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Pseudo Parton Distributions

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in collaboration with

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Introduction

- Goal: Compute hadron structure properties from QCD
  - Parton distribution functions (PDFs)
- Operator product: Mellin moments are local matrix elements that can be computed in Lattice QCD
  - Power divergent mixing limits us to few moments
- Few years ago X. Ji suggested an approach for obtaining PDFs from Lattice QCD
- First calculations already available

X. Ji, Phys.Rev.Lett. 110, (2013)

Y.-Q. Ma J.-W. Qiu (2014) 1404.6860

H.-W. Lin, J.-W. Chen, S. D. Cohen, and X. Ji, Phys.Rev. D91, 054510 (2015)

C. Alexandrou, et al, Phys. Rev. D92, 014502 (2015)

- A new approach for obtaining PDFs from LQCD introduced by A. Radyushkin

A. Radyushkin Phys.Lett. B767 (2017)

- Hadronic tensor methods

K-F Liu et al Phys. Rev. Lett. 72 (1994) , Phys. Rev. D62 (2000) 074501
Detmold and Lin 2005
M. T. Hansen et al arXiv:1704.08993.
UKQCD-QCDSF-CSSM Phys. Lett. B714 (2012), arXiv:1703.01153

Ma and Qiu : arXiv:1709.03018

PDFs: Definition

Light-cone PDFs:

$$f^{(0)}(\xi) = \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega^-}{4\pi} e^{-i\xi P^+ \omega^-} \left\langle P \left| T\, \overline{\psi}(0, \omega^-, \mathbf{0}_\mathrm{T}) W(\omega^-, 0) \gamma^+ \frac{\lambda^a}{2} \psi(0) \right| P \right\rangle_\mathrm{C} .$$

$$W(\omega^-, 0) = \mathcal{P} \exp\left[-ig_0 \int_0^{\omega^-} \mathrm{d}y^- A_\alpha^+(0, y^-, \mathbf{0}_\mathrm{T}) T_\alpha\right]$$

$$\langle P' | P \rangle = (2\pi)^3 2P^+ \delta\left(P^+ - P'^+\right) \delta^{(2)}\left(\mathbf{P}_\mathrm{T} - \mathbf{P}'_\mathrm{T}\right)$$

Moments:

$$a_0^{(n)} = \int_0^1 \mathrm{d}\xi\, \xi^{n-1} \left[f^{(0)}(\xi) + (-1)^n \overline{f}^{(0)}(\xi)\right] = \int_{-1}^1 \mathrm{d}\xi\, \xi^{n-1} f(\xi)$$

Local matrix elements:

$$\left\langle P | \mathcal{O}_0^{\{\mu_1 \ldots \mu_n\}} | P \right\rangle = 2a_0^{(n)} \left(P^{\mu_1} \cdots P^{\mu_n} - \mathrm{traces}\right)$$

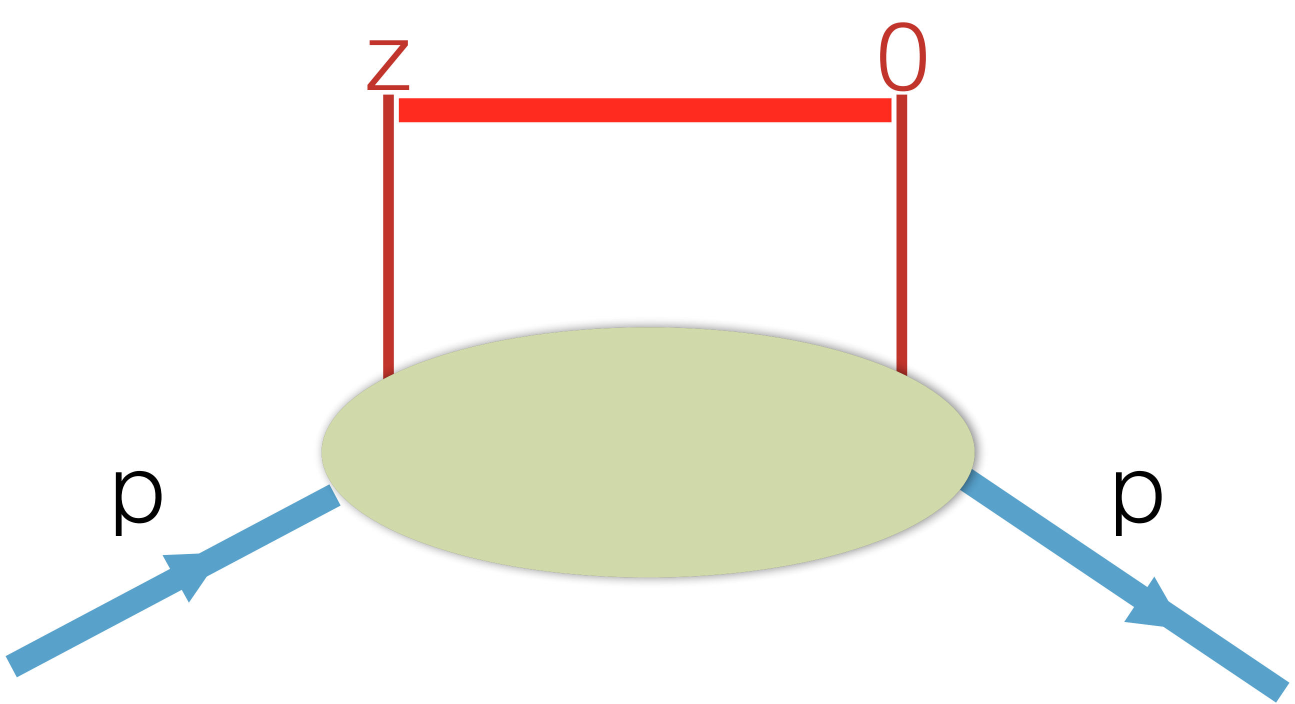
$$\mathcal{O}_0^{\{\mu_1 \cdots \mu_n\}} = i^{n-1} \overline{\psi}(0) \gamma^{\{\mu_1} D^{\mu_2} \cdots D^{\mu_n\}} \frac{\lambda^a}{2} \psi(0) - \mathrm{traces}$$

Pseudo-PDFs

Unpolarized PDFs proton:

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar{\psi}(0)\,\gamma^\alpha\,\hat{E}(0,z;A)\psi(z)|p\rangle$$

$$\hat{E}(0,z;A) = \mathcal{P}\exp\left[-ig\int_0^z \mathrm{d}z'_\mu\, A^\mu_\alpha(z')T_\alpha\right]$$



Lorentz decomposition:

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar\psi(0)\,\gamma^\alpha\,\hat{E}(0,z;A)\psi(z)|p\rangle$$

$$\mathcal{M}^\alpha(z,p) = 2p^\alpha \mathcal{M}_p(-(zp),-z^2) + z^\alpha \mathcal{M}_z(-(zp),-z^2)$$

Collinear PDFs: Choose

$$z = (0, z_-, 0)$$

$$p = (p_+, 0, 0)$$

$$\gamma^+$$

$$\mathcal{M}^+(z,p) = 2p^+ \mathcal{M}_p(-p_+ z_-, 0)$$

Definition of PDF:

$$\mathcal{M}_p(-p_+ z_-, 0) = \int_{-1}^{1} dx\, f(x)\, e^{-ixp_+ z_-}$$

$\mathcal{M}_p(-pz, -z^2)$ is a Lorentz invariant therefore computable in any frame

$\nu = -zp$ ν is called Ioffe time B. L. Ioffe, Phys. Lett. 30B, 123 (1969)

$$\mathcal{M}_p(\nu, -z^2) \equiv \int_{-1}^{1} dx\, \mathcal{P}(x, -z^2) e^{ix\nu}$$

It can be shown that the domain of x is [-1, 1]

A. Radyushkin Phys.Lett. B767 (2017)

$\mathcal{M}_p(\nu, -z^2)$ at small z^2 is called Ioffe time PDF

V. Braun, et. al Phys. Rev. D 51, 6036 (1995)

$-z^2 \leftrightarrow 1/\mu^2$ $\mathcal{P}(x, -z^2) \leftrightarrow f(x, \mu^2)$

Ji Quasi-PDF

Choose
$$p = (p_0, 0, 0, p_3)$$
$$z = (0, 0, 0, z_3)$$
$$\gamma^3$$

$$h(z_3, p_3) = \frac{1}{2p_3}\mathcal{M}^3 = \mathcal{M}_p(-z_3 p_3, -z_3^2) + \frac{z^3}{2p_3}\mathcal{M}_z(-z_3 p_3, -z_3^2)$$

$$Q(y, p_3) = \frac{p_3}{2\pi}\int_{-\infty}^{\infty} dz_3 h(z_3, p_3) e^{iyp_3 z_3}$$

$\mathcal{M}^3$ On shell time local matrix element computable in Euclidean space

Briceno *et al* arXiv:1703.06072

$$Q(y, p_3) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu \left[\mathcal{M}_p(\nu, \nu^2/p_3^2) - \frac{\nu}{2p_3^2} \mathcal{M}_z(\nu, \nu^2/p_3^2) \right] e^{-iy\nu}$$

$$\nu = -p_3 z_3$$

Range of ν is (-∞, +∞)

Artifacts scale as $\nu \cdot \frac{\Lambda_{qcd}^2}{p_3^2}$

at finite momentum the full range is not accessible resulting additional systematic error

$$Q(y, p_3) = \int_{-1}^{1} \frac{dx}{|x|} Z(\frac{y}{x}, \frac{\mu}{p_3}) f(x, \mu) + \mathcal{O}(\frac{\Lambda_{qcd}^2}{p_3^2})$$ **+ ….. ??**

Chen et al. arXiv:1711.07858

* Potential issue with power divergences?

G. Rossi and M. Testa 10.1103/PhysRevD.96.014507

A. Radyushkin Phys.Lett. B767 (2017)

Alternatively

Choose

$$p = (p_0, 0, 0, p_3)$$

$$z = (0, 0, 0, z_3)$$

$$\gamma^0$$

Obtaining only the relevant

$$\mathcal{M}_p(\nu, z_3^2) = \frac{1}{2p_0}\mathcal{M}^0(z_3, p_3)$$

$$\mathcal{M}_p(\nu, z_3^2) = \int_{-1}^{1} dx \mathcal{P}(x, z_3^2)$$

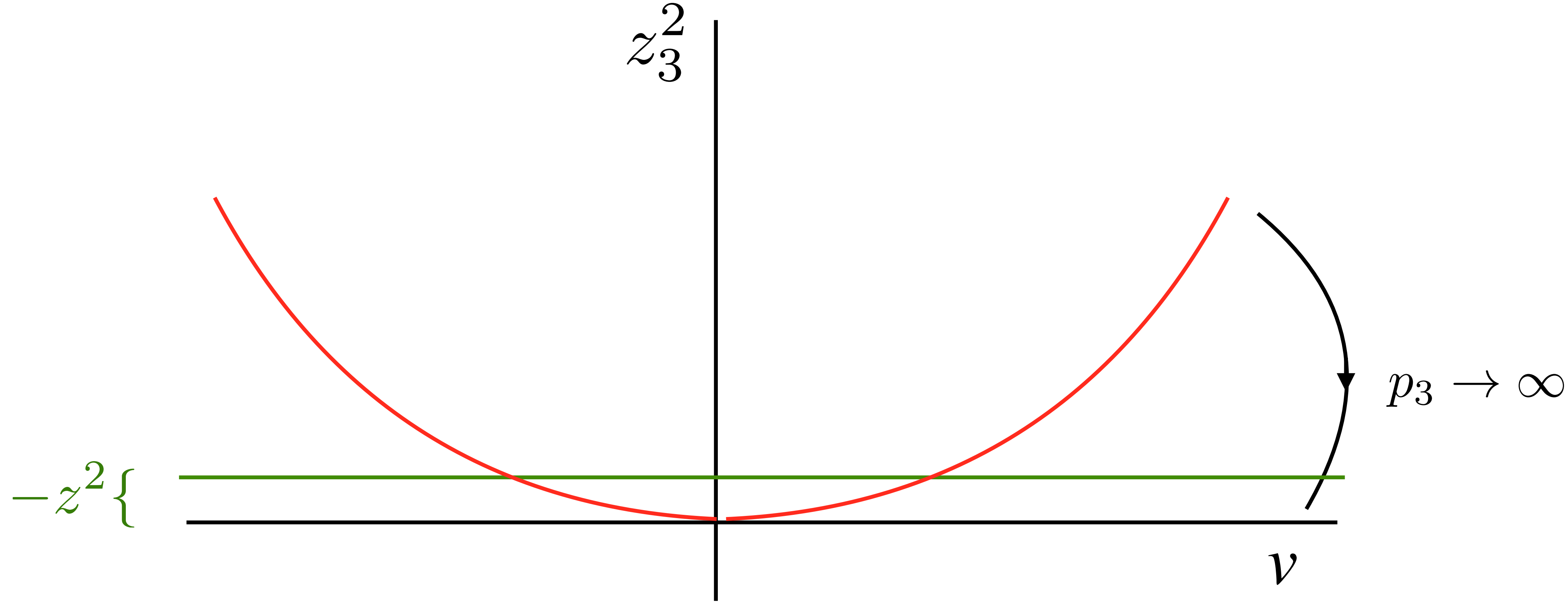
Chosing γ^0 was also suggested also by M. Constantinou at GHP2017 based on an operator mixing argument for the renormalized matrix element

Alexandrou *et al* arXiv:1706.00265

$$Q(y, p_3) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu \mathcal{M}_p(\nu, \nu^2/p_3^2) e^{-iy\nu}$$

Large values of $z_3 = \nu/p_3$ are problematic

Alternative approach to the light-cone:



$$\mathcal{P}(x, -z^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu \, \mathcal{M}_p(\nu, -z^2) e^{-ix\nu}$$

$-z^2 \to 0$ PDFs can be recovered

Lattice QCD requirements

$$aP_{max} = \frac{2\pi}{4} \sim \mathcal{O}(1)$$

$$a \sim 0.1fm \rightarrow P_{max} = 10\Lambda$$
$$a \sim 0.05fm \rightarrow P_{max} = 20\Lambda$$

$$\Lambda \sim 300MeV$$

For practical calculations large momentum is needed

*Higher twist effect suppression (qpdfs)

*Wide coverage of Ioffe time ν

P= 3 GeV is already demanding due to statistical noise

achievable with easily accessible lattice spacings

P= 6 GeV exponentially harder

requires current state of the art lattice spacing

Statistical noise

Nucleon with momentum P two-point function:

$$C_{2p}(P,t) = \langle O_N(P,t) O_N^\dagger(P,0) \rangle \sim \mathcal{Z} e^{-E(P)t}$$

Variance of nucleon two-point function:

$$\mathrm{var}\left[C_{2p}(P,t)\right] = \langle O_N(P,t) O_N(P,t)^\dagger O_N(P,0) O_N^\dagger(P,0) \rangle \sim \mathcal{Z}_{3\pi} e^{-3m_\pi t}$$

Variance is independent of the momentum

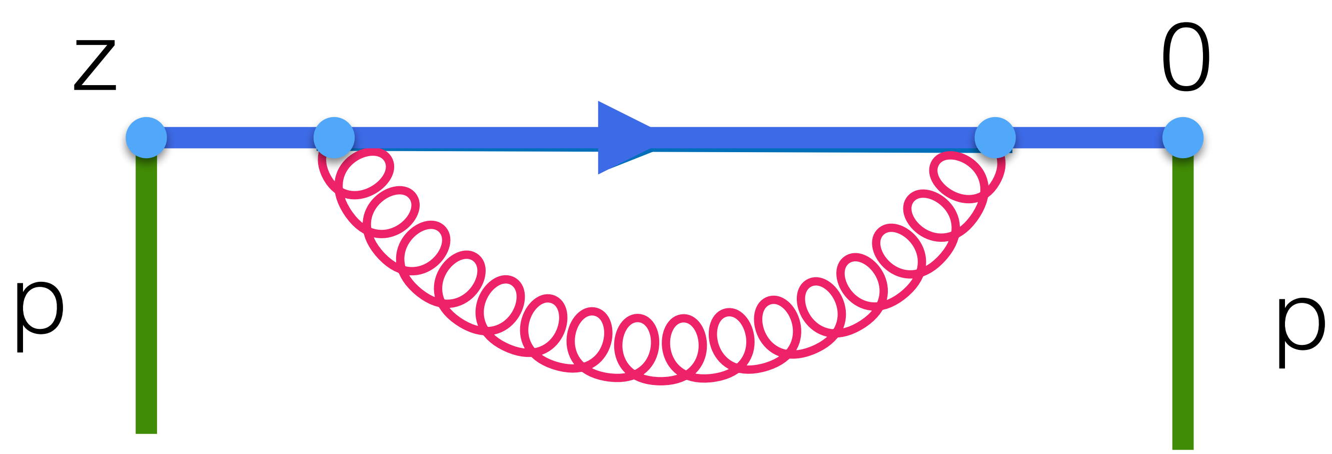
$$\frac{\mathrm{var}\left[C_{2p}(P,t)\right]^{1/2}}{C_{ap}(P,t)} \sim \frac{\mathcal{Z}}{\mathcal{Z}_{3\pi}} e^{-[E(P)-3/2 m_\pi]t}$$

Statistical accuracy drops exponentially with the increasing momentum limiting the maximum achievable momentum.

Continuum limit

$$\mathcal{M}^0_{ren}(z,p,\mu) = \lim_{a\to 0} Z_{\mathcal{O}}(z,\mu,a)\mathcal{M}^0(z,P,a)$$

Determine Z non-perturbatively in some scheme



One loop linear divergence needs to be re-summed

$$Z_{\mathcal{O}}(z,\mu,a) \sim e^{+\delta m|z|/a - c|z|}$$

Dotsenko Nucl.Phys. B169 (1980) 527
Chen et al. Nucl.Phys. B915 (2017)
Ishikawa et al. arXiv:1707.03107, arXiv:1609.02018
Radyushkin arXiv:1710.08813

RI’ MOM scheme

Alexandrou et al. Nucl.Phys. B923 (2017) 394

Use gauge fixed off-shell external quark states to compute:

$$\mathcal{M}^0(z,p) = \langle p|\bar{\psi}(0)\,\gamma^0\,\hat{E}(0,z;A)\psi(z)|p\rangle$$

Define

$$Z_{\mathcal{O}}(z,\mu) = \frac{Z_q}{\frac{1}{12}\mathrm{Tr}\left[\mathcal{M}^0(z,p)\left(\mathcal{M}^{0,\mathrm{Born}}(z,p)\right)^{-1}\right]\Big|_{p=\mu}}$$

Z_q is the quark wave function renormalization in RI’ MOM

Consider the ratio $\mathfrak{M}(\nu, z_3^2) \equiv \frac{\mathcal{M}_p(\nu, z_3^2)}{\mathcal{M}_p(0, z_3^2)}$

UV divergences will cancel in this ratio

Denominator is regular at $z_3^2 \to 0$

$\mathcal{M}_p(0,0) = 1$ Isovector matrix element

Polynomial corrections to the Ioffe time PDF may be suppressed

B. U. Musch, *et al* Phys. Rev. D 83, 094507 (2011)
M. Anselmino et al. 10.1007/JHEP04(2014)005
A. Radyushkin Phys.Lett. B767 (2017)

$$\mathcal{M}_p(\nu, z_3^2)) = \mathcal{Q}(\nu, z_3^2) + \mathcal{O}(z_3^2)$$

$$\mathfrak{M}(\nu, z_3^2) = \mathcal{Q}(\nu, z_3^2) + \mathcal{O}(z_3^2)$$ with smaller corrections

$$\mu^2 = (2e^{-\gamma_E}/z_3)^2 \qquad \mathcal{Q}(\nu, z_3^2) \xrightarrow{\text{F.T.}} f(x, \mu^2) \qquad \overline{MS}$$

Radyushkin arXiv:1710.08813

TMD factorization

A. Radyushkin Phys.Lett. B767 (2017)

$$\mathcal{M}_p(\nu, -z^2) \equiv \int_{-1}^{1} dx\, \mathcal{P}(x, -z^2) e^{ix\nu}$$

Taking $z = (0, z_-, z_\perp)$ we can identify $\mathcal{P}(x, z_\perp^2) = \int d^2k_\perp\, \mathcal{F}(x, k_\perp^2)$

$\mathcal{F}(x, k_\perp^2)$ the primordial TMD

Assuming $\mathcal{F}(x, k_\perp^2) = f(x) g(k_\perp^2)$ we obtain $\mathcal{P}(x, z_\perp^2) = f(x)\tilde{g}(z_\perp^2)$

M. Anselmino et al. 10.1007/JHEP04(2014)005

B. U. Musch, *et al* Phys. Rev. D 83, 094507 (2011)

Implying that $\mathcal{M}_p(\nu, -z^2) = \mathcal{Q}(\nu, -z^2)\mathcal{M}_p(0, -z^2)$

where $\mathcal{M}_p(0, -z^2) = \tilde{g}(-z^2)$

The TMD factorization assumption implies that the ratio

$$\mathfrak{M}(\nu, z_3^2) \equiv \frac{\mathcal{M}_p(\nu, z_3^2)}{\mathcal{M}_p(0, z_3^2)}$$

is the Ioffe time distribution with small polynomial corrections

This ratio has a well defined continuum limit

$$\mathfrak{M}(\nu, z_3^2) = \mathcal{Q}(\nu, z_3^2) + \mathcal{O}(z_3^2)$$

$\mu^2 = (2e^{-\gamma_E}/z_3)^2$ $\qquad$ $\mathcal{Q}(\nu, z_3^2) \xrightarrow{\text{F.T.}} f(x, \mu^2)$ $\qquad$ $\overline{MS}$

Radyushkin arXiv:1710.08813

$$\frac{d}{d\ln z_3^2}\,\mathcal{Q}(\nu, z_3^2) = -\frac{\alpha_s}{2\pi}\,C_F \int_0^1 du\, B(u)\,\mathcal{Q}(u\nu, z_3^2)$$

$$B(u) = \left[\frac{1+u^2}{1-u}\right]_+$$

DGLAP kernel in position space

V. Braun, et. al Phys. Rev. D 51, 6036 (1995)

Checked at 1-loop in Radyushkin arXiv:1710.08813

at small z_3^2

$$\mathcal{Q}(\nu, {z'}_3^2) = \mathcal{Q}(\nu, z_3^2) - \frac{2}{3}\frac{\alpha_s}{\pi}\ln({z'}_3^2/z_3^2)\int_0^1 du\, B(u)\,\mathcal{Q}(u\nu, z_3^2)$$

if power corrections are small

Numerical Tests

in collaboration with

J. Karpie, A. Radyushkin, S. Zafeiropoulos

arXiv:1706.05373

Numerical Tests

- Quenched approximation β=6.0

$32^3 \times 64 \quad m_\pi \sim 600 MeV$

- Need series of small z_3
- Need a range of momenta to scan ν
- Goals:
    - Check scaling violations
    - Understand the systematics of the approach

Matrix element calculation

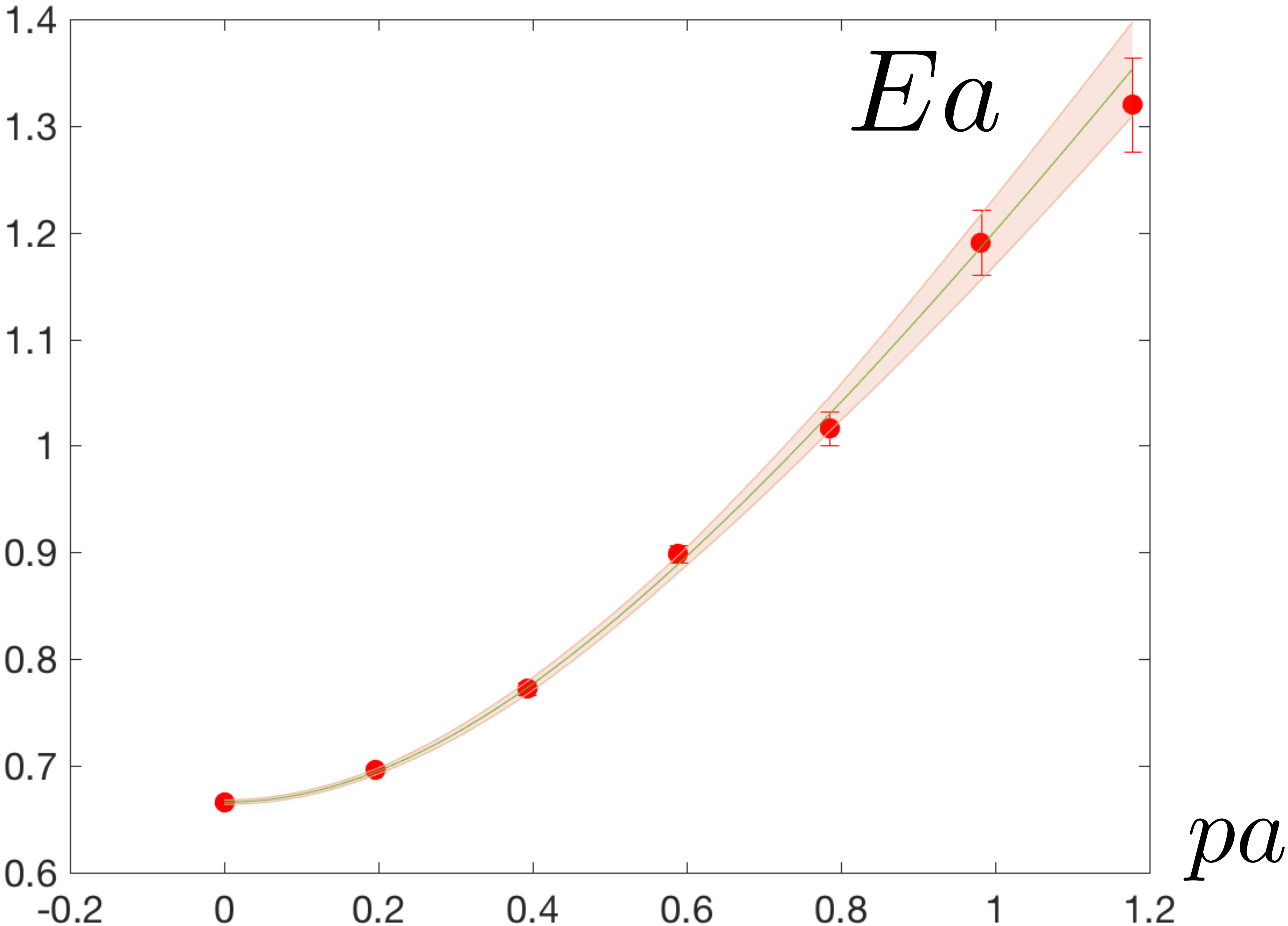
$$C_P(t) = \langle \mathcal{N}_P(t)\overline{\mathcal{N}}_P(0)\rangle \qquad C_P^{\mathcal{O}^0(z)}(t) = \langle \mathcal{N}_P(t)\mathcal{O}^0(z)\overline{\mathcal{N}}_P(0)\rangle$$

$$\mathcal{M}_{\rm eff}(z_3P, z_3^2; t) = \frac{C_P^{\mathcal{O}^0(z)}(t+1)}{C_P(t+1)} - \frac{C_P^{\mathcal{O}^0(z)}(t)}{C_P(t)}$$

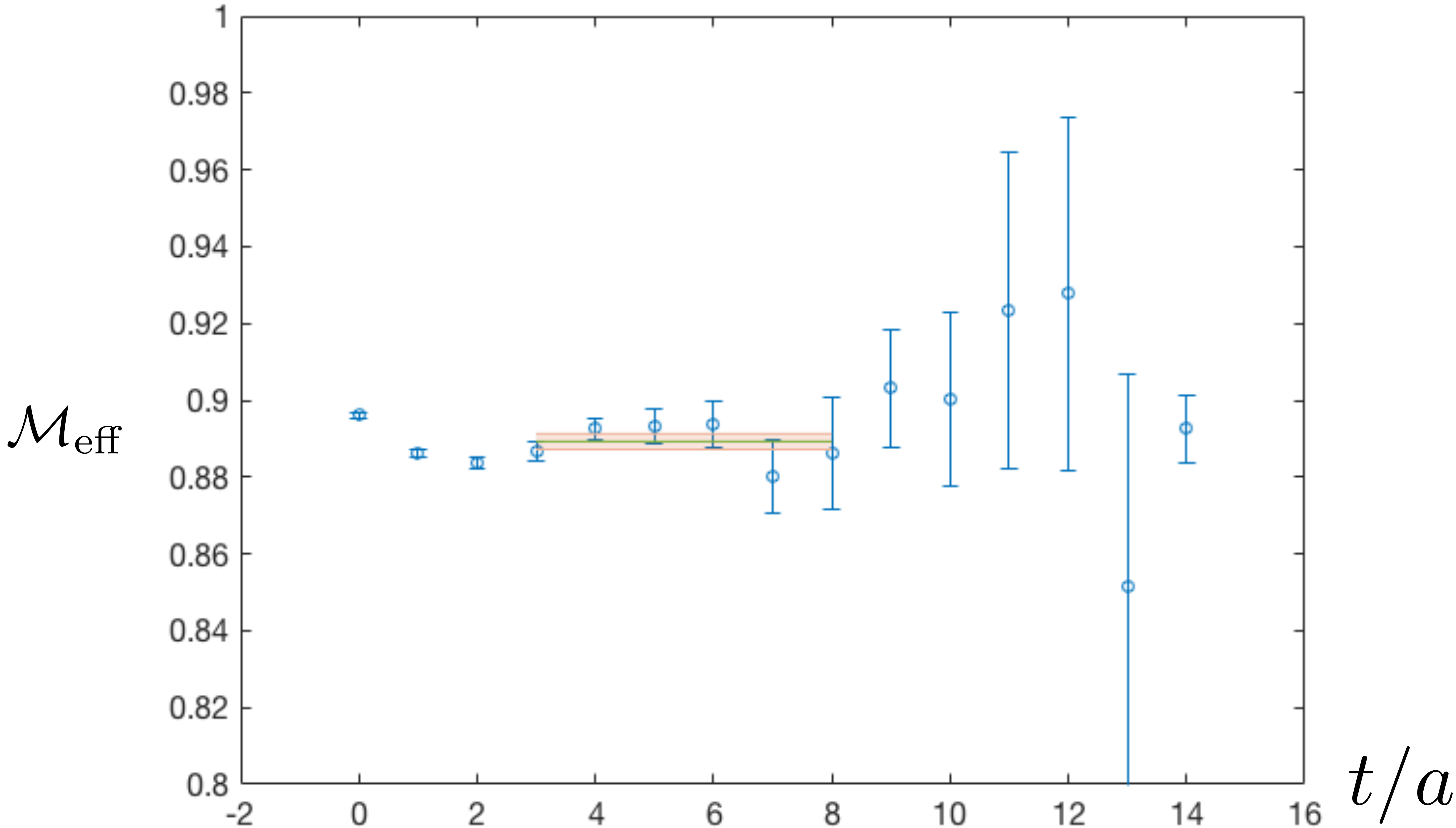
C. Bouchard, et al arXiv:1612.06963 [hep-lat]

$$\mathfrak{M}(\nu, z_3^2) = \lim_{t\to\infty} \frac{\mathcal{M}_{\rm eff}(z_3P, z_3^2; t)}{\mathcal{M}_{\rm eff}(z_3P, z_3^2; t)\big|_{z_3=0}} \times \frac{\mathcal{M}_{\rm eff}(z_3P, z_3^2; t)\big|_{z_3=0}}{\mathcal{M}_{\rm eff}(z_3P, z_3^2; t)\big|_{P=0}}$$

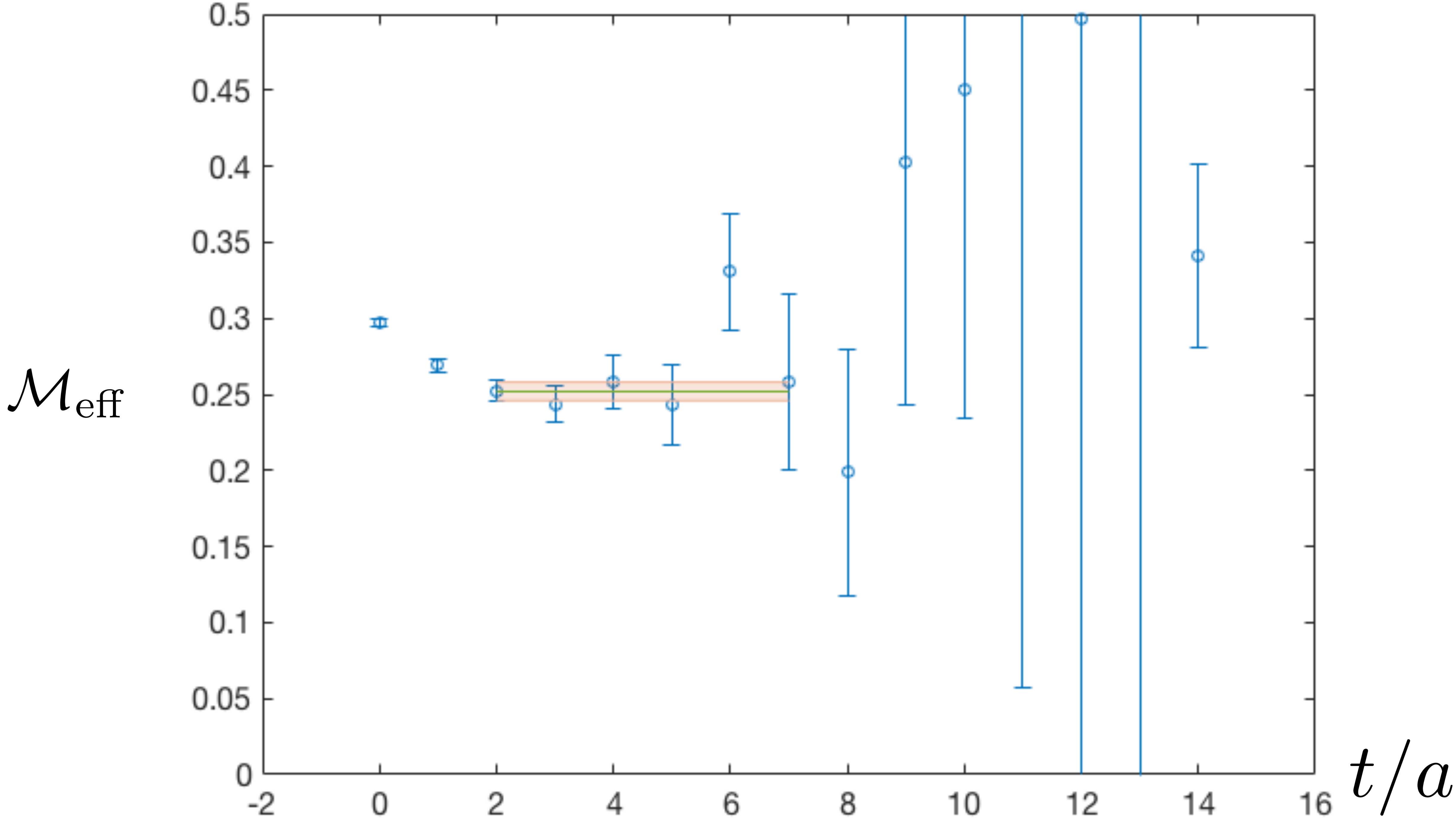
Constructed to remove lattice spacing errors



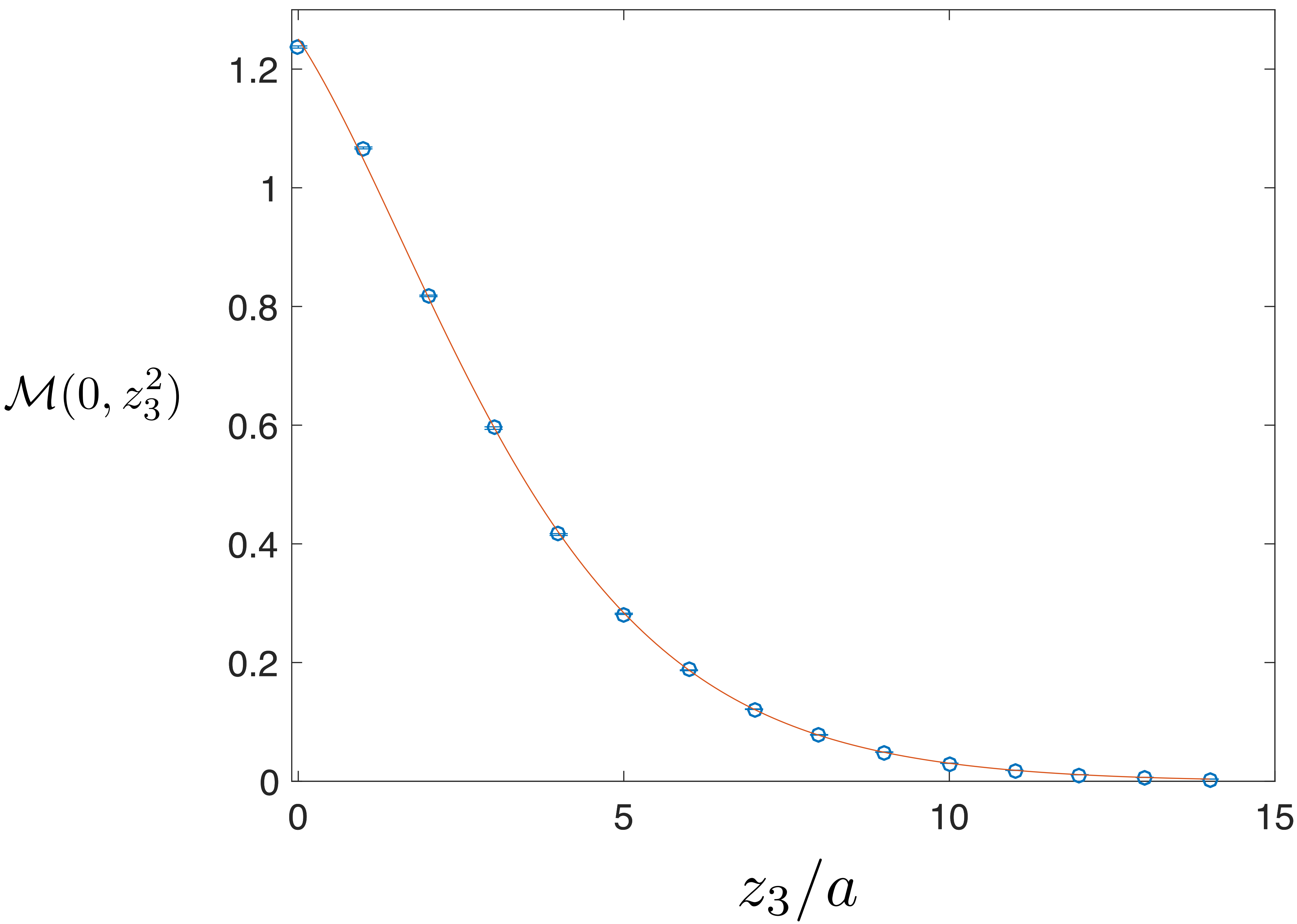
Gaussian smeared sources



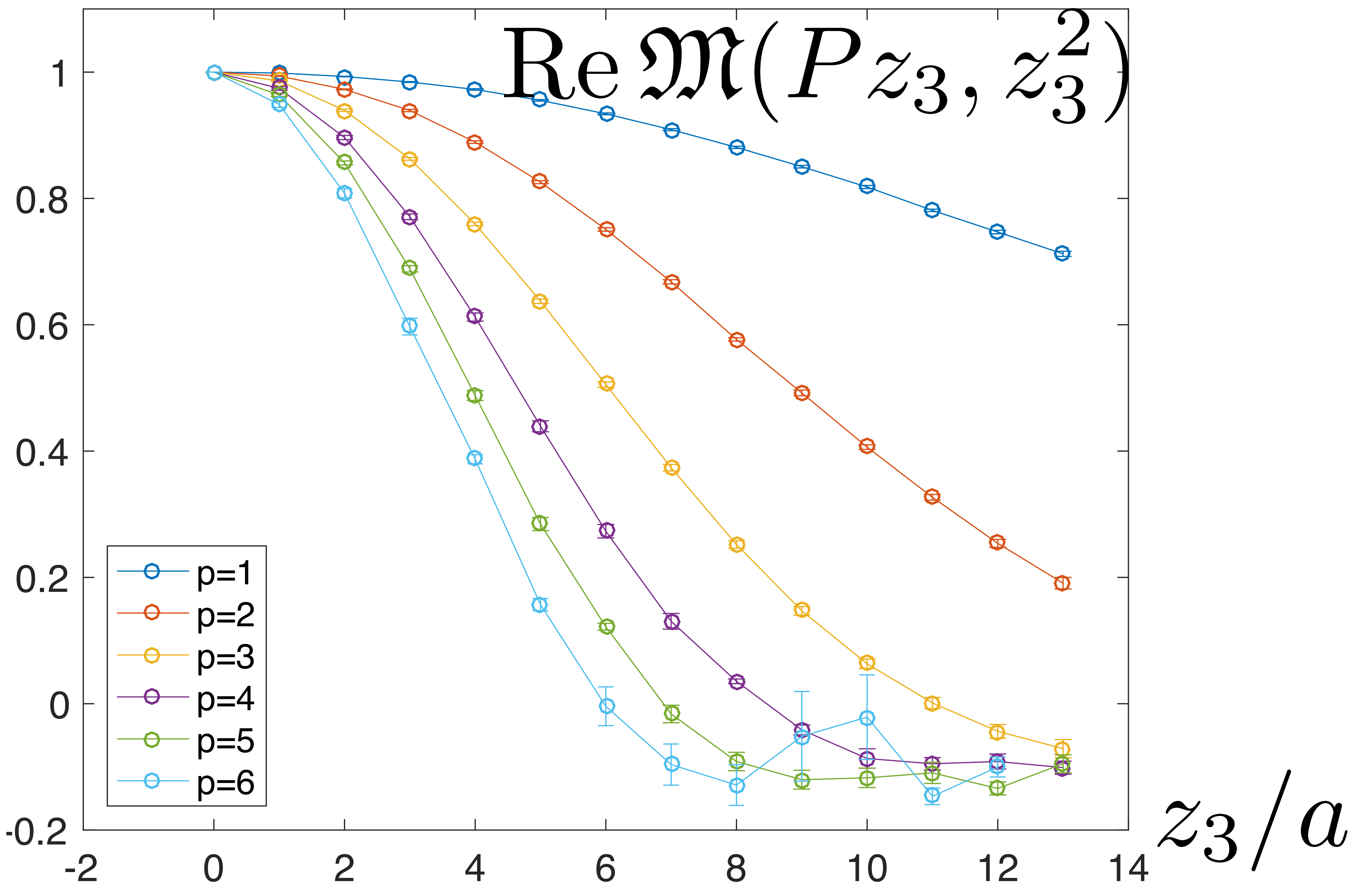
$\mathcal{M}_{\rm eff}$
t/a
1
0.98
0.96
0.94
0.92
0.9
0.88
0.86
0.84
0.82
0.8
-2
0
2
4
6
8
10
12
14
16



$\mathcal{M}_{\rm eff}$
t/a
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-2
0
2
4
6
8
10
12
14
16



Cusp indicates “linear” divergence of Wilson line



Ratio removes the linear” divergence of Wilson line

Real Part

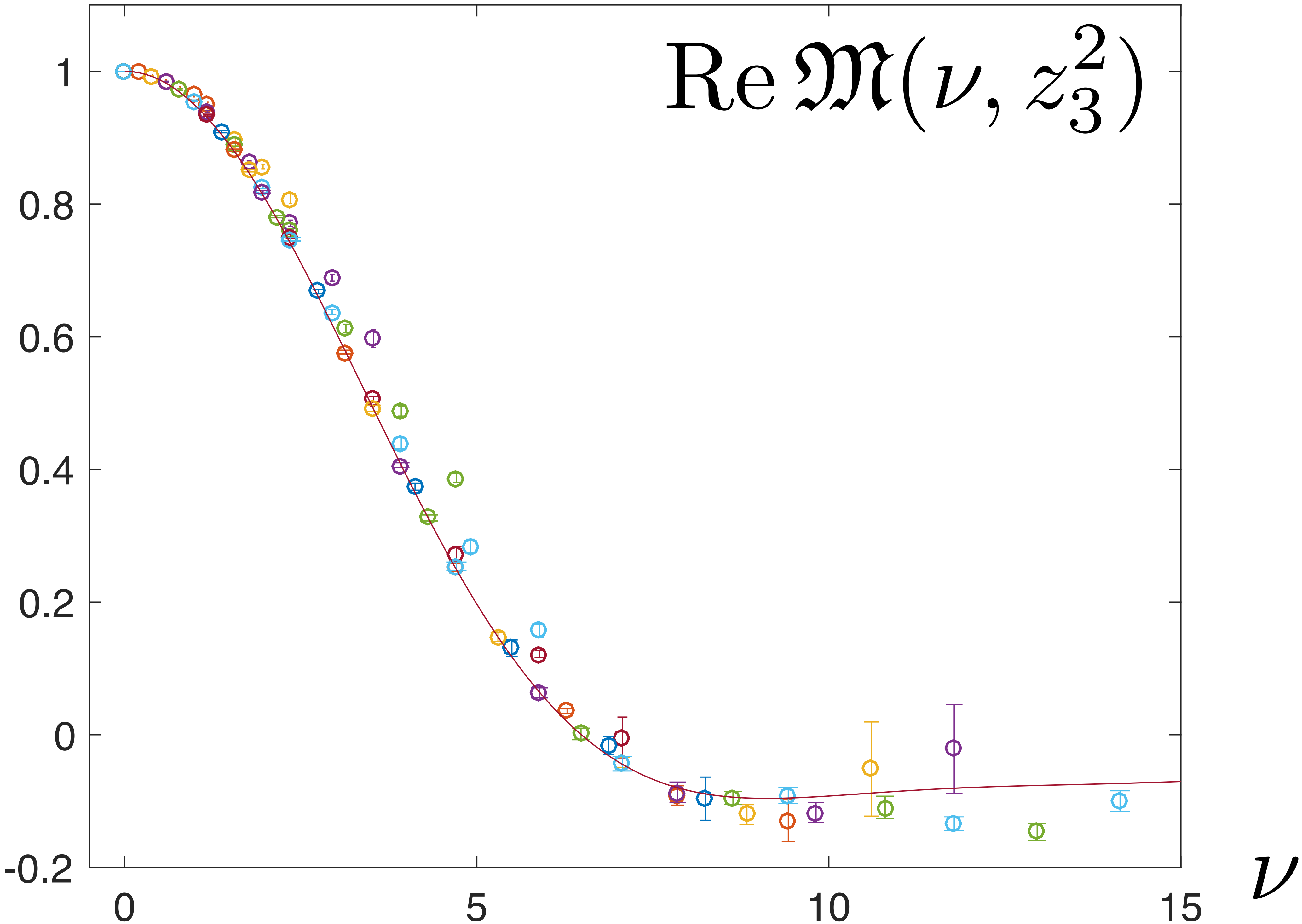
Isovector distribution

$$\mathfrak{M}_R(\nu, z^2 = 1/\mu^2) \equiv \int_0^1 dx\ \cos(\nu x)\, q_v(x, \mu^2)$$

$$q_v(x) = q(x) - \bar{q}(x) \qquad\qquad q(x) = u(x) - d(x)$$

$\overline{MS}$ $\quad \mu^2 = (2e^{-\gamma_E}/z_3)^2$

Radyushkin arXiv:1710.08813



Points almost collapse on a universal curve

$$q_v(x) = \frac{315}{32}\sqrt{x}(1-x)^3$$

Imaginary Part

Isovector distribution

$$\mathfrak{M}_I(\nu, z^2 = 1/\mu^2) \equiv \int_0^1 dx\, \sin(\nu x)\, q_+(x,\mu^2)\,.$$

$$q_+(x) = q(x) + \bar{q}(x)$$

$$q(x) = u(x) - d(x)$$

$$q_+(x) = q_v(x) + 2\bar{q}(x)$$

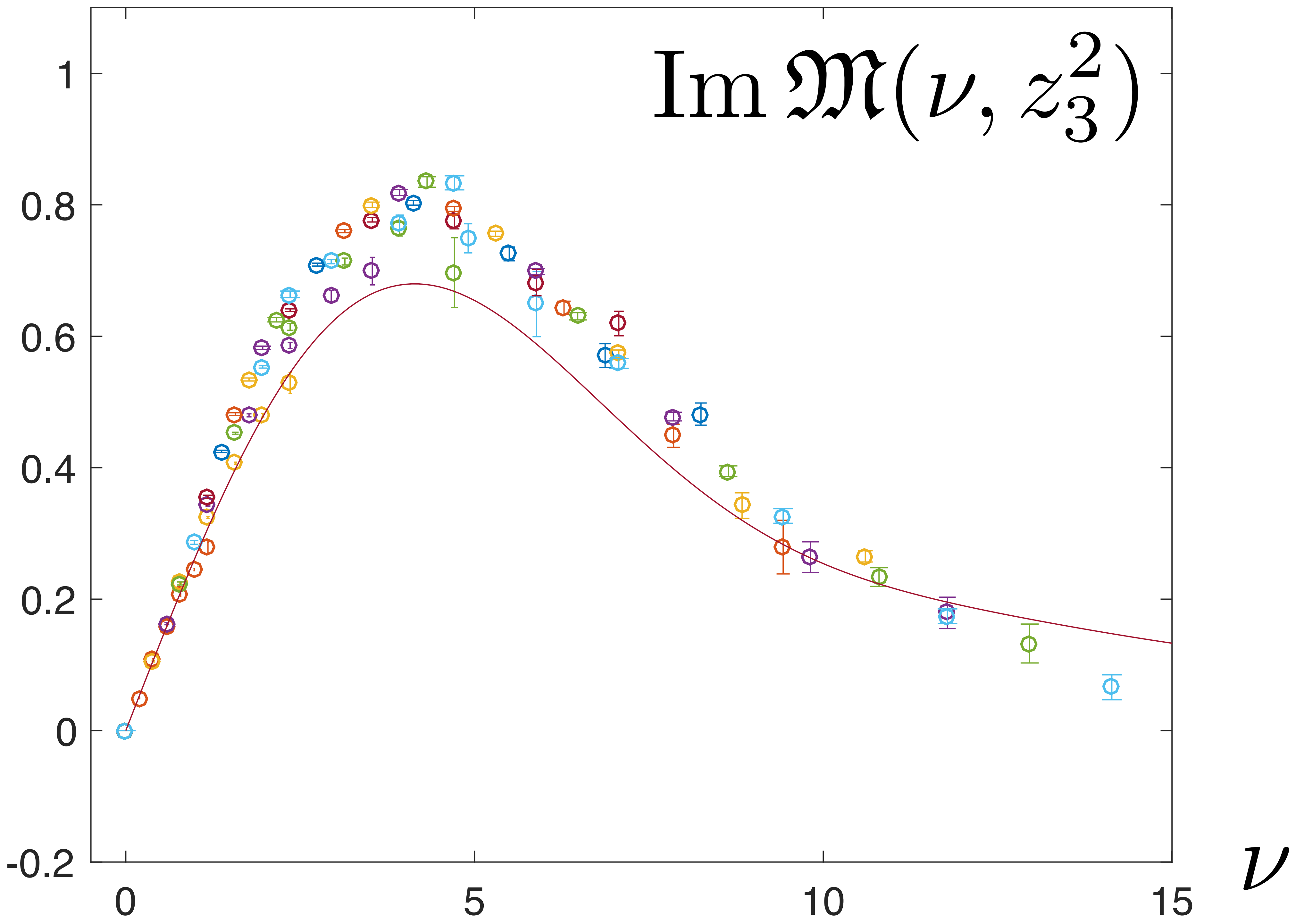
$$q_v(x) = q(x) - \bar{q}(x)$$

$\overline{MS}$ $\quad \mu^2 = (2e^{-\gamma_E}/z_3)^2$

Radyushkin arXiv:1710.08813

anti-quarks contribute to the imaginary part

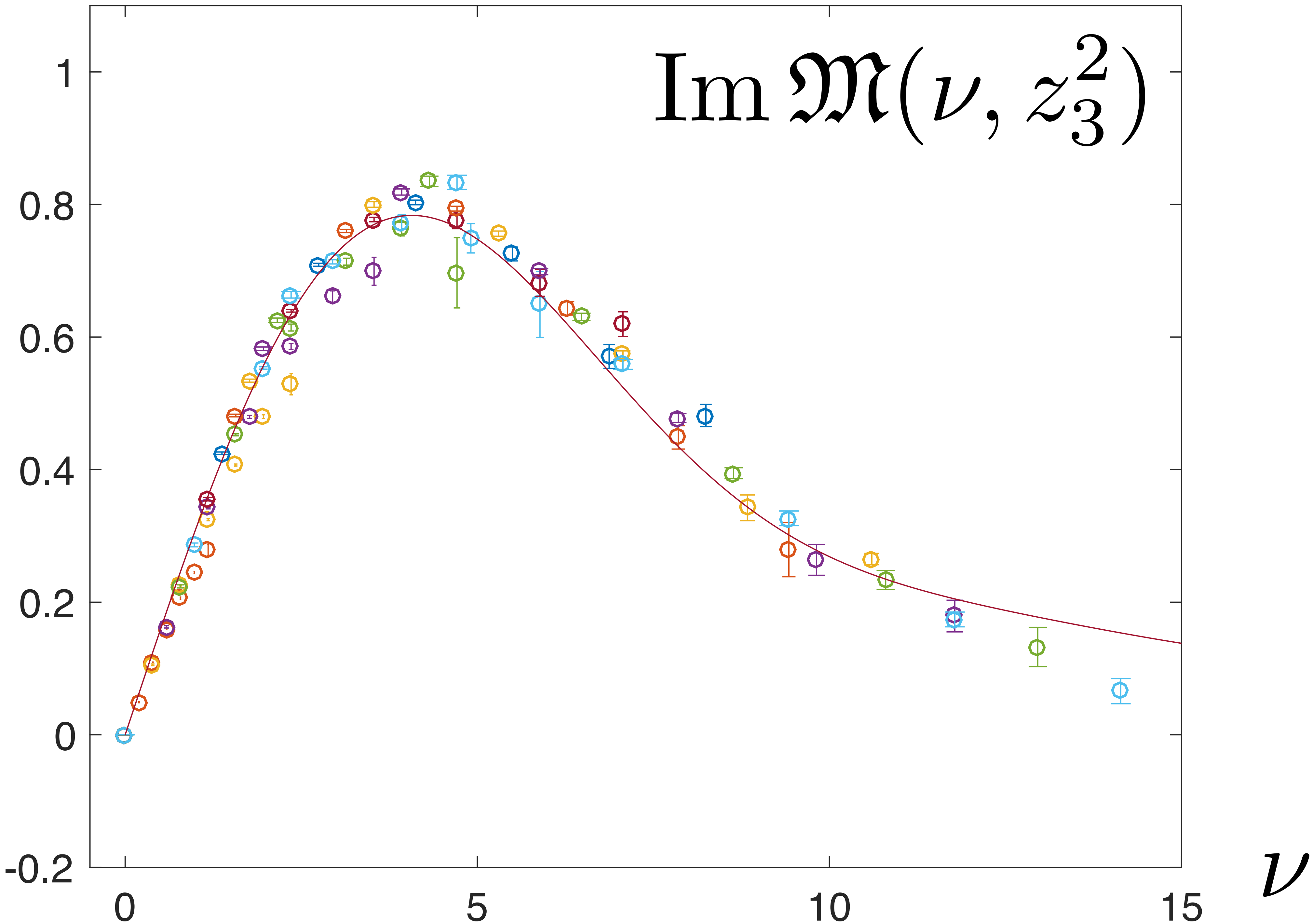
$$q_+(x) = q_v(x) + 2\bar{q}(x)$$



$$q_v(x) = \frac{315}{32}\sqrt{x}(1-x)^3 \qquad \bar{q}(x) = 0$$

anti-quarks contribute to the imaginary part

$$q_+(x) = q_v(x) + 2\bar{q}(x)$$



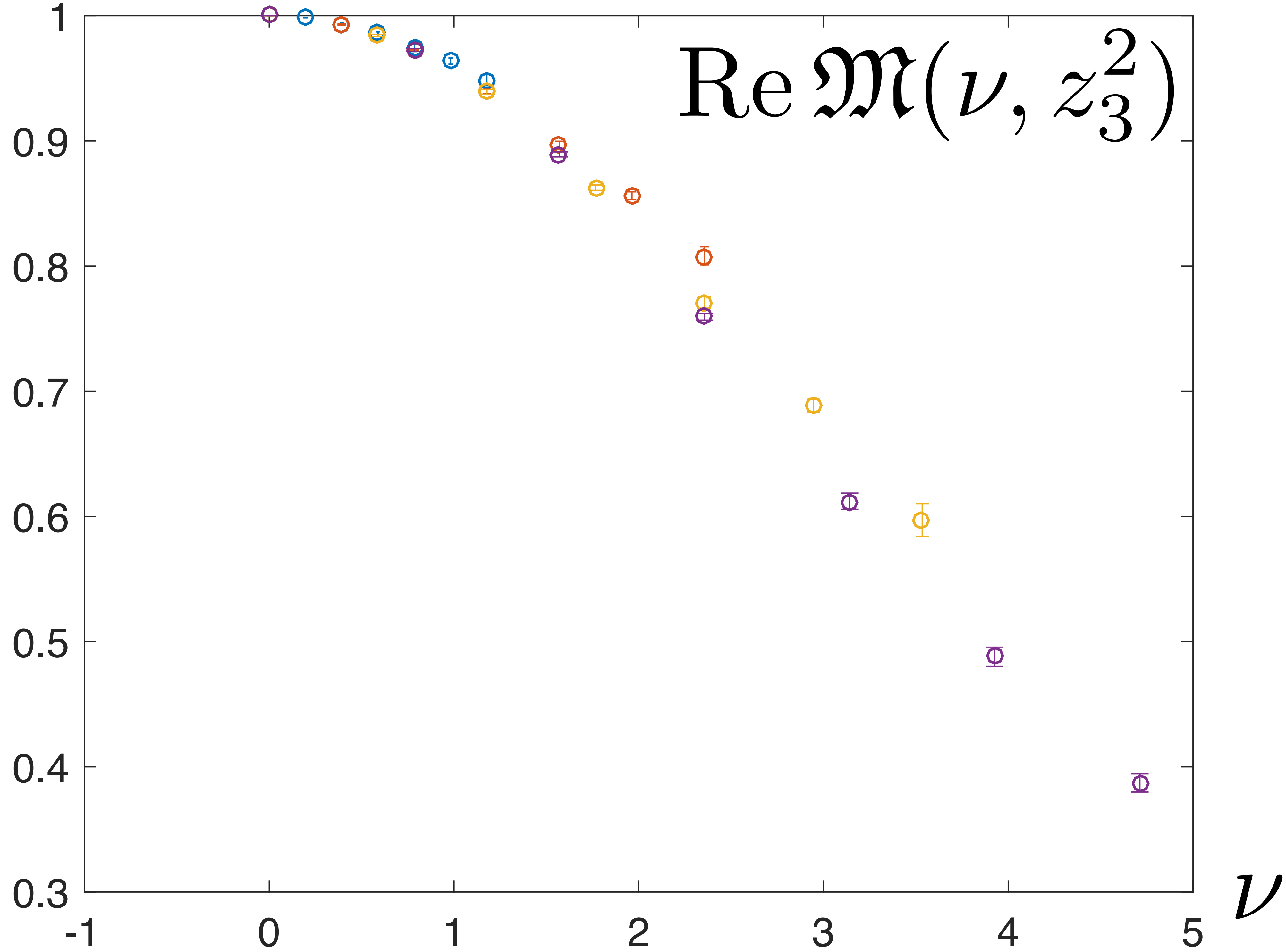
$$q_v(x) = \frac{315}{32}\sqrt{x}(1-x)^3 \qquad \bar{q}(x) = 1.4\,x\,(1-x)^3$$

Points in previous plots obtained in with different z/a
i.e. correspond to the Ioffe time PDF at different scales!

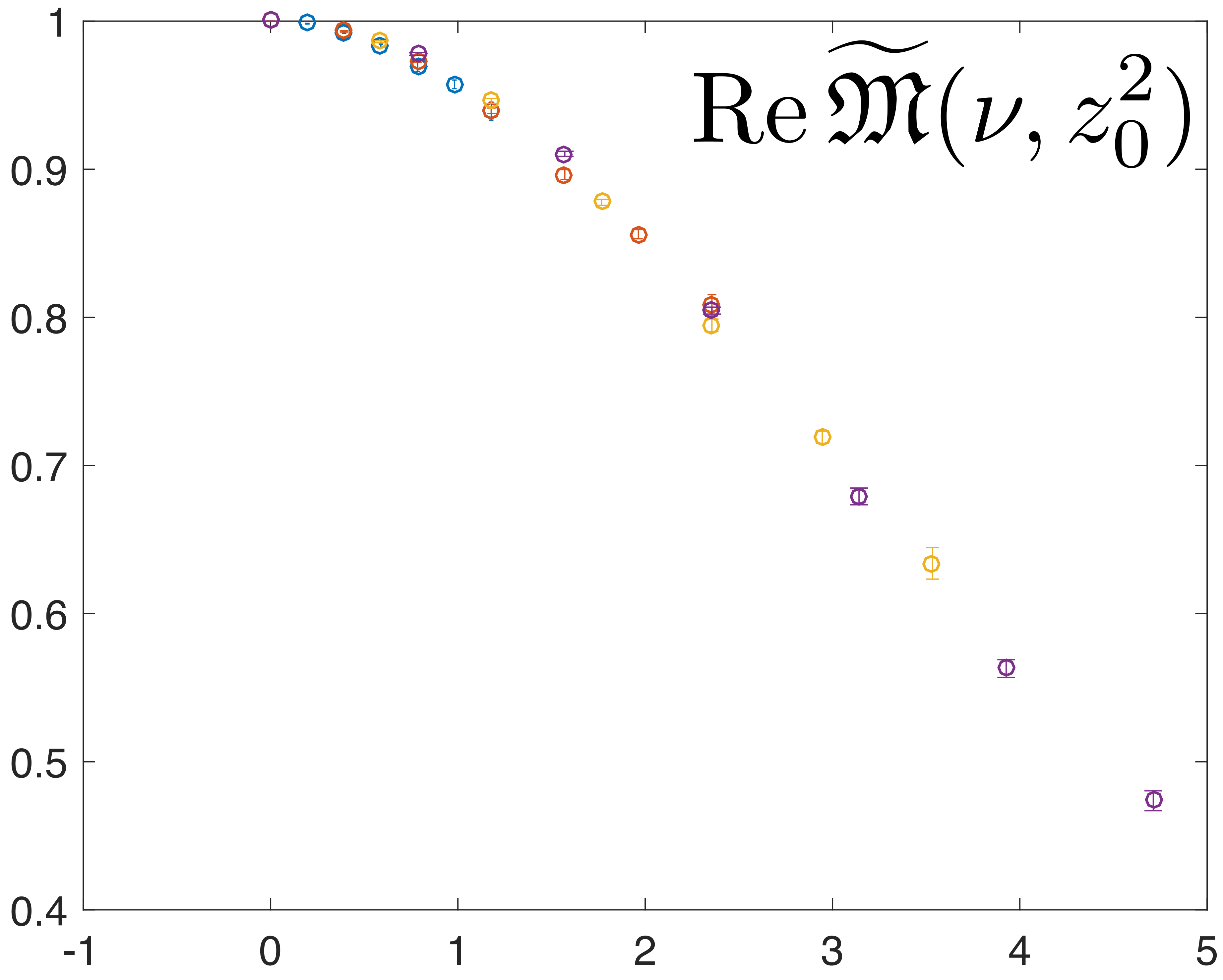
DGLAP evolution:

$$\mathcal{Q}(\nu, {z'}_3^2)=\mathcal{Q}(\nu, z_3^2) \; - \frac{2}{3}\frac{\alpha_s}{\pi} \ln({z_3'}^2/z_3^2) \int_0^1 du\, B(u)\, \mathcal{Q}(u\nu, z_3^2)$$

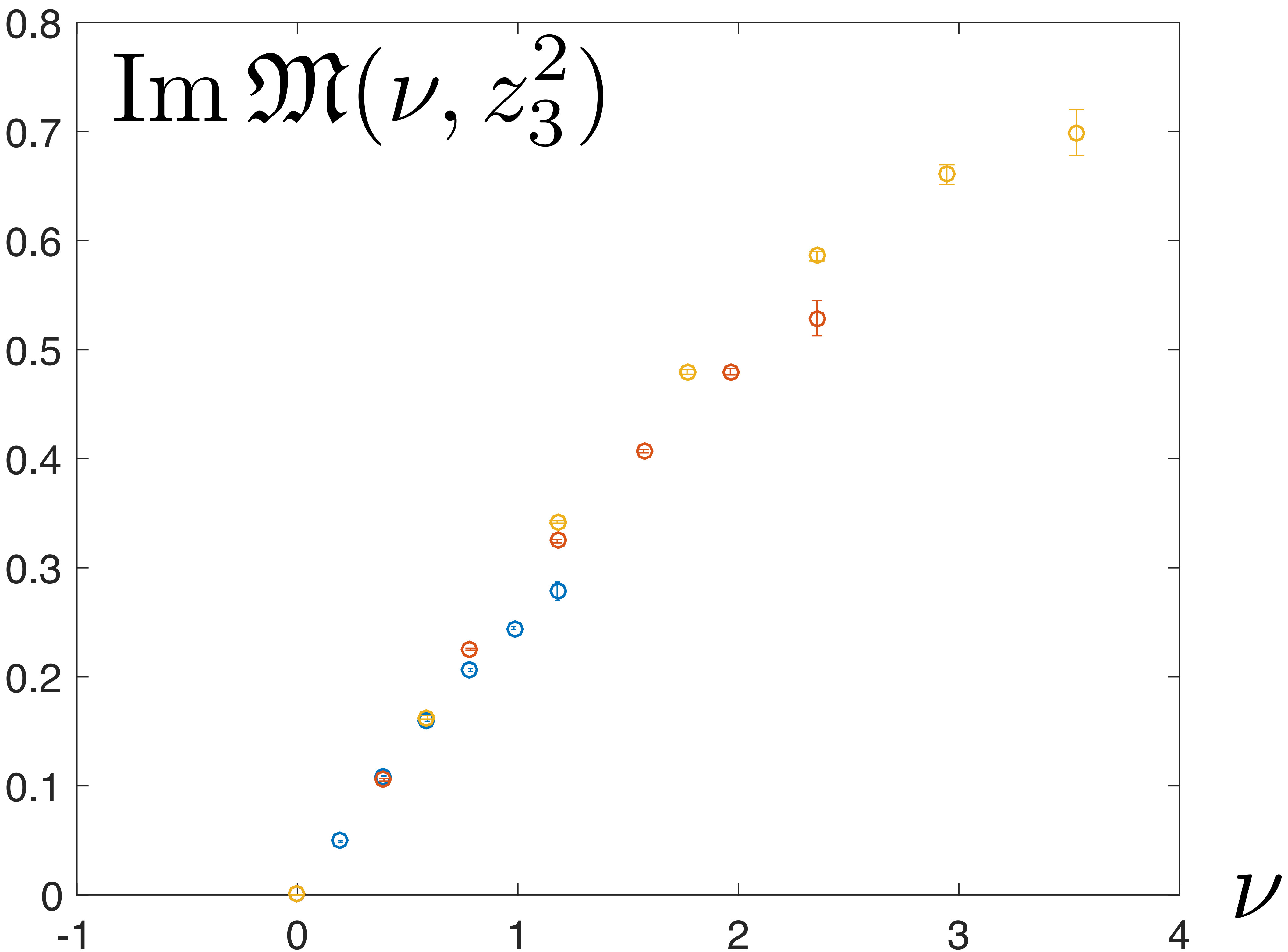
Apply evolution only at short distance points [~1GeV]



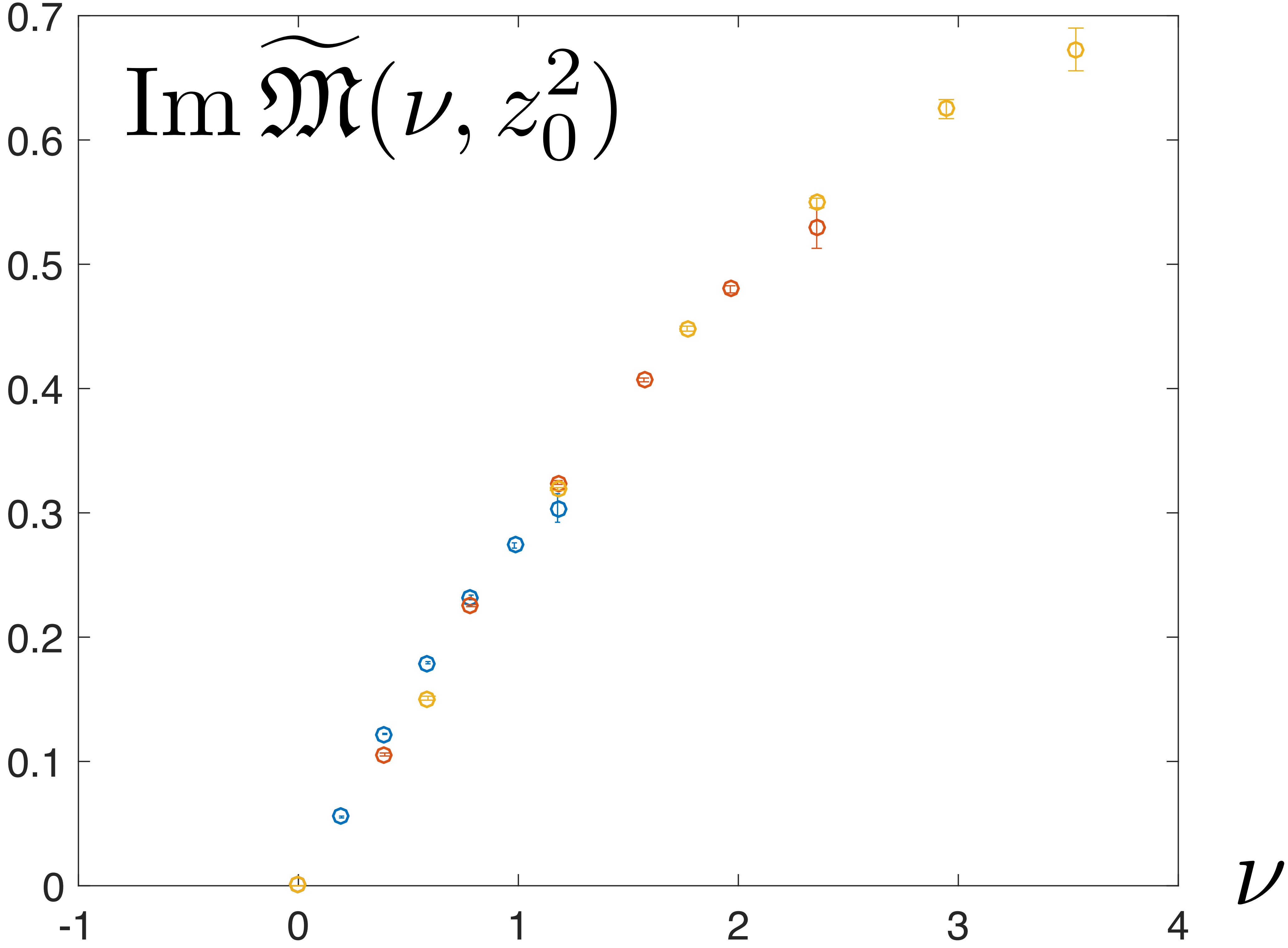
Data corresponding to z/a= 1, 2, 3, 4



Evolved to 1GeV

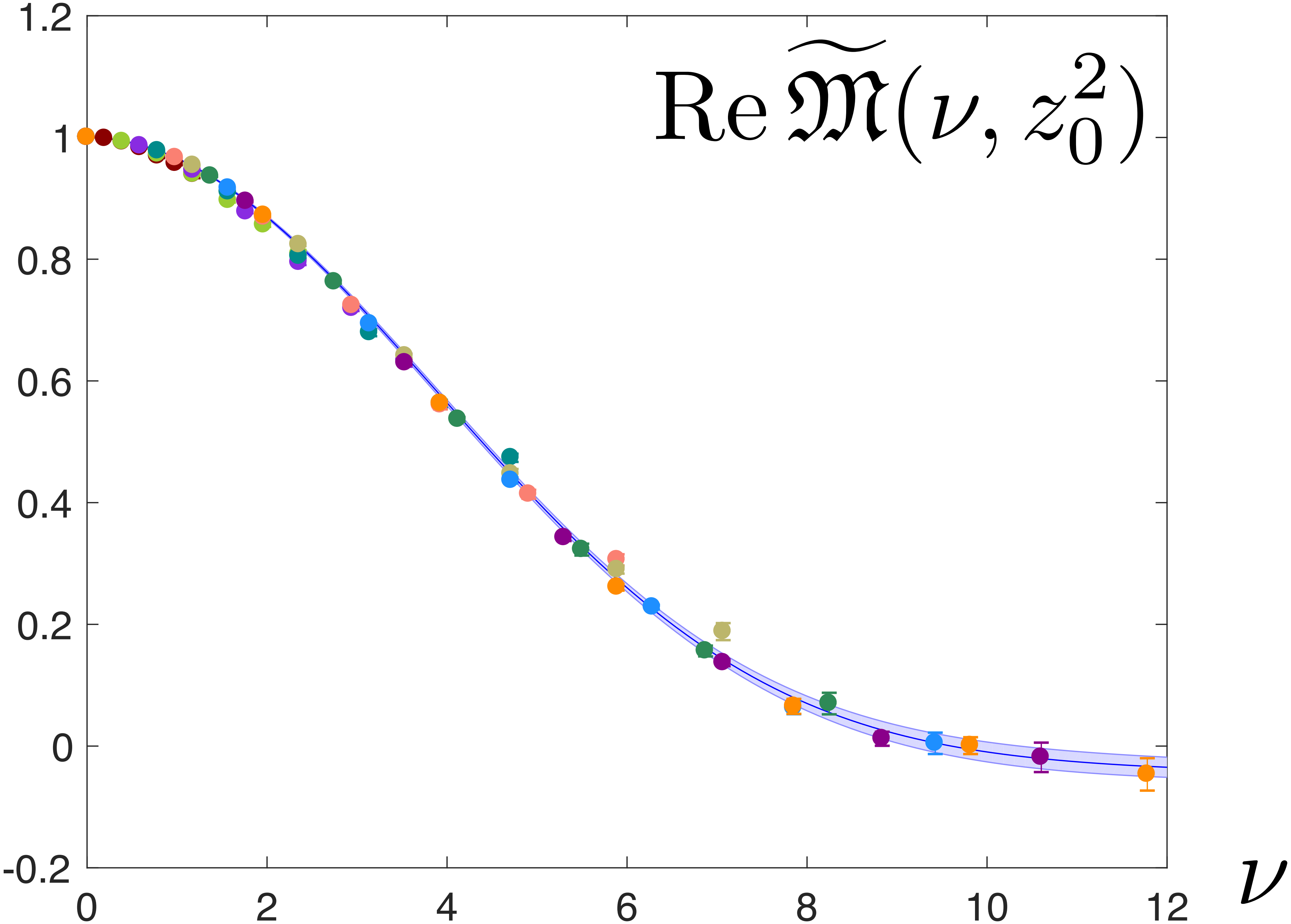


Data corresponding to z/a= 1, 2, 3, 4



Evolved to 1GeV

$$\mathfrak{M}_R(\nu, z^2 = 1/\mu^2) \equiv \int_0^1 dx\ \cos(\nu x)\, q_v(x, \mu^2)$$



Evolved to 1GeV

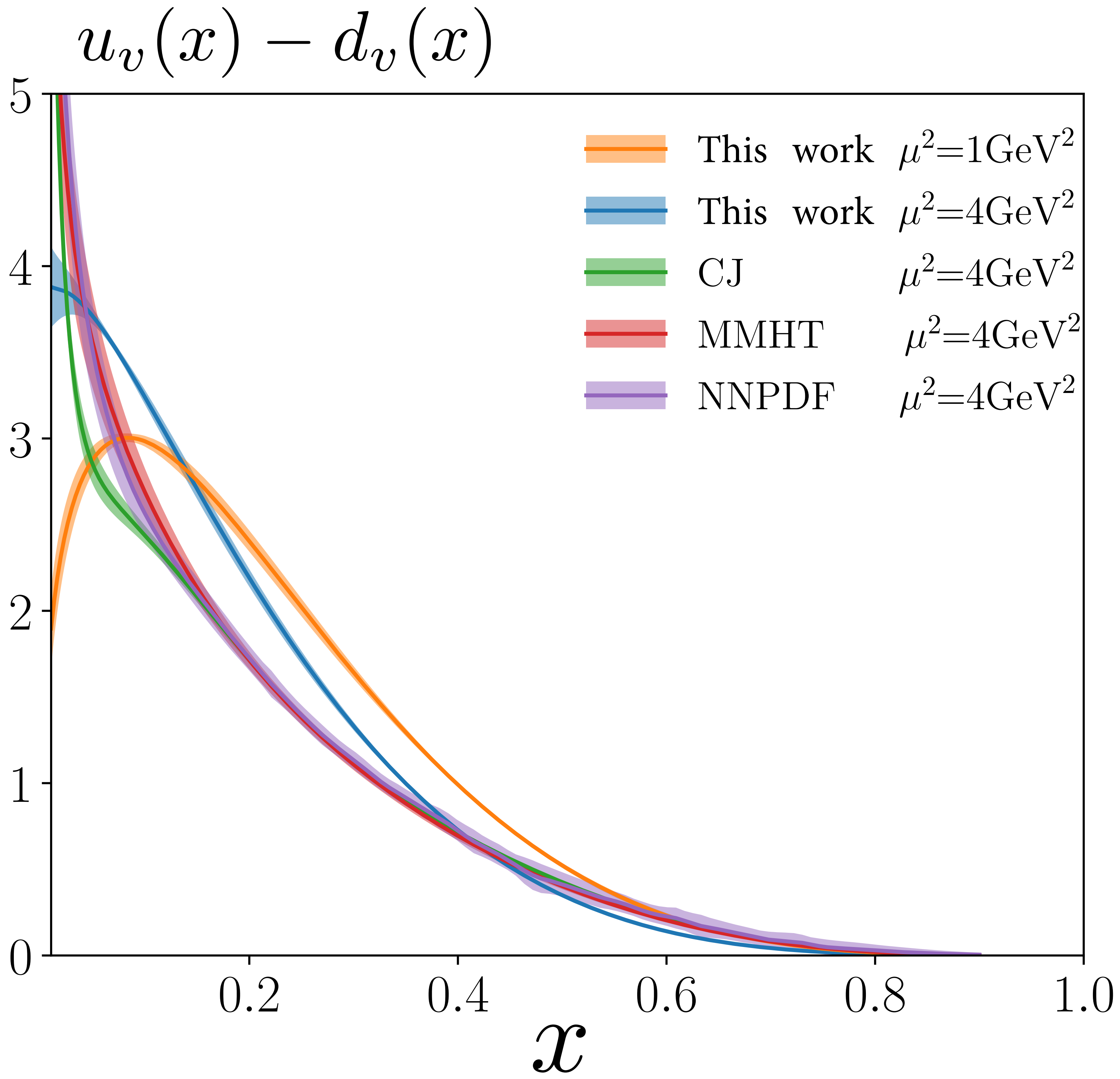
Fit to:

$$N(a,b)x^a(1-x)^b$$

$$a = 0.36(6)$$

$$b = 3.95(22)$$

Perturbative evolution only for z/a<6
above this scale evolution was assumed to be frozen



Thanks to N. Sato for making this figure

Summary

- Methods for obtaining parton distribution from Lattice QCD have now emerged
- An approach based on pseudo-PDFs has been proposed
  - Renormalization is handled in a simple way
  - Light cone limit is obtained by computing real space matrix elements at short Euclidean distances
  - All hadron momenta are useful in obtaining PDFs
- WM/JLab: first numerical tests are available in quenched approximation indicating the feasibility of the method
  - Results consistent with DGLAP evolution
- Dynamical fermion simulations are on the way
- Lattice spacing effect under study (quenched)
- Probing the small x region (or large Ioffe time) remains a challenge
  - Large Ioffe time may be probed with high momentum which requires a small lattice spacing (JLab anisotropic gauge ensembles?)
- Correctly applying evolution is essential for obtaining reliable results