Light-cone distributions from the Bethe Salpeter Eq. in Minkowski-space

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Motivation

Physical space-time = Minkowski space

Develop methods in continuous nonperturbative QCD within a given dynamical simple framework

Solve the Bethe-Salpeter bound state equation

Observables: spectrum, SL/TL momentum region

Relation BSA to LF Fock-space expansion of the hadron wf

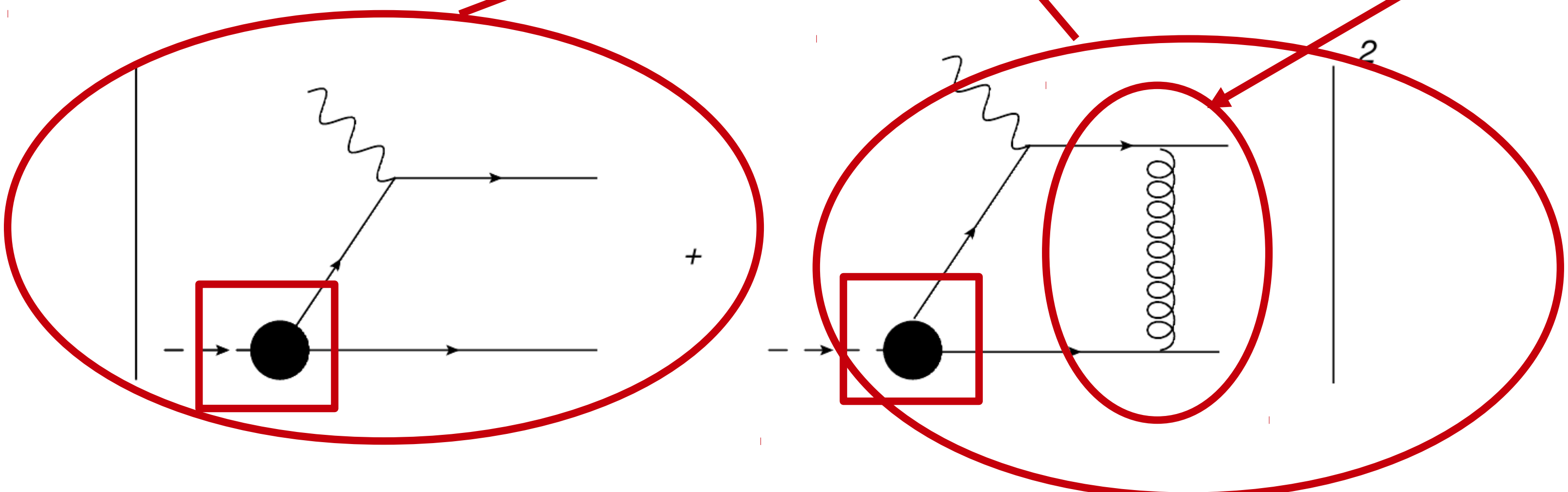
Problems to be addressed

Observables associated with the hadron structure in Minkowski Space obtainable from BSA

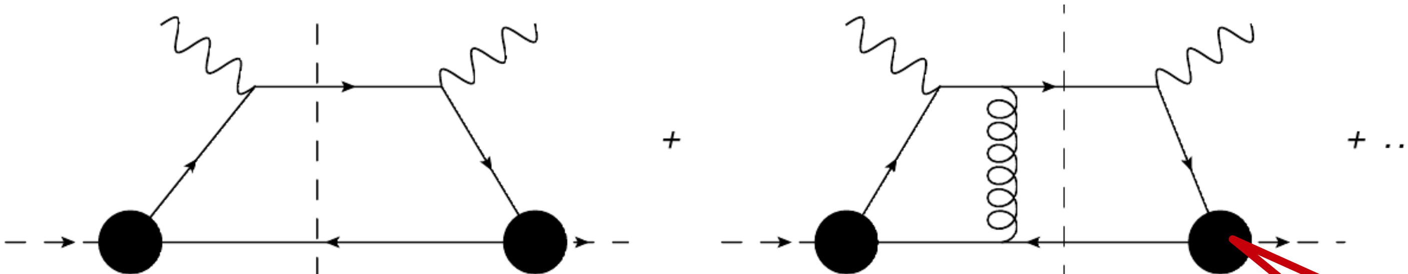
- parton distributions (pdfs)
- generalized parton distributions
- transverse momentum distributions (TMDs)
- Fragmentation functions
- SL and TL form factors ….

TMDs & PDFs

FSI gluon exchange: T-odd



TF & Miller PRD 50 (1994)210



$$q^2 = q^+q^- - q^2_T$$

$$q^+ = q^0+q^3 \quad q^- = q^0-q^3$$

$q^- \rightarrow$ *infty*
DIS

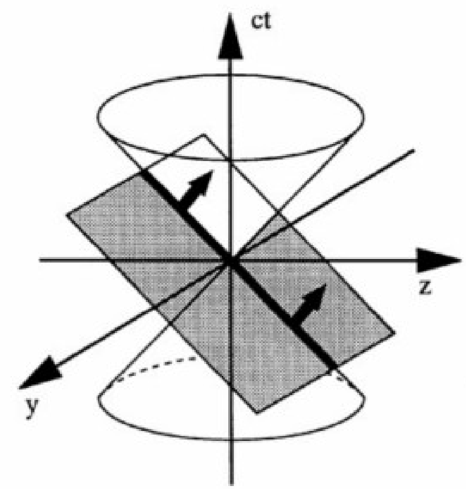
Bethe-Salpeter Amplitude @ $x^+=0$

Light-Front WF (LFWF)

basic ingredient in PDFs, GPDs and TMDs

$$\tilde{\Phi}(x,p) = \int \frac{d^4k}{(2\pi)^4} e^{ik\cdot x}\Phi(k,p)$$

$$p^\mu = p_1^\mu + p_2^\mu \qquad k^\mu = \frac{p_1^\mu - p_2^\mu}{2}$$



$$\tilde{\Phi}(x,p) = \langle 0|T\{\varphi_H(x^\mu/2)\varphi_H(-x^\mu/2)\}|p\rangle$$
$$= \theta(x^+)\langle 0|\varphi(\tilde{x}/2)e^{-iP^- x^+/2}\varphi(-\tilde{x}/2)|p\rangle e^{ip^- x^+/4} + \cdots$$
$$= \theta(x^+)\sum_{n,n'} e^{ip^- x^+/4}\langle 0|\varphi(\tilde{x}/2)|n'\rangle\langle n'|e^{-iP^- x^+/2}|n\rangle\langle n|\varphi(-\tilde{x}/2)|p\rangle + \cdots$$

$x^+ = 0$ only valence state remains! How to rebuilt the full BS amplitude?

Iterated Resolvents: Brodsky, Pauli, and Pinsky, Phys. Rep. 301, 299 (1998)

Yes! Sales, et al. PRC61, 044003 (2000)

Nakanishi Integral Representation (NIR)

"Parametric representation for any Feynman diagram for interacting bosons, with a denominator carrying the overall analytical behavior in Minkowski space" (Nakanishi 1962)

Bethe-Salpeter amplitude

$$\Phi(k,p) = \int_{-1}^{1} dz' \int_{0}^{\infty} d\gamma' \frac{g(\gamma',z')}{(\gamma' + \kappa^2 - k^2 - p.kz' - i\epsilon)^3}$$

BSE in Minkowski space with NIR for bosons

Kusaka and Williams, PRD 51 (1995) 7026;

Light-front projection: integration in k^-

Carbonell&Karmanov EPJA27(2006)1;EPJA27(2006)11;

TF, Salme, Viviani PRD89(2014) 016010,…

Equivalent to Generalized Stietjes transform

Carbonell, TF, Karmanov PLB769 (2017) 418

Two-Boson System: ground-state

Building a solvable model…

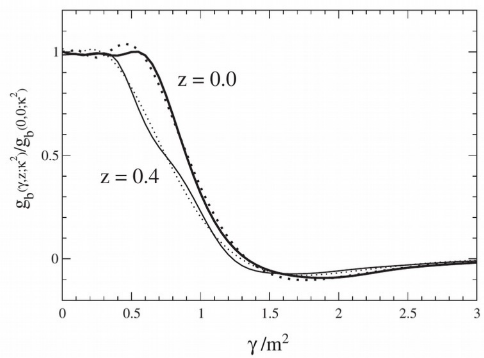
Nakanishi weight function

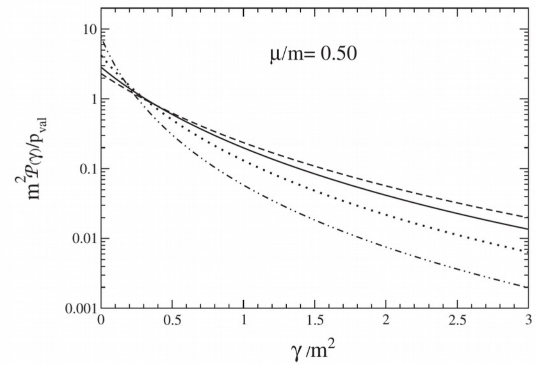
Valence wave function

3+1 n=1

LADDER KERNEL

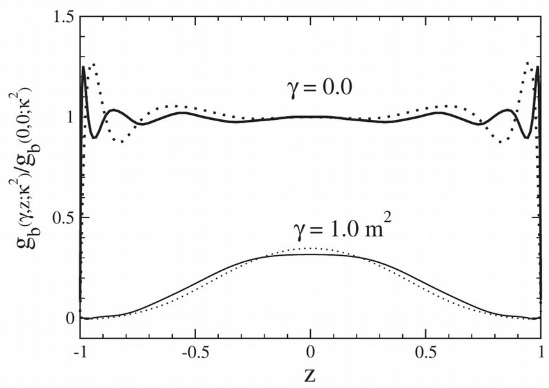
3+1 n=1





Ladder exchange

$\mu = 0.5 \;\; B/M = 1$



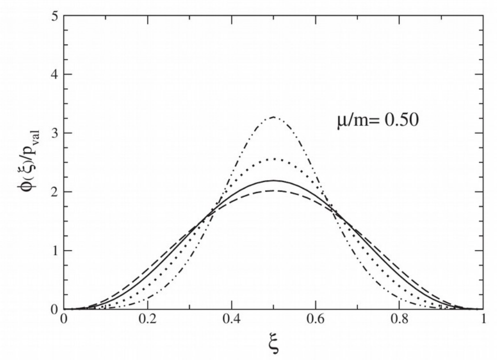


FIG. 3. The longitudinal LF distribution $\phi(\xi)$ for the valence component Eq. (34) vs the longitudinal-momentum fraction ξ for $\mu/m = 0.05$, 0.15, 0.50. Dash-double-dotted line: $B/m = 0.20$. Dotted line: $B/m = 0.50$. Solid line: $B/m = 1.0$. Dashed line: $B/m = 2.0$. Recall that $\int_0^1 d\xi \phi(\xi) = P_{val}$ (cf. Table III).

Karmanov, Carbonell, EPJA 27, 1 (2006)
Frederico, Salmè, Viviani PRD89, 016010 (2014)

Transverse distribution: Euclidean and Minkowski

$$\phi_M^T(\mathbf{k}_\perp) \equiv \int dk^0 dk^3 \Phi(k,p) = \frac{1}{2}\int dk^+ dk^- \Phi(k,p) \text{ and}$$

$$\phi_E^T(\mathbf{k}_\perp) \equiv \mathrm{i}\int dk_E^0 dk^3 \Phi_E(k_E,p),$$

136 C. Gutierrez et al. / Physics Letters B 759 (2016) 131–137

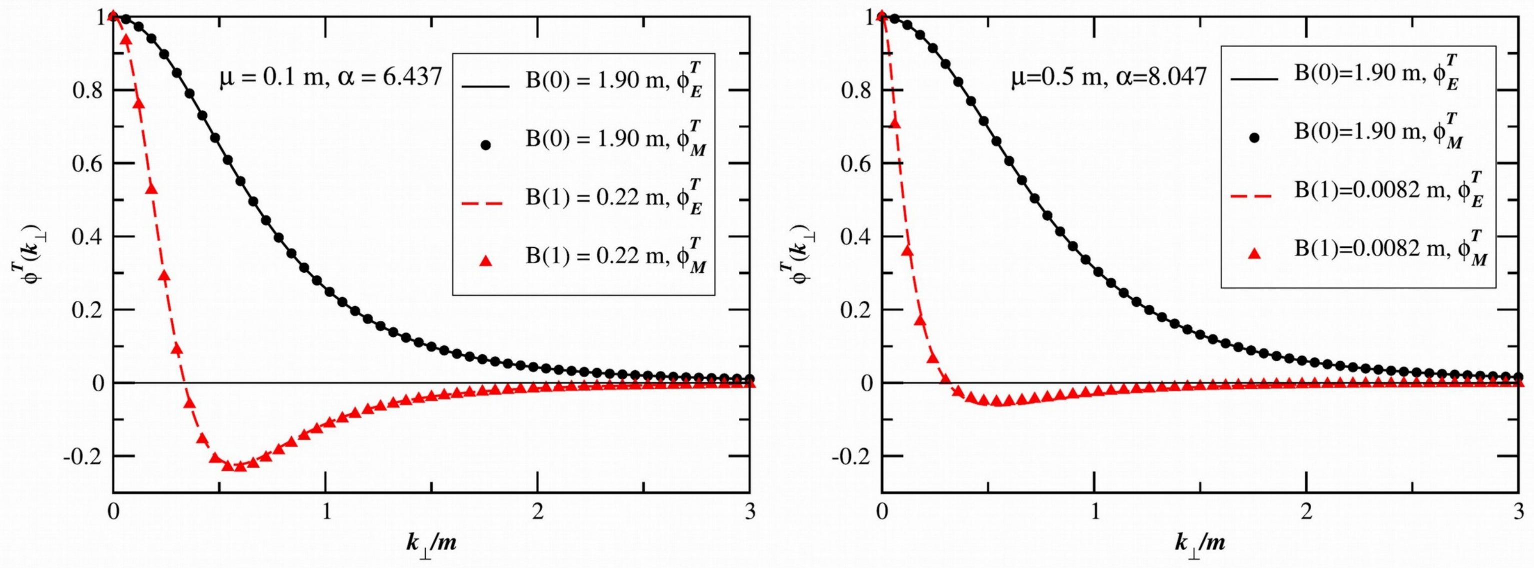


Fig. 6. Transverse momentum amplitudes *s*-wave states, in Euclidean and Minkowski spaces, vs $k_\perp$, for both ground- and first-excited states, and two values of μ/m and α_{gr} (as indicated in the insets). The amplitudes ϕ_E^T and ϕ_M^T, arbitrarily normalized to 1 at the origin, are not easily distinguishable.

Gutierrez, Gigante, TF, Salmè, Viviani, Tomio PLB759 (2016) 131

(II) Valence LF wave function in impact parameter space

$$F(\xi, b)|_{b\to\infty} \to e^{-b\sqrt{\kappa^2+(\xi-1/2)^2M^2}} f(\xi, b)$$

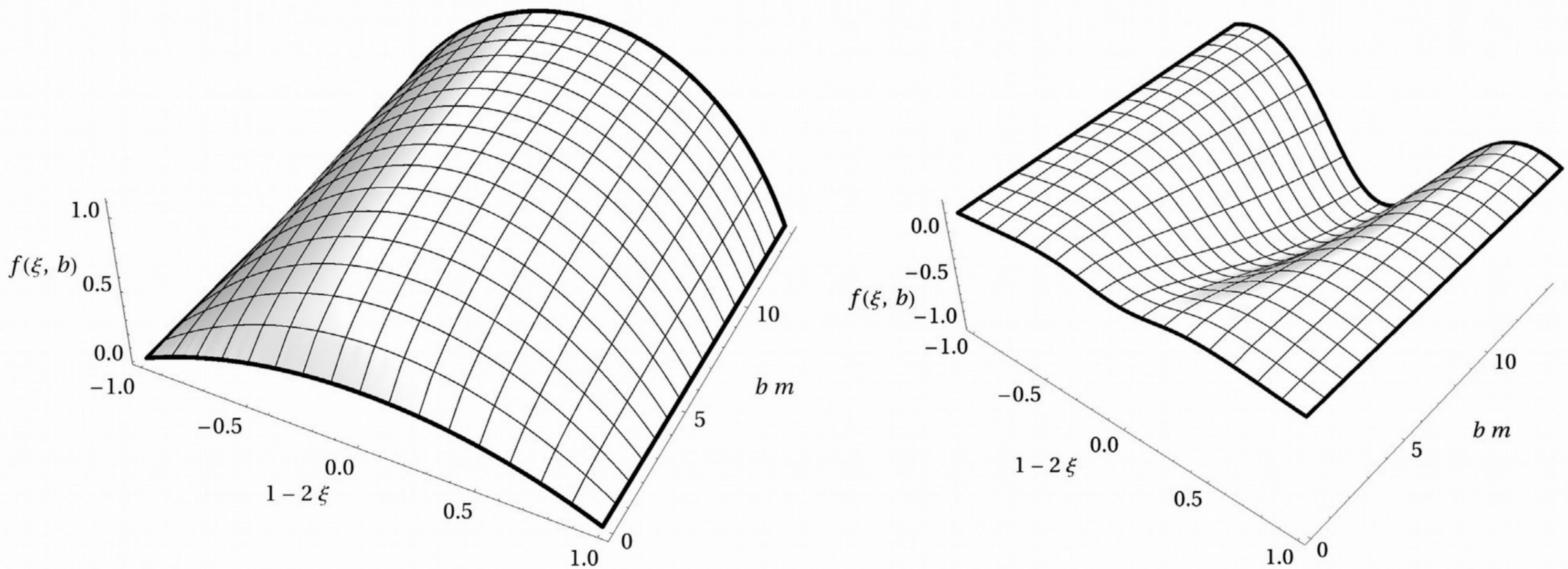


Fig. 7. The valence functions $f(\xi, b)$ in the impact parameter space. Left panel: the ground state, corresponding to $B(0) = 1.9m$, $\mu = 0.1m$ and $\alpha_{gr} = 6.437$. Right panel: first-excited state, corresponding to $B(1) = 0.22m$, $\mu = 0.1m$ and $\alpha_{gr} = 6.437$.

Light-front valence wave function L+XL

Gigante, Nogueira, Ydrefors, Gutierrez, Karmanov, TF, PRD95(2017)056012.

Large momentum behavior

$$\psi_{LF}(\gamma,\xi) \to \alpha\, \gamma^{-2}\, C(\xi)$$

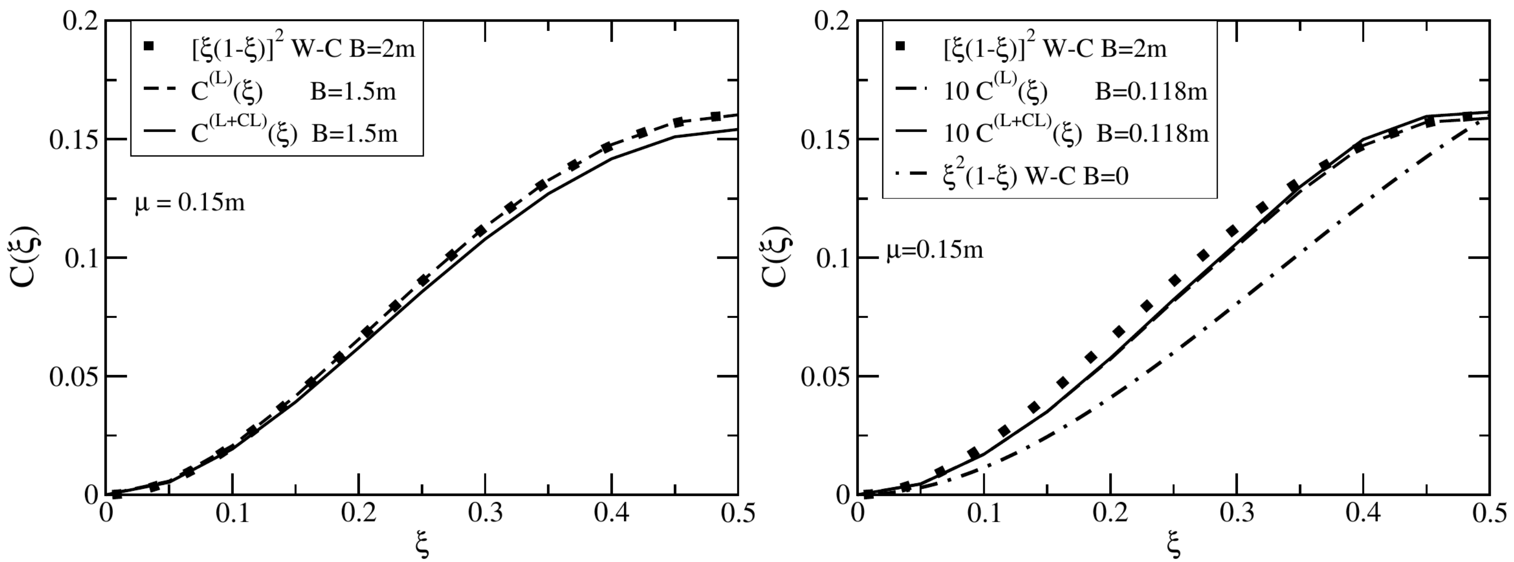
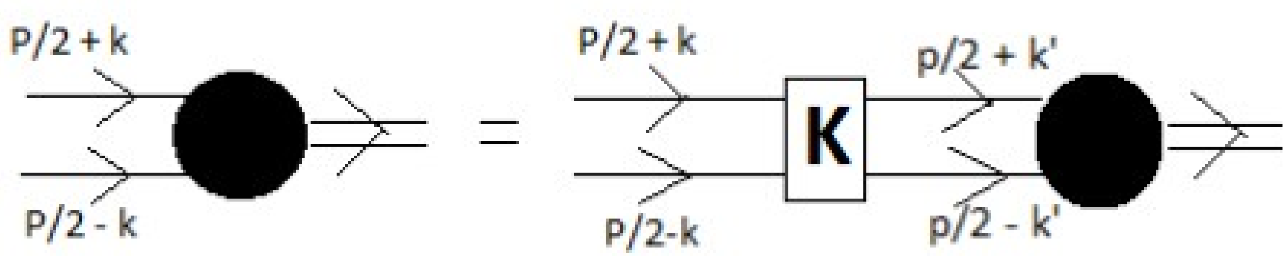


Fig. 2. Asymptotic function $C(\xi)$ defined from the LF wave function for $\gamma \to \infty$ (6) computed for the ladder kernel, $C^{(L)}(\xi)$ (dashed line), and ladder plus cross-ladder kernel, $C^{(L+CL)}(\xi)$ (solid line), with exchanged boson mass of $\mu = 0.15\,m$. Calculations are performed for $B = 1.5\,m$ (left frame) and $B = 0.118\,m$ (right frame). A comparison with the analytical forms of $C(\xi)$ valid for the Wick-Cutkosky model for $B = 2m$ (full box) and $B \to 0$ (dash-dotted line) both arbitrarily normalized.

BSE for qqbar: pion

Carbonell and Karmanov EPJA 46 (2010) 387;

de Paula, TF,Salmè, Viviani PRD 94 (2016) 071901;



$$\Phi(k,p) = S(k+p/2) \int \frac{d^4k'}{(2\pi)^4} F^2(k-k')\, i\mathcal{K}(k,k')\, \Gamma_1\, \Phi(k',p)\, \bar{\Gamma}_2\, S(k-p/2)$$

Ladder approximation (L): suppression of XL for Nc=3

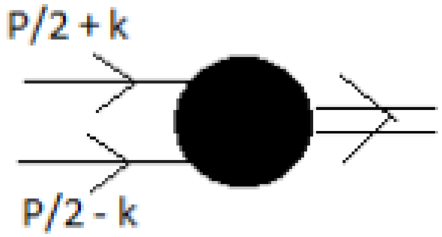
[A. Nogueira, CR Ji, Ydrefors, TF, PLB(2017) 1710.04398 [hep-th]]

Vector $\quad i\mathcal{K}_V^{(Ld)\mu\nu}(k,k') = -ig^2 \frac{g^{\mu\nu}}{(k-k')^2 - \mu^2 + i\epsilon}$

Vertex Form-Factor $\quad F(q) = \frac{\mu^2 - \Lambda^2}{q^2 - \Lambda^2 + i\epsilon}$

NIR for fermion-antifermion: 0^- (pion)

BS amplitude



$$\Phi(k,p) = S_1\phi_1 + S_2\phi_2 + S_3\phi_3 + S_4\phi_4$$

$$S_1 = \gamma_5 \quad S_2 = \frac{1}{M}\not{p}\gamma_5 \quad S_3 = \frac{k\cdot p}{M^3}\not{p}\gamma_5 - \frac{1}{M}\not{k}\gamma_5 \quad S_4 = \frac{i}{M^2}\sigma_{\mu\nu}p^\mu k^\nu \gamma_5$$

$$\phi_i(k,p) = \int_{-1}^{+1} dz' \int_0^{\infty} d\gamma' \frac{g_i(\gamma',z')}{\left(k^2 + p\cdot k\; z' + M^2/4 - m^2 - \gamma' + i\epsilon\right)^3}$$

Light-front projection: integration over k^- (LF singularities)

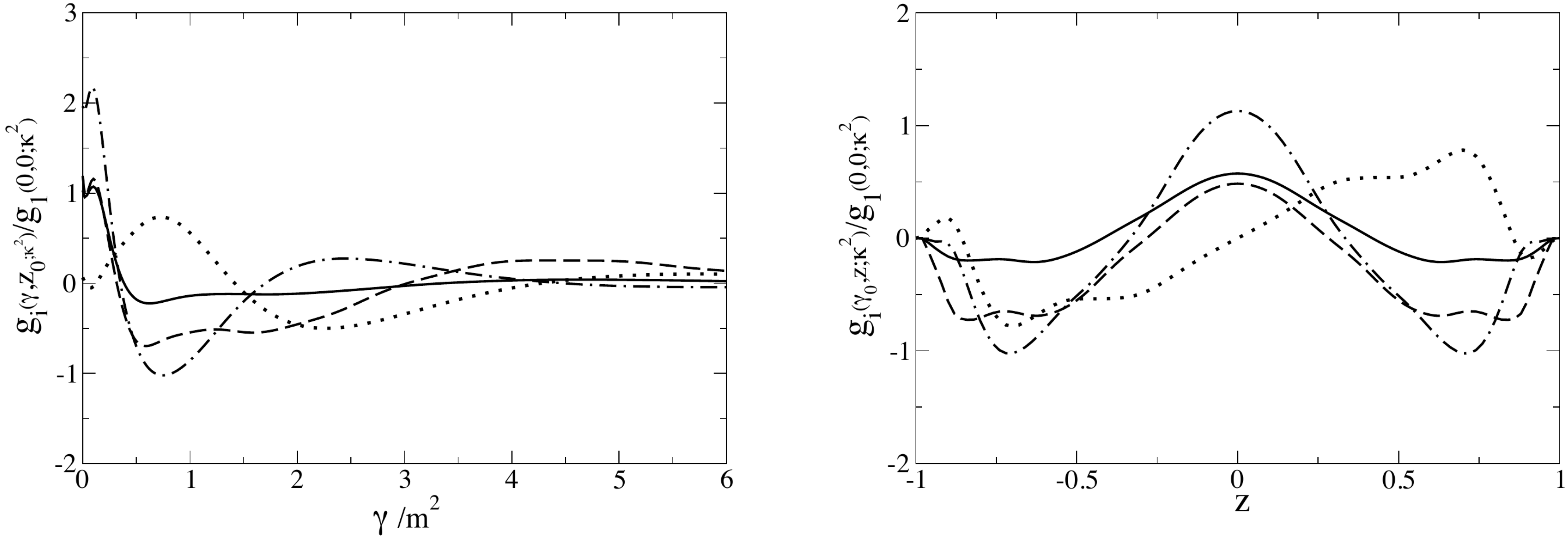
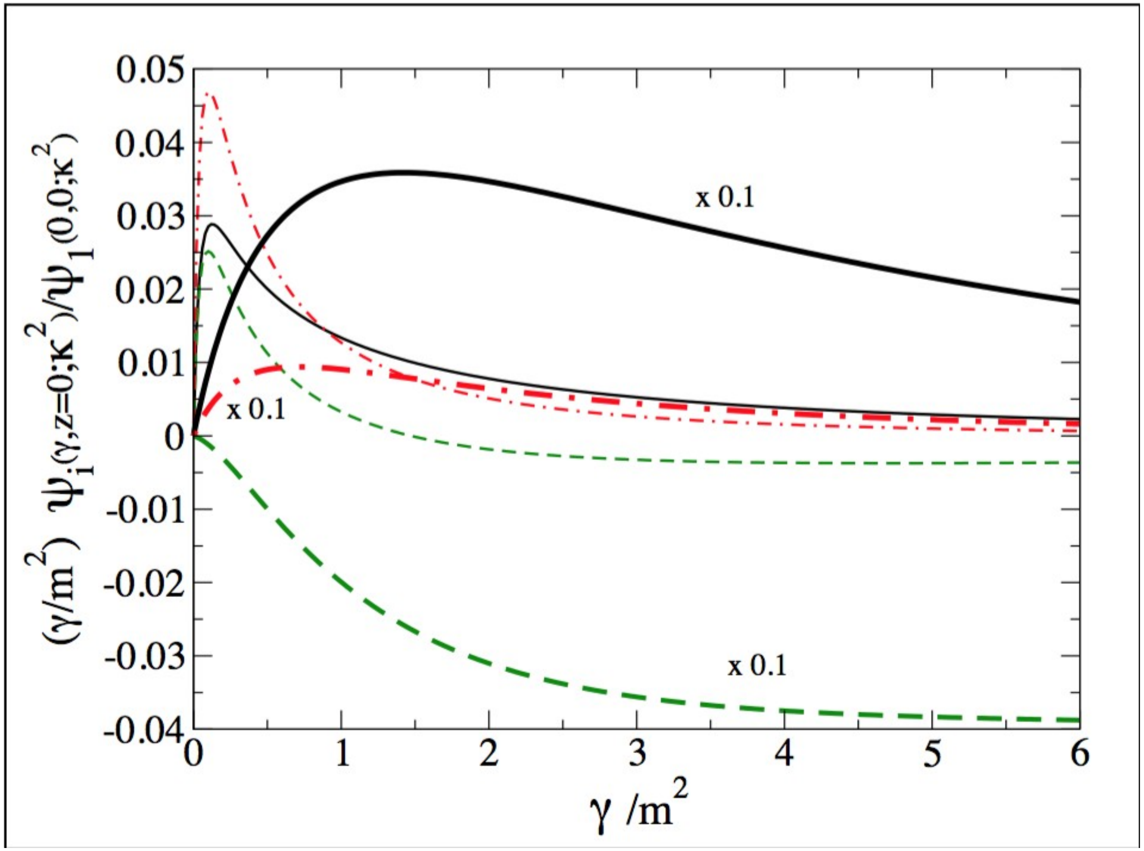


Figure 2. Nakanishi weight-functions $g_i(\gamma,\, z;\kappa^2)$, Eqs. 3.1 and 3.2 evaluated for the 0^+ two-fermion system with a scalar boson exchange such that $\mu/m = 0.5$ and $B/m = 0.1$ (the corresponding coupling is $g^2 = 52.817$ [17]). The vertex form-factor cutoff is $\Lambda/m = 2$. Left panel: $g_i(\gamma,\, z_0;\kappa^2)$ with $z_0 = 0.6$ and running γ/m^2. Right panel: $g_i(\gamma_0,\, z;\kappa^2)$ with $\gamma_0/m^2 = 0.54$ and running z, The Nakanishi weight-functions are normalized with respect to $g_1(0,0;\kappa^2)$. Solid line: g_1. Dashed line: g_2. Dotted line: g_3. Dot-dashed line: g_4.

de Paula, TF,Salmè, Viviani PRD 94 (2016) 071901;

Massless vector exchange: high-momentum tails

de Paula, TF,Salmè, Viviani PRD 94 (2016) 071901;



LF amplitudes ψ_i times γ/m^2 at fixed $z = 0$, for the vector coupling.
$B/m = 0.1$ (thin lines) and 1.0 (thick lines).

—— : $(\gamma/m^2)\ \psi_1$.
— —: $(\gamma/m^2)\ \psi_2$.
— • : $(\gamma/m^2)\ \psi_4$.
$\psi_3 = 0$ for $z = 0$

Power one is expected for the pion valence amplitude:

X Ji et al, PRL 90 (2003) 241601.

PION MODEL

W. de Paula, TF, Pimentel, Salmè, Viviani, EPJC 77 (2017) 764

- **Gluon effective mass ~ 500 MeV – Landau Gauge LQCD**

[Oliveira, Bicudo, JPG 38 (2011) 045003;
Duarte, Oliveira, Silva, Phys. Rev. D 94 (2016) 01450240]

- **Mquark = 250 MeV**

[Parappilly, et al, PR D73 (2006) 054504]

- **Λ/m =3 & 8**

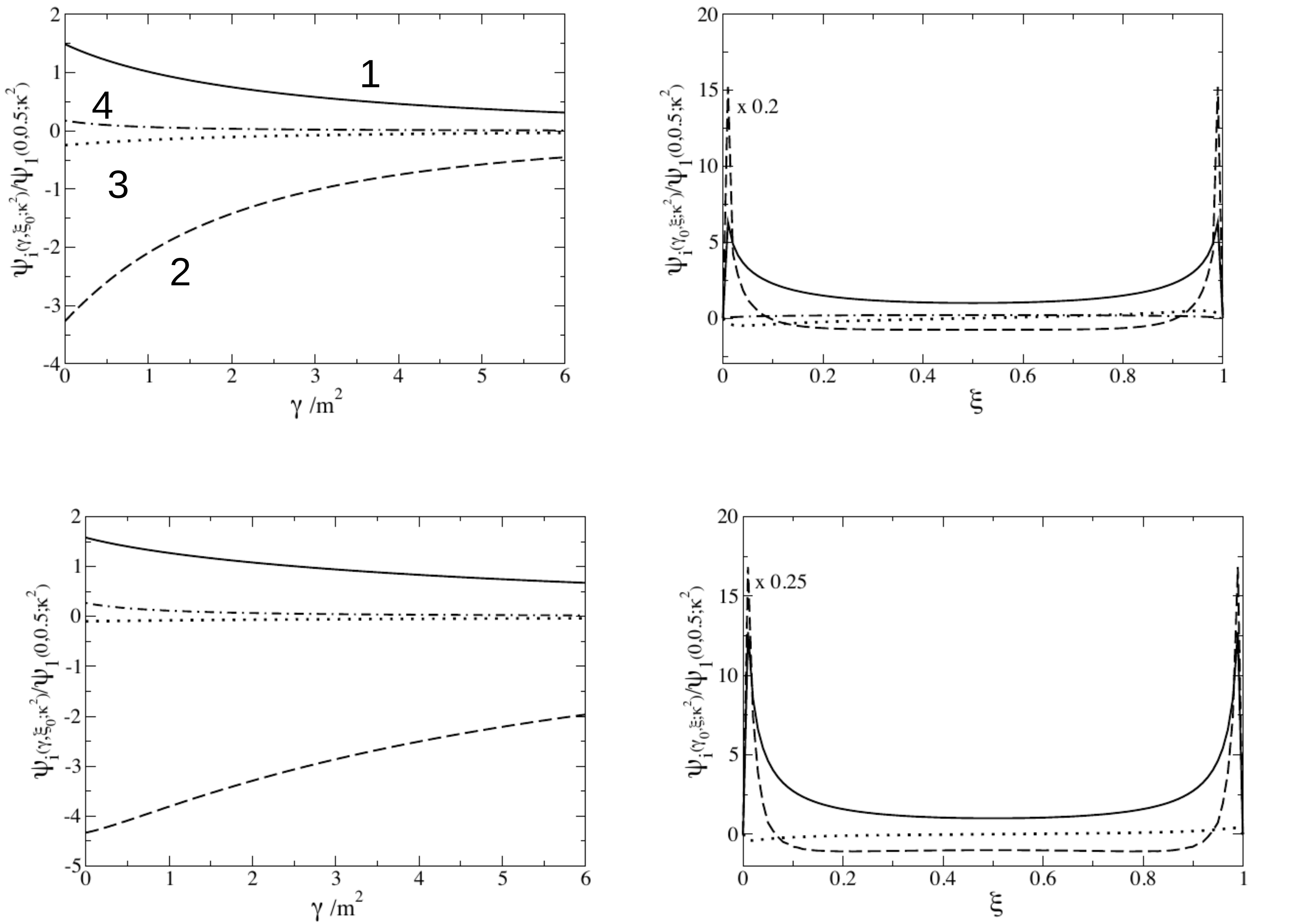


Figure 6. Light-front amplitudes $\psi_i(\gamma, \zeta)$, Eq. 3.11, for the pion-like system with a heavy-vector exchange ($\mu/m = 2$), binding energy of $B/m = 1.44$ and constituent mass $m = 250$ MeV. Upper panel: vertex form-factor cutoff $\Lambda/m = 3$ and $g^2 = 435.0$, corresponding to $\alpha_s = 10.68$ (see text for the definition of α_s). Lower panel: vertex form-factor cutoff $\Lambda/m = 8$ and $g^2 = 53.0$, corresponding to $\alpha_s = 3.71$. The value of the longitudinal variable is $\xi_0 = 0.2$ and $\gamma_0 = 0$. Solid line: ψ_1 . Dashed line: ψ_2. Dotted line: ψ_3. Dot-dashed line: ψ_4

Valence distribution functions

W. de Paula, TF, Salmè, Viviani, in preparation

Valence probability:

$$N_2 = \frac{1}{32\,\pi^2} \int_{-1}^{1} dz \int_0^\infty d\gamma \left\{ \tilde{\psi}_{val}(\gamma,\xi)\ \tilde{\psi}_{val}(\gamma,\xi) + \frac{\gamma}{M^2}\ \psi_{val;4}(\gamma,\xi)\ \psi_{val;4}(\gamma,\xi) \right\}$$

$$\begin{aligned}
\tilde{\psi}_{val}(\gamma,z) &= -\frac{i}{M} \int_0^\infty d\gamma' \frac{g_2(\gamma',z)}{[\gamma+\gamma'+m^2 z^2+(1-z^2)\kappa^2 - i\epsilon]^2} \\
&- \frac{i}{M}\frac{z}{2} \int_0^\infty d\gamma' \frac{g_3(\gamma',z)}{[\gamma+\gamma'+m^2 z^2+(1-z^2)\kappa^2 - i\epsilon]^2} \\
&+ \frac{i}{M^3} \int_0^\infty d\gamma' \ \frac{\partial g_3(\gamma',z)/\partial z}{[\gamma+\gamma'+z^2 m^2+(1-z^2)\kappa^2 - i\epsilon]}
\end{aligned}$$

$$\psi_{val;4}(\gamma,z) = -\frac{i}{M} \int_0^\infty d\gamma' \frac{g_4(\gamma',z)}{[\gamma+\gamma'+m^2 z^2+(1-z^2)\kappa^2 - i\epsilon]^2} \,.$$

Valence probability

Table 1 Valence probability for a massive vector exchange, with $\mu/m = 0.15$ and a cut-off $\Lambda/m = 2$ for the vertex form-factor. The number of gaussian points is 72.

B/m	Prob.
0.01	0.48
0.1	0.39
1.0	0.34

Table 2 Valence probability for a massive vector exchange, with $\mu/m = 0.5$ and a cut-off $\Lambda/m = 2$ for the vertex form-factor. The number of gaussian points is 72.

B/m	Prob.
0.01	0.48
0.1	0.42
1.0	0.34

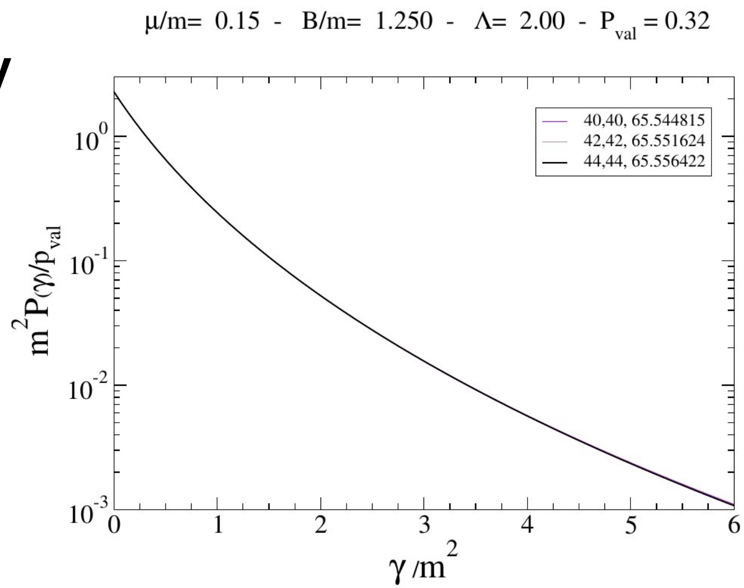
Lot of room for the higher LF Fock components of the wave function to manifest!

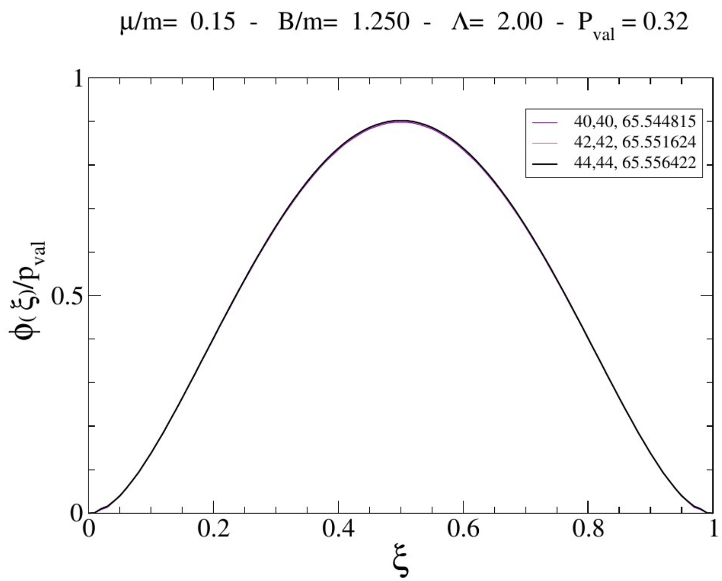
Valence distribution functions: longitudinal and transverse

Mquark 200 MeV

Mgluon 30 MeV

Λ/m =2

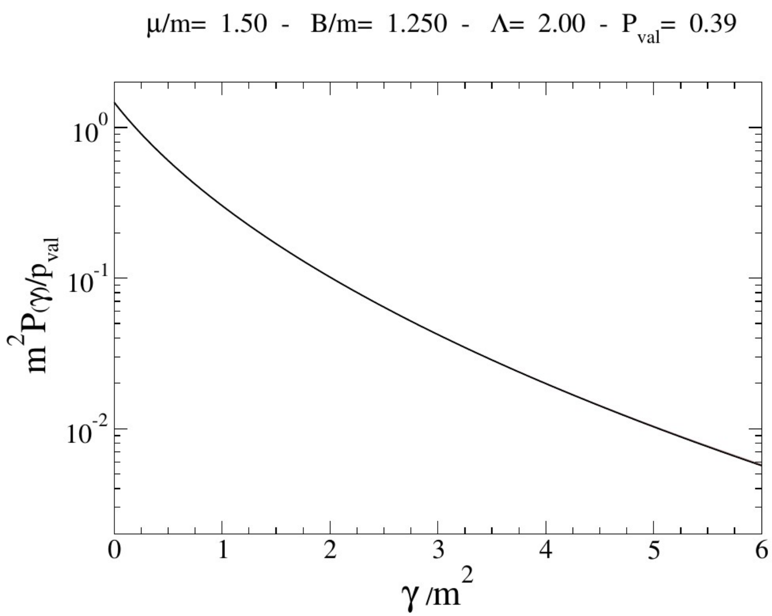


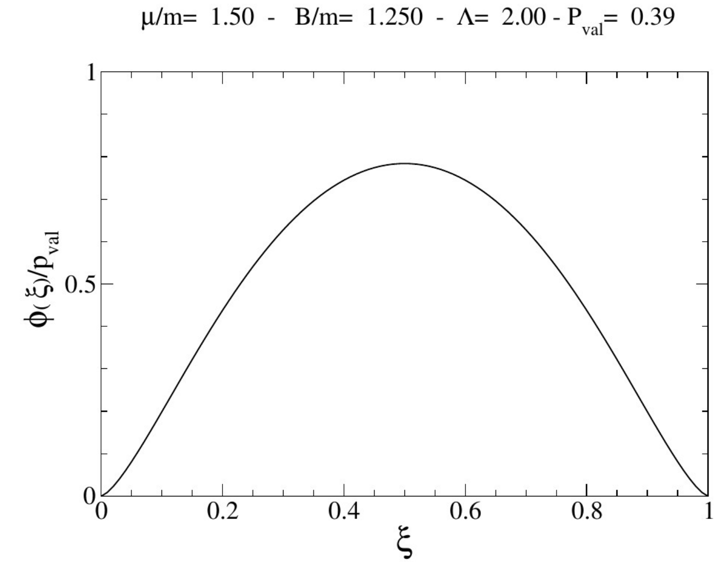


Mquark 200 MeV

Mgluon 300 MeV

Λ/m =2

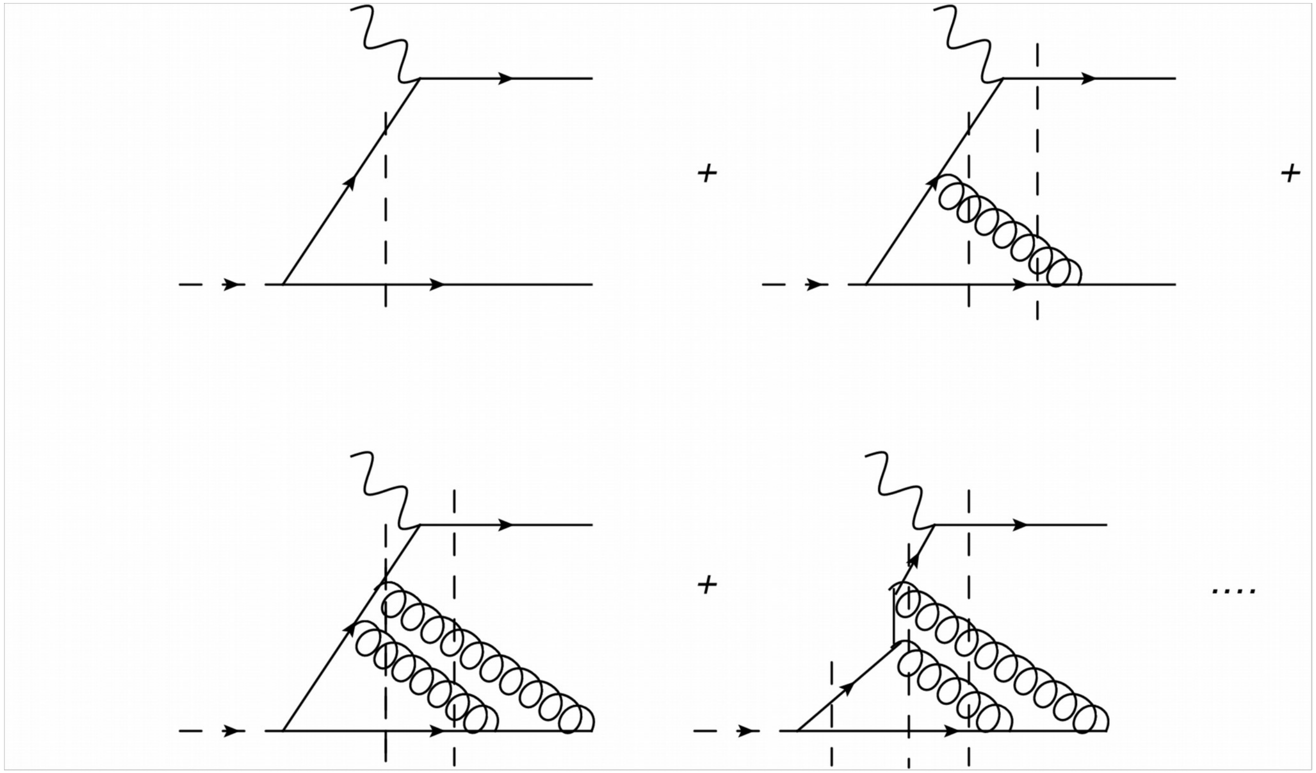




Beyond the valence ....

Sales, TF, Carlson,Sauer, PRC 63, 064003 (2001)

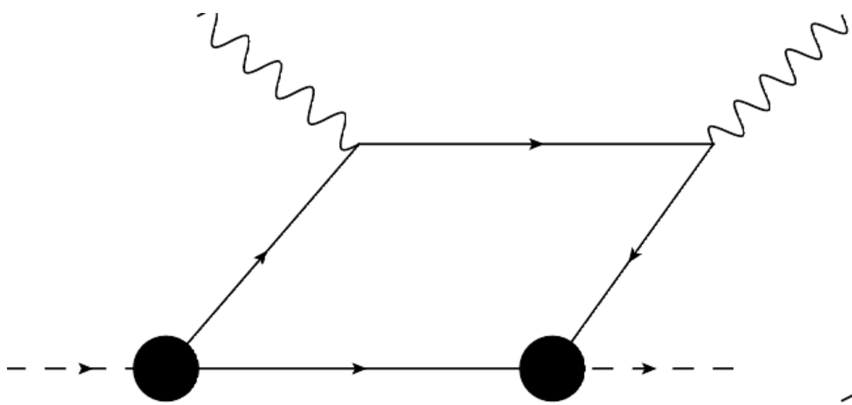
Marinho, TF, Pace,Salme,Sauer, PRD 77, 116010 (2008)

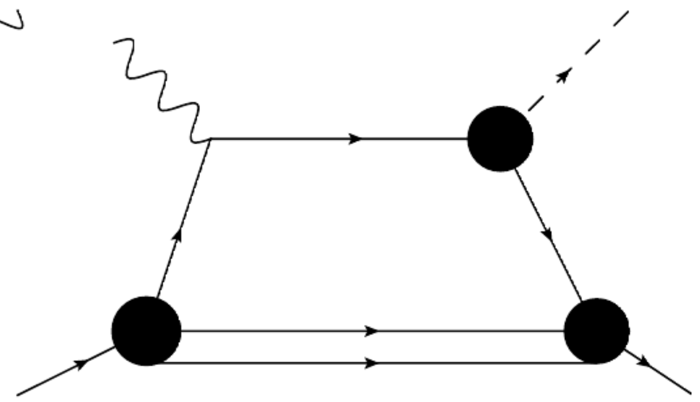


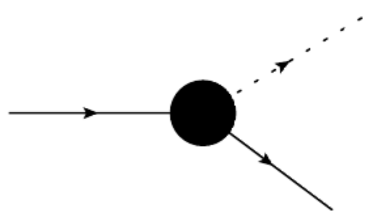
- **Population of lower x, due to the gluon radiation!**
- **Evolution?**

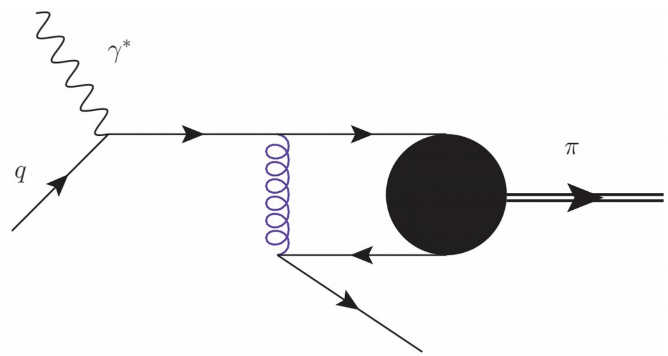
Beyond the valence ….

ERBL – DGLAP regions









Fragmentation function

Conclusions and Perspectives

- **A method for solving the fermionic BSE: singularities**

LF framework to investigate the fermionic bound state system

- **Our numerical results confirm the robustness of the Nakanishi Integral Representation for solving the BSE.**
- **More realism: self-energies, vertex corrections, Landau gauge, ingredients from LQCD….**
- **Confinement?**
- **Beyond the pion, kaon, D, B, rho…, and the nucleon**
- **Form-Factors, PDFs, TMDs, Fragmentation Functions...**

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THANK YOU!

65 anos CAPES

CNPq

FAPESP