# Overview of 3D structure of the nucleon from lattice QCD

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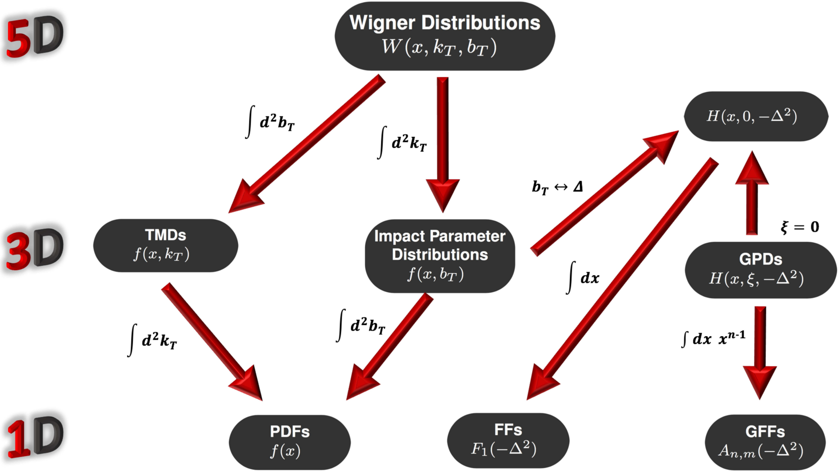
Temple University

Transversity 2017

*INFN - Frascati National Laboratories*

December 13, 2017

# Nucleon Characterization



5D

**Wigner Distributions**
$W(x, k_T, b_T)$

$\int d^2b_T$

$\int d^2k_T$

$b_T \leftrightarrow \Delta$

$H(x, 0, -\Delta^2)$

$\xi = 0$

3D

**TMDs**
$f(x, k_T)$

**Impact Parameter Distributions**
$f(x, b_T)$

$\int dx$

**GPDs**
$H(x, \xi, -\Delta^2)$

$\int d^2k_T$

$\int d^2b_T$

$\int dx\ x^{n\text{-}1}$

1D

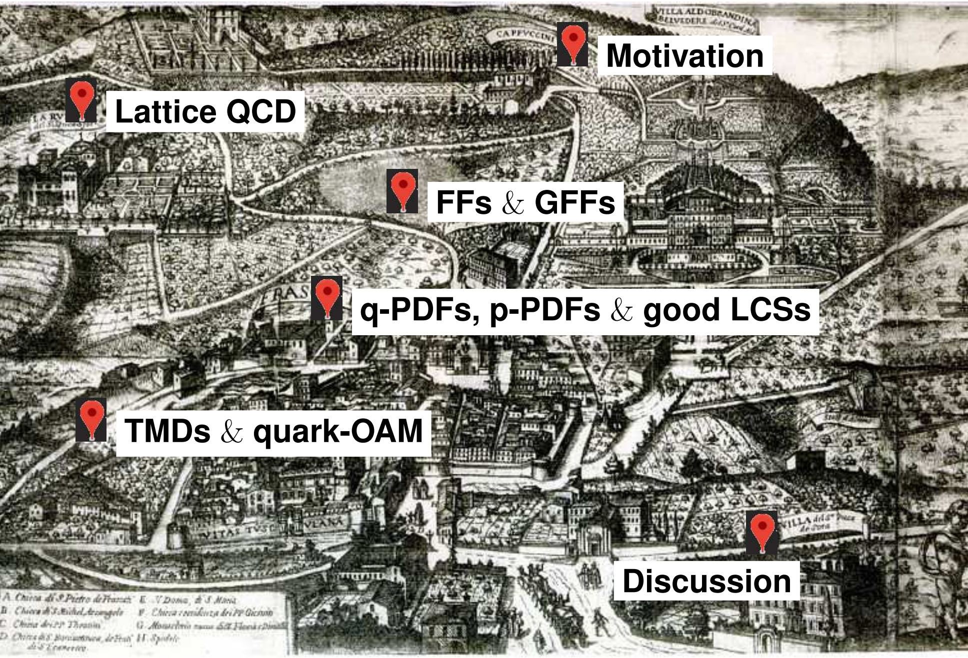
**PDFs**
$f(x)$

**FFs**
$F_1(-\Delta^2)$

**GFFs**
$A_{n,m}(-\Delta^2)$

[X. Ji, D. Mueller, A. Radyushkin]

# ROADMAP OF TALK



Motivation

Lattice QCD

FFs & GFFs

q-PDFs, p-PDFs & good LCSs

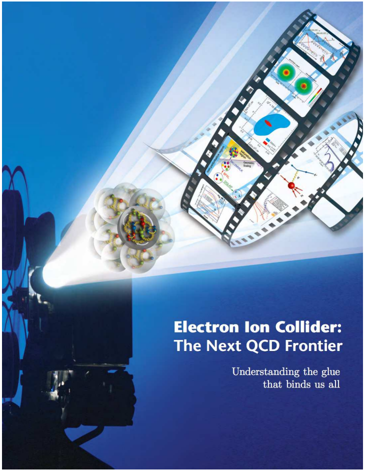
TMDs & quark-OAM

Discussion

# A

# Motivation

# Synergy of the EIC and LQCD



Electron Ion Collider:
The Next QCD Frontier

Understanding the glue
that binds us all

"*measurements at the EIC and lattice calculations will have a high degree of complementarity. For some quantities,... a precise determination will be possible both in experiment and on the lattice. Using this to validate the methods used in lattice calculations, one will gain confidence in computing quantities whose experimental determination is very hard, such as generalized form factors. Furthermore, one can gain insight into the underlying dynamics by computing the same quantities with values of the quark masses that are not realized in nature, so as to reveal the importance of these masses for specific properties of the nucleon.*"

**[A. Accardi et al., arXiv:1212.1701]**

# Where are we today?

★ **Long history of calculating moments of PDFs and GPDs**

- proton spin
- FFs and GFF vs momentum transfer
- proton radius
- Investigation of sea quark and gluon contributions

★ **Exploration of novel approaches to access PDFs and TMDs directly from the lattice**

- x-dependence of unpolarized, polarized and transversity quark distributions
- Sivers function, Boer-Mulders function, generalized tensor charge, Worm Gear function
- quark Orbital Angular Momentum in different decompositions

# DoE funded Topical Collaboration for theory

Slide from J.-W. Qiu and A. Prokudin

TMD Collaboration

18 institutions
Theory, phenomenology, lattice QCD
Several postdoc positions.
2 tenure track positions: Temple, NMSU
Support of undergraduates.

The TMD Collaboration
Spokespersons: William Detmold (MIT) and Jianwei Qiu (BNL)

Co-Investigators - (in alphabetical order of institutions):
Jianwei Qiu and Raju Venugopalan (Brookhaven National Laboratory)
Thomas Mehen (Duke University)
Ted Rogers (Jefferson Laboratory and Old Dominion University)
Alexei Prokudin (Jefferson Laboratory and Penn State University at Berks)
Feng Yuan (Lawrence Berkeley National Laboratory)
Christopher Lee and Ivan Vitev (Los Alamos National Laboratory)
William Detmold, John Negele and Iain Stewart (MIT)
Matthias Burkardt and Michael Engelhardt (New Mexico State University)
Leonard Gamberg (Penn State University at Berks)
Andreas Metz (Temple University)
Sean Fleming (University of Arizona)
Keh-Fei Liu (University of Kentucky)
Xiangdong Ji (University of Maryland)
Simonetta Liuti (University of Virginia)

- ◇ 5 years of funding
- ◇ 18 institutions
- ◇ Theory, phenomenology, lattice QCD
- ◇ Several postdoc and tenure track positions are created
- ◇ "To address the challenges of extracting novel quantitative information about the nucleon's internal landscape"
- ◇ "To provide compelling research, training, and career opportunities for young nuclear theorists"

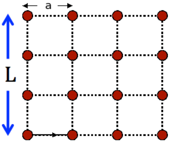
# B

# Lattice QCD

# Lattice formulation of QCD

★ **Space-time discretization on a finite-sized 4-D lattice**

- **Quark fields on lattice points**
- **Gluons on links**



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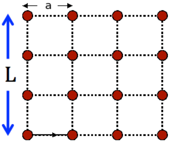


## Technical Aspects

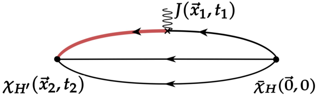
★ **Parameters (define cost of simulations):**

- **quark masses (aim at physical values)**
- **lattice spacing (ideally fine lattices)**
- **lattice size (need large volumes)**

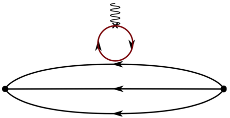
★ **Discretization not unique:**

- **Wilson, Clover, Twisted Mass,**
- **Staggered, Overlap, Domain Wall**

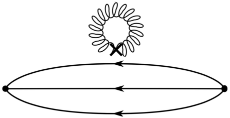
# Nucleon Structure



**Connected**



**Disconnected**
**Quark loop**



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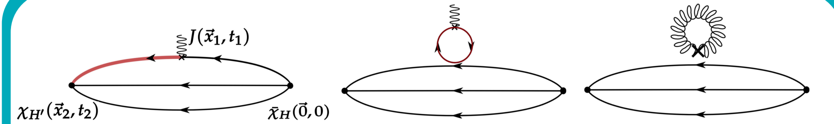

**Connected**

**Disconnected**
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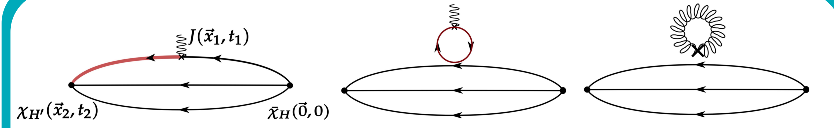
★ **Calculation of 2pt- and 3-pt functions**

$$G_{\mathcal{O}}(\Gamma^{\kappa}, \vec{q}, t) = \sum_{\vec{x}_f, \vec{x}} e^{i\vec{x}\cdot\vec{q}} e^{-i\vec{x}_f\cdot\vec{p}'} \Gamma^{\kappa}_{\beta\alpha} \langle J_\alpha(\vec{x}_f, t_f) \mathcal{O}(\vec{x}, t) \overline{J}_\beta(0) \rangle$$ **(3pt)**

# Nucleon Structure



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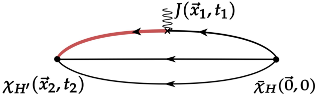
- ★ **Construction of optimized ratios**

$$R^\mu_{\mathcal{O}}(\Gamma,\vec q,t)=\frac{G_{\mathcal{O}}(\Gamma,\vec q,t)}{G(\vec 0,t_f)}\times\sqrt{\frac{G(-\vec q,t_f-t)G(\vec 0,t)G(\vec 0,t_f)}{G(\vec 0,t_f-t)G(-\vec q,t)G(-\vec q,t_f)}}$$ **(fit to a plateau)**

# Nucleon Structure



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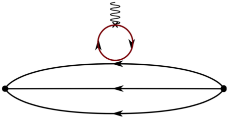

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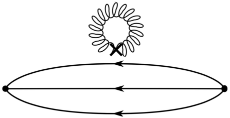

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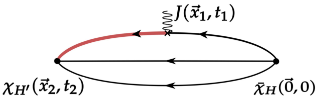
- ★ **Renormalization** $\Pi^R(\Gamma, \vec{q}) = Z_{\mathcal{O}}\,\Pi(\Gamma, \vec{q})$ **(Simpler case!)**

# Nucleon Structure



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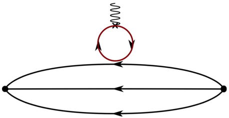

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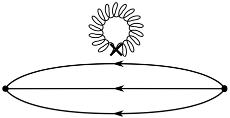

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★ **Renormalization** $\Pi^{R}(\Gamma,\vec{q})=Z_{\mathcal{O}}\,\Pi(\Gamma,\vec{q})$ **(Simpler case!)**

★ **Decomposition into form factors**

$$A^3_\mu\equiv\bar{\psi}\,\gamma_\mu\,\gamma_5\,\frac{\tau^3}{2}\,\psi\Rightarrow\bar{u}_N(p')\left[\mathbf{G_A(q^2)}\,\gamma_\mu\,\gamma_5+\mathbf{G_p(q^2)}\,\frac{q_\mu\,\gamma_5}{2\,m_N}\right]u_N(p)$$

# Systematic uncertainties: Challenges & Progress

1. **Cut-off Effects: finite lattice spacing**

2. **Finite Volume Effects**

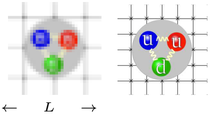
3. **Contamination from other hadron states**

4. **Not simulating the physical world**

5. **Renormalization and mixing**

# Systematic uncertainties: Challenges & Progress

1 **Cut-off Effects: finite lattice spacing**



- **Continuum limit $a \to 0$**
- **Simulations with fine lattices ($a < 0.1$ fm)**
- **Improve actions, algorithmic improvements**

2 **Finite Volume Effects**

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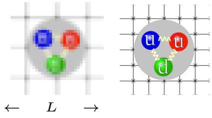

2. Finite Volume Effects
   - **Infinite volume limit $L \to \infty$**
   - **Simulating hadrons in large volumes (Rule of thumb: $L\,m_\pi > 3.5$)**
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# Systematic uncertainties: Challenges & Progress

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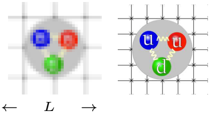

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- **Various methods for extracting information from lattice data**

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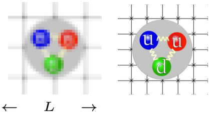

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    - Chiral extrapolation
    - Simulations at physical parameters are now feasible
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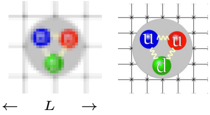

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- Chiral extrapolation
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## 5 Renormalization and mixing

- Subtraction of lattice artifacts, utilize perturbation theory

# C

# FFs & GFFs

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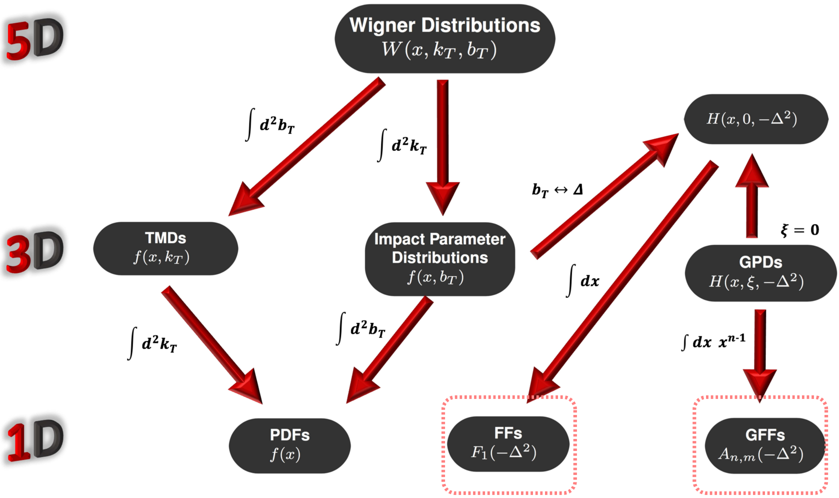

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# Spin Structure from First Principles

**DIS experiment (1988) show 20-30$\%$ of spin carried by valence quarks**

**Spin Sum Rule (Ji):**

$$\boxed{\frac{1}{2} = \sum_q J^q + J^G = \sum_q \left(L^q + \frac{1}{2}\Delta\Sigma^q\right) + J^G}$$

$L_q$: **Quark orbital angular momentum**

$\Delta\Sigma_q$: **intrinsic spin**

$J^G$: **Gluon part**

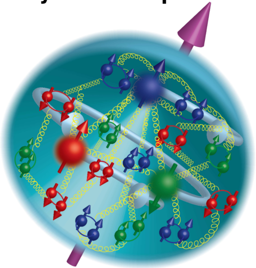

Image by Z.-E. Meziani

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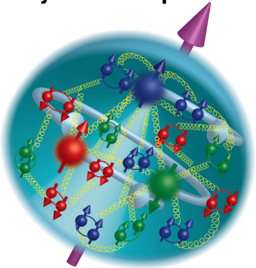


Image by Z.-E. Meziani

**Extraction from LQCD:**

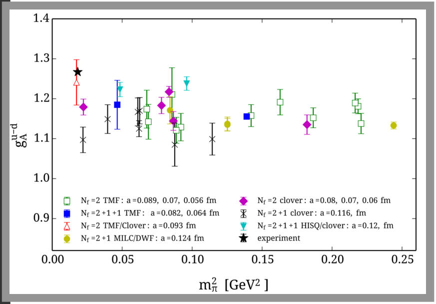
$$J^q = \frac{1}{2}\left(A_{20}^q + B_{20}^q\right)\,, \quad L^q = J^q - \Sigma^q\,, \quad \Sigma^q = g_A^q$$

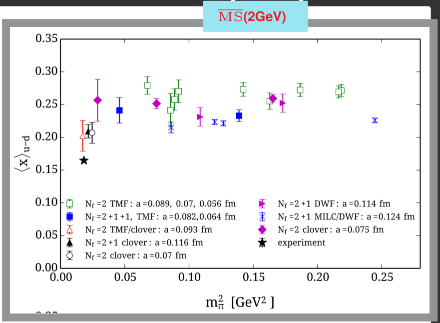
**We need a theoretical formulation to address the proton spin puzzle**

⇩

**Lattice QCD**

# Valence Quark Contributions (u-d)





## Investigation of systematic uncertainties

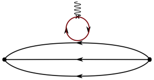
**Significant effort for addressing systematic uncertainties**

[ETMC: C. Alexandrou et al., Phys. Rev. Lett. 119, 142002 (2017)]

$N_f=2$ TM fermions, $m_\pi=130$MeV

- ★ **Excited states:** Mild for $g_A$, 10-15% for $\langle x \rangle$
- ★ **Volume effects:** negligible for $g_A$, non-zero for $\langle x \rangle$
- ★ **Renormalization:** elimination of lattice artifacts (up to 10%)

# Sea quark & gluon contributions



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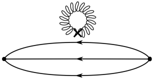
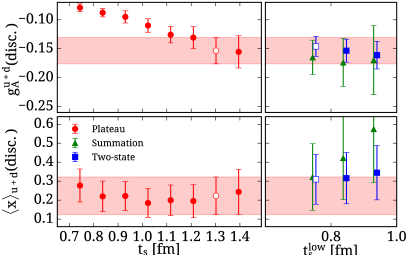
[C. Alexandrou et al. (ETMC), Phys. Rev. D 96, 054503 (2017)]

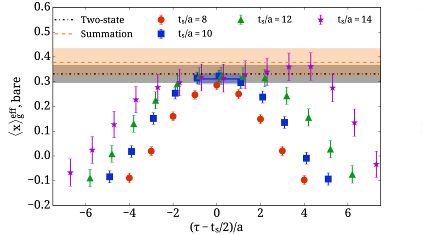




- ★ **Similar calculation of the strange and charm quark contribution**
- ★ **disconnected contributions is crucial for spin**

$$g_A^{u+d} = -0.153(23)(7)$$

$$\langle x_{u+d}\rangle = 0.215(113)(95)$$



- ★ **Mixing of $\langle x\rangle_g$ with $\langle x\rangle_{u+d}$**
- ★ **Computation of mixing coefficients in lattice pert. theory**
- ★ **Upon disentangling the gluon momentum fraction from the quark:**

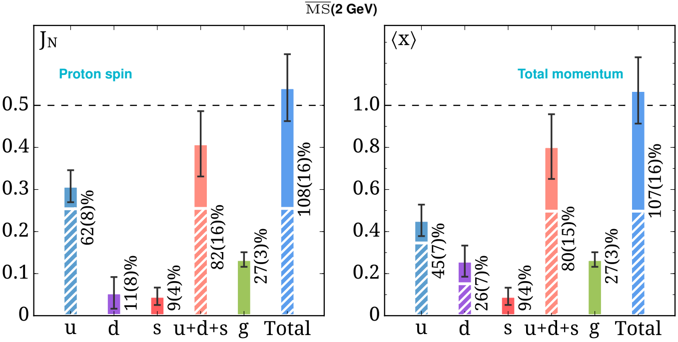
$$\langle x\rangle_g^R = 0.267(22)(19)(24)$$

# Collected Results

- ★ **Satisfaction of spin and momentum sum rule is not forced**

$\Downarrow$

- ★ **important check of results and the systematic uncertainties**



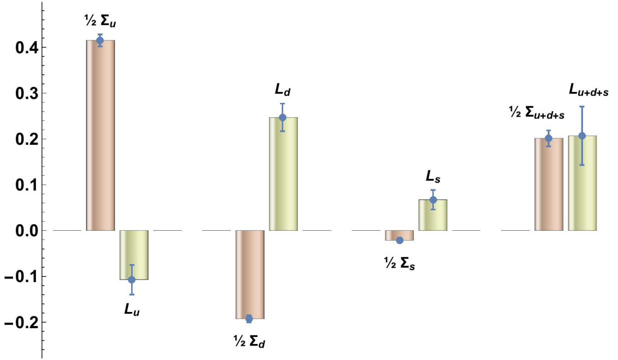
**Striped segments: valence quark contributions (connected)**

**Solid segments: sea quark & gluon contributions (disconnected)**

C. Alexandrou et al., Phys. Rev. Lett. 119, 142002 (2017)

# Collected Results

## Quark Orbital Angular momentum - Intrinsic spin



- ★ **Largest contribution from up-quark**
- ★ **d-quark:**
  **orbital angular momentum almost cancelled by its intrinsic spin**

# Alternative Spin Decomposition

$$\boxed{\frac{1}{2} = \sum_q \left(L^q + \frac{1}{2}\Delta\Sigma^q\right) + \Delta_G + L_G}$$

[R. Jaffe and A. Manohar, Nucl. Phys. B 337, 509 (1990)]

$\Delta_G$: **glue helicity**
$L_G$: **glue OAM**

**Cannot be computed directly on the lattice**

$$\Delta G = \int dx \frac{i}{2xP^+} \int \frac{d\xi^-}{2\pi} e^{-ixP^+\xi^-} \langle PS| F_a^{+\alpha}(\xi^-)\mathcal{L}^{ab}(\xi^-,0)\tilde{F}^{+}_{\alpha,b}(0)|PS\rangle$$

$$\Big\downarrow \int dx$$

$$\tilde{S}_G = \left[\vec{E}^a(0) \times (\vec{A}^a(0) - \frac{1}{\nabla^+}(\vec{\nabla}A^{+,b})\mathcal{L}^{ba}(\xi^-,0))\right]^z$$

**gauge-invariant gluon helicity operator**

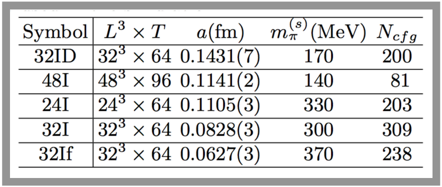
★ **In Coulomb gauge ($\vec{\partial}\cdot\vec{A} = 0$):**
**scale dependence is different with that of glue helicity**

★ $\tilde{S}_G$ **can be matched to $\Delta_G$ via a factorization formula in LaMET**

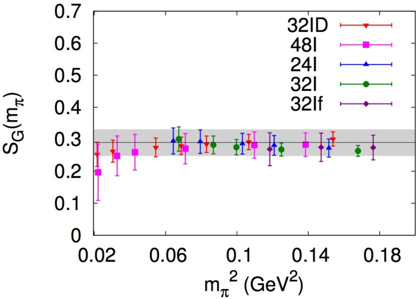
$$\vec{S}_G = 2\int d^3x \mathrm{Tr}[\vec{E}_c \times \vec{A}_c]$$

# Glue Spin

[$\chi$QCD: Y-B Yang et al., Phys. Rev. Lett. 118, 102001 (2017)]



| Symbol | $L^3 \times T$ | $a$(fm) | $m_\pi^{(s)}$(MeV) | $N_{cfg}$ |
|---|---|---|---|---|
| 32ID | $32^3 \times 64$ | 0.1431(7) | 170 | 200 |
| 48I | $48^3 \times 96$ | 0.1141(2) | 140 | 81 |
| 24I | $24^3 \times 64$ | 0.1105(3) | 330 | 203 |
| 32I | $32^3 \times 64$ | 0.0828(3) | 300 | 309 |
| 32If | $32^3 \times 64$ | 0.0627(3) | 370 | 238 |

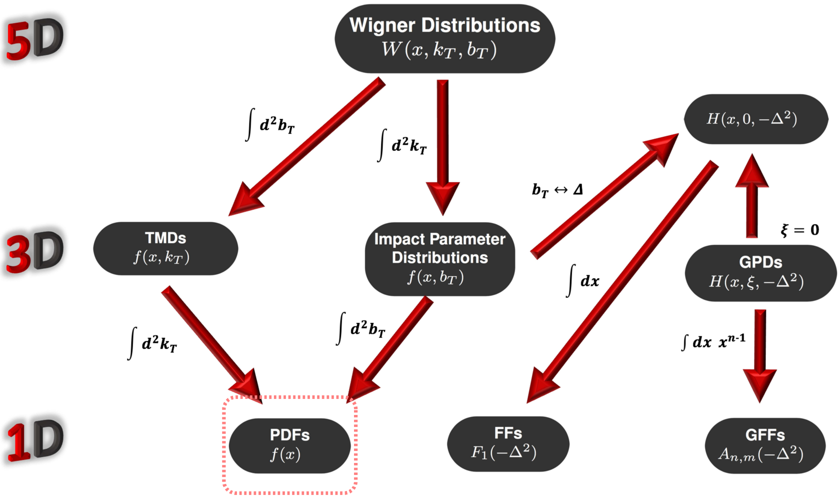


**Large momentum limit:** $S_G = 0.251(47)(16)$ **at 10 GeV$^2$**

## D

# PDFs directly from LQCD

# FFs & GFFs



5D

**Wigner Distributions**
$W(x, k_T, b_T)$

$\int d^2b_T$

$\int d^2k_T$

$H(x, 0, -\Delta^2)$

$b_T \leftrightarrow \Delta$

3D

**TMDs**
$f(x, k_T)$

**Impact Parameter Distributions**
$f(x, b_T)$

$\int dx$

$\xi = 0$

**GPDs**
$H(x, \xi, -\Delta^2)$

$\int d^2k_T$

$\int d^2b_T$

$\int dx\ x^{n\text{-}1}$

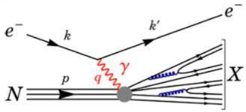
1D

**PDFs**
$f(x)$

**FFs**
$F_1(-\Delta^2)$

**GFFs**
$A_{n,m}(-\Delta^2)$

# Probing Nucleon Structure via PDFs



- ★ **powerful tool to describe the structure of a nucleon**
- ★ **Lattice QCD: long history of moments of PDFs**

  **rely on OPE to reconstruct the PDFs (difficult task):**
  - **signal-to-noise is bad for higher moments**
  - **n $>$ 3: operator mixing (unavoidable!)**

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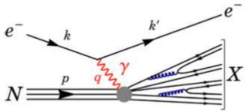

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- ★ **Alternative approaches to access PDFs:**

  **Purely spatial matrix elements that can be matched to PDFs**
  - **quasi-PDFs** [X. Ji, Phys. Rev. Lett. 110, (2013) 262002]
  - **pseudo-PDFs** [A. Radyushkin, Phys. Rev. D 96, 034025 (2017)]
  - **good lattice cross-sections** [Y-Q Ma&J. Qiu, PRL, arXiv:1709.03018]

# PDFs on the Lattice

**Various aspect of direct approaches have been investigated, e.g.:**

- ★ **Renormalization of lattice operators**
- ★ **Matching procedure (LaMET)**

# PDFs on the Lattice

## Various aspect of direct approaches have been investigated, e.g.:

- ★ **Renormalization of lattice operators**
- ★ **Matching procedure (LaMET)**

## Exploratory studies are maturing:

[X. Xiong et al., arXiv:1310.7471], [H-W. Lin et al., arXiv:1402.1462], [Y. Ma et al., arXiv:1404.6860],

[Y.-Q. Ma et al., arXiv:1412.2688], [C. Alexandrou et al., arXiv:1504.07455], [H.-N. Li et al., arXiv:1602.07575],

[J.-W. Chen et al., arXiv:1603.06664], [J.-W. Chen et al., arXiv:1609.08102], [T. Ishikawa et al., arXiv:1609.02018],

[C. Alexandrou et al., arXiv:1610.03689], [C. Monahan et al., arXiv:1612.01584], [A. Radyushkin et al., arXiv:1702.01726],

[C. Carlson et al., arXiv:1702.05775], [R. Briceno et al., arXiv:1703.06072], [M. Constantinou et al., arXiv:1705.11193],

[C. Alexandrou et al., arXiv:1706.00265], [J-W Chen et al., arXiv:1706.01295], [X. Ji et al., arXiv:1706.08962],

[K. Orginos et al., arXiv:1706.05373], [T. Ishikawa et al., arXiv:1707.03107], [J. Green et al., arXiv:1707.07152],

[Y-Q Ma et al., arXiv:1709.03018], [J. Karpie et al., arXiv:1710.08288, [J-W Chen et al., arXiv:1711.07858],

[C.Alexandrou et al., arXiv:1710.06408 ]

**Also talks by: K. Orginos and M. Testa in this session**

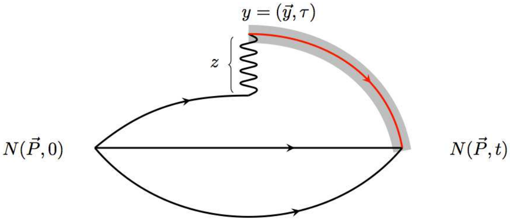
# Access of PDFs on a Euclidean Lattice

[X.Ji, Phys. Rev. Lett. 110, (2013) 262002]

★ **quasi-PDF purely spatial for nucleons with finite momentum**

$$\boxed{\tilde{q}(x,\mu^2,P_3) = \int \frac{dz}{4\pi} e^{-i\,x\,P_3\,z} \langle N(P_3)|\bar{\Psi}(z)\,\gamma^z\,\mathcal{A}(z,0)\Psi(0)|N(P_3)\rangle_{\mu^2}}$$

- $\mathcal{A}(z,0)$: **Wilson line from** $0 \to z$
- $z$: **distance in any spatial direction** (**momentum boost in** $z$ **direction**)

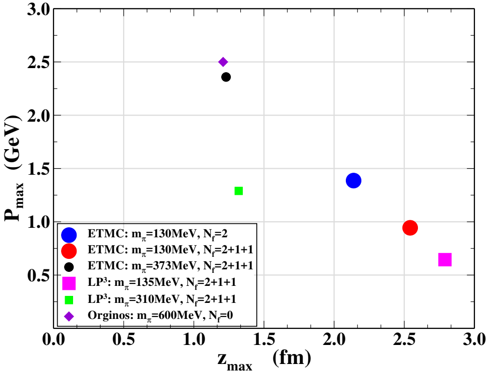


★ **At finite but feasibly large momenta on the lattice:**

**a large momentum EFT can relate Euclidean** $\tilde{q}$ **to PDFs through a factorization theorem**

★ **use of Perturbation Theory for the matching**

# Landscape of Simulations



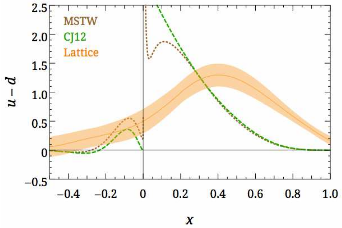
- ★ **Large values for $z_{\max}$ from large volumes**
- ★ $z_{\max} >> 1$**: not reliable region (affects small $x$ region)**
- ★ $P_{\max} >> 1$ **in quasi-PDFs: crucial for matching to physical PDFs**
- ★ **ETMC, LP$^3$: quasi-PDFs, Orginos: pseudo-PDFs**
- ★ **quasi-PDFs $\&$ pseudo-PDFs use same raw data**

# Bare Nucleon Matrix Elements (Unpolarized u-d)

[H-W. Lin, Phys. Rev. D 91, 054510 (2015)]

$N_f = 2{+}1{+}1$ Clover/HISQ

$m_\pi = 310\,\mathrm{MeV}$

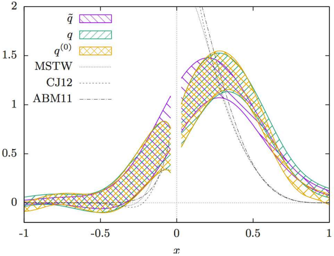


extracted from $P_3 = 2\pi/L * \{1, 2, 3\}$

[ETMC: C. Alexandrou et al., Phys. Rev. D 92, 014502 (2015)]

$N_f = 2{+}1{+}$ Twisted Mass

$m_\pi = 375\,\mathrm{MeV}$



$P_3 = 6\pi/L$, 5 HYP steps

- $-q(-x)$: anti-quark distribution

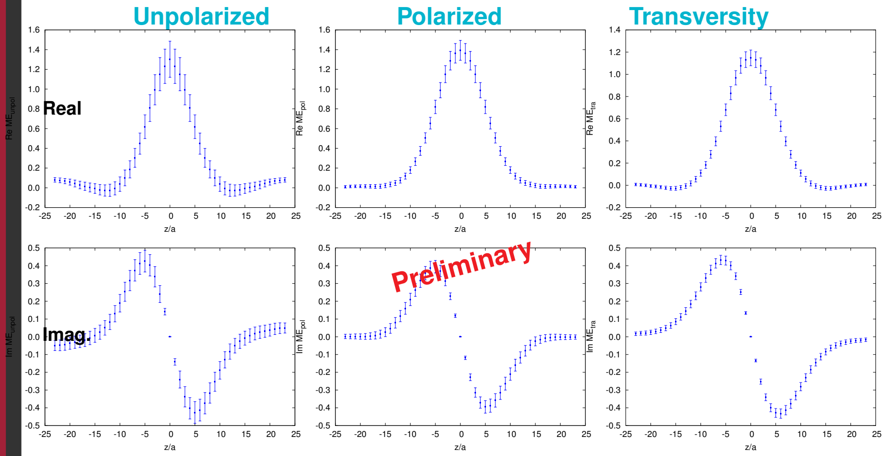
**Status until mid-2016**

- ★ Renormalization missing
- ★ Linear Divergence (from Wilson line) not subtracted
- ★ Mixing for unpolarized not known

# Bare Matrix Elements (Physical point!)

[C. Alexandrou et al. (ETMC), arXiv:1710.06408]

**Twisted Mass Fermions & clover term,** $m_\pi = 130\text{MeV}$ $P_3 = 6\pi/L$



- ★ **Momentum smearing allows to reach higher momenta**

# 2017: Renormalization... At last!

[M. Constantinou, H. Panagopoulos, Phys. Rev. D96, 054506 (2017), [arXiv:1705.11193] ]

## Exploration of renormalization in lattice Perturbation Theory

- ★ Computation of conversion factor between various schemes
- ★ Explore renormalization pattern
- ★ Mixing was revealed... not anticipated earlier
  Affects the computation of the unpolarized quasi-PDF

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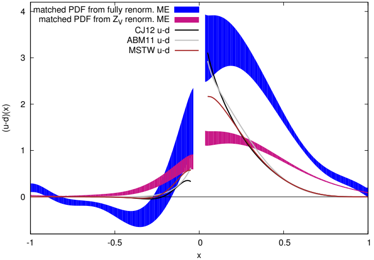
  **Affects the computation of the unpolarized quasi-PDF**

- ★ **Understanding renormalization led to development of non-pert. prescription (RI-type scheme):**
  [C. Alexandrou, et al. (ETMC), Nucl. Phys. B923 (2017) 394 (Frontiers Article)]
- ★ **Procedure followed in other works:**
  [J.-W. Chen et al, (LP$^3$) [arXiv:1706.01295]]
- ★ **Possibilities for matching:** $\overline{\rm MS} \to \overline{\rm MS}$ **or** $RI \to \overline{\rm MS}$

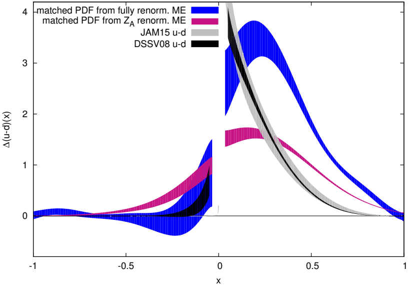
# Renormalized PDFs @ $P_z = 6\pi/L$

[C. Alexandrou, et al. (ETMC), Nucl. Phys. B923 (2017) 394]

## Unpolarized



## Polarized



**Mixing not included**

**Twisted Mass fermions:**

**Mixing with Pseudoscalar ($\mathcal{O}(a)$)**

★ **Results are promising**

- **Renormalization brings lattice data closer to the phenomenological estimates**
- **Need to reach higher momenta**

# pseudo-PDFs

[A. Radyushkin, Phys. Rev. D 96, 034025 (2017)]

Talk by: K. Orginos, Wed @ 3:40pm

★ **Same matrix elements as quasi-PDFs**

★ **Form the ratio**

$$\mathcal{M}(\nu, z_3^2) \equiv \frac{M_p(\nu, z_3^2)}{M_p(0, z_3^2)} \qquad \nu \equiv P_3\, z\text{: Ioffe time}$$

★ **UV divergences cancel in $\mathcal{M}(\nu, z_3^2)$**
**(Provided there is no mixing, e.g. $g_0$ for unpolarized)**

$$\mathcal{M}(\nu, z_3^2) = \mathcal{Q}(\nu, z_3^2) + \mathcal{O}(z_3^2) \underset{F.T.}{\rightarrow} f(x, \mu^2) + \mathcal{O}(z_3^2)$$

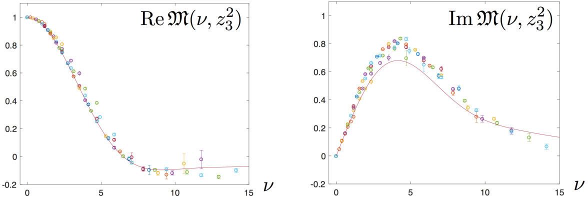
[K. Orginos et al., Phys. Rev. D96 (2017) 094503, J. Karpie et al., [arXiv:1710.08288] ]

$$\mathcal{M}(\nu, z_3^2) = \lim_{t\to\infty} \frac{M_{eff}(Z_3\, P, z_3^2; t)}{M_{eff}(Z_3\, P, z_3^2; t)|_{z_3=0}} \times \frac{M_{eff}(Z_3\, P, z_3^2; t)|_{z_3=0}}{M_{eff}(Z_3\, P, z_3^2; t)|_{P=0}}$$

**optimized to remove lattice spacing effects, where**

$$M_{eff}(Z_3\, P, z_3^2; t) = \frac{C_P^{3pt}(z; t+1)}{C_P^{2pt}(t+1)} - \frac{C_P^{3pt}(z; t)}{C_P^{2pt}(t)}$$

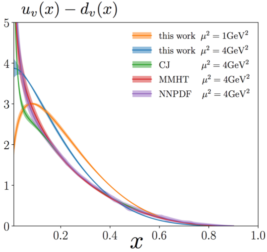
# pseudo-PDFs



★ **Pert. evolution of $z <= 10a$ data to $z = 2a$ to remove residual $z$-dependence**

$$\mathcal{M}(\nu, {z'}_3^2) = \mathcal{M}(\nu, z_3^2) = \frac{2}{3}\frac{\alpha_s}{\pi}\ln({z'}_3^2/z_3^2)\, B \otimes \mathcal{M}(\nu, z_3^2)$$

$B$**: evolution kernel**



**[J. Karpie et al., arXiv:1710.08288]**

# Good Lattice Cross-Sections

[Y. Q. Ma & J. Qiu, accepted in Phys. Rev. Lett., [arXiv:1709.03018] ]

Talk by: J. Qiu, Mon @ 9:15am

- ★ **LQCD: a tool to compute -directly- time-independent good "lattice cross sections"**
- ★ **Computation of current-current correlators (4pt-functions)**

$$\sigma_n(\omega, \xi^2, P^2) = \langle P|T\{\mathcal{O}_n(\xi)\}|P\rangle$$

$$\mathcal{O}_{j_1\, j_2}(\xi) \equiv \xi^{d_{j_1}+d_{j_2}-2}\, Z_{j_1}\, Z_{j_2}\, j_1(\xi)\, j_2(0)$$

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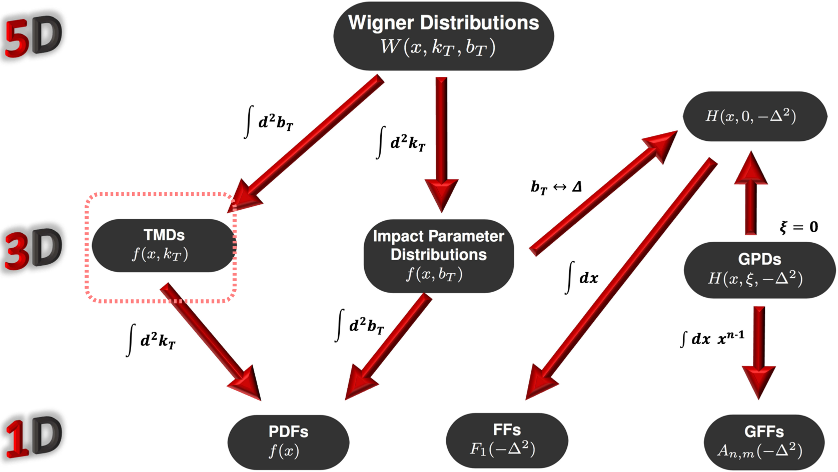
- ★ **Matching coefficients have been computed to LO**

# E

# TMDs from LQCD

# TMDs



5D

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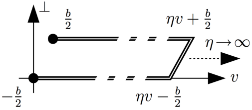
# TMDs from LQCD

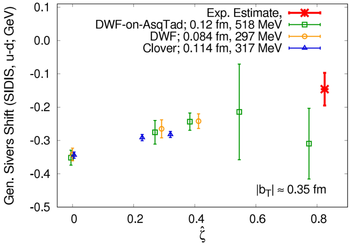
[B. Yoon et al., Phys. Rev. D 96, 094508 (2017), and earlier works of M. Engelhardt]

**Correlator studied on the lattice:**

$$\tilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}(b,P,S) \equiv \langle P,S|\bar{\psi}(-b/2)\Gamma\mathcal{U}[-b/2,b/2]\psi(b/2)|P,S\rangle$$



- ★ $\mathcal{U}$: **Staple of gauge links**
- ★ $\tilde{\Phi}^{[\Gamma]}_{\text{unsubtr.}}$ **includes ultraviolet and soft divergences**
- ★ $n = 0$ **may also be studied (straight wilson line)**
- ★ $|n| \to \infty$**: gluon exchange in SIDIS and DY**
- ★ $b$**: transverse to proton momentum (**$P$**)**
- ★ **different structures for** $\Gamma$ **give access to: Sivers ratio, Boer-Mulders ratio,** $h_1$ **,** $g_{1T}$



**Plot:**
**Collins-Soper parameter:** $\hat{\zeta} \equiv \frac{u \cdot P}{|u|\,|P|}$**, light cone:** $\hat{\zeta} \to \infty$
**Exp. value: global fit to HERMES, COMPASS and JLab data** [M. Echevarria et al., Phys. Rev. D 89 (2014)]

# TMDs and Orbital Angular momentum

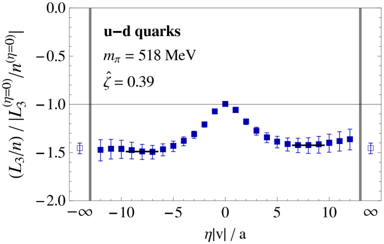
Talk by: M. Engelhardt, Wed @ 5:00pm

[Abha et al., Phys. Rev. D 94, 034041 (2016), M. Engelhardt, Phys. Rev. D 95, 094505 (2017)]

$$\frac{1}{2} = \frac{1}{2}\sum_q \Delta_q + \sum_q L_q + J_g \qquad \text{(Ji)}$$

$$\frac{1}{2} = \frac{1}{2}\sum_q \Delta_q + \sum_q \mathcal{L}_q + \Delta_g + \mathcal{L}_g \qquad \text{(Jaffe – Manohar)}$$

- ★ $L_q$ extracted indirectly in LQCD: $L_q = J_q - \frac{1}{2}\Delta_q$
- ★ $\mathcal{L}_q$ not accessible in LQCD
- ★ straight link operators related to $L_q$
- ★ staple-link operators related to $\mathcal{L}_q$
- ★ operator same as in TMD studies (off-forward matrix element)
- ★ Difference is torque accumulated due to final state interaction



Plot: $\mathcal{L}_q$ vs staple length parameter, in units of $L_q$

# F

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# DISCUSSION

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- ★ Calculation of more complicated quantities:
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  - pseudo-PDFs
  - TMDs
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  - Individual quark quasi-PDFs and pseudo-PDFs
  - Gluon distribution functions
  - Renormalization of staple-link operators (TMDs)

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1 3 5
2 4 ? →

# THANK YOU

U.S. DEPARTMENT OF ENERGY | Office of Science

**TMD Topical Collaboration**

**Grant No. PHY-1714407**

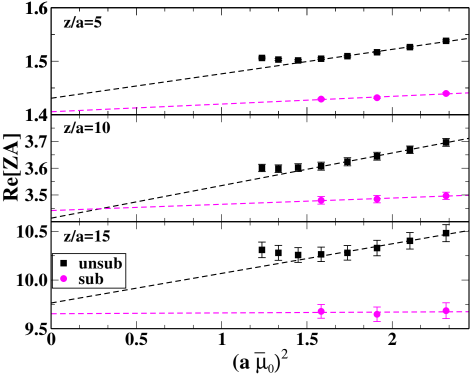
# BACKUP SLIDES

# Refining Renormalization

★ **Improvement Technique:**

- **Computation of 1-loop lattice artifacts to $\mathcal{O}(g^2\, a^\infty)$**
- **Subtraction of lattice artifacts from non-perturbative estimated**

★ **Application to the quasi-PDFs:** **PRELIMINARY**



# Quark Orbital Angular Momentum