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# Recent analysis of DY (and Z-boson) data 

Most recent results in collaboration with Alexey Vladimirov

## Outline \& Issues

* DY data: basic test of TMD factorization
* Theory status of unpolarized TMD's
* Scale prescriptions, convergence, models, theoretical errors,..
* The impact of LHC
* aufTCeMMilde


## ....TMD factorization ....

.. for DY and heavy boson production we have (Collins 2011, Echevarria, Idilbi, Scimemi (EIS) 2012 )

$$
\begin{aligned}
\frac{d \sigma}{d Q^{2} d q_{T} d y} & =\sum_{q} \sigma_{q}^{\gamma} H\left(Q^{2}, \mu^{2}\right) \int \frac{d^{2} \mathbf{b}}{4 \pi} e^{-i \mathbf{q}_{\mathbf{T}} \cdot \mathbf{b}} \Phi_{q / A}\left(x_{A}, \mathbf{b}, \zeta_{A}, \mu\right) \Phi_{q / B}\left(x_{B}, \mathbf{b}, \zeta_{B}, \mu\right) \\
\sqrt{\zeta_{A} \zeta_{B}} & =Q^{2}
\end{aligned}
$$

$\ldots$...and similar formulas are valid for SIDIS (EIC) and hadron production in ee colliders
The pathological behavior is associated to a particular kind of divergences: rapidity divergences
The renormalization of the rapidity divergences is responsible for the a new resummation scale We have new non-perturbative effects which cannot be included in PDFs.

TMD's factorization and Operator Product Expansion:

> general outlook

Factorized hadronic tensor

$$
q^{2}=Q^{2} \gg q_{T}^{2} \quad \mathrm{Q}=\mathrm{M}=\text { di-lepton invariant mass }
$$

Factorization
OPE

$\tilde{M}=H\left(Q^{2} / \mu^{2}\right) \tilde{F}_{n}\left(x_{n}, b ; Q^{2}, \mu^{2}\right) \tilde{F}_{\bar{n}}\left(x_{\bar{n}}, b ; Q^{2}, \mu^{2}\right)$

The factorization theorem predicts that each coefficient can be extracted on its own.
The evolution of TMD is universal (process independent)
Renomalons: power corrections are x-dependent

TMD's factorization and Operator Product Expansion:

## general outlook

Factorized hadronic tensor

$$
q^{2}=Q^{2} \gg q_{T}^{2} \quad \mathrm{Q}=\mathrm{M}=\text { di-lepton invariant mass }
$$

Factorization
OPE

$\tilde{M}=H\left(Q^{2} / \mu^{2}\right) \tilde{F}_{n}\left(x_{n}, b ; Q^{2}, \mu^{2}\right) \tilde{F}_{\bar{n}}\left(x_{\bar{n}}, b ; Q^{2}, \mu^{2}\right)$
important

TMDs are built joining a perturbative calculable part (in QCD) and a non-perturbative part (Models, Lattice): This separation is crucial for TMD extractions

## Status of unpolarized TMDs in perturbation theory



IT IS POSSIBLE TO MAKE A COMPLETE ANALYSIS OF UNPOLARIZED TMD IN DRELL-YAN AND SIDIS USING NNLO RESULTS

The study of polarized TMDs at the same precision is just started:
D. Gutierrez-Reyes, I.S., A. Vladimirov, arXiv:1702.06558

## Regions in $b$-space

The factorization theorem works in b-space.
The perturbative expansion does not work on the whole space...


EACH REGION NEEDS A PARTICULAR TREATMENT

## Regions in b-space

The factorization theorem works in b-space.
The perturbative expansion does not work on the whole space...


NOT ALL REGIONS ARE EOUALLY IMPORTANT FOR EACH EXPERMMENT

## Cross section and TMD structure



## Cross section and TMD structure



## Perturbative orders．．．

| Name | $\left\|C_{V}\right\|^{2}$ | $C_{f \leftarrow f^{\prime}}$ | $\Gamma$ | $\gamma_{V}$ | $\mathcal{D}$ | PDF set | $a_{s}$（run） | $\zeta_{\mu}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NLL／LO | $a_{s}^{0}$ | $a_{s}^{0}$ | $a_{s}^{2}$ | $a_{s}^{1}$ | $a_{s}^{2}$ | nlo | nlo | NLL |
| NLL／NLO | $a_{s}^{1}$ | $a_{s}^{1}$ | $a_{s}^{2}$ | $a_{s}^{1}$ | $a_{s}^{2}$ | nlo | nlo | NLO |
| NNLL／NLO | $a_{s}^{1}$ | $a_{s}^{1}$ | $a_{s}^{3}$ | $a_{s}^{2}$ | $a_{s}^{3}$ | nlo | nlo | NNLL |
| NNLL／NNLO | $a_{s}^{2}$ | $a_{s}^{2}$ | $a_{s}^{3}$ | $a_{s}^{2}$ | $a_{s}^{3}$ | nnlo | nnlo | NNLO | $\mathbb{N}$ NWWप【

．．．Theoretical uncertainties．．．

## MAルCVルNe SCAUES

In the implementation we must choose matching prescriptions such that the perturbative series is as convergent as possible，undesired power corrections are not introduced

$$
\begin{aligned}
& \text { Raplaty } \\
& \text { Evolution } \\
& \frac{d \sigma}{d Q^{2} d y d\left(q_{T}^{2}\right)}=\frac{4 \pi}{3 N_{c}} \frac{\mathcal{P}}{s Q^{2}} \sum_{G G^{\prime}} z_{l l^{\prime}}^{G G^{\prime}}(q) \sum_{f f^{\prime}} z_{f f^{\prime}}^{G G^{\prime}} \int \frac{d^{2} \vec{b}}{4 \pi} e^{i(\vec{b} \vec{q})}\left|C_{V}\left(Q, c_{2} Q\right)\right|^{2}\left\{R^{f}\left[\vec{b} ;\left(c_{2} Q, Q^{2}\right) \rightarrow\left(c_{3} \mu_{i}, \zeta_{c_{3} \mu_{i}}\right) ; c_{1} \mu_{i}\right]\right\} \\
& \times F_{f \leftarrow h_{1}}\left(x, \vec{b} ; c_{4} \mu_{\mathrm{OPE}}, \zeta_{c_{4} \mu_{\mathrm{OPE}}}\right) F_{f^{\prime} \leftarrow h_{2}}\left(x, \vec{b} ; c_{4} \mu_{\mathrm{OPE}}, \zeta_{c_{4} \mu_{\mathrm{OPE}}}\right) \\
& \text { Parameters and quality of the fits }
\end{aligned}
$$ depend strongly on the choices made for the implementation

## Details of scale variations

```
c
```

This uncertainty arises from the dependence (at the fixed perturbative order) on the initial evolution point and should be compensated between the Sudakov factor and the boundary term in the TMD evolution factor.

$$
c_{2} \sim \text { hard factorization scale }
$$

This uncertainty arises from the dependence (at the fixed perturbative order) on the hard factorization scale which is to be compensated between the hard coefficient function and the TMD evolution factor.
$c_{3} \sim$ TMD evolution tactor
This uncertainty arises from the dependence (at the fixed perturbative order) on initial scale of TMD evolution, which is to be compensated between the evolution integral and the mu-dependence of zeta_i.
$c_{4} \sim$ small- $b$ matching
This uncertainty arises from the dependence (at the fixed perturbative order) on the scale of the small-b matching mu_OPE which is to be compensated between the small-b Wilson coefficient function $\mathrm{C}_{-}\left\{\mathrm{f} / \mathrm{f}^{\prime}\right\}$ and the evolution of PDF.

## Details of scale variations

$c_{1} \sim$ perturbative matching of rapidity anomalous dimension


## TMD evolution and scale prescriptions

The perturbative expression for the evolution kernel work only up to a certain scale...

$$
R^{f}\left[\mathbf{b} ;\left(\mu_{f}, \zeta_{f}\right) \leftarrow\left(\mu_{i}, \zeta_{i}\right) ; \mu_{0}\right]=\exp \left[\int_{\mu_{i}}^{\mu_{f}} \frac{d \mu}{\mu} \gamma_{F}^{f}\left(\mu, \zeta_{f}\right)-\int_{\mu_{0}}^{\mu_{i}} \frac{d \mu}{\mu} \Gamma^{f}(\mu) \ln \left(\frac{\zeta_{f}}{\zeta_{i}}\right)\right]\left(\frac{\zeta_{f}}{\zeta_{i}}\right)^{-\mathcal{D}_{\text {perp }}^{f}\left(\mu_{0}, \mathbf{b}\right)-g_{K} \mathbf{b}^{2}} \ldots . .
$$

...and in principle we include some (renormalon Consistent) corrections

## What is the best prescription to choose scales?

b* prescription is not satisfactory (not fully inconsistent, but very confusing):

* It is not fully consistent with renormalon calculations (I.S., A. Vladimirov 2016)
* It introduces undesired power corrections (which alter model building):

Often parameters are due just to cancel induced power corrections

## $\zeta$-prescription



In practice we implement.. $\mu^{2} \frac{d F\left(x, \mathbf{b} ; \mu, \zeta_{\mu}\right)}{d \mu^{2}}=0$. ...and obtain iso-evolution curves..

$$
\begin{aligned}
\zeta_{\mu} & =\frac{2 \mu}{b} \exp \left(-\gamma_{E}+a_{s}\left[\frac{11 C_{A}-4 T_{F} N_{f}}{36} \mathbf{L}_{\mu}^{2}+C_{F}\left(-\frac{3}{4}+\pi^{2}-12 \zeta_{3}\right)+C_{A}\left(\frac{649}{108}-\frac{17 \pi^{2}}{12}+\frac{19}{2} \zeta_{3}\right)\right.\right. \\
& \left.\left.+T_{F} N_{f}\left(-\frac{53}{27}+\frac{\pi^{2}}{3}\right)\right]+\mathcal{O}\left(a_{s}^{2}\right)\right)
\end{aligned}
$$

## $\zeta$-prescription

In this prescription the structure of coefficient is much simpler

$$
C_{q \leftarrow q}\left(x, \mathbf{L}_{\mu} ; \mu, \zeta_{\mu}\right)=\delta(\bar{x})+a_{s}(\mu) C_{F}\left[-2 \mathbf{L}_{\mu}\left(\frac{2}{(1-x)_{+}}-1-x\right)+2 \bar{x}+\delta(\bar{x})\left(-3 \mathbf{L}_{\mu}-\frac{\pi^{2}}{6}\right)\right]+\cdots
$$

We do not introduce undesired power corrections
We have several proof of scale stability: TMD area, ..

$$
\int_{0}^{1} d x C_{q \leftarrow q}\left(x, \mathbf{L}_{\mu} ; \mu, \zeta_{\mu}\right)=1+a_{s}(\mu) C_{F}\left(1-\frac{\pi^{2}}{6}\right)+\cdots
$$

## Cancellation of logs

$$
\mu^{2} \frac{d}{d \mu^{2}} C_{f \leftarrow f^{\prime}}\left(x, \mathbf{b} ; \mu, \zeta_{\mu}\right) \otimes f_{f^{\prime} \leftarrow h}(x, \mu)=0
$$

We are left with the freedom to choose

$$
\mu=\mu_{b}=\frac{C_{0}}{b}+2 \mathrm{GeV}
$$

## DATA: Z-boson production....

|  | CDF run I | D0 run I |
| :---: | :---: | :---: |
| $\sqrt{s}$ | 1.8 TeV | 1.8 TeV |
| process | $p+\bar{p} \rightarrow Z \rightarrow e^{+} e^{-}$ | $p+\bar{p} \rightarrow Z \rightarrow e^{+} e^{-}$ |
| $M_{l l}$ range | $66-116 \mathrm{GeV}$ | $75-105 \mathrm{GeV}$ |
| y | y-integrated | y-integrated |
| Observable | $\frac{d \sigma}{d q_{T}}$ | $\frac{d \sigma}{d q_{T}}$ |
| Exp. $\sigma_{\text {tot }}[\mathrm{pb}]$ | $248 \pm 17$ | $\sigma=221 \pm 11$ |


|  | CDF run II | D0 run II |
| :---: | :---: | :---: |
| $\sqrt{s}$ | 1.96 TeV | 1.96 GeV |
| process | $p+\bar{p} \rightarrow Z \rightarrow e^{+} e^{-}$ | $p+\bar{p} \rightarrow Z \rightarrow e^{+} e^{-}$ |
| $M_{l l}$ range | $66-116 \mathrm{GeV}$ | $70-110 \mathrm{GeV}$ |
| y | y-integrated | y-integrated |
| Observable | $\frac{d \sigma}{d q_{T}}$ | $\frac{1}{\sigma} \frac{d \sigma}{d q_{T}}$ |
| Exp. $\sigma_{\text {tot }}[\mathrm{pb}]$ | $256 \pm 2.91$ | $\sigma=255$ |


|  | ATLAS | ATLAS |
| :---: | :---: | :---: |
| $\sqrt{s}$ | 7 TeV | 8 TeV |
| process | $p p \rightarrow Z \rightarrow e e+\mu \mu$ | $p p \rightarrow Z \rightarrow \mu \mu$ |
| $M_{l l}$ range | $66-116 \mathrm{GeV}$ | $66-116 \mathrm{GeV}$ |
| lepton cuts | $p_{T}>20 \mathrm{GeV}$ | $p_{T}>20 \mathrm{GeV}$ |
|  | $\|\eta\|<2.4$ | $\|\eta\|<2.4$ |
| $y$ | $-2.4<y<2.4$ | $-2.4<y<2.4$ |
| Observable | $\frac{1}{\sigma} \frac{d \sigma}{d q_{T}}$ | $\frac{1}{\sigma} \frac{d \sigma}{d q_{T}}$ |


|  | CMS | CMS |
| :---: | :---: | :---: |
| $\sqrt{s}$ | 7 TeV | 8 TeV |
| process | $p p \rightarrow Z \rightarrow e e+\mu \mu$ | $p p \rightarrow Z \rightarrow \mu \mu$ |
| $M_{l l}$ range | $60-120 \mathrm{GeV}$ | $60-120 \mathrm{GeV}$ |
| lepton cuts | $p_{T}>20 \mathrm{GeV}$ | $p_{T}>15 \mathrm{GeV}$ |
|  | $\|\eta\|<2.1$ | $\|\eta\|<2.1$ |
| y | $\|y\|<2.1$ | $\|y\|<2.1$ |
| Observable | $\frac{1}{\sigma} \frac{d \sigma}{d q_{T}}$ | $\frac{1}{\sigma} \frac{d \sigma}{d q_{T}}$ |


|  | LHCb | LHCb | LHCb |
| :---: | :---: | :---: | :---: |
|  | 7 TeV | 8 TeV | 13 TeV |
|  | $p p \rightarrow Z \rightarrow \mu \mu$ | $p p \rightarrow Z \rightarrow \mu \mu$ | $p p \rightarrow Z \rightarrow \mu \mu$ |
| $M_{l l}$ range | $60-120 \mathrm{GeV}$ | $60-120 \mathrm{GeV}$ | $60-120 \mathrm{GeV}$ |
| lepton cuts | $p_{T}>20 \mathrm{GeV}$ | $p_{T}>20 \mathrm{GeV}$ | $p_{T}>20 \mathrm{GeV}$ |
|  | $2<\eta<4.5$ | $2<\eta<4.5$ | $2<\eta<4.5$ |
| y | $2<y<4.5$ | $2<y<4.5$ | $2<y<4.5$ |
| Observable | $d \sigma\left(q_{T}\right)$ | $d \sigma\left(q_{T}\right)$ | $\frac{d \sigma}{d q_{T}}$ |
| Norm. exp. | $\sigma=76.0 \pm 3.1 \mathrm{pb}$ | $\sigma=95.0 \pm 3.2 \mathrm{pb}$ | $\sigma=198.0 \pm 13.3 \mathrm{pb}$ |

## DATA: and Drell-Yan....

|  | E288 200 | E288 300 | E288 400 |
| :---: | :---: | :---: | :---: |
| $\sqrt{s}$ | 19.4 GeV | 23.8 GeV | 27.4 GeV |
| process | $\mathrm{p}+\mathrm{Cu} \rightarrow \gamma \rightarrow \mu^{+} \mu^{-}$ | $\mathrm{p}+\mathrm{Cu} \rightarrow \gamma \rightarrow \mu^{+} \mu^{-}$ | $\mathrm{p}+\mathrm{Cu} \rightarrow \gamma \rightarrow \mu^{+} \mu^{-}$ |
| $Q$ range | $4-9 \mathrm{GeV}$ | $4-9 \mathrm{GeV}$ | $5-14 \mathrm{GeV}$ |
| $\Delta Q$-bin | 1 GeV | 1 GeV | 1 GeV |
| y | $\mathrm{y}=0.4$ | $\mathrm{y}=0.21$ | $\mathrm{y}=0.03$ |
| Observable | $E \frac{d^{3} \sigma}{d^{3} q}$ | $E \frac{d^{3} \sigma}{d^{3} q}$ | $E \frac{d^{3} \sigma}{d^{3} q}$ |


|  | ATLAS | ATLAS |
| :---: | :---: | :---: |
| $\sqrt{s}$ | 8 TeV | 8 TeV |
| process | $p p \rightarrow Z / \gamma^{*} \rightarrow \mu \mu$ | $p p \rightarrow Z / \gamma^{*} \rightarrow \mu \mu$ |
| $M_{l l}$ range | $46-66 \mathrm{GeV}$ | $116-150 \mathrm{GeV}$ |
| lepton cuts | $p_{T}>20 \mathrm{GeV}$ | $p_{T}>20 \mathrm{GeV}$ |
|  | $\|\eta\|<2.4$ | $\|\eta\|<2.4$ |
| $y$ | $-2.4<y<2.4$ | $-2.4<y<2.4$ |
| Observable | $\frac{1}{\sigma} \frac{d \sigma}{d q_{T}}$ | $\frac{1}{\sigma} \frac{d \sigma}{d q_{T}}$ |

## Lepton cuts...

Lepton cuts have implemented numerically for LHC.
However all experiments suffer from lepton cuts: they should always be reported!!

## Normalization of the cross sections

Not all experiments provide a value for total cross sections:

- $\mathrm{N}_{\text {E288 }}=0.8$ fixed
- For CDF, D0 we use DYNNLO
- for LHC we normalize areas of partially integrated cross sections. N=th/exp General agreement within errors with published results

| order | ATLAS <br> Z-boson <br> 7 TeV | ATLAS <br> Z-boson <br> 8 TeV | $\begin{gathered} \text { ATLAS } \\ 46-66 \\ 8 \mathrm{TeV} \\ \hline \end{gathered}$ | $\begin{gathered} \text { ATLAS } \\ 116-150 \\ 8 \mathrm{TeV} \\ \hline \end{gathered}$ | $\begin{aligned} & \mathrm{CMS} \\ & 7 \mathrm{TeV} \end{aligned}$ | $\begin{aligned} & \mathrm{CMS} \\ & 8 \mathrm{TeV} \end{aligned}$ | $\begin{gathered} \mathrm{LHCb} \\ 7 \mathrm{TeV} \end{gathered}$ | $\begin{gathered} \mathrm{LHCb} \\ 8 \mathrm{TeV} \end{gathered}$ | $\begin{aligned} & \mathrm{LHCb} \\ & 13 \mathrm{TeV} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NLL/NLO | 438 pb | 0.92 | 1.01 | 0.93 | 369 pb | 407 pb | 0.92 | 0.93 | 0.93 |
| NNLL/NLO | 438 pb | 0.92 | 1.01 | 0.93 | 368 pb | 407 pb | 0.92 | 0.93 | 0.93 |
| NNLL/NNLO | 461 pb | 0.97 | 1.08 | 0.98 | 387 pb | 429 pb | 0.97 | 0.99 | 0.98 |

## Models, data, stability

Data are sensitive to models for non-perturbative part of TMDs.
We explore models with

- Minimal set of parameters
- renormalon consistency
- Independent on number of data points (Stability)
- We do not include Y-terms: we should select qT/Q proper interval

To be checked on data!!


Theory prediction: very small or zero $g_{K}=0.01 \pm 0.03 \mathrm{GeV}^{2}$
I.S., A. Vladimirov: arXiv:1609.06147

$$
F_{q \leftarrow h}(x, \boldsymbol{b} ; \mu, \zeta)=\int_{x}^{1} \frac{d z}{z} \sum_{f} C_{q \leftarrow f}(z, \boldsymbol{b} ; \mu, \zeta) f_{f \leftarrow h}\left(\frac{x}{z}, \mu\right) \stackrel{\circ}{\circ} \cdot \stackrel{f_{N P}}{ }(z, \boldsymbol{b})
$$

Non-perturbative corrections to TMD-PDF matching

$$
f_{N P}=1\left|f_{N P}=e^{-\lambda_{1} b^{2}}\right| f_{N P}=e^{-\lambda_{1} b}\left|f_{N P}=1, g_{K} \neq 0\right|: ?
$$

## NNLO

| data $/ f_{N P}$ | $e^{-\lambda b}$ | $e^{-\lambda b^{2}}$ | $\cosh ^{-1}(\lambda b)$ |
| :---: | :---: | :---: | :---: |
| ATLAS | 4.78 | 1.43 | 1.42 |
| E288 | 2.70 | 5.68 | 3.64 |
| E288+ATLAS | 8.18 | 5.77 | 3.72 |

- High energy data favors Gaussian (also theoretically) - Low energy data favors Exponential (also theoretically) The difference between model is not much at NLO, but it is important at NNLO
* We need at least 2 parameters (+gK)


$$
\begin{aligned}
\frac{\chi^{2}}{\text { d.o.f. }}(\text { Total }) & \simeq 1.2 \\
\frac{\chi^{2}}{\text { d.o.f. }}(\text { High E. data }) & \simeq 1.0 \\
\frac{\chi^{2}}{\text { d.o.f. }}(\text { Low E. data }) & \simeq 1.4
\end{aligned}
$$

MODEL $1 \quad f_{N P}(b)=\frac{\cosh \left(\left(\frac{\lambda_{2}}{\lambda_{1}}-\frac{\lambda_{1}}{2}\right) b\right)}{\cosh \left(\left(\frac{\lambda_{2}}{\lambda_{1}}+\frac{\lambda_{1}}{2}\right) b\right)}$
MODEL $2 \quad f_{N P}(z, \boldsymbol{b})=\exp \left(\frac{-\lambda_{q} z \boldsymbol{b}^{2}}{\sqrt{1+z^{2} \boldsymbol{b}^{2} \frac{\lambda_{q}^{2}}{\lambda_{1}^{2}}}}\right)+$ ren. NEW (renormalon consistent) ansatz!


## auf Tre:Mrildie



## Stability

- Different models show different stability of chi^2: Check on high energy data
- For $\delta_{T}=q_{T} / Q \lesssim 0.2$ power corrections (Y-terms) are not needed



## Fitted constants



| Variation | $\frac{\chi^{2}}{\text { d.o.f. }}$ | $\lambda_{1}$ | $\lambda_{2}$ | $g_{K} \times 10^{-2}$ |
| :---: | :---: | :---: | :---: | :---: |
| Model 1 NNLL/NLO |  |  |  |  |
| $c_{1,2,3,4}=1$ | 1.17 | 0.189 | 0.425 | 2.31 |
| $c_{1}=2$ | 1.31 (+0.14) | $0.201(+0.012)$ | 0.316 (-0.109) | 3.00 (+0.69) |
| $c_{1}=0.5$ | 1.10 (-0.07) | $0.184(-0.005)$ | 0.308 (-0.117) | 1.60 (-0.71) |
| $c_{2}=2$ | 1.19 (+0.02) | $0.204(+0.015)$ | 0.223 (-0.202) | 2.12 (-0.19) |
| $c_{2}=0.5$ | 1.20 (+0.03) | 0.219 (+0.030) | 0.226 (-0.199) | 1.93 (-0.38) |
| $c_{3}=2$ | 1.23 (+0.06) | $0.251(+0.062)$ | 0.315 (-0.110) | 3.75 (+1.44) |
| $c_{3}=0.5$ | 1.13 (-0.04) | 0.160 (-0.029) | 0.220 (-0.205) | 1.12 (-1.19) |
| $c_{4}=2$ | 1.76 (+0.59) | 0.137 (-0.052) | 0.473 (+0.046) | 2.71 (+0.40) |
| $c_{4}=0.5$ | 2.49 (+1.32) | 0.303 (+0.114) | 0.175 (-0.250) | 1.15 (-1.16) |
| Result | $1.17{ }_{-0.07}^{+1.32}$ | $0.189_{-0.052}^{+0.114}$ | $0.425_{-0.250}^{+0.047}$ | $2.311_{-1.19}^{+1.44}$ |
| Model $1 \mathrm{~N}^{3} \mathrm{LL} / \mathrm{NNLO}$ |  |  |  |  |
| $c_{1,2,3,4}=1$ | 1.23 | 0.228 | 0.306 | 0.73 |
| $c_{1}=2$ | 1.40 (+0.17) | 0.242 (+0.014) | 0.296 (-0.010) | 1.21 (+0.48) |
| $c_{1}=0.5$ | $1.14(-0.09)$ | 0.221 (-0.007) | 0.346 (+0.020) | 0.12 (-0.61) |
| $c_{2}=2$ | 1.22 (-0.01) | 0.217 (-0.011) | 0.295 (-0.011) | 0.86 (+0.13) |
| $c_{2}=0.5$ | 1.26 (+0.03) | $0.252(+\mathbf{0 . 0 2 4 )}$ | 0.326 (+0.020) | 0.48 (-0.25) |
| $c_{3}=2$ | 1.27 (+0.04) | 0.260 (+0.032) | 0.344 (+0.038) | 1.82 (+1.09) |
| $c_{3}=0.5$ | $1.31(+0.08)$ | 0.198 (-0.030) | 0.358 (+0.052) | 0.00 (-0.73) |
| $c_{4}=2$ | 1.10 (-0.13) | 0.168 (-0.060) | 0.571 (+0.265) | 1.27 (+0.54) |
| $c_{4}=0.5$ | 1.53 (+0.30) | $0.262(+0.034)$ | 0.243 (-0.063) | 0.68 (-0.05) |
| Result | $1.23{ }_{-0.13}^{+0.30}$ | $0.228_{-0.060}^{+0.034}$ | $0.306_{-0.063}^{+0.265}$ | $0.73_{-0.73}^{+1.09}$ |

- Not much difference between models
- gK consistent with renormalons
- Renormalons effects small
- Error on fitted constants converges


## aur ${ }^{\text {The Mildide }: ~}$

Results for LHC in Z-production ....



## ...and Drell-Yan at NNLO



## Errors and orders: E288



## Errors and orders: LHCb



## Conclusions

A NNLO ANALYSIS IS NECESSARY FOR FITTING DATA AND EXTRACTING TMD (MANY ISSUES SOLVED JUST INCREASING THE PERTURBATIVE ORDER).

* LHC PROVIDES VERY PRECISE DATA THAT SHOULD BE INCLUDED IN FITS (ESPECIALLY DATA OFF THE Z-BOSON PEAK). ATLAS AND CMS COULD DO BETTER AT 13 TEV!!
WE HAVE DISCUSSED A NUMBER OF ISSUES WHICH ARE RELEVANT IN TMD ANALYSIS (DATA CHOICE, NORMALIZATIONS, PRESCRIPTIONS, SCALE CHOICES, STABILITY, THEORETICAL ERRORS,..ETC.)
All This is included in aule $]$ (e Miilde (VERSION 1.1)


Back up

## Results of the fit

| Data set | point | Model 1 |  |  | Model 2 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\begin{aligned} & \text { NLL/ } \\ & \text { NLO } \end{aligned}$ | $\begin{aligned} & \text { NNLL/ } \\ & \text { NLO } \end{aligned}$ | $\begin{aligned} & \text { NNLL// } \\ & \text { NNLO } \end{aligned}$ | $\begin{aligned} & \text { NLL/ } \\ & \text { NLO } \end{aligned}$ | $\begin{aligned} & \text { NNLL/ } \\ & \text { NLO } \end{aligned}$ | $\begin{aligned} & \text { NNLL/ } \\ & \text { NNLO } \end{aligned}$ |
| CDF run1 | 30 | 0.67 | 0.68 | 0.64 | 0.67 | 0.67 | 0.64 |
| D0 run1 | 14 | 0.50 | 0.52 | 0.60 | 0.49 | 0.51 | 0.62 |
| CDF run2 | 36 | 1.22 | 1.36 | 1.30 | 1.17 | 1.29 | 1.33 |
| D0 run2 | 7 | 2.52 | 2.69 | 2.75 | 2.45 | 2.64 | 2.79 |
| ATLAS (7TeV) Z-boson | 9 | 1.54 | 1.55 | 2.01 | 1.60 | 1.59 | 2.27 |
| ATLAS (8TeV) Z-boson | 9 | 2.32 | 2.48 | 2.69 | 2.46 | 2.70 | 2.79 |
| ATLAS (8TeV) $46-66 \mathrm{GeV}$ | 5 | 0.04 | 0.05 | 0.16 | 0.05 | 0.04 | 0.20 |
| ATLAS (8TeV) 116-150 GeV | 9 | 0.30 | 0.35 | 0.31 | 0.30 | 0.36 | 0.30 |
| CMS ( 7 TeV ) | 7 | 1.38 | 1.39 | 1.36 | 1.38 | 1.38 | 1.36 |
| CMS (8 TeV) | 7 | 1.38 | 1.38 | 1.54 | 1.38 | 1.37 | 1.58 |
| LHCb ( 7 TeV ) | 10 | 0.26 | 0.26 | 0.31 | 0.25 | 0.26 | 0.33 |
| LHCb (8 TeV) | 10 | 0.11 | 0.12 | 0.27 | 0.11 | 0.12 | 0.32 |
| LHCb (13 TeV) | 10 | 0.50 | 0.50 | 0.28 | 0.50 | 0.50 | 0.27 |
| High energy data | 163 | 0.95 | 1.00 | 0.94 | 0.94 | 1.00 | 1.04 |
| E288(200) 4-5 GeV | 5 | 3.86 | 4.28 | 3.86 | 4.25 | 4.59 | 4.30 |
| E288(200) 5-6 GeV | 6 | 3.00 | 3.03 | 1.92 | 3.05 | 3.07 | 1.92 |
| E288(200) 6-7 GeV | 7 | 1.68 | 1.68 | 0.84 | 1.66 | 1.67 | 0.79 |
| E288(200) $7-8 \mathrm{GeV}$ | 8 | 1.10 | 1.10 | 0.93 | 1.13 | 1.11 | 1.00 |
| E288(200) 8-9 GeV | 9 | 1.83 | 1.84 | 0.78 | 1.89 | 1.87 | 1.87 |
| E288(300) 4-5 GeV | 5 | 1.93 | 2.20 | 4.09 | 2.24 | 2.44 | 4.90 |
| E288(300) 5-6 GeV | 6 | 1.15 | 1.18 | 1.15 | 1.19 | 1.21 | 1.21 |
| E288(300) 6-7 GeV | 7 | 0.84 | 0.83 | 0.66 | 0.85 | 0.83 | 0.69 |
| E288(300) $7-8 \mathrm{GeV}$ | 8 | 1.18 | 1.17 | 0.90 | 1.16 | 1.17 | 0.86 |
| E288(300) 8-9 GeV | 9 | 1.13 | 1.14 | 1.13 | 1.11 | 1.36 | 1.10 |
| E288(300) 11-12 GeV | 12 | 1.08 | 1.08 | 1.00 | 1.11 | 1.10 | 1.04 |
| E288(400) 5-6 GeV | 6 | 2.11 | 2.04 | 1.12 | 1.94 | 1.92 | 1.01 |
| E288(400) 6-7 GeV | 7 | 2.59 | 2.68 | 2.55 | 2.59 | 2.64 | 2.55 |
| E288(400) $7-8 \mathrm{GeV}$ | 8 | 0.83 | 0.97 | 2.02 | 0.99 | 1.07 | 2.44 |
| E288(400) 8-9 GeV | 9 | 1.36 | 1.31 | 1.37 | 1.37 | 1.32 | 1.54 |
| E288(400) 11-12 GeV | 12 | 1.08 | 1.06 | 1.25 | 1.05 | 1.05 | 1.17 |
| E288(400) 12-13 GeV | 12 | 0.88 | 0.88 | 1.10 | 0.87 | 0.88 | 1.14 |
| E288(400) 13-14 GeV | 12 | 0.39 | 0.38 | 0.72 | 0.39 | 0.39 | 0.71 |
| Low energy data | 146 | 1.38 | 1.41 | 1.35 | 1.50 | 1.48 | 1.49 |
| Total | 309 | 1.17 | 1.21 | 1.23 | 1.18 | 1.22 | 1.29 |

## Tevatron Z-boson plots



