\mathrm{F.T.}\left\{\tilde{D}_{h/q}(z, z\,\boldsymbol{b}_\perp; Q)\;\tilde{f}_{q/P}(x, \boldsymbol{b}_\perp; Q)\right\}$$

$$+ \text{ large } q_T \text{ corrections} + \text{power suppressed terms}$$



	Q^2	x_B	P_{hT}	z_h
binned	●	●	●	●
integrated				

TMD

$$\mathcal{F}^{-1}\left\{\frac{d\sigma^h}{dz\,d^2\boldsymbol{p}_\perp}\right\} \propto \exp\left\{\left(\lambda_\Gamma(b_*) + g_K(b_\perp)\right)\log\left(\frac{Q}{Q_0}\right)\right\}\Bigg|_{b_\perp \to z\, b_\perp}$$

$$\lambda_\Gamma(b_*) \equiv \frac{32}{27}\log\left(\log\frac{2e^{-\gamma_E}}{\Lambda_{QCD}\, b_*}\right)$$

MODEL

$$h(p_\perp) = 2(\alpha-1)\mathrm{M}^{2\,(\alpha-1)}\frac{1}{\left(p_\perp^2 + \mathrm{M}^2\right)^\alpha}$$

$$\mathcal{F}^{-1}\left\{\frac{1}{\left(p_\perp^2+\mathrm{M}^2\right)^\alpha}\right\} \xrightarrow{\text{large}\, b_\perp} \frac{1}{2^\alpha\,\pi\,\Gamma(\alpha)}\left(\frac{b_\perp}{\mathrm{M}}\right)^{\alpha-1}\sqrt{\frac{\pi}{2}}\,\frac{e^{-b_\perp \mathrm{M}}}{\sqrt{b_\perp \mathrm{M}}}\left[1+\mathcal{O}\left(\frac{1}{b_\perp \mathrm{M}}\right)\right]$$

TMD

$$\mathcal{F}^{-1}\left\{\frac{d\sigma^h}{dz\, d^2\boldsymbol{p}_\perp}\right\} \propto \exp\left\{\Big(\lambda_\Gamma(b_*) + g_K(b_\perp)\Big)\log\left(\frac{Q}{Q_0}\right)\right\}\Bigg|_{b_\perp \to z\, b_\perp}$$

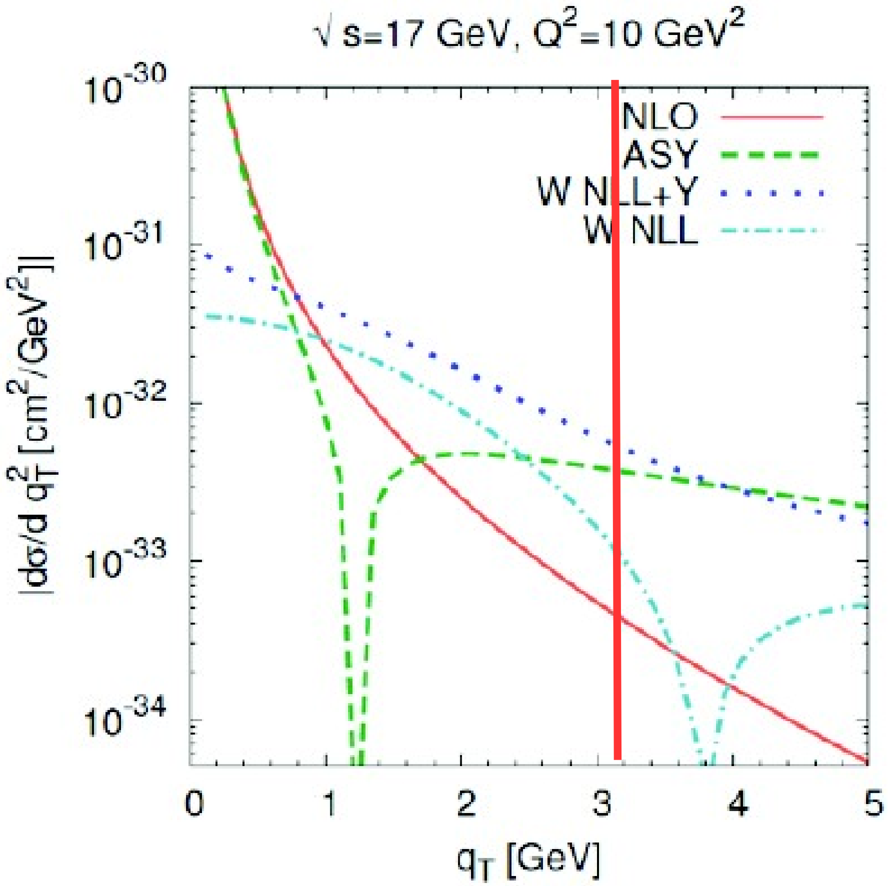
$$\lambda_\Gamma(b_*) \equiv \frac{32}{27}\log\left(\log\frac{2e^{-\gamma_E}}{\Lambda_{QCD}\, b_*}\right)$$

Logarithmic behavior of alpha may be interpreted as a consequence of the **Log** in the definition of the **TMD FF**.

$$\alpha = \alpha_0 + \tilde{\alpha}\log\left(\frac{Q}{Q_0}\right)$$

$$g_K(b_\perp) \xrightarrow{\text{large } b_\perp} \tilde{\alpha}\log(\nu\, b_\perp)$$

Large qT corrections are hard to implement.



- Large Y-term at small q_T
- Small cross section at large q_T
- No smooth matching
- Delicate kinematics

Delicate kinematics

Boglione, JOGH, Melis, Prokudin

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