TMDs at Small x

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Outline

- How to obtain small-x asymptotics of TMDs – the small-x evolution equations and their solution. What we mean by small-x asymptotics: small x (linear evolution) vs. very small x (saturation).

- Unpolarized nucleon: small-x asymptotics of
  - Unpolarized quark and gluon TMDs;
  - Linearly polarized gluon TMDs.

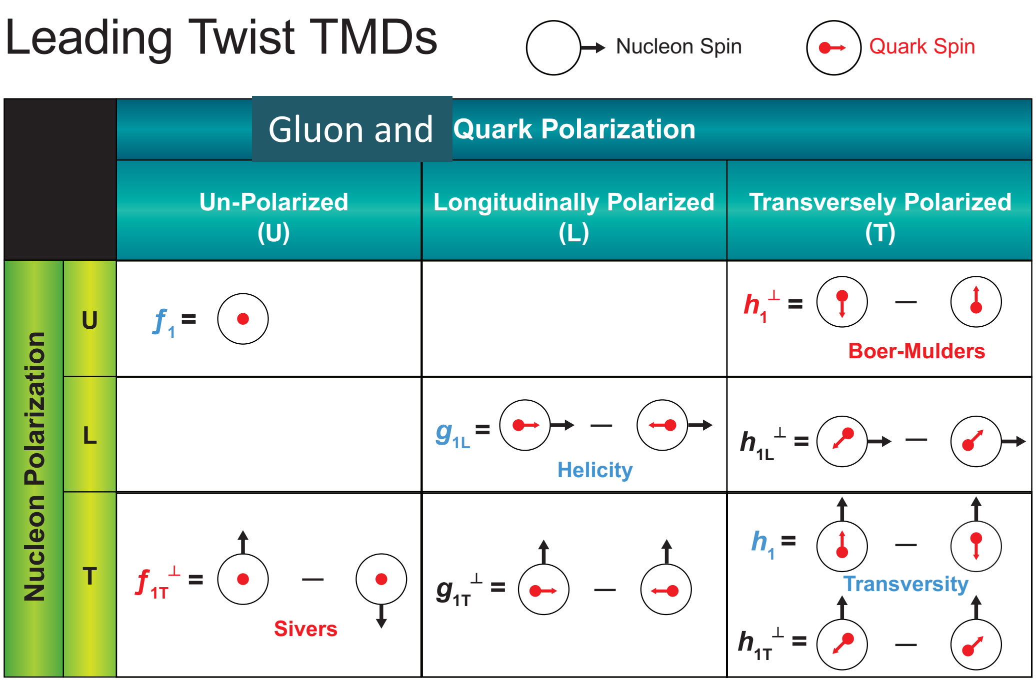
- Longitudinally polarized nucleon: small-x asymptotics of quark and gluon helicity TMDs.

- Transversely polarized nucleon: small-x asymptotics of transversity and Sivers distribution.

- Outlook:
  - Small-x asymptotics of TMDs beyond the ones listed above;
  - connecting small-x, CSS and DGLAP evolutions?

My goal here

- In this talk I will try to describe the small-x asymptotics of quark and gluon TMDs, in the cases where it is known.

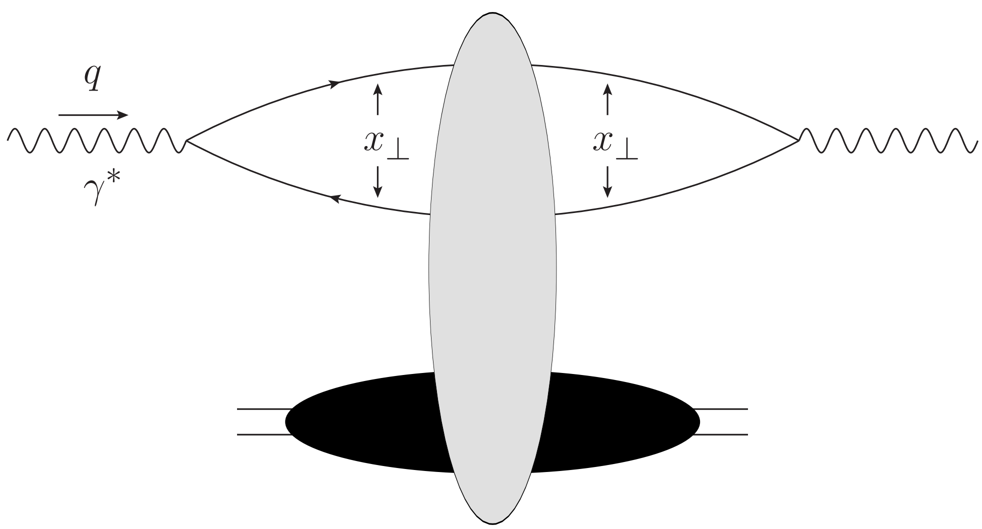


- Note that when small-x asymptotics is known, the k_T dependence is usually known as well (theoretically).

Small-x Evolution

Dipole picture of DIS

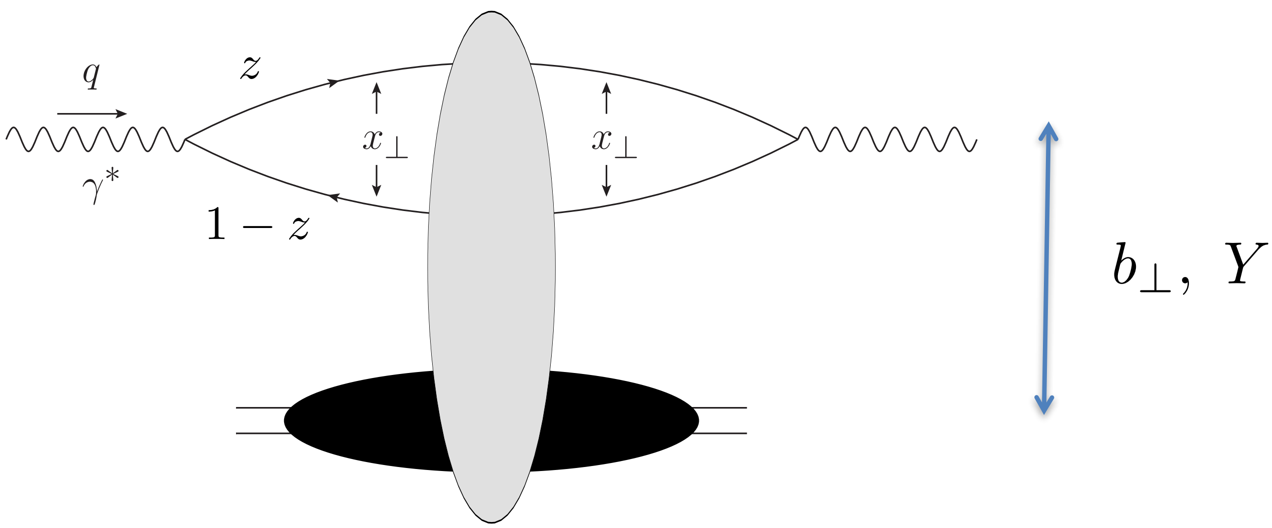
- In the dipole picture of DIS the virtual photon splits into a quark-antiquark pair, which then interacts with the target.
- The total DIS cross section and structure functions are calculated via:



Dipole Amplitude

- The total DIS cross section is expressed in terms of the (Im part of the) forward quark dipole amplitude N:

$$\sigma_{tot}^{\gamma^* A} = \int \frac{d^2 x_\perp}{2\pi} d^2 b_\perp \int\limits_0^1 \frac{dz}{z(1-z)} \, |\Psi^{\gamma^* \to q\bar{q}}(\vec{x}_\perp, z)|^2 \; N(\vec{x}_\perp, \vec{b}_\perp, Y)$$



Dipole Amplitude

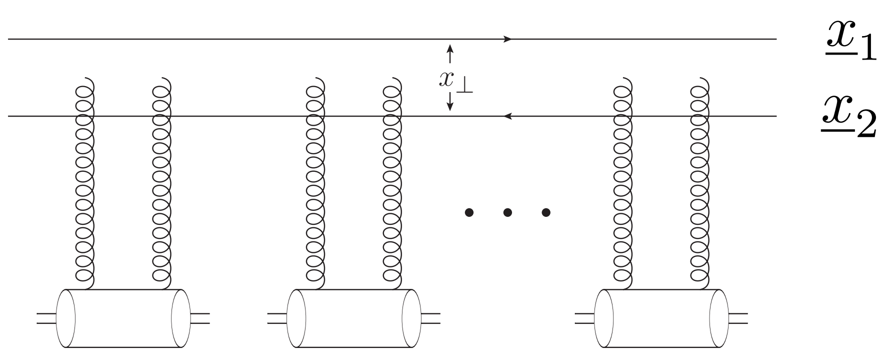
- The quark dipole amplitude is defined by

$$N(\underline{x}_1, \underline{x}_2) = 1 - \frac{1}{N_c} \left\langle \mathrm{tr} \left[V(\underline{x}_1)\, V^\dagger(\underline{x}_2) \right] \right\rangle$$

- Here we use the Wilson lines along the light-cone direction

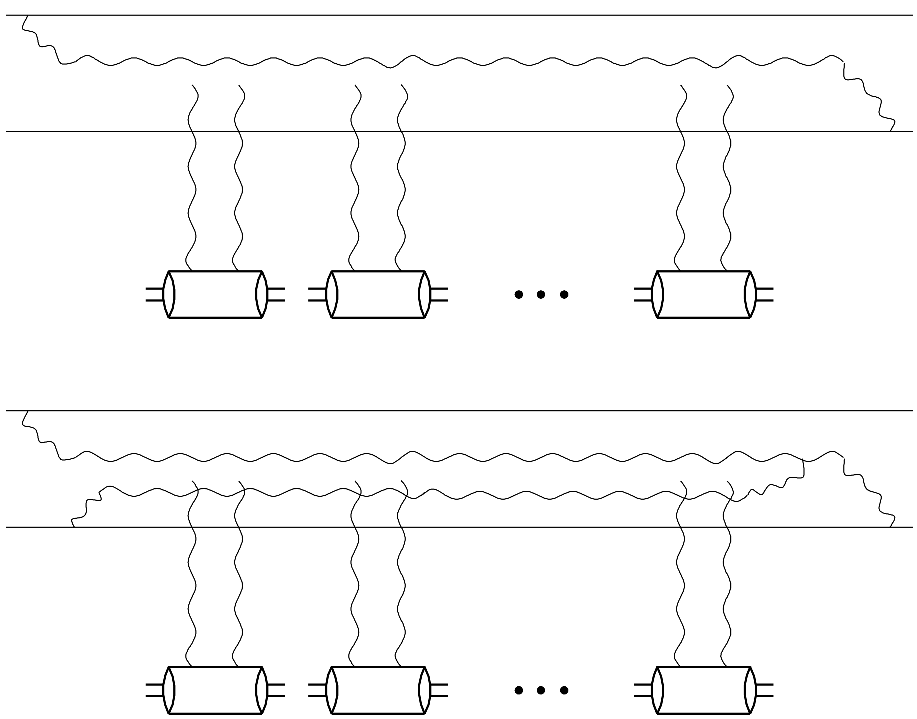
$$V(\underline{x}) = \mathrm{P} \exp \left[i\, g \int\limits_{-\infty}^{\infty} dx^+ \, A^-(x^+, x^- = 0, \underline{x}) \right]$$

- In the classical Glauber-Mueller/McLerran-Venugopalan approach the dipole amplitude resums multiple rescatterings:



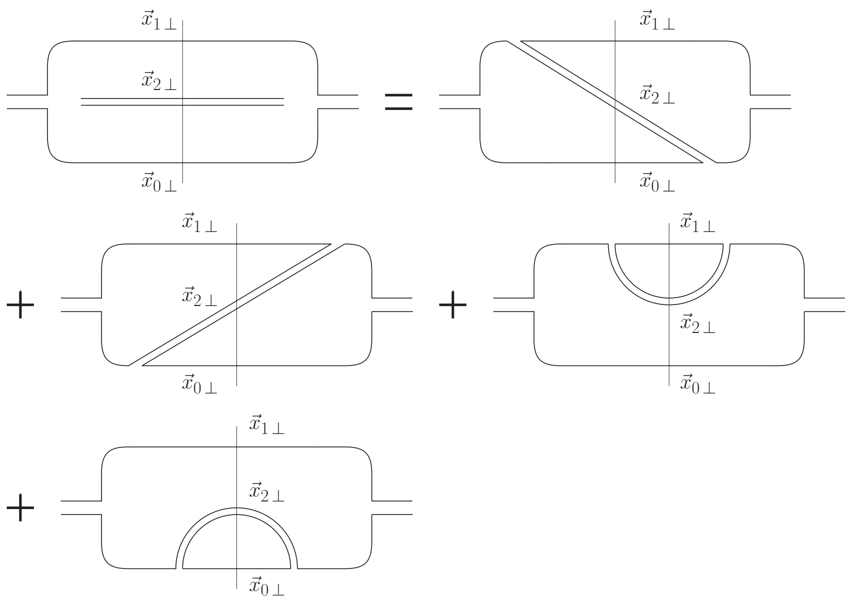
Dipole Amplitude

- The energy dependence comes in through nonlinear small-x BK/JIMWLK evolution, which resums the long-lived s-channel gluon corrections:



$$\alpha_s \ln \frac{1}{x} \sim \alpha_s Y \sim 1$$

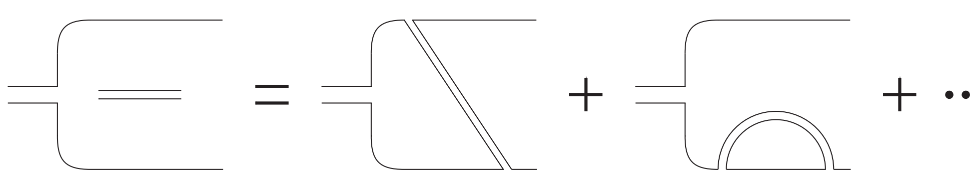
Notation (Large-N_C)



Real emissions in the amplitude squared

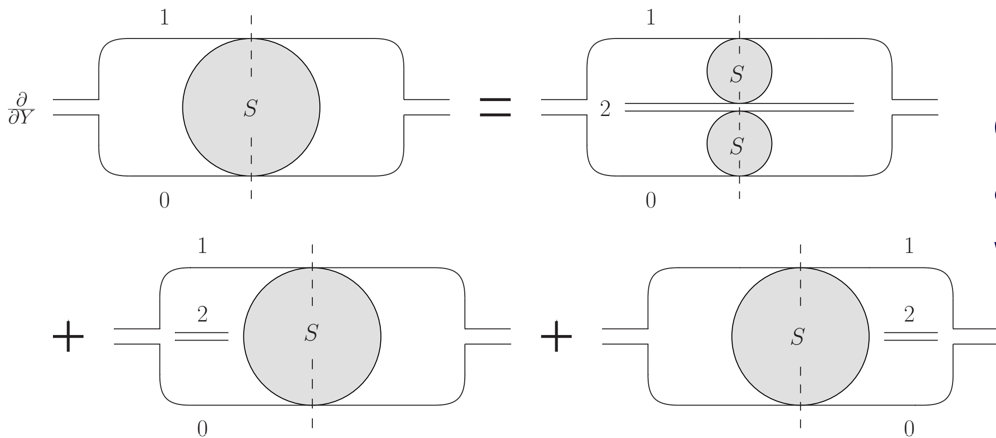
(dashed line – all Glauber-Mueller exchanges at light-cone time =0)

Virtual corrections in the amplitude (wave function)



Nonlinear Evolution

To sum up the gluon cascade at large-N_C we write the following equation for the dipole S-matrix:



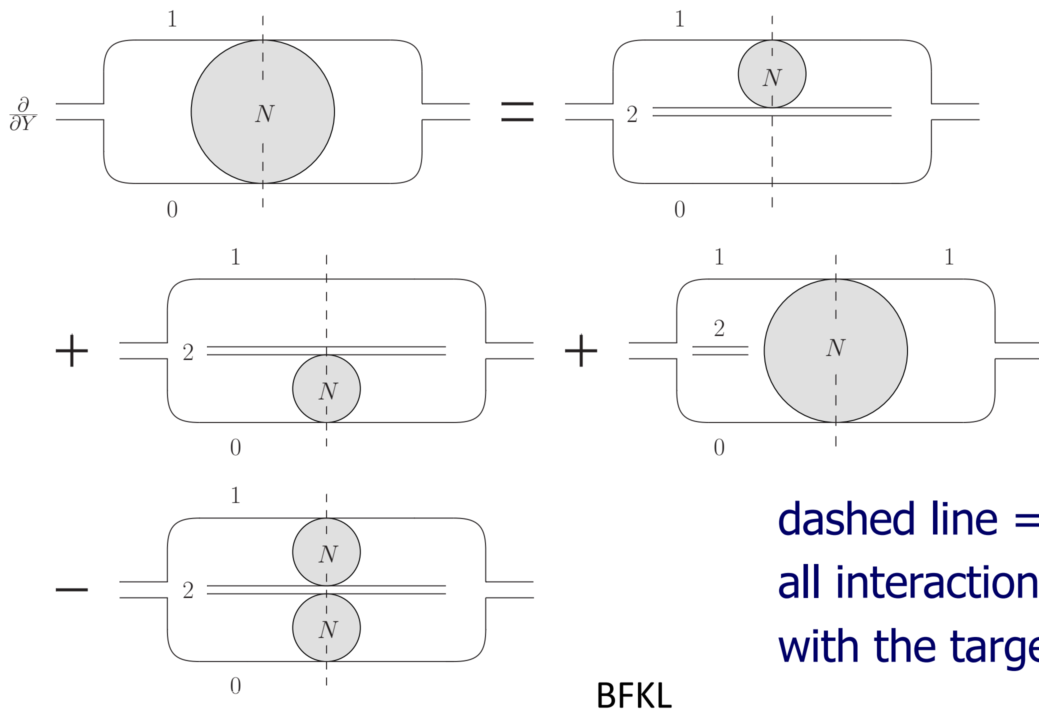
dashed line = all interactions with the target

$$\partial_Y S_{\mathbf{x}_0,\mathbf{x}_1}(Y) = \frac{\alpha_s N_c}{2\pi^2} \int d^2x_2 \frac{x_{01}^2}{x_{02}^2 \, x_{21}^2} \left[S_{\mathbf{x}_0,\mathbf{x}_2}(Y) \, S_{\mathbf{x}_2,\mathbf{x}_1}(Y) - S_{\mathbf{x}_0,\mathbf{x}_1}(Y)\right]$$

Remembering that S= 1-N we can rewrite this equation in terms of the dipole scattering amplitude N.

Nonlinear evolution at large N_c

As N=1-S we write



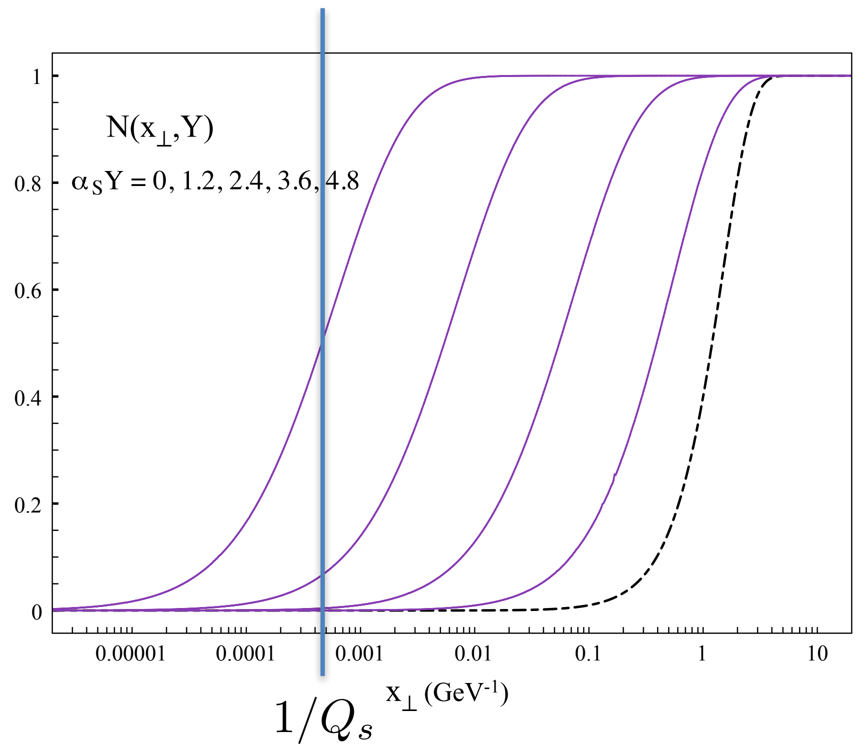
dashed line = all interactions with the target

BFKL

$$\partial_Y N_{\mathbf{x}_0,\mathbf{x}_1}(Y) = \frac{\alpha_s N_c}{2\pi^2} \int d^2x_2 \frac{x_{01}^2}{x_{02}^2 x_{21}^2} \left[N_{\mathbf{x}_0,\mathbf{x}_2}(Y) + N_{\mathbf{x}_2,\mathbf{x}_1}(Y) - N_{\mathbf{x}_0,\mathbf{x}_1}(Y) - N_{\mathbf{x}_0,\mathbf{x}_2}(Y)\, N_{\mathbf{x}_2,\mathbf{x}_1}(Y) \right]$$

Balitsky '96, Yu.K. '99

Solution of BK equation



numerical solution by J. Albacete ‘03 (earlier solutions were found numerically by Golec-Biernat, Motyka, Stasto, by Braun and by Lublinsky et al in ‘01)

BK solution preserves the black disk limit, N<1 always (unlike the linear BFKL equation)

$$\sigma^{q\bar{q}A} = 2 \int d^2b\, N(x_\perp, b_\perp, Y)$$

Small-x Asymptotics

- BFKL solution gives (x<<1) $N \sim \left(\frac{1}{x}\right)^{\alpha_P - 1}$

  with
  $$\alpha_P - 1 = \frac{4\,\alpha_s\,N_c}{\pi}\,\ln 2 \approx 2.77\,\frac{\alpha_s\,N_c}{\pi}$$

- NLO corrections are known (Fadin, Lipatov ’98; Ciafaloni, Camici ‘98).

- Full BK equation solution also leads to saturation at very small x (x<<<1):
  $$N \sim \text{const}$$

- Below we will refer to the BFKL-like linear regime as the “small-x asymptotics” of TMDs. It should be understood that at even smaller x saturation is expected to come in and stop the small x evolution.

Going Beyond Large N_C: JIMWLK

To do calculations beyond the large-N_C limit on has to use a functional integro-differential equation written by Iancu, Jalilian-Marian, Kovner, Leonidov, McLerran and Weigert (JIMWLK):

$$\frac{\partial Z}{\partial Y} = \alpha_s \left\{ \frac{1}{2} \frac{\delta^2}{\delta\rho(u)\,\delta\rho(v)} \left[Z\,\chi(u,v) \right] - \frac{\delta}{\delta\rho(u)} \left[Z\,\sigma(u) \right] \right\}$$

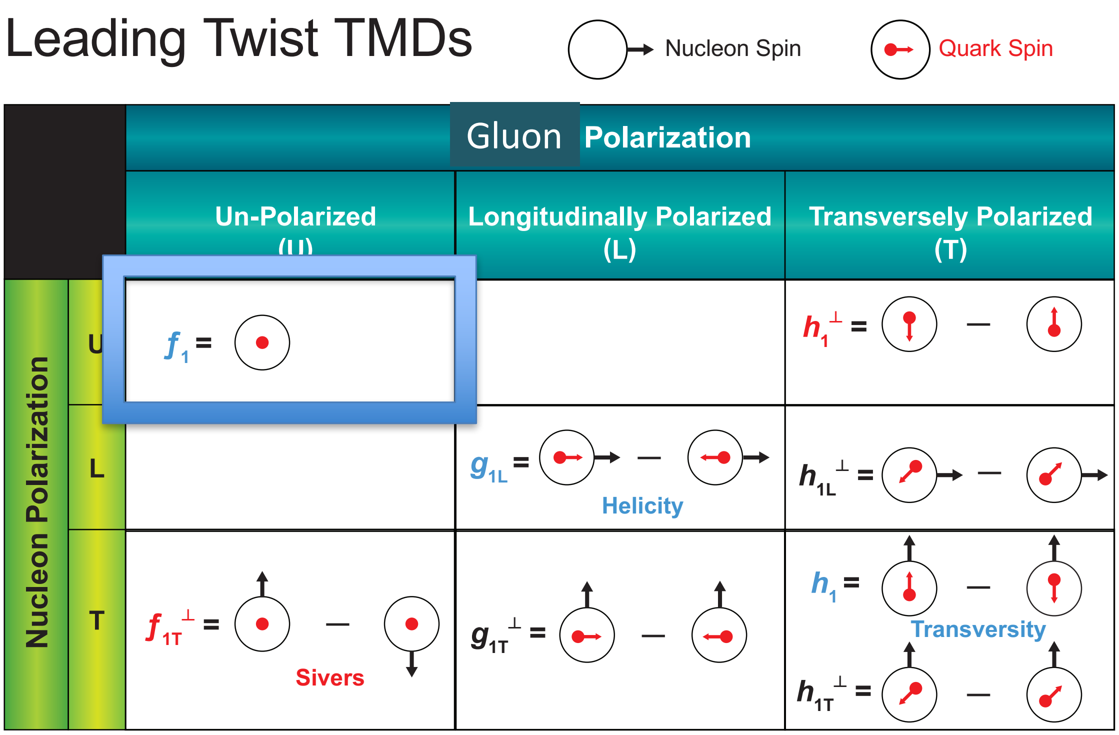
where the functional Z[r] can then be used for obtaining wave function-averaged observables (like Wilson loops for DIS):

$$\langle O \rangle = \int D\rho\, Z[\rho]\, O[\rho]$$

Unpolarized Nucleon TMDs

Unpolarized Gluon TMD

- We start with the unpolarized gluon TMD at small x:

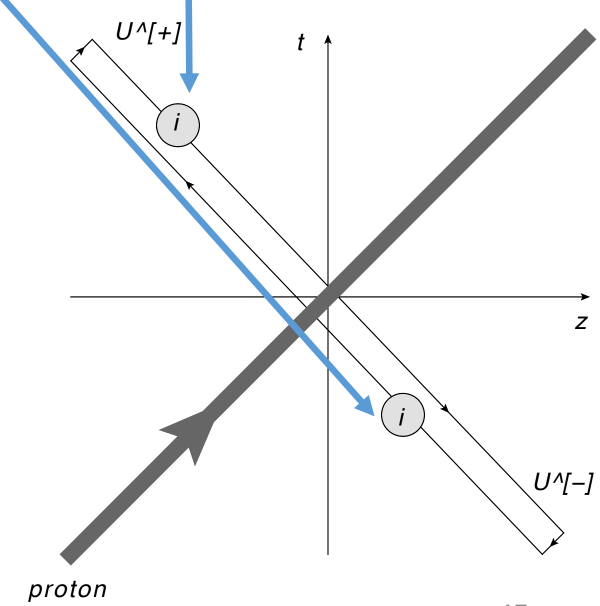


Dipole Gluon TMD

- We start with the gluon dipole TMD:

$$f_1^{G\,dip}(x, k_T^2) = \frac{2}{x\,P^+} \int \frac{d\xi^-\,d^2\xi}{(2\pi)^3}\, e^{ixP^+\,\xi^- - i\underline{k}\cdot\underline{\xi}} \,\langle P|\mathrm{tr}\left[F^{+i}(0)\,\mathcal{U}^{[+]}[0,\xi]\,F^{+i}(\xi)\,\mathcal{U}^{[-]}[\xi,0]\right]|P\rangle_{\xi^+=0}$$

- Here $U^{[+]}$ and $U^{[-]}$ are future and past-pointing fundamental Wilson line staples (hence the name `dipole' TMD – it looks like a quark dipole scattering on a proton)
- Dipole gluon TMD enters a number of cross sections: DIS, DY, SIDIS, hadron production in pA.
- Dominguez, Marquet, Xiao, Yuan ’11; M. Braun ’00; YK, Tuchin ‘01, Kharzeev, YK, Tuchin ’03.



Dipole Gluon TMD

- One can show that the gluon dipole TMD at small x is indeed related to the dipole amplitude N= 1-S (Dominguez et al, ’11; M. Braun ’00; YK, Tuchin ‘01, Kharzeev, YK, Tuchin ’03):

$$f_1^{G\,dip}(x, k_T^2) = \frac{k_T^2\, N_c}{(2\pi)^3\, \pi\, \alpha_s\, x} \int d^2b\, d^2r\, e^{-i\vec{k}_\perp \cdot \vec{r}_\perp}\, S(\vec{r}_\perp, \vec{b}_\perp, Y = \ln(1/x))$$
$$= -\frac{k_T^2\, N_c}{(2\pi)^3\, \pi\, \alpha_s\, x} \int d^2b\, d^2r\, e^{-i\vec{k}_\perp \cdot \vec{r}_\perp}\, N(\vec{r}_\perp, \vec{b}_\perp, Y = \ln(1/x))$$

- The resulting small-x asymptotics is given by the BFKL evolution,

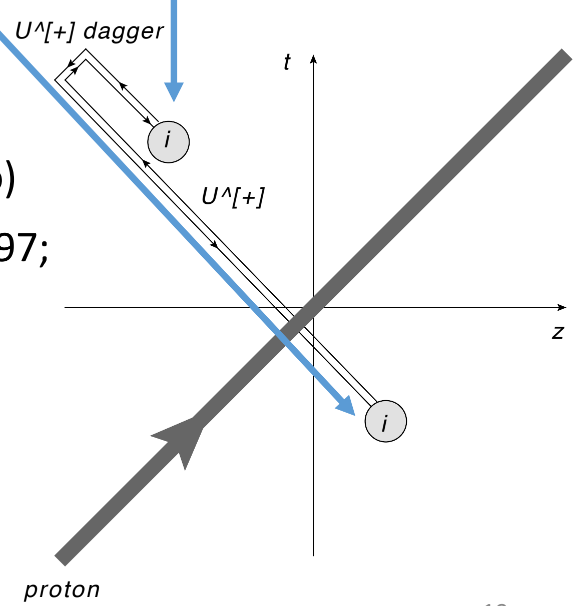
$$f_1^{G\,dip}(x, k_T^2) \sim \frac{1}{x}\, N(\vec{r}_\perp, \vec{b}_\perp, Y = \ln(1/x)) \sim \left(\frac{1}{x}\right)^{1+\frac{4\,\alpha_s\,N_c}{\pi}\,\ln 2+\mathcal{O}(\alpha_s^2)}$$

- The k_T dependence is also determined by the small-x evolution.

WW Gluon TMD

- Next consider the Weizsacker-Williams gluon TMD:

$$f_1^{G\,WW}(x, k_T^2) = \frac{2}{x\,P^+} \int \frac{d\xi^-\, d^2\xi}{(2\pi)^3}\, e^{ixP^+\,\xi^- - i\underline{k}\cdot\underline{\xi}}\, \langle P|\mathrm{tr}\left[F^{+i}(0)\,\mathcal{U}^{[+]}[0,\xi]\,F^{+i}(\xi)\,\mathcal{U}^{[+]\,\dagger}[\xi,0]\right]|P\rangle_{\xi^+=0}$$



- Here $U^{[+]}$ is the future-pointing fundamental Wilson line staple (can use past-pointing too)
- Jalilian-Marian, Kovner, McLerran, Weigert '97; Dominguez, Marquet, Xiao, Yuan '11.
- WW gluon TMD can be measured in dijet production in DIS and in pA

WW Gluon TMD

- At small x the WW gluon TMD is proportional to a different object, now made out of 4 Wilson lines, the quadrupole amplitude Q:

$$Q(x_1, x_2, x_3, x_4) = \frac{1}{N_c} \langle \text{tr}[V_1 \, V_2^\dagger \, V_3 \, V_4^\dagger] \rangle$$

- Small-x evolution for the quadrupole amplitude Q is given by an evolution equation different from BK. (Jalilian-Marian, YK '04; Dominguez, Mueller, Munier, Xiao '11.)

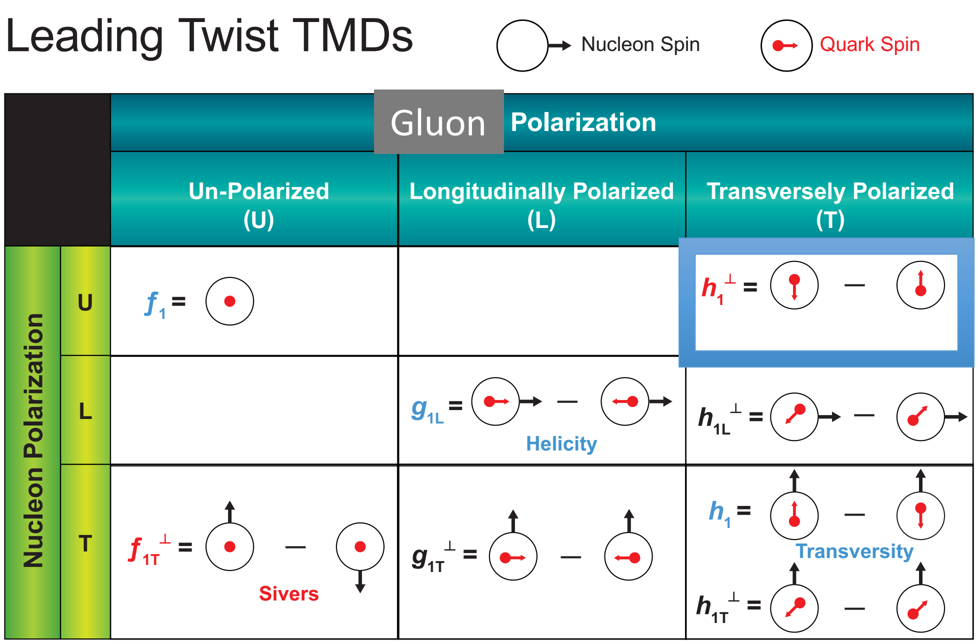
- In the linear regime the dipole amplitude Q obeys BFKL equation, such that the small-x asymptotics of the WW gluon TMD is the same as for the dipole gluon TMD:

$$f_1^{G\ WW}(x, k_T^2) \sim \frac{1}{x} \, Q \sim \left(\frac{1}{x} \right)^{1 + \frac{4 \, \alpha_s \, N_c}{\pi} \, \ln 2 + \mathcal{O}(\alpha_s^2)}$$

- The difference between the two TMDs is inside the saturation region.

Linearly Polarized Gluon TMD

- Let us discuss the linearly polarized (WW) gluon TMD $h_1^{\perp}$:



- Linearly polarized TMDs at small x can be measured from $\cos(2\,\varphi)$ modulation of the angles in dijet production in DIS (Dumitru, Lappi, Skokov ‘15).

Linearly Polarized Gluon TMD

- If we keep the indices of the two F^{+i} different, we get access to the linearly polarized (WW) gluon TMD $h_1^\perp$ (Metz, Zhou, '11):

$$\frac{1}{P^+}\int\frac{d\xi^-\,d^2\xi}{(2\pi)^3}\,e^{ixP^+\,\xi^- - i\underline{k}\cdot\underline{\xi}}\,\langle P|\mathrm{tr}\left[F^{+i}(0)\,\mathcal{U}^{[+]}[0,\xi]\,F^{+j}(\xi)\,\mathcal{U}^{[+]\,\dagger}[\xi,0]\right]|P\rangle_{\xi^+=0}$$
$$=\frac{1}{2}\,\delta^{ij}\,x\,f_1^{G\ WW}(x,k_T^2)+\frac{2k^ik^j-k_T^2\,\delta^{ij}}{4\,k_T^2}\,x\,h_{1,\ WW}^\perp(x,k_T^2)$$

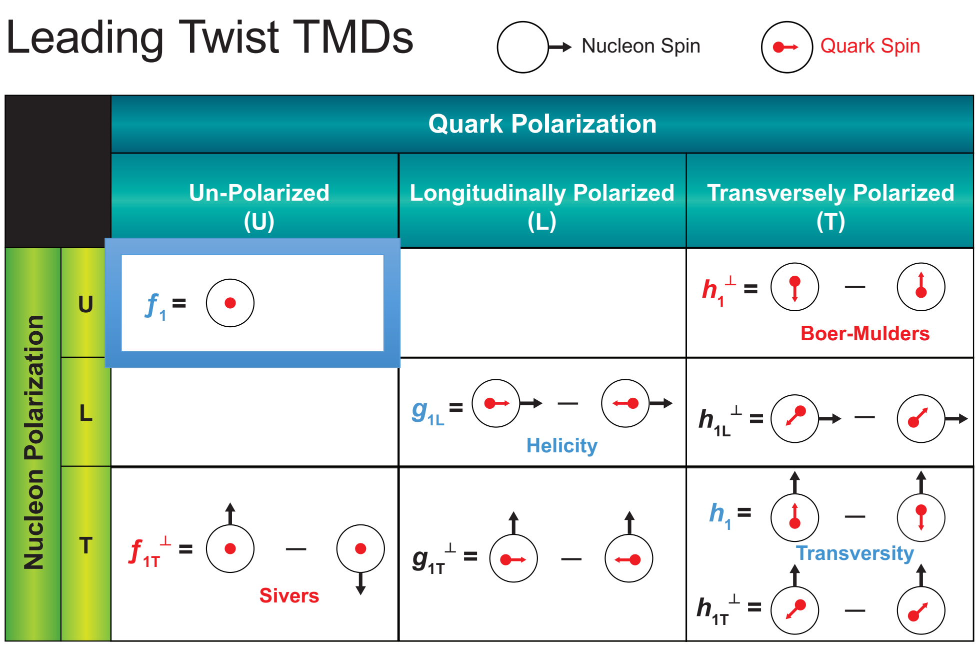
- The linearly polarized WW gluon TMD is thus also related to the color-quadrupole amplitude Q.
- In the linear (BFKL) regime the small-x asymptotics is the same,

$$h_{1,\ WW}^\perp(x,k_T^2)\sim\frac{1}{x}\,Q\sim\left(\frac{1}{x}\right)^{1+\frac{4\,\alpha_s\,N_c}{\pi}\ \ln 2+\mathcal{O}(\alpha_s^2)}$$

- For more on small-x evolution of the linear gluon polarization see recent work by Dumitru, Skokov '17.

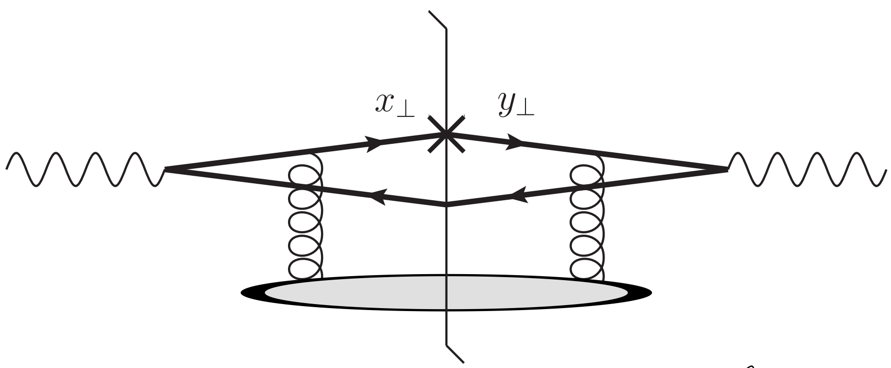
Unpolarized Quark TMD

- Next, let’s talk about the unpolarized quark TMDs:



Quark Production in SIDIS at Small-x

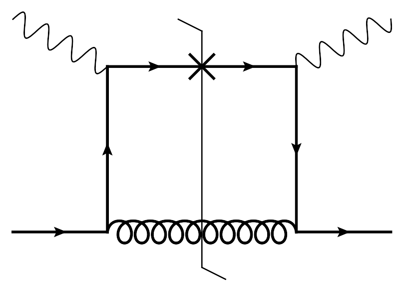
- To find the unpolarized-nucleon quark TMDs at small-x it is convenient to start by considering the quark production cross section for SIDIS on an unpolarized nucleon.
- The dominant process is due to gluon exchanges, even at the lowest order:



- Compared to the standard LO process, the one above comes in with an extra factor of

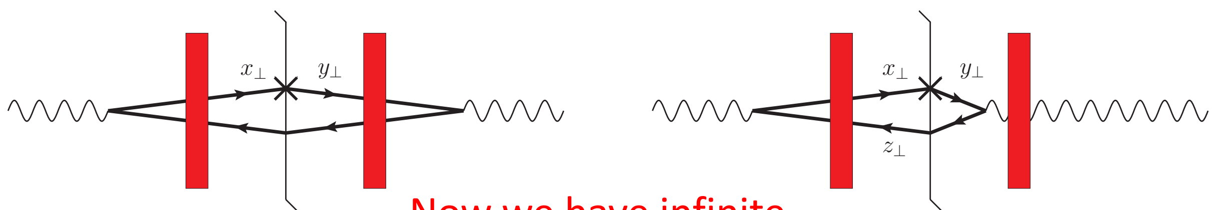
$$\sim \frac{\alpha_s}{x}$$

and is dominant at very low x.

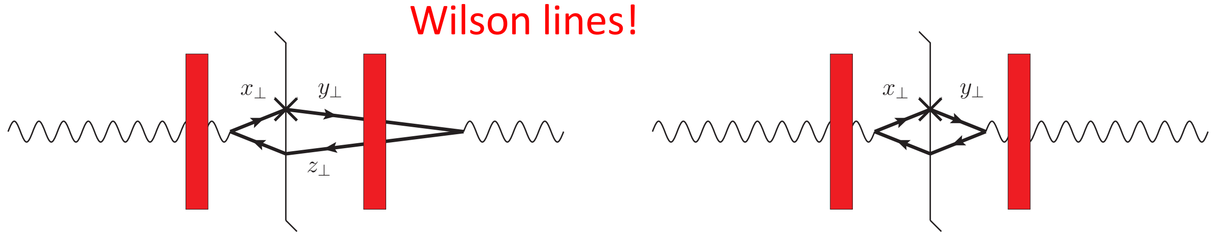


SIDIS to All Orders

- SIDIS process can now be easily generalized to include all-order interactions with the shock waves:



Now we have infinite Wilson lines!



- The SIDIS cross section is

$$\frac{d\sigma_{T,L}^{SIDIS}}{d^2 k_T} = \int\limits_0^1 \frac{dz}{z\,(1-z)} \int \frac{d^2 x_\perp \, d^2 y_\perp \, d^2 z_\perp}{2(2\pi)^3} \, e^{-i\underline{k}\cdot(\underline{x}-\underline{y})} \, \Psi_{T,L}^{\gamma^* \to q\bar{q}}(\underline{x}-\underline{z}, z) \, \left[\Psi_{T,L}^{\gamma^* \to q\bar{q}}(\underline{y}-\underline{z}, z)\right]^* \times \left[S_{x,y}^{[+\infty,-\infty]} - S_{x,z}^{[+\infty,-\infty]} - S_{z,y}^{[+\infty,-\infty]} + 1\right]$$

Quark TMD Evolution at Small-x

- Taking the large-Q^2 limit of the SIDIS cross section we can extract the unpolarized quark TMD out of it (A.H. Mueller '99; Marquet, Xiao and Yuan, '09; YK, Sievert '15):

$$f_1^A(x, k_T) = \frac{2\,N_c}{\pi^3\,x} \int \frac{d^2 x_\perp\, d^2 y_\perp\, d^2 z_\perp}{2(2\pi)^3}\, e^{-i\underline{k}\cdot(\underline{x}-\underline{y})}\, \frac{\underline{x}-\underline{z}}{|\underline{x}-\underline{z}|^2} \cdot \frac{\underline{y}-\underline{z}}{|\underline{y}-\underline{z}|^2}$$

$$\times\, \frac{|\underline{x}-\underline{z}|^4 - |\underline{y}-\underline{z}|^4 - 2\,|\underline{x}-\underline{z}|^2\,|\underline{y}-\underline{z}|^2\,\ln\frac{|\underline{x}-\underline{z}|^2}{|\underline{y}-\underline{z}|^2}}{(|\underline{x}-\underline{z}|^2 - |\underline{y}-\underline{z}|^2)^3}\, \left[S_{x,y}^{[+\infty,-\infty]} - S_{x,z}^{[+\infty,-\infty]} - S_{z,y}^{[+\infty,-\infty]} + 1\right]$$

- Since the Wilson lines are now infinite, we have infinite dipoles, whose evolution is given by the BK equation at large-N_c:

$$\partial_Y S_{\mathbf{x}_0,\mathbf{x}_1}(Y) = \frac{\alpha_s\,N_c}{2\pi^2} \int d^2 x_2 \frac{x_{01}^2}{x_{02}^2\,x_{21}^2}\, \left[S_{\mathbf{x}_0,\mathbf{x}_2}(Y)\,S_{\mathbf{x}_2,\mathbf{x}_1}(Y) - S_{\mathbf{x}_0,\mathbf{x}_1}(Y)\right]$$

Unpolarized Quark TMD

- We conclude that the small-x asymptotics of the unpolarized quark TMD is

$$f_1^q(x, k_T^2) \sim \frac{1}{x} N \sim \left(\frac{1}{x}\right)^{1+\frac{4\,\alpha_s\,N_c}{\pi}\,\ln 2+\mathcal{O}(\alpha_s^2)}$$

- One needs to re-check the above conclusions using the operator definition of the unpolarized quark TMD, but the x-dependence above will remain the same.

Mini-Summary

- So far, all the quark and gluon TMDs for an unpolarized nucleon had the same x-dependence at small x,

$$\mathrm{TMD}^{q,G}_{unpolarized}(x, k_T^2) \sim \left(\frac{1}{x}\right)^{1+\frac{4\,\alpha_s\,N_c}{\pi}\,\ln 2+\mathcal{O}(\alpha_s^2)}$$

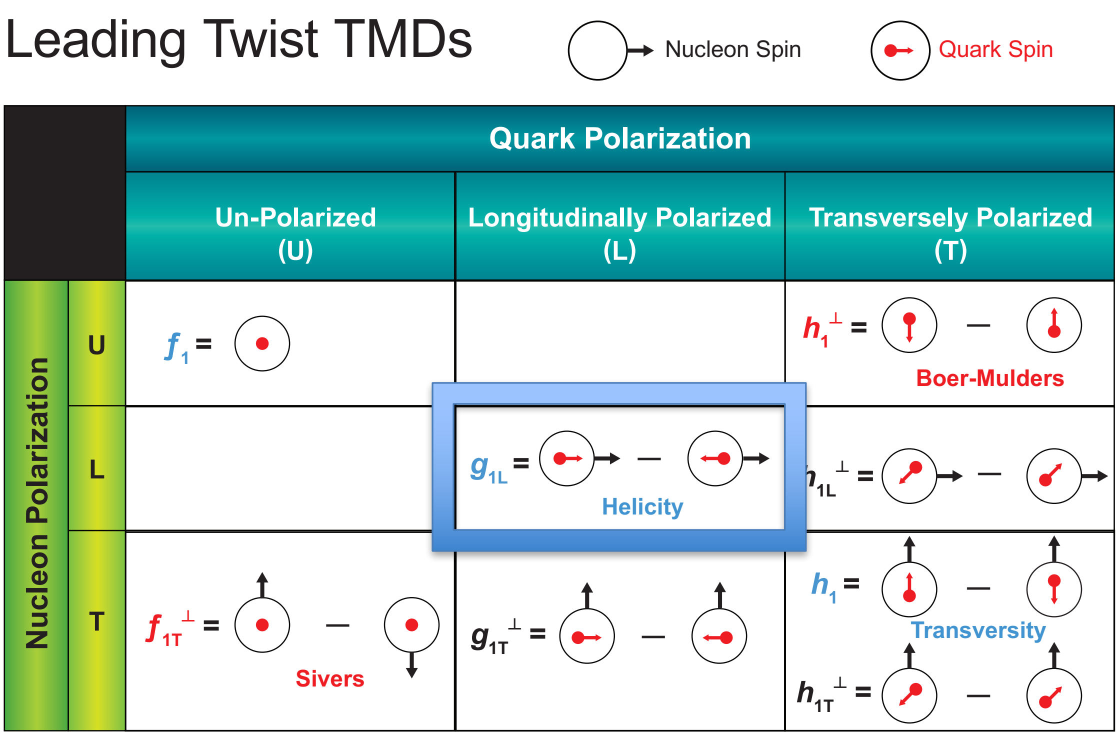
- Not all the unpolarized-nucleon TMDs have their small-x asymptotics derived yet -- it has not been derived for Boer-Mulders distribution to the best of my knowledge.

Longitudinally Polarized Nucleon TMDs

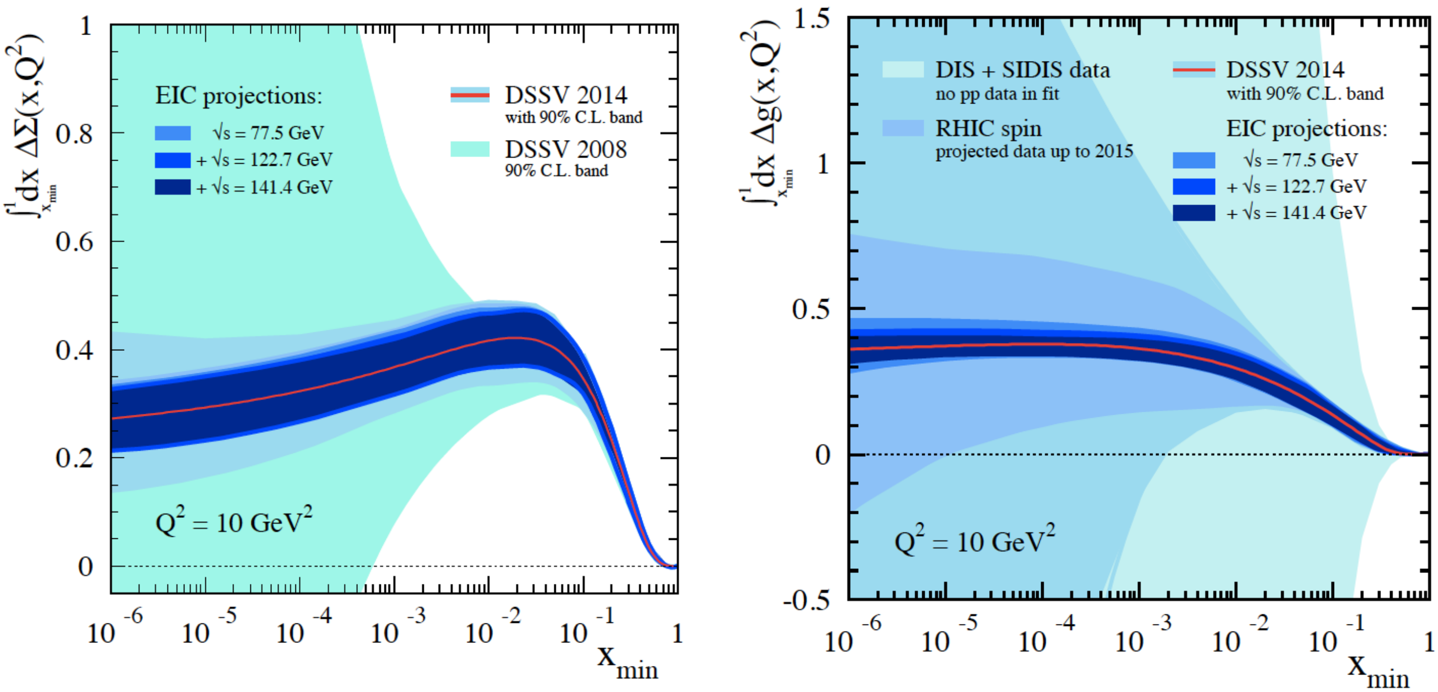
Yu.K., M. Sievert, arXiv:1505.01176 [hep-ph]
Yu.K., D. Pitonyak, M. Sievert, arXiv:1511.06737 [hep-ph], arXiv:1610.06197 [hep-ph], arXiv:1610.06188 [hep-ph], arXiv:1703.05809 [hep-ph], arXiv:1706.04236 [nucl-th]

Quark Helicity TMD

- We now want to calculate quark and gluon helicity TMDs at small x:

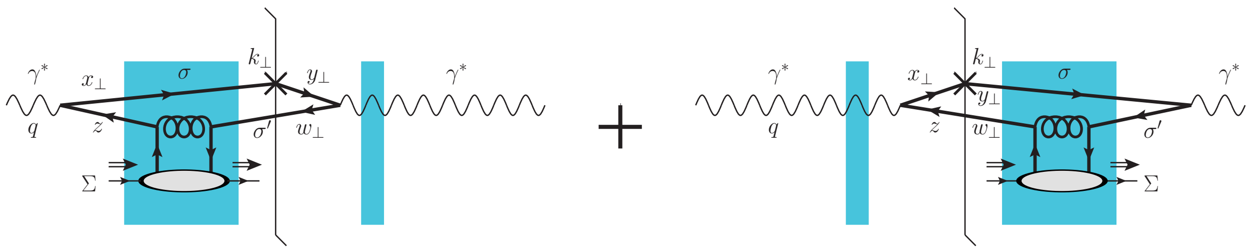


How much spin is at small x?



- E. Aschenaur et al, arXiv:1509.06489 **[hep-ph]**
- Uncertainties are very large at small x!

Quark Helicity TMD from SIDIS Cross Section



- One can show that the quark helicity TMD at small x can be expressed in terms of the polarized dipole amplitude (flavor singlet case):

$$g_1^S(x,Q^2) = \frac{N_c\, N_f}{2\,\pi^2 \alpha_{EM}} \int\limits_{z_i}^{1} \frac{dz}{z^2(1-z)} \int dx_{01}^2 \left[\frac{1}{2} \sum_{\lambda\sigma\sigma'} |\psi^T_{\lambda\sigma\sigma'}|^2_{(x_{01}^2,z)} + \sum_{\sigma\sigma'} |\psi^L_{\sigma\sigma'}|^2_{(x_{01}^2,z)} \right] G(x_{01}^2, z),$$

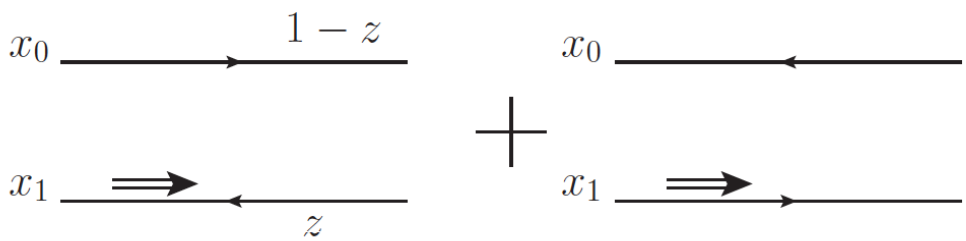
$$\Delta q^S(x,Q^2) = \frac{N_c\, N_f}{2\pi^3} \int\limits_{z_i}^{1} \frac{dz}{z} \int\limits_{\frac{1}{zs}}^{\frac{1}{zQ^2}} \frac{dx_{01}^2}{x_{01}^2}\, G(x_{01}^2, z),$$

$$g_{1L}^S(x,k_T^2) = \frac{8\, N_c\, N_f}{(2\pi)^6} \int\limits_{z_i}^{1} \frac{dz}{z} \int d^2 x_{01}\, d^2 x_{0'1}\, e^{-i\underline{k}\cdot(\underline{x}_{01} - \underline{x}_{0'1})}\, \frac{\underline{x}_{01} \cdot \underline{x}_{0'1}}{x_{01}^2 x_{0'1}^2}\, G(x_{01}^2, z)$$

- Here s is cms energy squared, $z_i = \Lambda^2/s$, $G(x_{01}^2, z) \equiv \int d^2 b\, G_{10}(z)$

Polarized Dipole

- All flavor singlet small-x helicity observables depend on one object, “polarized dipole amplitude”:



$$G_{10}(z) \equiv \frac{1}{2N_c} \left\langle\!\!\left\langle \mathrm{tr}\left[V_{\underline{0}} V_{\underline{1}}^{pol\,\dagger} \right] + \mathrm{tr}\left[V_{\underline{1}}^{pol} V_{\underline{0}}^{\dagger} \right] \right\rangle\!\!\right\rangle (z)$$

unpolarized quark

polarized quark (“polarized Wilson line”): eikonal propagation, non-eikonal spin-dependent interaction

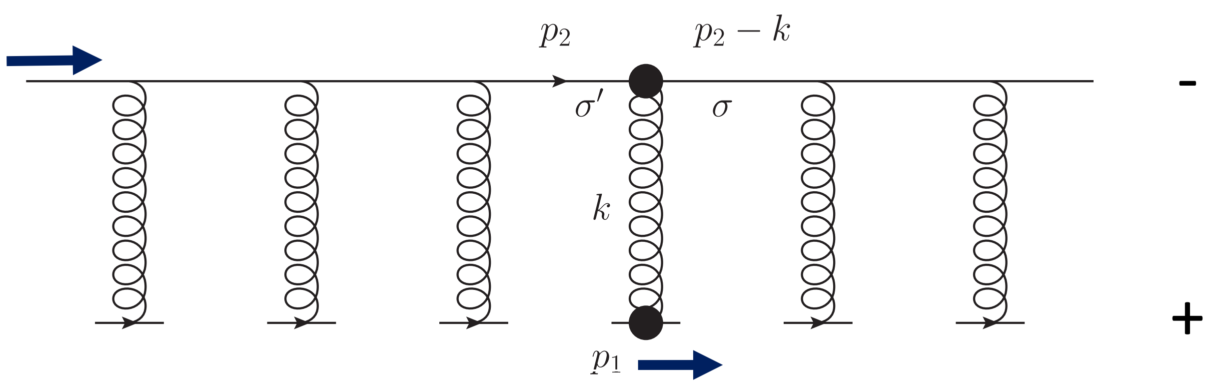
$$V_{\underline{x}} \equiv \mathcal{P} \exp \left[ig \int\limits_{-\infty}^{\infty} dx^+ \, A^-(x^+, 0^-, \underline{x}) \right]$$

- Double brackets denote an object with energy suppression scaled out (single brackets = averaging in the target state):

$$\left\langle\!\!\left\langle \mathcal{O} \right\rangle\!\!\right\rangle (z) \equiv zs \left\langle \mathcal{O} \right\rangle (z)$$

“Polarized Wilson line”

To obtain an explicit expression for the “polarized Wilson line” operator due to a sub-eikonal gluon exchange (as opposed to the sub-eikonal quarks exchange, which needs to be added as well), consider multiple Coulomb gluon exchanges with the target:



Most gluon exchanges are eikonal spin-independent interactions, while one of them is a spin-dependent sub-eikonal exchange. (cf. Mueller ‘90, McLerran, Venugopalan ‘93)

“Polarized Wilson line”

- A simple calculation in $A^-=0$ gauge yields the “polarized Wilson line”:

$$V_{\underline{x}}^{pol} = \frac{1}{2\,s} \int\limits_{-\infty}^{\infty} dx^- \, \mathrm{P} \exp \left\{ ig \int\limits_{x^-}^{\infty} dx'^- \, A^+(x'^-, \underline{x}) \right\} \, ig \, \underline{\nabla} \times \underline{\tilde{A}}(x^-, \underline{x}) \; \mathrm{P} \exp \left\{ ig \int\limits_{-\infty}^{x^-} dx'^- \, A^+(x'^-, \underline{x}) \right\}$$

where $\underline{A}_\Sigma(x^-, \underline{x}) = \frac{\Sigma}{2p_1^+} \, \underline{\tilde{A}}(x^-, \underline{x})$

is the spin-dependent sub-eikonal gluon field of the plus-direction moving target with helicity Σ.

(A^+ is the unpolarized eikonal field.)

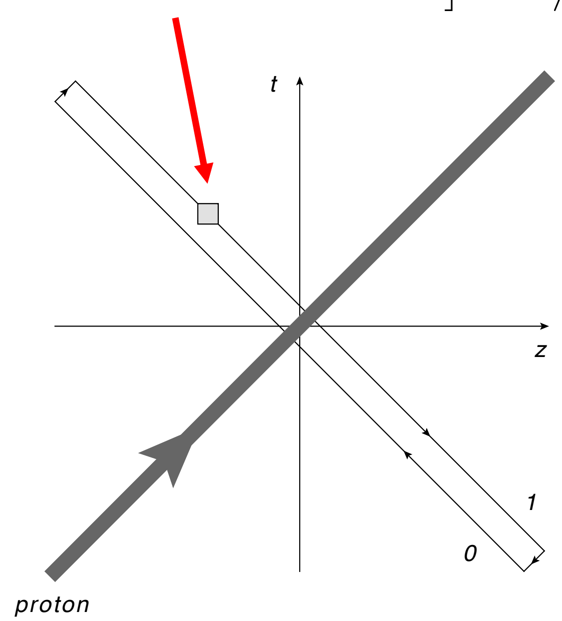
Polarized Dipole Amplitude

- The polarized dipole amplitude is then defined by

$$G_{10}(z) \equiv \frac{1}{4N_c} \int\limits_{-\infty}^{\infty} dx^- \left\langle \mathrm{tr} \left[V_{\underline{0}}[\infty, -\infty] V_{\underline{1}}[-\infty, x^-] \, (-ig) \, \underline{\nabla} \times \underline{\tilde{A}}(x^-, \underline{x}) \, V_{\underline{1}}[x^-, \infty] \right] + \mathrm{c.c.} \right\rangle (z)$$

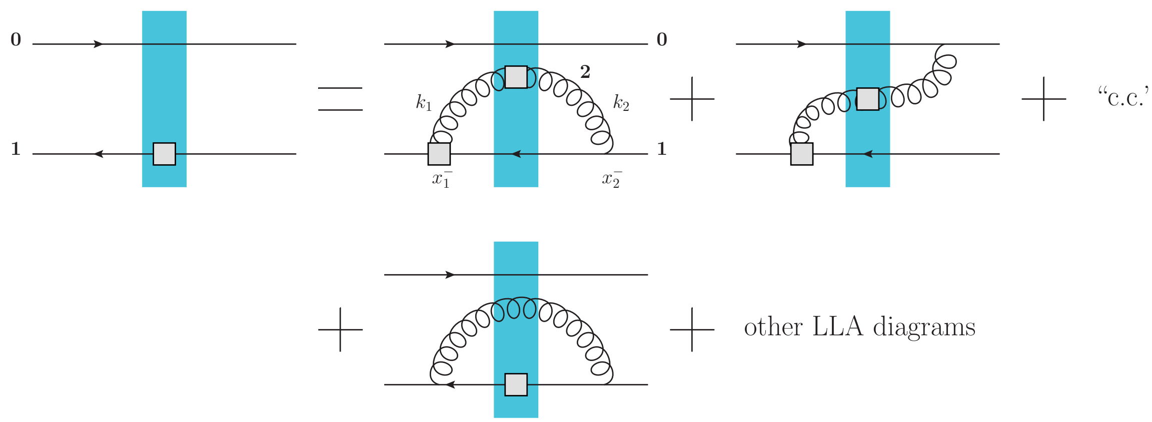
with the standard light-cone Wilson line

$$V_{\underline{x}}[b^-, a^-] = \mathrm{P} \exp \left\{ ig \int\limits_{a^-}^{b^-} dx^- \, A^+(x^-, \underline{x}) \right\}$$



Evolution for Polarized Quark Dipole

- We can evolve the polarized dipole operator and obtain its small-x evolution equation:

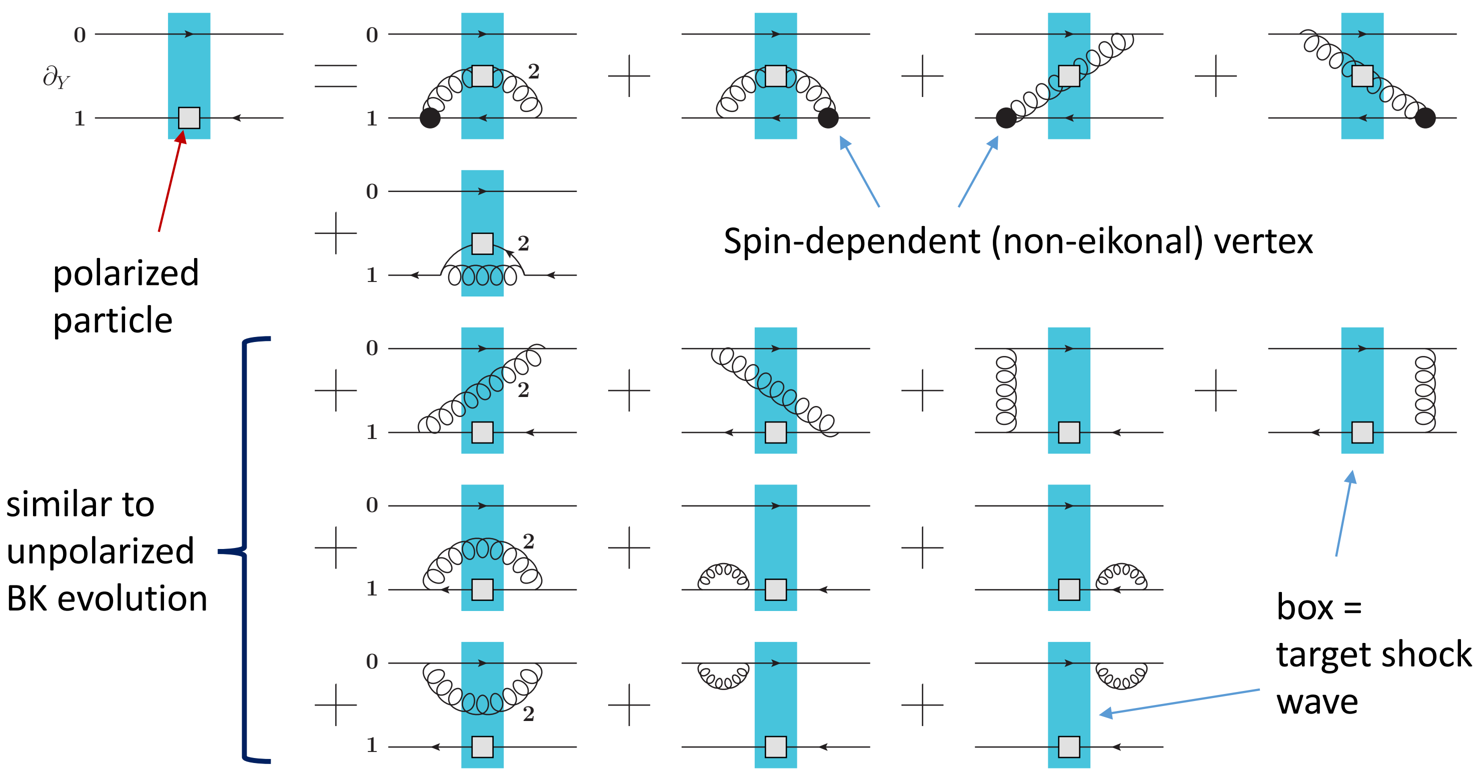


- From the first two graphs on the right we get

$$G_{10}(z) = G_{10}^{(0)}(z) + \frac{\alpha_s}{\pi^2} \int^{z} \frac{dz'}{z'} \int \frac{d^2 x_2}{x_{21}^2} \frac{1}{N_c} \left\langle\!\!\left\langle \mathrm{tr}\left[t^b V_0 t^a V_1^\dagger \right] U_2^{pol\, ba} \right\rangle\!\!\right\rangle + \ldots$$

Evolution for Polarized Quark Dipole

One can construct an evolution equation for the polarized dipole:



Resummation Parameter

- For helicity evolution the resummation parameter is different from BFKL, BK or JIMWLK, which resum powers of leading logarithms (LLA)

$$\alpha_s \ \ln(1/x)$$

- Helicity evolution resummation parameter is double-logarithmic (DLA):

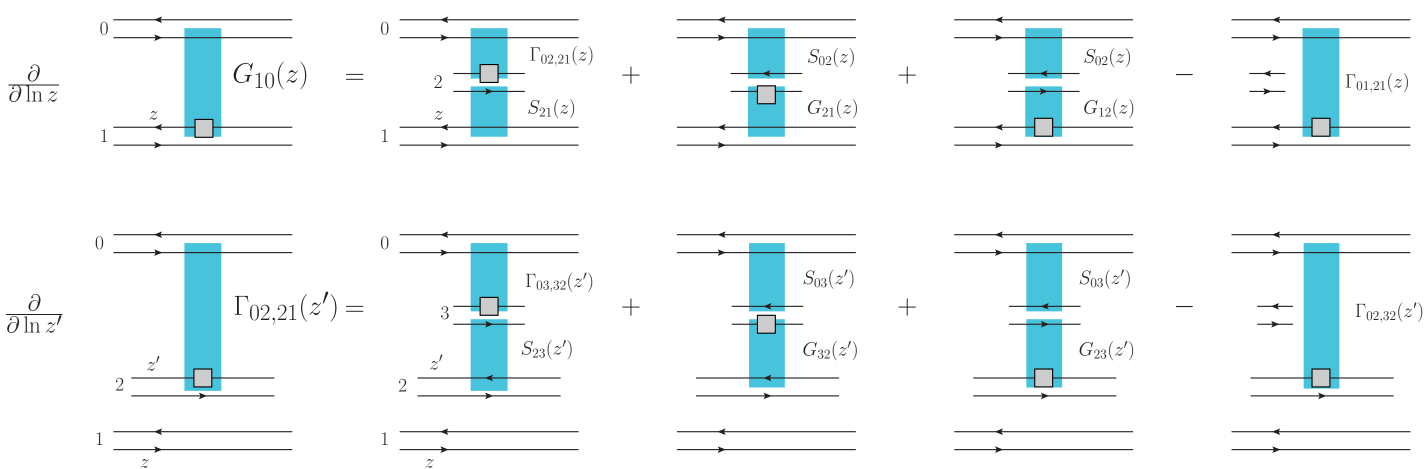
$$\alpha_s \ \ln^2 \frac{1}{x}$$

- The second logarithm of x arises due to transverse momentum (or transverse coordinate) integration being logarithmic both in UV and IR.

- This was known before: Kirschner and Lipatov ’83; Kirschner ’84; Bartels, Ermolaev, Ryskin ‘95, ‘96; Griffiths and Ross ’99; Itakura et al ’03; Bartels and Lublinsky ‘03.

Polarized Dipole Evolution in the Large-N_c Limit

In the large-N_c limit the equations close, leading to a system of 2 equations:

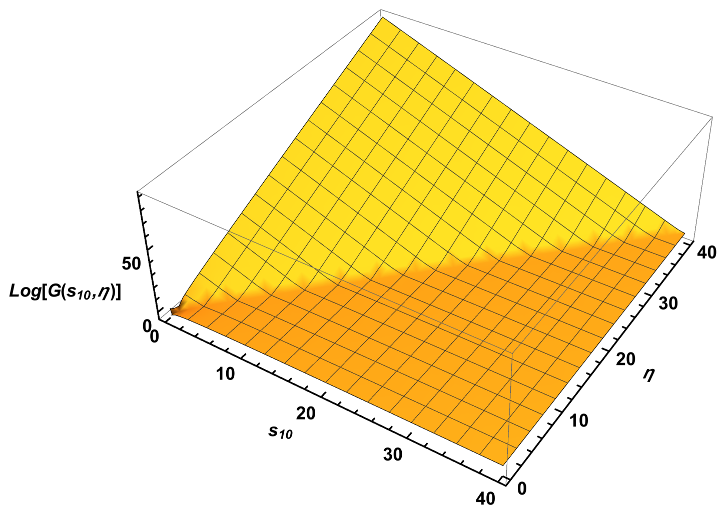


$$G_{10}(z) = G_{10}^{(0)}(z) + \frac{\alpha_s N_c}{2\pi} \int\limits_{z_i}^{z} \frac{dz'}{z'} \int\limits_{\rho'^2}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[2\,\Gamma_{02,\,21}(z')\, S_{21}(z') + 2\, G_{21}(z')\, S_{02}(z') + G_{12}(z')\, S_{02}(z') - \Gamma_{01,\,21}(z') \right]$$

$$\Gamma_{02,\,21}(z') = \Gamma_{02,\,21}^{(0)}(z') + \frac{\alpha_s N_c}{2\pi} \int\limits_{z_i}^{z'} \frac{dz''}{z''} \int\limits_{\rho''^2}^{\min\{x_{02}^2, x_{21}^2\, z'/z''\}} \frac{dx_{32}^2}{x_{32}^2} \left[2\,\Gamma_{03,\,32}(z'')\, S_{23}(z'') + 2\, G_{32}(z'')\, S_{03}(z'') + G_{23}(z'')\, S_{03}(z'') - \Gamma_{02,\,32}(z'') \right]$$

S = found from BK/JIMWLK, it is LLA

Solution of the large-N_C Equations



$$\alpha_h \approx 2.31 \sqrt{\frac{\alpha_s N_c}{2\pi}}$$

- Numerical solution results in the following small-x asymptotics:

$$g_1^S(x, Q^2) \sim \Delta q^S(x, Q^2) \sim g_{1L}^S(x, k_T^2) \sim \left(\frac{1}{x}\right)^{\alpha_h} \approx \left(\frac{1}{x}\right)^{2.31\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

Quark Helicity TMD: Small-x Asymptotics

- The above equations can be solved analytically too, giving the helicity intercept

$$\alpha_h^q = \frac{4}{\sqrt{3}} \sqrt{\frac{\alpha_s \, N_c}{2\pi}} \approx 2.3094 \sqrt{\frac{\alpha_s \, N_c}{2\pi}}$$

- This is in complete agreement with the numerical solution!

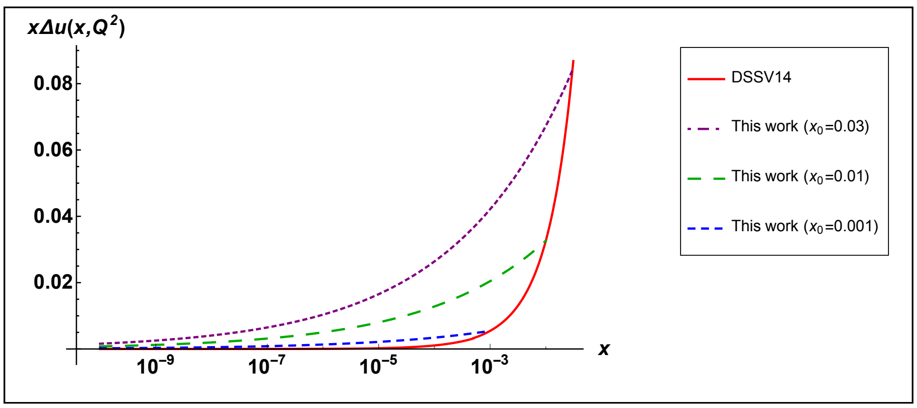
$$\alpha_h^q \approx 2.31 \sqrt{\frac{\alpha_s \, N_c}{2\pi}}$$

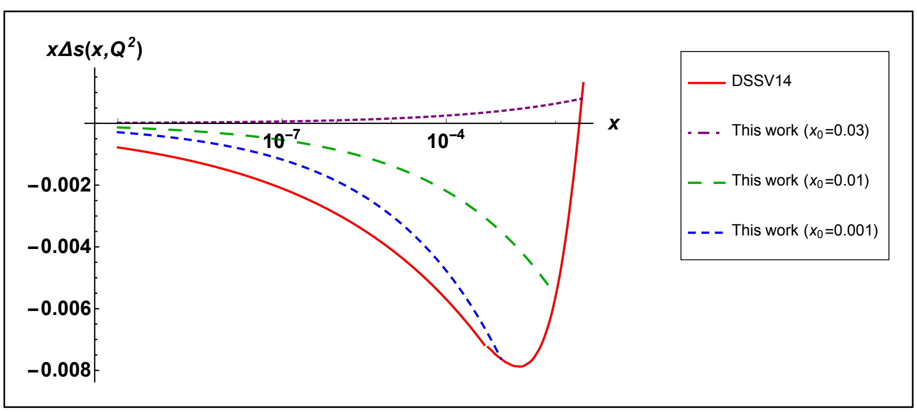
- The small-x asymptotics of quark helicity is (at large N_c)

$$g_{1L}^{S,\, quark} \sim \left(\frac{1}{x} \right)^{\frac{4}{\sqrt{3}} \sqrt{\frac{\alpha_s \, N_c}{2\pi}} + \mathcal{O}(\alpha_s)}$$

Impact of our ΔΣ on the proton spin

- We have attached a $\Delta\tilde{\Sigma}(x,Q^2) = N\,x^{-\alpha_h}$ curve to the existing hPDF's fits at some ad hoc small value of x labeled x_0 :

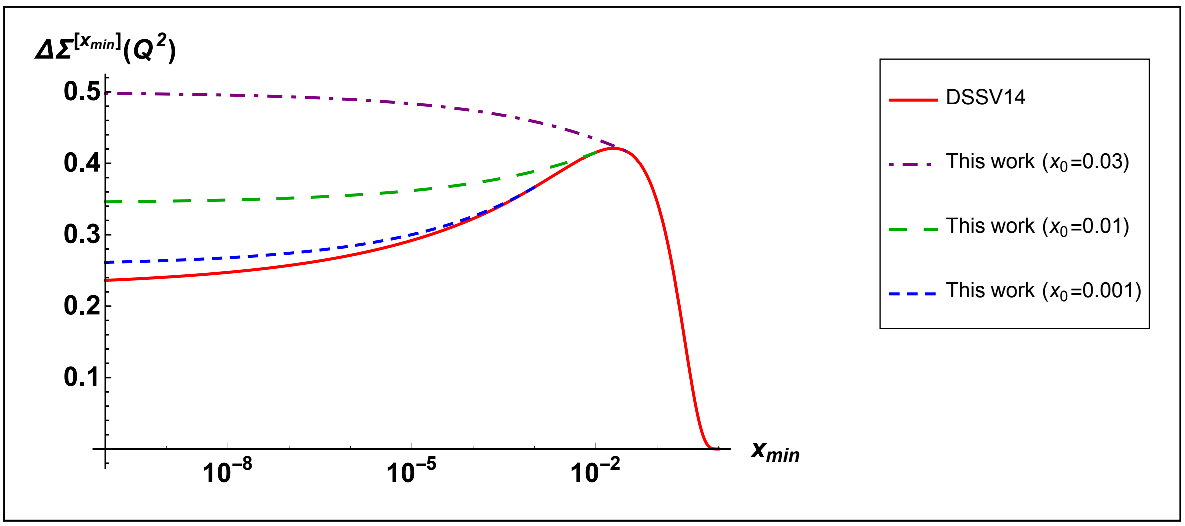




"ballpark" phenomenology

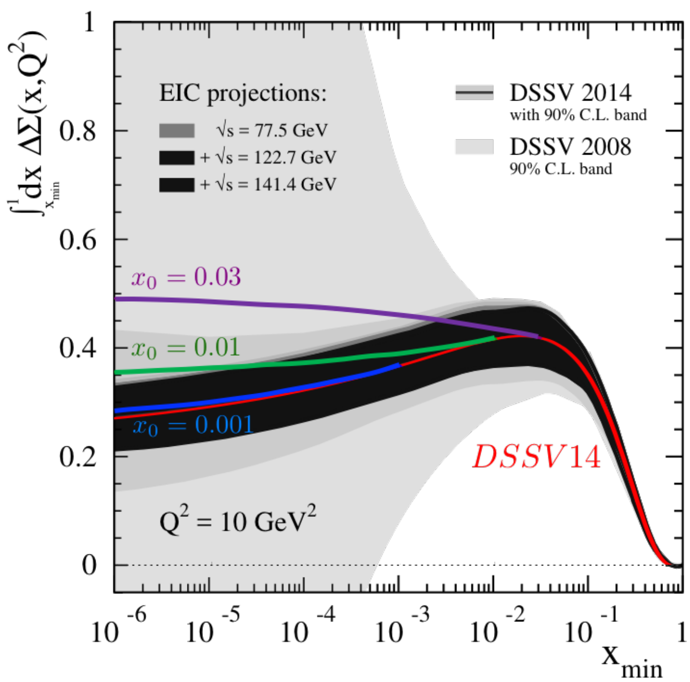
Impact of our ΔΣ on the proton spin

- Defining $\Delta\Sigma^{[x_{min}]}(Q^2) \equiv \int_{x_{min}}^{1} dx\, \Delta\Sigma(x, Q^2)$ we plot it for x_0=0.03, 0.01, 0.001:



- We observe a moderate to significant enhancement of quark spin.
- More detailed phenomenology is needed in the future.

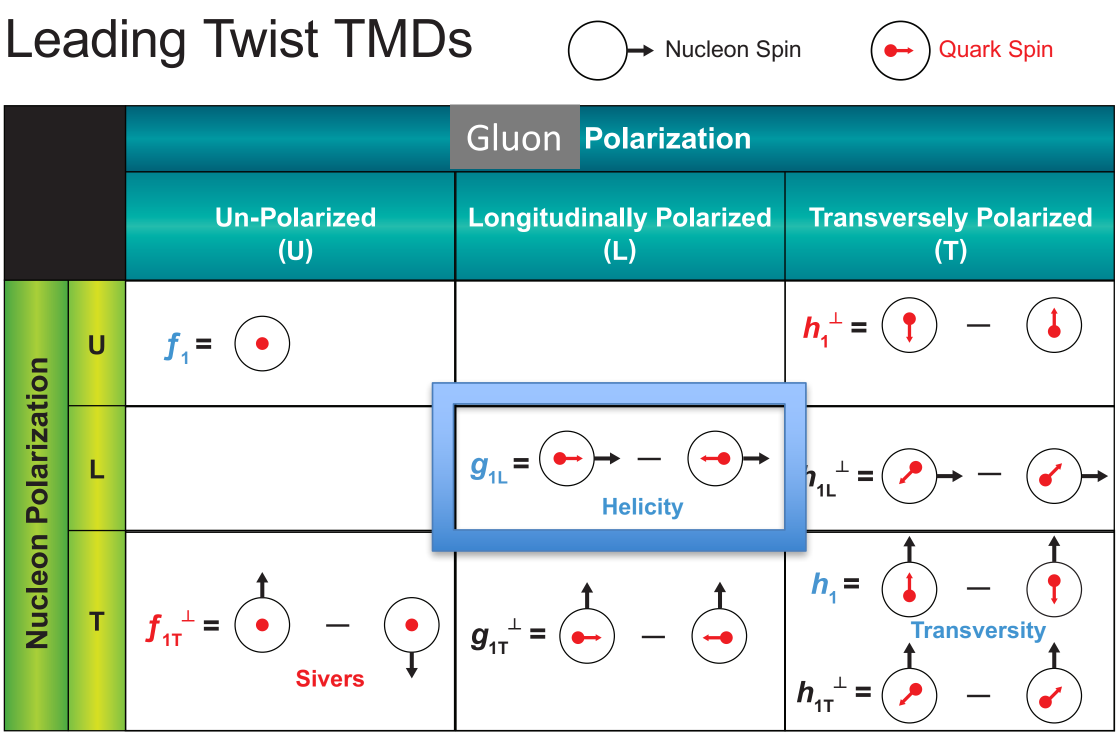
Impact on proton spin



- Here we compare our results with DSSV, now including the error band.
- We observe consistency of our lower two curves with DSSV.
- Our upper curve disagrees with DSSV, but agrees with NNPDF (Nocera, Santopinto, '16).
- Better phenomenology is needed. EIC would definitely play a role.

Gluon Helicity TMD

- We now want to calculate quark and gluon helicity TMDs at small x:



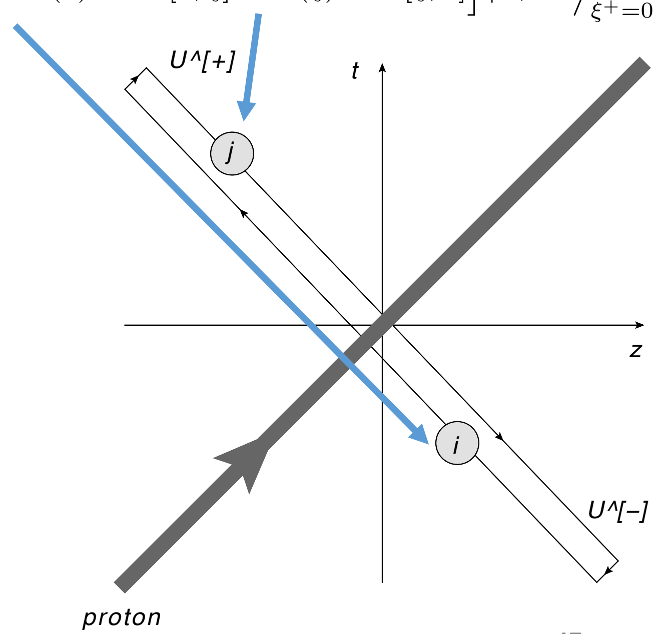
Dipole Gluon Helicity TMD

- Now let us repeat the calculation for gluon helicity TMDs.

- We start with the definition of the gluon dipole helicity TMD:

$$g_1^G(x, k_T^2) = \frac{-2i\, S_L}{x\, P^+} \int \frac{d\xi^- \, d^2\xi}{(2\pi)^3} \, e^{ixP^+ \xi^- - i\underline{k}\cdot\underline{\xi}} \left\langle P, S_L | \epsilon_T^{ij} \, \mathrm{tr} \left[F^{+i}(0) \, \mathcal{U}^{[+]\dagger}[0, \xi] \, F^{+j}(\xi) \, \mathcal{U}^{[-]}[\xi, 0] \right] |P, S_L \right\rangle_{\xi^+ = 0}$$

- Here $U^{[+]}$ and $U^{[-]}$ are future and past-poiting Wilson line staples



Dipole Gluon Helicity TMD

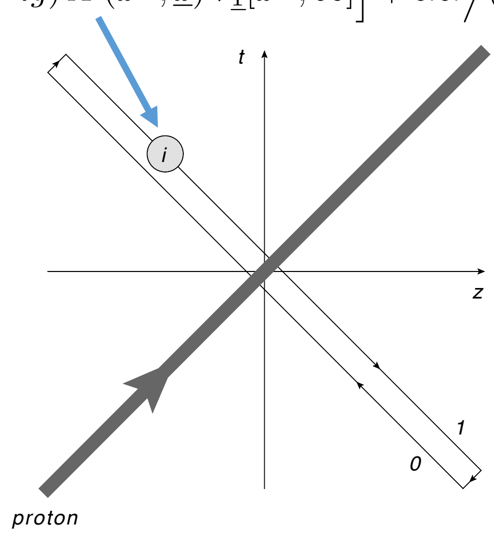
- At small x, the definition of dipole gluon helicity TMD can be massaged into

$$g_1^{G\,dip}(x, k_T^2) = \frac{8i\, N_c\, S_L}{g^2 (2\pi)^3} \int d^2 x_{10}\, e^{i \underline{k} \cdot \underline{x}_{10}}\, k_\perp^i \epsilon_T^{ij} \left[\int d^2 b_{10}\, G_{10}^j (zs = \frac{Q^2}{x}) \right]$$

- Here we obtain a new operator, which is a transverse vector (written here in A^-=0 gauge):

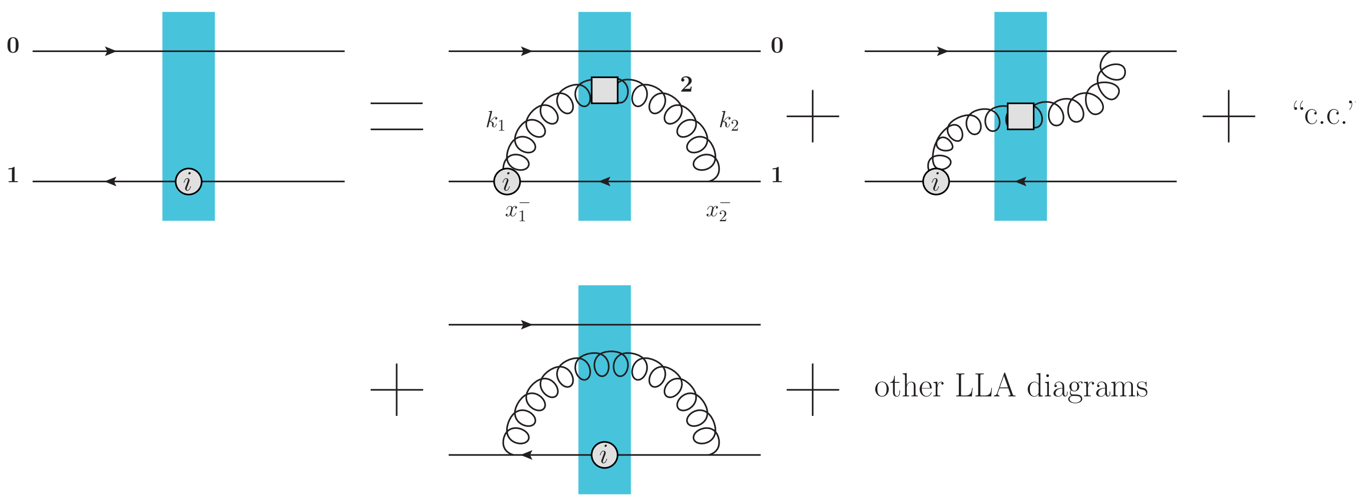
$$G_{10}^i(z) \equiv \frac{1}{4N_c} \int\limits_{-\infty}^{\infty} dx^- \left\langle \mathrm{tr} \left[V_{\underline{0}}[\infty, -\infty] V_{\underline{1}}[-\infty, x^-] \, (-ig) \, \tilde{A}^i(x^-, \underline{x}) \, V_{\underline{1}}[x^-, \infty] \right] + \mathrm{c.c.} \right\rangle (z)$$

- Note that $k_\perp^i \, \epsilon_T^{ij}$ can be thought of as a transverse curl acting on $G_{10}^i(z)$ and not just on $\tilde{A}^i(x^-, \underline{x})$ -- different from the polarized dipole amplitude!



Evolution Equation

- To construct evolution equation for the operator G^i governing the gluon helicity TMD we resum similar (to the quark case) diagrams:



Large-N_c Evolution: Equations

- This results in the following evolution equations:

$$G^i_{10}(zs) = G^{i\,(0)}_{10}(zs) + \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2 x_2 \, \ln \frac{1}{x_{21}\Lambda} \, \frac{\epsilon_T^{ij}\,(x_{21})^j_\perp}{x_{21}^2} \left[\Gamma^{gen}_{20\,,\,21}(z's) + G_{21}(z's)\right]$$

$$- \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2 x_2 \, \ln \frac{1}{x_{21}\Lambda} \, \frac{\epsilon_T^{ij}\,(x_{20})^j_\perp}{x_{20}^2} \left[\Gamma^{gen}_{20\,,\,21}(z's) + \Gamma^{gen}_{21\,,\,20}(z's)\right]$$

$$+ \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z's}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[G^i_{12}(z's) - \Gamma^i_{10\,,\,21}(z's)\right]$$

$$\Gamma^i_{10\,21}(z's) = G^{i\,(0)}_{10}(z's) + \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z'} \frac{dz''}{z''} \int d^2 x_3 \, \ln \frac{1}{x_{31}\Lambda} \, \frac{\epsilon_T^{ij}\,(x_{31})^j_\perp}{x_{31}^2} \left[\Gamma^{gen}_{30\,,\,31}(z''s) + G_{31}(z''s)\right]$$

$$- \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z'} \frac{dz''}{z''} \int d^2 x_3 \, \ln \frac{1}{x_{31}\Lambda} \, \frac{\epsilon_T^{ij}\,(x_{30})^j_\perp}{x_{30}^2} \left[\Gamma^{gen}_{30\,,\,31}(z''s) + \Gamma^{gen}_{31\,,\,30}(z''s)\right]$$

$$+ \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{1}{x_{10}^2 s}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z''s}}^{\min\left[x_{10}^2\,,\,x_{21}^2 \frac{z'}{z''}\right]} \frac{dx_{31}^2}{x_{31}^2} \left[G^i_{13}(z''s) - \Gamma^i_{10\,,\,31}(z''s)\right].$$

Large-N_c Evolution Equations: Solution

- These equations can be solved in the asymptotic high-energy region yielding the small-x gluon helicity intercept

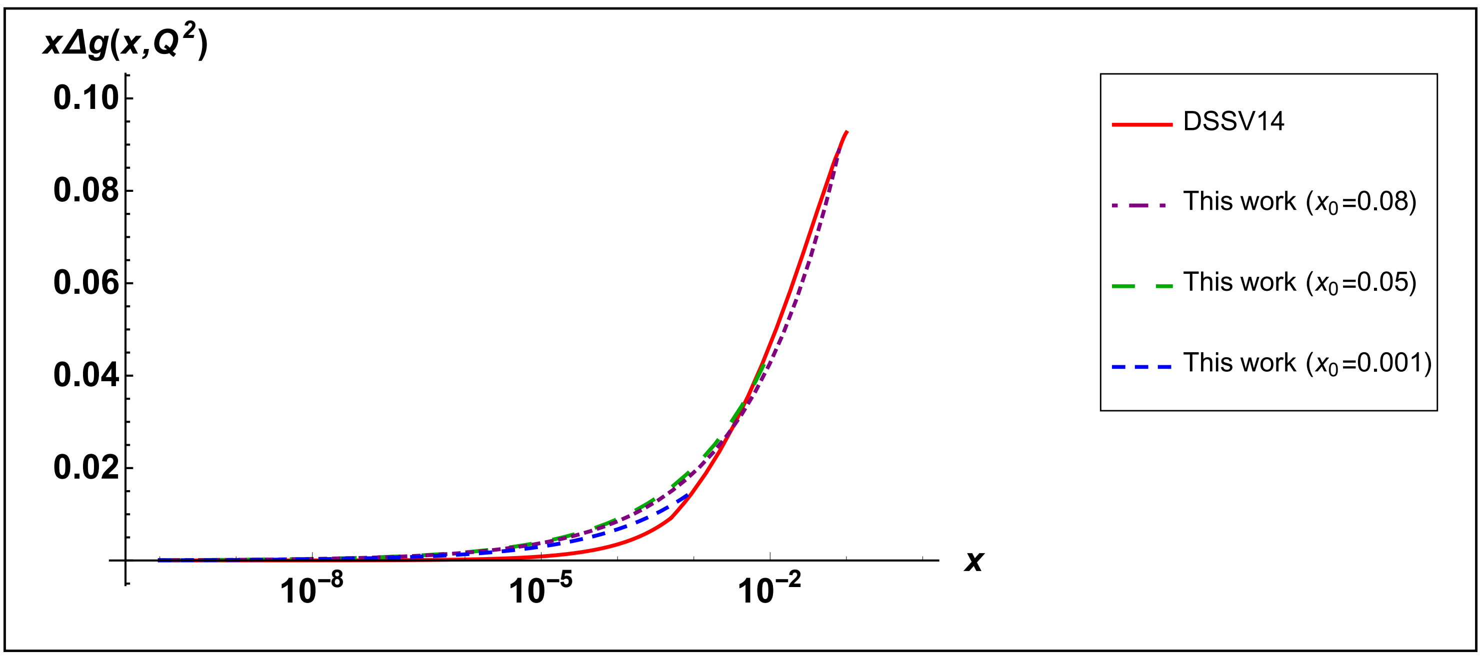
$$\alpha_h^G = \frac{13}{4\sqrt{3}} \sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 1.88 \sqrt{\frac{\alpha_s N_c}{2\pi}}$$

- We obtain the small-x asymptotics of the gluon helicity distributions:

$$\Delta G(x, Q^2) \sim g_{1L}^{G\, dip}(x, k_T^2) \sim \left(\frac{1}{x}\right)^{\frac{13}{4\sqrt{3}} \sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

Impact of our ΔG on the proton spin

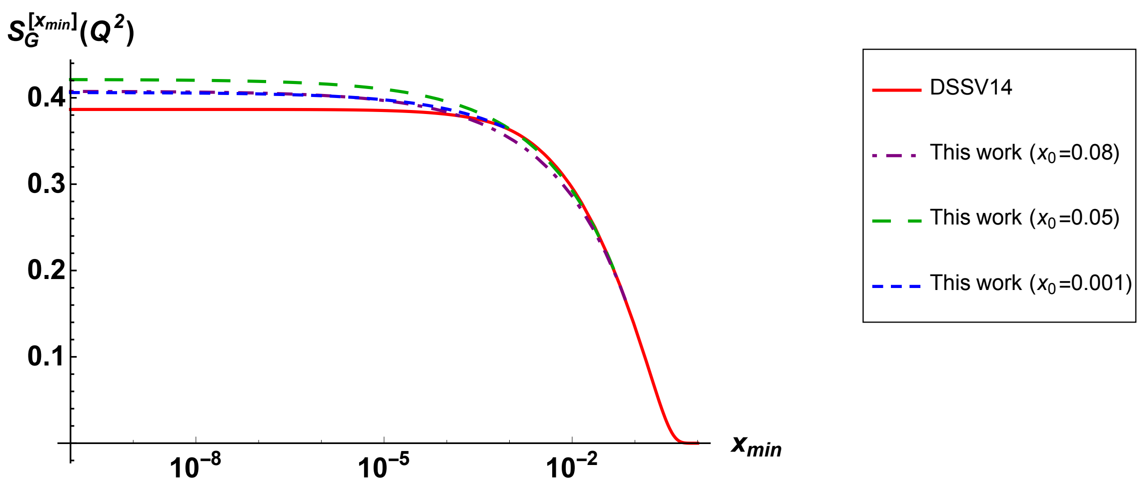
- We have attached a $\Delta\tilde{G}(x,Q^2) = N\,x^{-\alpha_h^G}$ curve to the existing hPDF's fits at some ad hoc small value of x labeled x_0 :



"ballpark"
phenomenology

Impact of our ΔG on the proton spin

- Defining $S_G^{[x_{min}]}(Q^2) \equiv \int_{x_{min}}^{1} dx\, \Delta G(x, Q^2)$ we plot it for x_0=0.08, 0.05, 0.001:

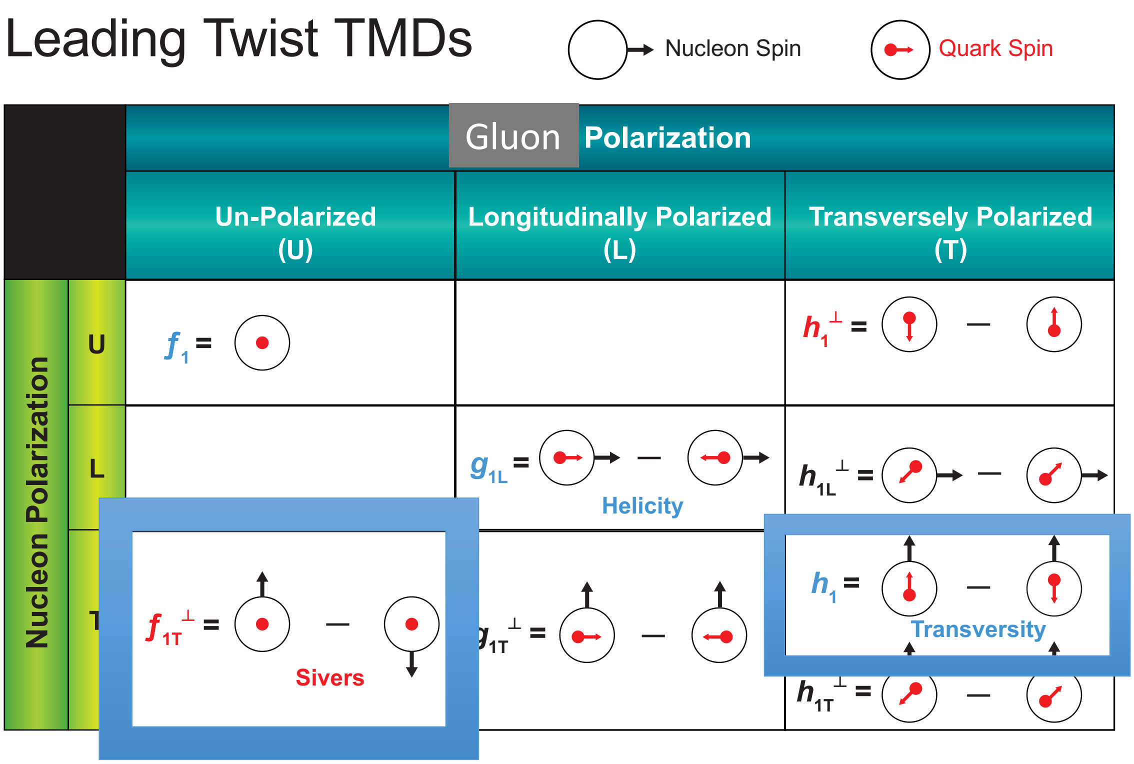


- We observe a moderate enhancement of gluon spin.
- More detailed phenomenology is needed in the future.

Transversely Polarized Nucleon TMDs

Transversely Polarized Nucleon TMDs

- Now let's consider transversity and the Sivers TMD function (for quarks and gluons). I will mainly talk about gluons, since we know more theoretically about their small-x asymptics.



Gluon TMDs of the Transversely Polarized Nucleon

- Boer, Echevarria, Mulders and Zhou '15 argued that one could relate the dipole-type T-odd gluon TMDs for a transversely polarized nucleon, such as the Sivers function and transversity, to the so-called spin-dependent odderon operator (Zhou, '14):

$$f_{1T}^{\perp\, G} = h_{1T}^{G} = h_{1T}^{\perp\, G} = -\frac{k_T^2\, N_c}{4\pi^2\, \alpha_s\, x}\, O_{1T}^{\perp}(x, k_T^2)$$

- The latter two TMDs give us transversity

$$h_1(x, k_T^2) = h_{1T}(x, k_T^2) + \frac{k_T^2}{2M^2}\, h_{1T}^{\perp}(x, k_T^2)$$

The Odderon

- To determine the small-x asymptotics of the Sivers function and transversity TMD we need to find the small-x asymptotics of the odderon.
- There is a rich history behind this question.
- The end result is that we know that (probably an exact result in QFT)

$$O \sim \left(\frac{1}{x}\right)^0 = \mathrm{const}(x)$$

- This is shown to be true in
  - LO QCD: Bartels, Lipatov, Vacca (‘00); YK, Szymanowski, Wallon ‘03 – dipole approach
  - NLO QCD: YK, Diffraction conference proceedings, ‘12
  - Large-coupling N=4 SYM: Brower, Djuric, Tan ‘08
  - Exact in large-N_c QCD: Caron-Huot and Herranen, ‘16.

Gluon and Quark TMDs of the Transversely Polarized Nucleon

- Combining the Boer, Echevarria, Mulders and Zhou '15 result with the odderon's small-x asymptotics, we conclude that

$$f_{1T}^{\perp\, G} = h_{1T}^{G} = h_{1T}^{\perp\, G} \sim \frac{1}{x}$$

- Unlike other asymptotic expressions I showed, this one could be exact! (It is exact at large N_c.)

- Quark transversity may have a similar asymptotics, but an unresolved controversy exists, whether the small-x asymptotics is given by the odderon with $h_1^q \sim \left(\frac{1}{x}\right)^0$

  or by the DLA small-x evolution (similar to helicity) with $h_1^q \sim \left(\frac{1}{x}\right)^{-1+\sqrt{\frac{4\,\alpha_s\,C_F}{\pi}}}$

  (Kirschner, Mankiewicz, Schafer, Szymanowski '96).

Outlook

Unifying Small-x, DGLAP and CSS Evolution

Evolution equation of gluon TMDs

$$
\begin{aligned}
&\frac{d}{d\ln\sigma}\langle p|\tilde{\mathcal{F}}_i^a(x_\perp,\beta_B)\mathcal{F}_j^a(y_\perp,\beta_B)|p\rangle \\
&= \; -\alpha_s\langle p|\mathrm{Tr}\Big\{\int \bar{d}^2k_\perp\theta\big(1-\beta_B-\frac{k_\perp^2}{\sigma s}\big)\Big[(x_\perp|\Big(\tilde{U}\frac{1}{\sigma\beta_B s+p_\perp^2}(\tilde{U}^\dagger k_k+p_k\tilde{U}^\dagger) \\
&\times \frac{\sigma\beta_B s g_{\mu i}-2k_\mu^\perp k_i}{\sigma\beta_B s+k_\perp^2} - \; 2k_\mu^\perp g_{ik}\tilde{U}\frac{1}{\sigma\beta_B s+p_\perp^2}\tilde{U}^\dagger - 2g_{\mu k}\tilde{U}\frac{p_i}{\sigma\beta_B s+p_\perp^2}\tilde{U}^\dagger\Big)\tilde{\mathcal{F}}^k\big(\beta_B+\frac{k_\perp^2}{\sigma s}\big)|k_\perp) \\
&\times (k_\perp|\mathcal{F}^l\big(\beta_B+\frac{k_\perp^2}{\sigma s}\big)\Big(\frac{\sigma\beta_B s\delta_j^\mu-2k_\perp^\mu k_j}{\sigma\beta_B s+k_\perp^2}(k_l U+Up_l)\frac{1}{\sigma\beta_B s+p_\perp^2}U^\dagger \\
&-2k_\perp^\mu g_{jl}U\frac{1}{\sigma\beta_B s+p_\perp^2}U^\dagger - \; 2\delta_l^\mu U\frac{p_j}{\sigma\beta_B s+p_\perp^2}U^\dagger\Big)|y_\perp) \\
&+ 2(x_\perp|\tilde{\mathcal{F}}_i\big(\beta_B+\frac{k_\perp^2}{\sigma s}\big)|k_\perp)(k_\perp|\mathcal{F}^l\big(\beta_B+\frac{k_\perp^2}{\sigma s}\big)\Big(\frac{k_j}{k_\perp^2}\frac{\sigma\beta_B s+2k_\perp^2}{\sigma\beta_B s+k_\perp^2}(k_l U+Up_l)\frac{1}{\sigma\beta_B s+p_\perp^2}U^\dagger \\
&+ 2U\frac{g_{jl}}{\sigma\beta_B s+p_\perp^2}U^\dagger - 2\frac{k_l}{k_\perp^2}U\frac{p_j}{\sigma\beta_B s+p_\perp^2}U^\dagger\Big)|y_\perp) \\
&+ 2(x_\perp|\Big(\tilde{U}\frac{1}{\sigma\beta_B s+p_\perp^2}(\tilde{U}^\dagger k_k+p_k\tilde{U}^\dagger)\frac{k_i}{k_\perp^2}\frac{\sigma\beta_B s+2k_\perp^2}{\sigma\beta_B s+k_\perp^2} + 2\tilde{U}\frac{g_{ik}}{\sigma\beta_B s+p_\perp^2}\tilde{U}^\dagger \\
&- 2\tilde{U}\frac{p_i}{\sigma\beta_B s+p_\perp^2}\tilde{U}^\dagger\frac{k_k}{k_\perp^2}\Big)\tilde{\mathcal{F}}^k\big(\beta_B+\frac{k_\perp^2}{\sigma s}\big)|k_\perp)(k_\perp|\mathcal{F}_j\big(\beta_B+\frac{k_\perp^2}{\sigma s}\big)|y_\perp)\Big] \\
&+ 2\tilde{\mathcal{F}}_i(x_\perp,\beta_B)(y_\perp| -\frac{p^m}{p_\perp^2}\mathcal{F}_k(\beta_B)(i\overleftarrow{\partial}_l+U_l)(2\delta_m^k\delta_j^l-g_{jm}g^{kl})U\frac{1}{\sigma\beta_B s+p_\perp^2}U^\dagger|y_\perp) \\
&+ 2(x_\perp|\tilde{U}\frac{1}{\sigma\beta_B s+p_\perp^2}\tilde{U}^\dagger(2\delta_i^k\delta_m^l-g_{im}g^{kl})(i\partial_k-\tilde{U}_k)\tilde{\mathcal{F}}_l(\beta_B)\frac{p^m}{p_\perp^2}|x_\perp)\mathcal{F}_j(y_\perp,\beta_B) \\
&- 4\int\frac{\bar{d}^2k_\perp}{k_\perp^2}\Big[\theta\big(1-\beta_B-\frac{k_\perp^2}{\sigma s}\big)\tilde{\mathcal{F}}_i\big(x_\perp,\beta_B+\frac{k_\perp^2}{\sigma s}\big)\mathcal{F}_j\big(y_\perp,\beta_B+\frac{k_\perp^2}{\sigma s}\big)e^{i(k,x-y)_\perp} \\
&- \frac{\sigma\beta_B s}{\sigma\beta_B s+k_\perp^2}\tilde{\mathcal{F}}_i(x_\perp,\beta_B)\mathcal{F}_j(y_\perp,\beta_B)\Big]\Big\}|p\rangle \; + \; O(\alpha_s^2)
\end{aligned}
$$

- Nonlinear evolution
- Valid for the whole range of β_B and $p_\perp^2$
- No infrared divergency
- Real emission part contains kinematical constraint $p_\perp^2 < \sigma(1-\beta_B)s$

Balitsky, Tarasov ‘15

Unifying Small-x, DGLAP and CSS Evolution: very promising, but complicated

Three limits of evolution equation

$$
\begin{aligned}
&\frac{d}{d\ln\sigma}\langle p|\tilde{\mathcal{F}}^a_i(x_\perp,\beta_B)\mathcal{F}^a_j(y_\perp,\beta_B)|p\rangle \\
&= \; -\alpha_s\langle p|\mathrm{Tr}\Big\{\int d^2k_\perp\theta\Big(1-\beta_B-\frac{k_\perp^2}{\sigma s}\Big)\Big[(x_\perp|\Big(\tilde{U}\frac{1}{\sigma\beta_B s+p_\perp^2}(\tilde{U}^\dagger k_k+p_k\tilde{U}^\dagger) \\
&\times \frac{\sigma\beta_B s g_{\mu i}-2k^\perp_\mu k_i}{\sigma\beta_B s+k_\perp^2} - \; 2k^\perp_\mu g_{ik}\tilde{U}\frac{1}{\sigma\beta_B s+p_\perp^2}\tilde{U}^\dagger - 2g_{\mu k}\tilde{U}\frac{p_i}{\sigma\beta_B s+p_\perp^2}\tilde{U}^\dagger\Big)\tilde{\mathcal{F}}^k\Big(\beta_B+\frac{k_\perp^2}{\sigma s}\Big)|k_\perp) \\
&\times (k_\perp|\mathcal{F}^l\Big(\beta_B+\frac{k_\perp^2}{\sigma s}\Big)\Big(\frac{\sigma\beta_B s\delta^\mu_j-2k^\mu_\perp k_j}{\sigma\beta_B s+k_\perp^2}(k_l U+Up_l)\frac{1}{\sigma\beta_B s+p_\perp^2}U^\dagger \\
&-2k^\mu_\perp g_{jl}U\frac{1}{\sigma\beta_B s+p_\perp^2}U^\dagger - \; 2\delta^\mu_l U\frac{p_j}{\sigma\beta_B s+p_\perp^2}U^\dagger\Big)|y_\perp) \\
&+ 2(x_\perp|\tilde{\mathcal{F}}_i\Big(\beta_B+\frac{k_\perp^2}{\sigma s}\Big)|k_\perp)(k_\perp|\mathcal{F}^l\Big(\beta_B+\frac{k_\perp^2}{\sigma s}\Big)\Big(\frac{k_j}{k_\perp^2}\frac{\sigma\beta_B s+2k_\perp^2}{\sigma\beta_B s+k_\perp^2}(k_l U+Up_l)\frac{1}{\sigma\beta_B s+p_\perp^2}U^\dagger \\
&+ 2U\frac{g_{jl}}{\sigma\beta_B s+p_\perp^2}U^\dagger - 2\frac{k_l}{k_\perp^2}U\frac{p_j}{\sigma\beta_B s+p_\perp^2}U^\dagger\Big)|y_\perp) \\
&+ 2(x_\perp|\Big(\tilde{U}\frac{1}{\sigma\beta_B s+p_\perp^2}(\tilde{U}^\dagger k_k+p_k\tilde{U}^\dagger)\frac{k_i}{k_\perp^2}\frac{\sigma\beta_B s+2k_\perp^2}{\sigma\beta_B s+k_\perp^2} + 2\tilde{U}\frac{g_{ik}}{\sigma\beta_B s+p_\perp^2}\tilde{U}^\dagger \\
&- 2\tilde{U}\frac{p_i}{\sigma\beta_B s+p_\perp^2}\tilde{U}^\dagger\frac{k_k}{k_\perp^2}\Big)\tilde{\mathcal{F}}^k\Big(\beta_B+\frac{k_\perp^2}{\sigma s}\Big)|k_\perp)(k_\perp|\mathcal{F}_j\Big(\beta_B+\frac{k_\perp^2}{\sigma s}\Big)|y_\perp)\Big] \\
&+ 2\tilde{\mathcal{F}}_i(x_\perp,\beta_B)(y_\perp| - \frac{p^m}{p_\perp^2}\mathcal{F}_k(\beta_B)(i\overleftarrow{\partial}_l+U_l)(2\delta^k_m\delta^l_j-g_{jm}g^{kl})U\frac{1}{\sigma\beta_B s+p_\perp^2}U^\dagger|y_\perp) \\
&+ 2(x_\perp|\tilde{U}\frac{1}{\sigma\beta_B s+p_\perp^2}\tilde{U}^\dagger(2\delta^k_i\delta^l_m-g_{im}g^{kl})(i\partial_k-\tilde{U}_k)\tilde{\mathcal{F}}_l(\beta_B)\frac{p^m}{p_\perp^2}|x_\perp)\mathcal{F}_j(y_\perp,\beta_B) \\
&- 4\int\frac{d^2k_\perp}{k_\perp^2}\Big[\theta\Big(1-\beta_B-\frac{k_\perp^2}{\sigma s}\Big)\tilde{\mathcal{F}}_i\Big(x_\perp,\beta_B+\frac{k_\perp^2}{\sigma s}\Big)\mathcal{F}_j\Big(y_\perp,\beta_B+\frac{k_\perp^2}{\sigma s}\Big)e^{i(k,x-y)_\perp} \\
&- \frac{\sigma\beta_B s}{\sigma\beta_B s+k_\perp^2}\tilde{\mathcal{F}}_i(x_\perp,\beta_B)\mathcal{F}_j(y_\perp,\beta_B)\Big]\Big\}|p\rangle \; + \; O(\alpha_s^2)
\end{aligned}
$$

$\beta_B = x_B \sim 1 \qquad k_\perp^2 \sim (x-y)_\perp^{-2} \sim s$

→ DGLAP (moderate x) limit

$\beta_B = x_B \sim 1 \qquad k_\perp^2 \sim (x-y)_\perp^{-2} \sim few\ GeV^2$

→ Sudakov limit (double log)

$\beta_B = x_B \sim \frac{(x-y)_\perp^{-2}}{s} \qquad k_\perp^2 \sim (x-y)_\perp^{-2} \ll s$

→ Small-x limit

Summary

Summary

- We have described what is theoretically known abuot the small-x asymptotics of TMDs:
- Unpolarized-nucleon TMDs all seem to scale as the BFKL solution,

$$\mathrm{TMD}^{q,G}_{unpolarized}(x, k_T^2) \sim \left(\frac{1}{x}\right)^{1+\frac{4\,\alpha_s\,N_c}{\pi}\;\ln 2+\mathcal{O}(\alpha_s^2)}$$

- Longitudinally polarized nucleon: Helicity TMDs (at large N_c) scale as

$$g_{1L}^{S,\,quark} \sim \left(\frac{1}{x}\right)^{\frac{4}{\sqrt{3}}\sqrt{\frac{\alpha_s\,N_c}{2\pi}}+\mathcal{O}(\alpha_s)}$$

$$g_{1L}^{G\,dipole} \sim \left(\frac{1}{x}\right)^{\frac{13}{4\,\sqrt{3}}\sqrt{\frac{\alpha_s\,N_c}{2\pi}}+\mathcal{O}(\alpha_s)}$$

- Transversely polarized nucleon: Transversity and the Sivers function are

$$f_{1T}^{\perp\,G} = h_{1T}^{G} = h_{1T}^{\perp\,G} \sim \frac{1}{x}$$

- While significant progress has been made, many more TMDs are left to explore!

INT Program on EIC Physics, Fall 2018

- **Probing Nucleons and Nuclei in High Energy Collisions** (INT-18-3)
  October 1 - November 16, 2018
  Y. Hatta, Y. Kovchegov, C. Marquet, A. Prokudin
- Institute for Nuclear Theory, Seattle, WA
- Please mark
  your calendars!