TMD Factorization Theory

Ted Rogers

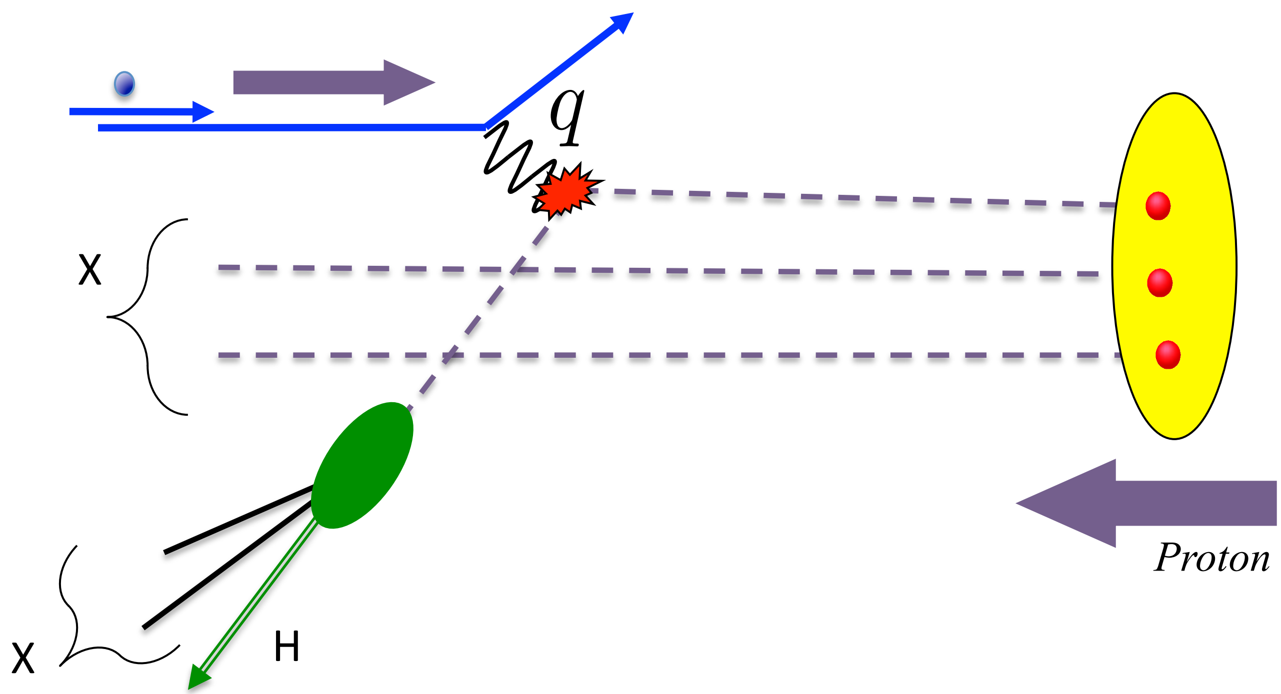
Jefferson Lab/Old Dominion University

Transversity 2017, December 11, Frascati

Outline

- Overview
- Kinematical cartography of a process
- Hadronization and fragmentation
- Evolution and perturbation theory

Example



Collinear Semi-Inclusive DIS

Correlation Function Taxonomy

Quark \ Proton	Unpolarized	Longitudinally polarized	Transversely polarized
Unpolarized	$f(x)$	✖	✖
Longitudinally polarized	✖	$g_{1L}(x)$	✖
Transversely polarized	✖	✖	$h_{1T}(x)$

Intrinsic Transverse Momentum

PHYSICAL REVIEW D VOLUME 2, NUMBER 9 1 NOVEMBER 1970

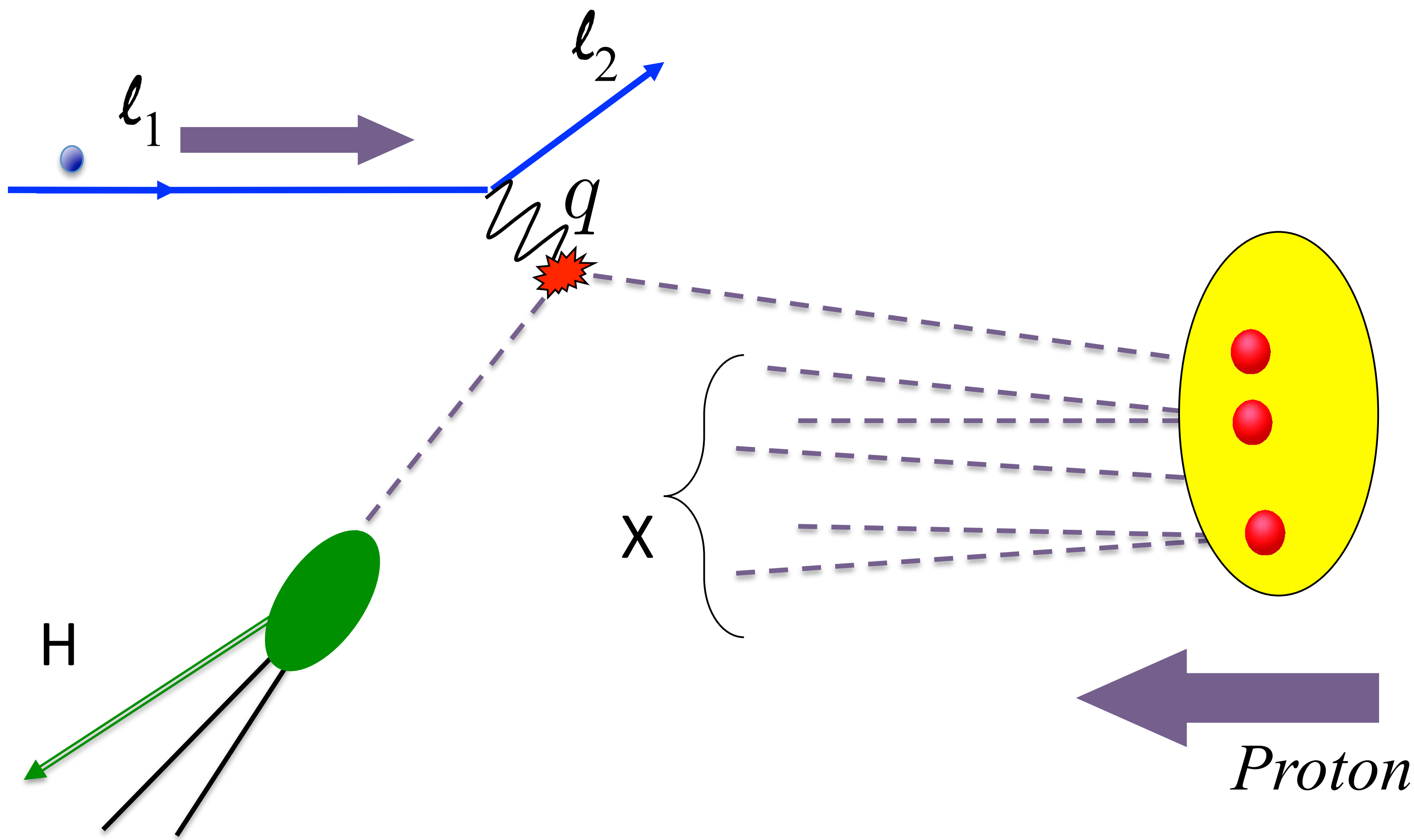
Effect of a Transverse Momentum Distribution in the Parton Model*

C. W. GARDINER AND D. P. MAJUMDAR
Physics Department, Syracuse University, Syracuse, New York 13210

(Received 24 June 1970)

"The parton model for the inelastic lepton-nucleon scattering is generalized to include a realistic momentum distribution of the partons. In this formalism each parton is given a component of momentum… to take into account the effect of this orthogonal (transverse) momentum distribution of the partons."

Example



Correlation Function Taxonomy

Quark \ Proton	Unpolarized	Longitudinally polarized	Transversely polarized
Unpolarized	$f(x)$	✗	✗
Longitudinally polarized	✗	$g_{1L}(x)$	✗
Transversely polarized	✗	✗	$h_{1T}(x)$

TMD Taxonomy

Quark \ Proton	Unpolarized	Longitudinally polarized	Transversely polarized
Unpolarized	$f_1(x, k_T)$	✗	$f_{1T}^{\perp}(x, k_T)$
Longitudinally polarized	✗	$g_{1L}(x, k_T)$	$g_{1T}(x, k_T)$
Transversely polarized	$h_1^{\perp}(x, k_T)$	$h_{1L}(x, k_T)$	$h_{1T}(x, k_T)$ $h_{1T}^{\perp}(x, k_T)$

Sivers

Zero in naive parton model by TP symmetry

Boer-Mulders

"Worm Gear"

"Pretzelocity"

Why Study Transverse Momentum

- Intrinsic Transverse momentum
  - Hadron bound state properties in terms of quark and gluon properties

- Very high energies
  - Multiple large but widely separated scales

TMD PDFs and Collinear PDFs

- Similarities
  - Correlation functions with universal (and np calculable) properties
  - Perturbatively calculable hard parts
  - Evolution

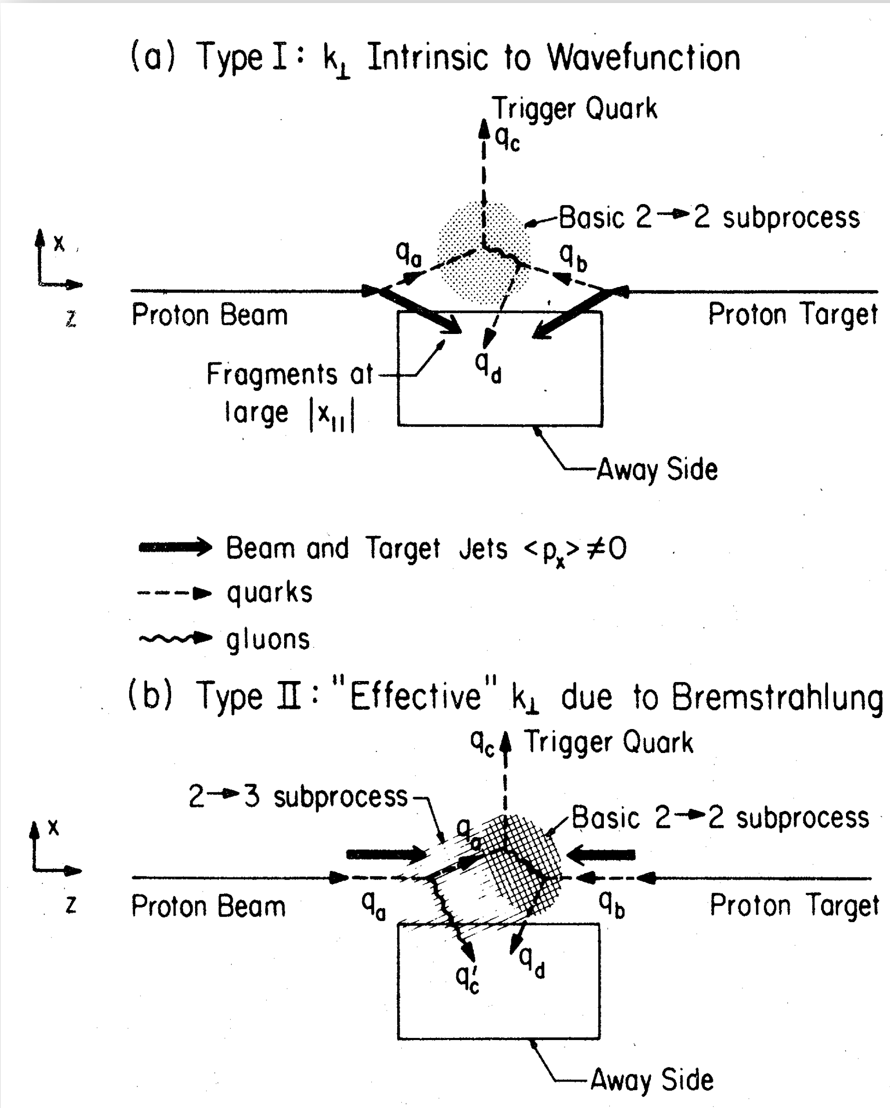
- Differences
  - m/Q ➔ 0, fixed x,z limit,   m/Q, P_T/Q ➔ 0, fixed x, z limit
  - Wilson lines and gauge invariance
  - Soft factors, etc
  - Regions of transverse momentum

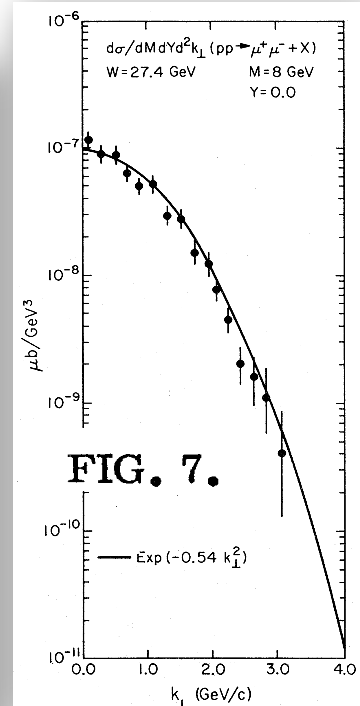
$$d\sigma_{\rm SIDIS} = \sum_f \mathcal{H}_{f,{\rm SIDIS}}(Q) \otimes F_{f/H_1}(x, k_{1T}, Q) \otimes D_{H_2/f}(z, k_{2T}, Q) \qquad + Y_{\rm SIDIS}$$

$$d\sigma_{\rm DY} = \sum_f \mathcal{H}_{f,{\rm DY}}(Q) \otimes F_{f/H_1}(x_1, k_{1T}, Q) \otimes F_{\bar{f}/H_2}(x_2, k_{2T}, Q) \qquad + Y_{\rm Drell-Yan}$$

$$d\sigma_{\rm e^+e^-} = \sum_f \mathcal{H}_{f,{\rm e^+e^-}}(Q) \otimes D_{H_1/\bar{f}}(z_1, k_{1T}, Q) \otimes D_{H_2/f}(z_2, k_{2T}, Q) \quad + Y_{\rm e^+e^-}$$

Intrinsic Transverse Momentum





Fermilab (1976)

"There has been much speculation about how much of the dimuon k_T spectra shown in Fig.7 is due to the wave function (Type I) and how much is explained by QCD perturbation calculations (Type II)."

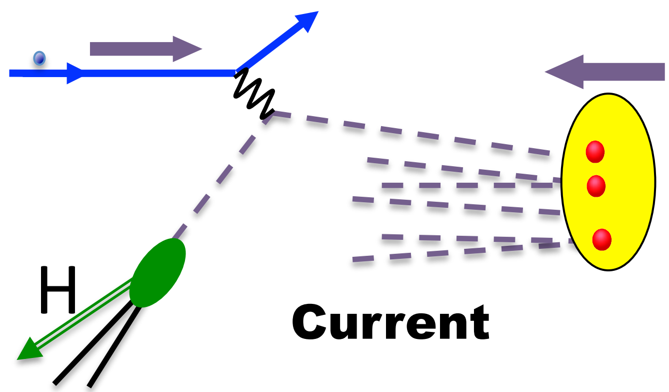
- R. Feynman, R. Field, G. Fox
Phys.Rev. D18 **(1978)** 3320

Cartography of a process

- Example: Semi-inclusive deep inelastic scattering

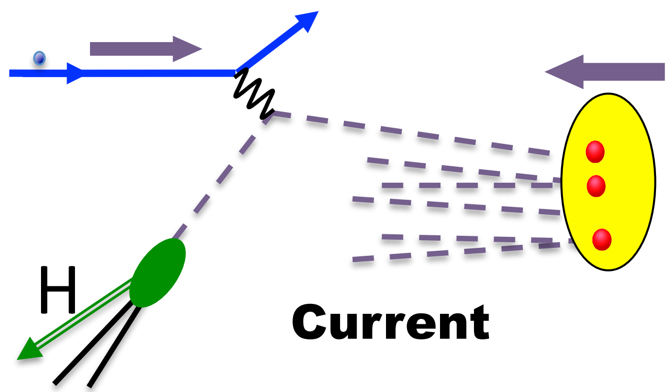
Cartography of a process

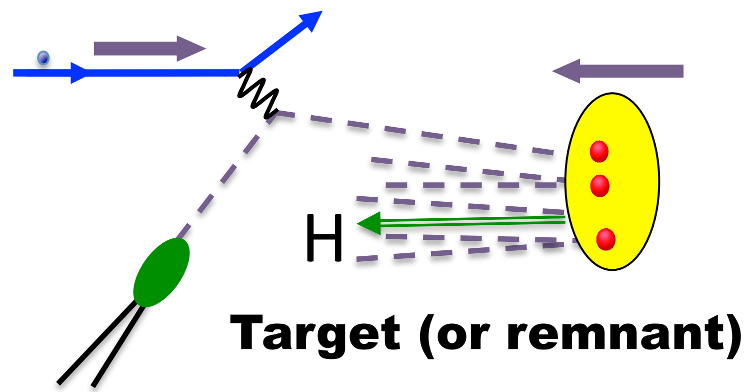
- Example: Semi-inclusive deep inelastic scattering



Cartography of a process

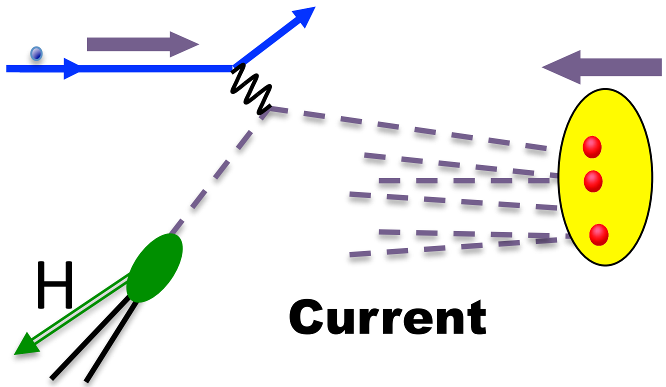
- Example: Semi-inclusive deep inelastic scattering



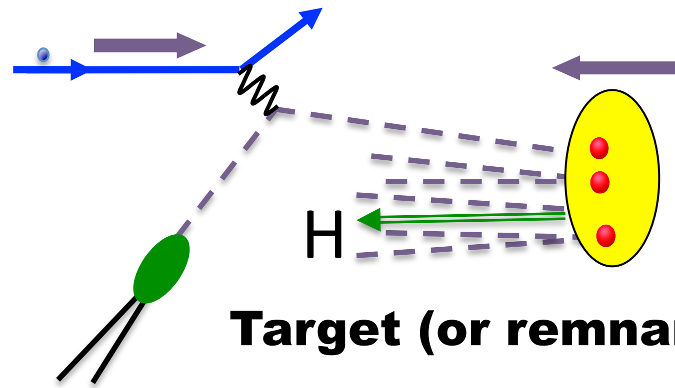


Cartography of a process

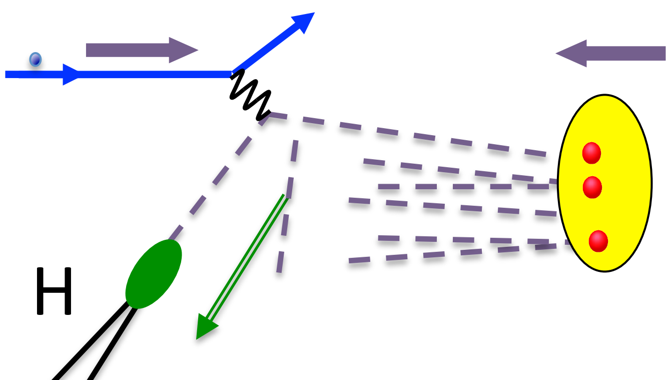
- Example: Semi-inclusive deep inelastic scattering



Current



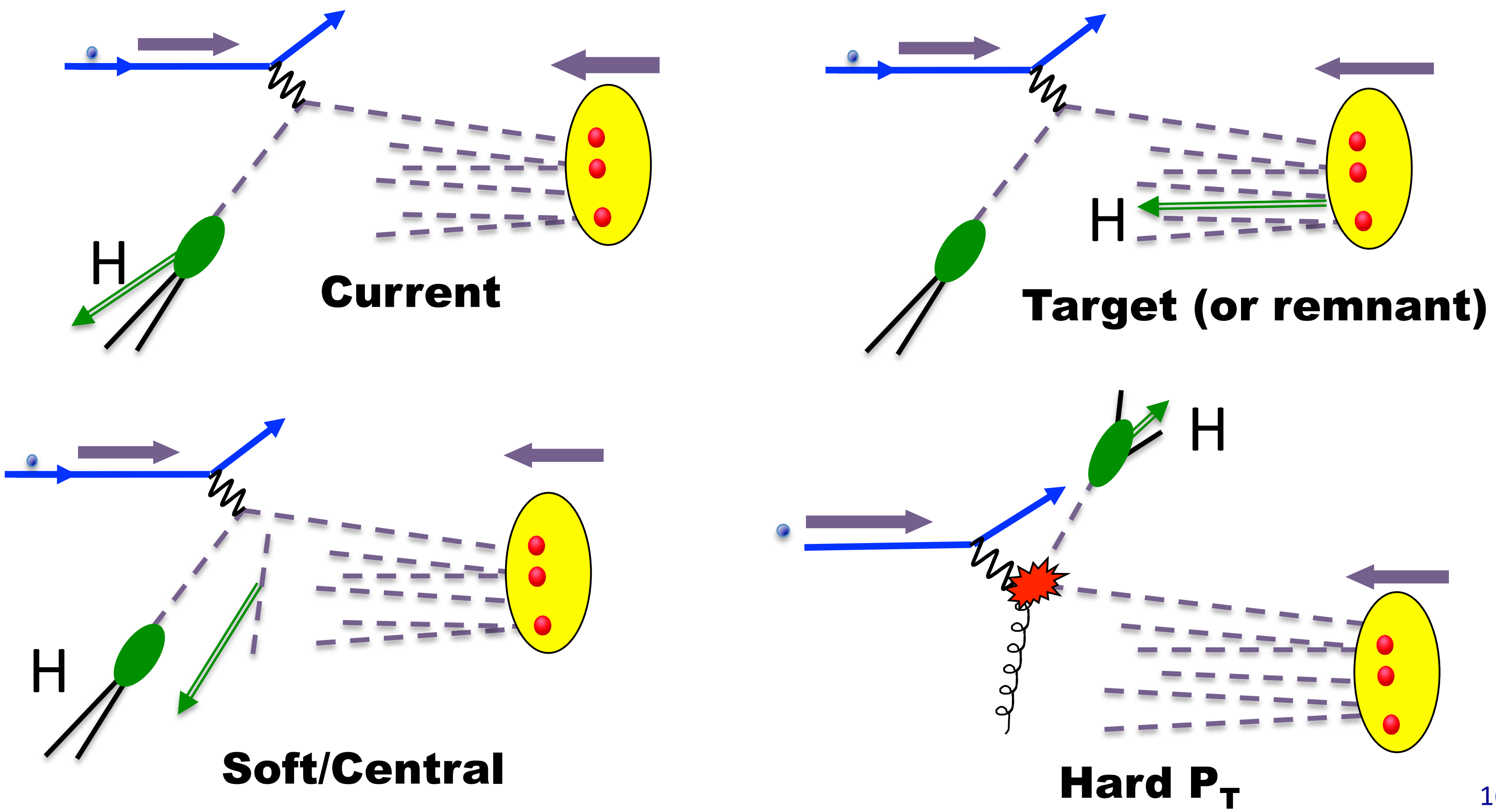
Target (or remnant)

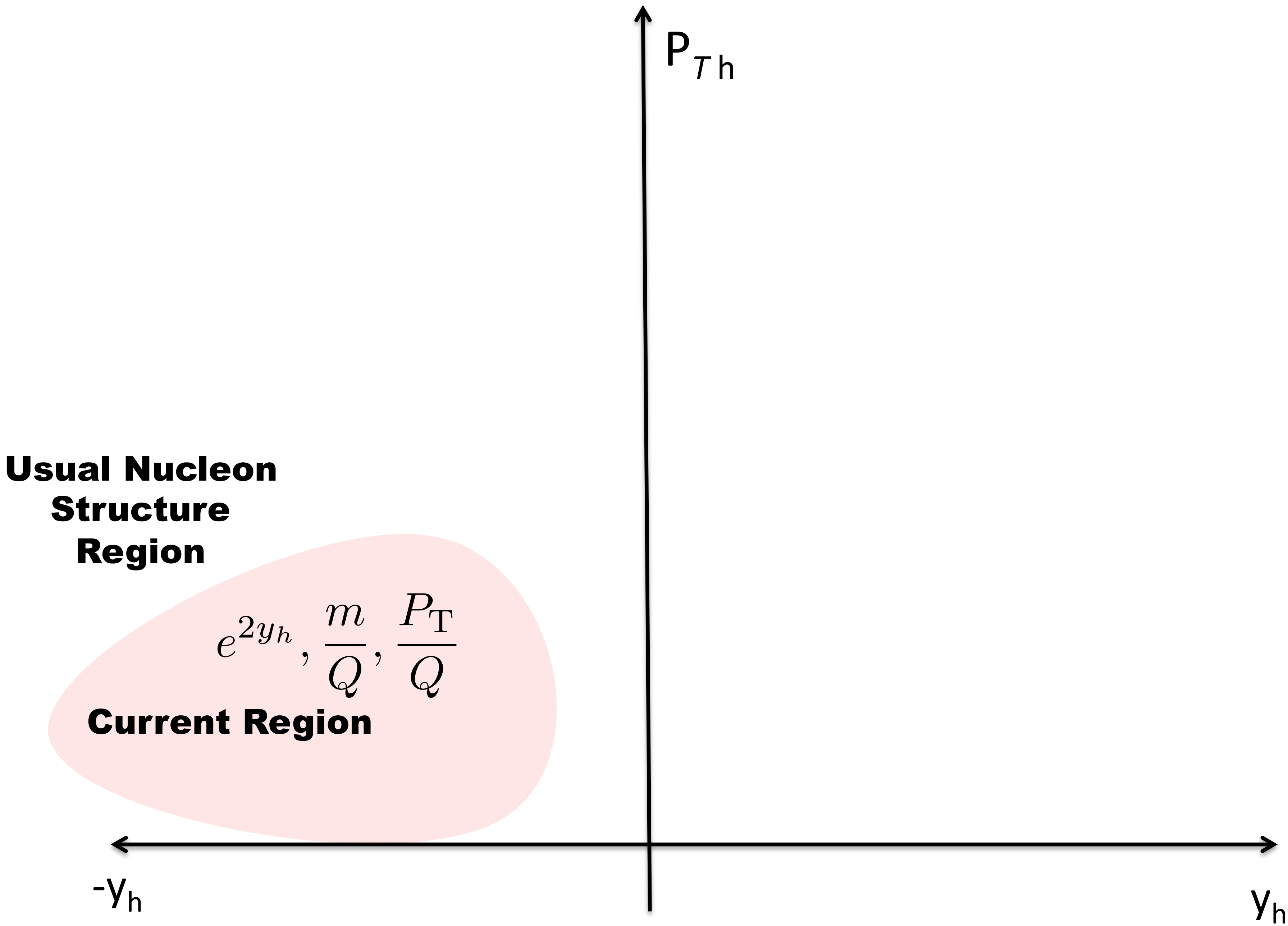


Soft/Central

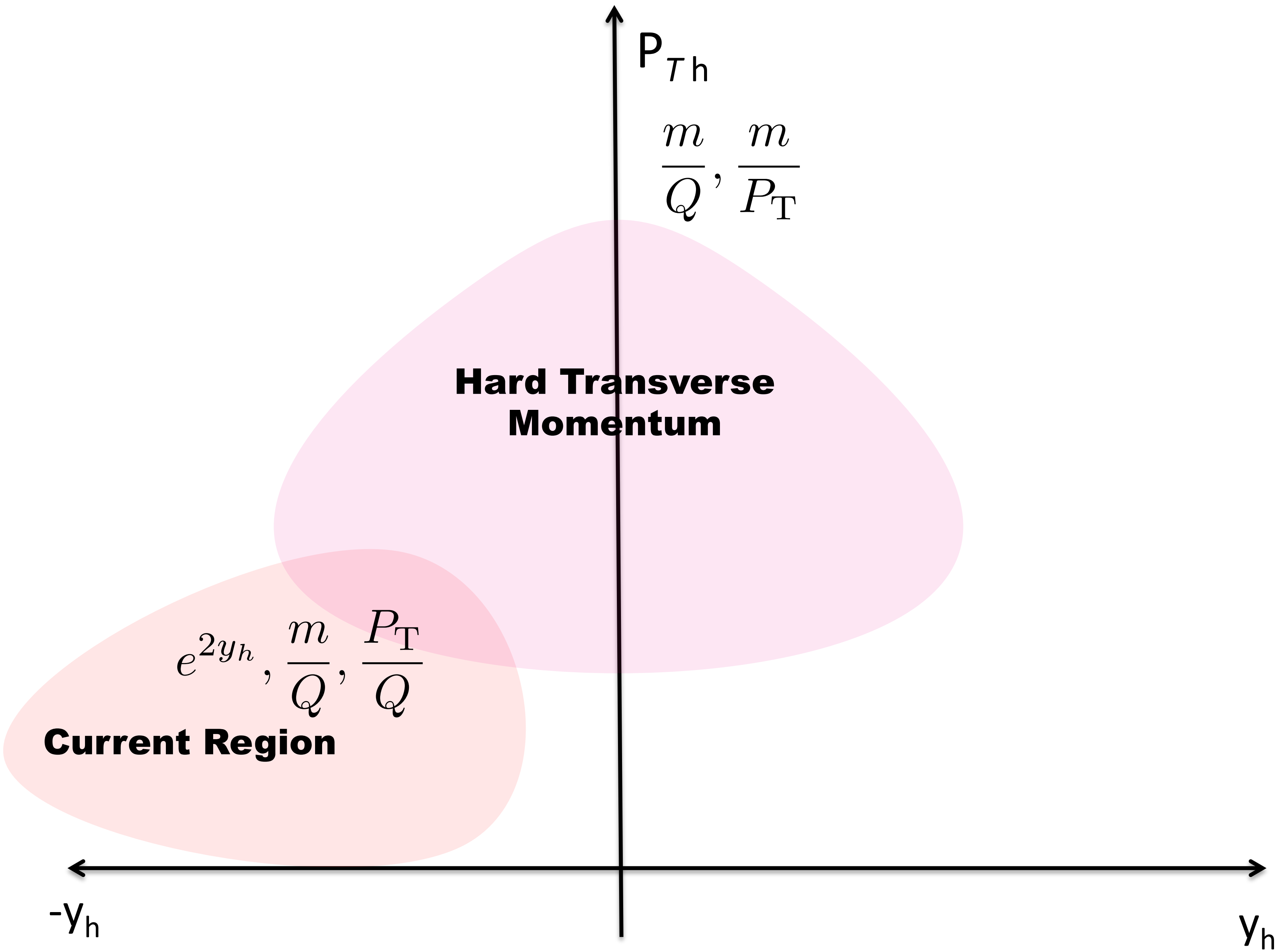
Cartography of a process

- Example: Semi-inclusive deep inelastic scattering

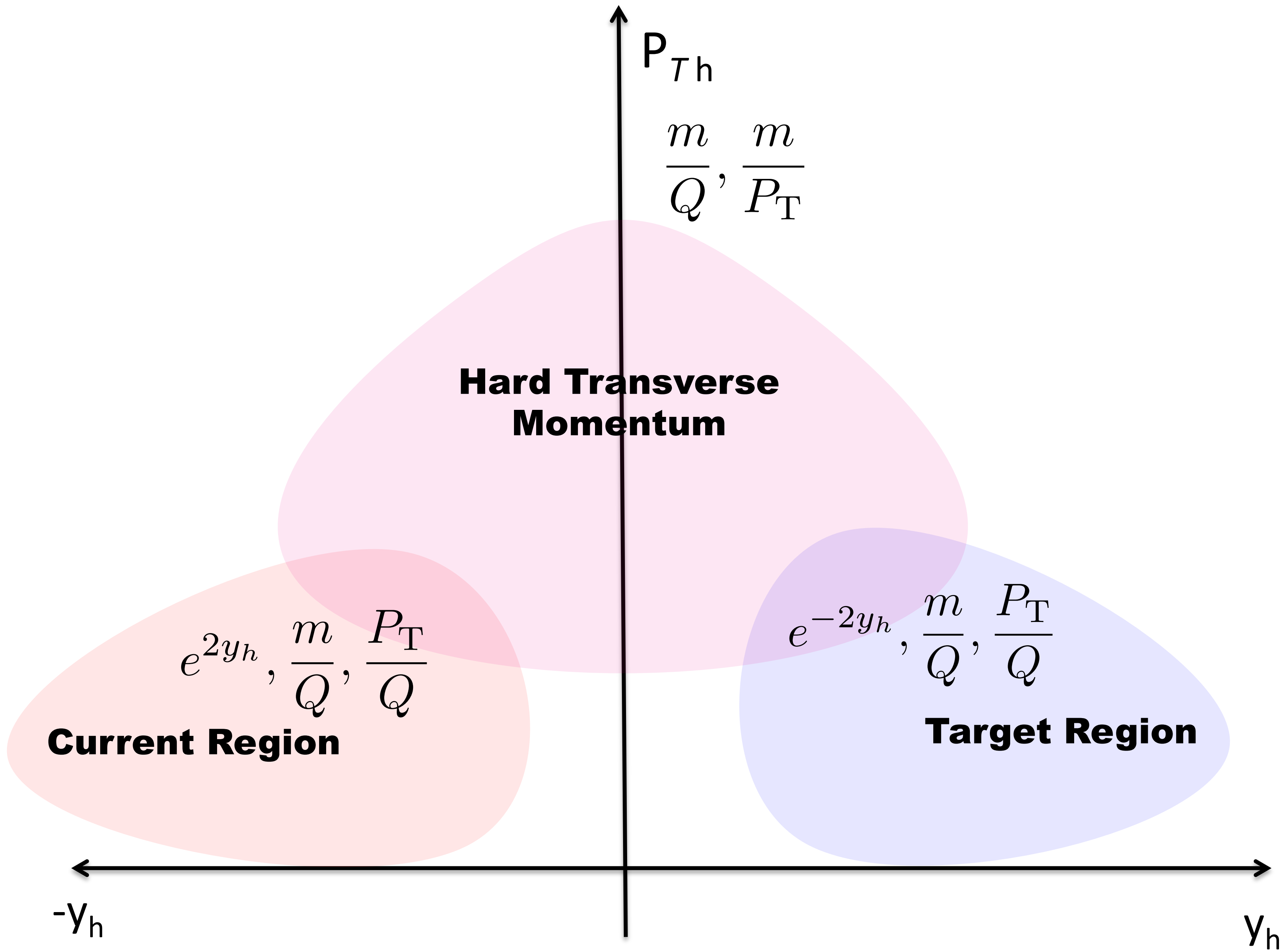




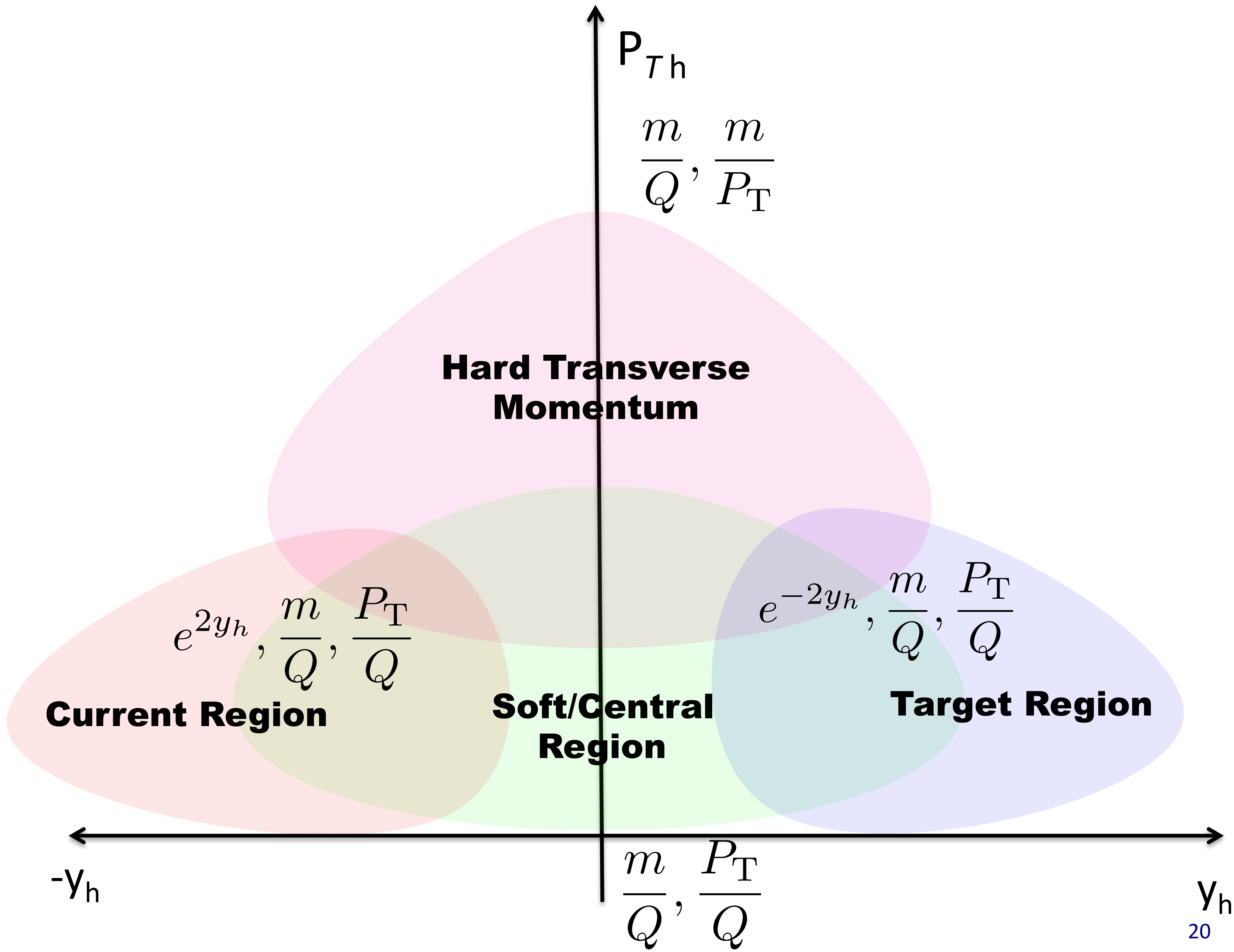
$P_{T\,h}$
Usual Nucleon Structure Region
$e^{2y_h}, \frac{m}{Q}, \frac{P_{\mathrm{T}}}{Q}$
Current Region
$-y_h$
y_h

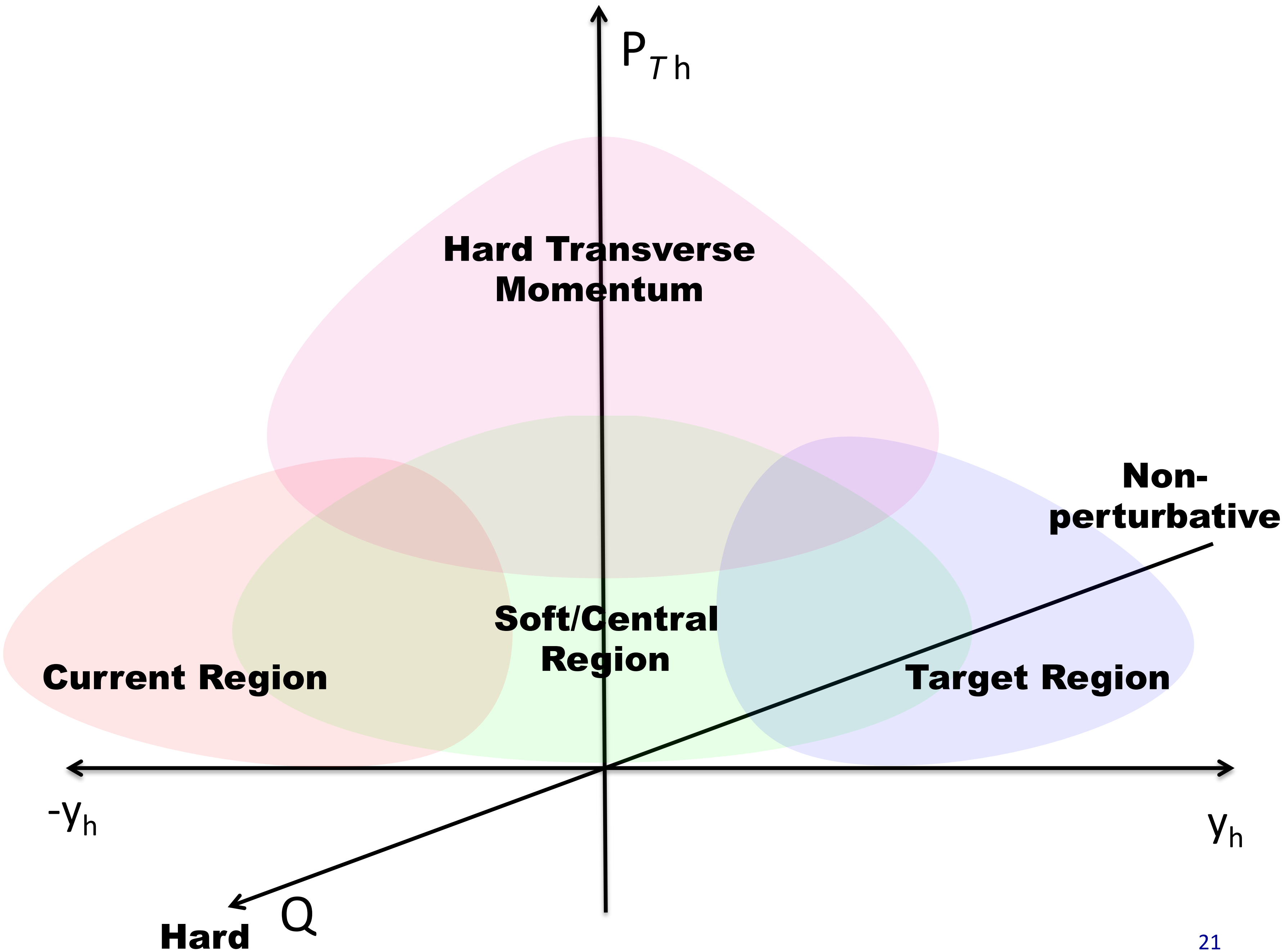


P_{Th}
$\frac{m}{Q}, \frac{m}{P_T}$
Hard Transverse Momentum
$e^{2y_h}, \frac{m}{Q}, \frac{P_T}{Q}$
Current Region
$-y_h$
y_h



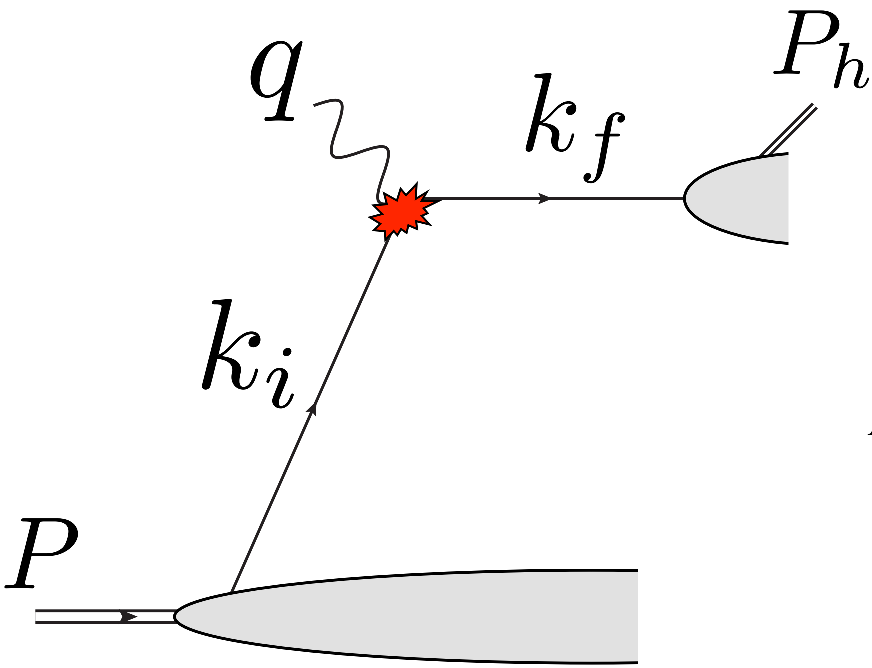
$P_{T\,h}$
$\frac{m}{Q}, \frac{m}{P_T}$
Hard Transverse Momentum
$e^{2y_h}, \frac{m}{Q}, \frac{P_T}{Q}$
Current Region
$e^{-2y_h}, \frac{m}{Q}, \frac{P_T}{Q}$
Target Region
$-y_h$
y_h


$P_{T\,h}$
$\frac{m}{Q}, \frac{m}{P_T}$
Hard Transverse Momentum
$e^{2y_h}, \frac{m}{Q}, \frac{P_T}{Q}$
Current Region
Soft/Central Region
$e^{-2y_h}, \frac{m}{Q}, \frac{P_T}{Q}$
Target Region
$-y_h$
$\frac{m}{Q}, \frac{P_T}{Q}$
y_h



P$_{Th}$
Hard Transverse Momentum
Non-perturbative
Soft/Central Region
Current Region
Target Region
-y$_h$
y$_h$
Hard
Q

Kinematics of Small P_T



$$k_\mathrm{i} = \left(\frac{M_\mathrm{iT}}{\sqrt{2}} e^{y_\mathrm{i}}, -\frac{M_\mathrm{iT}}{\sqrt{2}} e^{-y_\mathrm{i}}, \mathbf{k}_\mathrm{T} \right)$$

$$k_\mathrm{f} = \left(\frac{M_\mathrm{fT}}{\sqrt{2}} e^{y_\mathrm{i}}, \frac{M_\mathrm{fT}}{\sqrt{2}} e^{-y_\mathrm{f}}, \mathbf{k}_\mathrm{T} \right)$$

$$P = \left(P^+, \frac{M_p^2}{2P^+}, \mathbf{0}_\mathrm{T} \right) = \left(\frac{Q}{x_\mathrm{n}\sqrt{2}}, \frac{x_\mathrm{n} M_p^2}{Q\sqrt{2}}, \mathbf{0}_\mathrm{T} \right),$$

$$q = \left(-x_\mathrm{n} P^+, \frac{Q^2}{2x_\mathrm{n} P^+}, \mathbf{0}_\mathrm{T} \right) = \left(-\frac{Q}{\sqrt{2}}, \frac{Q}{\sqrt{2}}, \mathbf{0}_\mathrm{T} \right),$$

$$P_h = \left(\frac{M_{h\mathrm{T}}}{\sqrt{2}} e^{y_\mathrm{h}}, \frac{M_{h\mathrm{T}}}{\sqrt{2}} e^{-y_\mathrm{h}}, \mathbf{P}_{h\mathrm{T}} \right),$$

Quantify proximity to collinear regions

$$R(y_\mathrm{h}, z_\mathrm{h}, x_\mathrm{bj}, Q) \equiv \frac{P_h \cdot k_\mathrm{f}}{P_h \cdot k_\mathrm{i}}\,, \qquad \overset{\frac{m}{Q} \to 0}{=} \quad e^{2 y_h}$$

Effect of target, final state masses?

M. Boglione , J. Collins, L. Gamberg , J. O. Gonzalez-Hernandez , TCR , N. Sato (2017)

Quantify proximity to collinear regions

$$R(y_{\rm h}, z_{\rm h}, x_{\rm bj}, Q) \equiv \frac{P_h \cdot k_{\rm f}}{P_h \cdot k_{\rm i}}, \qquad \overset{\frac{m}{Q}\to 0}{=} \quad e^{2y_h}$$

Effect of target, final state masses?

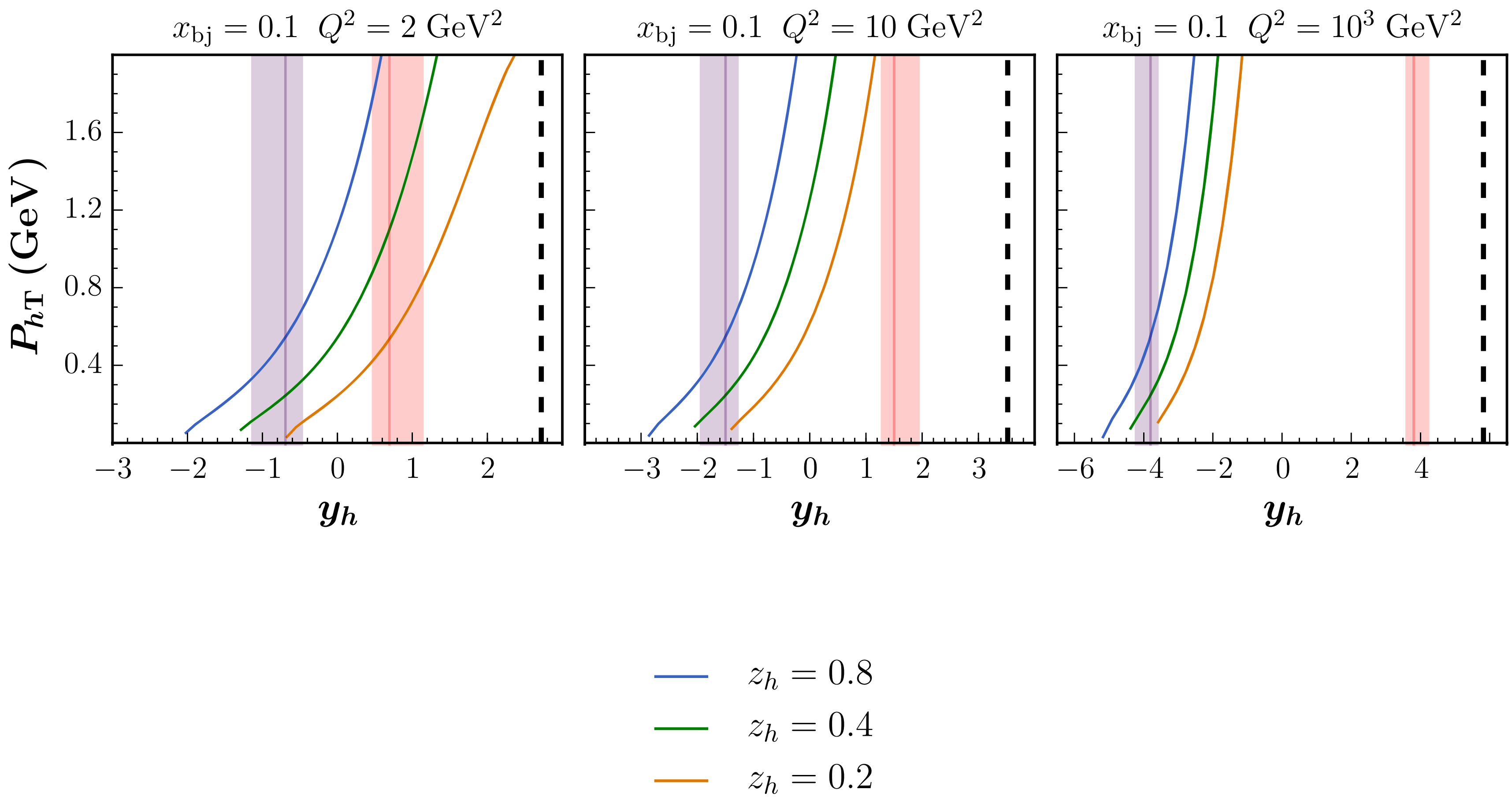
- Need estimates of non-perturbative scales:

$$y_i = \ln \frac{Q}{M_{i,\rm T}}; \qquad y_f = -\ln \frac{Q}{M_{f,\rm T}}$$

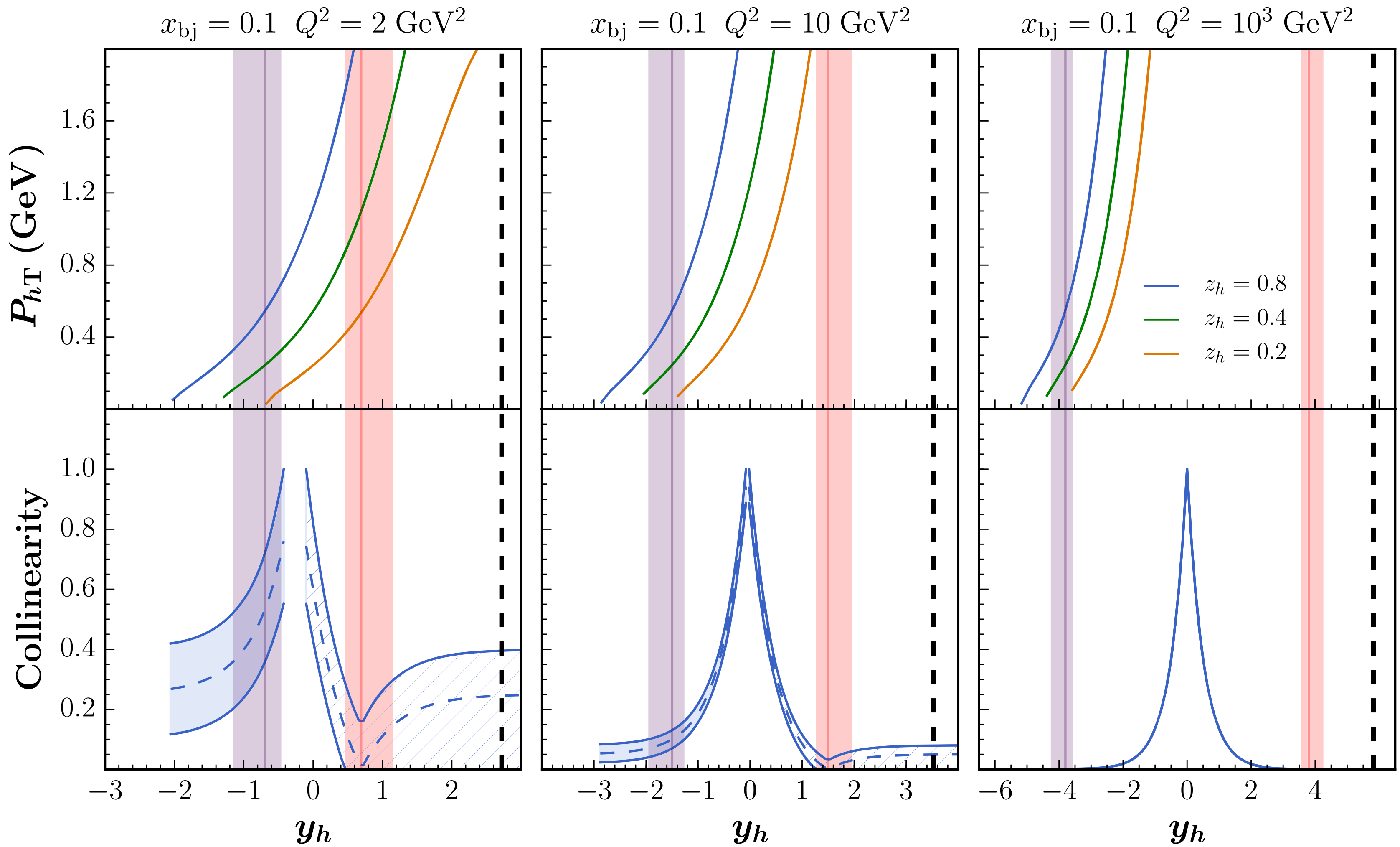
$$M_{i,\rm T} \approx M_{f,\rm T} \approx 0.5 \pm 0.3 \text{ GeV}$$

M. Boglione , J. Collins, L. Gamberg , J. O. Gonzalez-Hernandez , TCR , N. Sato (2017)

Rapidity Regions



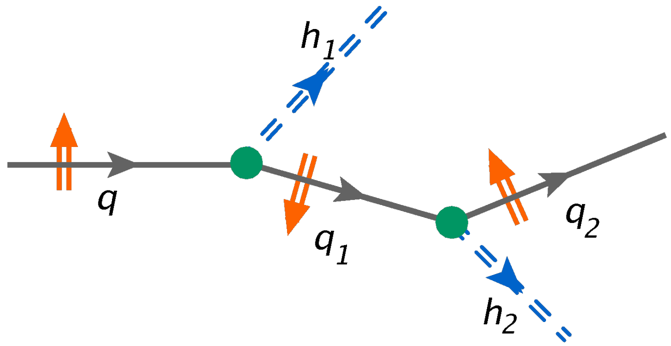
Rapidity Regions

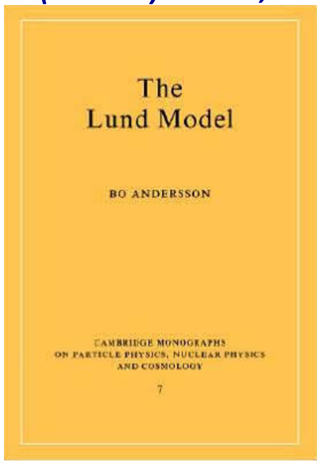


Hadronization

- Example: Spin in a MC event generator

(Matevosyan, Kotzinian, Thomas, Phys.Rev. D95 (2017) no.1, 014021)





Bo Andersson, 2005

(Bentz, Matevosyan, Kotzinian, Ninomiya, Thomas, Yazaki, Phys.Rev. D94 (2016) no.3, 034004)
(Ito, Bentz, Cloët, Thomas, Yazaki, Phys.Rev. D80 (2009))
(A. Kerbizi, X. Artru, Z. Belghobsi, F. Bradamante, A. Martin, E. Redouane Salah, arXiv:1701.08543)

- More dynamics
- Interface with factorization theory?

(J. Collins, TCR: In preparation)

TMD Factorization and Evolution

- Many results exist, but in different languages
  - Resummation in collinear factorization
  - CSS
  - SCET
  - Sudakov Factors

- Results can appear different on the surface

- Map old style to new
  - Is there convergence toward a standardized set of definitions?
  - Bring all results together in TMD-style language

- Nonperturbative parts?

Older Language: Examples

- CSS1 - Multiple redefinitions of factors (starting from TMD definitions) No explicit hard part.

  (Collins, Soper, Sterman (1981-1985))

  – Match to collinear for $\Lambda_{QCD} << q_T << Q$ and $q_T \approx Q$.

- Catani, de Florian, Grazzini et al.

  (Catani, de Florian, Grazzini (2001))

  – Resummation scheme dependence; no uniquely defined hard part.

Newer (TMD) methods: Examples

- Improved TMD functions: Eg:
  - Definitions (e.g., CSS2) *(J. Collins textbook, (2011))*
  - SCET-based approaches
    - Main differences from CSS2: Implementation of regulators.
    - At least two are equivalent to CSS2
      (Echevarria, Idilbi, Scimemi (2012); Collins, TCR (2013))
      (Li, Neill, Zhu, (2016); Collins, TCR (2017) App. B)
  - Structurally matches TMD phenomenology

$$H_f \int \mathrm{d}^2\boldsymbol{k}_\mathrm{T}\, F_{f/p}(x, \boldsymbol{k}_\mathrm{T} - \boldsymbol{q}_\mathrm{T})\, D_{h/f}(z, z\boldsymbol{k}_\mathrm{T})$$

  - Well-oriented for NP hadron structure studies (e.g. lattice QCD)
  - Hard parts are fixed by factorization of operator structures.

$$\frac{\text{Cross Section}}{\int \mathrm{d}^2\boldsymbol{k}_\mathrm{T}\, F_{f/p}(x, \boldsymbol{k}_\mathrm{T} - \boldsymbol{q}_\mathrm{T})\, D_{h/f}(z, z\boldsymbol{k}_\mathrm{T})} = H_f$$

New (i.e., TMD-based) methods

- TMD parton model structure + evolution equations.

Ex: CSS2

$$\frac{d\sigma}{dQ^2\, dy\, dq_T^2} = \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_j \underline{H_{j\bar{j}}^{DY}(Q, \mu_Q, a_s(\mu_Q))} \int \frac{d^2\boldsymbol{b}_T}{(2\pi)^2}\, e^{i\boldsymbol{q}_T\cdot\boldsymbol{b}_T}\, \underline{\tilde{f}_{j/A}(x_A, b_T; Q^2, \mu_Q)}\, \underline{\tilde{f}_{\bar{j}/B}(x_B, b_T; Q^2, \mu_Q)}$$

$$+ \text{suppressed corrections,}$$

$$\frac{\partial \ln \tilde{f}(x, b_T; \mu, \zeta)}{\partial \ln \sqrt{\zeta}} = \tilde{K}(b_T; \mu)$$

$$\tilde{f}_{j/H}(x, b_T; \zeta; \mu) = \sum_k \int_{x-}^{1+} \frac{d\xi}{\xi}\, \tilde{C}_{j/k}^{PDF}(x/\xi, b_T; \zeta, \mu, a_s(\mu))\, f_{k/H}(\xi; \mu) \;+\; O[(m b_T)^p]$$

$$\frac{d\tilde{K}(b_T; \mu)}{d \ln \mu} = -\gamma_K(a_s(\mu))$$

$$\frac{d \ln \tilde{f}(x, b_T; \mu, \zeta)}{d \ln \mu} = \gamma_j(a_s(\mu)) - \frac{1}{2}\gamma_K(a_s(\mu)) \ln \frac{\zeta}{\mu^2}$$

$$\mu_Q \equiv C_2 Q$$

$$\mu_b \equiv C_1/b_T$$

$$\mu_{b_*} \equiv C_1/b_*$$

Translation to new TMD methods

(J. Collins, TCR (2017))

$$A_{\text{CSS1}}(a_s(\mu_{b_*}); C_1) = -\frac{\mathrm{d}\tilde{K}(b_*; \mu_{b_*})}{\mathrm{d}\ln b_*^2} + \frac{1}{2}\gamma_K(a_s(\mu_{b_*})) = -\frac{\partial \tilde{K}(b_*; \mu)}{\partial \ln b_*^2}\bigg|_{\mu \mapsto \mu_{b_*}}$$

$$B_{\text{CSS1, DY}}(a_s(\mu_Q); C_1, C_2) = -\tilde{K}(C_1/\mu_Q; \mu_Q) - \frac{\partial \ln H^{\text{DY}}_{j\bar{\jmath}}(Q, \mu_Q, a_s(\mu_Q))}{\partial \ln Q^2}$$

$$g_K^{\text{CSS1}}(b_{\text{T}}; b_{\max}) = g_K(b_{\text{T}}; b_{\max}) \qquad \gamma_{\text{PDF}} = \gamma_{\text{FF}}$$

$$|e_j| \tilde{C}^{\text{CSS1, DY}}_{j/k}\left(\frac{x}{\xi}, b_*; \mu^2_{b_*}, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right) = \tilde{C}^{\text{PDF}}_{j/k}\left(\frac{x}{\xi}, b_*; \mu^2_{b_*}, \mu_{b_*}, a_s(\mu_{b_*})\right) \sqrt{H^{\text{DY}}_{j\bar{\jmath}}(\mu_{b_*}/C_2, \mu_{b_*}, a_s(\mu_{b_*}))} \exp\left[-\tilde{K}(b_*; \mu_{b_*}) \ln C_2\right]$$

$$g^{\text{CSS1}}_{j/H}(x, b_{\text{T}}; b_{\max}) = g_{j/H}(x, b_{\text{T}}; b_{\max})$$

Combining Results in TMD Factorization, order ≈ α_s^3 (Drell-Yan)

Sudakov Form Factor: (Moch,Vermaseren (2005), Vogt, Gehrmann et al (2014))

α_s^2 Wilson Coefficients from Collinear Factorization: (Catani et al, (2012)), and SCET (Echevarria, Scimemi, Vladimirov (2016))

$$
\begin{aligned}
\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_\mathrm{T}^2} &= \frac{4\pi^2\alpha^2}{9Q^2 s}\sum_{j,j_A,j_B} \underline{H^{\mathrm{DY}}_{j\bar{j}}(Q,\mu_Q,a_s(\mu_Q))} \int \frac{\mathrm{d}^2\boldsymbol{b}_\mathrm{T}}{(2\pi)^2} e^{i\boldsymbol{q}_\mathrm{T}\cdot\boldsymbol{b}_\mathrm{T}} \\
&\times \underline{e^{-g_{j/A}(x_A,b_\mathrm{T};b_\mathrm{max})}} \int_{x_A}^1 \frac{\mathrm{d}\xi_A}{\xi_A} \underline{f_{j_A/A}(\xi_A;\mu_{b_*})}\, \underline{\tilde{C}^{\mathrm{PDF}}_{j/j_A}\left(\frac{x_A}{\xi_A}, b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)} \\
&\times \underline{e^{-g_{\bar{j}/B}(x_B,b_\mathrm{T};b_\mathrm{max})}} \int_{x_B}^1 \frac{\mathrm{d}\xi_B}{\xi_B} \underline{f_{j_B/B}(\xi_B;\mu_{b_*})}\, \underline{\tilde{C}^{\mathrm{PDF}}_{\bar{j}/j_B}\left(\frac{x_B}{\xi_B}, b_*;\mu_{b_*}^2,\mu_{b_*},a_s(\mu_{b_*})\right)} \\
&\times \exp\left\{ \underline{-g_K(b_\mathrm{T};b_\mathrm{max})} \ln\frac{Q^2}{Q_0^2} + \underline{\tilde{K}(b_*;\mu_{b_*})} \ln\frac{Q^2}{\mu_{b_*}^2} + \int_{\mu_{b_*}}^{\mu_Q} \frac{\mathrm{d}\mu'}{\mu'} \left[2\underline{\gamma_j(a_s(\mu'))} - \ln\frac{Q^2}{(\mu')^2} \underline{\gamma_K(a_s(\mu'))} \right] \right\} \\
&+ \text{suppressed corrections.}
\end{aligned}
$$

Ex: Konychev, Nadolsky (2006) ResBos extractions (and others)

Li, Zhu (2017) Vladimirov (2017)

From Sudakov Form Factor: (Moch,Vermaseren (2005), Vogt, Gehrmann et al (2014))

Summary/Conclusions

- Unify Pictures
  - Large and small P_T *(Gamberg talk)*
  - Hadron masses, etc *(Accardi talk)*

- Theory of non-perturbative physics
  - Lattice *(Qiu talk)*
  - Fragmentation and Hadronization
  - Input for evolution

Summary/Conclusions

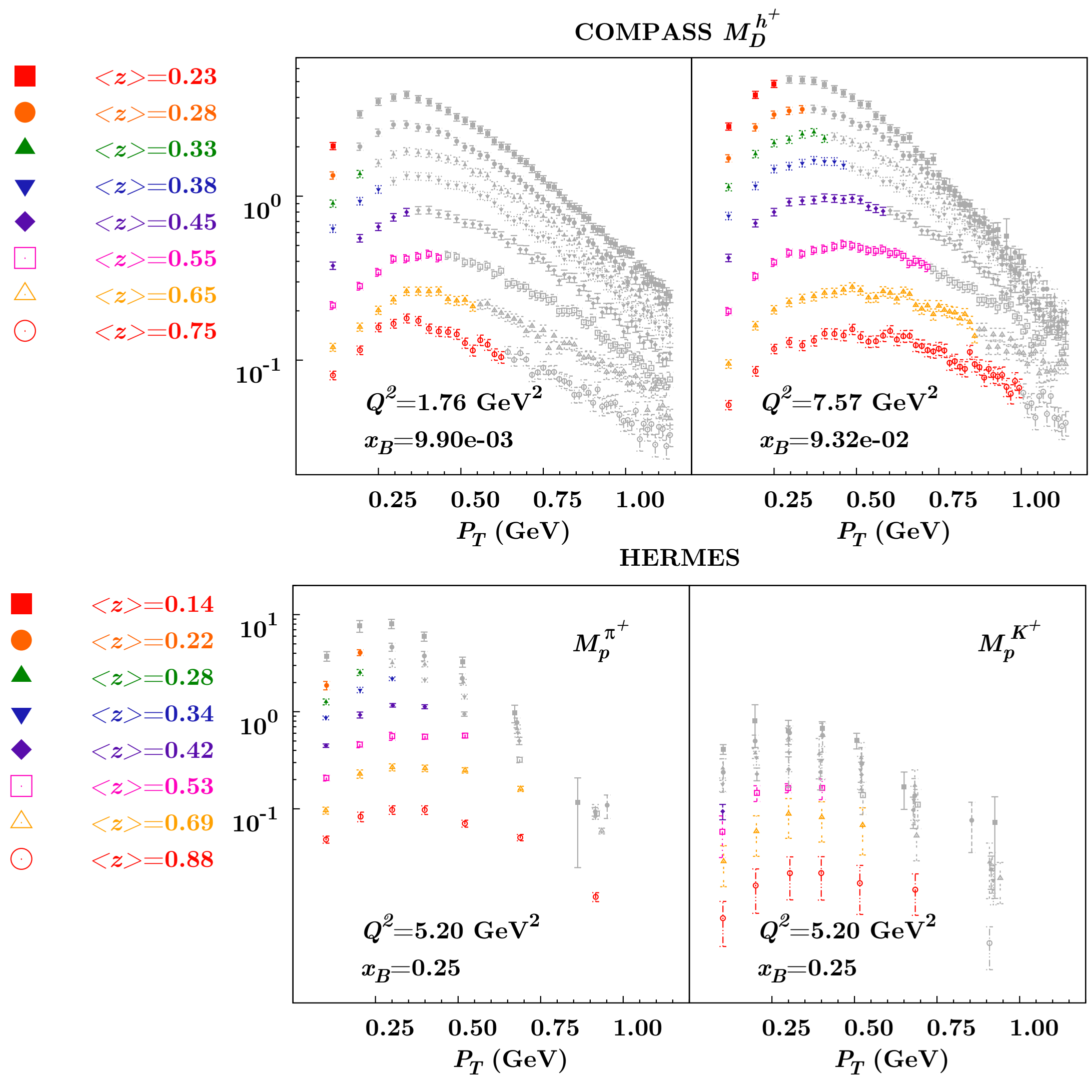
- Unify Pictures

Thank you!

  - Fragmentation and Hadronization
  - Input for evolution

Backup

Effect of restricting data

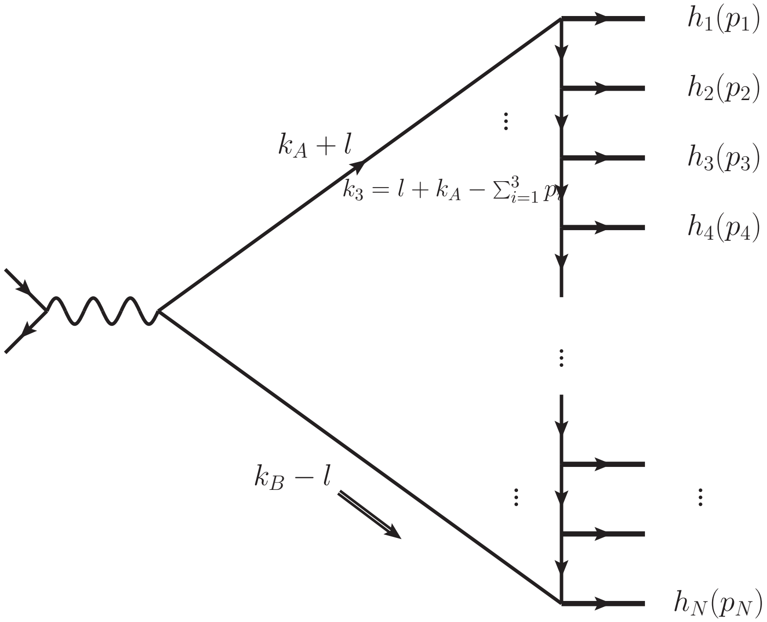


- Colored points: R < .25
- Transition to gray needs large q_T, central, target…

Hadronization

(J. Collins, TCR: In preparation)

$e^+ + e^- \rightarrow h_1 + h_2 + X$

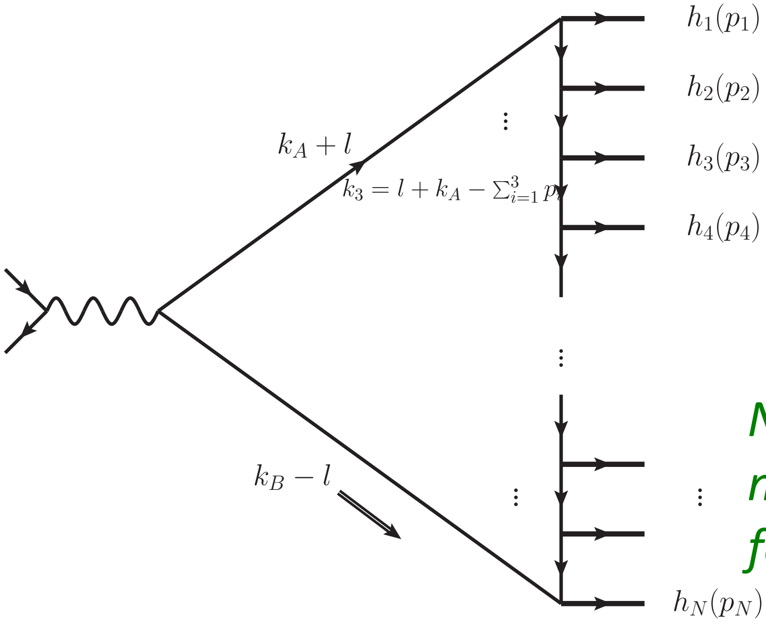


Where are the dominant momentum regions?

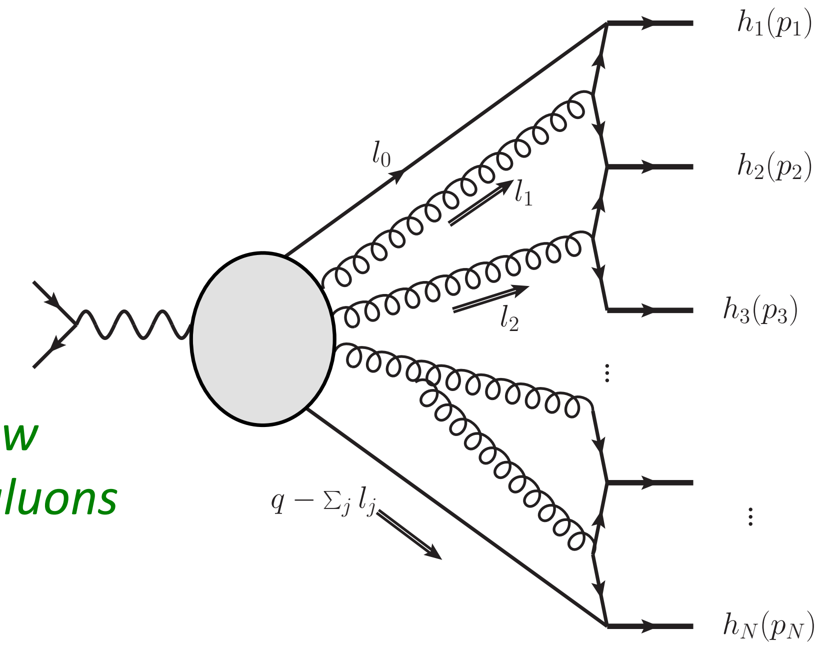
Hadronization

(J. Collins, TCR: In preparation)

$e^+ + e^- \rightarrow h_1 + h_2 + X$



Need extra low momentum gluons for a trap

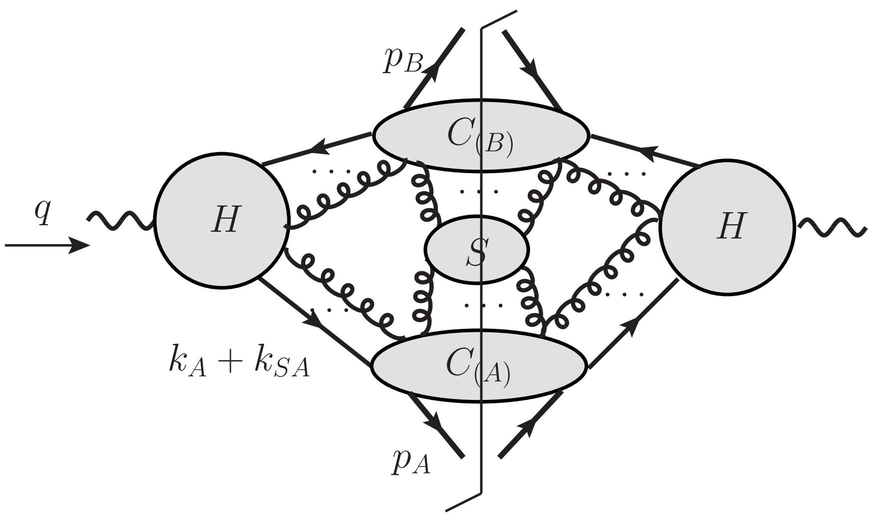


Where are the dominant momentum regions?

Hadronization & Factorization Proofs

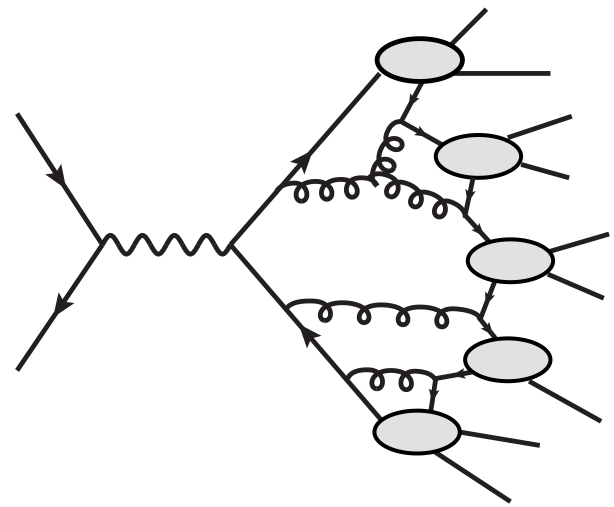
(J. Collins, PoS QCDEV2016, 003 (2017), arXiv:1610.09994)

Factorization Derivation Diagrams



Blobs widely separated in rapidity

Hadronization Models



Blobs close in rapidity

Drell-Yan

CSS1

$$\mu_Q \equiv C_2 Q$$
$$\mu_b \equiv C_1/b_{\rm T}$$
$$\mu_{b_*} \equiv C_1/b_*$$

$$\begin{aligned}
\frac{d\sigma}{dQ^2\,dy\,dq_{\rm T}^2} = \frac{4\pi^2\alpha^2}{9Q^2 s} \sum_{j,j_A,j_B} e_j^2 \int \frac{d^2\boldsymbol{b}_{\rm T}}{(2\pi)^2} e^{i\boldsymbol{q}_{\rm T}\cdot\boldsymbol{b}_{\rm T}} \\
\times \int_{x_A}^{1} \frac{d\xi_A}{\xi_A} f_{j_A/A}(\xi_A;\mu_{b_*})\ \tilde{C}_{j/j_A}^{\rm CSS1,\ DY}\left(\frac{x_A}{\xi_A}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right) \\
\times \int_{x_B}^{1} \frac{d\xi_B}{\xi_B} f_{j_B/B}(\xi_B;\mu_{b_*})\ \tilde{C}_{\bar{j}/j_B}^{\rm CSS1,\ DY}\left(\frac{x_B}{\xi_B}, b_*; \mu_{b_*}^2, \mu_{b_*}, C_2, a_s(\mu_{b_*})\right) \\
\times \exp\left\{ - \int_{\mu_{b_*}^2}^{\mu_Q^2} \frac{d\mu'^2}{\mu'^2} \left[A_{\rm CSS1}(a_s(\mu'); C_1) \ln\left(\frac{\mu_Q^2}{\mu'^2}\right) + B_{\rm CSS1,\ DY}(a_s(\mu'); C_1, C_2) \right] \right\} \\
\times \exp\left[-g_{j/A}^{\rm CSS1}(x_A, b_{\rm T}; b_{\max}) - g_{\bar{j}/B}^{\rm CSS1}(x_B, b_{\rm T}; b_{\max}) - g_K^{\rm CSS1}(b_{\rm T}; b_{\max}) \ln(Q^2/Q_0^2) \right] \\
+ \text{suppressed corrections.}
\end{aligned}$$

No explicit hard part here

Old Schemes and New Schemes

- Questions:

  – CSS1 involves “A” and “B” functions not explicit in CSS2.

  – Non-perturbative parts in CSS1 and in TMD functions? Where do they go?

  – Anomalous dimension of PDFs vs. FFs?

  – Higher order calculations in, for example, old resummation, SCET, etc… how to utilize in, for example, CSS2?

Fast translation to new TMD methods

- CSS1 and CSS2 drop same subleading powers:

$$\left.\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_{\mathrm{T}}^2}\right|_{\mathrm{DY}}^{\mathrm{CSS1}} = \left.\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_{\mathrm{T}}^2}\right|_{\mathrm{DY}}^{\mathrm{CSS2}} \quad ; \quad \left.\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_{\mathrm{T}}^2}\right|_{\mathrm{SIDIS}}^{\mathrm{CSS1}} = \left.\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}y\,\mathrm{d}q_{\mathrm{T}}^2}\right|_{\mathrm{SIDIS}}^{\mathrm{CSS2}}$$

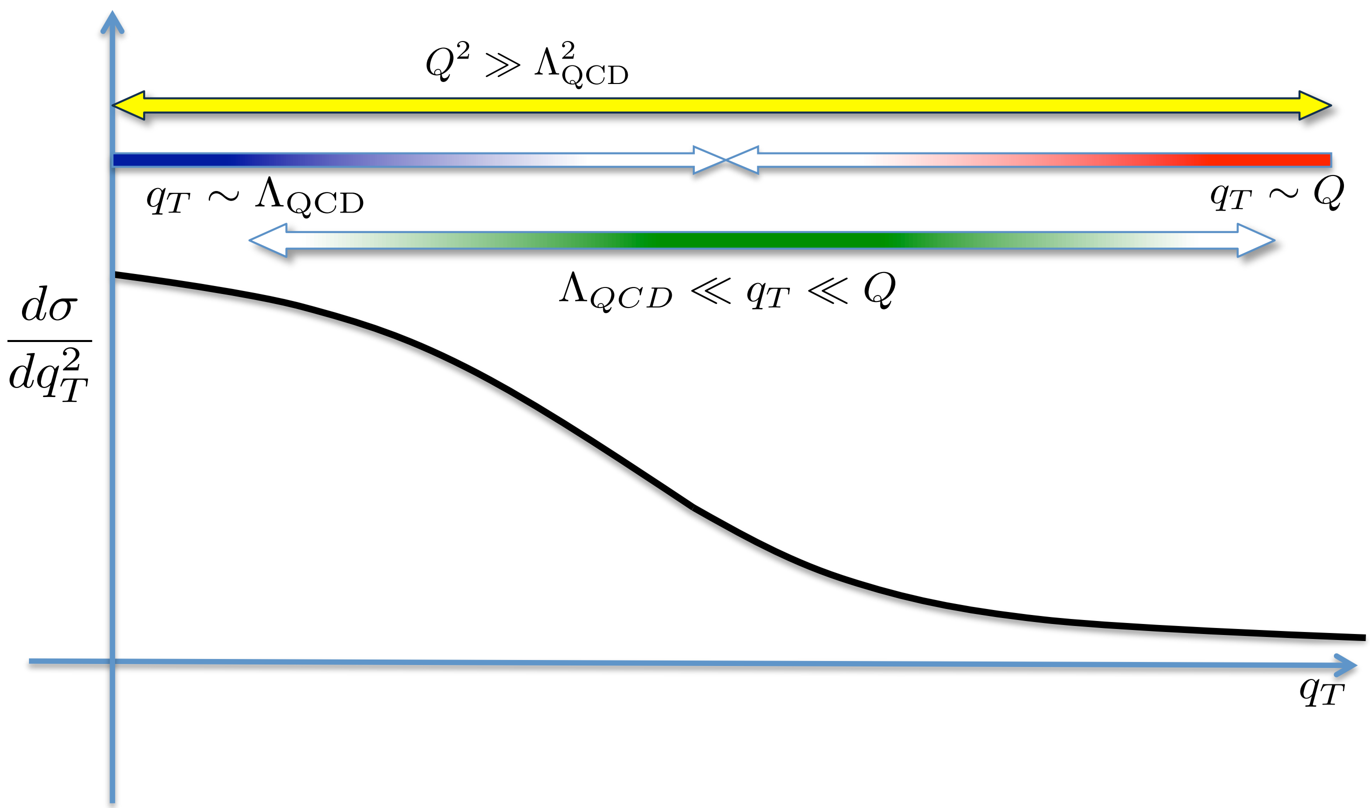
- Derivatives given by evolution equations.
  (anomalous dimensions)

- b_{max} independence.

- Charge conjugation invariance.

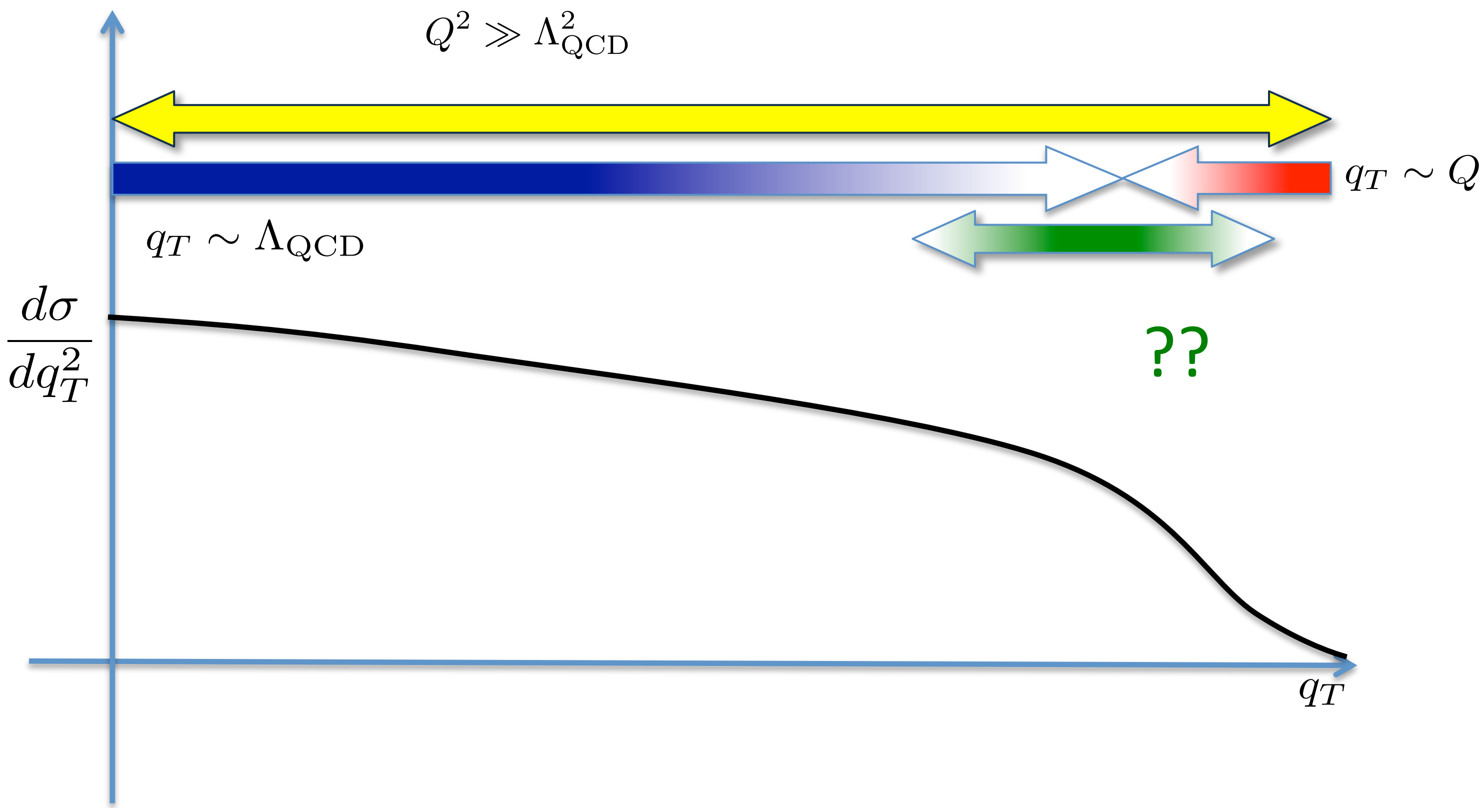
Very Large Transverse Momentum

- Merging large and small transverse momenta



Very Large Transverse Momentum

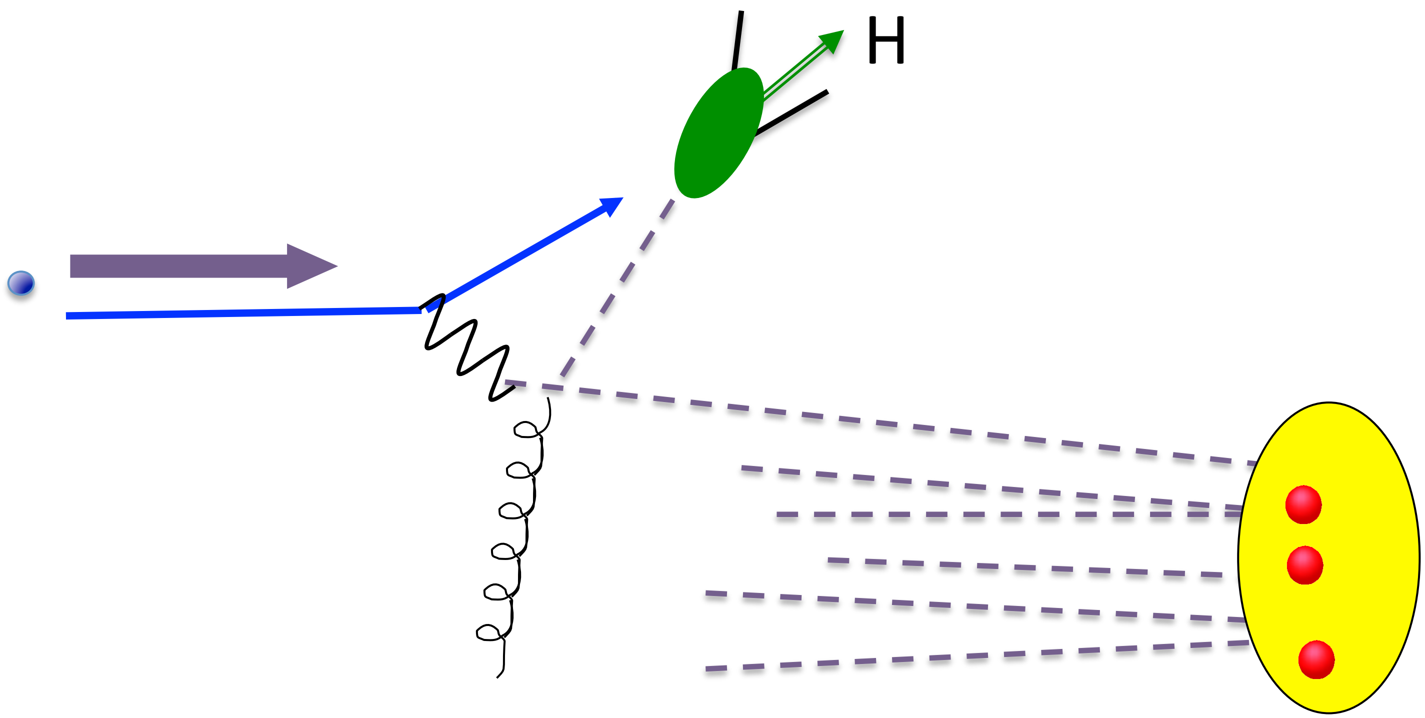
- Merging large and small transverse momenta



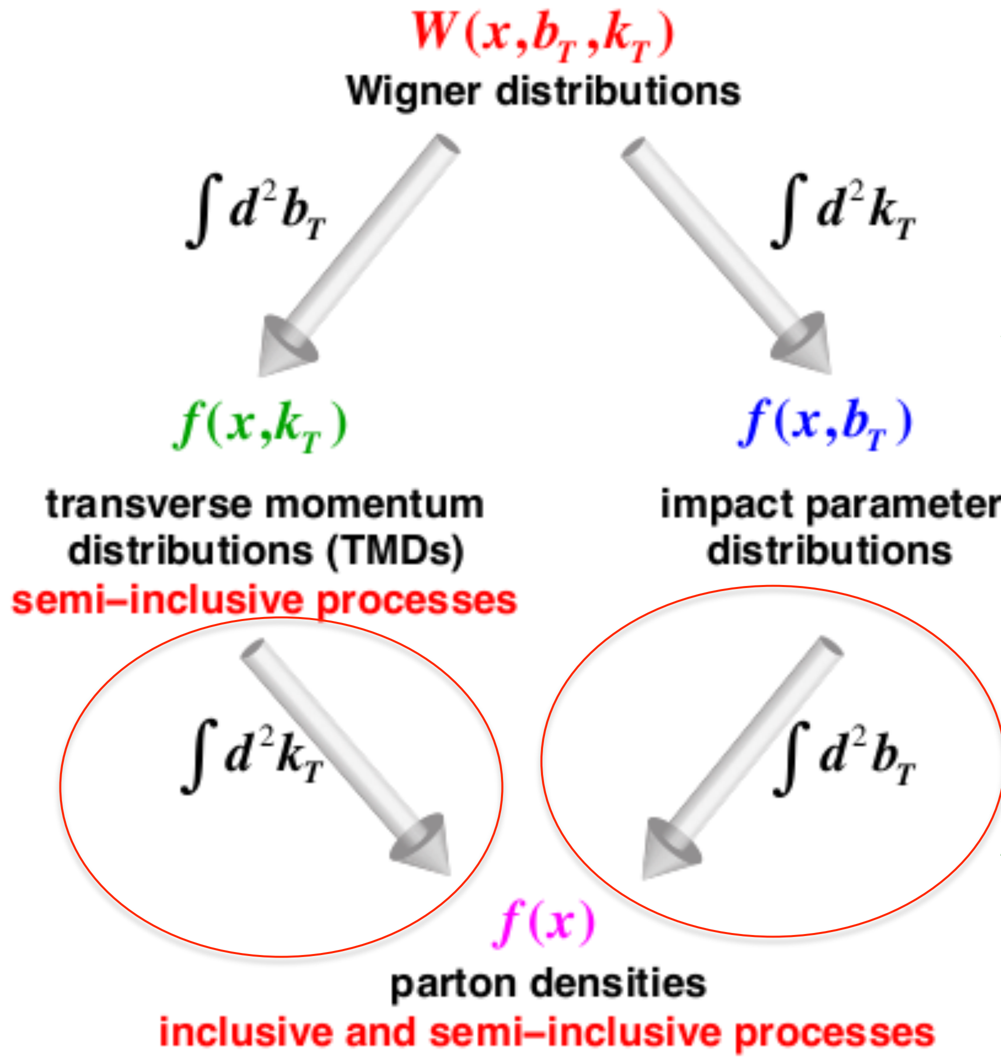
(See Gamberg Talk)

Very Large Transverse Momentum

- Merging large and small transverse momenta



Very Large Transverse Momentum



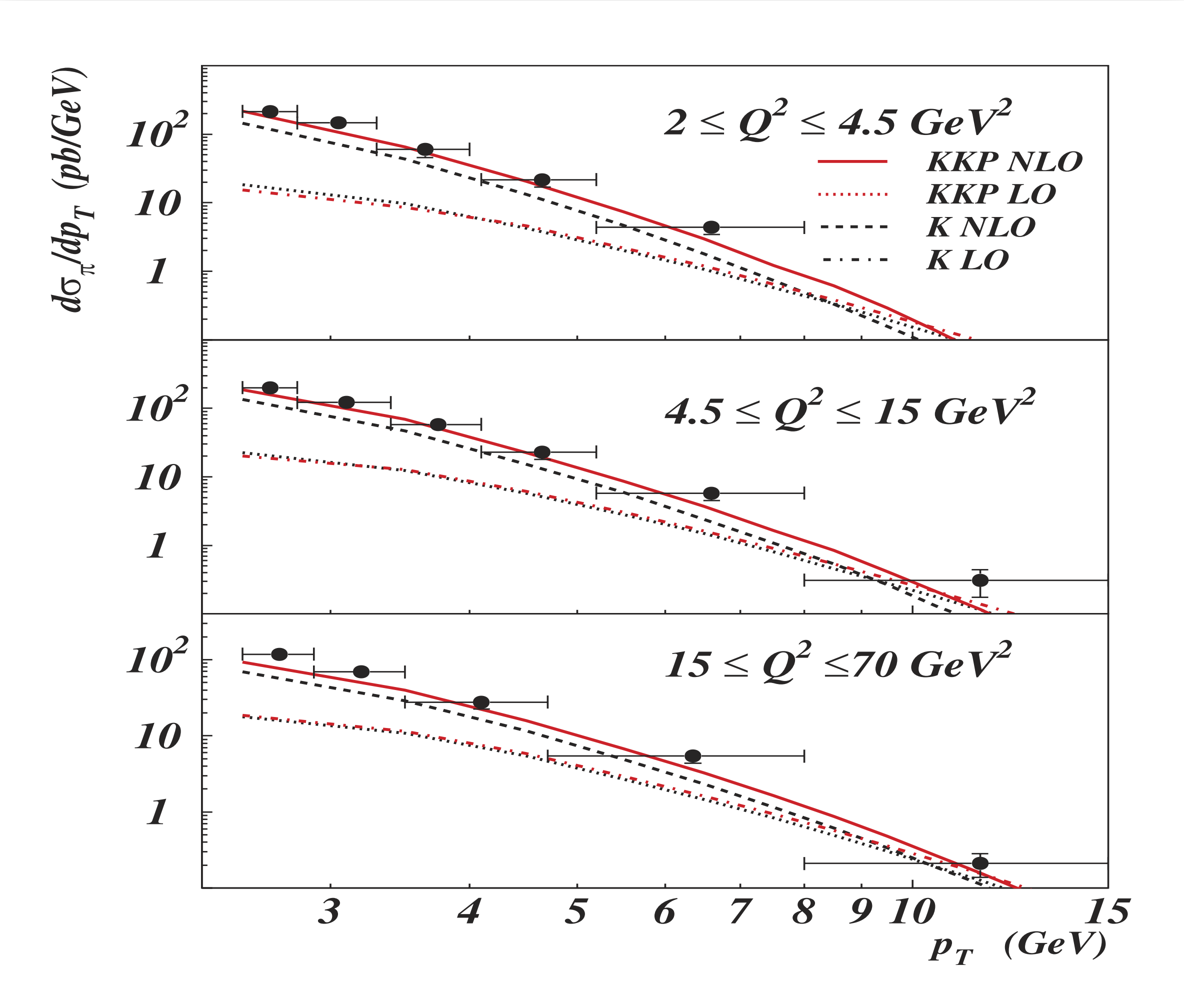
TMD Factorization Derived for $P_T/Q \rightarrow 0$

(See Gamberg Talk)

From EIC White Paper (2014)

Need to address

- Large transverse momentum.



Daleo, de Florian, Sassot (2005)
Phys.Rev. D71 (2005) 034013

Data: H1 (2004)
Eur.Phys.J.C36:441-452,2004